Term Structure Persistence

Mirko Abbritti∗  Luis Gil-Alana‡  Yuliya Lovcha‡
Antonio Moreno§

Abstract

Stationary I(0) models employed in yield curve analysis typically imply an unrealistically low degree of volatility in long-run short-rate expectations due to fast mean reversion. In this paper we propose a novel general multivariate affine term structure model with a two-fold source of persistence in the yield curve: Long-memory and short-memory. Our model, based on an I(d) specification, nests the I(0) and I(1) models as special cases and the I(0) model is decisively rejected by the data. Our model estimates imply both mean reversion in yields and quite volatile long-distance short-rate expectations, due to the higher persistence imparted by the long-memory component. Our implied term premium estimates differ from those of the I(0) model during some relevant periods by more than 4 percentage points and exhibit a realistic countercyclical pattern.

JEL Classification: C3, E4, G1

Keywords: Fixed Income Securities, Yield Curve, Affine Term Structure, Fractional Integration, Term Premium

∗Economics Department, University of Navarra, Spain. He acknowledges financial support from the Spanish Ministry of Education grant ECO2009-11151.
†Economics Department and ICS, University of Navarra, Spain. He acknowledges financial support from the Spanish Ministry of Education grant ECO2011-2014 ECON Y FINANZAS, and from the Jerónimo de Ayanz Project.
‡Economics Department, University of Navarra, Spain. She acknowledges financial support from the Spanish Ministry of Education grant ECO2011-2014 ECON Y FINANZAS.
§Corresponding Author: Economics Department, University of Navarra, Pamplona, Spain. Phone #: +34 948425600 Ext. 2330. Fax #: +34 948425626. e-mail: antmoreno@unav.es. He acknowledges financial support from the Spanish Ministry of Education grant ECO2009-11151.
1 Introduction

The yield curve contains a wealth of information about key current and expected macro-
finance developments. As a result, academics and policy makers continue to be intrigued
about the source of its changes and dynamics. In particular, the yield curve contains
relevant information about the future expected short-term interest rates as well as the
investors’ risk attitudes towards the different maturities along the yield curve. In fact,
researchers often decompose yields at longer maturities into a weighted average of current
and expected future short-rates, also known as the risk-neutral rate, and a risk premium,
also known as term premium.

In models of the term structure of interest rates, there are typically three crucial
elements determining short-rate expectations and the associated term premiums on long-
term bonds: The short-term process itself, the dynamics followed by the factors in the
short-rate process and the risk compensations required by the investors’ exposures to the
risk factors. In this paper we propose a flexible multivariate state process characterizing
the dynamics of the factors: A Vector Auto-Regressive Fractionally Integrated Moving
Average (VARFIMA) process which generalizes standard stationary and non-stationary
processes and which can accommodate two alternative kinds of persistence, long-memory
and short-memory.

Our proposed model provides several economic and methodological advantages with
respect to previous studies. Economically, our long-memory model has the potential to
generate more volatile and realistic long-horizon expectations of the short-rate without
implying explosive dynamics for the short-rate. As was originally shown by Shiller (1979),
purely stationary I(0) models imply unrealistically flat distant expectations of the short-
rate because of the fast mean reversion.\textsuperscript{1} This is a very unfortunate feature of these models as it implies that changes in distant forward rates are mechanically attributed to term premium changes, even when there is evidence that distant expectations of the short-rate can be instrumental for these changes. In this sense, our model can derive a more realistic measure of the term premium, a variable of extraordinary importance for policy makers (see Bernanke (2006)).

On the methodological front, our model allows for a rich two-fold persistence structure (long and short-memory) characterizing term structure factors and is able to impart a very high persistence to term structure dynamics. Moreover, our model nests I(0) and I(1) term structure models, is more flexible than standard approaches based on integer degrees of differentiation, and simultaneously estimates the order of integration of the factors and identifies term premium. In this way, we do not have to choose the orders of integration of the term structure factors -and yields themselves- ex-ante.

We embed our VARFIMA model for factors in a standard affine term structure model and perform a battery of comparisons with respect to a simple I(0) VAR structure for factors. We estimate our model with U.S. quarterly data using the short-term interest rate and the unemployment rate as factors and the I(0) model is soundly rejected. We show that the volatility of 10-year short-rate expectations is between 60\% and 100\% higher in the VARFIMA term structure model, depending on the sample chosen. This wedge brings about important differences in term premium identification, sometimes involving more than 4 percentage points. In our estimation, the I(0) long-maturity term premium exhibits a clear downward trend for most of the sample, whereas that implied by the VARFIMA appears clearly countercyclical. These two term premiums have very

\textsuperscript{1}This point has later been illustrated in a variety of contexts by Kozicki and Tinsley (2001), Gürkaynak, Sack, and Swanson (2005), Backus and Wright (2007), Cochrane and Piazzesi (2008) and Gil-Alana and Moreno (2012).
different policy implications.

Two recent papers relating term structure models with fractional integration are Golinski and Zaffaroni (2011) and Gil-Alana and Moreno (2012). The first paper estimates two univariate fractionally integrated processes in order to separately identify the real rate and expected inflation components in the term structure, whereas the second one explores the consequences of univariate long memory in the short-rate in order to uncover the relation between the term premium and macro dynamics. The present paper is more general as it shows a novel methodology to estimate multivariate fractionally integrated models of the term structure. As extensively shown in the term structure literature, dynamic multivariate relations among factors are key in order to characterize yield curve dynamics. While the factors used are the short-term interest rate and the unemployment rate, our methodology can flexibly accommodate any set of macroeconomic, international or financial factors. Moreover, it can be used to estimate non-affine term structure models, as those estimated in Duffee (2011). It can also accommodate alternative estimation techniques for the prices of risk, such as those recently proposed by Joslin, Singleton, and Zhu (2011) and Hamilton and Wu (2012).

As recently shown by Bauer, Rudebusch, and Wu (2011, 2012), implied persistence in estimated short-memory-only models can exhibit severe downward biases due to small-sample problems, thus justifying bias-correction adjustment. While both fractional integration and bias-correction in I(0) models both impinge more persistence to term structure dynamics than estimated I(0) models, these two approaches are conceptually very different. Bias-correction is based on a well-known statistical problem, the downward bias in estimated persistence, whereas fractional integration is a statistical approach which can be economically motivated on aggregation grounds. For instance, the aggregation of inflation sub-indexes can give rise to a fractionally integrated inflation rate, as shown
in Altissimo, Mojon, and Zaffaroni (2009). Since interest rates are intimately related to the inflation rate, they can exhibit long-memory, as we show in the paper. We also compare both empirical approaches and show that while fractional integration delivers more persistence than the I(0)-bias corrected model, bias correction closes an important part of the persistence gap between the I(0) and the fractionally integrated model. We show however that the fractional approach delivers more similar degrees of persistence across subsamples, thus providing a very stable identification of the term premium.

In a related article, Jardet, Monfort, and Pegoraro (2011) apply the “averaging estimators” technique devised by Hansen (2007) to identify the term premium. This method, based on local-to-unity asymptotics, involves performing weighted averages of an I(0) VAR and a cointegrated I(1) model based on the prediction performance of each model. Therefore, this approach is also based on the I(0)/I(1) dichotomy, not taking into account fractional alternatives. Interestingly, and despite the clear methodological differences, our estimated term premium displays quite similar countercyclical dynamics to the ones derived by Jardet, Monfort, and Pegoraro (2011) and Bauer, Rudebusch, and Wu (2012).

The paper proceeds as follows. In Section 2, we illustrate some of the implications of stationary I(0) dynamics for factors in the context of an affine term structure model. In Section 3, we derive our term structure model with factors displaying short and long-memory persistence. Section 4 shows the empirical results, emphasizing the relevant differences between our fractionally integrated model and the I(0) counterpart. Section 5 concludes.
2 A Stationary Affine Term Structure Model

In the term structure literature, factors determining yield curve dynamics are typically assumed to be stationary I(0) (see Ang and Piazzesi (2003) and Bekaert, Cho, and Moreno (2010), among others).\(^2\) In fact, researchers routinely reject models with unit roots for interest rates because they may imply negative interest rates and explosive interest rate dynamics. To understand the influence of factor dynamics on the entire term structure of interest rates, we consider a discrete-time, no-arbitrage, affine term structure model (ATSM) of the sort employed by Cochrane and Piazzesi (2008).

ATSMs typically display stationary dynamics through the \(q \times 1\) state process vector \((Y_t)\), which can be modeled as an I(0) VAR(1) with constant variance without loss of generality (a companion matrix form can nest higher VAR orders):\(^3\)

\[
Y_t = \mu + \Omega Y_{t-1} + \Xi \epsilon_t \quad \epsilon_t \sim (0, I).
\]

Let \(p_t^{(n)}\) represent the price at time \(t\) of an \(n\)–period zero-coupon bond, and let \(i_t^{(n)} = -\log \left( p_t^{(n)} \right) / n\) denote its yield. If \(m_{t+1}\) denotes the nominal pricing kernel, bond prices can be recursively recursively as:

\[
p_t^{(n)} = E_t \left( m_{t+1} p_{t+1}^{(n)} \right).
\]

\(^2\)There are of course some exceptions. An incomplete list includes Campbell and Shiller (1987), who assume that yields are I(1), or Ang and Bekaert (2002) and Ang, Bekaert, and Wei (2008), who assume regime switches for interest rates.

\(^3\)An I(0) vector process is defined as a covariance stationary vector with spectral density matrix that is finite and positive definite.
Under conditional log-normality in the pricing kernel \( (m_{t+1}) \), we have that:

\[
m_{t+1} = \exp \left( -r_t - \frac{1}{2} \lambda_t' \lambda_t - \lambda_t' \varepsilon_{t+1} \right),
\]

(3)

where \( \lambda_t = \lambda_0 + \lambda_1 Y_t \), the set of prices of risk, is an affine function of the vector of state variables. In turn, \( \lambda_0 \) and \( \lambda_1 \) are a \( q \times 1 \) vector and a \( q \times q \) matrix, respectively.

The short-rate process can be expressed as:

\[
i_t = \delta_0 + \delta_1' Y_t,
\]

(4)

where \( \delta_0 \) is a scalar and \( \delta_1 \) is a \( q \times 1 \) vector. Bond prices are then found as in Wright (2011):

\[
p_t^{(n)} = \exp \left( A_n + B_n' Y_t \right),
\]

(5)

where \( A_n \) is a scalar and \( B_n \) is a \( q \times 1 \) vector, satisfying the recursive equations:

\[
A_{n+1} = -\delta_0 + A_n + B_n' (\mu - \Xi \lambda_0) + \frac{1}{2} B_n' B_n
\]

(6)

\[
B_{n+1}' = B_n' (\Omega - \Xi \lambda_1) - \delta_1'
\]

(7)

with \( A_1 = -\delta_0 \), \( B_1 = -\delta_1 \). As a result, this model characterizes the entire yield curve as:

\[
i_t^{(n)} = - \frac{A_n}{n} - \frac{B_n'}{n} Y_t.
\]

(8)

As the discussions in Cochrane and Piazzesi (2008), Jardet, Monfort, and Pegoraro (2011) and Gil-Alana and Moreno (2012) illustrate, the stationarity I(0) assumption for the factors is at all not innocuous and carries controversial implications for the characterization of term structure dynamics. In particular, one discomforting feature of the
stationary model is its relatively rapid mean reversion towards the interest rate mean in forecasting exercises. As a result, long-term expectations of the short-rate -needed to compute both the risk-neutral rate and the implied term premium- tend to display too little volatility.

To make this point transparent, in Figure 1 we plot the expectations of the short-rate (3-month T-Bill rate) 10 years out implied by our estimated I(0) VAR model for factors and compare it with the one-quarter ahead expectations. We estimated the model with the quarterly unemployment rate, the three and six-month T-Bill rates, downloaded from the FRED database, and with the quarterly zero-coupon bond (one to ten year) rates retrieved from the updated Gürkaynak, Sack, and Wright (2007) database from 1971:3Q to 2011:2Q (the data available when we started writing the article). We assume that the vector of state variables $Y_t$ consists of the unemployment rate and the short-term (3-month) rate. Thus, $Y_t = [u_t, r_t]'$. The short-rate captures the level of the yield curve, whereas the unemployment rate closely follows the yield slope dynamics. As a result, $\delta_0 = 0$ and $\delta_1 = [0, 1]'$. The estimation strategy of the ATSM closely follows the two-step procedure outlined in Wright (2011). We first estimate both the optimally selected VAR for the demeaned factors (VAR(3) according to the Schwarz BIC criterion) and then minimize the square difference between actual and model-implied yields to identify the prices of risk (see also subsection 4.1). Table 1 shows the term structure model parameter estimates.

Figure 1 shows that while the one-quarter-ahead expectations are very similar to the prevailing 3-month rate at each point in time, substantial mean reversion is present in the 10-year-ahead short-rate expectations. Indeed, long-run expectations are much closer to the historical mean than their short-run counterparts and are significantly less volatile. In Figure 2 we plot the standard deviation of the short-rate expectations across
forecast horizons. There is an exponential decay intrinsically related to the rapid mean reversion of short-term forecasts. It is important to note the radical difference between these forecasts and those which would be implied by an I(1) model for the short-rates, whose standard deviations are essentially fixed across forecast-horizons.

The previous figures clearly show that as the forecasting horizons lengthen, expectations of the I(0) and I(1) models diverge. So then, which one is the correct modeling assumption? One alternative is to formally test for the order of integration of the short-rate. Most tests in fact fail to reject the null of a unit root, but they are however plagued with small-sample problems and lack of power.\footnote{It is well-known that in small samples unit root tests have very low power against alternative such as trend-stationary models (DeJong, Nankervis, Savin, and Whiteman (1992)), structural breaks (Campbell and Perron (1991)), regime-switching (Nelson, Piger, and Zivot (2001)), or fractional integration (Diebold and Rudebusch (1991), Hassler and Wolters (1994), Lee and Schmidt (1996)).} Faced with this arbitrary choice, researchers typically feel more comfortable with the mean-reverting non-explosive I(0) model. As we show in the next section, there is another way out of this cross-roads: Let the data simultaneously determine the (possibly fractional) order of integration of the term structure factors and simultaneously identify both the short-rate expectations and the term premium.

3 A Fractionally Integrated Affine Term Structure Model (FIATSM)

In this section, we first describe a type of long-memory process, denoted fractional integration and then embed this structure into a standard affine term structure model. As we show below, an important advantage of our modeling strategy is that we can potentially avoid the lack of variability of long-horizon interest rate expectations while retaining
interest rate mean reversion.

3.1 Fractional Integration

Fractional integration lets the data decide the order of integration of macro-finance variables. This order could be zero, a fraction between zero and one, one or even above one. In the fractional integration setting, if a demeaned variable $x_t$ has an order of integration $d, (d \in \mathbb{R})$, it is denoted as $x_t \sim I(d)$ and can be expressed as:

$$(1 - L)^d x_t = \mu_t \quad t = 1, 2, \ldots, \quad (9)$$

with $x_t = 0, t \leq 0$. $\mu_t$ is assumed to be an I(0) process, defined as a covariance stationary process, with a spectral density function that is positive and finite at the zero frequency. Thus $\mu_t$ can be a stationary ARMA process. We can express $(1 - L)^d$ as the following binomial expansion:

$$(1 - L)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j = \left(1 - dL + \frac{d(d-1)}{2!} L^2 - \frac{d(d-1)(d-2)}{3!} L^3 \ldots\right). \quad (10)$$

The representation of $x_t$ in (9) can then be approximated for any real $d$, as:

$$\left(1 - dL + \frac{d(d-1)}{2!} L^2 - \frac{d(d-1)(d-2)}{3!} L^3 \ldots\right) x_t = \mu_t, \quad (11)$$

which is the infinite auto-regressive representation of the process. Alternatively, the process can also be expressed in terms of an infinite moving average process. While $d$ captures the long-memory component of the series, $\mu_t$ describes the short-run dynamics through its ARMA structure. The literature on fractional models like (9) has recently
emerged in macroeconomics and finance. Some examples are Diebold and Rudebusch (1989), Baillie and Bollerslev (1994) and Gil-Alana and Robinson (1997). If $d = 0$, the series is a covariance stationary process and possesses ‘short memory’, with the autocorrelations decaying fairly rapid. If $d = 1$, the series is a non-stationary I(1) process. But in a fractional framework there are more alternatives available for the order of integration of $x_t$. If $d$ belongs to the interval $(0, 0.50)$, $x_t$ is still covariance stationary, but both the autocorrelations and the response of a variable to a shock take much longer to disappear than in the standard ($d = 0$) stationary case. If $d = 0$ and $\mu_t$ follows an AR process, the decay in the autocorrelations is exponentially rapid compared with the I($d, d > 0$) case, where the decay is hyperbolic. If $d \in [0.50, 1)$, the series is no longer covariance stationary, but is still mean reverting, with the effect of the shocks dying away in the long-run.

In our case, the fractional differencing parameter $d$ plays a crucial role for our understanding of the dynamics of the short-term rate expectations. In particular, an I(0) model for the factors typically implies long-run expectations which are very close to the historical mean, thus making the term premium dynamics very similar to the actual long-rate. In contrast, if at least one of the factors follows an I(1) process, then the long-rate expectations of the short-rate mimic the current short-rate. While there may be economic reasons to postulate each one of these two alternatives, this choice is always essentially arbitrary. In this paper, we circumvent this problem by estimating the actual integration order of the factors from the data, and thus the implied yield curve dynamics, allowing it to be of a fractional order.

Even though most of the methodologies developed for fractional integration focus on univariate models, multivariate methods have also been developed, and, though they have been much less used than their univariate counterparts, they are extremely useful in our
context as will be shown below. Multivariate I(d) models have been mainly developed in the context of fractional cointegration (see, e.g., Robinson and Hualde (2003)), requiring, in our bi-variate context, that at least two of the series display the same degree of integration. However, in our work, we do not impose a priori the existence of any long-run equilibrium relationship among the factors, and consider a simple fractional VAR specification. To our knowledge, this is the first paper to introduce a VARFIMA model into a term structure model, as we show in the next subsection.

3.2 Affine Structure

We first introduce the process for the state vector and then derive implications for affine bond pricing in an ATSM. As a new contribution to the affine term structure literature, we let the vector of factors follow a VARFIMA (Vector Auto-Regressive Fractionally Integrated Moving Average) process of form:

$$DY_t = \mu + \zeta_t,$$

where \(D\) is a diagonal matrix of form:

$$D = \begin{pmatrix} (1 - L)^{d_1} & 0 \\ 0 & (1 - L)^{d_2} \end{pmatrix},$$

and \(d_1\) and \(d_2\) are the (potentially) fractional orders of integration of the factors. \(\zeta_t\) is a 2 × 1 stationary I(0) vector of errors. We can further assume that \(\zeta_t\) follows a stationary VAR(1) process, such as:

$$\zeta_t = \Omega \zeta_{t-1} + \Theta \eta_t,$$
where \( \eta_t \sim N(0, I) \). Notice that we can rewrite our model for the factors as:

\[
DY_t = (I - \Omega)\mu + \Omega DY_{t-1} + \Theta \eta_t. \tag{15}
\]

Applying the fractional integration filter, we can rewrite the system as an infinite sum of lags of the \( Y_t \) vector:

\[
Y_t = (I - \Omega)\mu + \sum_{i=1}^{\infty} \Upsilon_i(d, \Omega) Y_{t-i} + \Theta \eta_t, \tag{16}
\]

where, importantly, the coefficient matrices \( \Upsilon_i \) depend on the vector of long-memory \( d \) and short-memory \( \Omega \) parameters. As a result, our model for the factors combines both long and short-memory dynamics and nests the purely I(0) and I(1) models as special cases of interest. If we truncate the sequence of coefficient matrices \( \Upsilon_i \) at \( i = k \), we can express the factor dynamics in companion form as:

\[
Y_t = \tilde{\mu} + \Gamma Y_{t-1} + \Sigma \nu_t, \tag{17}
\]

where \( Y_t \) is a \( 2k \times 1 \) vector \( (Y_t = [Y'_t, Y'_{t-1}, \ldots, Y'_{t-k+1}]') \), \( \nu_t \) is a \( 2k \times 1 \) vector \( (\nu_t = [\eta'_t, 0_{1 \times 2(k-1)}]') \), and the \( 2k \times 1 \) \( \tilde{\mu} \) vector and the \( 2k \times 2k \) \( \Gamma \) matrix are, respectively:

\[
\tilde{\mu} = \begin{bmatrix}
(I - \Omega)\mu \\
0 \\
\vdots \\
0
\end{bmatrix}, \quad \Gamma(d, \Omega) = \begin{bmatrix}
\Upsilon_1 & \Upsilon_2 & \Upsilon_3 & \ldots & \Upsilon_k \\
I & 0 & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \ldots & \vdots \\
0 & 0 & \ldots & I & 0
\end{bmatrix}. \tag{18}
\]

Finally, \( \Sigma \) is also a \( 2k \times 2k \) matrix with zeros everywhere except in the upper-left \( 2 \times 2 \) block, which is equal to \( \Theta \).
Analogously to the I(0) model, the short-rate process can be expressed as:

\[ i_t = \delta_0 + \delta_1'Y_t, \quad (19) \]

where \( \delta_0 \) is a scalar and \( \delta_1 \) is now a \( 2k \times 1 \) vector. Bond prices are then:

\[ p_t^{(n)} = \exp (A_n + B_n'Y_t), \quad (20) \]

where \( A_n \) is a scalar and \( B_n \) is a \( 2k \times 1 \) vector that satisfy the recursion:

\[
\begin{align*}
A_{n+1} &= -\delta_0 + A_n + B_n' (\mu - \Sigma \lambda_0) + \frac{1}{2} B_n' B_n \\
B_{n+1}' &= B_n' (\Gamma(d, \Omega) - \Sigma \lambda_1) - \delta_1' 
\end{align*} \quad (21) \]

with \( A_1 = -\delta_0, \ B_1 = -\delta_1 \). Notice that \( \lambda_0 \) is now a \( 2k \times 1 \) vector whereas \( \lambda_1 \) is a \( 2k \times 2k \) matrix. In terms of yields:

\[ i_t^{(n)} = a_n - b_n'Y_t, \quad (23) \]

where \( a_n = -\frac{A_n}{n} \) and \( b_n = -\frac{B_n'}{n} \).

4 Data and Estimation Strategy

We work in the quarterly frequency with the unemployment rate and the end-of-quarter three and six-month T-Bill rates, downloaded from the FRED database, and with the one to ten-year zero-coupon rates, from the updated database elaborated by Gürkaynak, Sack, and Wright (2007). Our full sample covers the 1971:Q3 - 2011:Q2 period.

As mentioned above, we choose the demeaned 3-month T-Bill rate and the unem-
ployment as the two factors because they capture the level and slope of the yield curve, two key factors in the identification of the term structure. We also note that our FI-ATSM setting can accommodate essentially any set of factors but principal components, because these are constructed on the basis of contemporaneous yields alone, which is not consistent with the no-arbitrage framework, as shown in equation (23).

The vector of prices of risk is defined as $\lambda_t = \lambda_0 + \lambda_1 Y_t$. To avoid parameter proliferation, the time invariant part of prices of risk is also restricted to the first two elements:

$$
\lambda_0 = \begin{bmatrix} \lambda_0^{(1)} & \lambda_0^{(2)} & 0 & \ldots & 0 \end{bmatrix}'.
$$

Furthermore, price of risk sensitivities are restricted to contemporaneous components, i.e., the $\lambda_1$ matrix becomes:

$$
\lambda_1 = \begin{bmatrix}
\tilde{\lambda}_1 & 0_{2\times2(k-1)} \\
0_{2(k-1)\times2} & 0_{2(k-1)\times2(k-1)}
\end{bmatrix}.
$$

Our two-step estimation approach is similar to that employed by Wright (2011). Key to our strategy is the first-step estimation of the VARFIMA model outlined in the previous section. The elements in $\Gamma$, containing long-memory ($D$) and short-memory ($\Omega$) parameters, and $\Sigma$ are estimated following the maximization of the Whittle function, which is an approximation of the likelihood function in the frequency domain, as outlined in the Appendix. Since the short-rate is a factor itself, $\delta_0 = 0$ and the vector $\delta_1$ is a vector of zeros except for the contemporaneous response to the short-rate, where there is a one entry.

The remaining parameters, the prices of risk, are estimated in a second step by
minimizing the sum of squared differences between actual and fitted yields, that is:

\[
\{\hat{\lambda}_0, \hat{\lambda}_1\} = \arg \min_{\lambda_0, \lambda_1} \sum_t \sum_n \left( \tilde{i}^{(n)}_t - \tilde{\tilde{i}}^{(n)}_t \right)^2,
\]

where \(\tilde{\tilde{i}}^{(n)}_t = a_n + b'_n Y_t\) are the model implied yields, and where \(Y_t\) is truncated at lag \(k = 20\). Alternative high values of \(k\) yielded almost identical results, although this may not be the case in every subsample, as we discuss in the last subsection. Having estimated the model parameters, short-term expectations and term premiums at various horizons can be easily computed.

5 Results

In the first part of this section, we present results for the full sample of the FIATSM model and compare them with those obtained in the I(0) exercise. We then show the relation between the persistence implied by the FIATSM and the I(0) ATSM with and without bias-correction for the full sample and alternative subsamples.

5.1 Full Sample Results

Table 2 shows the estimates of our Fractionally Integrated ATSM (FIATSM) for the full sample (1971:3Q to 2011:2Q). The order of integration of the unemployment rate is 0.76, significantly different from zero and one, whereas that of the short-term interest rate is 0.91, significantly different from 0, but not from 1. Thus, while the factors exhibit significant long-memory, the implied process for the term structure of interest rate is mean reverting. Interestingly, the I(0) model for factors is clearly rejected in the data. Because the long-memory part exhibits great persistence, the short-memory component
of the FIATSM displays less persistence than its I(0) ATSM counterpart. As a result, much of the persistence previously shown by the I(0) model is actually due to the long-memory component and our flexible FIATSM captures this fact.

The model fits the yield curve very well with a low root-mean-square fitting error (0.0069), lower than the I(0) ATSM (0.0074). Indeed, predicted yields track actual yields very closely across the whole term structure of interest rates. This is a well-known common finding in the ATSM literature, which our model shares with its I(0) counterparts. Figure 3 compares the implied intercepts and factor loadings of our FIATSM with those implied by the I(0) ATSM. These coefficients are quite similar with the small difference that the I(0) ATSM loadings on the short-term rate are slightly higher across maturities.\footnote{Notice that our FIATSM implies factor loadings on both contemporaneous and past values of the factors in $\mathbf{Y}_t$. Thus, we add the contemporaneous factor loadings and the associated ones for all lags when graphing the three factor loadings in our FIATSM. If we only account for the contemporaneous factor loadings in the FIATSM, results are still quite similar to the I(0) ATSM.} The mean term structure is upward sloping and loads positively on both the unemployment rate and the short-term rate. However, while the loadings on the short-rate negatively depend on the maturity, the loadings on the unemployment rate are higher at the long end of the yield curve. This is an interesting finding, showing that uncertainty about the state of the economy translates into higher long-term yields. Given the similarity of the estimated coefficients, a first conclusion emerges: Both the I(0) and fractionally integrated models fit equally well the cross-section of yields. So where are the differences?

The two ATSMs clearly differ in the persistence implied by their state processes. Table 3 shows that while the first eigenvalue is 0.9951 for the VARFIMA, it is 0.9822 for the I(0) VAR. Moreover, the second eigenvalues are 0.9877 and 0.7525 respectively. Thus, the VARFIMA imparts substantially more persistence to state dynamics than the I(0) model and this brings up some stark differences in terms of the implications between
the I(0) and FIATSM models, as we will now show.

Figure 4 performs an analogous exercise to Figure 1, but now with our FIATSM. It compares the one-quarter-ahead and 10-year-ahead expectations of the short-rate. Unlike the I(0) model, the two FIATSM expectation processes are not too different and this is due to the persistence implied by the FIATSM, which incorporates long-memory to yield dynamics. To emphasize this important point, Figure 5 compares the standard deviations of short-rate expectations across forecast horizons. It shows that short-rate expectations are clearly more volatile in the FIATSM across forecast horizon, especially at longer ones. This implies expectations which are much closer to actual rates than those implied by the I(0) ATSM.

Figure 6 shows the impulse response functions of both the 1-year and 10-years interest rates, as well as the spread between the 10-year and 3-month interest rate— to the unemployment and the interest rate shocks for both the I(0) ATSM and the FIATSM. It shows important differences. In particular, following the unemployment shock, the short-rate decrease is stronger and considerably more persistent under the FIATSM, whereas the long-rate increases persistently, in contrast to the I(0) ATSM. As a result, the spread notably and persistently increases following the unemployment shock. Responses are more similar following the interest rate shock, with the spread experiencing an initial decline but reverting back to normal after 10 quarters.

Figure 7 plots the five-year-ahead-five-year term premium for the I(0) ATSM and the FIATSM. This is computed as the difference between the model-implied five-to-ten-year forward rate and the average expected three-month interest rate from five to ten years hence. While both term premiums are mildly positively correlated (0.32), there are some significant departures across term premiums in relevant periods of time. For instance, in the late 70s and early 80s, the term premium implied by the FIATSM
becomes negative and sometimes 4 percentage points below its I(0) counterpart. Also, during the recent credit crisis, the FIATSM term premium is more than two percentage points above the I(0). Importantly, the FIATSM produces a term premium significantly more countercyclical-and thus more reasonable- than the I(0) model.

This figure captures a key idea of the paper: The I(0) model implies long-run short-rate expectations which are very close to the historical mean, producing term premiums very much correlated with the forward rates. This is very clear all throughout the sample, until the 2008 crisis; see for instance the downward trend starting in the early 80s. In stark contrast, the FIATSM does not exhibit this trend. Since it fully accounts for the high persistence embedded in the whole term structure, it produces volatile expectations of the short-rate even many years ahead. In this respect, our approach achieves the same goal as Bauer, Rudebusch, and Wu (2011) but through a very different route, as they impart more persistence to the term structure through bias-correction in an I(0) model. We explore the relation between the two approaches at the end of this section.

In order to shed intuition on the sources of term premium dynamics, Figure 8 compares the impulse response functions of the term premiums implied by the FIATSM and the I(0) ATSM following the unemployment and interest rates shocks, respectively. Structural shocks are recursively identified assuming that the short-rate does not contemporaneously affect unemployment. The figure shows that in the FIATSM, the term premium increases persistently after an unemployment shock, reflecting the idea that a more uncertainty about the real economy makes investors more risk averse to long-term bonds. In the I(0) ATSM, the effect is statistically lower, less persistent and disappears after ten quarters. In contrast, the effect of the short-rate shock on the FIATSM term premium is small, negative and not-statistically significant, whereas the response of its I(0) counterpart is positive and statistically higher. Our results thus imply that term
premium dynamics are mostly driven by real factors, especially at longer horizons.

5.2 Relation with Bias-Correction in I(0) Models

In recent papers, Bauer, Rudebusch and Wu (2011, 2012) have shown that bias-correcting in an I(0) ATSM imparts more persistence to fitted term structure dynamics and implies more realistic and counter-cyclical term premiums. In this subsection we explore the relation between our fractional integration approach and bias correction in an I(0) framework.

As shown in the previous subsection, the FIATSM imparts considerably more persistence to the state-vector dynamics than the I(0) model. The VARFIMA embedded in the FIATSM nests the I(0) VAR as a special case and clearly rejects this special case. As shown above, the maximum eigenvalue implied by the VARFIMA, which governs the term structure persistence, is considerably higher than that implied by the I(0) VAR. This result has important economic implications as the FIATSM implied term premium exhibits a clear countercyclical and more volatile dynamic pattern, in contrast to its I(0) counterpart.

At this point, it is important to point out that the persistence implied by the VARFIMA model is dependent on the truncation lag, as seen in the definition of the matrix $\Gamma$. This can again be seen through the lens of the maximum eigenvalue of the state-system. In Table 3 we show the maximum eigenvalues of the VARFIMA implied by two truncation lags, 20 and 100. As can be seen, for the full sample, the maximum eigenvalues are similar and thus the implied term structure dynamics also are. For alternative subsamples, the eigenvalues differ a bit more. As a result, the decision on the truncation lag can be a non-trivial one and has important implications on term structure persistence and term
premium identification.

We now compare relate the results implied by the VARFIMA with the recent work on bias-correction in bond pricing I(0) state-vector dynamics. We first perform bias-correction to the I(0) VAR for the full sample dynamics. We first compare the corresponding maximum eigenvalues for the VARFIMA for the two truncation lags, the I(0) VAR and the I(0) bias-corrected VAR. We then perform analogous exercises for alternative subsamples in order to uncover a pattern independently of the sample chosen.

In order to estimate the bias-corrected I(0) VAR, we perform the following bootstrap exercise. We construct 1,000 synthetic samples bootstrapping from the original error terms of the I(0) model. We then re-estimate the model 1,000 times, yielding 1,000 sets of parameter estimates. Given the mean of this set, we correct the original estimates and compute the maximum eigenvalue of the system.

As shown in Table 3, there is a consistent downward bias in the implied persistence of the originally estimated I(0) model. Indeed, in the full sample, the maximum eigenvalue of the I(0)-bias corrected VAR practically converges towards that of the fractionally integrated model. To graphically illustrate this point, Figure 9 shows the impulse responses of unemployment and the short-term interest rate following the respective structural shocks under the three modeling frameworks: I(0) VAR, VARFIMA (with 100 lags) and bias-corrected-I(0) VAR. Shocks are identified recursively, assuming that unemployment does not react contemporaneously to interest rate shocks. The figure clearly shows across impulse responses that the VARFIMA model provides more persistence to macro-finance dynamics in comparison to the I(0) model. The bias-correction in the I(0) model closes an important part of this gap.

We further elaborate this point by estimating the state variable systems (I(0) VAR,
I(0)-bias corrected VAR and VARFIMA with two truncation lags) for alternative sub-
samples. In particular, we estimate the subsamples starting at six different times (1st 
quarter of 1985, 1986, 1987, 1988, 1989 and 1990) and ending in the second quarter of 
2011). Starting with the second row, Table 3 shows the implied maximum eigenvalues 
of alternative systems. Across samples, the results appear very robust: the VARFIRMA 
consistently implies a very high degree of persistence (for both truncation lags), signif-
icantly higher than the I(0) VAR, whereas the I(0)-bias corrected VAR closes some of 
this gap. In any case, this bias correction never reaches the level of persistence in the 
VARFIMAs (especially that with 100 lags). As a result, the VARFIMA will exhibit great 
stability in the term premium identification than its counterparts, given that it delivers 
a very stable degree of persistence across samples. This is an important point for policy 
makers because they are likely to use alternative datasets and sample periods in the 
identification of the key term premium variable.

6 Conclusions and Extensions

In a well-known speech, Bernanke (2006) warned economic analysts that the monetary 
policy implications of a change in long-term yields crucially depend on its source. An 
increase in the long-rates due to higher expected short-rates would have very different 
implications for policy than if it is due to a higher term premium. In short, when a long-
rate hike follows an increase in the term premium, the situation calls for a monetary policy 
expansion, whereas the opposite is true if it follows an increase in expected future short 
rates. While both expected future short-rates and the term premium are unobservable, 
their dynamics can be extracted from a well-defined term structure model. In this paper, 
we have proposed an affine model based on a dynamic multivariate state space combining
both long and short-memory components. Our model estimates imply that term structure persistence is well characterized by such combination, with long-memory providing a great deal of volatility to the expectations of the short-rate, even in a context of mean-reversion. Interestingly, the persistence implied by our fractionally integrated multivariate model is very stable across subsamples, which is key for stability of term premium identification. We also show that standard stationary models severely underpredict the term premium in the years following the 2008 credit crisis.

The methodology presented in this paper is quite general and we intend to apply it in several directions. First, we can include almost any set of additional relevant macro-finance variables in order to examine the structural shocks affecting term premium dynamics as well as the effects of term premium shocks on these macro-finance variables. Second, while the application presented here focuses on an affine term structure model, it can be easily applied to non-affine models, such as linear models. We can then assess the differences across models and test their relative forecasting ability. Finally, we intend to extend the analysis in this paper to an international setting. While several papers have studied the relation between the home term structure and international factors (see, for instance, Diebold, Li, and Yue (2008)), all these applications suffer from the fast-mean-reversion problem in short-rate expectations, associated with stationary I(0) contexts.
Appendix: VARFIMA Estimation Procedure

The frequency domain approach to estimating the model parameters is based on the maximization of the Whittle function, which is an approximation to the likelihood function. The approximation was originally proposed by Whittle (1952) for scalar-valued stationary processes; see also Dunsmuir and Hannan (1976). The discussion of the multivariate version can be found in Hosoya (1996).

To derive the frequency domain log-likelihood function, assume for the moment that the process for $Y_t = [u_t, r_t]'$, $t = 1, ..., T$ is Gaussian. Denote the vector of parameters to estimate by $\theta$. It contains the parameters of fractional integration in the two factors, short memory parameters and the parameters from the variance-covariance matrix of the error terms. The $2 \times 2$ spectral density matrix of the process $Y_t$, $f_Y(\omega, \theta)$ is given by:

$$ f_Y(\omega, \theta) = (2\pi)^{-1} D(e^{-i\omega})^{-1} f_\zeta(\omega) D(e^{i\omega})^{-1}, \quad (25) $$

where $D(e^{i\omega})$ is a $2 \times 2$ diagonal matrix with the diagonal elements given by $(1 - e^{i\omega})^{d_k}$, $d_k$ is a fractional order of integration of the $k-th$ factor, and where the complex conjugate of $D(e^{i\omega})$ is $D(e^{-i\omega})$; $f_\zeta(\omega)$ is defined as:

$$ f_\zeta(\omega, \theta) = (I - F(e^{-i\omega}))^{-1} \Sigma (I - F(e^{i\omega}))^{-1} \quad (26) $$

where $F(e^{i\omega}) = F_1 e^{i\omega} + F_2 e^{2i\omega} + ... + F_p e^{pi\omega}$ and $p$ is a number of lags in the short-memory polynomial.

The finite Fourier transforms $w_i(\omega_j, Y_{i,t})$ based on finite observations $Y_{i,t}$, $i = 1, 2$ and $t = 1, 2, \ldots, T$, are given by:
\[ w(\omega_j, Y_i) = \frac{1}{\sqrt{2\pi T}} \sum_{t=1}^{T} Y_{i,t} e^{-i\omega_j(t-1)} \]  

(27)

where the frequencies \( \omega_j = \frac{2\pi j}{T} \), \( j = 0, 1, ..., T/2 - 1 \) are chosen equispaced on \([0, \pi]\) in such a way that \( f_Y(\omega, \theta) \) is continuous at \( \omega = \omega_j \).

The approximate log-likelihood function of \( \theta \) based on \( Y \) is given up to constant multiplication, by:

\[
\ln L(\omega, \theta) = - \sum_{j=0}^{T/2 - 1} \left[ \ln \det f_Y(\omega_j, \theta) + tr f_Y^{-1}(\omega_j, \theta) I_T(\omega_j, Y) \right]
\]

(28)

with the \( 2 \times 2 \) cross-periodogram matrix \( I_T(\omega_j, Y) \). Each element \( kl, k, l = 1, 2 \) of this matrix is given by:

\[
I_T^{kl}(\omega_j, Y) = w(\omega_j, Y_k) w(\omega_j, Y_l)^*,
\]

(29)

where \( w(\omega_j, Y_i) \) is the tapered Fourier transform based on finite observations \( Y_{i,t}, i = 1, 2, t = 1, ..., T \) and \( w(\omega_j, Y_i)^* \) is its complex conjugate.\(^6\)

In such a way, for each \( j \), the elements of the main diagonal \( (k = l = i) \) are points of the periodogram of \( y_{i,t} \) at frequency \( \omega_j \), which are real numbers. The off-diagonal elements \( (k \neq l) \) are points of the cross-periodogram, which are complex. Note, that the element \( (k, l) \) is the complex conjugate of the element \( (l, k) \).

\(^6\)We use taper with Parzen weights because we suspect non-stationarity (in levels) or anti-persistence (after taking the first difference) of the data. As shown by Velasco and Robinson (2000), the estimates from the Whittle maximum likelihood are still consistent and asymptotically normal for long range dependence if the periodogram is computed from the tapered Fourier transform.
References


Cochrane, John, and Monika Piazzesi, 2008, Decomposing the Yield Curve, Mimeo, University of Chicago.


Duffee, Gregory R., 2011, Forecasting with the term structure: the role of no-arbitrage restrictions, Mimeo, Johns Hopkins University.


Table 1: **Parameter Estimates: VAR I(0) ATSM**

<table>
<thead>
<tr>
<th>$\Omega_{11}$</th>
<th>1.4675 (0.0839)</th>
<th>$\sigma_{11}$</th>
<th>0.2966</th>
<th>$\lambda_{0,1}$</th>
<th>0.1827 (0.0741)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_{12}$</td>
<td>0.2127 (0.0920)</td>
<td>$\sigma_{21}$</td>
<td>-0.1080</td>
<td>$\lambda_{0,2}$</td>
<td>-1.1488 (0.0257)</td>
</tr>
<tr>
<td>$\Omega_{21}$</td>
<td>-0.2201 (0.0749)</td>
<td>$\sigma_{22}$</td>
<td>0.2418</td>
<td>$\lambda_{1,11}$</td>
<td>-0.1166 (0.0307)</td>
</tr>
<tr>
<td>$\Omega_{22}$</td>
<td>0.6615 (0.0822)</td>
<td></td>
<td></td>
<td>$\lambda_{1,12}$</td>
<td>0.2355 (0.0177)</td>
</tr>
<tr>
<td>$\Omega_{13}$</td>
<td>-0.2964 (0.1443)</td>
<td></td>
<td></td>
<td>$\lambda_{1,21}$</td>
<td>-0.2280 (0.0089)</td>
</tr>
<tr>
<td>$\Omega_{14}$</td>
<td>0.0197 (0.1176)</td>
<td></td>
<td></td>
<td>$\lambda_{1,22}$</td>
<td>0.1161 (0.0127)</td>
</tr>
<tr>
<td>$\Omega_{23}$</td>
<td>0.2009 (0.1289)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Omega_{24}$</td>
<td>0.0213 (0.1050)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Omega_{15}$</td>
<td>-0.2205 (0.0839)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Omega_{16}$</td>
<td>-0.1733 (0.0935)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Omega_{25}$</td>
<td>0.0062 (0.0749)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Omega_{26}$</td>
<td>0.2990 (0.0839)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This table shows the full-sample (1971:3Q-2011:2Q) point estimates of the I(0) ATSM (Affine Term Structure Model) parameters. The first two columns show the coefficient estimates of the state process for factors, a VAR(3): $Y_t = \Omega_1 Y_{t-1} + \Omega_2 Y_{t-2} + \Omega_3 Y_{t-3} + \Xi \varepsilon_t$. The $\sigma_{ij}$'s are the elements in the variance-covariance matrix of the reduced-form error terms, $E[\Xi \cdot \Xi']$. The last three columns show the price of risk estimates associated with the factors.
Table 2: Parameter Estimates: FIATSM (Full Sample)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_1$</td>
<td>0.7624</td>
<td>(0.1269)</td>
</tr>
<tr>
<td>$d_2$</td>
<td>0.9116</td>
<td>(0.0882)</td>
</tr>
<tr>
<td>$\Omega_{11}$</td>
<td>0.7962</td>
<td>(0.0943)</td>
</tr>
<tr>
<td>$\Omega_{12}$</td>
<td>-0.1401</td>
<td>(0.0592)</td>
</tr>
<tr>
<td>$\Omega_{21}$</td>
<td>-0.1793</td>
<td>(0.1080)</td>
</tr>
<tr>
<td>$\Omega_{22}$</td>
<td>0.2199</td>
<td>(0.1017)</td>
</tr>
<tr>
<td>$\sigma_{11}$</td>
<td>0.3133</td>
<td>(0.0176)</td>
</tr>
<tr>
<td>$\sigma_{21}$</td>
<td>-0.0362</td>
<td>(0.0157)</td>
</tr>
<tr>
<td>$\sigma_{22}$</td>
<td>0.2782</td>
<td>(0.0076)</td>
</tr>
<tr>
<td>$\Omega_{11}$</td>
<td>0.7962</td>
<td>(0.0943)</td>
</tr>
<tr>
<td>$\Omega_{12}$</td>
<td>-0.1401</td>
<td>(0.0592)</td>
</tr>
<tr>
<td>$\Omega_{21}$</td>
<td>-0.1793</td>
<td>(0.1080)</td>
</tr>
<tr>
<td>$\Omega_{22}$</td>
<td>0.2199</td>
<td>(0.1017)</td>
</tr>
<tr>
<td>$\lambda_{0,1}$</td>
<td>0.0333</td>
<td>(0.0636)</td>
</tr>
<tr>
<td>$\lambda_{0,2}$</td>
<td>-1.0935</td>
<td>(0.0164)</td>
</tr>
<tr>
<td>$\lambda_{1,11}$</td>
<td>0.0609</td>
<td>(0.0337)</td>
</tr>
<tr>
<td>$\lambda_{1,12}$</td>
<td>0.0615</td>
<td>(0.0173)</td>
</tr>
<tr>
<td>$\lambda_{1,21}$</td>
<td>-0.1226</td>
<td>(0.0090)</td>
</tr>
<tr>
<td>$\lambda_{1,22}$</td>
<td>0.0595</td>
<td>(0.0377)</td>
</tr>
</tbody>
</table>

This table shows the full-sample (1971:3Q-2011:2Q) estimates and standard errors of the FIATSM (Fractionally Integrated Affine Term Structure Model) parameters. Associated standard errors appear in parentheses. The first three columns show the coefficient estimates of the state process for factors: $DY_t = \zeta_t$, $\zeta_t = \Omega_{\zeta_t-1} + \Theta \eta_t$. The $\sigma_{ij}$’s are the elements in the variance-covariance matrix of the reduced-form error terms, $E[\Theta \cdot \Theta']$. The last three columns show the price of risk estimates associated with the factors.
Table 3: **System Eigenvalues**

<table>
<thead>
<tr>
<th>Sample</th>
<th>VARFIMA(100)</th>
<th>VARFIMA(20)</th>
<th>I(0) VAR</th>
<th>I(0) VAR-bc</th>
</tr>
</thead>
<tbody>
<tr>
<td>1971:3-2011:2</td>
<td>0.9990</td>
<td>0.9951</td>
<td>0.9822</td>
<td>0.9947</td>
</tr>
<tr>
<td>1985:1-2011:2</td>
<td>0.9935</td>
<td>0.9792</td>
<td>0.9412</td>
<td>0.9538</td>
</tr>
<tr>
<td>1986:1-2011:2</td>
<td>0.9951</td>
<td>0.9819</td>
<td>0.9368</td>
<td>0.9558</td>
</tr>
<tr>
<td>1987:1-2011:2</td>
<td>0.9955</td>
<td>0.9831</td>
<td>0.9495</td>
<td>0.9794</td>
</tr>
<tr>
<td>1988:1-2011:2</td>
<td>0.9955</td>
<td>0.9837</td>
<td>0.9407</td>
<td>0.9778</td>
</tr>
<tr>
<td>1989:1-2011:2</td>
<td>0.9968</td>
<td>0.9867</td>
<td>0.9267</td>
<td>0.9678</td>
</tr>
<tr>
<td>1990:1-2011:2</td>
<td>0.9969</td>
<td>0.9871</td>
<td>0.9200</td>
<td>0.9638</td>
</tr>
</tbody>
</table>

This table shows the largest eigenvalues associated with the estimation of the three state-systems for the full sample and alternative subsamples: the Fractionally Integrated VAR (VARFIMA) under two truncation schemes (20 and 100 lags), the I(0) Vector Auto-Regression (VAR) and the bias-corrected I(0) VAR.
Figure 1: Short-term rate expectations (I(0) model)

Note: This figure plots the expectations of the short-rate (3-month) one-quarter and 10-years ahead implied by the I(0) ATSM (Affine Term Structure Model).
Figure 2: Volatility of short-rate expectations as a function of the forecast horizon (I(0) model)

Note: This figure plots the volatility of the short-rate (3-month) expectations series implied by the I(0) ATSM (Affine Term Structure Model) as a function of the forecast horizon measured in quarters.
Figure 3: Factor loadings of the ATSMs

Note: This figure compares the factor loadings of the I(0) ATSM (Affine Term Structure Model) and the FIATSM (Fractionally Integrated Affine Term Structure Model). The FIATSM determines the yields as: $i_t^{(n)} = a_n + b_n Y_t$, with $(Y_t = [Y_t', Y_{t-1}', ..., Y_{t-k+1}]')$ and $Y_t = [u_t, r_t]'$. In the case of the I(0) ATSM, $Y_t = Y_t$. The maturity of the yield is measured in quarters and shown in the x-axis.
Note: This figure plots the expectations of the short-rate (3-month) one-quarter and 10-years ahead implied by the FIATSM (Fractionally Integrated Affine Term Structure Model).
Figure 5: Volatility of short-rate expectations as a function of the forecast horizon

Note: This figure compares the volatility of the short-rate (3-month) expectations series implied by the I(0) ATSM (Affine Term Structure Model) and the FIATSM (Fractionally Integrated Affine Term Structure Model) as a function of the forecast horizon measured in quarters.
Figure 6: Yield Curve Impulse Response Functions

Note: This figure shows the impulse response functions of the 1-year rate, 10-year rate and interest rate spread (10-year minus 3-month) to a positive unemployment (left) and interest rate (right) for both the FIATSM (Fractionally Integrated Affine Term Structure Model) and the I(0) ATSM.
Figure 7: Term Premiums

Note: This figure plots the implied five-year, five-year ahead forward term premiums implied by the I(0) ATSM (Affine Term Structure Model, in thin line) and the FIATSM (Fractionally Integrated Affine Term Structure Model, in thick line) models. The term premium is computed as the difference between the model-implied five-to-ten-year forward rate and the average expected three-month interest rate from five to ten years hence.
Figure 8: Term Premium Impulse Response Functions

Note: This figure shows the impulse response functions of the term premium responses following unemployment and interest rate shocks for both the FIATSM (Fractionally Integrated Affine Term Structure Model) and the I(0) ATSM.
Note: This figure shows the impulse response functions of the two term structure factors following the two structural shocks using the VARFIMA, I(0) VAR and I(0) VAR-bias corrected models in the full sample (1971:3Q-2011:2Q).