DYNAMICAL SYSTEMS APPROACH TO MODELING CHILD MORTALITY, FERTILITY RATE, FEMALE EDUCATION AND ECONOMIC GROWTH

By Shyam Ranganathan* ,Richard P. Mann, Stamatios C.Nicolis, Ranjula Bali Swain and David J.T. Sumpter

ABSTRACT

The increasing availability of detailed socio-economic data, in the form of indicators measured at different points in time, provides new challenges and possibilities for data analysis. For instance, a key aim of development economics is to investigate the relationship between various economic, health and social indicators. The question here is to identify, interpret and predict dynamic changes in these indicators, with an aim to setting goals for future development and making informed policy decisions. We present a dynamical systems approach to visualizing socio-economic data and for fitting differential equation models directly to this data. We employ a Bayesian methodology to identify non-linear interactions between indicators and assign likelihoods to the differential equation models. We focus our approach on four development indicators available for different countries: child mortality, fertility rate, female education and log GDP. We show that our approach can be used to predict future changes in these indicators, and to set country specific development goals. We discuss how to use the model to elucidate the relationship between the four indicators. The data-driven approach we propose here is relevant not only to questions of development, but also to the analysis of other data rich socio-economic systems.

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* This work was funded by ERC grant 1DC-AB. Ranganathan: Department of Mathematics, Uppsala University, Shyam.Ranganathan@math.uu.se. Mann: Department of Mathematics, Uppsala University, Richard.Mann@math.uu.se. Nicolis:Department of Mathematics, Uppsala University, Stamatios.Nicolis@math.uu.se. Swain: Department of Economics, Uppsala University, ranjula@nek.uu.se. Sumpter: Department of Mathematics, Uppsala University, David.Sumpter@math.uu.se.

I. Introduction

Human social and economic development is a complex process taking place across multiple dimensions. To study this process methodically, even in terms of the available statistics on economic indicators, is immensely difficult (Sagar and Najam, 1998; Sen, 1992). In the context of the Human Development Index (HDI), Srinivasan (1994) identifies a number of problems, including incompleteness of coverage, measurement errors, and biases in the collection of statistics. Moreover, development indicators are correlated and non-orthogonal, making them difficult to analyse. On the other hand, the consistent formulation of ambitious policy goals such as the Millenium Development Goals (MDGs) requires mathematical justification. With the availability of reliable aggregate socio-economic data over many years across many countries (UNDP, 2011; WorldBank, 2009) it is imperative to try and come up with mathematical models which can be used to understand interactions between development indicators and to predict future changes in these indicators. These models will then inform policy making intelligently and result in consistent decisions.

In development economics, usually relationships between indicator variables are established on the basis of survey data (Hobcraft, Mcdonald and Rutstein, 1984; Cleland and van Ginneken, 1988). Using survey data is the most appropriate method for evaluating whether particular intervention methods, such as increasing female education to reduce fertility, are likely

* This work was funded by ERC grant 1DC-AB. Ranganathan: Department of Mathematics, Uppsala University, Shyam.Ranganathan@math.uu.se. Mann: Department of Mathematics, Uppsala University, Richard.Mann@math.uu.se. Nicolis:Department of Mathematics, Uppsala University, Stamatios.Nicolis@math.uu.se. Swain: Department of Economics, Uppsala University, ranjula@nek.uu.se. Sumpter: Department of Mathematics, Uppsala University, David.Sumpter@math.uu.se.

to have an effect. Unfortunately, survey data is usually collected for a small number of countries to answer specific research questions, and as such is not sufficient to identify dynamic interactions between development indicators over many years.

Growth econometrics uses aggregate data to construct models of GDP. Its aim is "to investigate whether or not particular hypotheses [about the causes of economic growth] have any support in the data" (Durlauf, Johnson and Temple, 2005). In practice, the goal is to identify systematic patterns in economic growth and assess the degree of confidence in these patterns. Thus, while grounded in the Solow one-sector growth model (Barro, 1991), growth econometrics has employed regression models to relate per capita GDP growth rate to a whole range of factors. Countries are treated as independent observations upon which to perform this regression and the question is which country-level indicators predict growth. For example, Sala-i Martin (1997) chooses a set of 59 aggregate indicators and concludes that at least 22 of these are correlated with growth across different countries. Fernandez, Ley and Steel (2001) describe a Bayesian framework to give the relative importance of different variables. GDP level in 1960, fraction of Confucians in the population, life expectancy and equipment investment are identified as the four most important variables.

Fernandez, Ley and Steel (2001) is part of a growing trend in growth econometrics to use the Bayesian approach, where all available data are used and prior assumptions are clearly stated before model fitting is carried out, as an important tool in growth econometrics. They advocate the use of the Bayesian posterior probability (which is greater than 0.9 for the four variables mentioned in the previous paragraph) as a weighting when building a model by which to infer economic growth. In general, Bayesian model averaging is prescribed to avoid model uncertainty, with studies also focusing on the role of prior distributions, i.e. pre-conceived notions about models (Ley and Steel, 2009).

One approach to extending growth economics to address questions about development is to build on the neo-classical literature. Under this approach, dynamic interactions of indicators such as education and fertility can be included, by making assumptions about the micro-level interactions of economic agents. Causation is implied by the assumption that each economic agent attempts to maximize their own utility, subject to some constraints about how they can act. A dynamical model of the macro-level economy is then derived. For example, the basic relationship between human and technological capital and economic growth is captured by the Solow one-sector growth model (Solow, 1956; Basu, 2003). To understand the relationship between child mortality, fertility, human capital, technology, and economic growth, further models have been developed in which individuals attempt to optimise the number of children they produce (Barro and Becker, 1989; Galor, 2005, 2010). For example, Barro and Becker (1989) look at how tradeoffs in quantity and quality of children lead to relationships between education and fertility. More ambitiously, Galor (2005) formulates a unified growth model which captures the transition from "Malthusian" population growth and economic stagnation to sustained economic growth and population stabilization.

When confronting models with data, there are both strengths and limitations in the above approach to development economics. The strength lies in the micro foundations, which can be tested against data. A limitation is that it remains unclear if other alternative models may have the same or better predictive power. This point is crucial since as a rule, many models with different and correct micro-level assumptions make similar macro-level predictions. Studies on model uncertainty show that any dogmatic adherence to a particular model may not be the best solution in policy analysis (Brock, Durlauf and West, 2007). Moreover, there is also some question as to the extent to which economic models capture appropriate non-linearities. Complex systems theory suggests that we need to better appreciate the different kinds of non-linearities possible in economic systems and these need to be carefully accounted for in models (May, Levin and Sugihara, 2008; Helbing, 2012). These limitations of the neoclassical approach all relate to the ability of economic models to give a succinct and accurate empirical match to data, rather than a critique of the underlying model assumptions (see Rogeberg and Melberg (2011) for a discussion of this distinction).

The aim of this paper is to present an approach which focusses on matching data and suggesting relationships between indicators from macro-level, aggregate data, provided by, for example, the World Bank. We present a rigorous Bayesian framework for data-driven dynamic modeling of interactions in economic and social development. The approach is inspired by tools such as Gapminder (www.gapminder.org), which provide interesting visualizations of the datasets. For example, one can plot GDP against child mortality in a two-dimensional graph and then animate how different countries change position in this space through time. The animated changes then give a picture of the demographic transition that has characterised development through history. In this paper we show how to choose, using a Bayesian approach, a small number of differential equations which best explain such data in terms of interactions between the development indicators. We also show how these models can be used to predict future changes in indicators.

II. Two variables

A. Methods

Our basic approach to understanding interactions between pairs of indicators is to create a phase portrait of the two variables. A phase portrait in dynamical systems theory refers to a plot of the time evolution of two variables related to each other, usually through coupled differential equations, starting with different initial conditions (Strogatz, 2000). The available time series data on different development indicators can be converted into a phase portrait by plotting how different countries pass through the set of development states at different points in time. We let $x_i(t)$ and $y_i(t)$ be the value of the two studied indicators for country i at time t. The yearly change in x with respect to the current x and y values averaged over all countries that had this particular combination of x and y is written as

$$dx(x, y) = \frac{1}{|N(x, y)|} \sum_{(i,t)\in N(x,y)} (x_i(t + 1) - x_i(t))$$

where $N(x, y) = \{(i, t) : x < x_i(t) < x + \delta_x, y < y_i(t) < y + \delta_y\}$. $dx(x, y)$ gives the length of the vector in the x-direction in the phase portrait. A similar expression can be given for $dy(x, y)$, and this gives the length of the vector in the y-direction. The values δ_x and δ_x are chosen to give a good visualization and to obtain a sufficient number of observations at each lattice point.

The above method of "boxing" data is purely for visualization purposes. In order to quantify the patterns seen in the phase portrait, we fit a model to the average yearly changes in x and y as

$$\frac{dx}{dt} = f(x, y) \tag{1}$$

$$\frac{dy}{dt} = g(x, y) \tag{2}$$

In making a fitting, we first assume that the yearly changes are polynomial functions of x and y. Each term of the polynomial is one of the variables with power -1, 0, 1 or is the product of such powers of the variables. For instance, the model for change in x is

$$\begin{aligned} f(x, y) &= a_0 + \frac{a_1}{x} + \frac{a_2}{y} + a_3 x + a_4 y + \frac{a_5}{xy} + \frac{a_6 y}{x} + \frac{a_7 x}{y} + \\ &\quad + a_8 xy + a_9 x^2 + a_{10} y^2 + \frac{a_{11}}{x^2} + \frac{a_{12}}{y^2} \end{aligned}$$

These terms allow sufficient flexibility to capture possible non-linearities in the system. There are 13 models with one term and, in general $\binom{13}{m}$, models with m terms. In the first stage of our fitting process, we aim to rapidly narrow our search by finding the maximum-likelihood model for each possible number of terms, m. We fit the yearly samples of the yearly changes in the indicator variables using multiple linear regression over all $8,192$ possible functions $f(x, y)$. For each possible number of terms we find the model with the greatest likelihood (equivalently the model that minimises the sum of squared errors with the observed data). We repeat the same process to obtain the best possible $g(x, y)$.

The log-likelihood of the best fit for dx models with m terms is

$$L(m) = \log P(dx|x, y, m, \phi^*_m) \tag{3}$$

where ϕ^*_m is the set of unique parameter values obtained from the best fit regression out of all of the $\binom{13}{m}$ models with m terms. Assuming that the actual observations are due to the underlying model with additional Gaussian noise, $L(m)$ is the logarithm of the error sum of squares scaled by the variance (Bishop, 2006).

A key question about the robustness of particular models is why we choose a particular number of terms. For polynomial function fitting $L(m) \geq L(m-1)$, that is the maximum likelihood is monotonically increasing with additional terms, since each term allows an extra degree of freedom on curve fitting. For a finite data set this extra degree of freedom can fit artifactual patterns due to noise. As a result, reliance on $L(m)$ alone can lead to over-fitting the data by selecting too many terms and thus accepting a model that accurately fits the existing data but that generalises poorly to unseen data and has little predictive power.

To address this problem and evaluate the fit of these models we adopt a Bayesian approach. We calculate the Bayesian marginal-likelihood or *evidence* $B(m)$ for the set of models which have the largest log-likelihood within their respective number of terms. The Bayesian evidence compensates for the increase in the dimensions of the model search space by integrating over all parameter values, i.e.,

$$B(m) = \int_{\phi_m} P(dx|x, y, m, \phi_m^*)\pi(\phi_m)d\phi_m \tag{4}$$

The Bayesian evidence is thus the likelihood averaged over the parameter space with a prior distribution defined by $\pi(\phi_m)$. We choose a non-informative prior distribution (Ley and Steel, 2009). For example, $\pi(\phi_m)$ can be chosen to be uniform over the range of parameter values. This range of values is chosen to include all feasible values but to be small enough for the integral to be computed using Monte Carlo methods. $B(m)$ is computationally expensive to calculate, even for models with a small number of terms. Therefore we first identify the best fit model for each number of terms using maximum-likelihood, since models of equal complexity can be more fairly evaluated in terms of their maximum likelihood. We then compare those selected in terms of the Bayesian evidence to fairly compare models of varying complexity.

Note that 'Bayes factor,' which refers to a *ratio* of model likelihoods is used in Bayesian literature to compare pairs of models (Robert, 1994). We use the same term in this paper to refer to the marginal likelihood as defined above, with the understanding that this quantity would have the same function as the Bayes factor when comparing between more than two models.

Calculating the best fit regressions for dx and dy independently, as we do above, is equivalent to assuming that the errors in the two equations are uncorrelated. In fact ,there is a possibility that the two errors are correlated due to any systematic reason causing the errors in both cases, for example the same omitted variable. In this case we have to include an error covriance matrix in our approach and use a generalised least squares approach to

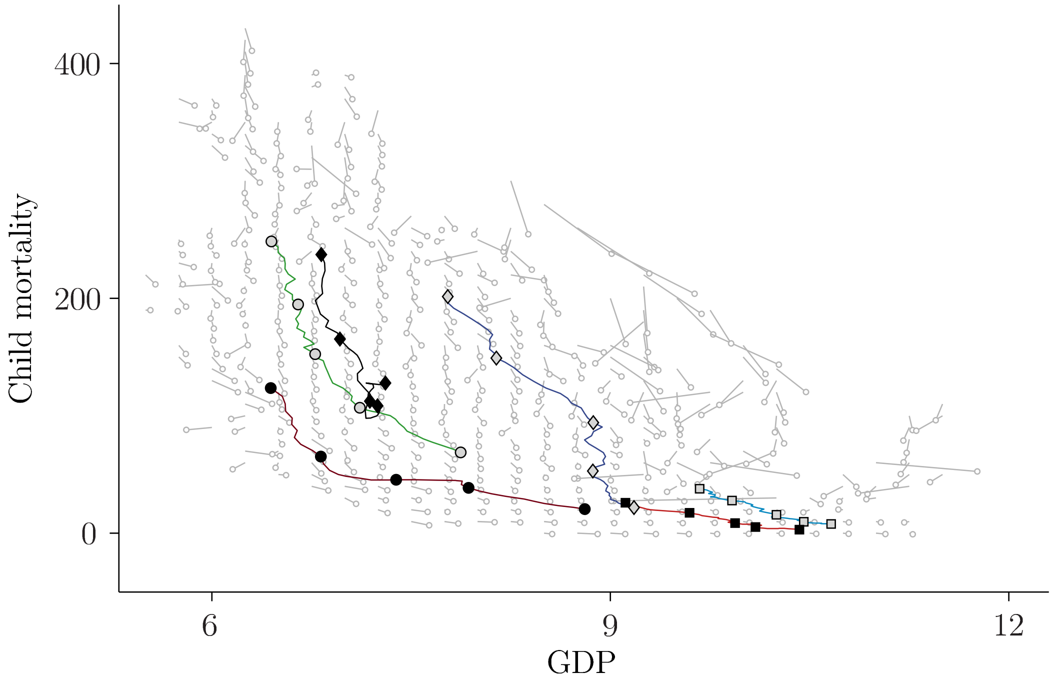

Figure 1. Data Phase portrait for child mortality and GDP. Dots represent development states and lines show average yearly change in indicators. Development statistics show child mortality decreasing and GDP increasing almost throughout. The continuous lines represent trajectories for different countries over the last fifty years. The country code is: solid circle - China, hollow circle - India, solid diamond - Kenya, hollow diamond - Brazil, solid square - Sweden, hollow square - USA

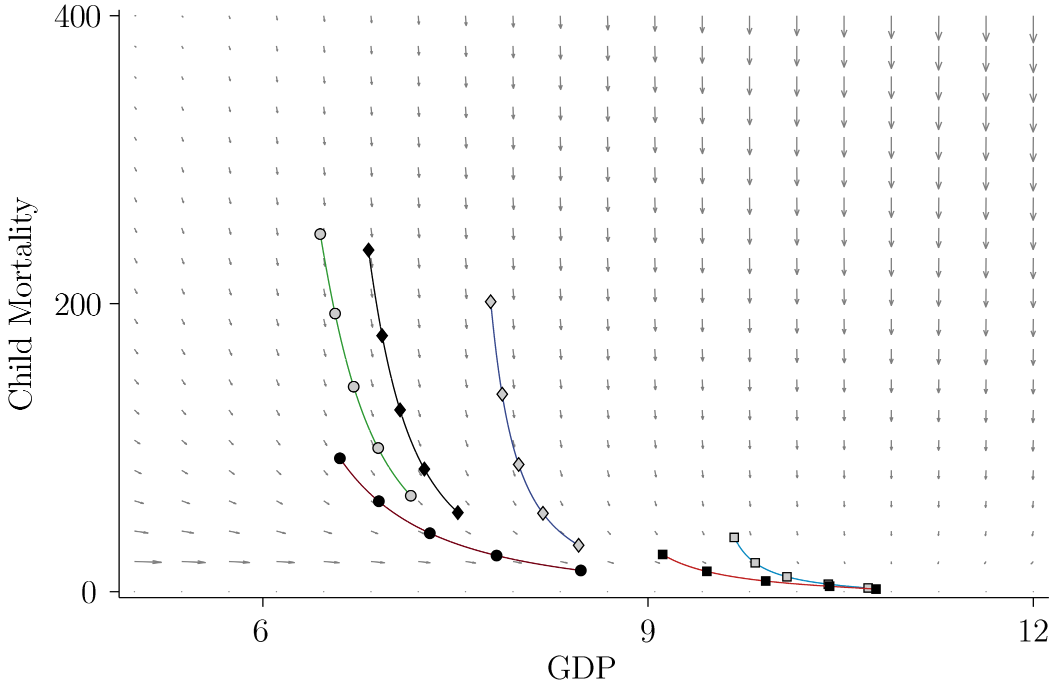

Figure 2. Model Phase portrait for child mortality and GDP. Lines and dots represent the development states and predicted changes based on the models. The dark, continuous lines represent trajectories for different countries over the last fifty years. The country code is: solid circle - China, hollow circle - India, solid diamond - Kenya, hollow diamond - Brazil, solid square - Sweden, hollow square - USA

finding the regression coefficients. If the error covariance matrix is diagonal, this reduces to the ordinary least squares approach used here.

To test if the errors are in fact significantly correlated, we use the "seemingly unrelated regressions" approach (Amemiya, 1985), where the two regressions for dx and dy are first performed under the assumption that the errors are in fact uncorrelated. We then estimate an error covariance matrix from the model suggested by this first step and the data, and use it to estimate the parameters based on a generalised least squares approach. This process may be iterated until the true parameters are obtained. If the covariance matrix is "almost" diagonal, indicating that error terms are uncorrelated, the parameters estimated by the "seemingly unrelated regressions" approach will not differ significantly from the parameters obtained assuming uncorrelated errors. If not, we have to account for the difference in our calculation of log-likelihood values and Bayes factor.

B. Application to child mortality and GDP

To illustrate the above procedure, we now apply it to two important indicators of development: Child mortality (C) and log GDP per capita (G). Child mortality refers to the number of children not surviving to age 5 per 1,000 live births and is a strong indicator of child health. GDP per capita is a measure of the average economic output in a country and is used widely to measure economic well-being. We use reported GDP in purchasing power parity (PPP) dollars and transform it to the logarithm scale. Data for C, G and a number of other development indicators is available for a large number of countries and for a number of years from the World Bank dataset (WorldBank, 2009), and we use the 2009 dataset in our work.

Fig. 1 shows the phase portrait of the C and G data. We use the data illustrated in the phase portrait to fit a model to the interactions between C and G, and obtain the following models when we allow two terms:

$$\frac{dC}{dt} = -0.0028C(1.6G - 0.02C) \tag{5}$$

$$\frac{dG}{dt} = \frac{2.5}{CG}(10.9 - G) \tag{6}$$

Fig. 2 shows the model fit. These equations summarize a number of important facts about how these indicators have changed over time. Firstly, we note that child mortality declines on average, with the mean fractional decrease per year equal to

$$0.0028(1.6G - 0.02C)$$

Percentage decrease in child mortality is therefore larger when GDP is high and when child mortality is low. G is log GDP, and as a result the right hand side of equation 6 gives the percentage change in GDP. Equation 6 implies that GDP increases faster when child mortality and GDP are low, and decreases when $G > 10.9$, a value similar to that of developed countries in 2008.

The above discussion simply tells us the best model for fitting the data and tells us nothing about how reliable the model is compared to alternatives. Figure 3 gives log-likelihood $L(m)$ and Bayes factor $B(m)$ as a function of the number of terms m in the model for child mortality. Here only the best possible models with m terms obtained from linear regression analysis are

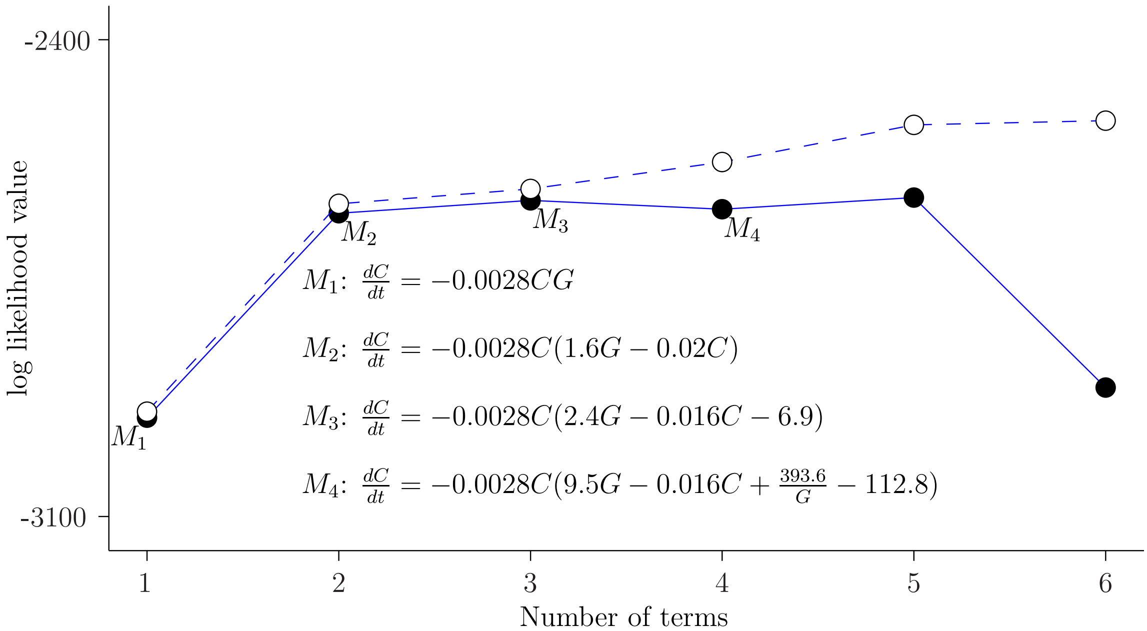

Figure 3. The log-likelihood (hollow circles) and log-Bayes factor (solid circles) for dC models. Log-likelihood value increases with number of terms but Bayes factor decreases after $m = 3$

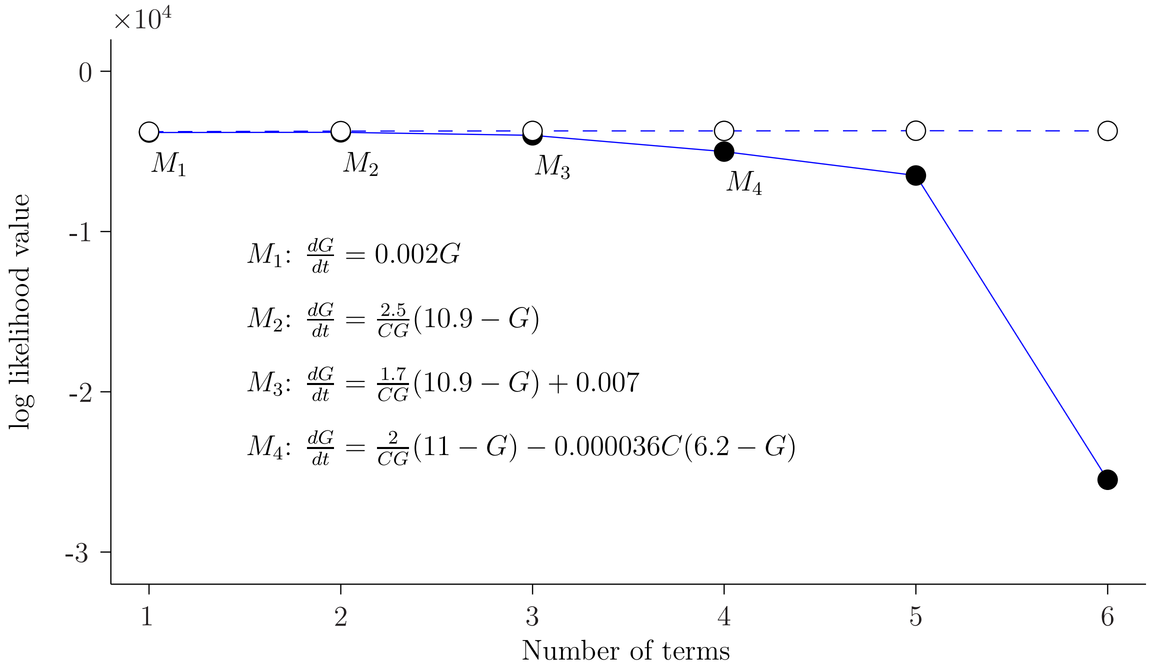

FIGURE 4. THE LOG-LIKELIHOOD (HOLLOW CIRCLES) AND LOG-BAYES FACTOR (SOLID CIRCLES) FOR dG MODELS. LOG-LIKELIHOOD VALUE INCREASES WITH NUMBER OF TERMS BUT BAYES FACTOR DECREASES CONTINUOUSLY

considered. While $L(m)$ increases with number of terms, $B(m)$ increases very little or decreases for $m > 2$. Looking at the plot, we see that the three term model for $\frac{dC}{dt}$ involves adding a constant term and changing the constants related to the effect of G and C. Importantly, the additional term does not change the role of G and C in determining $\frac{dC}{dt}$. As in the two term model, higher GDP and lower child mortality still cause greater decreases in child mortality. Similarly the four term model adds another term $(1/G)$ to the three term model and slightly alters the parameter values for the other terms. The overall effect of this addition remains the same, so that the percentage change in child mortality is high when GDP is high and child mortality is low. This conclusion reassures us about the robustness of equation 5 as a model. If by adding an extra term we had completely changed the interpretation of the model, then we would have less confidence that this model provides a reflection of an underlying reality.

The fit of the two term model for $\frac{dC}{dt}$ improves only slightly relative to the one term model (Figure 4). This result reduces our confidence in equation 6 as a good model. The one term model with the highest log-likelihood is

$$\frac{dG}{dt} = 0.002G$$

implying that child mortality is not an important factor in economic growth. The exponential growth model, where $\frac{dG}{dt} = 0.018$, is also reasonably close to the best model in terms of Bayes factors. These results are consistent with earlier conclusions that child mortality does not directly impact economic growth (Fernandez, Ley and Steel, 2001; Kalemli-Ozcan, 2002).

Finally, we estimate the error covariance matrix for these models. We find that the off-diagonal terms in the covariance matrix (scaled such that the diagonal elements are 1) are -0.1261. This indicates that the error terms are largely uncorrelated. Applying the "seemingly unrelated regression" approach outlined above, we see that the best parameters also change only slightly compared to our original estimate. On this basis, we retain the models that we obtained using the independence assumption.

III. Three variables

A. Methods

The key idea in our approach to modeling additional development indicators is to look at how model fit improves as we add further indicators. Model complexity depends now both on the number of terms and the number of variables. For three variable models we would like to determine whether or not we require all of these variables to model their rates of change. To do this, we calculate Bayes factor for models including all three indicators and compare them to those including just pairs of indicators. For three indicators there are now $\binom{33}{m}$ models with m terms and we generally restrict our analysis to those with up to $m = 5$ terms. By plotting $B(m)$ for three variable models as a function of m and comparing this to $B(m)$ for two variable models we can assess the utility of adding a third explanatory variable to the model.

B. Child Mortality, GDP and Total Fertility Rate

The natural next step in modeling the demographic transition is to add fertility rate to our two variable model of child mortality and GDP (Galor, 2005; Doepke, 2005; Strulik and Weisdorf, 2008; Becker, Cinnirella and Woessmann, 2010). The total fertility rate, A, is defined as the average number of children a woman has in the course of her lifetime. We again fit models for the yearly changes of $\frac{dC}{dt}$, $\frac{dG}{dt}$ and $\frac{dA}{dt}$ using polynomial functions of the three variables, C, G and A. Figure 5 gives Bayes factor values for the best model for $\frac{dC}{dt}$ as a function of all three variables (solid circles), as a function of just C and G (hollow circles), and as a function of C and A (crosses). Here, the addition of a fifth term makes little improvement in the Bayes factor for the three variable model, and we conclude that a model with four terms is sufficiently good. We note also that of the models with only two indicators, the model with only C and G has a Bayes factor much closer to the three variable model than the model with only C and A. This observation suggests that fertility rate is less important than GDP as a predictor of changes in child mortality.

In the case of GDP (figure 5), the two variable A and G model has a higher Bayes factor than the three variable model which includes C. This provides further support for the conclusion at the end of the last section that child mortality is not a good predictor of changes in GDP. For fertility (figure 7), the two variable A and C model has Bayes factor close to the three variable model, while the A and G model has a much lower Bayes factor.

The Bayes factor approach provides a measure of likelihood of various models and figures 5-7 allow us to weigh up the relative value of particular models. For GDP we see that the model

$$\frac{dG}{dt} = \frac{0.043}{A}(16 - G - \frac{51}{G}) \tag{7}$$

has highest Bayes factor. In this model a high fertility rate slows economic

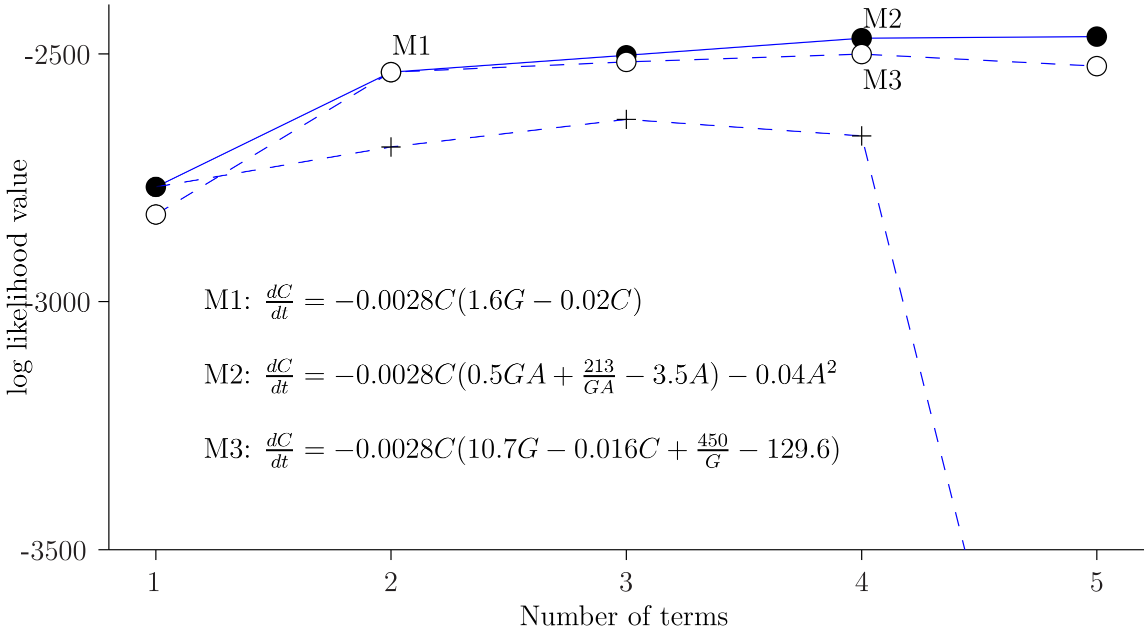

FIGURE 5. BAYES FACTOR PLOT FOR $\frac{dC}{dt}$ MODELS. SOLID CIRCLES CORRESPOND TO MODELS WITH ALL THREE VARIABLES INCLUDED, HOLLOW CIRCLES TO MODELS WITH ONLY C AND G AND $+$ TO MODELS WITH ONLY C AND A

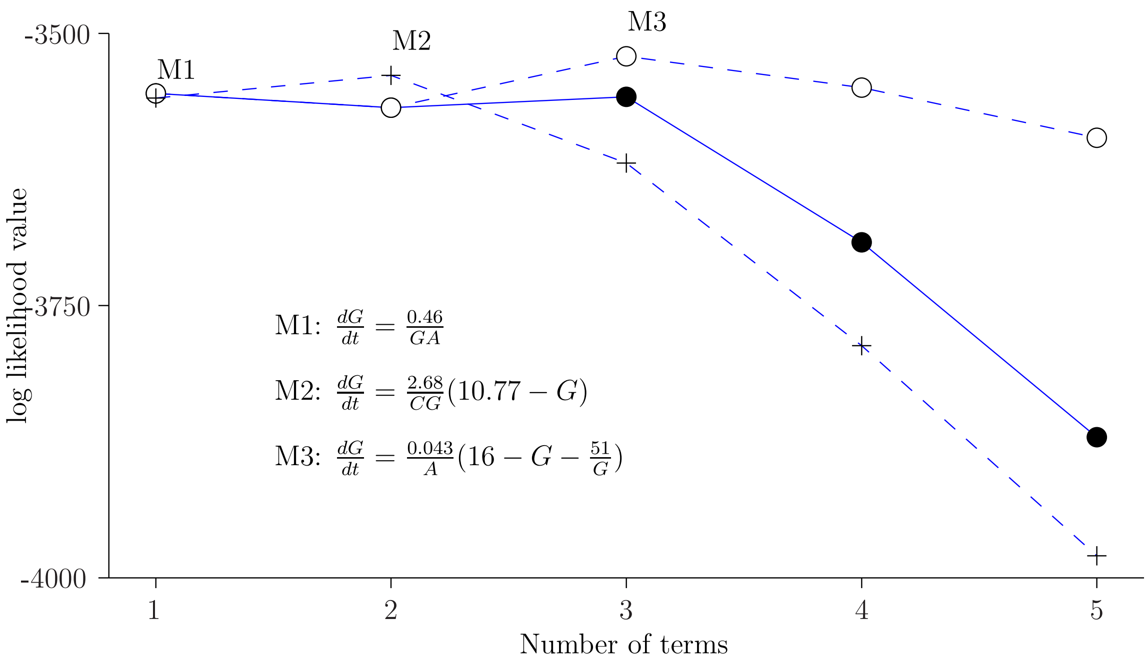

FIGURE 6. BAYES FACTOR PLOT FOR $\frac{dG}{dt}$ MODELS. SOLID CIRCLES CORRESPOND TO MODELS WITH ALL THREE VARIABLES INCLUDED, HOLLOW CIRCLES TO MODELS WITH ONLY G AND A AND $+$ TO MODELS WITH ONLY G AND C

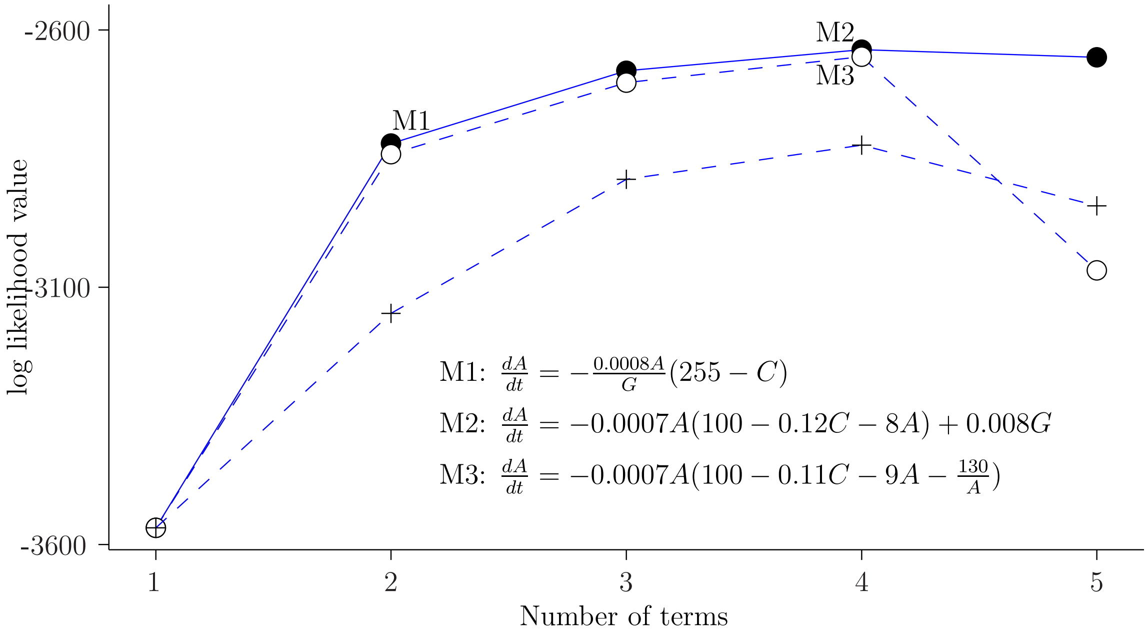

Figure 7. Bayes factor plot for $\frac{dA}{dt}$ models. Solid circles correspond to models with all three variables included, hollow circles to models with only A and C and $+$ to models with only A and G

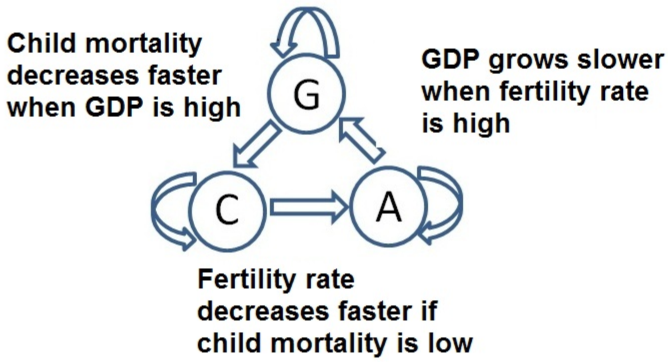

FIGURE 8. MOST SIGNIFICANT PREDICTOR FOR GDP, CHILD MORTALITY AND FERTILITY RATE - TWO VARIABLES CAPTURE MOST OF THE INFORMATION IN THE DATA FOR CHANGES IN THE INDICATORS

growth. Solving $\frac{dG}{dt} = 0$ gives two equilibrium points, $G = 4.4$ and $G = 11.6$, suggesting that there is a slowdown in growth at both low and high GDP.

For the child mortality and fertility rate models, the Bayes factor for models with only two variables is relatively close to that of the three variable models, suggesting that two variable models provide a good description of changes in these indicators. Thus the following equations capture the essential nature of the interactions of these indicators.

$$\frac{dC}{dt} = -0.0028C(1.6G - 0.02C) \tag{8}$$

$$\frac{dA}{dt} = -0.0007A(100 - 0.11C - 9A - \frac{130}{A}) \tag{9}$$

These equations, combined with equation 7, provide an overall structure for how the indicators interact. This is illustrated digramatically in Fig.

8. The overall cycle illustrated here is that child mortality decreases faster with higher GDP, fertility rate decreases faster when child mortality is low and decreased fertility rate is associated with growth in GDP.

Just as in the two variable case, we check if the errors are uncorrelated and obtain similar results for the three variable case also. Hence we are justified in assuming that the errors across variables are almost uncorrelated and we use the models obtained using this assumption.

In addition to pointing to the basic structure of interactions, the models above also show the non-linearities involved. For example, we see that the fertility rate decreases faster when it is itself high, but this decrease is slowed if child mortality is also high. There is also a secondary effect which slows the percentage decrease in the fertility rate when it is very low. Thus, the model shown above has two equilibrium points (at roughly $A_* = 10$ and $A_* = 1.5$) obtained by solving the equation $\frac{dA}{dt} = 0$.

IV. Four and more variables

A. Methods

In order to extend our approach to include more indicators we need to develop ways of dealing with increasing computational complexity. We already compare millions of models using three indicators, and for more indicators the computational cost of calculating Bayes factor for all models is prohibitive. One approach to get around this problem is to employ an algorithm which chooses which combination of indicators to investigate further. For instance, it may be computationally practical to calculate the best three term model, M_3, but not a four term model. In this case, we could compute likelihood values only for those four term models which have the same first three terms as M_3. For a more thorough search of the model space, instead of fixating on the best model, we may set up a table of, say, the 10 best three term models and then search for the best four term models from the tree structure that is created with these 10 models as the roots. Then we can compare the Bayes factors for the models with different number of terms and arrive at the best possible model that explains the data.

With more than three economic variables, the brute force search task rapidly becomes implausible due to the combinatorial complexity of the model space. An expert-driven search may be able to ameliorate this problem. A sociologist or applied economist may select specific effects that either must or must not be represented by mathematical terms, and thus limit the number of possible models to evaluate. This would be equivalent to placing a binary prior probability on the potential models, classing them as either possible or impossible before assessing the evidence from the data, ensuring that a selected model fits the constraints of the data and prior socioeconomic knowledge.

B. Fertility Rate and Education

Female education has been suggested as an important component of reduced fertility rate (UN, 2002; Cochrane, 1979). International organizations such as the United Nations Population Fund and the World Bank advocate better schooling for girls as a means of achieving lower child mortality and fertility rate. School enrolments potentially transform inter-generational relationships by raising the direct cost of child-bearing, reducing the avail-

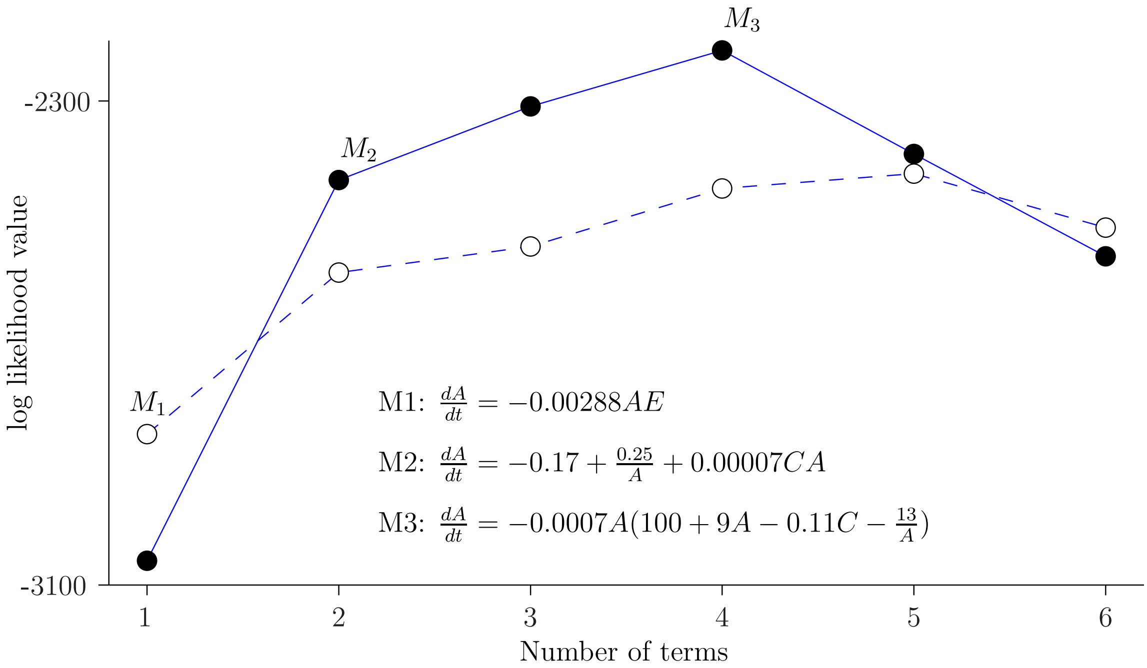

FIGURE 9. THE LOG-BAYES FACTOR PLOTS FOR A-C MODELS (SOLID CIRCLES) AND A-E MODELS (HOLLOW CIRCLES) SHOWING THAT CHILD MORTALITY IS MORE IMPORTANT THAN AVERAGE YEARS OF SCHOOLING AS AN EXPLANATORY VARIABLE FOR FERTILITY RATE. BUT FOR THE SIMPLEST ONE-TERM MODELS, THE EDUCATION INDICATOR SEEMS MORE CRUCIAL.

ability of household production and allowing parents to invest in the quality of offspring at the expense of quantity (Becker, 1981; Caldwell, 1982). However, significant fertility declines have occurred without noticeable changes in female education (Basu, 2002). Cleland (2000) argues that fertility declines have occurred under vastly different settings with improving or deteriorating economic conditions, high or low standards of living, varying political regimes. Changing educational composition in India, for instance, explains only about 20 per cent of fertility change. Most of the change is due to fertility decline among illiterate women. We now test the role of female education as a general predictor of decreases in fertility.

We define the educational indicator E to be the average years of schooling for female population as collected in the Barro-Lee dataset (Barro and Lee, 2010). Since the data is available only on a five-yearly basis in that dataset, we use linear interpolation to obtain the yearly data points in order to apply our method to the data. We then ask if we can improve the fit of the model in equations 7 to 9 by including educational attainment. We expect, based on the studies cited above, female educational attainment to be more likely to have a direct influence on fertility rate than on either child mortality or GDP. To test this hypothesis we compare our models for the fertility rate ($\frac{dA}{dt}$ models) containing only the two variables A and C with $\frac{dA}{dt}$ models obtained using A and E. We plot the Bayes factor values for these two models for different numbers of terms in Fig. 9. We see that when a single term model is required to explain decrease in fertility rate, education is the best single explanatory variable. However, for models with $2, 3$ and 4 terms, we see that the models with C out-perform models with E as the explanatory variable. If we go on to calculate $\frac{dA}{dt}$ models with three variables (A, E and C) we find that the $2, 3$ and 4 term models with the highest Bayes factor involve only $A$and C. In conclusion, while higher female educational attainment does predict decreases in fertility rate, child mortality remains the most effective predictor.

V. Discussion

The innovation of the approach we have presented here lies in identifying dynamic interactions between development indicators. This paper has focussed primarily on the methodology for identifying the interactions that best fit the data. Our approach provides (i) an immediate visualization of the data; (ii) an emphasis on yearly changes, i.e. on an underlying dynamical system; (iii) the appearance of non-linearities in indicator interactions; and (iv) a method for comparing models in terms of goodness of fit.

The application of our approach to human development is motivated by the long time series available from multiple countries and the importance of this process in development economics. However, the methodology itself is in no way limited to this application. The method can be applied to any economic or sociological process where we have temporal panel data of indicators for multiple entities, be they countries, local authorities or companies. The question we now discuss is how to interpret such models. We separate this discussion into two parts, 'prediction' and 'causation' interpretations of models. Making this distinction helps us delineate the different situations in which our approach may be useful in development economics and elsewhere.

A. Prediction using models

Given the two-variable model (equations 5 and 6) for the evolution of GDP and child mortality we can integrate them forward in time to make predictions about future changes. One of the Millenium Development Goals (MDGs) is to reduce child mortality levels by two-thirds before 2015. We can use our approach to look at how reasonable the MDG is. Figure 10 shows Kenya and Brazil's development during the last 50 years. The solid lines are predictions of their development starting from 1990 (the year the MDGs take as the starting point) based on the models derived above. Here, the model is fit for data over all countries prior to 1990 and then predictions are made on the basis of this model fit.

The most striking aspect of Figure 10 is that, while Brazil has followed

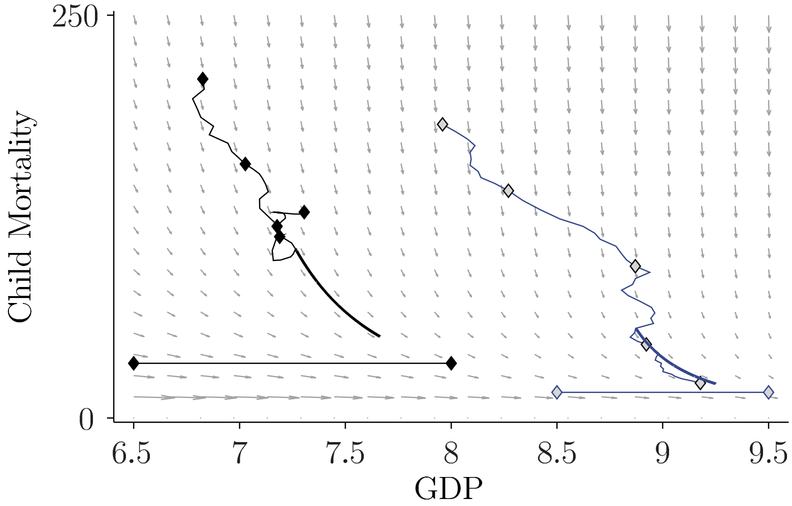

Figure 10. Development of Kenya (solid diamond) and Brazil (hollow diamond) between 1959 and 2009. The solid lines are the model predictons, based on data of all countries in the world prior to 1990. The prediction trajectory uses 1990 data for the two countries as initial conditions and integrates forward to 2015 (target year for MDG). The MDG target was to reduce by two-thirds the child mortality from 1990 levels by 2015, i.e. down to 34 for Kenya and 16 for Brazil. These are shown as horizontal lines.

the model prediction reasonably well, Kenya has fallen back to higher child mortality levels and lower economic growth. The 'failure' of the model with respect to Kenya can be explained in a post-hoc manner by turbulent politics until large-scale reforms around 1997. These events were obviously not part of the model. Indeed, we should rather think of the model prediction as describing how indicators should develop assuming the same circumstances which characterized the average changes of other countries up until 1990. These circumstances prevailed in Brazil which continued on a path of democratization and from the early 1990s embarked on economic plans for growth. In cases, like Kenya, where the model does not fit the data we can start to ask what differences there were in Kenya which meant that this type of “normal circumstances” assumption did not apply.

Our analysis suggests that the 2015 MDG for Kenya was, even if we ignore political instability, overly optimistic. The target of child mortality of 34 per 1,000 in Kenya was below our prediction of 46 per 1,000 based on our model. On the other hand, the MDG target of 16 per 1,000 for Brazil was reasonably consistent with our model (the prediction is 19). Our approach would allow us to set country by country targets, based on the simple assumption that data from other countries on previous years is relevant for predicting the future change of other countries. For example, our model shows that a country with high starting GDP is likely to reduce child mortality faster than a country with low GDP, a fact that could be incorporated into targets.

Our approach can be extended in a straightforward manner to provide confidence intervals to these predictions. The error in the model fit can provide a stochastic term which can be used on the model iterations. For example, the mean of square errors for the $\frac{dC}{dt}$ and $\frac{dG}{dt}$ models were respectively, $\sigma_C^2 = 4.2$ and $\sigma_G^2 = 0.003$. This gives us the stochastic model for yearly changes as

$$dC(t) = -0.0028C(t)\left(1.6G(t) - 0.02C(t)\right) + \sigma_C \epsilon_C(t) \tag{10}$$

$$dG(t) = \frac{2.5}{C(t)G(t)}\left(10.9 - G(t)\right) + \sigma_G \epsilon_G(t) \tag{11}$$

where $\epsilon_C(t)$ and $\epsilon_G(t)$ are normally distributed random variables with mean zero and standard deviation of unity. Repeated simulations of this model will give a confidence interval for changes given our assumption of ‘normal circumstances. This can be useful in setting upper and lower bounds for development targets.

B. Causation implied by models

The link between changes in child mortality, fertility rate and economic growth has previously been presented as a unified model of modern growth (for instance, Barro and Becker (1989)). Many of these studies are focussed on the fertility aspect of the demographic transition (Galor and Weil, 1999; Greenwood and Seshadri, 2002). Becker (1981) suggest that the demographic transition occurs at high levels of income, where the adverse effect of the opportunity cost of raising children dominates the positive income effect. A large body of literature explains the demographic transition as a consequence of increased investments in human capital due to technological change (Becker, 1981; Tamura, 1996; Galor and Weil, 1999). Increased returns on education are thought to initiate the demographic transition and therefore a decline in fertility (Galor and Weil, 1999; Galor and Moav, 2002). Other researchers attribute the demographic transition to the decline in the gender wage gap (Galor and Weil, 1996; Heckman and Walker, 1990a).

Our analysis and the cycle presented in Figure 8 emphasizes lowered child mortality over increased economic opportunities as the more immediate cause of drops in fertility rates. While higher GDP lowers child mortality, probably as a result of improvements in economic and social conditions, the best single predictor of decreases in fertility rate is then child mortality. From an individual mother's point of view, if the probability of children surviving is lower, then having more children increases biological fitness. Similarly, we find that while higher female education does predict decreases in fertility, child mortality remains a better predictor of these decreases. The decision whether or not to have a child may well involve a tradeoff against other economic and education opportunities (Becker, Murphy and Tamura, 1990; Ehrlich and Lui, 1991; Kalemli-Ozcan, Ryder and Weil, 2000; Boldrin and Jones, 2002; Eckstein, Mira and Wolpin, 1999), but it is changes in the costs of child bearing which have the greatest role in decreasing fertility.

Although the emphasis on child mortality in the causal cycle implied by figure 8 is different from that emphasized in earlier work, the change in focus is relatively small. Importantly, none of our findings shift us a long way from those hypotheses previously proposed about human development. Instead, our analysis aims to sharpen the picture by finding those models that are closest to all the available aggregate data. By fitting rate of change of indicators to their current state we have looked explicitly at how the state of the world in one year leads to the state of the world the following year. However, if we are interested in establishing the causes of interactions we have to check, as we did in the preceding two paragraphs, whether our model is consistent with the already available literature. A model that doesn't make causal sense should not be accepted. In Bayesian language, such a model would have a low prior probability.

The approach we have taken in this paper can be contrasted with one that starts from the point of view of underlying micro-level interactions of economic agents. There are a number of limitations to this latter approach, with respect to providing succinct and empirically accurate models of data. Firstly, although based on observations, such models do not necessarily provide the best fit to the existing data. Instead, correlational evidence is provided for particular assumptions or predictions of the model. For example, Doepke (2005) tests the Barro-Becker model against fertility and child mortality data for various European countries. In such examples, the data shows how the model fits the data within a certain error range, but this does not rule out the existence of a large number of alternative models each of which has some degree of empirical support in the available data. The advantage of the Bayes factor based analyses we have performed here is that they provide a likelihood measure over all plausible models.

A second limitation is that economic models usually involve specific mathematical forms that are stipulated mainly on the basis of convenience and are less suited to capturing interactions in data. Equilibrium analysis plays a big role and hence the range of models which can be studied formally using the available tools is limited. While these restrictions help mathematical analysis, they are not necessarily feasible and restrict the degree to which non-linearities in the data can be captured by the model. A third limitation is that, despite their mathematical tractability, the statement of neoclassical economic models is often very complicated in comparison to a set of differential equations such as equations 7 to 9.

The data-driven approach we have outlined can match data well but with no underlying assumptions and thus no *a priori* causal basis. This criticism must be taken seriously, but it does not prevent us from discussing, in a post-analysis stage as we have done above, how the derived models relate to the micro-level motives of economic actors. Indeed, looking at the interactions we have identified and evaluating whether they make sense in light of what is already known is an essential part of the process of model selection. Models which do not make sense should be discarded in favor of those that do. A process of understanding interactions is not possible using statistical analysis alone, but it can be incorporated by our changing the prior probabilities of certain models. Such a post-fitting analysis stage is necessary if we are to use our model to inform policy decisions.

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