Date-stamping multiple bubble regimes*

David I. Harvey[a], Stephen J. Leybourne[a] and Emily J. Whitehouse[b]

[a]School of Economics, University of Nottingham

[b]Newcastle University Business School

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Abstract

Identifying the start and end dates of explosive bubble regimes has become a prominent issue in the econometric literature. Recent research has demonstrated the advantage of a model-based minimum sum of squared residuals estimator, combined with Bayesian Information Criterion model selection, over recursive unit root testing methods in providing accurate date estimates for a single bubble. However, in the context of multiple bubbles, a large number of models are possible, making such a model-based method unattractive to practitioners. In this paper, we propose a two-step procedure for dating multiple bubbles. First, recursive unit root tests are used to identify a 'date window' in which we believe a bubble starts and ends. Second, a model-based BIC approach is used to estimate the regime change points within each window. Monte Carlo simulations highlight the effectiveness of our procedure. In addition, our method allows us to distinguish between different types of bubble behaviour, such as whether or not each bubble crashes before reverting back to a unit root process, and date any crash regimes. The advantages of our procedure over existing methods of bubble dating is demonstrated through empirical application to housing markets.

Keywords: Explosive autoregression; Break date estimation

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*Correspondence to: Emily Whitehouse, Newcastle University Business School, Newcastle University, Newcastle upon Tyne, NE1 4SE, UK. E-mail: emily.whitehouse@newcastle.ac.uk

1 Introduction

The role that asset price bubbles play in financial instability has become increasingly apparent in recent years as a result of events such as the US housing price bubble of the mid-2000s which triggered the financial crisis that followed. Indeed, as Bernanke (2013) notes, whilst it is unavoidable that there will periodically be bubbles in the financial system, now that the damage these bubbles can cause has been realised, it is crucial that central banks and other regulators take their emergence seriously.

This increasing interest in asset price bubbles has corresponded with a renewed focus within the econometric literature and there is now a substantial body of work regarding econometric detection and dating of asset price bubbles. Much of this literature concerns rational bubbles, where investors are assumed to have rational expectations, such that investing in an asset can be a rational choice even when the price is not justified by the underlying fundamental value of the asset, due to a belief that its price will continue to increase beyond the price paid. In the rational bubble literature, the price of an asset is assumed to be equal to the present value of the fundamentals of that asset plus a bubble component which grows at the real interest rate in expectation. Consider a stock, with real stock price, P_t, real dividends, D_t, and with r denoting the real interest rate used for discounting expected future cash flows. We can define prices as

$$P_t = P_t^f + B_t$$

where the fundamentals component is given by

$$P_t^f = \sum_{i=1}^{\infty}(1+r)^{-i}E_t(D_{t+i})$$

If the bubble component, B_t, satisfies the following stochastic difference equation:

$$B_{t+1} = (1+r)B_t + u_{t+1}$$

and $E_{t-i}(u_{t+1}) = 0$ for all $i \geq 0$, then a rational bubble is said to exist. It is evident from the above that as the bubble component grows at an explosive rate, if a bubble is present in prices, it will manifest itself as a statistically explosive process. Therefore detection of explosive behaviour in a price series, and the absence of explosive behaviour in the fundamental component, is sufficient to conclude that a bubble exists.

In light of the above, Diba and Grossman (1988) proposed detecting a bubble by applying standard, left-tailed Dickey-Fuller unit root tests to both the levels and first differences of a price series, as well as the fundamental series. As an explosive series will not be stationary in first differences, if the null hypothesis of a unit root is rejected for the first difference series, and not rejected for the series in levels, they conclude that the series is not explosive. As suggested by Evans (1991), explosive bubbles will typically be of a temporary nature, and price series which contain bubbles which collapse at some point in time may give the appearance of mean-reversion. Full sample tests for explosive behaviour may therefore fail to identify these periodically collapsing bubbles due to the appearance of stationarity. To address the issue of detecting temporary explosive regimes, Phillips et al. (2011) [PWY] propose a test that takes the supremum of forward recursive *right-tailed* unit root tests applied to the levels of the price series and fundamental series. The PWY test is designed for circumstances in which a maximum of one asset bubble is present in a series. Of course, in practice, it is possible that in a given time period a series may contain more than one explosive regime. Phillips et al. (2015) [PSY] therefore propose a bubble detection methodology capable of detecting multiple bubbles based on forward and backward recursions of right-tailed unit root tests.

An equally important issue to detecting the presence of bubbles, is dating the timing of those bubbles. Accurate identification of the start and end points of historical bubble regimes is vital for a number of reasons. First, by understanding the timing of a bubble in a given price series we can reconcile this with other macroeconomic events. Next, by analysing the start and end dates of bubble regimes, we are able to construct timelines of bubble behaviour across countries or across different classes of assets. These timelines can provide useful information about the evolution of bubble behaviour throughout the economy. Finally, if we know the dates over which a price series is and is not explosive, we can create a bubble indicator for that asset which can then be used as a variable in regression models, to analyse the determinants of explosive behaviour, or the effect that explosiveness in a particular asset has on other variables of interest. Clearly the validity of any of the above analyses requires precise date estimation of bubble regimes.

Both PWY and PSY propose methods of dating the start and end points of explosive regimes using sequences of recursive unit root test statistics which are compared to a threshold critical value. Using this approach, PWY find evidence of a stock market bubble in the NASDAQ index from February 1973 - June 2005. PSY apply there dating procedure to the S&P 500 stock market index and identify multiple periods of bubble behaviour corresponding to macroeconomic events such as black Monday, the dot-com bubble, and the sub-prime mortgage crisis. The dating procedure of PSY, in particular, is now widely used to estimate the timing of bubbles in a variety of asset markets, *inter alia*, housing (Anundsen et al., 2016; Pavlidis et al., 2016), commodities (Alexakis et al., 2017; Figuerola-Ferretti and McCrorie, 2016), energy markets (Sharma and Escobari, 2018), exchange rates (Hu and Oxley, 2017), and cryptocurrencies (Corbet et al., 2018).

Harvey et al. (2017) [HLS] propose an alternative method of bubble date-stamping, where regime change points are estimated based on minimum sum of squared residual estimators and a Bayesian Information Criterion (BIC) model selection procedure. Specifically, the HLS procedure allows for different identification amongst a number of different types of bubble processes, where the bubble may be followed by a stationary collapse regime, or may still be ongoing at the end of the sample. For a fixed bubble magnitude, the estimated dates of regime change are found to be consistent, and, it is demonstrated that this BIC-based procedure delivers much improved dating accuracy in comparison to the recursive unit root based method of PSY.

Whilst the BIC procedure of HLS offers superior dating accuracy, a disadvantange of the procedure is that is designed to date a single bubble in a time series, whereas the PSY procedure is equipped to date multiple bubble regimes. The model-based nature of the BIC procedure produces a challenge for multiple bubble regimes, as when the number of bubbles is unknown, and when each of those bubbles can follow different patterns of behaviour, that behaviour also being unknown, a huge number of potential models are possible. Testing between such a large number of models would become infeasible in practice.

In this paper, we consider a date-stamping methodology based on minimum sum of squared residual estimators and BIC model selection, which is capable of accurately dating multiple bubble regimes. We propose a two-step procedure, where, assuming that pre-testing for the presence of explosive behaviour has occurred, in the first stage we apply the PSY dating procedure, and use these preliminary date estimates to split our sample into a number of sub-sample 'date-windows' in which we believe that only a single bubble exists. In the second stage, we apply the HLS procedure to each date-window, to obtain an accurate estimate of the regime change points. Monte Carlo simulations demonstrate the superior accuracy delivered by our two-step procedure in comparison to the dating methodology of PSY. In particular, we observe that whereas PSY has a tendency to date the regime change points later than they occur, our methodology is often able to identify the exact start and end point of an explosive regime, especially when considering end dates.

To demonstrate the effectiveness of our proposed date-stamping procedure, we undertake an analysis of explosive behaviour in the housing markets of 23 countries from 1975:Q1-2018:Q2. We find that 20 of these countries contain at least one explosive episode within the time period, with a maximum of four explosive regimes detected in some countries. We show that our BIC date-stamping procedure typically found earlier start dates and end dates for the explosive regimes than the PSY procedure. Our procedure also finds that many of the detected explosive regimes subsequently collapse, and is able to estimate the end point of the collapse. Overall, we note three key phases in the behaviour of house prices across countries. The first phase corresponds to the house price bubble and crash seen in many countries in the late 1980s, the second phase describes the housing bubbles in the mid-2000s whose collapse preceded the global financial crisis, and finally we observe the emergence of bubble behaviour in a number of countries that is still ongoing at the end of the sample.

The next section outlines the basic multiple bubble framework that we consider. Section 3 presents our proposed date-stamping procedure. Section 4 discusses details regarding the practical implementation of the procedure. In Section 5 we present a finite sample Monte Carlo simulation comparison of the accuracy of our proposed methodology to the recursive unit root based method of PSY. An empirical application of our date-stamping procedure to housing markets is considered in Section 6. Section 7 concludes.

The following notation is used throughout the paper. $\lfloor . \rfloor$ denotes the integer part and 1 denotes the indicator function.

2 Multiple Bubble Model

Consider the following model for a time series $\{y_t\}$, $t = 1, ..., T$, which allows for multiple changes in the AR(1) parameter:

$$\begin{aligned} y_t &= \mu + u_t \\ u_t &= (1 + \rho_t) u_{t-1} + v_t, \quad t = 2, ..., T \end{aligned}$$

with $y_1 = O_p(1)$ and while the innovation process $\{v_t\}$ satisfies the following linear process assumption:

Assumption 1. *The stochastic process* $\{v_t\}$ *is such that*

$$v_t = C(L)\eta_t \qquad C(L) := \sum_{j=0}^{\infty} C_j L^j$$

with $C(1)^2 > 0$ *and* $\sum_{i=0}^{\infty} i|C_i| < \infty$ *and where* $\{\eta_t\}$ *is an iid sequence with mean zero, unit variance and finite fourth moment. The short run variance of* η_t *is defined as* $\sigma_\eta^2 = \sum_{j=0}^{\infty} C_j^2$.

With regard to the AR coefficient $(1 + \rho_t)$ we let $\rho_t = \rho(t/T)$, with $\rho(\tau)$, $\tau \in [0, 1)$ being a piecewise function of the following form:

$$\rho(\tau) = \sum_{j=1}^{J} \rho_{j1}^* \mathbb{I}(\tau_{j1}^* < \tau \leqslant \tau_{j2}^*) + \sum_{j=1}^{J} \rho_{j2}^* \mathbb{I}(\rho_{j1}^* > 0) \mathbb{I}(\tau_{j2}^* < \tau \leqslant \tau_{j3}^*) \tag{1}$$

where $j = 1, ..., J$, with $\tau_{11}^* > 0$, $\tau_{J3}^* \leq 1$, $\tau_{(j+1)1}^* > \tau_{j3}^*$ to ensure an ordering of explosive regimes, together with $\rho_{j1}^* \geq 0$ and $\rho_{j2}^* \leq 0$. The specification in (1) implies that y_t follows an underlying unit root process but is subject to J explosive regimes, each of which can terminate with a stationary collapse regime. Specifically, y_t starts as a unit root process, then if $\rho_{11}^* > 0$, a bubble exists over $\tau_{11}^* < \tau \leqslant \tau_{12}^*$, with AR(1) parameter $1 + \rho_{11}^*$. If $\rho_{12}^* < 0$, then this is followed by a stationary collapse over $\tau_{12}^* < \tau \leqslant \tau_{13}^*$, with AR(1) parameter $1 + \rho_{12}^*$; after the collapse regime, y_t returns to a unit root process. If, instead, $\rho_{j2}^* = 0$, then no collapse regime occurs and y_t returns to a unit root process after τ_{12}^*. Then if $\rho_{21}^* > 0$, a second bubble exists over $\tau_{21}^* < \tau \leqslant \tau_{22}^*$, followed by potential collapse over $\tau_{22}^* < \tau \leqslant \tau_{23}^*$, before unit root dynamics return, and so on until the last (Jth) bubble/collapse pair of regimes has occurred. Note that the indicator $\mathbb{I}(\rho_{j1}^* > 0)$ in (1) ensures that stationary regimes do not exist without an immediately prior bubble regime. We do not assume knowledge of J or any of the corresponding $\tau_{j1}^*, \tau_{j2}^*, \tau_{j3}^*$, hence we allow for an unknown number of explosive bubble regimes at unknown times, with or without collapse. The model also permits a bubble (or collapse) regime to run to the end of the sample period, by letting $\tau_{j2}^* = 1$ (or $\tau_{j3}^* = 1$).

Indeed, for each explosive regime j, and following HLS, we can consider two specifications of bubble behaviour: the bubble terminates and unit root behaviour resumes; or the bubble terminates and is followed by a stationary collapse regime before unit root behaviour resumes. For the J^{th} bubble in the sample, we can consider two additional specifications: the bubble continues until the end of the sample; or the bubble terminates and is followed by a stationary collapse regime which continues until the end of the sample. We can define these DGP specifications as follows:

DGP 1: $0 < \tau^*_{j1} < 1$, $\tau^*_{j2} = 1$

(unit root, then explosive until sample end)

DGP 2: $0 < \tau^*_{j1} < \tau^*_{j2} < 1$, $\tau^*_{j2} = \tau^*_{j3}$

(unit root, then explosive, then unit root until sample end)

DGP 3: $0 < \tau^*_{j1} < \tau^*_{j2} < 1$, $\tau^*_{j3} = 1$

(unit root, then explosive, then stationary collapse until sample end)

DGP 4: $0 < \tau^*_{j1} < \tau^*_{j2} < \tau^*_{j3} < 1$

(unit root, then explosive, then stationary collapse, the unit root until sample end)

The J^{th} explosive regime in the sample may take any of the specifications DGP 1 - DGP 4. We assume that each temporary explosive process is preceded by a unit root process, and therefore we restrict bubble j, where $j = 1, ..., J-1$, to follow either DGP 2 or DGP 4 only, to ensure that unit root behaviour is restored before the emergence of the next bubble.

3 Date-Stamping Procedure

Before we begin to estimate the regime change points of the multiple bubble process y_t, we must first confirm that y_t does contain explosive behaviour. PSY propose a method for detecting temporary explosiveness based on forward and backward recursions of right-tailed Dickey-Fuller unit root tests. Let r_1 and r_2 denote fractions of the sample, such that $r_2 > r_1$, then $ADF^{r_2}_{r_1}$ denotes the OLS Augmented DF test statistic, computed over $t = \lfloor r_1 T \rfloor, ..., \lfloor r_2 T \rfloor$. The GSADF test of PSY considers the supremum of a series of $ADF^{r_2}_{r_1}$ statistics where the end point over which the sub-sample statistics are computed varies over $r_2 \in [r_0, 1]$ where r_0 is the minimum window width, and the start point varies over $r_1 \in [0, r_2 - r_0]$. The GSADF statistic is given by

$$GSADF(r_0) = \underset{r_2 \in [r_0,1], r_1 \in [0, r_2 - r_0]}{sup} ADF^{r_2}_{r_1}$$

If the GSADF test statistic exceeds the relevant critical value then we assume that y_t exhibits at least one explosive regime.

Once the presence of explosive behaviour has been confirmed, our concern is the precise estimation of τ^*_{j1}, τ^*_{j2} and τ^*_{j3} for each explosive regime j. We propose a two-step dating procedure, which capitalises on the accuracy advantages offered by HLS in date-stamping a single explosive regime, and the capabilities of PSY to date multiple explosive episodes.

The first step of our procedure is to apply the BSADF dating procedure of PSY, to obtain a preliminary estimate of bubble start and end dates. The BSADF statistic is given by:

$$BSADF_{r_2}(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} ADF_{r_1}^{r_2}$$

The date estimates are then given by:

$$\begin{aligned} \widehat{\tau}_{j1}^{PSY} &= \inf_{r_2 \in [r_0, 1]} \{r_2 : \; BSADF_{r_2}(r_0) > scv_{r_2}\} \\ \widehat{\tau}_{j2}^{PSY} &= \inf_{r_2 \in [\widehat{\tau}_{j1}^{PSY} + log(T)/T, 1]} \{r_2 : \; BSADF_{r_2}(r_0) < scv_{r_2}\} \end{aligned}$$

where scv_{r_2} is the critical value of the sup ADF statistic based on $\lfloor r_2 T \rfloor$ observations, and where $\widehat{\tau}_{j1}^{PSY}$ and $\widehat{\tau}_{j2}^{PSY}$ are estimates of τ_{j1}^{*} and τ_{j2}^{*} and with the necessary restriction that $\widehat{\tau}_{(j+1)1}^{PSY} > \widehat{\tau}_{j2}^{PSY}$. That is, the start date of the first ($j = 1$) explosive episode is the first value of r_2 where $BSADF_{r_2}(r_0)$ exceeds its critical value. The end date of the explosive regime is then given by the first value of r_2 where $BSADF_{r_2}(r_0)$ drops below the threshold critical value. The start date of the second ($j = 2$) explosive regime is then given by the first value of r_2, after τ_{12}^{*}, where $BSADF_{r_2}(r_0)$ exceeds the critical value, and so on.

Once the estimates $\widehat{\tau}_{j1}^{PSY}$ and $\widehat{\tau}_{j2}^{PSY}$ have been obtained for $j = 1, ..., \widehat{J}$, where $\widehat{J}$ is the number of explosive regimes detected by PSY, we can use these estimates to split our full sample into $\widehat{J}$ sub-sample date-windows. The start and end points of these date windows ($start_j$ and end_j) are given by

$$start_j = \begin{cases} 1 & j = 1 \\ end_{j-1} + 1 & j > 1 \end{cases}$$

and

$$end_j = \begin{cases} \lfloor \widehat{\tau}_{j2}^{PSY} T \rfloor + (\lfloor \widehat{\tau}_{(j+1)1}^{PSY} T \rfloor - \lfloor \widehat{\tau}_{j2}^{PSY} T \rfloor)/2 & j < \widehat{J} \\ T & j = \widehat{J} \end{cases}$$

such that we use the mid-point between the end of one explosive regime and the start of the next as the sample splitting point. In fact, once we have more information about the regime change points for bubble j, we can adjust $start_{j+1}$ in order to provide more accurate estimation of the regime change points of bubble $j + 1$, as we will discuss later.

By splitting y_t into sub-samples on the basis of PSY date estimation, we can now assume that each date-window contains a single explosive regime only, and therefore, in the second step of our date-stamping procedure, HLS date estimation can be applied to each date-window in turn. As the four model specifications, DGP 1 - DGP 4, that we consider all contain a period of explosiveness, this two-step procedure will always find the same number of bubbles, $\widehat{J}$, as implementation of the PSY procedure only. As a result, our procedure does not concern itself with detection of explosive processes, and focuses solely on improving the accuracy of date estimation and allowing for different model specifications for each bubble process.

Following HLS, if we assume knowledge about which of DGP 1 - DGP 4 is the true DGP for each of the $\widehat{J}$ explosive regimes in the full sample, then we can estimate the relevant change-points for each bubble j by minimising the sum of squared residuals across all candidate dates. We do this using the fitted OLS regression models below for the relevant DGP:

$$\begin{aligned}
&\text{Model 1:} && \Delta y_t = \widehat{\mu}_1 D_t(\tau_{j1}^*, 1) + \widehat{\rho}_{j1}^* D_t(\tau_{j1}^*, 1) y_{t-1} + \widehat{v}_{1t} \\
&\text{Model 2:} && \Delta y_t = \widehat{\mu}_1 D_t(\tau_{j1}^*, \tau_{j2}^*) + \widehat{\rho}_{j1}^* D_t(\tau_{j1}^*, \tau_{j2}^*) y_{t-1} + \widehat{v}_{2t} \\
&\text{Model 3:} && \Delta y_t = \widehat{\mu}_1 D_t(\tau_{j1}^*, \tau_{j2}^*) + \widehat{\mu}_2 D_t(\tau_{j2}^*, 1) + \widehat{\rho}_{j1}^* D_t(\tau_{j1}^*, \tau_{j2}^*) y_{t-1} \\
& && + \widehat{\rho}_{j2}^* D_t(\tau_{j2}^*, 1) y_{t-1} + \widehat{v}_{3t} \\
&\text{Model 4:} && \Delta y_t = \widehat{\mu}_1 D_t(\tau_{j1}^*, \tau_{j2}^*) + \widehat{\mu}_2 D_t(\tau_{j2}^*, \tau_{j3}^*) + \widehat{\rho}_{j1}^* D_t(\tau_{j1}^*, \tau_{j2}^*) y_{t-1} \\
& && + \widehat{\rho}_{j2}^* D_t(\tau_{j2}^*, \tau_{j3}^*) y_{t-1} + \widehat{v}_{4t}
\end{aligned}$$

where $t = start_j, ..., end_j$. The dummy variables are defined as $D_t(a,b) = 1(\lfloor aT \rfloor < t \leq \lfloor bT \rfloor)$, with the constant dummy variables associated with μ_1 and μ_2 ensuring invariance of the residuals of each model to the series mean μ. The regime change-point estimators obtained from these four models are then given by:

$$\begin{aligned}
&\text{Model 1:} && \widehat{\tau}_{j1}^* = \arg \min_{0<\tau_{j1}^*<1, y_{T_j} > y_{\lfloor \tau_{j1}^* T_j \rfloor}} SSR_{j1}(\tau_{j1}^*) \\
&\text{Model 2:} && (\widehat{\tau}_{j1}^*, \widehat{\tau}_{j2}^*) = \arg \min_{0<\tau_{j1}^*<\tau_{j2}^*<1, y_{\lfloor \tau_{j2}^* T_j \rfloor} > y_{\lfloor \tau_{j1}^* T_j \rfloor}} SSR_{j2}(\tau_{j1}^*, \tau_{j2}^*) \\
&\text{Model 3:} && (\widehat{\tau}_{j1}^*, \widehat{\tau}_{j2}^*) = \arg \min_{0<\tau_{j1}^*<\tau_{j2}^*<1, y_{\lfloor \tau_{j2}^* T_j \rfloor} > y_{\lfloor \tau_{j1}^* T_j \rfloor}, y_{\lfloor \tau_{j2}^* T_j \rfloor} > y_{T_j}} SSR_{j3}(\tau_{j1}^*, \tau_{j2}^*) \\
&\text{Model 4:} && (\widehat{\tau}_{j1}^*, \widehat{\tau}_{j2}^*, \widehat{\tau}_{j3}^*) = \arg \min_{0<\tau_{j1}^*<\tau_{j2}^*<\tau_{j3}^*<1, y_{\lfloor \tau_{j2}^* T_j \rfloor} > y_{\lfloor \tau_{j1}^* T_j \rfloor}, y_{\lfloor \tau_{j2}^* T_j \rfloor} > y_{\lfloor \tau_{j3}^* T_j \rfloor}} \\
& && SSR_{j4}(\tau_{j1}^*, \tau_{j2}^*, \tau_{j3}^*)
\end{aligned}$$

where $SSR_{ji}(.) = \Sigma_{t=2}^{T_j} \widehat{v}_{it}^2$ for $i = 1, ..., 4$ for Models 1-4. HLS show that for correct pairings of the true DGP and the estimated model, the estimated start dates, end dates, and end of collapse dates are consistent.

Of course accurate date estimation requires the selection of the correct model for each explosive regime j. We denote the number of observations in each date-window as T_j. HLS propose using a BIC based model selection procedure in which the chosen model is given by

$$m_j^{opt} = \arg \min_{m_j \in \{1,2,3,4\}} BIC_{jm}$$

where m_j is the true DGP model, and

$$\begin{aligned} BIC_{j1} &= T_j ln\{T_j^{-1} SSR_{j1}(\widehat{\tau}_{j1}^*, 1)\} + (2+1)ln(T_j) \\ BIC_{j2} &= T_j ln\{T_j^{-1} SSR_{j2}(\widehat{\tau}_{j1}^*, \widehat{\tau}_{j2}^*)\} + (2+2)ln(T_j) \\ BIC_{j3} &= T_j ln\{T_j^{-1} SSR_{j3}(\widehat{\tau}_{j1}^*, \widehat{\tau}_{j2}^*, 1)\} + (4+2)ln(T_j) \\ BIC_{j4} &= T_j ln\{T_j^{-1} SSR_{j4}(\widehat{\tau}_{j1}^*, \widehat{\tau}_{j2}^*, \widehat{\tau}_{j3}^*)\} + (4+3)ln(T_j) \end{aligned}$$

Asymptotic results in HLS establish that model selection based on BIC will lead to the selection of the true model in the limit.

In a multiple bubble dating context, one challenge that presents itself is ensuring that our method of partitioning the sample separates the explosive regimes into different date-windows. Due to the recursive nature of the PSY dating method, HLS note that their date estimates tend to occur later than the true regime change points. When two explosive episodes (j and $j+1$) occur in quick succession, taking the mid-point of $\widehat{\tau}_{j2}^{PSY}$ and $\widehat{\tau}_{(j+1)1}^{PSY}$ may be problematic. This is because, if $\widehat{\tau}_{j2}^{PSY} > \tau_{j2}^*$ and/or $\widehat{\tau}_{(j+1)1}^{PSY} > \tau_{(j+1)1}^*$, then it is possible that $start_{j+1} > \tau_{(j+1)1}^*$ making accurate date-stamping of this regime change point impossible. To overcome this issue, we require each date-window to start with at least one observation of a unit root regime. Recalling that, if $j < \widehat{J}$, we only allow Model 2 or Model 4 to be fitted, we can therefore adjust the starting point of the next date-window, $start_{j+1}$, to be equal to the first observation of the post-explosive unit root regime for bubble j, i.e. $\lfloor \widehat{\tau}_{j2} T \rfloor + 1$ for Model 2 or $\lfloor \widehat{\tau}_{j3} T \rfloor + 1$ for Model 4.

4 Practical Implementation of the PSY and BIC Dating Procedures

When implementing the GSADF detection procedure and BSADF dating procedure, we adopt PSY's recommended minimum window width of $r_0 = \lfloor 0.01 + 1.8/\sqrt(T) \rfloor$, and impose their restriction that an explosive regime must last $log(T)$ observations. One feature we observed when employing the BSADF test was that a long series of rejections may be interrupted by a very small number of non-rejections, giving the appearance of multiple explosive regimes, when in fact, the true DGP was a single bubble. In empirical work, this phenomenon may be obvious from visual inspection of a price series, but in simulations we are not able to make this manual adjustment. Instead, we mitigate this issue by assuming that if up to 3 non-rejections are surrounded on either side by an explosive regime of length $log(T)$, then they can be treated as a single bubble episode, in essence joining these two strings of rejections together.

Following HLS, we require that $\widehat{\tau}_{j1} \geq s$, that $\widehat{\tau}_{j2} - \widehat{\tau}_{j1} \geq s$ and that $\widehat{\tau}_{j3} - \widehat{\tau}_{j2} \geq s/2$ imposing minimum length requirements on the initial unit root regime, bubble length, and length of collapse respectively. However, as we split our sample into $\widehat{J}$ date-windows, it is possible that the number of observations in each date-window, T_j, is small enough that $\lfloor sT_j \rfloor < 2$ and

$\lfloor sT_j/2 \rfloor < 2$. In this multiple bubble context, we therefore impose instead that $\lfloor \widehat{\tau}_{j1}T \rfloor \geq \max(\lfloor sT_j \rfloor, 2)$, $\lfloor \widehat{\tau}_{j2}T \rfloor \geq \max(\lfloor sT_j \rfloor, 2)$ and $\lfloor \widehat{\tau}_{j3}T \rfloor \geq \max(\lfloor sT_j/2 \rfloor, 2)$.

5 Finite Sample Simulations

To assess the performance of our two-step BIC procedure at accurately date-stamping multiple explosive regimes, we conduct a finite sample Monte Carlo simulation exercise. For a single bubble episode, $J = 1$, our two-step procedure becomes identical to the BIC procedure of HLS, so we don't examine this case, instead focusing on $J = \{2, 3\}$. For the case of $J = 2$, we set the sample size to $T = 200$, and for $J = 3$ we have $T = 300$. We consider six DGPs which together demonstrate the performance of our two-step procedure over all model specifications, across different lengths of explosive and stationary regimes, across different locations of the regimes within the sample, and for different magnitudes of explosive behaviour and stationary collapse. The exact settings for the six DGPs we examine are given below:

DGP_a : $J = 2,\ m_1 = 4,\ m_2 = 4$

$\rho^*_{11} = 0.08,\ \rho^*_{12} = -0.06,\ \tau^*_{11} = 0.3,\ \tau^*_{12} = 0.5,\ \tau^*_{13} = 0.6$

$\rho^*_{21} = 0.02,\ \rho^*_{22} = -0.02,\ \tau^*_{21} = 0.7,\ \tau^*_{22} = 0.8,\ \tau^*_{23} = 0.85$

DGP_b : $J = 2,\ m_1 = 2,\ m_2 = 1$

$\rho^*_{11} = 0.08,\ \tau^*_{11} = 0.3,\ \tau^*_{12} = 0.5,\ \tau^*_{13} = 0.6$

$\rho^*_{21} = 0.01,\ \tau^*_{21} = 0.95,\ \tau^*_{22} = 1$

DGP_c : $J = 2,\ m_1 = 2,\ m_2 = 4$

$\rho^*_{11} = 0.06,\ \tau^*_{11} = 0.15,\ \tau^*_{12} = 0.3$

$\rho^*_{21} = 0.02,\ \rho^*_{22} = -0.01,\ \tau^*_{21} = 0.7,\ \tau^*_{22} = 0.8,\ \tau^*_{23} = 0.9$

DGP_d : $J = 2,\ m_1 = 4,\ m_2 = 3$

$\rho^*_{11} = 0.04,\ \rho^*_{12} = -0.02,\ \tau^*_{11} = 0.5,\ \tau^*_{12} = 0.6,\ \tau^*_{13} = 0.65$

$\rho^*_{21} = 0.02,\ \rho^*_{22} = -0.01,\ \tau^*_{21} = 0.7,\ \tau^*_{22} = 0.9,\ \tau^*_{23} = 1$

DGP_e : $J = 3,\ m_1 = 4,\ m_2 = 4,\ m_3 = 2$

$\rho^*_{11} = 0.08,\ \rho^*_{12} = -0.04,\ \tau^*_{11} = 0.3,\ \tau^*_{12} = 0.4,\ \tau^*_{13} = 0.5$

$\rho^*_{21} = 0.04,\ \rho^*_{22} = -0.02,\ \tau^*_{21} = 0.6,\ \tau^*_{22} = 0.7,\ \tau^*_{23} = 0.75$

$\rho^*_{31} = 0.02,\ \tau^*_{31} = 0.85,\ \tau^*_{32} = 0.95$

DGP_f : $J = 3,\ m_1 = 4,\ m_2 = 2,\ m_3 = 1$

$\rho^*_{11} = 0.08,\ \rho^*_{12} = -0.08,\ \tau^*_{11} = 0.2,\ \tau^*_{12} = 0.35,\ \tau^*_{13} = 0.4$

$\rho^*_{21} = 0.01,\ \tau^*_{21} = 0.6,\ \tau^*_{22} = 0.65,\ \tau^*_{23} = 0.75$

$\rho^*_{31} = 0.01,\ \tau^*_{31} = 0.9,\ \tau^*_{32} = 1$

Figures 1-6 display histograms of the estimated start dates, $\hat{\tau}_{j1}^{PSY}$ and $\hat{\tau}_{j1}^{*}$, and end dates, $\hat{\tau}_{j2}^{PSY}$ and $\hat{\tau}_{j2}^{*}$ of the PSY dating procedure and our two-step BIC procedure respectively. We report results only for the replications where at least one explosive regime was detected according to the GSADF test. When the test procedures found more explosive regimes than J, we report results for the estimated dates that are closest to the true DGP dates. Monte Carlo simulations were carried out in Gauss 18 using 10,000 replications.

Consider first DGP_a in figure 1. Here, both bubbles are followed by a stationary collapse period, before unit root behaviour is restored. For the first explosive regime, both the explosive and collapse phases last for 0.1 of the sample, although the magnitude of the collapse is smaller in absolute value than the magnitude of the explosive process. Examining the estimated start dates of the first explosive regime, it is clear that BIC correctly identifies the true start date more often than it estimates any other date, with a symmetric distribution around this date. In contrast, PSY estimates of the start date fall somewhat later than the true date. Turning our attention to the estimated end date of the first explosive regime, the BIC procedure shows excellent accuracy, with the estimated end date equalling the true end date in almost every replication. The variance of PSY end date estimates is also shown to be lower compared to its start date estimates, but as with the start dates, it dates the end point late in most replications. Considering now the second explosive regime in the DGP, the magnitude of explosion and collapse are both smaller than in the first regime. Given that the process y_t has already undergone an explosive regime, which did not collapse all the way back to its starting value, when the second explosive regime takes place, the starting value of y_t is higher and therefore does not require as large a value of ρ_{j1}^{*} to achieve the same pattern of growth. Again, examining both the start and end dates, the distribution of BIC estimates is centred around the true DGP value, whereas the PSY estimates are generally late.

DGP_b features no stationary collapses, with the first explosive regime reverting back to a unit root process, and the second explosive regime continuing to the end of the sample. Examining figure 2, for the first regime, despite there being no stationary collapse following the bubble, BIC still estimates the true end date in almost all replications. However, we note that the PSY end dates are noticeably more inaccurate without this collapse phase. Considering the second explosive regime, setting $\tau_{21}^{*} = 0.95$ such that the regime doesn't occur until very close to the end of the sample does not seem to have unduly affected BIC's accuracy. Figure 3 displays the date estimates for DGP_c where the first explosive regime is located closer to the beginning of the sample, with $\tau_{11}^{*} = 0.15$. The magnitude of explosion is also smaller compared with DGP_a and DGP_b. The effect of this lower magnitude bubble and the earlier location in the sample is a more leptokurtic distribution of estimated start dates for the BIC procedure, although it is still centred around the true start date. Again, PSY date estimates are late in comparison. DGP_d features two explosive regimes which are located close to each other in the sample, with only 0.05 of the sample between the end of the collapse regime of the first bubble, and the start of explosive behaviour in the second. Despite this, as we see in figure 4, the distribution of BIC start date estimates for the second regime is still centred around the true start date.

Considering now our three bubble DGPs, figure 5 displays the estimated dates for DGP_e where the first two explosive regimes contain a stationary collapse period, and the third reverts back to a unit root process with no collapse. A similar pattern of behaviour is observed as in the two bubble DGPs, with the distribution of BIC start date estimates being centred around the true value, and with BIC end date estimates being almost exclusively equal to the true end date. Meanwhile, PSY date estimates are most often later than the true start and end values. Finally, examining DGP_f in figure 6, we now consider three different models of bubble behaviour, with the first explosive regime containing a stationary collapse before reverting back to a unit root process, the second reverting back to unit root with no collapse, and the third explosive process continuing until the end of the sample. The first regime is placed early in the sample, with $\tau_{11}^* = 0.2$, and the third regime is towards the end of the sample, with $\tau_{31}^* = 0.9$. The comparative performance of the PSY and BIC estimates is found to be qualitatively similar to DGP_e.

As well as examining the accuracy of date estimates, we also consider the extent to which BIC is able to select the correct model for each explosive regime. Of course, as HLS note, identification of the correct model is not essential for accurate date estimation. For example, if BIC were to mistakenly select Model 2 instead of Model 4, this would still provide estimates of τ_{j1}^* and τ_{j2}^*. Table 1 displays the frequencies with which the BIC procedure selects each model across 10,000 replications. For most of the explosive regimes considered, BIC selects the correct model more frequently than it selects any other model. The most noticeable exceptions to this are the second explosive regime for DGP_c, where the collapse is of a comparatively small magnitude, and so remains undetected in many replications; and the first regime of DGP_d, where, again, the collapse is not detected in many replications, resulting in the selection of Model 2 over Model 4. Overall our results demonstrate a clear accuracy advantage of our proposed BIC date-stamping procedure over recursive unit root based methods of date estimation.

6 Empirical Application

To demonstrate the practical value of our proposed date-stamping procedure, we examine the historical bubble behaviour of house prices for a set of 23 countries. The sub-prime crisis and subsequent financial distress of the late 2000s has led to increased scrutiny of the dynamics of the housing market. Several recent studies have implemented the PSY detection and dating procedures to investigate the timing of explosive episodes in house prices. Anundsen et al. (2016) consider the effect of the interaction between credit and the housing market on financial stability. They apply the BSADF dating procedure of PSY to quarterly house price data for 20 OECD countries from 1975Q1-2014:Q2, and use the estimated dates to construct 'exuberance' indicators which are employed as explanatory variables in a logit regression of financial instability. Their findings suggest that the probability of a financial crisis increases substantially when bubbles in house prices coincide with similar exuberance in credit. Pavlidis et al. (2016) also consider the role of house price bubbles on financial stability. Applying the GSADF detection procedure and BSADF dating procedure of PSY to the International House Price Database of the Federal Reserve Bank of Dallas (Mack and Martinez-Garcia, 2011) for 22 countries, they find significant evidence of house price exuberance in 20 countries in the early 2000s, preceding the global financial crisis. The date estimates are used to construct a 'chronology of exuberance' tracking the evolution of house price bubbles across countries and time. Using these date estimates to construct an exuberance indicator, they estimate a probit model to assess the predictive ability of a number of financial and macroeconomic variables on house price exuberance. The Dallas Federal Bank has provided house price exuberance indicators from 2013:Q2 onwards based on the methodology of Pavlidis et al. (2016). Whilst these papers provide very interesting results, the validity of their analysis is clearly dependent on accurate start and end date estimates of bubbles in housing markets.

As Pavlidis et al. (2016) do, we consider the International House Price Database of the Federal Reserve Bank of Dallas (Mack and Martinez-Garcia, 2011) to detect and date house price bubbles in 22 countries.[1] Our sample period runs from 1975:Q1-2018:Q2. First, considering the GSADF detection procedure of PSY, we find evidence of explosive behaviour in all but two countries (Croatia and South Korea), and therefore proceed to apply both the PSY BSADF dating procedure and our BIC procedure to date-stamp the detected explosive episodes. Of the 20 countries where at least one explosive regime is detected, PSY finds only one period of explosiveness in four countries (Denmark, France, Ireland and South Africa), two explosive regimes are detected in eight countries (Australia, Belgium, Switzerland, Germany, UK, Japan, Netherlands and US), three explosive regimes are detected in six countries (Canada, Spain, Italy, Norway, New Zealand and Sweden), and four explosive regimes are detected in two countries (Finland and Luxembourg).

Figure 7 displays the house price index and the corresponding start and end date estimates of the PSY and BIC dating procedures for the 20 countries where explosive behaviour is detected. We note the following key features from our analysis. First, when PSY and BIC are dating the same explosive episode, BIC tends to find start date estimates earlier than PSY in line with our simulation results. See, for example, the two explosive regimes detected in Australia, displayed in figure 7a, where BIC estimates start dates of 1996:Q4 and 2012:Q3 compared to PSY start date estimates of 1998:Q4 and 2016:Q2. It is also evident that the BIC procedure typically provides earlier end dates than PSY. Considering Ireland, in figure 7k, for example, BIC estimates the end date of the single explosive regime as 1995:Q2, corresponding to the maximum value of the house price index, whereas PSY finds an estimated end date of 1995:Q4, after the turning point of the price series. Across all countries, out of the approximately 36

[1]The database now contains information for an additional country, Israel, than in the analysis of Pavlidis et al. (2016) but, in unreported results, we find no evidence of bubble behaviour in this country

occasions where BIC and PSY appear to date the same explosive regime, BIC finds an earlier (or identical) start date to PSY in 34 of these regimes, and an earlier (or identical) end date in 29 of these regimes. Another advantage of the BIC dating procedure is its ability to date a crash regime following an explosive bubble. This is evident in our application, see, for example, the first explosive regime detected for Switzerland, figure 7r.

Another interesting feature of the analysis is that BIC and PSY do not always date the same regime. We have deliberately constructed our BIC procedure to provide the same *number* of detected explosive regimes as PSY, but within a given date window there is no guarantee that the two procedures will date the same regime. Consider Canada in figure 7c. PSY dates the first explosive regime from 1988:Q3-1990:Q3 and the second explosive regime from 2002:Q4-2008:Q4. Therefore the first sub-sample date window that the BIC procedure uses to date runs from the start of the sample until 1996:Q3 (the mid-point of the estimated end date of the first explosive regime and the estimated start date of the second). However, within this date window, BIC fits a bubble regime much earlier in the sample from 1979:Q1-1981:Q3, with a crash then lasting until 1982:Q4. From visual inspection of the house price series, this first explosive regime detected by BIC appears more pronounced than the later regime detected by PSY, but remains undetected by the PSY procedure. As a consequence of this, and due to the adjustment made in the BIC procedure in which the the start date of the subsequent sub-sample date window is rolled back to the first unit root observation that follows the previous explosive episode, it is possible for BIC to detect the start of two explosive regimes before PSY has detected any, see Belgium, for example, in figure 7b.

Whilst we restrict the BIC procedure to detect only positive explosive regimes, no such restriction applies to the PSY procedure and therefore it is possible for PSY to detect negative explosive episodes. In our simulation exercise, we generated only positive explosive processes, and as such did not encounter this issue. However, we note several occasions in our empirical application where this occurs. Take Japan for example, in figure 7m. The second regime detected by PSY is clearly one of negative explosivity. As a consequence, the BIC procedure is forced to date a positive explosive regime in a date window where one may not exist. In practice, we would expect it to be easy to identify whether a dated explosive episode is positive or negative through visual inspection of the price series. If PSY detects a negative period of explosiveness, then it may be sensible to exclude this regime (and consequently the regime detected by BIC) from our analysis.

The application of the BIC procedure to house price data allows us to build up a chronology of events. Focusing on periods of positive explosivity only, we observe that many countries experienced explosive episodes in house prices during the 1980s, with this explosiveness originating as early as 1977:Q1 for Switzerland, and emerging in the UK in 1983:Q1. For the UK, this explosiveness ended in 1988:Q4, and was followed by a collapse in prices which continued until 1992:Q4. Other countries took longer to recover from this housing bubble, with the collapse continuing until 1994:Q2 in Japan, and 1997:Q2 in Switzerland. The second key phase of activity is the house price exuberance and subsequent crash that occurred in the majority of countries studied during the mid-2000s. Our analysis shows that this bubble behaviour emerged as early as 1997:Q2 in the US before prices collapsed in 2006:Q4, with this crash lasting until 2011:Q2. In the UK, house price exuberance emerged only two quarters later than in the US, before collapsing in 2007:Q4. The UK housing market suffered a shorter period of distress than the US, with the collapse in prices ending much sooner in 2009:Q1. Finally, another interesting observation from our analysis is that explosive behaviour has emerged in many countries (Switzerland, Germany, Spain, Italy, Japan, Luxembourg, Norway, and the US) which is still ongoing at the end of our sample, suggesting that another phase of house price boom and bust may have begun.

7 Conclusion

In this paper we have proposed a date-stamping methodology for asset price bubbles, that delivers precise date estimates for multiple bubbles within a series. We have demonstrated the superior accuracy of our procedure in comparison to the recursive unit root based date-stamping approach of PSY. An additional advantage of our procedure, is its ability to characterise the specific bubble behaviour of each bubble episode within a sample, and date the timing of bubble crashes when they are present. An empirical application of our methodology to housing markets demonstrated that the house price exuberance in the mid-2000s which preceded the global financial crisis originated as early as 1997 in the US and the UK.

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Figure 1: Histogram of date estimates for DGP_a

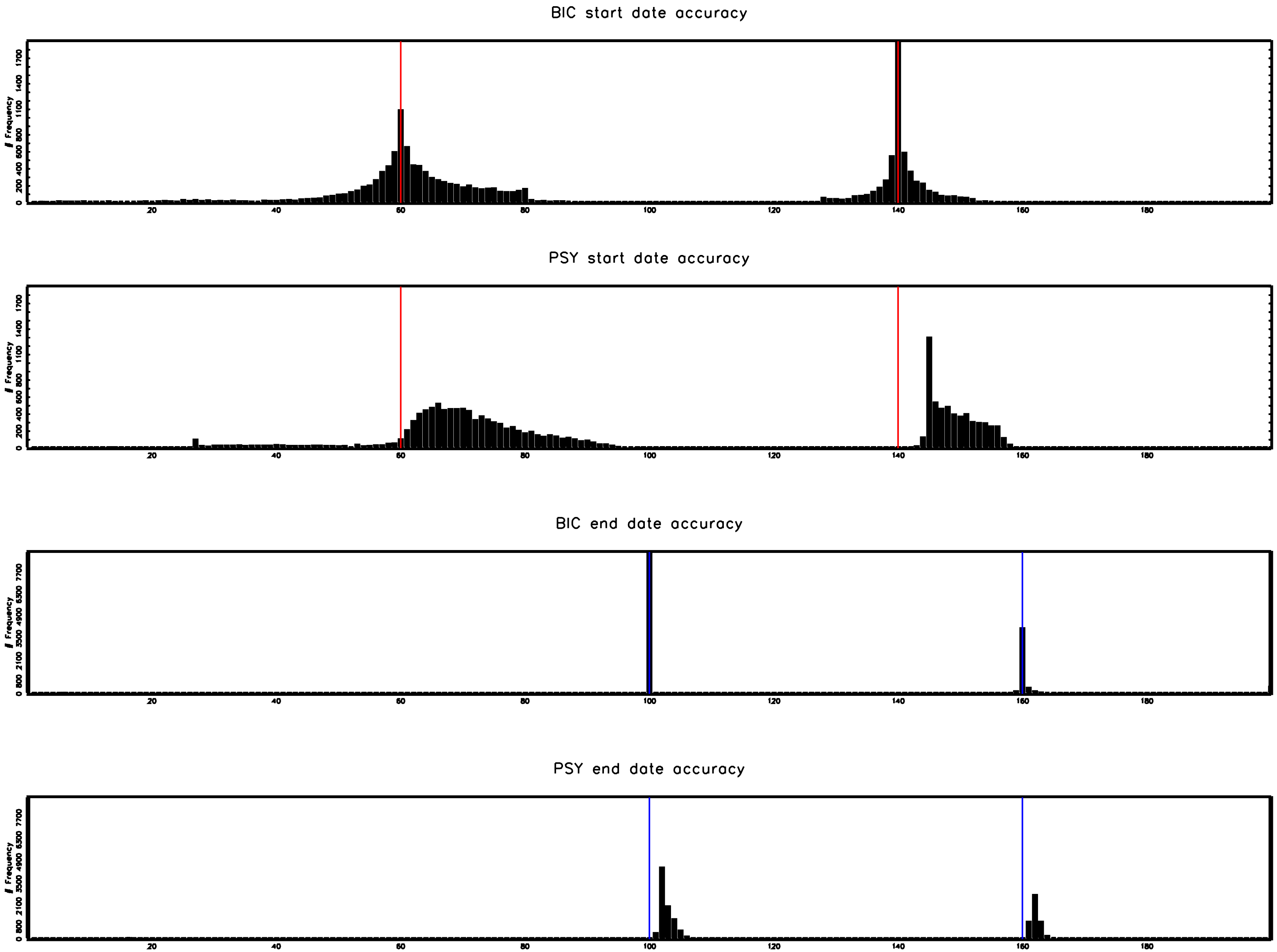


Figure 2: Histogram of date estimates for DGP_b

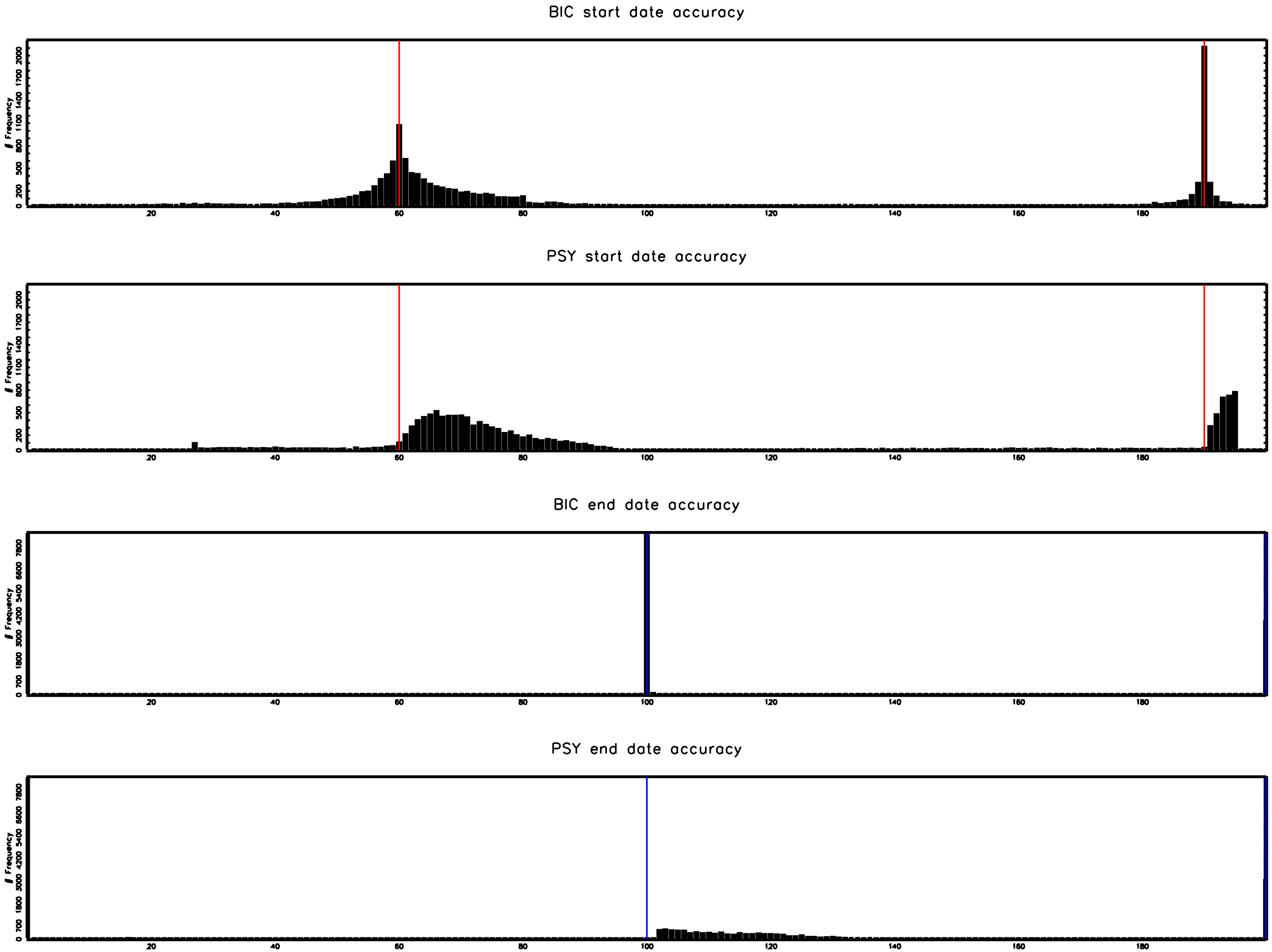


Figure 3: Histogram of date estimates for DGP_c

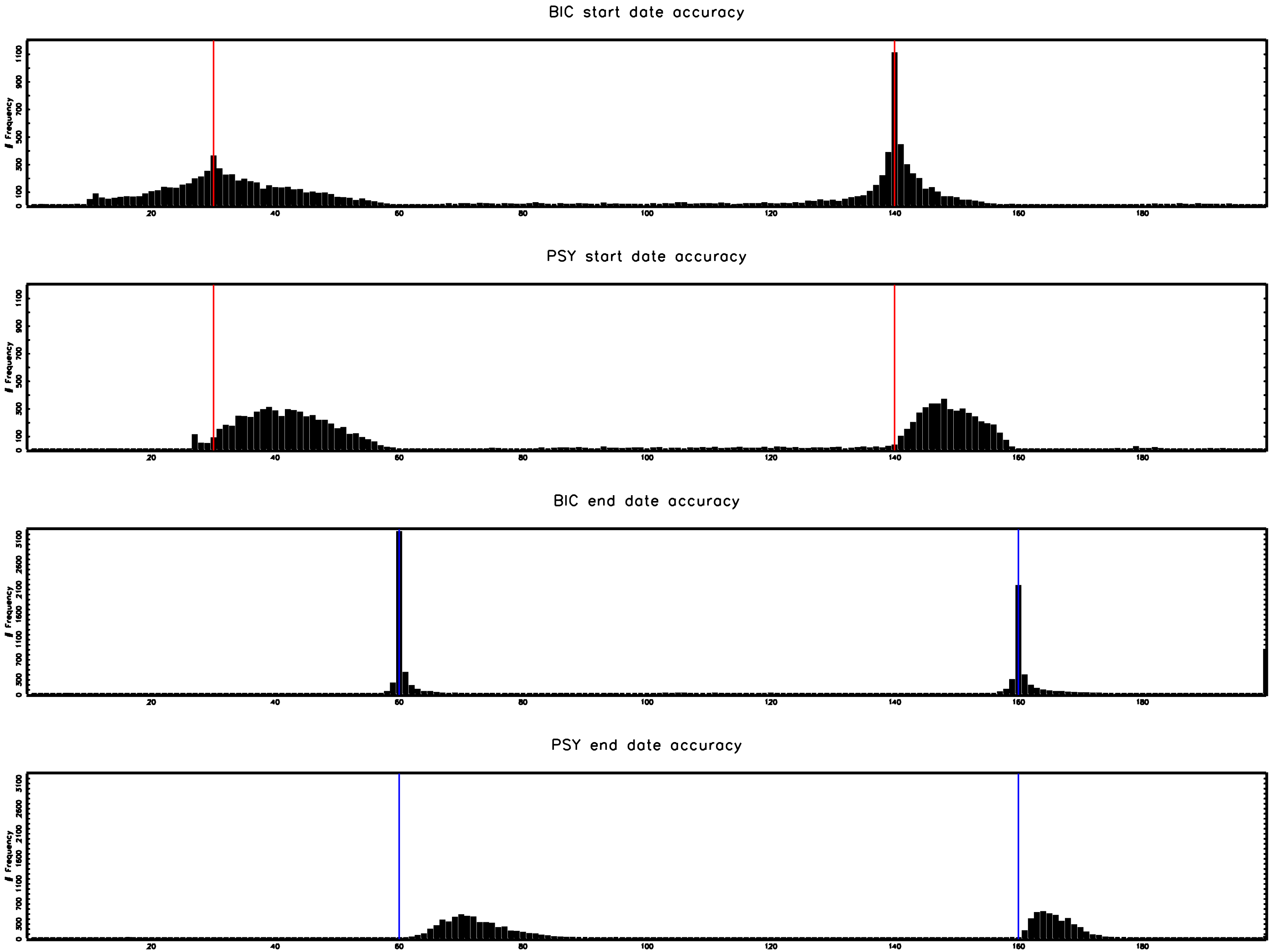


Figure 4: Histogram of date estimates for DGP_d

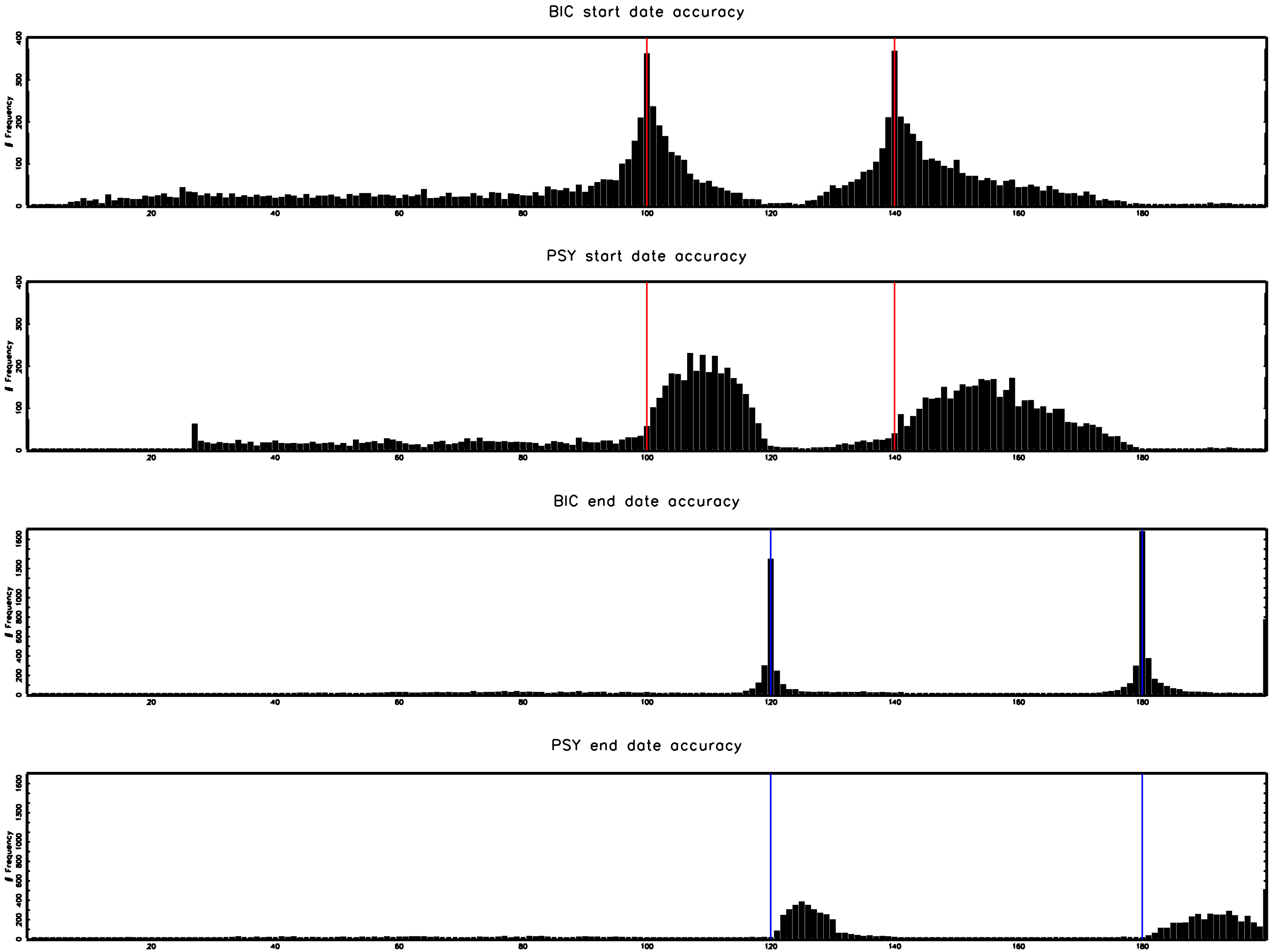


Figure 5: Histogram of date estimates for DGP_e

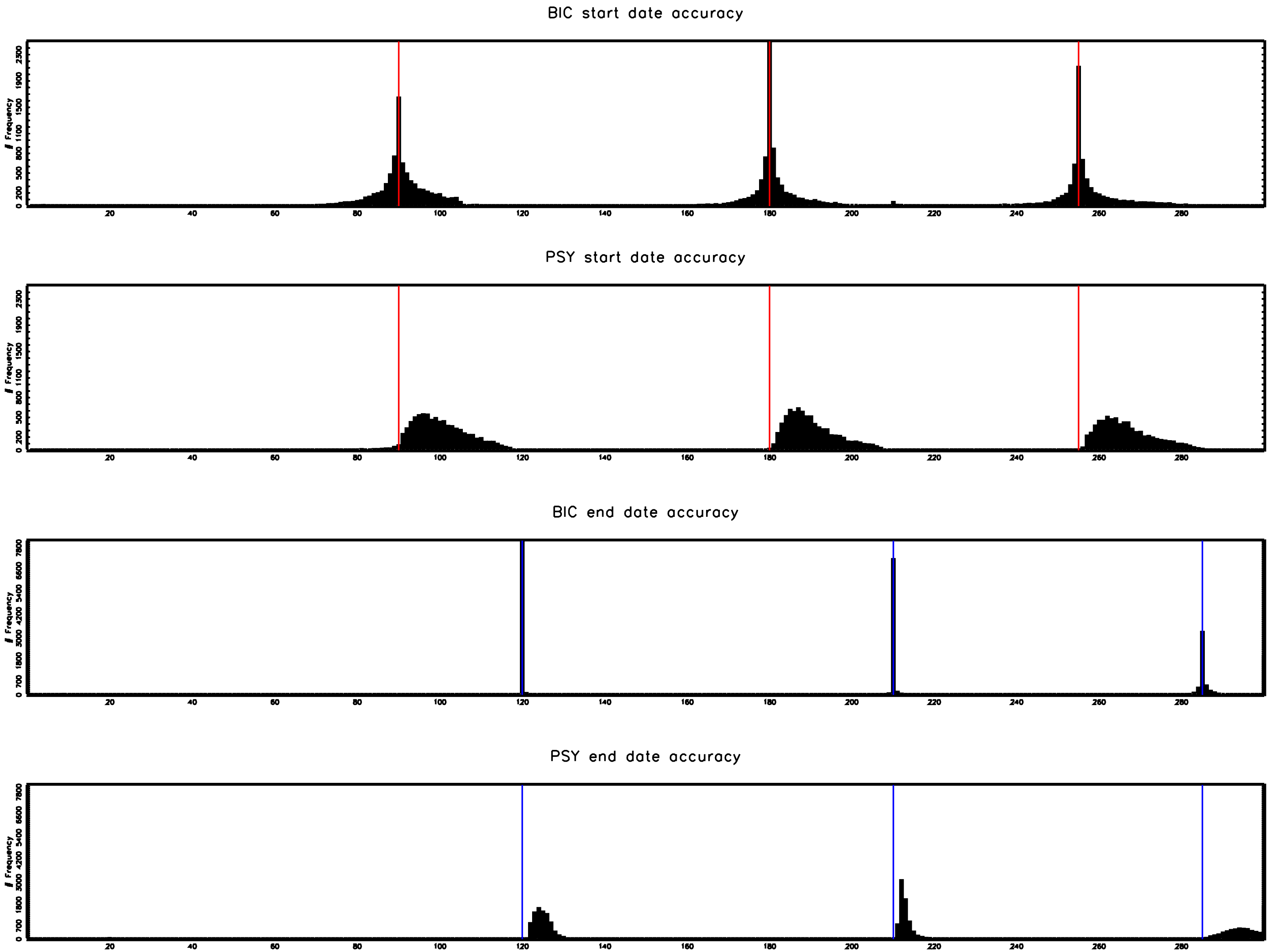


Figure 6: Histogram of date estimates for DGP_f

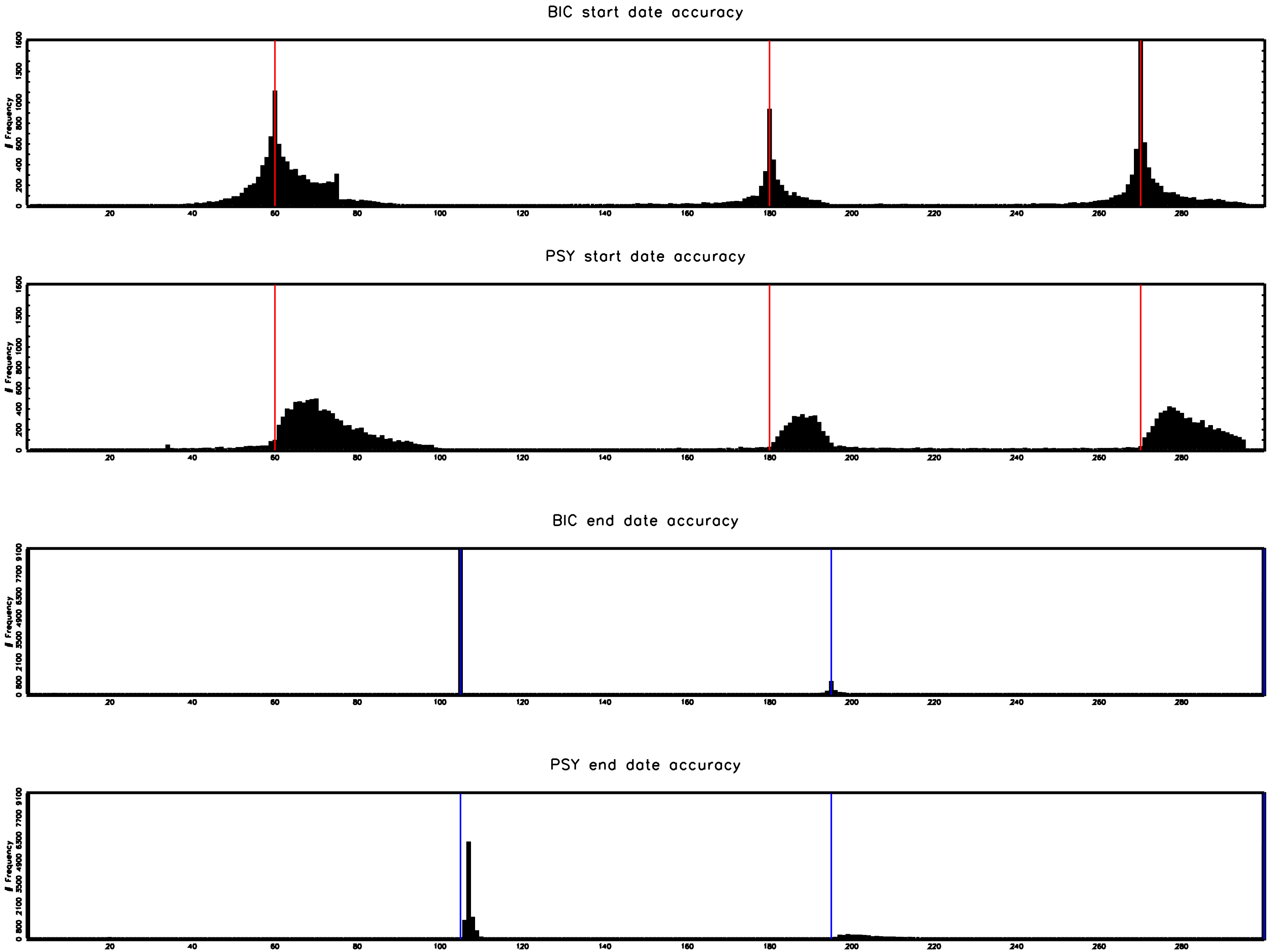
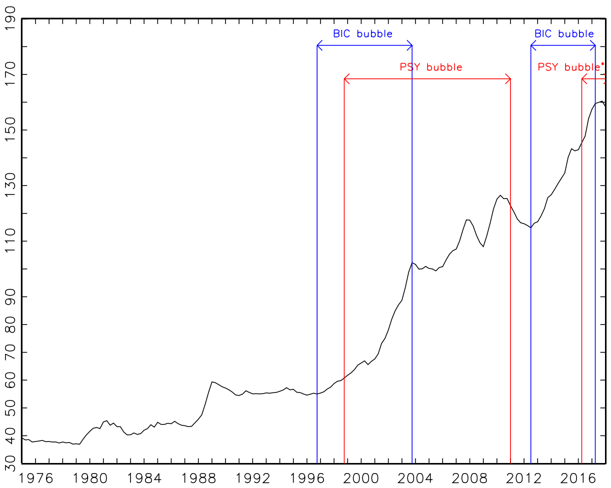


Table 1: BIC model selection frequencies for $DGP_a - DGP_f$

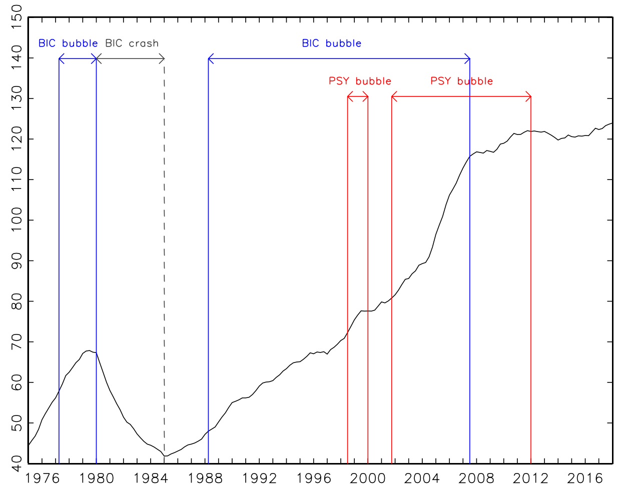
	Model 1	Model 2	Model 3	Model 4
DGP_a				
$j = 1$	0.004	0.009	0.079	**0.908**
$j = 2$	0.048	0.048	0.352	**0.553**
DGP_b				
$j = 1$	0.020	**0.965**	0.007	0.009
$j = 2$	**0.427**	0.555	0.012	0.006
DGP_c				
$j = 1$	0.049	**0.902**	0.016	0.033
$j = 2$	0.144	0.375	0.379	**0.102**
DGP_d				
$j = 1$	0.092	0.709	0.065	**0.134**
$j = 2$	0.176	0.365	**0.427**	0.032
DGP_e				
$j = 1$	0.000	0.080	0.025	**0.895**
$j = 2$	0.051	0.058	0.065	**0.895**
$j = 3$	0.156	**0.640**	0.084	0.120
DGP_f				
$j = 1$	0.002	0.008	0.015	**0.974**
$j = 2$	0.287	**0.447**	0.020	0.246
$j = 3$	**0.411**	0.024	0.018	0.547

bold indicates the true bubble DGP

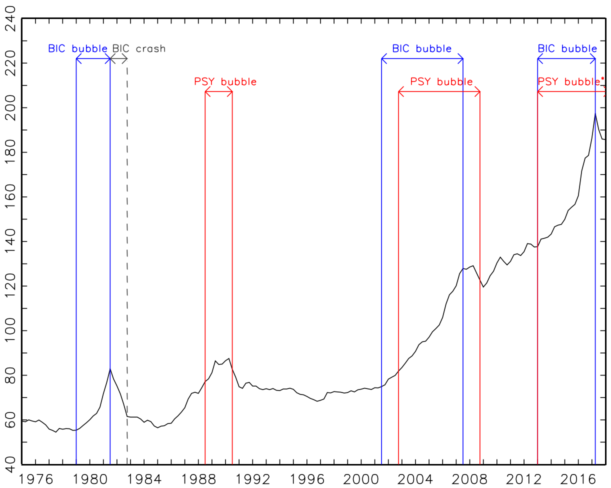
Figure 7: House price index: BIC and PSY bubble date estimates



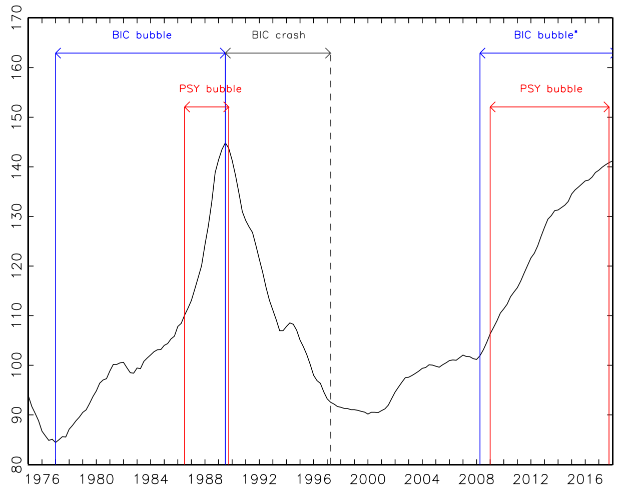
(a) Australia



(b) Belgium



(c) Canada

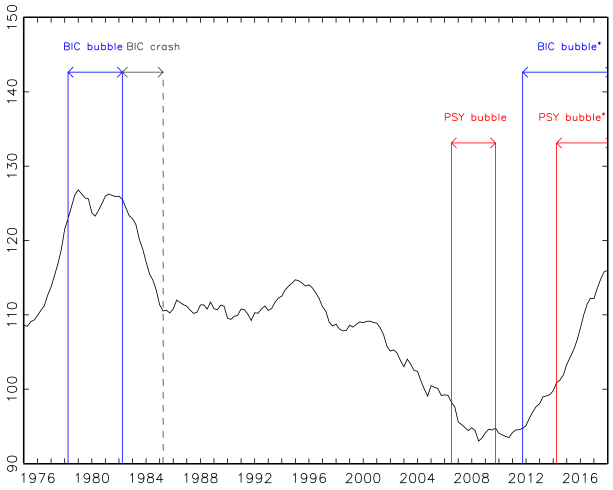


(d) Switzerland

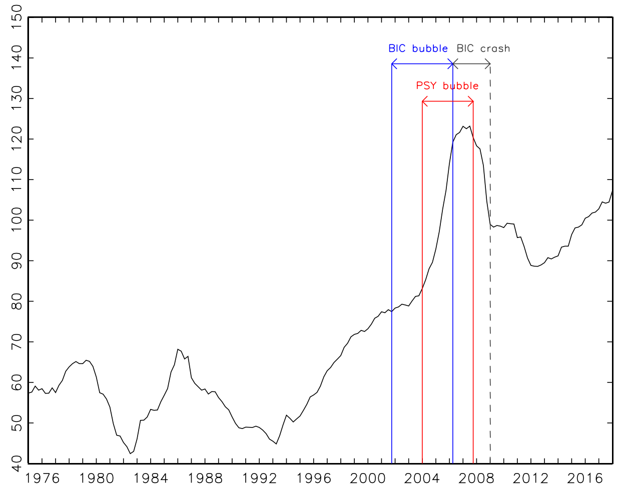
— BIC bubble start and end dates, - - BIC crash end date, — PSY bubble start and end dates

* indicates that the regime continues to the end of the sample

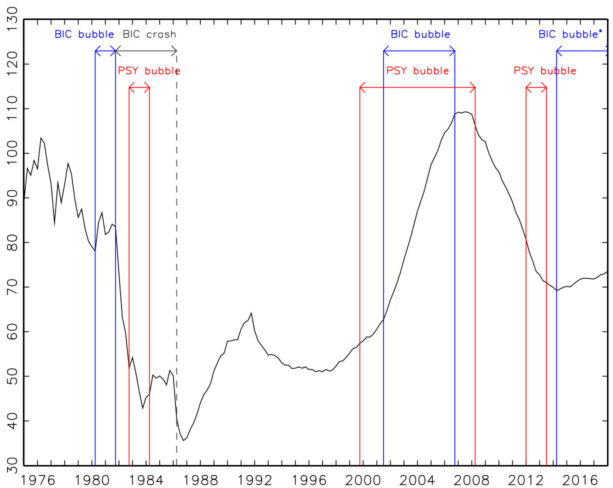
Figure 7: House price index: BIC and PSY bubble date estimates (continued)



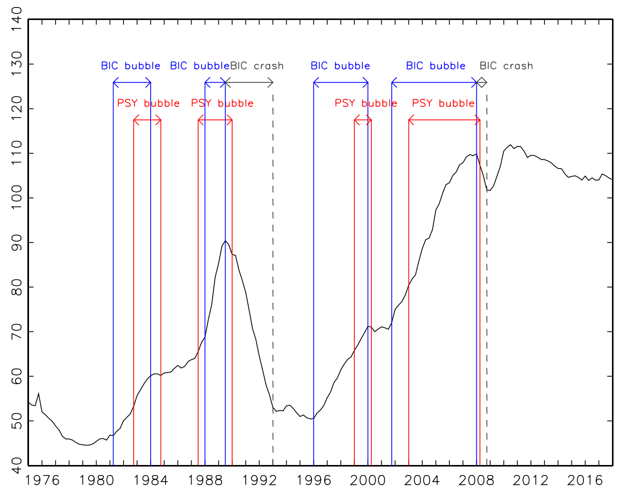
(e) Germany



(f) Denmark



(g) Spain

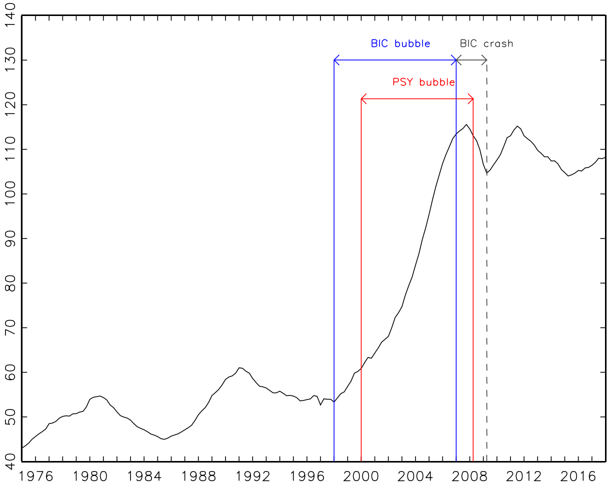


(h) Finland

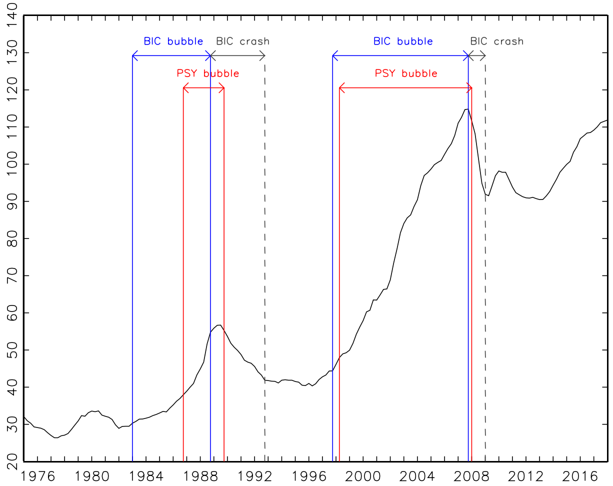
— BIC bubble start date, — BIC bubble end date, - - BIC crash end date, — PSY bubble start date, — PSY bubble end date

* indicates that the regime continues to the end of the sample

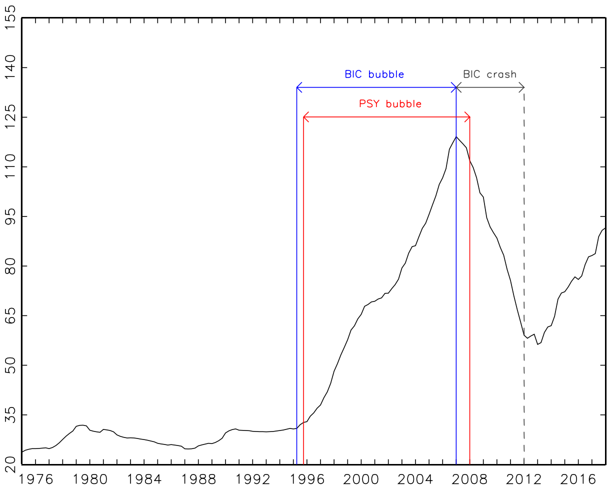
Figure 7: House price index: BIC and PSY bubble date estimates (continued)



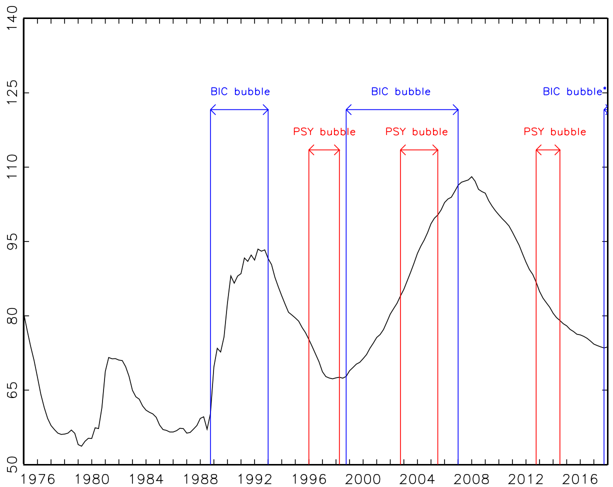
(i) France



(j) United Kingdom



(k) Ireland

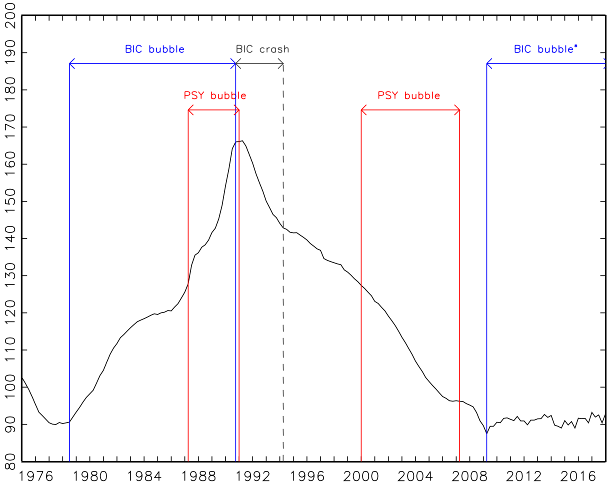


(l) Italy

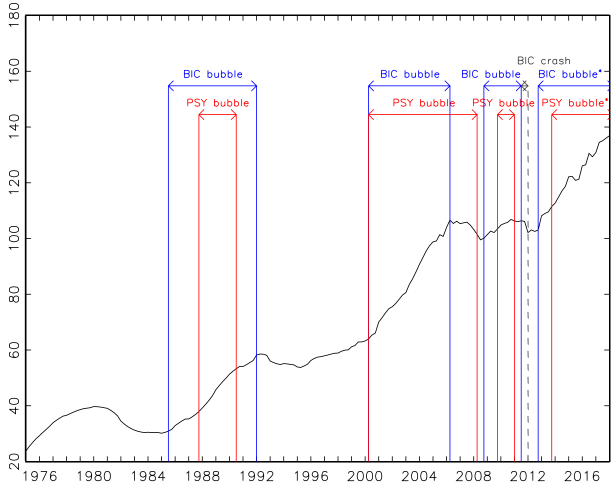
— BIC bubble start date, — BIC bubble end date, - - BIC crash end date, — PSY bubble start date, — PSY bubble end date

* indicates that the regime continues to the end of the sample

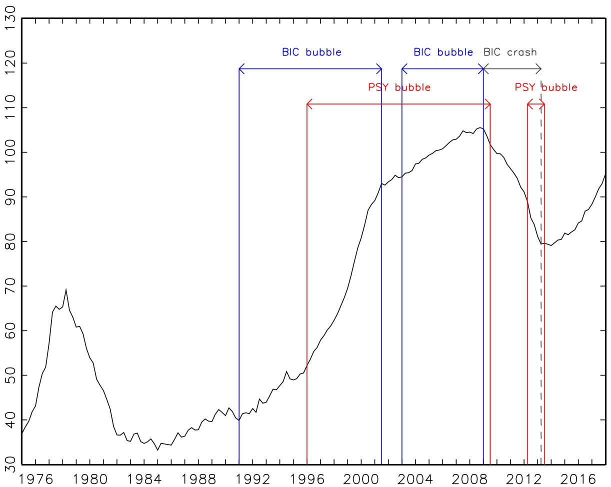
Figure 7: House price index: BIC and PSY bubble date estimates (continued)



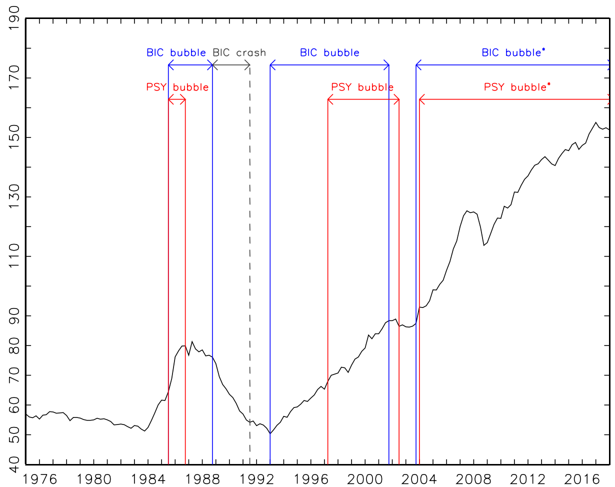
(m) Japan



(n) Luxembourg



(o) Netherlands

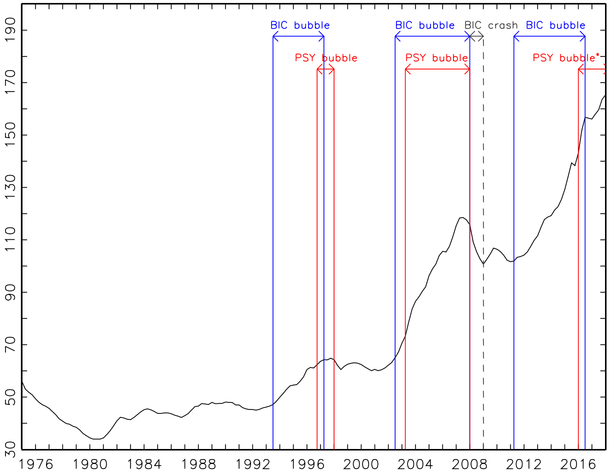


(p) Norway

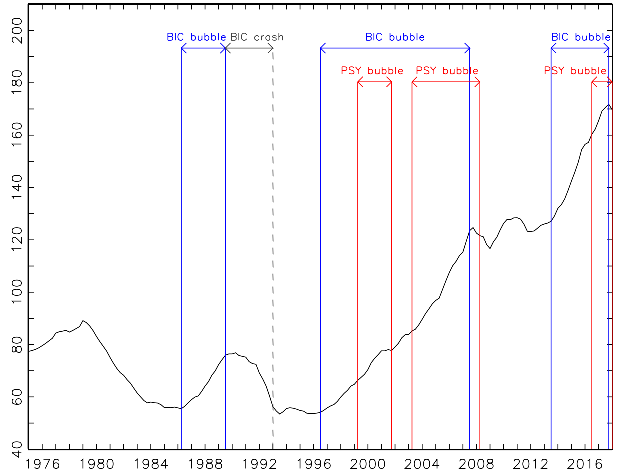
— BIC bubble start date, — BIC bubble end date, - - BIC crash end date, — PSY bubble start date, — PSY bubble end date

* indicates that the regime continues to the end of the sample

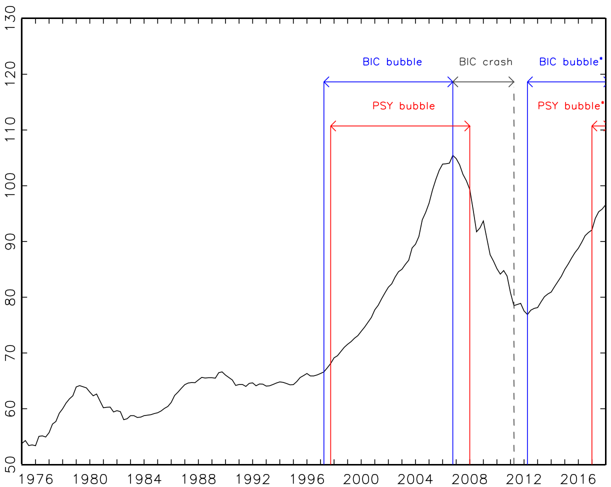
Figure 7: House price index: BIC and PSY bubble date estimates (continued)



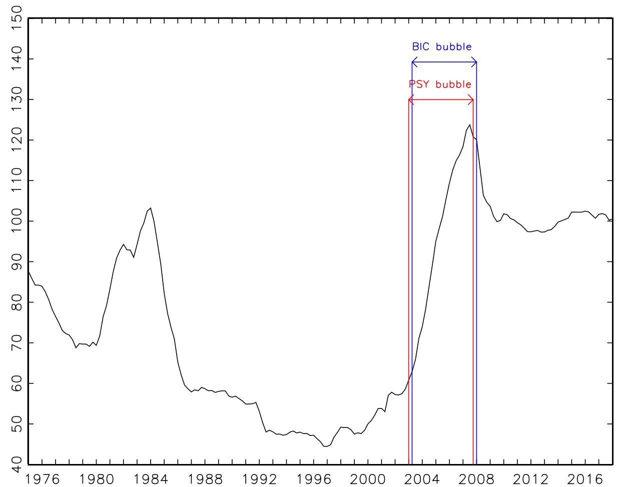
(q) New Zealand



(r) Sweden



(s) United States



(t) South Africa

— BIC bubble start date, — BIC bubble end date, - - BIC crash end date, — PSY bubble start date, — PSY bubble end date

* indicates that the regime continues to the end of the sample