What does the Eurodollar futures market tell us about the effects of credit shocks and monetary policy at the lower bound?

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Abstract

The effects of credit and monetary policy shocks at the lower bound are analyzed using a shadow rate Gaussian term structure model of the Euro-Dollar futures and Treasury bond markets, facilitated by the observation that the shadow rate model gives a simple closed-form for an interest rate futures price. In addition to the usual insights that term structure models provide about monetary policy, this model shows that as intended, the policy initiatives that followed the Lehman default in 2008 had were much more effective in restraining the rise in risk premia and prices in banking markets than in the Treasury bond market. We also find that the Treasury–Eurodollar spread conventionally used as an indicator of default risk in the economy should be split into two components distinguishing transitory and semi-permanent default risk factors.

Keywords: ZLB, Term Structure, TED spread, Global Financial Crisis, Monetary Policy.

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1 Introduction

The effects of credit and monetary policy shocks at the lower bound are analyzed using a shadow rate Gaussian term structure model of the Euro-Dollar futures and Treasury bond markets. This research was facilitated by the observation that the shadow rate model gives a simple closed-form for an interest rate futures price, while, in contrast, bond yields and forward rates need to be approximated. More important from a policy perspective, the use of data for Euro-Dollar and Treasury markets allows one to study the relationship between these markets and distinguish between credit and policy shocks, yielding new insights into the effects of the recent financial crisis and the policy response.

The links between monetary policy and yield curves have been extensively studied by academics, practitioners and especially central bank economists (Gurkaynak and Wright (2012)). The slope of the yield curve provides a standard indicator of the thrust of monetary policy, while spreads between the yields on Treasury inflation-protected and conventional bonds provide simple measures of inflationary expectations and risk premia. The TED spread, the difference between the 3-month Euro-Dollar and Treasury bill rates, provides a similar measure of default risk (Cornett, McNutt, Strahan, and Tehranian (2011)).

Central banks largely relied upon short-maturity market operations and interest rates to implement their policies between the early post-war years and the financial crisis, but the Treasury bond yield curve effectively became a monetary policy instrument once interest rates approached their zero lower bound (ZLB). Central bank polices were then designed to lower longer term yields through forward interest rate guidance and open market operations. The severe problems experienced in the banking markets meant that these were a major focus of policy post-Lehman. Support for these markets began with the rescue of the American Insurance Group (AIG), the opening of the Fed discount window to the two surviving investment banks and announcement of the US Treasury's Troubled Assets Relief Program (TARP). This was followed in November by the Federal Reserve's Large Scale Asset Purchases (LSAP) program, which involved purchases in the mortgage markets offset by sales of Treasury securities (Rai (2013), Guidolin, Orlov, and Pedio (2104)). The chronology is set out in Table 1.

The aim of these unconventional policies, as set out in the minutes of the 12 December 2008 FOMC meeting for example, was to 'support overall market functioning' and ultimately to 'reduce borrowing costs for a range of private borrowers'. The Euro-Dollar rate and associated forward and futures markets are used to price a wide variety of banking and other private financial contracts and clearly reflected the problems that troubled the money and banking markets during the financial crisis. These markets offer a measure of the efficacy of policy in reaching the ultimate objective of policy after the crisis, one that is missed by the usual focus on government bond markets. This paper argues that these policies were in fact more successful in restraining the rise in risk premia post-Lehman in the banking markets than in the Treasury bond market.

The first piece of evidence for this proposition is the event analysis shown in Table 2. The top panel of this table extends the results of Table 1 in Gagnon, Raskin, Remache, and Sack (2011), which reports the changes in key interest rates on the day of the announcement of the first LSAP and subsequent FOMC meetings. I add the changes in the 10-year US Treasury (UST) forward rate and the 10-year future written on the 3-month Eurodollar (ED) rate as well as the effects of the key policy announcements made before the LSAP programme. This table shows that the effect of these initiatives on the ED future was consistently larger than the effect on the UST forward. The cumulative effect of the announcements was to depress the future by 1.24% and the forward by 0.37%.

Term structure models offer another way of examining the effectiveness of policy initiatives. In thus context, their ability to decompose a forward rate into an interest rate expectation and a term premium provides a way of distinguishing the effect of the Fed's forward interest rate guidance on expectations from the effect of its open market operations on term premiums. For example, Gagnon et al. (2011) find that these purchases lowered long yields by reducing term premia rather than expectations of future interest rates. However, they use the Gaussian Term Structure Model (GTSM) of Kim and Wright (2005), which like any GTSM, is handicapped in this task by its failure to allow for the ZLB.

This problem has naturally spurred recent research into shadow rate term structure models (SRTSMs), an endeavour that has yielded important insights into the effect of monetary policy on the bond market. For example, Wu and Xia (2016), Wu and Zhang (2016) and others have suggested that estimates of the shadow policy rate from a SRTSM provide a better indicator of the thrust of monetary policy in a macro-econometric model than the observed policy rate. They argue that the shadow rate shows the rate the authorities would like in the absence of the ZLB, and that they try to implement this using unconventional monetary policies (UMPs).

In focussing on the UST bond market, all of these studies are silent on the effects of such policies on credit spreads and the banking and equity markets. Other studies suggest that the wider stimulative effects were crucial in supporting the economy (Rai (2013), Guidolin, Orlov, and Pedio (2104)). The force of this point is apparent from the behavior of the well-known Treasury-Euro-Dollar (TED) spread. Figure 1 shows these two 3-month rates alongside the 3-month AA non-financial Commercial Paper rate. The Euro-Dollar and Commercial Paper (CP) rates are very close, both exceeding the T-bill rate, due to deposit and default risk respectively. These spreads are both about 25-50bp in normal times, but moved up to between 100-200 basis points (b.p.) as the interbank markets became stressed in 2007 (Afonso, Kovner, and Schoar (2011)). Although T-bill rates came down to around $1\frac{1}{2}\%$ prior to the Lehman bankruptcy, ED and CP rates remained stuck at around 3%. The ED rate then spiked up to 5% and CP rate to 6% in the aftermath of the collapse, while T-bill rates fell back close to zero as the Fed Funds rate was cut. ED and CP rates eased back as the UMPs relieved the stresses in the banking markets, allowing the spreads to normalize.

I build a model of the UST and ED markets to throw light on these developments. The paper begins with an outline of the shadow rate model, and the way that it can be used to model to futures and forward prices. The model assumes that the economy and the shadow policy rate (the rate that would be set in the absence of the lower bound constraint) are Gaussian under the risk-neutral probability measure used for asset pricing (Appendix 1). When the shadow rate is below the lower bound, the SRTSM assumes that the policy rate is set at the lower bound and the authorities use UMPs to support the economy. The policy rate is thus the censored shadow rate: the maximum of the shadow rate and the lower bound and so future policy rates have a truncated normal distribution. Other spot rates, such as Libor, equal the policy rate plus a non-negative spread, that can similarly be modelled by a shadow rate specification. If the spread factors are Gaussian, they also have a truncated normal distribution.

As Appendix 1 explains, the continuous settlement feature of a futures position stemming from its margin deposit means that it has a negligible cost of carry, so that its present value is given by the risk-neutral expectation of the terminal value of the interest rate (or other price) specified in the contract. Since this rate has a truncated normal distribution, this expectation follows immediately from the well-known formula (Greene, (2000)) for the expected value of a truncated normal variable. This gives a simple closed-form for a futures price. So far, shadow rate modelers like Wu and Xia (2016) and Priebsch (2013) have analyzed forward rates, which do not share this convenient property and are much more difficult to represent if rates are close to the lower bound (Appendix 1).

I then follow the multi-market literature (Egorov, Li, and Ng (2011)) and build a joint model of the futures and forward markets to analyze the links between the Treasury bond and Eurodollar futures markets. The approximation of Wu and Xia (2016) is used to model the UST bond forwards. This model extracts five latent factors from of the joint cross-section of futures and forward prices. I find that these two markets are driven by three common factors, allowing me to pin these down more precisely using data for both markets rather than just one. Additional spread factors are used to allow for the differences between these instruments. The model is identified by making the spot (i.e. 3-month) T-bill rate the sum of the common factors and the spot ED rate the sum of these factors plus the spread factors. The risk-neutral dynamics of the T-bill rate are also restricted in a way that ensures that shocks to the spread factors do not affect prices in the UST bond market immediately, only with a lag, so that they are not spanned by this market.

The econometric estimates show that the common factors are similar to those revealed by bond market studies, reflecting level, slope and curvature effects. The novelty lies in the spread factors, which model the TED spread and the differences between the forward and futures prices. The TED spread is frequently used as an indicator of default risk in the economy (Cornett, McNutt, Strahan, and Tehranian (2011)), but I find that it is necessary to use two factors to represent it, reflecting shocks with different durations. The first component is a spread-slope factor, which reflects credit risk shocks that are thought to be temporary. This increases short spreads relative to long maturity spreads. The second is a spread-level factor that reflects more persistent credit shocks that tend to increase all spread maturities.

A vector autoregression model is used to analyze the dynamic behavior of these factor dynamics under the time-series (or real-world) probability measure. The spread shocks do not have any immediate effect on the T-bill rate, but tend to depress this rate over the next two years, indicating a temporary relaxation of monetary policy. These spread shocks cause the T-bill and ED rates to behave differently in the short-run, but the model suggests that these differential effects largely disappear after two or three years. Indeed, expectations of T-bill and ED rates over longer horizons behave in a remarkably similar way. This means that differences in the risk-neutral dynamics embedded in longer-maturity future and forward prices are almost entirely due to differences in the behavior of their respective term premia rather than expectations. This immediately suggests that the marked differential responses to the policy announcements shown in Table 2, amounting to 87bp in total, are almost entirely due to their differential effects on term premia rather than their effect on long-term expectations.

The term structure model throws more light on the behavior of the markets over this period. The time-series model can be used to obtain a forecast of the T-bill and ED rates at (say) the 10-year horizon. Using these to represent market expectations and subtracting them from the observed 10-year forwards and futures gives residuals that can be interpreted as term premiums. The bottom panel of Table 2 uses this to analyze the price movements seen from mid-August to mid-March, which encompasses the events reported in the top panel. This suggests that the expected value of the two rates both fell by about $\frac{3}{4}$% over this period. This was reinforced by a similar fall in the term premium on the future, but it seems the term premium on the UST forward increased, perhaps because of uncertainties about the long run effects of expansionary monetary policies on inflation. Whatever the reason for this difference, prices and, it seems, term premia fell back by about $1\frac{1}{4}\%$ more in the future than in the forward market over this period. Putting this together with the difference shown in the top panel, suggests that most of this difference can be explained the effect of policy announcements, leaving the remaining 40bp to be explained by unidentified policy and other influences.

Tables 9 and 10, discussed in Appendix 2, provide a month by month analysis of changes in the respective expectations and term premia over the first few months following the Lehman bankruptcy and correlates these with the key policy announcements analyzed in Table 2. This evidence points consistently to the remarkable success that the policy initiatives following the Lehman bankruptcy had in supporting banking markets and the Eurodollar futures prices used in pricing credit contacts. Figure 7 shows the 10-year expected future spot rates and term premiums for the whole estimation period. This shows that the two premia normally behave in a similar way and that the post-Lehman episode was exceptional in this respect.

The paper is set out as follows. The next section derives the closed-form for the futures price, and shows that it has the same functional form as the approximation for the forward rate used in several recent studies. These formulae form the backbone of the joint model developed in section 3. The empirical results are discussed in section 4. The final section provides some concluding remarks, with a possible agenda for future research on multi-market asset-pricing models for monetary policy analysis.

2 Pricing interest rate futures

This section derives the closed-form for the futures price and shows that it resembles the approximation obtained by Wu and Xia (2016) for the forward rate. Let s_t be the shadow spot rate, $\underline{r}$ the lower bound and assume that the *Black* spot rate is the censored shadow rate $r_t = \max(s_t,\underline{r}) = \underline{r} + \max(s_t - \underline{r}, 0)$, (Black (1995)). As Appendix 1 explains, the futures price $g_{n,t}$ at time t for the spot rate in period $t+n$ is the risk-neutral expectation of the future spot rate r_{t+n} (Cox, Ingersoll, and Ross (1981)). In the Gaussian model, this is the expectation of the censored shadow future spot rate s_{t+n}:

$$\begin{aligned} g_{n,t} &= E_t\left[r_{t+n}\right] \\ &= \underline{r} + E_t\left[\max(s_{t+n} - \underline{r}, 0)\right] \end{aligned} \tag{1}$$

In this paper, E_t denotes the risk-neutral expectation conditional on information at time t. I assume that the shadow spot rate is Gaussian, which means that the spot rate itself has a truncated normal distribution. The forward rate $f_{n,t}$ for period $t+n$ corresponding to the future $g_{n,t}$ is the logarithm of the n-period discount bond price less that of the $(n+1)$ period price. The relationship between the forward rate and the distribution of shadow rates is less straightforward and discussion of this is deferred to section 2.3 below.

2.1 Shadow future and forward rates

Suppose that the shadow spot rate is driven by K homoscedastic latent factors that are collected in the vector $\mathbf{z}_t$.

$$s_t = \delta_0 + \boldsymbol{\delta}_1' \mathbf{z}_t \tag{2}$$

Also suppose that the dynamics of these factors under the risk-neutral and time-series measures $\mathcal{Q}$ and $\mathcal{P}$ are described respectively by the models:

$$\mathbf{z}_t = \mathbf{k} + \mathbf{K}\mathbf{z}_{t-1} + \mathbf{w}_t, \tag{3}$$

$$= \mathbf{k}^P + \mathbf{K}^P\mathbf{z}_{t-1} + \mathbf{w}_t^P, \tag{4}$$

where: $\mathbf{w}_t, \mathbf{w}_t^P \sim N(\mathbf{0}, \mathbf{\Omega})$ under the respective measures. Absent arbitrage, it follows that the term structures of shadow futures and forward rates are both linear in these factors and Gaussian. The risk-neutral expectation $p_{n,t} = E_t[s_{t+n}]$ of the shadow rate at $t+n$, which we may regard as a shadow $futures$ price, is given by setting the error term in (3) to zero, solving forward and substituting into (2).

$$p_{n,t} = E_t[s_{t+n}] = a_n + \mathbf{b}_n\mathbf{z}_t$$

where the coefficients are determined by standard recursion relations, with the solution:

$$\mathbf{b}_n = \boldsymbol{\delta}_1'\mathbf{K}^n\mathbf{z}_t, \tag{5}$$

$$a_n = \boldsymbol{\delta}_0 + \boldsymbol{\delta}_1'(\mathbf{I} - \mathbf{K}^{n+1})(\mathbf{I} - \mathbf{K})^{-1}\mathbf{k}. \tag{6}$$

Thus:

$$s_{t+n} = p_{n,t} + e_{n,t} \tag{7}$$

and:

$$P(s_{t+n}|p_{n,t}, \sigma_n^2) = \phi(e_{n,t}/\sigma_n), \tag{8}$$

where: $P(.|.)$ is the risk-neutral probability density, $\phi(.)$ is the standard normal density function and the variance is:

$$\sigma_n^2 = \sum\nolimits_{i=0}^{n-1} (\mathbf{K}^i)' \mathbf{\Omega} \mathbf{K}^i. \tag{9}$$

Similarly, it can be shown (Cochrane and Piazzesi (2010), Wu and Xia (2016)) that the shadow *forward* rate $q_{n,t}$ for period $t+n$ equals the shadow future or risk-neutral expectation of the future spot rate, less a convexity term due to the use of a logarithmic bond price transform:

$$q_{n,t} = p_{n,t} - \frac{1}{2}\sigma_n^2. \tag{10}$$

This Gaussian model is standard in research on the SRTSM. However, the spread between ED and T-bill rates should remain non-negative and the Gaussian model does not ensure this. This could be assured by modeling them using the square root specification of Cox, Ingersoll, and Ross (1985) to keep the spread factors non-negative, while preserving the linear characteristics of the shadow rate model. Two-market models of this type are discussed by Egorov, Li, and Ng (2011). However, we found that in practice negative spreads were not a problem, only occurring in 2014, when the shadow rates are negative, with negligible effect on the observed rates.

2.2 The Black futures price

Adapting the standard argument due to Black (1995), the *future* for any period $t+n$ can be seen as the shadow future plus an option to hold cash instead of investing at the future spot rate when this is negative. To value this we substitute (7) into (1):

$$g_{n,t} = \underline{\mathrm{r}} + E_t \left[\max(p_{n,t} - \underline{\mathrm{r}} + e_n, 0) \right] \tag{11}$$

and evaluate this expectation using (8) to get the futures price:

$$\begin{aligned} g_{n,t} &= h(p_{n,t}, \sigma_n, \underline{\mathrm{r}}) \\ &= \underline{\mathrm{r}} + \int_{-(p_{n,t}-\underline{\mathrm{r}})}^{\infty} eP(e, \sigma_n^2)de + \int_{-(p_{n,t}-\underline{\mathrm{r}})}^{\infty} P(e, \sigma_n^2)de \\ &= \sigma_n \phi(x) + p_{n,t}\Phi[x] + \underline{\mathrm{r}}(1 - \Phi[x]). \end{aligned} \tag{12}$$

where: $x = (p_{n,t} - \underline{\mathrm{r}})/\sigma_n$ and $\Phi[x]$ is the cumulative normal density function. The value of the Black option to hold cash follows by subtracting $p_{n,t}$. It follows that $d(\sigma_n \phi(x) + p_{n,t}\Phi[x])/dx = 0$, and hence:

$$\frac{dh(p_{n,t}, \sigma_n, \underline{\mathrm{r}})}{dp_{n,t}} = \Phi[x] \tag{13}$$

Similarly, the variance of any future can be computed using Theorem 20.3 of Greene (2000):

$$V_t\left[r_{t+n}\right] = \sigma_n^2\{\Phi[x] - 2x\phi(x) - \phi(x)^2 + x^2(1 - \Phi[x]) + x\phi(x)/\Phi[x]\}.$$

The function $h(p_{n,t}, \sigma_n, \underline{\mathrm{r}})$ maps the shadow futures price into the Black futures price. This function is identical to the function used to map the shadow forward rate $q_{n,t}$ (10) into the approximation of the forward rate by Wu and Xia (2016).[1] Their model approximates the corresponding *forward* rate $f_{n,t}$ by substituting (10) into

[1] This is the discrete time analogue of the model proposed by Krippner (2013). This represents the forward rate as the censored value of the future shadow spot rate under the shadow rather than the true forward measure. Although this makes little difference at short maturities, the bias is likely to be significant for longer maturities. The simulations of Priebsch (2013) suggest that the approximation could bias the 10-year yield upwards by around 4 basis points over the period since the financial crisis, with the bias being larger for the 10-year forward rate.

this function:

$$f_{n,t} = h(q_{n,t}, \sigma_n, \underline{\mathrm{r}}). \tag{14}$$

3 A joint model of the futures and forward markets

It is unfortunate that the forward rates in the shadow rate model can only be represented as an approximation. Nevertheless, the tractability of this approach means that it has been followed by Christiansen and Rudebusch (2013), Bauer and Rudebusch (2015), Wu and Xia (2016), Coroneo and Pastorello (2017) and many others. I follow these authors in using this approximation to model forward rates in a joint model of the futures and forward markets. I also follow them in using the Extended Kalman filter (EKF) to handle the latent variables $\mathbf{z}_t$. The research strategy was to begin by fitting futures data using (12), then adding conformable forward data and trying to fit them using (14). These two-market models follow the literature on the international bond markets in allowing for common and market-specific factors (Egorov, Li, and Ng (2011)).

3.1 The model of the risk-neutral dynamics in the cross-section

In order to identify the model of the cross-section, I adopted a variant of the identification scheme originally proposed by Dai and Singleton (2000) and used in multi-market models by Egorov, Li, and Ng (2011). They assume that the factors are mean-independent and mean-zero under $\mathcal{Q}$, so that $\mathbf{\Omega} = \mathbf{I}_J$ and $\mathbf{k} = \mathbf{0}_J$ and specify a lower triangular structure for the response matrix $\mathbf{K}$. The spot rate factor loadings are unrestricted in their scheme. My scheme is equivalent, but designed to allow for restrictions on these spot factor loadings. This uses a diagonal form for $\mathbf{\Omega}$, but with elements that are not set to unity. This allows me to set the spot ED rate factor loadings to unity instead. The spot T-bill rate loadings remain unrestricted in the unrestricted model (M1).

$$s_t^g = \delta_0^g + \mathbf{j}^{g\prime}\mathbf{z}_t, \tag{15}$$

$$s_t^f = \delta_0^f + \boldsymbol{\delta}^{f\prime}\mathbf{z}_t,$$

where, in M1:

$$\mathbf{j}^{g\prime} = [1\ 1\ 1\ 1\ ...\ \ 1] \text{ and } \boldsymbol{\delta}^{f\prime} = [\delta_1\ \ \delta_2\ \ \delta_3\ \ \delta_4\ ...\ \ \delta_K]. \tag{16}$$

This is an unrestricted $K-$factor model in which all factors are common to both markets. Preliminary checks suggested a model (labelled M1) with $K = 5$. However, as in the literature on international bond markets, some of these could be idiosyncratic, i.e. affect just one market. I followed the term structure literature in assuming that three factors were sufficient to model the UST bond market, but found that that two additional factors were necessary to handle the differences between the two markets[2]. These include bank default risk, which is clearly reflected in the TED spread. They also include differential tax & liquidity characteristics as well as regulatory requirements. Jermann (2017) and others have argued for example that forward bond positions need to be backed by regulatory capital, putting a price wedge between them and derivatives like interest rate swaps and futures. These developments may also have weakened arbitrage activity between the two markets. Finally, the use of an approximation to handle the forward prices may affect the spreads between these and the futures prices.

Econometric tests reported in Table 3 show that the model can be restricted by

[2] There are four possible ways of introducing two additional factors, but this is the one that worked best empirically. It is also the only one which retains the standard three factor specification for the Treasury forwards.

assuming that the spot ED and T-bill rates load equally on the first three factors, so that the loadings on the last two model the difference between the two markets . This gives the spot rate loadings for the restricted model (M2):

$$\mathbf{j}^{g\prime} = [1\ 1\ 1\ 1\ 1] \text{ and } \boldsymbol{\delta}^{f\prime} = \mathbf{j}^{f\prime} = [1\ 1\ 1\ 0\ 0]. \tag{17}$$

Thus, by subtraction, the loadings of the TED spread (the difference between the spot ED and T-bill rates) are [0 0 0 1 1] and so the last two factors can be interpreted as TED spread or 'spread' factors for short.

Since the risk-neutral response matrix $\mathbf{K}$ is lower triangular, this means that none of the UST forwards load on the last two factors.[3] Shocks to these factors have no contemporaneous effect on that market and in that sense are not spanned. However, because the time-series factor dynamics represented in (4) are not restricted, they can have a lagged effect and are known as 'unspanned' (Joslin, Priebsch, and Singleton (2014)) or 'hidden' (Duffee (2012)) factors in the affine term structure literature . They are of course spanned by the futures market.

Finally, I normalize δ_0^g and δ_0^f by setting them to zero and compensate for this by relaxing the restriction $\mathbf{k} = \mathbf{0}_J$. Specifically, I set $\mathbf{k} = [k_1\ 0\ 0\ k_4\ 0]$, which means that the first common and first spread factors may not be mean-zero under $\mathcal{Q}$. Empirically, k_4 was not significant and is excluded from model M2. The restrictions in (17) can be tested by comparing the fit of this model with that of M1 (Table 3). Given their

[3] That is because if $\mathbf{K}$ is lower triangular (or has real Jordan form, as in the Joslin, Singleton, and Zhu (2011) normalization) then $\mathbf{K}^n$, has the same structure. So with $\mathbf{j}^{f\prime} = [1\ 1\ 1\ 0\ 0]$, the longer-term loadings on the spread factors in $\mathbf{j}^{f\prime}\mathbf{K}^n$ are all zero.

respective structures, the shadow prices are:

$$p_{n,t} = E_t[s^g_{t\ t+n}] = a^g_n + \mathbf{b}^g_n \mathbf{z}_t \tag{18}$$

$$q_{n,t} = E_t[s^f_{t\ t+n}] = a^f_n + \mathbf{b}^f_n \mathbf{z}_t \tag{19}$$

and the coefficients are:

$$\mathbf{b}^g_n = \mathbf{j}^{g\prime} \mathbf{K}^n \mathbf{z}_t; \tag{20}$$

$$\mathbf{b}^f_n = \boldsymbol{\delta}^{f\prime} \mathbf{K}^n \mathbf{z}_t \tag{21}$$

$$a^g_n = \mathbf{j}^{g\prime}(\mathbf{I} - \mathbf{K}^{n+1})(\mathbf{I} - \mathbf{K})^{-1}\mathbf{k}, \tag{22}$$

$$a^f_n = \boldsymbol{\delta}^{f\prime}(\mathbf{I} - \mathbf{K}^{n+1})(\mathbf{I} - \mathbf{K})^{-1}\mathbf{k}. \tag{23}$$

Allowing for additive measurement and other errors, which can be different for forwards and futures, the data are modelled as:

$$g_{n,t} = h(p_{n,t}, \sigma_n, \underline{\mathrm{r}}_g) + u_{n,t} \tag{24}$$

$$f_{n,t} = h(q_{n,t}, \sigma_n, \underline{\mathrm{r}}_f) + v_{n,t}, \tag{25}$$

$$u_{n,t} \sim N(0, \zeta^2); \quad v_{n,t} \sim N(0, \xi^2). \tag{26}$$

This error specification means that the likelihood function gives equal weight to each observation. This specification was also found to be acceptable when tested against an unrestricted diagonal error covariance specification. However, in order to allow the measurement error variances to change after the Lehman crisis, separate variances were allowed for in the pre- and post-September 2008 sub-periods (denoted respectively as ζ_1^2, ξ_1^2 and ζ_2^2, ξ_2^2). The factor prediction error variances were also allowed to shift (from $\boldsymbol{\Omega}_1 = diag[\delta_{11}\ \delta_{12}\ \delta_{13}\ \delta_{14}\ \delta_{15}]^2$ to $\boldsymbol{\Omega}_2 = diag[\delta_{21}\ \delta_{22}\ \delta_{23}\ \delta_{24}\ \delta_{25}]^2$ in that month.

3.2 The model of the time-series dynamics

Recall that the dynamics are represented under the time-series probability measure $\mathcal{P}$ by (4). This is a first-order vector autoregression (VAR), which, in contrast to (3), can be unrestricted. Importantly, any non-zero elements appearing in the top right 3×2 block of $\mathbf{K}^P$ mean that shocks to the spread factors have dynamic effects on the common factors and hence the evolution of prices in the UST market. However, the unrestricted VAR model employs a large number of parameters, many of which are normally found to be insignificant. In order to avoid these problems, while preserving the identification restrictions on the risk-neutral parameters, it is usual to specify the model in terms of differences between the two sets of parameters:

$$\mathbf{k}^P = \mathbf{k} + \mathbf{l}; \quad \mathbf{K}^P = \mathbf{K} + \mathbf{L}. \tag{27}$$

and then test the significance of the differences $\mathbf{l}$ and $\mathbf{L}$, eliminating any that are insignificant. With the restrictions on the spot rate loadings in place, this makes the resulting model M3 a special case of both M2 and M1.

4 The empirical model

4.1 Data sources

Rich data sets are available for various US interest rate futures. Given my interest in the wider financial impact of unconventional monetary policies, I use the contract written on the 3-month (90 day) ED rate, which is traded on both the Chicago

Mercantile Exchange and NYSE-LIFFE.[4]. A useful description of this market is available in section 6.3 of Hull (2009), while an analysis of trading costs in this market is provided by Locke and Venkatesh (1997). These futures are available for settlement on the third Wednesday of the end-quarter months March, June, September and December and have a maximum maturity of ten years. Thus at any one time, 39 or 40 quarterly contracts are available.

I obtained futures prices for NYSE-LIFFE-ICE contracts from Datastream.[5] The shortest maturity is for the 3-month-ahead 3-month ED rate and the longest is the 116 month ahead rate, taken as a proxy for the 120-month-ahead rate (on the assumption of a flat yield curve in the 10-year area). Intra-quarter monthly maturity prices are interpolated linearly to give a data set with 120 monthly maturities. I used a matching mid-month time-series structure, taking observations were for the close on the 16th day of the month, or the following working day. These data were augmented by observations on the 3-month ED rate on the same dates, also from Datastream. The estimation period began in January 1999 when the 10-year contract stated trading[6] and ended in June 2018. The various models were fitted to the 3-; 12-; 24-; 36-; 48-; 60-; 84- and 120-month-ahead prices, as well as the 3-month ED interest rate, all expressed as quarterly interest rates in decimal fractions. A matching dataset of 3-month forward rates for the same dates and maturities used for the futures observations was calculated using the Gürkaynak, Sack, and Wright (2007) data set, augmented by matching observations on the 3-month T-bill rate from Datastream. The sample summary statistics are reported in Table 4.

[4] These contracts migrated in September 2014 to the Intercontinental Commodity Exchange (ICE).

[5] As Hull (1997) explains, the contract prices are expressed as annual discounts to 100. So for example, a contract price of 96 represents an annual interest rate of 4 percent and a quarterly decimal fractional rate of 0.01. These data were used to construct the implied rates $g_{n,t}$ used in the empirical model.

[6] The 3-year Euro-Dollar contract started trading in 1988 and the 5-year in 1993.

These data reflect the spreads between ED and T-bill rates and are complementary in terms of coverage, liquidity and heterogeneity. Although the near-end futures markets are very heavily traded, markets in short maturity UST bonds are not very active. This is essentially because, as their maturity reduces, long-term investment institutions sell their holdings on to banks and money market funds that then tend to hold them until maturity. Also, government bonds are heterogeneous in terms of maturity date, callability, coupon and tax exposure, while futures are designed to be homogeneous, to enhance liquidity by focusing trade on a few standard instruments. Last but not least, discount bond prices (and implied forwards) have to be backed out from coupon bond yields by bootstrap methods or by fitting a polynomial or similar function. In practice, bootstrap methods (as used by Le (2013) for example) tend to make estimates of long-term forward rates very volatile, while curve fitting methods (as used, for example, by Gürkaynak, Sack, and Wright (2007)) are problematic for short maturities.

4.2 The econometric estimates

Recall that model M2 specializes M1 by adopting the restrictions (17), so that the three common factors have an equal impact on prices in both markets and the two spread factors only affect the UST market with a lag. M3 further restricts the time-series dynamics by sequential elimination of insignificant parameters in $\mathbf{l}$ and $\mathbf{L}$ in (27). Since M3 is nested within M2, which is nested within M1, these restrictions can be tested using standard loglikelihood ratio tests. The results are reported in Table 3 and show that model M3 is an acceptable simplification. M0 is the Gaussian model M1, which may be considered the limit of M2 as $\underline{r}_g$ and $\underline{r}_f$ become very large negative numbers. All of these models allow the variance parameters to change between the pre- and post September 2008 sub-periods.

This section reports in detail the results obtained using model M3.[7] The data summary statistics are reported in Table 4 and the residual summary statistics in Table 6. Figure 2 compares the prices for representative maturities with the model predictions of the previous month. The fit is generally tight, although the model only explains a third of the jump in the ED rate seen (top left panel) in the four weeks following the Lehman bankruptcy in September 2008. It also over predicts the 10-year forward in this period (bottom right). Appendix 2 shows in detail how the model represents this extraordinary episode and the way that the subsequent policy initiatives appear to have affected the two markets. Figure 5 depicts the shadow spot rates. Recall that the shadow spot T-bill rate is the sum of the first three factors, while the shadow ED rate is the sum of all five factors. Deposit risk (and the effect of convexity on the forward rate shown in (10)) normally keeps the shadow ED rate above the T-bill rate.[8]

4.2.1 Key parameter estimates

The parameters are reported in Table 5. The variance parameters are shown in the final column. The $\delta-$values indicate a fall in the volatility of the first factor over this break; an increase in the volatility of factors 2 and 5 and a very large increase in the volatility of factors 3 and 4. The increased volatility of the two spread factors over the break in September 2008 clearly reflects the scale of the shocks impacting the banking markets in the second sub-period. The $\zeta-$ and $\xi-$values indicate that the variance of the measurement errors for the futures increases, while that for the forwards falls.

The behavior of the futures and forwards follow from the dynamics of the shadow

[7] The results for the other models are available upon request.

[8] Although the shadow ED rate falls below the shadow T-bill rate in 2014, these rates are poorly identified since they are negative and have a negligible effect on the predicted rates.

rates and the way in which the Black model maps these into predicted rates. The relationship between the risk-neutral and time-series dynamics depend upon the risk-adjustment parameters in $\mathbf{l}$ and $\mathbf{L}$ in (27). As usual, many of these were insignificant, making them very sparse in M3:

$$\mathbf{l} = \begin{bmatrix} (-) & 1.007 \times 10^{-5} & 8.855 \times 10^{-3} & (-) & 6.149 \times 10^{-5} \end{bmatrix} \tag{28}$$

$$\mathbf{L} = \begin{bmatrix} (-) & 0.1109 & 6.006{\times}10^{-3} & (-) & (-) \\ (-) & (-) & (-) & 8.534{\times}10^{-3} & (-) \\ (-) & (-) & (-) & (-) & -6.703{\times}10^{-2} \\ 0 & (-) & (-) & (-) & (-) \\ 0 & (-) & 6.209{\times}10^{-3} & (-) & -4.135{\times}10^{-2} \end{bmatrix}$$

Given these parameters the risk-neutral the time-series dynamic parameters can be arranged using (**??**) and (28) as:

$$\mathbf{K} = \begin{bmatrix} 0.9999 & (-) & (-) & (-) & (-) \\ -5.954{\times}10^{-3} & 0.9135 & (-) & (-) & (-) \\ 0 & -0.8845 & 0.9697 & (-) & (-) \\ 0 & -3.600{\times}10^{-2} & -4.697{\times}10^{-3} & 0.9894 & (-) \\ -1.360{\times}10^{-3} & -1.560{\times}10^{-2} & -3.343{\times}10^{-4} & -2.655{\times}10^{-2} & 0.9915 \end{bmatrix} \tag{29}$$

$$\mathbf{K}^P = \begin{bmatrix} 0.9999 & 0.1109 & 6.006{\times}10^{-3} & (-) & (-) \\ -5.954{\times}10^{-3} & 0.9135 & (-) & 8.534{\times}10^{-3} & (-) \\ 0 & -0.8845 & 0.9697 & (-) & -6.703{\times}10^{-2} \\ 0 & -3.600{\times}10^{-2} & -4.697{\times}10^{-3} & 0.9894 & (-) \\ -1.360{\times}10^{-3} & -1.560{\times}10^{-2} & 5.874{\times}10^{-3} & -2.655{\times}10^{-2} & 0.9501 \end{bmatrix}$$

The respective eigenvalues are: $eig(\mathbf{K})$=[0.9999 0.9915 0.9894 0.9697 0.9136] and

$eig(\mathbf{K}^P)$=[0.9999 0.9947 0.9513 0.9384 $\pm$ 0.0293i]. As we would expect from the affine term structure literature, the first factor exhibits a unit root under the risk-neutral measure and (in model M3 the time-series measure $\mathcal{P}$), while the other two common factors are mean-reverting. Figure 3 shows the estimates of the latent factors.

4.2.2 Model dynamics

These dynamic characteristics are clearly reflected in Figure 4, which reports the factor loadings. The risk-neutral loadings in the top panel are calculated using (20) and (21) show the impact of unit changes in each factor on shadow futures and forward rates of different maturities (which are the changes in the risk-neutral expectations of the future shadow rates). The first is clearly a 'level' factor since it has a similar effect on all rates, reflecting inflation and other persistent shocks. The second is a 'slope' factor since its effect declines gradually with maturity. The third is a curvature factor that depresses medium-term relative to both short and long term rates, reflecting monetary policy and other temporary effects. Panel (a) of Table 8, discussed in Appendix 2 shows how these behaved during the Lehman crisis. To get the time-series loadings shown in the central panel I repeat this exercise using $\mathbf{K}^P$ in place of $\mathbf{K}$ in (20) and (21). These loadings are known as impulse response functions (IRFs) in the macro-econometric literature, since they show the effect of each factor on the expected values of the future shadow spot interest rates under the time-series probability measure. The lower panel reports the differences between the risk-neutral and time-series expectations, which show the impact of unit changes in each factor on the respective term premiums.

The novelty here lies in the two spread factors, which model the difference between prices in the two markets. Panel (b) of Table 8 shows how these behaved during the Lehman crisis. I found that it was necessary to use two factors to model the spreads, reflecting shocks with different dynamic effects. The first type of shock is picked up by the fourth factor, and models the effect of shocks that are expected to be transitory. The risk-neutral factor loadings in Figure 4 show that this behaves like a spread-slope factor, increasing short spreads relative to the long spreads. It increases sharply in the three months following the Lehman bankruptcy but then falls back. This factor leads to an inversion in the spread curve immediately following the Lehman bankruptcy, similar to the inversion seen in US bank Credit Default Swap curves at the time. In contrast, the fifth factor reflects semi-permanent shocks and thus increases all spreads, particularly at the short end, behaving more like a level factor. The bottom panel of Figure 3 shows that it indicates an increase in risk during the dot-com boom in the final years of the last millennium, and a subsequent decline during the dot-com bust. It moved back up to another peak in the month following the Lehman default, before falling back sharply over the following three months.

The central panel of Figure 4, which shows the time-series responses, suggest that shocks to the spread factors have the effect of depressing T-bill rates in the short-term, suggesting that credit shocks tend to tighten monetary policy temporarily. However, the spillover effects of these spread shocks fade after two or three years, and they have little effect on the longer-run time-series dynamics of either rate. Indeed, the longer-run time-series responses of these two rates to all five factors are remarkably similar. In other words, the differential price responses in the medium- to long-term maturities revealed in the top panel are almost entirely due to the differential behavior of the respective term premia shown in the bottom panel, and in particular the differential effects of the spread factors. This is clear from table 7, which shows the three sets of 10-year loadings.

4.2.3 The impact of the UMPs

With short-term rates moving close to the lower bound following the Lehman bankruptcy, policy initiatives in these markets were largely aimed at moving prices in the longer-term maturities. The model allows me to decompose these into movements in expectations and term premia. Since their effects on longer-term expectations in the ED and UST markets are similar, differential policy and other effects are revealed more clearly by the behavior of the respective term premia than the behavior of prices themselves.

The effects of these movements on the predicted Black spot rates and term premia are non-linear, depending upon the initial state of the system. Figure 6 shows the estimates of the coefficients (13), that show the effect of changes in the shadow prices on the Black predictions of the corresponding prices at the 3-month 1-, 3-, 5- and 10-year horizons. These coefficients represent the fraction of the probability mass of the future ED and UST rate that is in the non-negative range under each measure. Figure 7 decomposes the estimated 10-year future and forward into the respective expected future spot rates and term premiums. I do this by first calculating the time-series expectations shown in the figure using $\mathbf{K}^P$ in place of $\mathbf{K}$ in (20) to (24). The term premiums follow by subtracting these from the respective model estimates (which, recall, use $\mathbf{K}$ in these equations).

The first factor is persistent under both measures and induces a strong downward tendency in the expected rates, making the term premiums stationary. As noted, the 10-year T-bill and ED rate expectations behave in a similar way, allowing the term premia to reveal policy and other differential effects. The estimates of the term premium in the 10-year UST market are similar to those found by researchers using the Gaussian model and a such, consistent with the conventional view that movements in the premium tend to offset the effect of monetary policy on expectations. The UMPs that followed the move to the lower bound in 2008 could be seen as an attempt by the authorities to restrain this tendency. Nevertheless, reflecting the results shown in the bottom panel of Table 2, Figure 7 shows a marked increase in the UST forward premium over this period. However, the increase in the ED premium was much more subdued. This again suggests that, as intended, UMPs were more effective in reducing risk premia in the ED futures market (and perhaps other private credit markets) than in the UST market. Further evidence of this is provided by Appendix 2, which keys in these monthly premium and price movements with the main policy initiatives as they were announced. In addition to the effect of the LSAPs analyzed by Gagnon et al. (2011) and others, the effect the TARP and the extension of direct support by the Federal Reserve to the two surviving investment banks in the month following the Lehman default stands out.

5 Conclusion

This paper follows Egorov, Li, and Ng (2011) and others in developing a multi-market term structure model. These models use prices from several markets to identify and inform the common pricing factors and their dynamics with greater precision than can be obtained using data for a single market. In particular, the addition of financial futures, which unlike UST bonds, are heavily traded at the short end, should also help inform the short-term factor dynamics.

The use of ED futures also allows us to identify the effect of credit and monetary policy shocks on spreads of different maturities. The model suggests that we need two spread factors to account for the effect of credit shocks, reflecting their differential effects on long- and short-term default risks. In particular, these two factors move in opposite directions in the months following the initial Lehman shock (Table 8), an effect that is obscured by the use of the simple TED spread indicator of default risk.

It would be interesting to add these two indicators to a macroeconometric model such as that used by Wu and Xia (2016) to analyze the effect of shadow policy rates on the economy.

The model suggests that, as intended, unconventional monetary policies that followed the crisis had more traction in ED than in the UST market. Although several researchers have analyzed the effect of these policies on the equity market and other financial variables, as far as I am aware, none have looked at the effect on corporate credit spreads. This suggests that the model should be extended to incorporate term structures of Credit Default Swap prices and the like. It would be also be interesting to model the behavior of futures written on the Federal Funds rate.

Of course, the US is not the only country that has experienced policy rates at the effective lower bound in recent years. Although futures markets for other countries are not as rich or as liquid as they are for the US, they surely warrant investigation. Quarterly maturity data for the Short Sterling futures contract, written on the 3-month London inter-bank offer rate (Libor) have been available up to a 3-year maturity since 1987, and with a 5-year maturity since 1999. Quarterly maturity data for the 3-month Euro inter-bank offer rate (Euribor) rate has been available since the Euro currency was launched in 2000 and there is a similar data series for the Tokyo inter-bank offer rate (Tibor). These datasets could be modelled jointly with the longer-maturity data government bond forwards. These data offer potentially important insights into the interaction between credit and policy shocks during a financial crisis.

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Key findings

The Gaussian shadow rate term structure specification gives a convenient closed-form for a futures price, while bond yields and forward rates need to be approximated.

The Treasury–Eurodollar (TED) spread conventionally used as an indicator of default risk in the economy should be split into two components, reflecting transitory and semi-permanent risk factors.

We show that policy announcements in the aftermath of the Lehman default had a bigger impact in the Eurodollar than in the Treasury market. Moreover, the authorities prevented a rise in the term premium in the Eurodollar market offsetting the fall in 10-year interest rate expectations as policy rates were moved to the lower bound, but not in the Treasury bond market, where 10-year yields remained broadly unchanged.

6 Appendix 1: Pricing futures and forward contracts

Asset pricing theory is based on the assumption that prices eliminate arbitrage opportunities (Harrison and Kreps (1979)). This means that allowing for risk premia by using the risk-free probability measure, all assets are expected to appreciate in line with their cost of carry, which is the risk free rate, available on an asset like a money market fund (MMF), less its dividend or coupon income yield. This risk neutral measure adjusts the probabilities given to each state when calculating risk-neutral expectations to allow for risk aversion.

Futures and forward contracts are similar in that they are both written at present time t on the difference between the value of an underlying instrument at a specified future date $t+n$, $n > 0$ and its present value. However, the institutional arrangements of these markets and in particular their settlement procedures differ in a way that can cause their values to differ if interest rates are stochastic. Specifically, a futures contract is typically taken out with a futures trading exchange. The exchange requires an initial margin deposit to help protect it from client default. Then, the client's position is 'marked to market' every day, and the margin account is credited or debited with any gains or losses. The margin is small relative to the size of the contract/ It has to be replenished if losses cause it to fall significantly, while gains can be withdrawn. Thus, the contract is settled continuously.

A future is priced on the assumption that capital gains and losses are effectively held in a MMF. Thus its cost of carry is zero and its price at time t is the risk-neutral expectation of prices at all future times $t+n$, including its terminal value, as shown in equation (1). A rigorous proof is provided by Cox, Ingersoll, and Ross (1991)

In contrast, a forward contract is an agreement between two parties at time t to buy or sell an asset for a specified price at time $t+n$. The contract is typically held until it expires, when settlement takes place. The arbitrage-free price of an interest rate forward follows from the relationship between interest rates and bond prices. Specifically, an investor can take a forward position directly by agreeing at time t to buy a one period bond at time $t+n$ in advance at the forward price $F_{n,t} = \exp[-f_{n,t}]$, where $f_{n,t}$ is the forward rate, or take this position indirectly by buying an $(n+1)$ period zero-coupon bond for $D_{n+1,t}$ and selling an n period bond for $D_{n,t}$. Equating these known costs gives the standard relationship:

$$\exp[-f_{n,t}] = F_{1,t+n} = D_{n+1,t}/D_{n,t} \Longleftrightarrow f_{n,t} = \ln D_{n,t} - \ln D_{n+1,t}. \qquad (30)$$

Thus the problem of valuing forward interest rate contract is the same as the problem of valuing zero-coupon bonds.

The problem in valuing a forward is that in contrast to the equivalent future, which is settled on a continuous basis, this contract is settled when the contract matures. Similarly, a zero-coupon bond does not pay anything until maturity. The cost of carry of both instruments is thus the safe rate, which is normally positive, and terminal values must be discounted to obtain their present values. For example, the terminal value of a forward written on $r_{1,t+n}$ is $F_{1,t+n} = D_{1,t+n} = \exp[-r_{1,t+n}]$ and its present value is:

$$D_{n+1,t} = D_{n,t} F_{1,t+n} \tag{31}$$

Since this has the same payoff as the $(n + 1)$ period bond, they must have the same present value in an arbitrage-free market. The Gaussian model, being arbirtrage-free, does satisfy this relationship. However, as Krippner (2013) and others explain, the problem of developing a shadow rate pricing model that satisfies this relationship becomes intractable with more than one or two factors.

Nevertheless, several researchers have proposed approximations that give a valuation formula (14) closely resembling (12). For example, Wu and Xia (2016) use the arbitrage-free bond price formula:

$$D_{n,t} = e^{-r_t} E_t[e^{-\Sigma_{j=1}^{n-1} r_{t+j}}] \tag{32}$$

and then approximate this as:

$$\ln D_{n,t} \simeq E_t[-\Sigma_{j=0}^{n-1} r_{t+j}] - \frac{1}{2} Var[-\Sigma_{j=1}^{n-1} r_{t+j}] \tag{33}$$

The Gaussian model makes the bond price a lognormal variable and this formula follows from the expected value of a lognormal variable and holds exactly in that case. Substituting into (30):

$$
\begin{aligned}
f_{n,t} &= \ln D_{n,t} - \ln D_{n+1,t} \\
&\simeq \underbrace{E_t[r_{t+n}]}_{\text{price of future}} + \underbrace{\frac{1}{2}\left\{Var[-\Sigma_{j=1}^{n} r_{t+j}] - Var[-\Sigma_{j=1}^{n-1} r_{t+j}]\right\}}_{\text{convexity}}
\end{aligned}
$$

This shows that the Wu and Xia (2016) approximation for the price of the forward is the price of the corresponding future (1) plus a convexity term, which depends on the variance of future interest rates. Further approximations simplify the latter and allow Wu and Xia (2016) to write the value of the forward as (14).

Appendix 2: Modeling the Lehman crisis

Tables 8 to 11 show how the model uses the model factors to represent the effect of the Lehman crisis and the measures taken to relieve the money and banking markets in its aftermath. They analyze the mid-month to mid-month changes in the 3-month T-bill & ED rates, the 10-year forwards & futures prices and risk premia and the changes in the factors used to model these movements. These periods and policy announcements are listed in Table 1.

Table 8 shows the monthly changes in the factors and hence the predicted shadow rates, given the simple loading structure shown in (17). Panel (a) shows a large increase in he first common factor, perhaps reflecting market worries about inflation. However, this was more than offset by a fall in the third factor, perhaps reflecting expectations of a temporary relaxation of monetary policy. Panel (b) shows the spread factors discussed in section 4.2.3. The last two panels show the Black model predictions and the observed changes. The large fall in the shadow T-bill rate in the fourth period took it below its lower bound and so the Black model prediction fell by only 13.5bp.

Table 9 shows how these factors represent the 10-year ahead expectations of these shadow interest rates. These use the time-series loadings shown in the middle panel of Table 7. As noted factors in section 4.2.3, the 10-year time-series loadings of the T-bill and ED rates on the common factors are close, so these expectations behave in a similar way. They are strongly influenced by the first common factor, since this is persistent. These expectations move up sharply in the month following the Lehman bankruptcy, before moving back as policy rates reached the lower bound in the fourth period and the first LSAP was announced.

Movements in the two term premia are analyzed in Table 10. By removing the common expectations components, this table provides a much impression of the differential effects of policy on the two markets than do the forward and futures prices themselves (shown in Table 11). Table 10 uses the respective premium loadings reported in the bottom panel of Table 7. It reveals a sharp fall in the premium in the 10-year ED future price as support for the troubled asset markets and the two investment banks was announced in the second period, followed by a further sharp fall in the fourth period when the first LSAP was announced. In contrast, the term premium on the 10-year UST forward increased sharply in the second period. It then remained stable over the rest of the year. Again, it seems clear that these policy initiatives had a much greater impact in the ED futures markets than in the UST market. Finally, Table 11 sums the time-series expectations and risk premia to get the shadow 10-year forwards and futures, and hence the Black Φ weights and model predictions. The observed changes are shown in the bottom panel of this table.

Table 1: Major policy events surrounding the Lehman crisis

1. 19 August 2008 to 16 September 2008

This covers the period from the market open on Tuesday 19 August 2008 to the close on Tuesday 16 September 2008. This was a period of increasing pressure in the banking and money markets. Lehman Brothers filed for bankruptcy on Monday 15th September after becoming distressed in the previous week and attempts to rescue it over the weekend were abandoned.The American Insurance Group was rescued on the same day. Reflecting these developments. the one-year subordinated CDS rates for the two remaining investment banks Goldman Sachs and Morgan Stanley moved up from 1.70% and 2.97% respectively at the beginning of the period to 2.1% and 3.42% on Friday 12 September, before reaching 6.46% and 15.29% by the close on 16th.

2. 17 September to 16 October

Goldman Sachs and Morgan Stanley converted into bank holding companies that were eligible for Federal Reserve support on 22 September. The TARP emergency stabilization package was signed into law on 3 October. These initiatives had the effect of reducing rates on bank CDS. The one-year subordinated CDS rates for Goldman Sachs and Morgan Stanley fell back to 3.65% and 8.0% respectively by the close on 16th October. These CDS rates remained close to these levels until the Autumn of 2009.

3 17 October to 17 November

The FOMC meeting of 28-29 October further reduced the Fed Funds target range to 1%

4. 18 November to 16 December

The Federal Reserve announced the first LSAP on November 25, saying that it would purchase up to $600 billion in agency mortgage-backed securities (MBS) and agency debt, sterilized by sales of Treasury bonds. Chairman Bernanke said that the Fed could purchase long-term USTs in substantial quantities in his speech of 1 December. The FOMC meeting of 15-16 December cut the Fed Funds target range to 0-25bp and stated that further asset purchases were being considered.

5. 17 December to 16 January 2009

This period saw the publication of the minutes of the December FOMC meeting.

6. 17 January to 16 February

This period encompassed the FOMC meeting of 28 January. No new policy measures were announced, which disappointed the markets.

Table 2: Daily interest rate changes around extended event set announcements (Basis points)

Date	**Event**	**10y UST forward**	**10y ED future**
9/15/2008	Lehman bankruptcy AIG rescue	-10	-17
9/22/2008	Investment banks eligible for FED discount window support	8	4
10/ 3/2008	Congress passes TARP bill	7	-2
10/29/2008	FOMC Statement	16	-1
11/25/2008	LSAP announced	-12	-36
12/1/2008	Chairman speech	-14	-27
12/16/2008	FOMC Statement	-13	-22
1/28/2009	FOMC Statement	16	14
3/18/2009	FOMC Statement	-35	-37
Baseline event set: Total		-37	-124
Cumulative change 8/16/2008-4/16/2009		-19	-146
of which:	Expectation	-75	-71
	Term premium	56	-75

The top panel of this Table extends the results of table 1 in Gagnon et al. (201i), which reports the changes in key interest rates on the day of the announcement of the first LSAP and subsequent FOMC meetings. I add the changes in the 10-year US Treasury (UST) forward rate and the 10-year future written on the 3-month Eurodollar (ED) rate as well as the effects of the key policy announcements made in September 2008. The table shows that the effect of these initiatives on the ED future was consistently larger than the effect on the UST forward. The cumulative effect of the announcements was to depress the future by 1.24% and the forward by 0.37%, a difference of 87bp.. The bottom panel uses model M3 to split the movements over the period into movements in expectations and term premiums. Comparing this difference with the one shown in the top panel, suggests that most of this 127bp difference can be explained the effect of policy announcements, leaving the remaining 40bp to be explained by unidentified policy and other influences.

Table 3: Residual summary statistics

Model	Param -eters	Loglike -lihood	(-)BIC	LR Test vv M1 $\chi^2(5)$
M0: Affine	66	37,997.0	75,433	
M1: Encompassing	68	38,482.5	76,382	
M2: Common and idiosyncratic factors	63	38,478.2	76,416	$\chi^2 = 8.6$ $p = 0.13$
M3: Parsimonious risk adjustment	39	38,468.5	76,602	$\chi^2 = 19.4$ $p = 0.91$

This Table reports the likelihood and BIC statistics for the four models described in section 3, and the results of likelihood ratio tests of models M2 and M3 against the encompassing model M1. Model M2 is described in section 3.1, and uses restrictions to distinguish common and spread factors. Model M3 is described in section 3.2 and eliminates redundant price of risk parameters.

Table 4: Data summary statistics

Maturity	**spot**	**3**	**12**	**24**	**36**	**48**	**60**	**84**	**120**
Eurodollar futures									
Mean	2.875	2.956	3.330	3.896	4.339	4.698	4.996	5.413	5.787
SD	2.251	2.304	2.272	2.137	1.981	1.857	1.772	1.716	1.765
Kurtosis	1.370	1.427	1.599	1.704	1.818	1.906	1.986	2.063	2.0418
Skewness	0.218	0.232	0.196	0.022	-0.081	-0.131	-0.169	-0.241	-0.302
ADF	-1.368	-1.653	-1.655	-1.673	-1.712	-1.721	-1.719	-1.724	-1.713
p-value	0.159	0.093	0.0919	0.089	0.082	0.081	0.081	0.081	0.082
KPSS	0.11	0.118	0.139	0.146	0.126	0.104	0.096	0.124	0.153
p-value	<0.01	<0.01	0.0621	0.049	0.086	<0.01	<0.01	0.092	0.044
Treasury forwards									
Mean	2.386	2.689	3.028	3.488	3.908	4.282	4.601	5.076	5.455
SD	2.153	2.289	2.256	2.038	1.821	1.665	1.566	1.487	1.469
Kurtosis	1.395	1.444	1.5761	1.760	1.965	2.112	2.173	2.222	2.367
Skewness	0.266	0.277	0.235	0.188	0.133	0.04.7	-0.06	-0.263	-0.419
ADF	-2.032	-1.82	-1.736	-1.804	-1.845	-1.829	-1.792	-1.764	-1.845
p-value	0.048	0.066	0.079	0.068	0.062	0.064	0.069	0.074	0.062
KPSS	0.123	0.132	0.167	0.19	0.16	0.092	0.060	0.156	0.27
p-value	0.099	0.075	0.032	0.022	0.038	<0.01	<0.01	0.042	0.012

Mean reports sample arithmetic mean and SD standard deviation. ADF shows the Augmented Dickey Fuller test statistic under the null hypothesis of non-stationarity. KPSS shows the Kwiatkowski Phillips Schmidt Shin (1992) test statistic under the null hypothesis of stationarity. The lag lengths for ADF and KPSS are determined by the Akaike information criterion. Maturity is in months.

Table 5: Parameter estimates for model M3

K_{11}	1.0	K_{55}	0.991	δ_{11}	5.841×10^{-4}
	(8980)		(413)		(34.67_
K_{21}	-5.954×10^{-3}	L_{12}	0.110	δ_{12}	1.076×10^{-4}
	(-7.99)		(3.59)		(63.58)
K_{22}	0.913	L_{13}	6.006×10^{-3}	δ_{13}	6.044×10^{-4}
	(397)		(-2.029)		(28.9)
K_{31}	(-)	L_{24}	8.534×10^{-3}	δ_{14}	1.558×10^{-4}
	(-)		(-2.083)		(17.60)
K_{32}	-0.884	L_{35}	-6.703×10^{-2}	δ_{15}	1.476×10^{-4}
	(-27.6)		(-3.39)		(-11.15)
K_{33}	0.969	L_{53}	6.209×10^{-3}	δ_{21}	3.09×10^{-4}
	(144)		(-3.123)		(17.86)
K_{41}	(-)	L_{55}	-4.135×10^{-2}	δ_{22}	1.756×10^{-4}
	(-)		(-5.72)		(21.56)
K_{42}	-3.599×10^{-2}	k_1	8.150×10^{-5}	δ_{23}	1.174×10^{-3}
	(-2.03)		(-4.06)		(27.440
K_{43}	-4.697×10^{-3}	k_4	(-)	δ_{24}	5.557×10^{-4}
	(-9.345)		(-)		(9.61)
K_{44}	0.989	l_2	1.012×10^{-5}	δ_{25}	2.32×10^{-4}
	(-382)		(-4.38)		(1.65)
K_{51}	-1.360×10^{-2}	l_3	8.859×10^{-6}	ζ_1	2.545×10^{-4}
	(-1.97)		(-1.98)		(136)
K_{52}	-1.560×10^{-2}	l_5	6.150×10^{-5}	ξ_1	3.351×10^{-4}
	(-1.55)		(-9.36)		(119)
K_{53}	-3.343×10^{-4}	$\underline{\mathrm{r}}_f$	1.204×10^{-3}	ζ_2	3.025×10^{-4}
	(-0.74)		(-19.86)		(102)
K_{54}	-2.655×10^{-2}	$\underline{\mathrm{r}}_g$	3.104×10^{-3}	ξ_2	2.551×10^{-4}
	(-8.02)		(-17.15)		(114)

Parameter estimates, with t-statistics in parenteses,

Table 6: Residual summary statistics for model M4

Maturity	spot	3	12	24	36	48	60	84	120
Eurodollar futures									
Mean$\times 10^{-4}$	1.163	-0.219	-1.486	0.333	-0.316	-1.540	-0.806	0.239	-0.461
SD$\times 10^{-4}$	6.288	6.596	8.137	8.193	8.358	8.394	8.220	7.942	8.0227
Kurtosis	64.412	5.158	3.887	3.707	4.24	5.812	7.527	8.974	10.507
Skewness	4.956	-0.834	-0.049	0.329	0.296	-0.083	-0.359	-0.491	-0.595
ADF	-4.363	-3.290	-3.954	-6.139	-6.206	-5.177	-6.012	-6.573	-5.479
p-value	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
Treasury forwards									
Mean$\times 10^{-4}$	-2.121	2.017	0.594	-0.850	-1.469	-1.262	-0.59	0.682	-0.615
SD$\times 10^{-4}$	5.541	6.368	7.711	8.234	8.108	7.974	7.924	7.756	7.662
Kurtosis	8.321	5.484	3.972	3.831	3.911	4.115	5.009	8.503	8.063
Skewness	-1.831	0.141	0.295	0.137	-0.019	-0.065	0.053	0.474	-0.215
ADF	-2.681	-2.802	-4.618	-5.868	-5.389	-4.996	-4.897	-4.952	-5.421
p-value	0.080	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01

This Table reports residual sample statistics for the preferrred model M3. See notes to Table 4.

Table 7: 10-year factor loadings

1. Under risk-neutral measure (Q)					
forward	2.865	-0.392	2.494×10^{-2}	0	0
future	2.770	-0.859	0.105	-0.742	0.359
2. Under time-series measure (P)					
forward	2.590	-0.767	0.402	4.246×10^{-2}	-0.546
future	2.636	-0.733	0.402	0.103	-0.544
3. Term premium loadings (Q-P)					
forward	0.274	0.374	-0.378	4.246×10^{-2}	0.546
future	0.134	-0.126	-0.2978	-0.844	0.902

This Table shows the three sets of 10-year loadings, which are used to weight the factors in calculating the expected shadow rates under the various probability measures. Changes in these shadow rates around the Lehman crisis are shown in the next three tables.

Table 8: Shocks to 3 month rates during the Lehman crisis

Change (b.p.) during:	18 Aug 08 16 Sep	17 Sep 16 Oct	17 Oct 17 Nov	18 Nov 16 Dec 08	17 Dec 08 16 Jan 09	17 Jan 09 16 Feb
(a) Common factors						
1	-10.7	84.9	-5.0	-62.0	-13.3	18.7
2	6.5	23.1	1.5	4.4	-5.2	-3.5
3	-26.2	-133.7	-91.7	-79.2	-16.9	48.6
(b) Spread factors						
4	22.2	144.3	0.2	69.3	7.4	-90.3
5	6.0	18.5	-24.9	-29.0	-21.1	28.1
Predicted shadow rates						
T-bill=total 1-3	-30.5	-25.6	-95.2	-136.8	-35.5	63.9
ED=total 1-5	-2.2	137.2	-119.9	-96.5	-49.2	1.6
Black model predictions						
T-bill Φ-weights	1	1	1	0.05	0	0
T-bill	-30.5	-25.6	-95.1	-13.5	0	0
ED Φ-weights	1	1	1	1	1	1
ED	-2.2	137.2	-119.9	-96.5	-49.2	1.6
Memo: Observed changes						
TB	-97	-39	-31	-10	8	17
ED	25	255	-275	-85	-40	-10

This Table is discussed in Appendix 2 and shows how the model uses the common and spread factors to represent the effect of the Lehman crisis and the measures taken to relieve the money and banking markets in its aftermath. It reports the mid-month to mid-month changes in the 3 month T-bill and ED rates and the changes in the factors used to model them,

Table 9: Shocks to 10-year ahead 3-month shadow interest rate expectations

Change (b.p.) during:		18 Aug 08 16 Sep	17 Sep 16 Oct	17 Oct 17 Nov	18 Nov 16 Dec 08	17 Dec 08 16 Jan 09	17 Jan 09 16 Feb
1. Treasury bill rate							
Factor contributions	1	-27.8	220.2	-12.9	-160.6	-34.5	48.5
	2	-5.0	-17.8	-1.2	-3.4	4.0	2.7
	3	-10.5	-53.9	-36.9	-31.9	-6.8	19.6
	4	0.9	6.1	8.5×10^{-3}	2.9	0.3	-3.8
	5	-3.3	-10.1	13.6	15.8	11.5	-15.3
Total		-45.7	144.6	-37.4	-177.1	-25.5	51.6
2. Eurodollar rate							
Factor contributions	1	-28.3	224.0	-13.1	-163.4	-35.2	49.3
	2	-4.7	-17.0	-1.1	-3.2	3.8	2.6
	3	-10.5	-53.9	-36.9	-31.9	-6.8	19.6
	4	2.3	14.8	2.1×10^{-2}	7.1	0.8	-9.3
	5	-3.3	-10.1	13.5	15.8	11.5	-15.3
Total		-44.6	157.9	-37.6	-175.7	-25.9	46.9
T-bill less ED rate		1.1	13.4	-0.2	1.4	-0.4	-4.6

This Table is discussed in Appendix 2 and shows how the model uses the factors, weighted by the loadings shown in Table 5, to represent the effect of the Lehman crisis on 10-year ahead rate expectations.

Table 10: Shocks to 10-year shadow term premiums

Change (b.p.) during:		18 Aug 08 16 Sep	17 Sep 16 Oct	17 Oct 17 Nov	18 Nov 16 Dec 08	17 Dec 08 16 Jan 09	17 Jan 09 16 Feb
1. Treasury bill rate							
Factor contributions	1	-3.0	23.3	-1.4	-17.0	-3.7	5.1
	2	2.4	8.7	0.6	1.7	-2.0	-1.3
	3	9.9	50.5	34.6	29.9	6.4	-18.4
Total		11.7	86.5	20.2	-4.2	-11.0	4.6
2. Eurodollar rate							
Factor contributions	1	-1.4	11.4	-0.7	-8.3	-1.8	2.5
	2	-0.8	-2.9	-0.2	-0.8	0.7	0.4
	3	7.8	39.7	27.2	23.5	5.0	-14.5
	4	-18.8	-121.9	-0.2	-58.5	-6.2	76.3
	5	5.4	16.7	-22.5	-26.2	-19.0	25.3
Total		-7.9	-57.0	3.8	-70.1	-21.3	90.1
T-bill less ED		-19.5	-143.5	-16.5	-65.8	-10.3	85.5

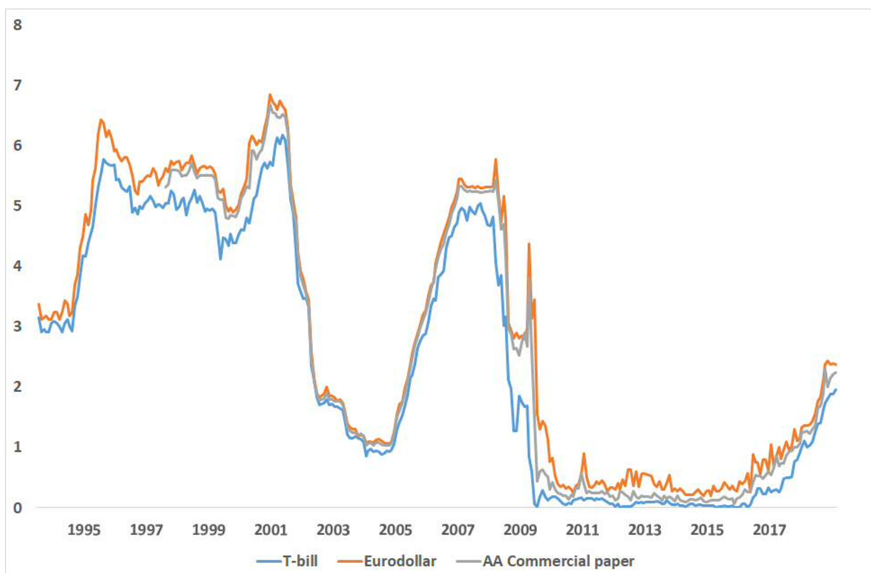
This Table shows how the model uses the factors, weighted by the loadings shown in Table 5, to represent the effect of the Lehman crisis on the term premia in the 10-year forwards & futures prices.

Table 11: Shocks to 10-year forwards and futures durung the Lehman crisis

Change (b.p.) during:		**18 Aug 08 16 Sep**	**17 Sep 16 Oct**	**17 Oct 17 Nov**	**18 Nov 16 Dec 08**	**17 Dec 08 16 Jan 09**	**17 Jan 09 16 Feb**
10-year forwards							
Factor contributions	1	-30.8	243.5	-14.3	-177.7	-38.2	53.6
	2	-2.5	-9.1	-0.6	-1.7	2.1	1.4
	3	-0.7	-3.3	-2.3	-2.0	-0.4	1.2
Forward=total 1-3		-34.0	231.1	-17.2	-181.4	-36.6	56.2
10-year futures							
Factor contributions	1	-29.8	235.4	-13.8	-171.8	-36.9	51.8
	2	-5.6	-19.9	-1.3	-3.8	4.5	3.0
	3	-2.8	-14.1	-9.7	-8.4	-1.8	5.1
	4	-16.5	-107.1	-0.1	-51.4	-5.5	67.0
	5	2.1	6.7	-8.9	-10.4	-7.6	10.1
Future=total 1-5		-52.4	101.0	-33.9	-245.8	-47.3	137.0
Future-forward		-18.5	-130.1	-16.7	-64.4	-10.7	80.8
Black model predictions							
Forward Φ-weights		0.940	0.846	0.870	0.840	0.666	0.675
Forward		-32.0	195.6	-14.9	-152.4	-24.4	37.9
Future Φ-weights		0.940	0.846	0.870	0.840	0.667	0.675
Future		-49.3	85.8	-29.5	-206.5	-31.5	92.5
Future-forward		-17.3	-110.1	-14.5	-54.1	-7.1	54.6
Memo: Observed changes							
Forward		-28	112	8.9	-154	-17	56
Future		-40	65.5	-55.5	-195	10	104

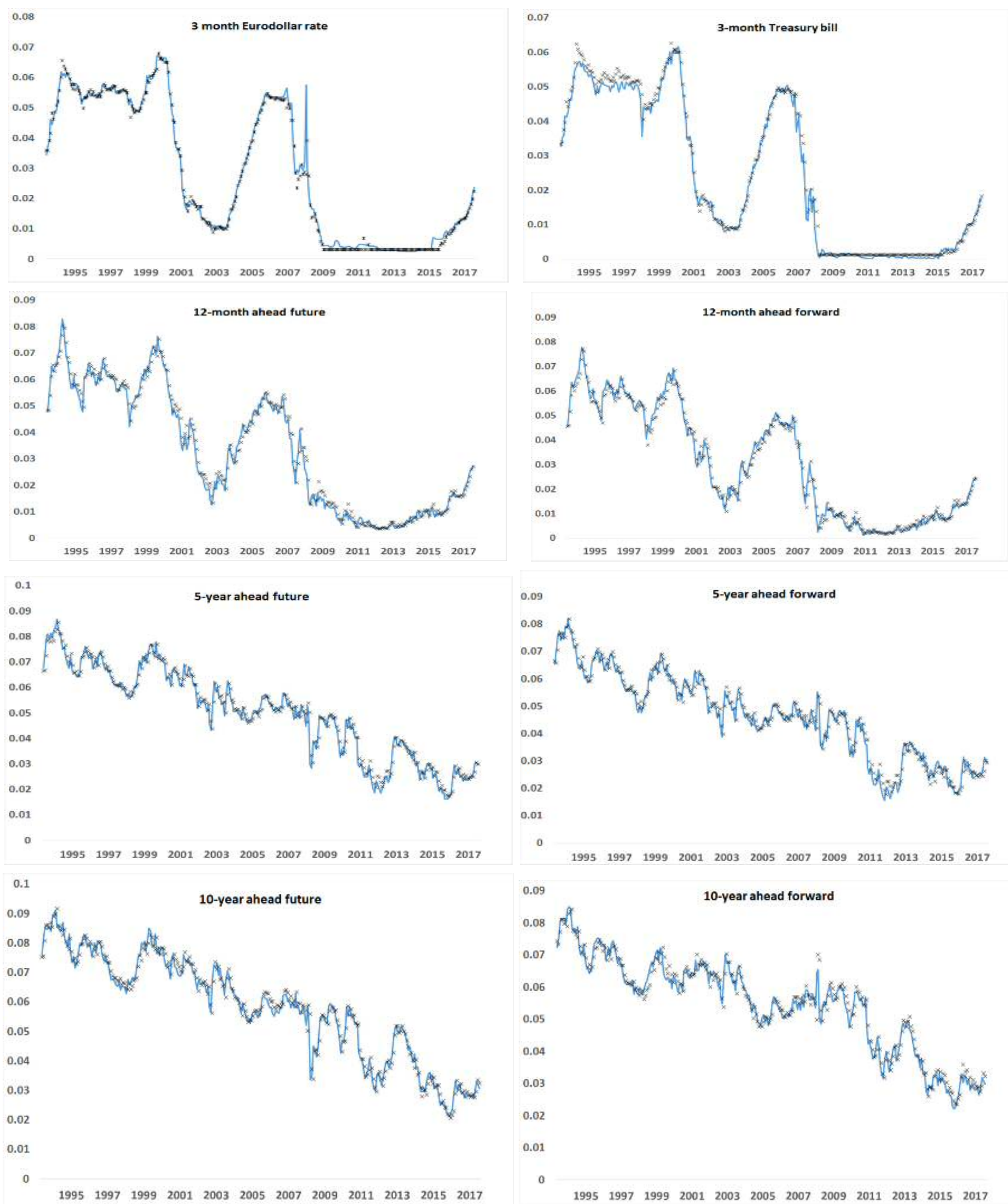
This Table is discussed in Appendix 2 and shows how the model uses the factors, weighted by the loadings shown in Table 5, to represent the effect of the Lehman crisis and the measures taken in its aftermath. It reports the monthly changes in the 10-year forwards & futures (which are decomposed into expectations and term premia in the previous two tables); the Black model predictions and the observed changes.

Figure 1: 3-month T-bill, Eurodollar and Commercial paper rates



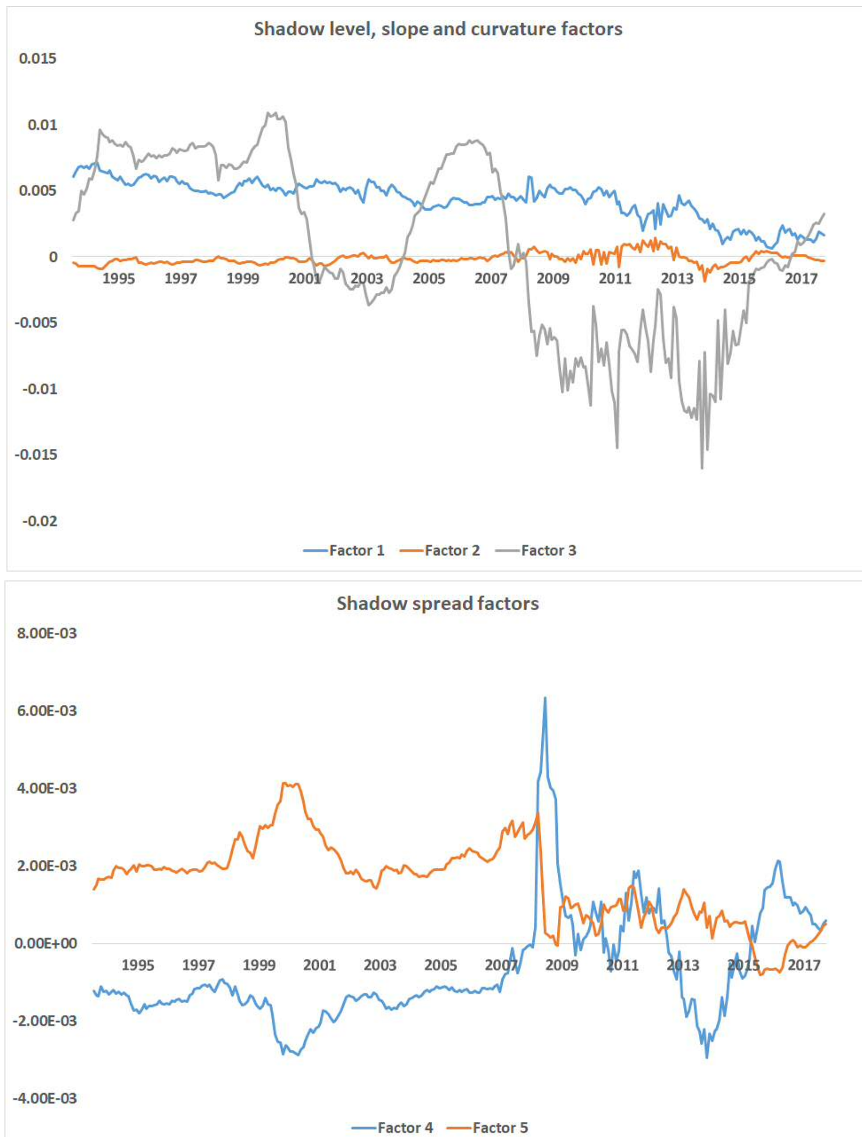
This Figure shows that the spreads of ED and CP rates over T-bill rates are both about 25-50bp in normal times, but moved up to between 100-200bp as the interbank markets became stressed in 2007. Although T-bill rates came down to around 150 bp prior to the Lehman collapse, ED and CP rates remained stuck at 300bp. The ED rate then spiked up to 500 and CP rate to 600bp in the aftermath of the collapse, while T-bill rates fell back close to zero as the Fed Funds rate was cut. ED and CP rates remained elevated until the first few months of 2009 when the TARP, LSAP and associated initiatives relieved the stresses in the money markets, allowing the spreads to normalize. The Appendix shows how the model represents these events. Data is from Datastream.

Figure 2: Representative data series and fitted values



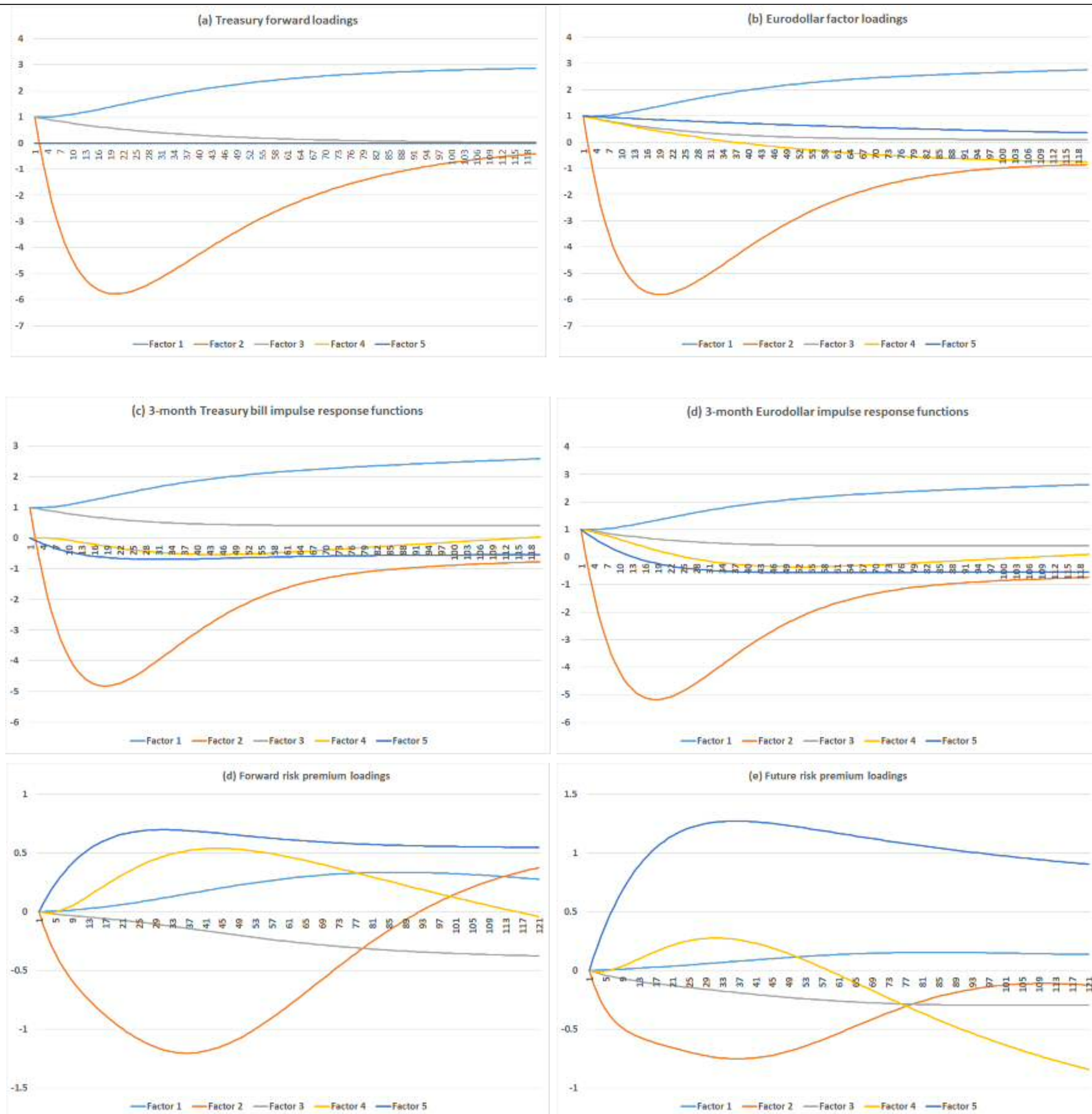
This Figure compares the fitted values (black crosses) with the data (shown in blue) for representative maturities.

Figure 3: The latent factors



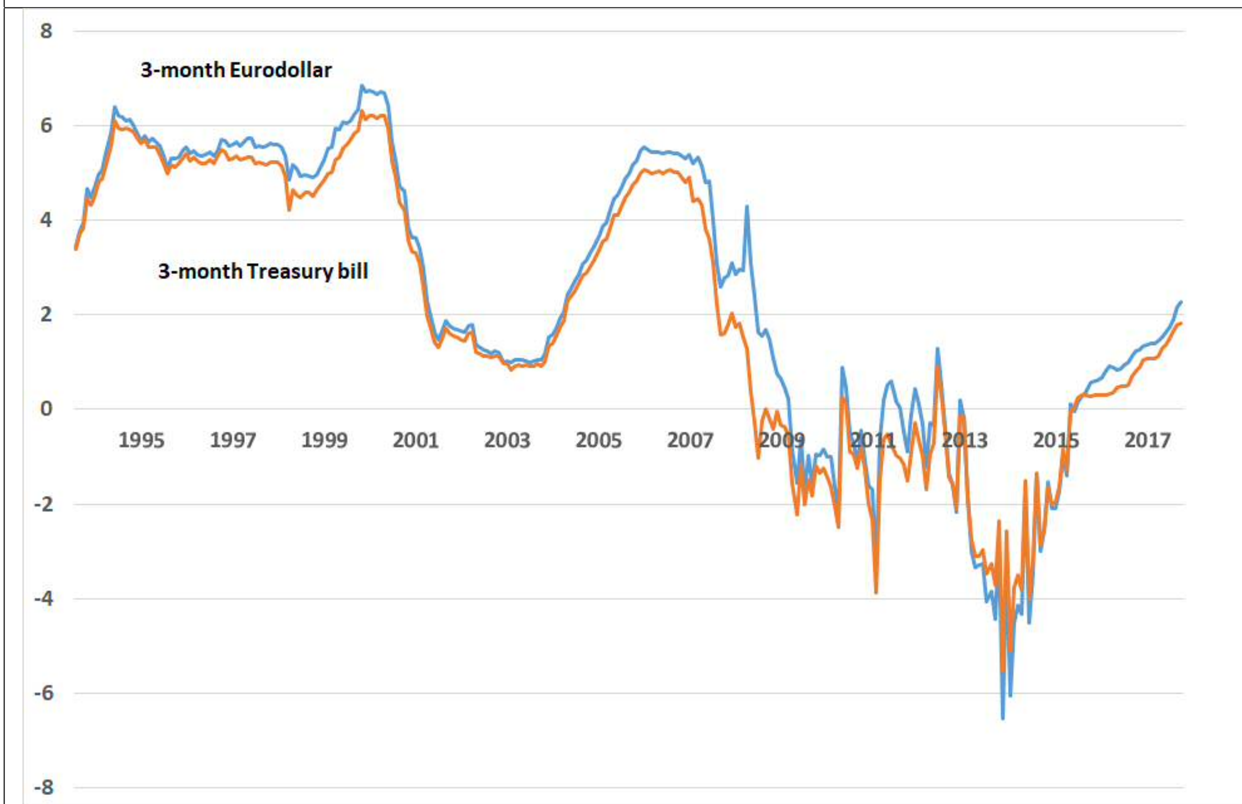
The first panel in this figure shows the common and the second the spread factors. The first factor largely accounts for the downward trend in long futures and forwards seen over this period while the other two common factors reflect swings in the economy and the slope of the shadow yield curve. The first spread factor reflects near-term deposit risk, increasing over the period of stressed money markets that began in the spring of 2007 and intensified in August. before spiking up sharply in response to the Lehman crisis. It then eased back as the TARP, LSAP and other support measures took effect early in 2009. The second spread factor seems to reflect more general default risk effects, for example those associated with the dot-com boon of the early years of the millennium.

Figure 4: Factor loadings and impulse response functions



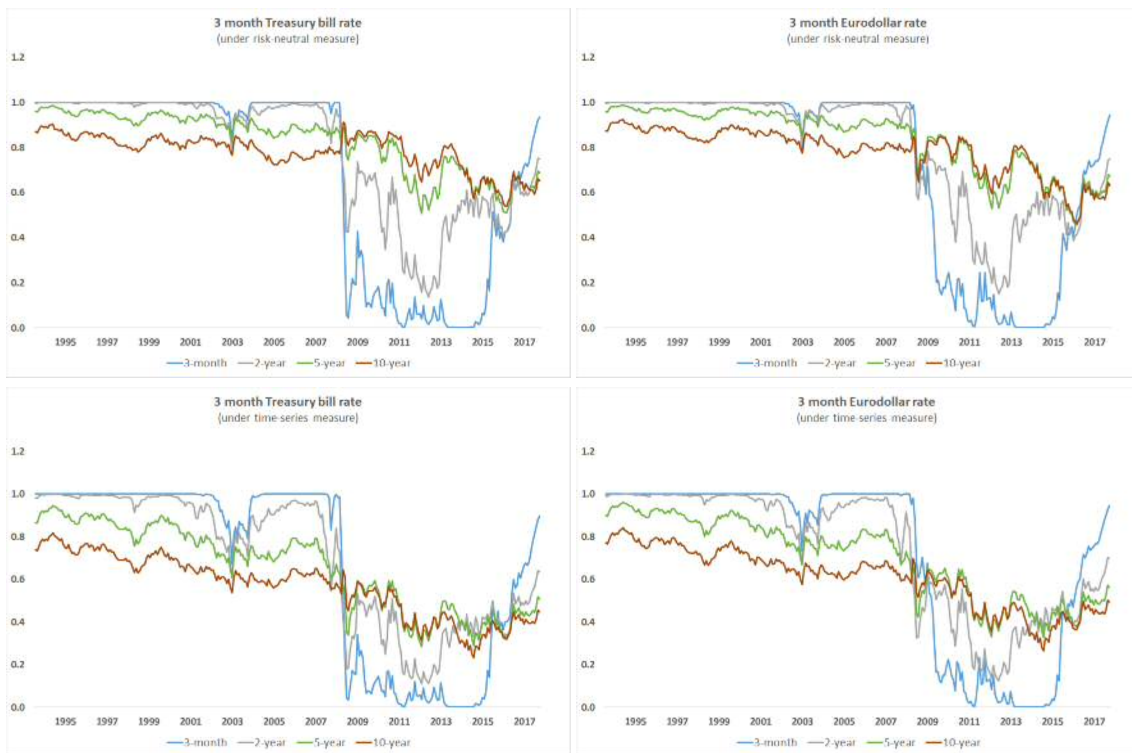
The charts in the top panel of this Figure show the estimates of the factor loadings and those in the middle panel show the associated IRFs. The loadings on the first three factors identify them as level, slope and curvature factors. The ED rate has a unit loading on the two spread factors, but the first of these loadings quickly turns negative, making this a spread-slope factor. The time series dynamics are unrestricted and suggest that the spread factors have the effect of depressing policy rates temporarily. The bottom panel represents the term premium loadings, which are the difference between the values shown in the top and middle panels.

Figure 5: Shadow interest rates



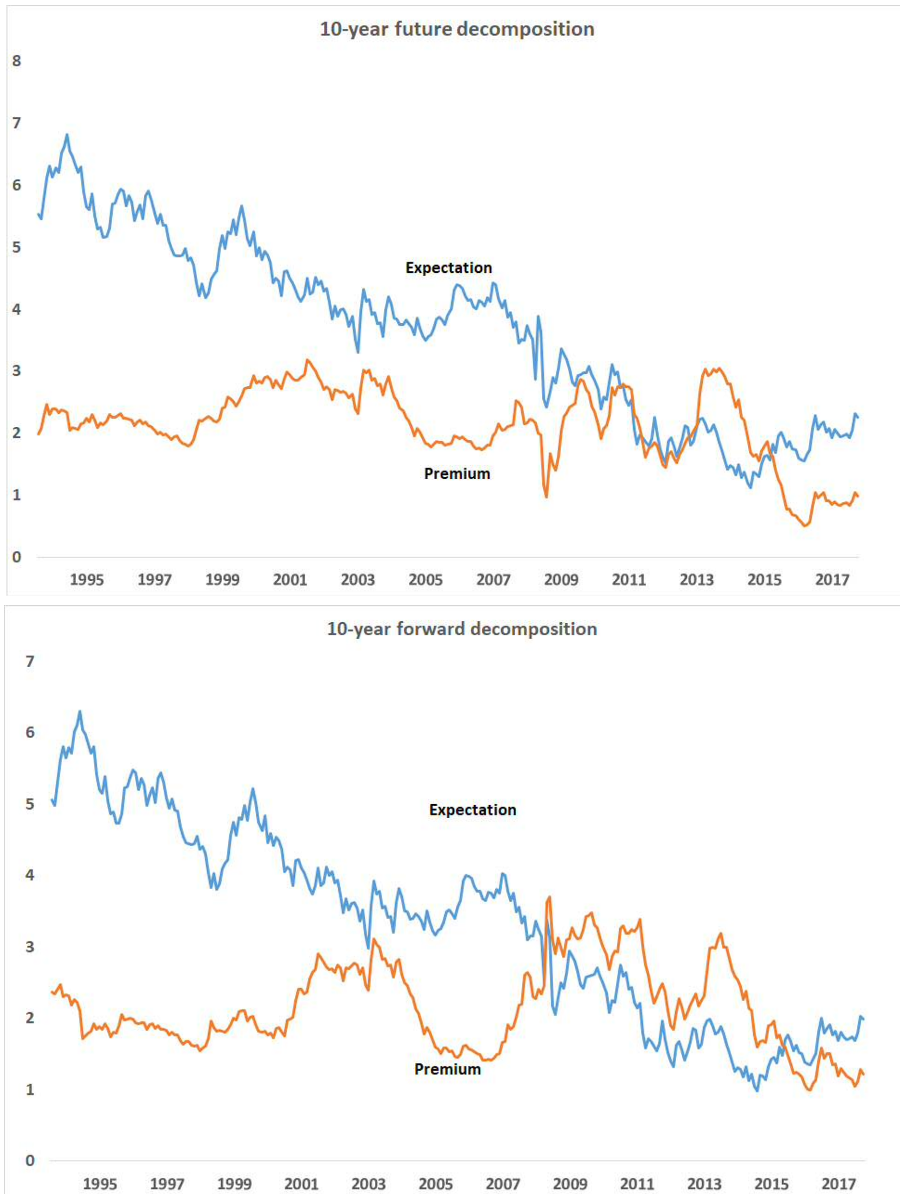
This Figure compares the two estimates of the shadow spot rates. The effect on the observed rate is damped by the multiplier shown in figure 6 for the Eurodollar rate. This normally exceeds the Treasury bill rate due to the effect of deposit risk, modeled by the two spread factors. These turn negative in 2014, reversing this effect, but have no practical effect on the observed rates since the Φ multiplier is effectively zero at the short end at this time.

Figure 6: The likelihood of positive future interest rates



This Figure shows the estimates of the cumulative densities (Φ) that map the shadow prices into the Black prices at different horizons. These show the fraction of the probability mass of the future shadow rate that is in the non-negative range under the measures Q and P at these horizons. They indicate that most of the probability mass was in the positive range before 2008, making the Gaussian model a reasonable approximation, but that these probabilities were much lower when rates were at the lower bound.

Figure 7: Rate expectations and term premiums in future and forward markets



The two panels of this Figure decompose the 10-year future and forwards into the risk premia (shown in amber) and respective expected future spot rates (shown in blue). The first factor dominates the expected rates and explains the downward tendency in the observed rates, rendering the risk premia stationary. The risk premiums both fall back as interest rates are increased in 2004 and 2005. The Treasury premium increases sharply as interest rates are reduced following the Lehman collapse, consistent with the view that this premium tends to offset the effect of changes in monetary policy. However, the premium in the ED futures market does not increase over this period, perhaps as a consequence of the measures taken to relieve the stress in the banking markets. Both premia increase in response to the Euro crisis in 2013 but fall back as this eases in 2014 and 2015.