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Equilibrium real exchange rate estimates across time and space*

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Abstract

Equilibrium real exchange rate and corresponding misalignment estimates differ tremendously depending on the panel estimation method used to derive them. Essentially, these methods differ in their treatment of the time-series (time) and the cross-section (space) variation in the panel. The study shows that conventional panel estimation methods (pooled OLS, fixed, random, and between effects) can be interpreted as restricted versions of a correlated random effects (CRE) model. It formally derives the distortion that arises if these restrictions are violated and uses two empirical applications from the literature to show that the distortion is generally very large. This suggests the use of the CRE model for the panel estimation of equilibrium real exchange rates and misalignments.

Keywords: Equilibrium real exchange rate, panel estimation method, correlated random effects model, productivity approach, BEER, price competitiveness

JEL-Classification: F31, C23

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1. Introduction

The present study proposes to use a correlated random effects (CRE) model in the spirit of Mundlak (1978) for the panel estimation of equilibrium real exchange rates. The proposed CRE model takes account of the fact that cross-section or between group estimates of the impact of various explanatory variables on real exchange rates often differ quite substantially from the corresponding time series or fixed effects estimates. This is of particular importance in the case of equilibrium real exchange rate estimates because the residuals of the corresponding panel regressions are regularly used to compute currency misalignments. The present study shows that conventional (i.e. fixed effects, pooled OLS, random effects or between effects) panel estimation methods can be interpreted as restricted versions of the CRE model and that conventional misalignment estimates are distorted if the implicit restrictions are violated. The study formally derives the distortion in the misalignment estimate for each of the conventional panel estimation methods. Moreover, it computes the distortion for two empirical misalignment estimation applications adapted from the literature (a Balassa-Samuelson-type panel regression and a regression related to the International Monetary Fund's (IMF) external balance approach real exchange rate level regression). The distortion is regularly found to be very large which implies that a CRE instead of a conventional model should normally be used for the panel estimation of equilibrium real exchange rates.

The long-run determination of real exchange rates is one of the perennial issues in economic research. In the 1960s, the classic idea of purchasing power parity that the real exchange rate is a constant at least in the long run had already given way to theories such as Balassa (1964) and Samuelson's (1964) hypothesis, which proposes that total factor productivity is a long-run determinant of the real exchange rate. The Balassa-Samuelson model suggests that an equilibrium real exchange rate can be obtained by a low-frequency regression of real exchange rates on a productivity variable. Berka et al. (2018), Chong et al. (2012), Hassan (2016) and Kakkar and Yan (2012) are only four prominent studies which demonstrate that the extent of the empirical validity of the Balassa-Samuelson theory is presently still being analysed.

Estimates of equilibrium real exchange rates play also an important role in economic policy because the residual of the underlying regression is often used to compute an effective misalignment of the currency in question. The most prominent among such estimates is probably the IMF's approach (cf. Phillips et al., 2013, and IMF, 2017) because it is employed in the IMF's annual assessment of price competitiveness and potential misalignments in individual countries (cf. e.g. IMF, 2018).[1] The IMF's approach and many similar real exchange rate regressions go beyond productivity and, referring to various models and economic considerations, include a set of explanatory variables in the regression.

Today, the large majority of applications uses panel data for real exchange rate regressions because the panel improves inference (cf. e.g. Bussière et al., 2010). The relevant studies differ, however, in the panel estimation technique applied. To give some examples, Couharde et al. (2017) and Fidora et al. (2017) use fixed effects estimators, Fischer and Hossfeld (2014) present fixed effects and pooled OLS estimates, Cheung et al. (2007) and Berka et al. (2018) compute fixed effects, between effects, random effects and pooled OLS estimates and, in their external balance assessment (EBA) framework, the IMF provides both, a fixed effects-based regression (cf. Phillips et al., 2013) and a 2SLS regression (cf. IMF, 2017), which boils down to a pooled OLS-based one if the instrumentation of some variables is omitted.

The multitude of panel estimation techniques applied prompts the question of whether the different methods generate economically significant differences in the equilibrium real exchange rate and misalignment estimates. Adler and Grisse (2017) have already shown that the inclusion of fixed effects can alter misalignment estimates substantially. The present study uses all of the conventional estimation methods to compute misalignments according to two alternative applications adapted from the literature and regularly finds economically very large deviations between misalignments obtained by different methods.

This result implies that it is essential to determine which panel estimation method should be used for the estimation of equilibrium real exchange rates, a question which has apparently not yet been investigated systematically. To answer the question, the present study first notes that the reason for the differences in the estimates is the fact that the alternative panel estimation methods treat the time-series variation and the cross-section variation in the data differently.[2] This generates differing coefficient and misalignment estimates.

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[1] Apart from reduced-form real exchange rate regressions, the IMF also obtains equilibrium real exchange rate values by following an alternative strategy. This strategy consists in a current account regression and, subsequently, a derivation of the equilibrium real exchange rate from the equilibrium current account and a trade elasticity of the real exchange rate. While this second strategy for obtaining equilibrium real exchange rates is beyond the scope of the present study, the study's fundamental considerations for optimally estimating equilibrium real exchange rates in a panel apply to panel estimates of equilibrium current accounts in the same manner.

[2] As will be discussed below, the existence of a meaningful cross-section level variation requires that real exchange rates are not computed from price indices but as relative price levels.

It is shown that the differing treatment of the time-series and the cross-section variation in the data is the consequence of imposing restrictions on a more general model, the CRE model introduced by Mundlak (1978). This model is well known in econometrics (cf. e.g. Wooldridge, 2010) and variously employed in non-economics Social Sciences, but apparently hardly ever used in economic applications. It provides separate estimates for the impact of the time-series and the cross-section variation of each explanatory variable. The fixed effects model results from the CRE model if it is assumed that there is no impact of the cross-section level variation in the explanatory variables. The assumption that the time-series variation in the explanatory variables does not affect the real exchange rate leads to the between effects model, and the assumption of an identical impact of the cross-section level and the time-series variation in the variables can either result in a pooled OLS or in a random effects model.

Depending on the restriction imposed, each of the conventional panel estimation methods generates a specific distortion in the misalignment estimate if the corresponding restriction is violated. This is the phenomenon which is explored by the present study. It formally derives the distortion of each conventional panel estimation method and it shows in two empirical applications adapted from the literature that the distortion is mostly very large and none of the restrictions are met. This result suggests the use of a CRE model to obtain equilibrium real exchange rate or misalignment estimates in a panel.

The results also support a more general conclusion. In applied economics, the fixed effects approach is the dominant linear panel estimation method. Its use is clearly warranted if the researcher is specifically interested in a coefficient estimate based predominantly on the time-series variation in the panel. In any other case, however, a careful consideration of the alternatives including a CRE model is advisable even if the null of a Hausman test is rejected.

Section 2 of the study presents two alternative strategies to determine equilibrium real exchange rate and misalignment estimates using panel data. Section 3 introduces the CRE model, shows how restrictions turn it into conventional panel estimation procedures, and derives the potential distortion in misalignment estimates that arises if these restrictions are violated. Section 4 uses two empirical applications from the literature to assess the extent of the distortion, and section 5 concludes.

2. Setting the stage: two alternative strategies for the determination of misalignments using panel data

Let us define (a) an equilibrium real effective exchange rate quite generally as the value of the effective real exchange rate that would hold for a given set of realizations of some fundamental variables abstracting from the effects of other factors and (b) the misalignment of a currency as the deviation of the actual from the equilibrium real exchange rate.[3] Before considering estimation issues, it is necessary to clarify how equilibrium real exchange rates or real exchange rate misalignments are derived in a panel data framework. This section presents two alternative strategies for determining such misalignments. The first essentially estimates the impact of explanatory variables on real exchange rates in a setting in which all the variables are defined bilaterally against a specific base country. Afterwards, a weighting matrix is used to calculate effective, i.e. multilateral, misalignments for all countries in the sample except the base country. The second strategy uses effective instead of bilateral variables already in the estimation. An example for misalignments derived according to strategy 1 is Fischer and Hossfeld (2014), an example for strategy 2 is the current IMF's official assessment method for exchange rates as described in Phillips et al. (2013) and IMF (2017).

For a formal description of **strategy 1**, assume that effective misalignments are to be derived from a panel of real exchange rates and explanatory variables by applying, for example, a fixed effects (FE) estimator. The FE estimation is based on the econometric model

$$q_{it} = x_{it}\beta + \omega_{it} \tag{1}$$

$$\omega_{it} = \mu_i + \varepsilon_{it}$$

where q_{it} denotes the log real exchange rate of country i relative to a base country at time t with an increase representing a domestic real appreciation, x_{it} a $(1 \times K)$ vector of explanatory variables, β is a $(K \times 1)$ vector of coefficients and μ_i a country-specific unobserved effect. The error term ε_{it} is assumed to be i.i.d. Let us assume, for ease of exposition, that the overall sample mean of all the variables is zero, or, equivalently, that all variables are demeaned, i.e. $1/NT \cdot \sum_{i=1}^{N}\sum_{t=1}^{T} q_{it} = 0$ and $1/NT \cdot \sum_{i=1}^{N}\sum_{t=1}^{T} x_{it} = 0$ for each of the K explanatory variables x_{it}.

[3] For a discussion of alternative definitions and concepts of equilibrium exchange rates see Driver and Westaway (2005).

The fixed effects regression consists in performing a pooled OLS estimation of $(q_{it} - \bar{q}_i)$ on $(x_{it} - \bar{x}_i)$ where $\bar{x}_i = 1/T \cdot \sum_{t=1}^{T} x_{it}$ and $\bar{q}_i$ is defined analogously. It yields a coefficient vector $\hat{\beta}_{FE}$ and a number of estimates of the unobserved effects $\hat{\mu}_{i(FE)} = \bar{q}_i - \bar{x}_i\hat{\beta}_{FE}$ where the *FE* subscript indicates that the coefficients are derived from a fixed effects regression. Given the definition above, the bilateral log equilibrium real exchange rate is, in a fixed effects framework, usually computed as

$$\hat{q}^*_{it}(FE, \varepsilon) = x_{it}\hat{\beta}_{FE} + \hat{\mu}_{i(FE)} \tag{2}$$

and the corresponding bilateral misalignment as

$$\hat{m}_{it}(FE, \varepsilon) = \hat{q}_{it}(FE, \varepsilon) - \hat{q}^*_{it}(FE, \varepsilon) = \hat{\varepsilon}_{it(FE)}, \tag{3}$$

where (*FE*, *ε*) indicates that the misalignment is computed from the residuals *ε* of a fixed effects regression. It is generally accepted that only effective real exchange rate measures can convey reliable information on issues such as the competitiveness of an economy or the misalignment of a currency (cf. e.g. Chinn, 2006). In order to obtain an effective log misalignment, $\breve{m}_{it}$, each bilateral misalignment is related to a weighted average of all the partner countries bilateral misalignments,

$$\breve{m}_{it}(FE, \varepsilon) = \breve{\varepsilon}_{it(FE)} = \hat{\varepsilon}_{it(FE)} - \sum_{j=1}^{N} w_{ij}\hat{\varepsilon}_{jt(FE)}, \tag{4}$$

where w_{ij} is the weight of partner country *j* for country *i*, $w_{ii} = 0$ and $\sum_{j=1}^{N} w_{ij} = 1$. The weighting matrix is usually derived from trade data.[4] According to strategy 1, the final effective misalignment is then defined as

$$\breve{M}_{it}(FE, \varepsilon) = e^{\breve{m}_{it}(FE,\varepsilon)}. \tag{5}$$

[4] Since, in policy applications, interest is usually focussed on current effective misalignment values, strategy 1 can be confined to use the most recent vintage of trade data (in $t = T$) instead of applying time-varying trade weights.

Expressed in percentage terms, the misalignment is given by $100 * \left[\breve{M}_{it}(FE, \varepsilon) - 1\right]$%.

As opposed to strategy 1, **strategy 2** uses effective data in the estimation equation. Effective real exchange rates, $\breve{q}_{it}$, and the vector of effective explanatory variables, $\breve{x}_{it}$, are defined as

$$\breve{q}_{it} = q_{it} - \sum_{j=1}^{N} w_{ijt} q_{jt} \quad \text{(6a)}$$

$$\breve{x}_{it} = x_{it} - \sum_{j=1}^{N} w_{ijt} x_{jt}, \quad \text{(6b)}$$

where the w_{ijt} are trade weights specifically for period t with $w_{iit} = 0$ und $\sum_{j=1}^{N} w_{ijt} = 1$. The econometric model

$$\breve{q}_{it} = \breve{x}_{it}\beta + \breve{\omega}_{it}, \quad \text{(7)}$$

where $\breve{\omega}_{it} = \breve{\mu}_i + \breve{\varepsilon}_{it}$, could be estimated using a fixed effects approach, which would yield the unadjusted effective log misalignment

$$\hat{\breve{m}}_{it}(FE, \varepsilon) = \hat{\breve{\varepsilon}}_{it(FE)}. \quad \text{(8)}$$

As elaborated in Faruqee (1998), the unadjusted effective log misalignments in (8) suffer from a redundancy problem which arises from the fact that the N multilateral real exchange rates used in the estimation of (7) are computed from only N-1 independent bilateral real exchange rates. Faruqee (1998) suggests a procedure for rectifying the problem by first observing that one of the eigenvalues of the time-specific matrix of trade weights, W_t, must equal unity because the columns of W_t sum to unity. Denote the elements of the corresponding eigenvector as $\widetilde{w}_{jt}$ where the eigenvector is normalized such that $\sum_{j=1}^{N} \widetilde{w}_{jt} = 1$. The correct (adjusted) effective log misalignments, $\widetilde{m}_{it}$, are then obtained as

$$\widetilde{m}_{it}(FE, \varepsilon) = \tilde{\varepsilon}_{it(FE)} = \hat{\breve{\varepsilon}}_{it(FE)} - \sum_{j=1}^{N} \widetilde{w}_{jt} \hat{\breve{\varepsilon}}_{jt(FE)} \quad \text{(9)}$$

(see also Adler and Grisse, 2017). The final effective misalignment according to strategy 2, $\widetilde{M}_{it}$, is then defined as

$$\widetilde{M}_{it}(FE,\varepsilon) = e^{\widetilde{m}_{it}(FE,\varepsilon)}. \tag{10}$$

Expressed in percentage terms, the effective misalignment is given by $100 * \left[\widetilde{M}_{it}(FE,\varepsilon) - 1\right]\%$.

3. Conventional panel estimators of misalignments as restricted versions of a correlated random effects model

3.1. Conventional panel estimation methods: the contenders

The description of the two strategies for determining an equilibrium real exchange rate and a currency misalignment in the previous section has been cast in the context of an FE estimation. Couharde et al. (2017), Fidora et al. (2017), and Phillips et al. (2013) are recent examples from the literature where such an estimation method has been chosen. Given the definition of an equilibrium exchange rate above, however, the misalignments could, in principle, have been calculated using alternative conventional panel estimation methods (cf. Table 1), such as the random effects (RE) or the between effects (BE) model, for instance.

Table 1: Estimators and misalignment measures considered

Estimator		Misalignment derived from	
		ε	ω
Pooled OLS	(LS)	-	$M_{it}(LS,\omega)$
Fixed effects (within group)	(FE)	$M_{it}(FE,\varepsilon)$	$M_{it}(FE,\omega)$
Between effects	(BE)	$M_{it}(BE,\varepsilon)$	$M_{it}(BE,\omega)$
Random effects	(RE)	$M_{it}(RE,\varepsilon)$	$M_{it}(RE,\omega)$
Correlated random effects	(CR)	$M_{it}(CR,\varepsilon)$	$M_{it}(CR,\omega)$

The RE model estimates equation (1) (or (7))[5] by applying a specific FGLS estimator with a block-diagonal covariance matrix of ω_{it} where the main diagonal elements of each block for a given country i are $\left(\hat{\sigma}_{\varepsilon}^{2} + \hat{\sigma}_{\mu}^{2}\right)$, the off-diagonal elements are $\hat{\sigma}_{\mu}^{2}$, the parameter $\hat{\sigma}_{\varepsilon}^{2}$ is the estimated variance of ε in (1), and $\hat{\sigma}_{\mu}^{2}$ is the estimated variance of μ in (1). The RE estimator can also be obtained by a pooled OLS regression of $\left(q_{it} - \hat{\theta}_{RE}\bar{q}_{i}\right)$ on $\left(x_{it} - \hat{\theta}_{RE}\bar{x}_{i}\right)$ where

$$\hat{\theta}_{RE} = 1 - \sqrt{\frac{\hat{\sigma}_{\varepsilon}^{2}}{\hat{\sigma}_{\varepsilon}^{2} + T\hat{\sigma}_{\mu}^{2}}}. \tag{11}$$

The residuals $\hat{\varepsilon}_{it(RE)}$ (or $\hat{\hat{\varepsilon}}_{it(RE)}$) of the RE regression could be used to compute the misalignment measure $M_{it}(RE, \varepsilon)$ in the same manner as it is done with $\hat{\varepsilon}_{it(FE)}$.[6]

If instead the BE estimator is applied, the time-average $\bar{q}_{i}$ is regressed on $\bar{x}_{i}$ using OLS. If strategy 1 is applied, for instance, the resulting estimates of $\hat{\beta}_{BE}$ and $\hat{\mu}_{i(BE)}$ can be used to first compute time and country-specific residuals $\hat{\varepsilon}_{it(BE)}$ via equation (1) and from that a misalignment measure $M_{it}(BE, \varepsilon)$.

The uncertainty over the question of which panel estimation method to use for the estimation of misalignments is well reflected in Cheung et al. (2007) and Berka et al. (2018). They compute estimates for all three methods, FE, BE, and RE, additionally applying a fourth, a pooled OLS estimate of equation (1). The real effective exchange rate level model of the IMF (2017) essentially uses a pooled OLS-type estimation as well, albeit in the context of strategy 2. By definition, pooled OLS does not split the residual ω_{it} into a country-specific unobserved effect μ_{i} and the error ε_{it}. Therefore, the misalignment is derived from ω_{it} such that the bilateral log misalignment in strategy 1, for instance, is defined as

$$\widehat{m}_{it}(LS, \omega) = \widehat{\omega}_{it(LS)} \tag{12}$$

[5] The following considerations apply regardless of whether $\breve{M}_{it}$ or $\widetilde{M}_{it}$ is used. An assessment of the relative strengths and weaknesses of the two strategies is beyond the scope of the present study.

[6] The random effects procedure yields an estimate of ω_{it}. This expression needs to be split into the country-specific unobserved effect μ_{i} and the error ε_{it}. Following Cameron and Trivedi (2005), p. 738, the unobserved effect is computed as $\hat{\mu}_{i(RE)} = \left[T\hat{\sigma}_{\mu}^{2} / \left(T\hat{\sigma}_{\mu}^{2} + \hat{\sigma}_{\varepsilon}^{2}\right)\right]\bar{\widehat{\omega}}_{i(RE)}$, because this is the best linear unbiased predictor of μ_{i}. Then, the residuals are simply obtained as $\hat{\varepsilon}_{it(RE)} = \widehat{\omega}_{it(RE)} - \hat{\mu}_{i(RE)}$.

where LS indicates that the estimate is obtained by pooled OLS. Using ω_{it} instead of ε_{it} implies that the misalignment measure $M_{it}(LS, \omega)$ – contrary to the previously presented measures – treats the unexplained differences in the relative price levels explicitly as being part of the misalignment. In a similar fashion, misalignment measures based on ω_{it} instead of ε_{it} can be computed for all the other conventional estimation methods: $M_{it}(RE, \omega)$, $M_{it}(BE, \omega)$ and $M_{it}(FE, \omega)$. Misalignment measures based on ω implicitly assume that fundamentals do not explain the unobserved effects at all, while misalignment measures based on ε implicitly assume that the unobserved effects are completely explained by fundamentals.

3.2. Data requirements and the case for an estimation method beyond the conventional ones

The alternative panel estimation methods principally differ in their treatment of the time-series variation and the cross-section variation in the data. The FE method does not consider the cross-section variation of the average levels of the series in the panel, while the BE method exclusively considers this cross-section level variation; the RE and the LS method, finally, use alternative ways to combine both. In most applications of misalignment estimation, panels of real exchange rates have been used that are based on price index data, such as consumption price indices. The broad availability of high-quality price index data at a relatively high frequency has probably contributed to the wide distribution of this approach. Price index-based real exchange rates obviously do not contain any meaningful cross-section level information. In such a case, it is quite natural to compute the misalignment measure $M_{it}(FE, \varepsilon)$ which is based on the residuals $\hat{\varepsilon}_{it}$ of an FE estimation. The FE estimation eliminates the meaningless relative level information from the estimation process, and the use of the residuals eliminates it from the computation of the misalignment.

In contrast, some recent applications such as Cheung et al. (2007), Fischer and Hossfeld (2014), Adler and Grisse (2017), IMF (2017) and Berka et al. (2018) use data containing meaningful cross-section level variation. In particular, they employ relative price level data to measure the real exchange rate q_{it}. Accordingly, the list of regressors x_{it} includes relative level variables such as relative productivity levels, net foreign asset levels and relative old age dependency ratios. Such data "allows one to exploit information from the cross-section" (Adler and Grisse, 2017), such as "the well-known positive cross-sectional relationship between relative price ... and relative per capita income levels" (Cheung et al., 2007). That theory suggests such a relationship in levels, given certain assumptions, is shown inter alia in Fischer and Hossfeld (2014). Yet also from a purely econometric point of view, the inclusion of a meaningful cross-section level variation is potentially important because, in relative price levels and commonly used explanatory variables, this variation is often much higher than their time-series variation.[7] Misalignment estimates based on FE approaches which necessarily exclude the cross-section level variation can potentially suffer from being imprecise (cf. e.g. Wooldridge, 2010, p. 326). Finally, Fischer and Hossfeld (2014) argue that an equilibrium real exchange rate is basically a cross-country concept, and that a predominantly time-series based assessment forgoes potentially essential information. In fact, it must be assumed obligatorily for misalignment measures calculated from real exchange rates based on price index data that, on average over time, there is no misalignment. Putting it differently, misalignment measures derived from data without cross-country level information content must make the rather unrealistic assumption that, on average, the price competitiveness of all countries in the sample is identical.

In sum, the use of relative price levels as an endogenous variable which contains meaningful cross-section level information for determining equilibrium exchange rates is obviously highly advisable. It prompts the question, however, of which panel estimation method should be used. The misalignment measure $M_{it}(FE,\varepsilon)$ is obviously no solution because the cross-country level information in the data is neither used in the estimation of the coefficients β, nor in the computation of the misalignment. In general, the presence of meaningful cross-section level information in the series of the panel implies that, using conventional panel estimation methods, a decision needs to be taken on whether the estimated unobserved effects are treated as being part of the equilibrium real exchange rate (misalignments based on ε) or part of the misalignment (misalignments based on ω).

There is, however, a panel estimation method which allows to isolate the effects of the economic fundamentals on the real exchange rate long-run levels, and thus to split each unobserved effect into two parts, one of which is explained by the fundamentals and the other is not. This is the correlated random effects estimator. In fact, it will be shown that each of the conventional panel estimation methods' different treatments of the time series and cross-section level variation can be interpreted as a testable restriction on this estimator, and that a violation of the restriction leads to a distorted misalignment estimate even if the coefficient estimates are unbiased.

[7] To give an example, the fraction of the between variance of a given variable in the sum of its between and its within variance exceeds 0.7 in all the variables used in the panel of application 2 below (cf. section 4.3 and the Data Appendix), and it exceeds 0.9 in the majority of the variables. The terms of trade, being an index and having a between variance of zero by construction, constitute the only exception.

3.3. The correlated random effects estimator and its properties

A simple panel estimator which explicitly models the impact of both the time series variation of the variables and the levels of the explanatory variables is the correlated random effects (CRE) estimator.[8] The CRE model estimates the equation

$$q_{it} = (x_{it} - \bar{x}_i)\beta_{1CR} + \bar{x}_i\beta_{2CR} + \omega_{it(CR)}$$
$$\omega_{it(CR)} = \mu_{i(CR)} + \varepsilon_{it(CR)} \qquad (13)$$

or, equivalently,

$$q_{it} = x_{it}\beta_{1CR} + \bar{x}_i(\beta_{2CR} - \beta_{1CR}) + \omega_{it(CR)}$$
$$\omega_{it(CR)} = \mu_{i(CR)} + \varepsilon_{it(CR)} \qquad (14)$$

where CR refers to the CRE estimator, and q_{it} and x_{it} necessarily contain level information (i.e. q_{it} and at least some of the x_{it} must not be based on index data), such that q_{it} denotes – more narrowly than in chapter 2 of the study – the log price level of country *i* relative to a base country.[9]

The CRE model goes back to Mundlak (1978). It is extensively covered in several advanced econometrics textbooks (cf. Wooldridge, 2010, chapter 10, or Biørn, 2017, chapter 6). To my knowledge, however, it is apparently hardly ever used in applied econometrics, especially as far as linear applications are concerned.[10] Instead, linear applications of the CRE model are more widely found in non-economics Social Sciences, see, for example, the references given in Bell and Jones (2015) and in Schunck (2013).

The properties of the CRE estimator include the following (see, for instance, Biørn, 2017): 1) Regardless of whether (13) or (14) is estimated, the estimates of β_{1CR}, β_{2CR},

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[8] The term "correlated random effects model" is used, for instance, in Wooldridge (2010), p. 286, or in Cameron and Trivedi (2005), p. 786. Note that the correlated random effects model considered here and the common correlated effects models used, for instance, by Bussière et al. (2010) and Fidora et al. (2017) for the calculation of exchange rate misalignments are quite different concepts. The common correlated effects models augment the explanatory variables by cross section averages in order to address the issue of cross-sectional dependency.

[9] For ease of notation, the variables in equations (13) and (14) are written in terms of bilateral measures as in strategy 1. However, all of these and the following equations also apply to effective measures as in strategy 2, of course.

[10] For non-linear applications in economics, see Jakubson (1988) and Goldbach et al. (2018).

and $\omega_{it(CR)} = \mu_{i(CR)} + \varepsilon_{it(CR)}$ are the same. 2) Regardless of whether a pooled OLS or an RE procedure is used to estimate the CRE model, the estimates of β_{1CR}, β_{2CR}, and $\omega_{it(CR)}$ are the same. 3) Since the (implicit) unobserved effects $\mu_{i(CR)}$ induce autocorrelation in the model, the pooled OLS estimator should only be used if it is computed with robust standard errors. 4) The orthogonality of the between and the within variation in equation (13) implies that $\hat{\beta}_{FE} = \hat{\beta}_{1CR}$ and $\hat{\beta}_{BE} = \hat{\beta}_{2CR}$.

3.4. Conventional panel methods as restricted versions of the CRE approach and biased coefficient estimates

The introduction of the CRE model suggests a natural econometric implementation of the definition of an equilibrium real exchange rate given at the start of section 2: Let us define the (log) equilibrium real exchange rate as the expected value of the (log) real exchange rate conditional on the fundamentals

$$q_{it}^{*} = E(q_{it}|\boldsymbol{x_i}) \tag{15}$$

where $\boldsymbol{x_i} = \{x_{i1}, x_{i2}, \dots, x_{iT}\}$. Within the CRE model, we obtain

$$q_{it}^{*} = E(q_{it}|\boldsymbol{x_i}) = x_{it}\beta_{1CR} + \bar{x}_i(\beta_{2CR} - \beta_{1CR}), \tag{16}$$

which implies that the (log) misalignment is

$$\begin{aligned} m_{it} &= q_{it} - q_{it}^{*} \\ &= q_{it} - x_{it}\beta_{1CR} - \bar{x}_i(\beta_{2CR} - \beta_{1CR}) \\ &= \omega_{it(CR)} \\ &= m_{it}(CR, \omega) \end{aligned} \tag{17}$$

These econometric definitions of an equilibrium real exchange rate and a misalignment suggest themselves because they are the most general measures for these economic concepts. In fact, misalignments computed from the conventional estimators can be interpreted as being restricted versions of misalignments computed from the CRE model: $M_{it}(CR, \omega)$ results from the estimates of ω_{it} either in equation (13) or in equation (14), regardless of whether pooled OLS or random effects is used to estimate the CRE model. Because of $q_{it} = (q_{it} - \bar{q}_i) + \bar{q}_i$ and the orthogonality property mentioned above, $M_{it}(FE, \omega)$ results if, in a pooled OLS estimation of the CRE model (13) or (14), β_{2CR} = 0 is imposed. Similarly, $M_{it}(BE, \omega)$ results if, in this estimation, β_{1CR} = 0 is imposed, and $M_{it}(LS, \omega)$ if β_{2CR} - β_{1CR} = 0 is imposed. Finally, $M_{it}(RE, \omega)$ results if, in a random effects estimation of (13) or (14), β_{2CR} - β_{1CR} = 0 is imposed.

The CRE model combines the FE and the BE model, where, in spite of the omitted variables, $\hat{\beta}_{FE}$ is an unbiased estimator of β_{1CR}, and $\hat{\beta}_{BE}$ is an unbiased estimator of β_{2CR}. The derivation in the Appendix shows, however, that the pooled OLS estimator of β in equation (1), $\hat{\beta}_{LS}$, suffers from an omitted variable bias which is the second term on the right-hand side of

$$\hat{\beta}_{LS} = \hat{\beta}_{1CR} + \hat{\rho}_{LS}\left(\hat{\beta}_{2CR} - \hat{\beta}_{1CR}\right) \tag{18}$$

where the ($K \times K$)-matrix $\hat{\rho}_{LS}$ is defined as

$$\hat{\rho}_{LS} = \left(\sum_{i=1}^{N}\sum_{t=1}^{T} x_{it}{}' x_{it}\right)^{-1}\left(T\sum_{i=1}^{N} \bar{x}_i{}'\bar{x}_i\right). \tag{19}$$

Similarly, the RE estimator of β in equation (1), β_{RE}, suffers from an omitted variable bias, too (for the derivation, see again the Appendix):

$$\hat{\beta}_{RE} = \hat{\beta}_{1CR} + \hat{\rho}_{RE}\left(\hat{\beta}_{2CR} - \hat{\beta}_{1CR}\right) \tag{20}$$

where the ($K \times K$)-matrix $\hat{\rho}_{RE}$ is defined as

$$\hat{\rho}_{RE} = \left[\sum_{i=1}^{N}\sum_{t=1}^{T}\left(x_{it} - \hat{\theta}_{RE}\bar{x}_i\right)'\left(x_{it} - \hat{\theta}_{RE}\bar{x}_i\right)\right]^{-1} \times \left[T\left(1 - \hat{\theta}_{RE}\right)^2 \sum_{i=1}^{N} \bar{x}_i{}'\bar{x}_i\right] \tag{21}$$

and $\hat{\theta}_{RE}$ is defined in equation (11).

3.5. Conventional panel methods as restricted versions of the CRE approach: distorted misalignment estimates

This section derives the effect on misalignment estimates that arises if the restrictions which convert the CRE model into one of the conventional panel estimators are imposed and/or if meaningful cross-section level variation is ignored in the computation of the misalignment. For this purpose, the CRE-based misalignment measure $M_{it}(CR,\omega)$ is taken as the benchmark. If an alternative misalignment estimate is obtained by imposing a restriction on the CRE model and/or the ignorance of the cross-section level variation in the data, the deviation of this misalignment measure from the benchmark misalignment is called a distortion.[11]

As a first step, Table 2 presents the formally derived distortions with respect to bilateral log misalignments. Thus, it shows the differences between the bilateral log misalignment based on each of the alternative estimates and $\widehat{m}_{it}(CR,\omega)$. In the upper part of the table (equations (22a) to (22d)), bilateral log misalignment measures based on the residual $\hat{\varepsilon}_{it}$ are considered. Equations (23a) to (23d) refer to bilateral log misalignment measures based on $\widehat{\omega}_{it} = \hat{\mu}_i + \hat{\varepsilon}_{it}$. To obtain the distortion when using effective variables instead of bilateral ones, i.e. strategy 2 instead of 1, just replace each variable z in the equations by $\check{z}$ such that the bilateral log misalignment $\widehat{m}_{it}$, for instance, becomes the effective log misalignment $\widehat{\check{m}}_{it}$.

In the table, each first line of the equations defines the distortion, while the following line(s) give(s) the distortion formula as derived in the Appendix. Trivially, if a CRE model is estimated, but the estimated unobserved effect which is not explained by the fundamentals, $\hat{\mu}_{i(CR)}$, is erroneously considered as being part of the equilibrium real exchange rate as in (22a), the deviation from the baseline case is simply $\hat{\mu}_{i(CR)}$. As a remark, the correction for $\hat{\bar{\varepsilon}}_{i(CR)}$ is necessary in (22a), for instance, because, in a random effects regression, it is not guaranteed that $\hat{\bar{\varepsilon}}_i = 0$ exactly, such that

$$\bar{q}_i - \bar{x}_i \hat{\beta}_{2CR} = \hat{\mu}_{i(CR)} + \hat{\bar{\varepsilon}}_{i(CR)}. \quad (24)$$

[11] The section considers differences of the form $\widehat{m}_{it}(\cdot) - \widehat{m}_{it}(CR,\omega)$ or $\widehat{M}_{it}(\cdot) - \widehat{M}_{it}(CR,\omega)$, respectively, because it aims at deriving the potential error obtained by imposing a restriction on the estimator. In the study, these differences are called distortions instead of biases. The term bias rather refers to differences of the form $E[\hat{\varphi}] - \varphi$ where φ is usually a parameter and not an expression such as m_{it} or M_{it} which potentially include explanatory variables.

Table 2: Distortions of bilateral log misalignment measures compared to $\hat{m}_{it}(CR, \omega)$

(22a)	$\hat{m}_{it}(CR,\varepsilon) - \hat{m}_{it}(CR,\omega)$	$= \hat{\varepsilon}_{it(CR)} - (\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)})$ $= -(\bar{q}_i - \bar{x}_i\hat{\beta}_{2CR}) + \hat{\bar{\varepsilon}}_{i(CR)}$
(22b)	$\hat{m}_{it}(FE,\varepsilon) - \hat{m}_{it}(CR,\omega)$	$= \hat{\varepsilon}_{it(FE)} - (\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)})$ $= -(\bar{q}_i - \bar{x}_i\hat{\beta}_{2CR})$
(22c)	$\hat{m}_{it}(BE,\varepsilon) - \hat{m}_{it}(CR,\omega)$	$= \hat{\varepsilon}_{it(BE)} - (\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)})$ $= -(\bar{q}_i - \bar{x}_i\hat{\beta}_{2CR}) + (x_{it} - \bar{x}_i)(\hat{\beta}_{1CR} - \hat{\beta}_{2CR})$
(22d)	$\hat{m}_{it}(RE,\varepsilon) - \hat{m}_{it}(CR,\omega)$	$= \hat{\varepsilon}_{it(RE)} - (\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)})$ $= -(\bar{q}_i - \bar{x}_i\hat{\beta}_{2CR}) + (x_{it} - \bar{x}_i)\hat{\rho}_{RE}(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}) + \hat{\bar{\varepsilon}}_{i(RE)}$
(23a)	$\hat{m}_{it}(FE,\omega) - \hat{m}_{it}(CR,\omega)$	$= \hat{\omega}_{it(FE)} - \hat{\omega}_{it(CR)}$ $= (\hat{\mu}_{i(FE)} + \hat{\varepsilon}_{it(FE)}) - (\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)})$ $= -\bar{x}_i(\hat{\beta}_{1CR} - \hat{\beta}_{2CR})$
(23b)	$\hat{m}_{it}(BE,\omega) - \hat{m}_{it}(CR,\omega)$	$= \hat{\omega}_{it(BE)} - \hat{\omega}_{it(CR)}$ $= (\hat{\mu}_{i(BE)} + \hat{\varepsilon}_{it(BE)}) - (\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)})$ $= (x_{it} - \bar{x}_i)(\hat{\beta}_{1CR} - \hat{\beta}_{2CR})$
(23c)	$\hat{m}_{it}(RE,\omega) - \hat{m}_{it}(CR,\omega)$	$= \hat{\omega}_{it(RE)} - \hat{\omega}_{it(CR)}$ $= (\hat{\mu}_{i(RE)} + \hat{\varepsilon}_{it(RE)}) - (\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)})$ $= (x_{it}\hat{\rho}_{RE} - \bar{x}_i)(\hat{\beta}_{1CR} - \hat{\beta}_{2CR})$
(23d)	$\hat{m}_{it}(LS,\omega) - \hat{m}_{it}(CR,\omega)$	$= \hat{\omega}_{it(LS)} - \hat{\omega}_{it(CR)}$ $= \hat{\omega}_{it(LS)} - (\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)})$ $= (x_{it}\hat{\rho}_{LS} - \bar{x}_i)(\hat{\beta}_{1CR} - \hat{\beta}_{2CR})$

Note: Parameters $\hat{\rho}_{RE}$ and $\hat{\rho}_{LS}$ are given by equations (21) and (19), respectively; parameters and variables marked by "*CR*" refer to the RE (GLS) estimates of the CRE model (14). To obtain the distortions when using effective variables in the regression (strategy 2) instead of strategy 1, just replace each variable z by $\check{z}$. For derivations, see the Appendix.

The distortion from computing the misalignment as the residual of an FE regression is again primarily the unobserved effect obtained in the corresponding CRE regression (cf. (22b)).[12] This is the error that is inherent in conventional estimates of the equilibrium

[12] Recall that the unobserved effect obtained from the CRE regression is usually different from the unobserved effect obtained in a FE regression because the CRE regression corrects the FE regression unobserved effect for the impact of the levels of the explanatory variables – insofar as these are different from the impact of the time series components.

real exchange rates which ignore cross-sectional level information, for instance, because they use real exchange rates computed from indices.

If instead the time series variation is eliminated from the regression by employing a BE estimator, the distortion of a misalignment measure based on the residual ε additionally depends on $\hat{\beta}_{1CR} - \hat{\beta}_{2CR}$ (cf. (22c)). The term $\hat{\beta}_{1CR} - \hat{\beta}_{2CR}$ also affects the distortion of the misalignments computed from residuals of conventional RE regressions in (22d), where the last equation of (22d) follows from (20).

Equations (23a) – (23d) show the distortions that arise if cross-section level variation is used in the calculation of the misalignment by assigning the country-specific unobserved effect to the misalignment but a conventional panel estimation method is applied instead of the CRE model. Each of the last lines in equations (23a) – (23d) shows that it is again $\hat{\beta}_{1CR} - \hat{\beta}_{2CR}$ that dominates the distortion. In fact, if $\hat{\beta}_{1CR} = \hat{\beta}_{2CR}$, no distortion arises.

Table 3: Distortions of final effective misalignment measures compared to $\breve{M}_{it}(CR,\omega)$

(25a)	$\breve{M}_{it}(CR,\varepsilon) - \breve{M}_{it}(CR,\omega)$	$= exp(\breve{\varepsilon}_{it(CR)})[1 - exp(\breve{\mu}_{i(CR)})]$
(25b)	$\breve{M}_{it}(FE,\varepsilon) - \breve{M}_{it}(CR,\omega)$	$= exp(\breve{\varepsilon}_{it(CR)})[exp(-\bar{\breve{\varepsilon}}_{i(CR)}) - exp(\breve{\mu}_{i(CR)})]$
(25c)	$\breve{M}_{it}(BE,\varepsilon) - \breve{M}_{it}(CR,\omega)$	$= exp(\breve{\varepsilon}_{it(CR)})\{exp[(\breve{x}_{it} - \bar{\breve{x}}_i)(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}) - \bar{\breve{\varepsilon}}_{i(CR)}] - exp(\breve{\mu}_{i(CR)})\}$
(25d)	$\breve{M}_{it}(RE,\varepsilon) - \breve{M}_{it}(CR,\omega)$	$= exp(\breve{\varepsilon}_{it(CR)})\{exp[(\breve{x}_{it} - \bar{\breve{x}}_i)\hat{\rho}_{RE}(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}) + \bar{\breve{\varepsilon}}_{i(RE)} - \bar{\breve{\varepsilon}}_{i(CR)}] - exp(\breve{\mu}_{i(CR)})\}$
(26a)	$\breve{M}_{it}(FE,\omega) - \breve{M}_{it}(CR,\omega)$	$= exp(\breve{\omega}_{it(CR)})\{exp[-\bar{\breve{x}}_i(\hat{\beta}_{1CR} - \hat{\beta}_{2CR})] - 1\}$
(26b)	$\breve{M}_{it}(BE,\omega) - \breve{M}_{it}(CR,\omega)$	$= exp(\breve{\omega}_{it(CR)})\{exp[(\breve{x}_{it} - \bar{\breve{x}}_i)(\hat{\beta}_{1CR} - \hat{\beta}_{2CR})] - 1\}$
(26c)	$\breve{M}_{it}(RE,\omega) - \breve{M}_{it}(CR,\omega)$	$= exp(\breve{\omega}_{it(CR)})\{exp[(\breve{x}_{it}\hat{\rho}_{RE} - \bar{\breve{x}}_i)(\hat{\beta}_{1CR} - \hat{\beta}_{2CR})] - 1\}$
(26d)	$\breve{M}_{it}(LS,\omega) - \breve{M}_{it}(CR,\omega)$	$= exp(\breve{\omega}_{it(CR)})\{exp[(\breve{x}_{it}\hat{\rho}_{LS} - \bar{\breve{x}}_i)(\hat{\beta}_{1CR} - \hat{\beta}_{2CR})] - 1\}$

Note: Expression $exp(z) = e^z$. Parameters $\hat{\rho}_{RE}$ and $\hat{\rho}_{LS}$ are given by equations (21) and (19), respectively; parameters and variables marked by "*CR*" refer to the RE (GLS) estimates of the CRE model (14). To obtain the distortions when using effective variables in the regression (strategy 2) instead of strategy 1, just replace each variable $\breve{z}$ by $\tilde{z}$. For derivations, see the Appendix.

The distortions of bilateral log misalignments given in Table 2 can be used to compute distortions of the final effective misalignments. In fact, these are of primary interest from an economic policy perspective. Table 3 provides the formulae for the distortions of the final effective misalignments compared to $\breve{M}_{it}(CR,\omega)$, again for strategy 1. The corresponding distortions of misalignment measures which result from strategy 2 are obtained if, in each formula, each variable $\breve{z}$ is replaced by $\tilde{z}$, respectively.

4. Applications

4.1. General remarks

This section considers two misalignment computation procedures from the literature, and applies all the conventional panel estimation methods as well as the CRE model to compute misalignment measures for the countries in the panel. A comparison of the alternative misalignment measures reveals the impact of the panel estimation method on the estimated misalignment. The comparison is based on the difference in the estimated final effective misalignments shown in equations (25a)-(25d) and (26a)-(26d) in Table 3. These values refer, however, to a single observation in the panel, i.e. the deviation of the conventionally estimated effective misalignment from the unrestricted full-information one for a given country at a given year measured in percentage points. Some aggregation or selection is therefore required to obtain meaningful metrics.

The first and most representative metric that will be shown is the mean absolute difference, which aggregates the differences over the entire sample; for equation (25b) and strategy 1, for example, this is computed as

$$MAD(FE,\varepsilon)_{NT} = \frac{1}{NT}\left[\sum_{i=1}^{N}\sum_{t=1}^{T}\left|\breve{M}_{it}(FE,\varepsilon) - \breve{M}_{it}(CR,\omega)\right|\right]. \quad (27)$$

However, policymakers are typically more interested in the current value of a misalignment than in historical ones. Therefore, the mean absolute difference for all the values at time T is shown as a second metric, i.e. for equation (25b) and strategy 1

$$MAD(FE,\varepsilon)_{T} = \frac{1}{N}\left[\sum_{i=1}^{N}\left|\breve{M}_{iT}(FE,\varepsilon) - \breve{M}_{iT}(CR,\omega)\right|\right]. \quad (28)$$

Finally, they may also be interested in the maximum absolute difference at time T, again for equation (25b) and strategy 1

$$AD(FE,\varepsilon)_{Tmax} = \max_i \left| \breve{M}_{iT}(FE,\varepsilon) - \breve{M}_{iT}(CR,\omega) \right|. \tag{29}$$

This value expresses the largest error that has been calculated for current values of all the countries in the sample if, for the computation of a misalignment, a given conventional panel estimation method (in (29) the residuals of a fixed effects estimation) has been used instead of the sum of the unobserved effect and the residual from the unrestricted CRE model.

To be able to estimate the CRE model, the applications considered must not employ index data, but rather data that contain relative level information for the endogenous and at least some of the explanatory variables. Annual relative price level data has therefore been used as the endogenous variable in both applications.

Because the fundamental source of the distortions in misalignments based on conventional panel methods is the correlation between the unobserved effects and the time averages of the explanatory variables, applying a test to this correlation would be helpful in this context. As Wooldridge (2010) points out, the null hypothesis of the Hausman test on the difference between the random and the fixed effects estimates is in fact equivalent to positing that this correlation is zero.[13] Hausman and Taylor (1981) have shown that the Hausman test can alternatively be computed by comparing the between and the fixed effects estimates. This amounts to estimating the CRE model (14) and testing H_0: $\left(\hat{\beta}_{2CR} - \hat{\beta}_{1CR}\right) = 0$ for all variables that vary over i and t. In fact, Wooldridge (2010) highly recommends applying this procedure by using a fully robust Wald statistic because – in contrast to a conventional Hausman test – it is robust against violations of the assumption of the usual random effects block-diagonal covariance matrix.

Therefore, this version of the Hausman test is regularly computed in the applications below. If the null hypothesis is rejected, however, the usual conclusion to turn to a fixed effects regression is generally only valid if the researcher is exclusively interested in the coefficient estimates derived under ignorance of the cross-section level variation in the data. If the analysis aims at broader insights, a rejection of the null suggests the use of a CRE model for estimation. In the present case, which focuses as much on the

[13] See Wooldridge (2010), p. 331. As a caveat, this requires that no time-constant explanatory variables are included in the regression equation.

misalignment estimate as on the coefficient estimate and where cross-section level variation is expected to contribute to the outcome, a rejection implies that the potential distortion in the misalignment is large (see Table 3), which makes it important to use a CRE model for estimation. If, by contrast, the null cannot be rejected there is less of a case for the CRE model.

4.2. Application 1: misalignments according to the productivity approach

As a first application, consider Fischer and Hossfeld's (2014) estimation procedure to obtain misalignments for a panel of advanced and emerging economies.[14] They employ a simple model-oriented equilibrium real exchange rate concept based on the productivity approach which is mostly associated with Balassa (1964) and Samuelson (1964). Therefore, they use only a single explanatory variable which is a country's relative productivity level. In the present application, their preferred measure, labour productivity per hour worked, is used because it is hardly biased by different levels of part-time work across countries.[15] The panel consists of annual data for log relative price and log relative productivity levels in $N = 53$ countries. It is an unbalanced panel in which the series for most countries span the period from 1980 to 2015 ($T_1 = 36$), while for some the observation period starts as late as 1995 ($T_2 = 21$).[16] Fischer and Hossfeld (2014) use strategy 1, a fixed effects regression and the estimated sum of the unobserved effect and the residual for the computation of the misalignment, i.e. they compute the misalignment as $\breve{M}_{it}(FE, \omega)$.

Table 4 compiles results for this application. Coefficient estimates of the various panel estimators are shown in Table 4a. The first thing to note is the wide range of values that is covered by the estimates. The fixed effects estimate of the elasticity amounts to 0.24 and is thus less than half the size of the between effects estimate of 0.52 or the OLS estimate of 0.50. This result contrasts with Fischer and Hossfeld (2014), who use a slightly smaller sample and present estimates for $\hat{\beta}_{FE}$ and $\hat{\beta}_{LS}$, both of which are close to 0.5. Especially the time-series relying fixed effects estimate seems to be relatively unstable. So, the panel estimation method clearly matters for the elasticity estimates.

[14] Low income economies which are mainly responsible for the non-linearity result in the income price level relationship found by Hassan (2016) are not included in the sample so that there is no reason to deviate from a linear specification of the model.
[15] See the Data Appendix for further information.
[16] The results for the linear CRE model also apply to the case of unbalanced panels, as is shown in Woodridge (2009). In unbalanced panels, parameters such as θ in equation (11) differ, however, across countries according to the respective value of T, of course.

Table 4: Misalignment estimation according to the productivity approach
a) Coefficient estimates obtained by alternative panel estimators

Variable	$(\hat{\beta}_{FE} = \hat{\beta}_{1CR})$	$(\hat{\beta}_{BE} = \hat{\beta}_{2CR})$	$(\hat{\beta}_{2CR} - \hat{\beta}_{1CR})$	$\hat{\beta}_{RE}$	$\hat{\beta}_{LS}$
Productivity	0.24**	0.52***	0.29***	0.30***	0.50***

b) Hausman test

Hausman (χ_1^2) = 12.00***

c) Deviation from unrestricted misalignment measure $\breve{M}_{it}(CR, \omega)$ in percentage points

	FE, ε	*BE, ε*	*RE, ε*	*CR, ε*	*FE, ω*	*BE, ω*	*RE, ω*	*LS, ω*
MAD_{NT}	18.80	19.20	18.31	18.23	12.39	2.95	9.50	2.68
MAD_{T}	20.13	22.37	19.96	19.44	12.95	4.31	10.62	4.82
AD_{Tmax}	57.79	82.89	61.54	55.83	71.79	23.57	62.00	28.70

d) Misalignments estimates for the four largest economies in the sample in *T* = 2015 in % (a positive value indicates an overvaluation)

	FE, ε	*BE, ε*	*RE, ε*	*CR, ε*	*FE, ω*	*BE, ω*	*RE, ω*	*LS, ω*	*CR, ω*
USA	-1.67	2.74	-0.32	-2.00	20.93	-5.62	14.09	-3.48	-9.67
China	39.05	20.85	34.19	40.78	8.49	56.71	18.28	51.57	80.28
Japan	-32.77	-27.39	-30.72	-32.37	16.42	1.77	12.80	3.02	-5.78
Germany	-6.89	-4.36	-6.04	-6.98	4.65	-8.62	1.37	-7.49	-11.02

Note: ***, ** denote significance at the 1, 5% level according to country cluster robust standard errors, respectively. Hausman test computed for H_0: $(\hat{\beta}_{2CR} - \hat{\beta}_{1CR}) = 0$. Coefficients $\hat{\beta}_{1CR}$ and $\hat{\beta}_{2CR}$ and the corresponding variances are obtained by estimating the CRE model using equation (13), coefficient $(\hat{\beta}_{2CR} - \hat{\beta}_{1CR})$ and the corresponding variance is obtained by estimating the CRE model using equation (14). The choice of the CRE equation does not affect the misalignment estimates.

Using robust standard errors, all the estimates are significant independently of the panel estimation method used.[17] Since all the coefficient estimates $\hat{\beta}_{1CR}$, $\hat{\beta}_{2CR}$, and $(\hat{\beta}_{2CR} - \hat{\beta}_{1CR})$ are significantly different from zero, all the restrictions considered above are rejected, which implies that the use of none of the conventional panel estimation methods is appropriate. In line with the significance of $(\hat{\beta}_{2CR} - \hat{\beta}_{1CR})$, the Hausman test is clearly rejected. This suggests that, instead of any conventional panel estimation technique, a CRE model should be used to obtain misalignment estimates.

[17] The standard errors are computed as being country-cluster robust. Specific time series issues are not additionally covered in this study, and are beyond its scope. For a corresponding discussion, see, for instance, Bussière et al. (2010).

As illustrated in Table 4c, misalignments computed conventionally (by applying a conventional panel estimation method and/or by ignoring the cross-country level variation in the data) differ substantially on average from those determined in a CRE model where the unobserved effect is included in the misalignment measure. The most widespread procedure, a conventional fixed effects estimation without the use of cross-section level information, yields misalignment estimates that deviate from them by 19 percentage points on average over the whole sample, by 20 percentage points on average in the politically sensitive most recent period and by 58 percentage points for the most seriously affected economy in the most recent period.

As long as the misalignment is computed merely from the residual, *ε*, the average deviation from the unrestricted misalignment measure is large but it is hardly affected by the panel estimation method. The only cases in which the average distortion is relatively small are the between effects estimates in which the unobserved effects are assigned to the misalignment (*BE*, *ω*) and the pooled OLS estimates (*LS*, *ω*). Yet even if these estimation methods are used, the maximum deviation from $\breve{M}_{iT}(CR, \omega)$ in 2015 ranged between 20 and 30 percentage points.

As an illustrative example, Table 4d shows the estimated misalignments for the four largest economies in nominal US dollar terms in 2015, the most recent year of the sample. The substantial distortion from imposing restrictions on a CRE model and/or ignoring the impact of cross-section level variation also arises in the case of the specific four countries considered. A comparison of columns (*FE*, *ε*), (*RE*, *ε*) and (*CR*, *ε*) illustrates again that it does not make much of a difference whether a fixed effects, a random effects or a correlated random effects panel estimator is used for the computation of the misalignment, as long as one thinks that the cross-section level variation in a panel is irrelevant for the assessment of an equilibrium exchange rate.

As soon as it is accepted that cross-section level variation is relevant for an equilibrium exchange rate assessment (i.e. *ω* is used for computation), however, the differences between the misalignment estimates of alternative panel estimators become economically large. Just considering the available conventional misalignment estimates (i.e. all the columns except "*CR*, *ε*" and "*CR*, *ω*"), the misalignment estimates for the USA range between an undervaluation of 3½% and an overvaluation of 21%, for China between an overvaluation of 8½% and 56½%, and for Japan between an undervaluation of 33% and an overvaluation of 16½%. Answering the question raised in the introduction, these differences are large to the point of making the assessment entirely arbitrary. They underline the importance of the present investigation.

4.3. Application 2: misalignments according to an IMF (2017)-type real effective exchange rate level model

Instead of computing a strictly model-based equilibrium exchange rate such as the one based on the productivity approach in the previous section, many applications pursue a more eclectic econometric approach in the sense that the estimate of the equilibrium value includes a multitude of regressors based on a variety of economic models and thoughts. The equilibrium exchange rates derived in such approaches are often termed behavioural equilibrium exchange rates (BEERs, cf. e.g. Clark and MacDonald, 1999).

In the following, a BEER approach will be presented as a second application which is close to but not identical with the IMF's (2017) real effective exchange rate (REER) level model. This model has obtained a significant political importance because, since 2015, the IMF employs it for the regular assessment of the external balances of 40 economies in the framework of the EBA methodology (see Phillips et al., 2013). As in the IMF's REER level model, the present application uses strategy 2 to compute misalignments for a balanced panel of the N = 40 economies exogenously given by the IMF. The present application differs from the IMF's REER model, however, in employing a reduced number of widely available and commonly used explanatory variables. The avoidance of both instrumental variables and interaction terms simplifies the analysis. In such a simplified setting, the IMF REER level model boils down to a pooled OLS estimate of the misalignment, i.e. to $\widetilde{M}_{it}(LS, \omega)$.[18]

The following explanatory variables are used in the analysis:[19] in accordance with the first application, a productivity variable should be included. Since the preferred measure, productivity per hour worked, is not available for South Africa prior to 2001, per capita GDP measured in PPP terms has been used as a proxy. Apart from that, the list of explanatory variables comprises the old age dependency ratio, government consumption per GDP, net foreign assets per GDP, trade openness, the change in reserves per GDP capturing foreign exchange intervention and finally the terms of trade. The endogenous variable is the relative price level based on the same data as in application 1. The observation period of the balanced panel spans 1995-2015, i.e. T = 21.

[18] As already mentioned, the IMF uses a two-stage least squares regression in fact in order to account for the instrumental variables.

[19] The Data Appendix provides more information on the data, especially about sources and comparisons with the data employed in IMF (2017) and Phillips et al. (2013).

Table 5: Misalignment estimation according to an IMF (2017)-type BEER approach: the unrestricted model

a) Coefficient estimates obtained by alternative panel estimators

Variable	$(\hat{\beta}_{FE} = \hat{\beta}_{1CR})$	$(\hat{\beta}_{BE} = \hat{\beta}_{2CR})$	$(\hat{\beta}_{2CR} - \hat{\beta}_{1CR})$	$\hat{\beta}_{RE}$	$\hat{\beta}_{LS}$
GDP per capita	0.65***	0.44***	-0.21***	0.64***	0.51***
Old age dependency ratio	0.002	0.01**	0.01	0.0004	0.006*
Gov. consumption per GDP	0.02***	0.007	-0.01	0.02***	0.009
Net foreign assets per GDP	-0.07**	0.19***	0.26***	-0.06*	0.10**
Openness	-0.003**	-0.0006	0.002	-0.003***	-0.001*
Reserves per GDP	0.05	-0.81*	-0.86*	0.05	-0.38
Terms of trade	0.54***	-	-	0.53***	0.53***

b) Hausman test

Hausman (χ^2_6) = 19.45***

c) Deviation from unrestricted misalignment measure $\widetilde{M}_{it}(CR, \omega)$ in percentage points

	FE, ε	*BE, ε*	*RE, ε*	*CR, ε*	*FE, ω*	*BE, ω*	*RE, ω*	*LS, ω*
MAD_{NT}	12.29	14.43	12.15	12.14	9.76	7.64	8.64	4.74
MAD_{T}	12.55	14.65	12.29	12.40	10.00	12.04	8.97	6.68
AD_{Tmax}	50.95	62.11	49.53	50.12	33.48	101.88	28.20	46.84

d) Misalignments estimates for the four largest economies in the sample in *T* = 2015 in % (a positive value indicates an overvaluation)

	FE, ε	*BE, ε*	*RE, ε*	*CR, ε*	*FE, ω*	*BE, ω*	*RE, ω*	*LS, ω*	*CR, ω*
USA	11.49	14.75	10.76	11.31	-16.12	2.18	-15.04	-3.08	-0.72
China	-3.08	15.73	-1.87	-2.80	37.41	42.09	32.45	33.16	19.00
Japan	-18.77	-32.41	-18.01	-18.65	-6.41	-24.90	-2.91	-13.07	-9.73
Germany	-1.64	-14.96	-1.99	-1.78	-3.51	-23.42	-2.74	-15.81	-11.42

Note: ***, **, * denote significance at the 1, 5, 10% level according to country cluster robust standard errors, respectively. Hausman test computed for H_0: $(\hat{\beta}_{2CR} - \hat{\beta}_{1CR}) = 0$. Coefficients $\hat{\beta}_{1CR}$ and $\hat{\beta}_{2CR}$ and the corresponding variances are obtained by estimating the CRE model using equation (13), coefficient $(\hat{\beta}_{2CR} - \hat{\beta}_{1CR})$ and the corresponding variance is obtained by estimating the CRE model using equation (14). The choice of the CRE equation does not affect the misalignment estimates.

Table 6: Misalignment estimation according to an IMF (2017)-type BEER approach: the restricted model
a) Coefficient estimates obtained by alternative panel estimators

Variable	$(\hat{\beta}_{FE} = \hat{\beta}_{1CR})$	$(\hat{\beta}_{BE} = \hat{\beta}_{2CR})$	$\hat{\beta}_{RE}$	$\hat{\beta}_{LS}$
GDP per capita	0.65***	0.43***	0.64***	0.47***
Old age dependency ratio	-	0.01***	0.002	0.01**
Gov. consumption per GDP	0.02***	-	0.02***	0.004
Net foreign assets per GDP	-0.07**	0.19***	-0.06*	0.10*
Openness	-0.003**	-	-0.003**	-0.002
Reserves per GDP	-	-0.98***	0.03	-0.61*
Terms of trade	0.53***	-	0.54***	0.52***

b) Deviation from unrestricted misalignment measure $\widetilde{M}_{it}(CR, \omega)$ in percentage points

	FE, ε	*BE, ε*	*RE, ε*	*CR, ε*	*FE, ω*	*BE, ω*	*RE, ω*	*LS, ω*
MAD_{NT}	12.93	14.96	12.78	12.78	10.42	8.14	10.10	5.45
MAD_{T}	13.14	15.90	13.11	12.98	10.27	13.22	10.06	8.26
AD_{Tmax}	49.55	72.71	49.13	48.76	26.94	119.20	28.06	63.48

c) Misalignments estimates for the four largest economies in the sample in *T* = 2015 in % (a positive value indicates an overvaluation)

	FE, ε	*BE, ε*	*RE, ε*	*CR, ε*	*FE, ω*	*BE, ω*	*RE, ω*	*LS, ω*	*CR, ω*
USA	11.16	13.42	11.12	11.00	-10.05	1.03	-8.66	-1.05	-0.99
China	-3.53	17.54	-1.81	-3.25	31.84	48.35	32.54	39.58	21.76
Japan	-17.31	-33.16	-18.85	-17.20	11.71	-25.80	6.93	-14.30	-8.21
Germany	-1.88	-16.29	-2.00	-2.04	-2.42	-26.29	-3.04	-19.00	-13.61

Note: ***, ** denote significance at the 1, 5% level according to country cluster robust standard errors, respectively. Coefficients $\hat{\beta}_{1CR}$ and $\hat{\beta}_{2CR}$ and the corresponding variances are obtained by estimating the CRE model using equation (13).

Table 5a shows that the coefficient estimates and significance values differ substantially across estimation methods. Apart from the terms of trade, only the coefficient of GDP per capita is always statistically significant and consistently signed.[20] This is in line with application 1 and underscores the importance of including a proxy for productivity in real exchange rate regressions. While the statistical significance and the sign of the other explanatory variables depend on the estimator, it should be noted that the CRE approach, which estimates both the effect of the time-series and the cross-section level variation in the variables as $\hat{\beta}_{1CR}$ and $\hat{\beta}_{2CR}$, respectively, yields evidence of a significant influence for each of the variables considered.

With one exception, all the significant coefficient estimates have the expected sign: a rise in relative productivity proxied by GDP per capita raises relative price levels as is implied by the productivity approach. A higher old age dependency ratio increases the share of the economically inactive population, which decreases net savings and appreciates real exchange rates. Accelerating government consumption per GDP exerts the same effect on net savings justifying the positive coefficient. A higher value of net foreign assets per GDP generates c.p. larger capital inflows which appreciate the domestic currency in nominal and real terms. The expected positive sign is found in the cross-section level variation of the data (i.e. $\hat{\beta}_{BE} = \hat{\beta}_{2CR}$) and by using OLS. The fixed effects (i.e. $\hat{\beta}_{FE} = \hat{\beta}_{1CR}$) and the simple random effects approach suggest a significantly negative sign. In the CRE model, the total effect will be dominated by the cross-section level variation because the corresponding coefficient is larger and because the between variation of the NFA variable is nearly double that of the within variation. However, the result once more illustrates the importance of accounting for the cross-section level variation in the data instead of ignoring it as is done by the fixed effects estimator.

The IMF considers trade openness as proxy for trade liberalisation, a greater degree of which lowers the domestic price of tradable goods and thus depreciates the real exchange rate (cf. Phillips et al., 2013). As far as an increasing build-up of reserves per GDP is a reflection of intensified foreign exchange intervention, this should exert a downward pressure on the domestic exchange rate and thus result in a real depreciation. An improvement in the terms of trade, finally, should c.p. also improve the trade balance and thus tend to appreciate the domestic currency in real terms as in IMF (2017).

As in application 1, Table 5b shows that the Hausman test is safely rejected. This suggests that the CRE model is the appropriate method for obtaining misalignment estimates without severe distortions. Table 5c gives evidence that misalignments based on conventional estimators again deviate considerably on average from misalignments

[20] The terms of trade are an index variable. Therefore, their time averages are meaningless, and $\hat{\beta}_{BE} = \hat{\beta}_{2CR}$ cannot be sensibly calculated. See the Data Appendix.

determined in a CRE model, where the unobserved effect is included in the misalignment measure. While still displaying substantial deviations, pooled OLS-based misalignments are closest to the CRE-based ones, although they are computed from biased coefficient estimates.

The exemplary misalignments provided in Table 5d illustrate once again the tremendous influence of the chosen panel estimator on the misalignment estimates. Compared to the estimates of application 1 shown in Table 4d, the CRE model using ω (last column) assesses the US dollar less favourably and the renminbi and the yen more so.[21]

Table 6 gives the results of a general-to-specific exercise where the least significant variables have been eliminated from the CRE model in successive steps. The insignificance of the entire set of all the eliminated variables has been confirmed by a Wald test. Columns two ($\hat{\beta}_{1CR}$) and three ($\hat{\beta}_{2CR}$) of Table 6a comprise the estimated coefficients of the resulting CRE model. Only for GDP per capita and net foreign assets per GDP, do both the time as well as the cross-section level variation exert a (statistically significant) influence on relative price levels. Only the within variation in the variables government consumption per GDP, trade openness and the terms of trade affects the real exchange rates significantly, while for the old age dependency ratio and the change in reserves per GDP, it is only the cross-section level variation.[22] Interestingly, the figures in Table 6a also imply that conventional panel estimators would have detected the significance of only a subset of the variables considered. While both a fixed effects and a random effects estimator assess the variables with a pure cross-section level effect as insignificant, the between group and the pooled OLS estimator ignore the significance of at least two variables with a pure within variation effect. This implies that a CRE model does not only provide estimates of the misalignment without distortions, it can also be superior to conventional panel estimators in identifying significant relationships for the set of explanatory variables.

5. Conclusions

The present study considers two questions. First, which panel estimation method should be used to estimate an equilibrium real effective exchange rate (REER) or a

[21] This illustrates that, apart from the panel estimation method, the choice of the equilibrium concept and the ensuing selection of explanatory variables can also have a substantial impact on a misalignment estimate. This topic is, however, not further explored in the present study.

[22] Interestingly, Phillips et al. (2013) already argue that it is a challenge “that the cross-sectional (between country) variation of reserve accumulation is twice its time variation within countries, making the effect difficult to detect under fixed effects estimation” (p. 25).

corresponding misalignment? Second, are the deviations between the misalignment estimates of alternative panel estimators economically significant?

In response to question 1, the study suggests to use a correlated random effects (CRE) model in the spirit of Mundlak (1978) for the panel estimation of an equilibrium REER. This model estimates separate coefficients for the deviation from the time average of a variable and for its time average (the cross-section level variation). Conventional panel estimation methods such as the fixed effects, between effects, random effects or pooled OLS estimator, which estimate only a single coefficient per variable, can be interpreted as restricted versions of the CRE model. If these restrictions are empirically violated, the resulting deviation of the conventionally estimated misalignment from the CRE model-based one is distorted. The study derives such distortions for each of the conventional panel estimation methods.[23]

In response to question 2, two applications of equilibrium REER estimates adapted from the literature show that the distortion is often very large; the deviation between the misalignment assessments derived from a conventional panel estimation method and from the CRE model typically exceeds 10 percentage points, on average. The conventional model with the smallest distortion is mostly the simple pooled OLS approach.

To give examples for the reasons behind the large distortions, the two coefficients for the relative productivity levels typically differ significantly from each other, and many variables significantly affect the real exchange rate either only in the cross-section level or only through the time variation. Therefore, using a CRE model can also help identifying significant explanatory variables.

Finally, distortions such as the ones derived here can generally occur, in principle, if cross-section level information in the panel is supposed to play a role. Therefore, CRE models may not only be recommendable in the case of REER misalignment computation but also in the panel estimation in a broad range of economic models where cross-section level variation is supposed to play a role. This applies, for instance, to equilibrium current account assessments such as the ones in Phillips et al. (2013) and IMF (2017). A corresponding exploration is, however, left for future research.

[23] The case for the CRE model depends, of course, on the acceptance of the idea put forward by Cheung et al. (2007), Fischer and Hossfeld (2014) IMF (2017) and Berka et al. (2018) among others that cross-section information should not be ignored in equilibrium real exchange rate panel regressions, which implies, for instance, that relative price levels instead of relative price indices should be used as endogenous variable.

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Appendix

1. Mathematical Appendix

Derivation of the bias in the coefficient estimates of the pooled OLS and the RE models compared with the CRE model

Suppose equation (14) represents the true model to be estimated by pooled OLS. Instead of (14), however, equation (1) is erroneously estimated by pooled OLS. Then, $\bar{x}_i$ must be treated as an omitted variable, $(\beta_{2CR} - \beta_{1CR})$ is its unbiased coefficient, and β_{1CR} is the unbiased coefficient that one tries to obtain by estimating β_{LS}. Now apply the general omitted variable formula (cf. e.g. Greene, 2017, p. 59) and recall that, for pooled OLS, $\sum_{i=1}^{N}\sum_{t=1}^{T} x_{it}'\widehat{\omega}_i = 0$. This yields

$$\begin{aligned}\hat{\beta}_{LS} &= \hat{\beta}_{1CR} + \left(\sum_{i=1}^{N}\sum_{t=1}^{T} x_{it}'x_{it}\right)^{-1}\Big[\left(\sum_{i=1}^{N}\sum_{t=1}^{T} x_{it}'\bar{x}_i\right)\left(\hat{\beta}_{2CR} - \hat{\beta}_{1CR}\right) + \\ &\quad \sum_{i=1}^{N}\sum_{t=1}^{T} x_{it}'\widehat{\omega}_i\Big] \\ &= \hat{\beta}_{1CR} + \left(\sum_{i=1}^{N}\sum_{t=1}^{T} x_{it}'x_{it}\right)^{-1}\left(\sum_{i=1}^{N}\sum_{t=1}^{T} x_{it}'\bar{x}_i\right)\left(\hat{\beta}_{2CR} - \hat{\beta}_{1CR}\right) \\ &= \hat{\beta}_{1CR} + \left(\sum_{i=1}^{N}\sum_{t=1}^{T} x_{it}'x_{it}\right)^{-1}\left(\sum_{i=1}^{N}\sum_{t=1}^{T} \bar{x}_i'\bar{x}_i\right)\left(\hat{\beta}_{2CR} - \hat{\beta}_{1CR}\right). \qquad \text{(A1)}\end{aligned}$$

From (A1) and

$$\sum_{t=1}^{T} \bar{x}_i'\bar{x}_i = T\bar{x}_i'\bar{x}_i \qquad \text{(A2)}$$

equations (18) and (19) follow.

Since RE and pooled OLS estimates of the CRE model (14) yield the same coefficients, pooled OLS and RE estimates of (1) can be treated as restricted versions of the same CRE model where, in both cases, the restriction $(\beta_{2CR} - \beta_{1CR}) = 0$ is imposed. An RE estimate of (14) is equivalent to the pooled OLS estimate of

$$\begin{aligned}q_{it} - \hat{\theta}_{CR}\bar{q}_i \\ &= \left(x_{it} - \hat{\theta}_{CR}\bar{x}_i\right)\beta_{1CR} + \left(\bar{x}_i - \hat{\theta}_{CR}\bar{x}_i\right)(\beta_{2CR} - \beta_{1CR}) + \omega_{it(CR)} - \hat{\theta}_{CR}\bar{\omega}_{i(CR)} \qquad \text{(A3)}\end{aligned}$$

where $\hat{\theta}_{CR}$ is defined as $\hat{\theta}_{RE}$ in equation (11). If instead of (A3), equation (1) is erroneously estimated by an RE procedure, this would amount to the pooled OLS estimation of

$$q_{it} - \hat{\theta}_{RE}\bar{q}_i = \left(x_{it} - \hat{\theta}_{RE}\bar{x}_i\right)\beta_{RE} + \omega_{it(RE)} - \hat{\theta}_{RE}\bar{\omega}_{i(RE)}. \qquad \text{(A4)}$$

Again, the unbiased coefficient of the omitted variable is $(\beta_{2CR} - \beta_{1CR})$, and β_{1CR} is the unbiased coefficient that one tries to obtain by estimating β_{RE}. Applying the omitted variable formula yields

$$\hat{\beta}_{RE} = \hat{\beta}_{1CR} + \left[\sum_{i=1}^{N}\sum_{t=1}^{T}(x_{it} - \hat{\theta}_{RE}\bar{x}_i)'(x_{it} - \hat{\theta}_{RE}\bar{x}_i)\right]^{-1}$$
$$\left[\sum_{i=1}^{N}\sum_{t=1}^{T}(x_{it} - \hat{\theta}_{RE}\bar{x}_i)'(\bar{x}_i - \hat{\theta}_{RE}\bar{x}_i)\right](\hat{\beta}_{2CR} - \hat{\beta}_{1CR}). \quad \text{(A5)}$$

$$\left[\sum_{i=1}^{N}\sum_{t=1}^{T}(x_{it} - \hat{\theta}_{RE}\bar{x}_i)'(\bar{x}_i - \hat{\theta}_{RE}\bar{x}_i)\right]$$
$$=\left[\sum_{i=1}^{N}\sum_{t=1}^{T}(x_{it} - \hat{\theta}_{RE}\bar{x}_i)'(1 - \hat{\theta}_{RE})\bar{x}_i\right]$$
$$=\left[\sum_{i=1}^{N}\sum_{t=1}^{T} x_{it}{}'\bar{x}_i - \sum_{i=1}^{N}\sum_{t=1}^{T}\hat{\theta}_{RE}\bar{x}_i{}'\bar{x}_i\right](1 - \hat{\theta}_{RE})$$
$$=\left[\sum_{i=1}^{N} T\bar{x}_i{}'\bar{x}_i - \sum_{i=1}^{N} T\hat{\theta}_{RE}\bar{x}_i{}'\bar{x}_i\right](1 - \hat{\theta}_{RE})$$
$$=\left(\sum_{i=1}^{N} T\bar{x}_i{}'\bar{x}_i\right)(1 - \hat{\theta}_{RE})^2. \quad \text{(A6)}$$

Combining (A5) and (A6) yields the equations (20) and (21).

Derivation of equations (22a)-(22d) and (23a)-(23d) in Table 2

Equation (22a) results from (24).

For the derivation of equation (22b), given (22a) it is sufficient to show that

$$\hat{\varepsilon}_{it(CR)} - \hat{\bar{\varepsilon}}_{i(CR)} = \hat{\varepsilon}_{it(FE)}. \quad \text{(A7)}$$

The residual $\hat{\varepsilon}_{it(FE)}$ of an FE regression of equation (1) is given by

$$\hat{\varepsilon}_{it(FE)} = q_{it} - \bar{q}_i - (x_{it} - \bar{x}_i)\hat{\beta}_{FE}. \quad \text{(A8)}$$

It is well known that $\hat{\beta}_{FE} = \hat{\beta}_{1CR}$ (cf. section 3.3 and Biørn, 2017), such that

$$\hat{\varepsilon}_{it(FE)} = q_{it} - \bar{q}_i - (x_{it} - \bar{x}_i)\hat{\beta}_{1CR}. \quad \text{(A9)}$$

Extending (A9) by $\bar{x}_i\hat{\beta}_{2CR}$ yields

$$\hat{\varepsilon}_{it(FE)} = q_{it} - (x_{it} - \bar{x}_i)\hat{\beta}_{1CR} - \bar{x}_i\hat{\beta}_{2CR} - (\bar{q}_i - \bar{x}_i\hat{\beta}_{2CR}) \quad \text{(A10)}$$

Combining equation (A10) with equation (24) yields

$$\hat{\varepsilon}_{it(FE)} = q_{it} - (x_{it} - \bar{x}_i)\hat{\beta}_{1CR} - \bar{x}_i\hat{\beta}_{2CR} - (\hat{\mu}_{i(CR)} + \hat{\bar{\varepsilon}}_{i(CR)}). \quad \text{(A11)}$$

Equations (A11) and (13) result in (A7).

For the derivation of equation (22c), start with the definition of $\hat{\varepsilon}_{it(BE)}$ according to (1),

$$\hat{\varepsilon}_{it(BE)} = q_{it} - x_{it}\hat{\beta}_{BE} - \hat{\mu}_{i(BE)} \quad \text{(A12)}$$
$$= q_{it} - x_{it}\hat{\beta}_{BE} - (\bar{q}_i - \bar{x}_i\hat{\beta}_{BE}).$$

It is well known that $\hat{\beta}_{BE} = \hat{\beta}_{2CR}$ (cf. section 3.3 and Biørn, 2017), such that

$$\hat{\varepsilon}_{it(BE)} = q_{it} - \bar{q}_i - (x_{it} - \bar{x}_i)\hat{\beta}_{2CR}. \quad \text{(A13)}$$

Inserting the CRE estimate of equation (13) into (A13) yields

$$\hat{\varepsilon}_{it(BE)} = (x_{it} - \bar{x}_i)\hat{\beta}_{1CR} + \bar{x}_i\hat{\beta}_{2CR} + \left(\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)}\right) - \bar{q}_i - (x_{it} - \bar{x}_i)\hat{\beta}_{2CR}. \quad \text{(A14)}$$

Equation (A14) is equivalent to (22c).

For the derivation of (22d), note first that the residual of an RE estimate of equation (1) is

$$\hat{\varepsilon}_{it(RE)} = q_{it} - x_{it}\hat{\beta}_{RE} - \hat{\mu}_{i(RE)}. \quad \text{(A15)}$$

As has already been noted in relation with equation (24), it is not guaranteed, in a random effects regression, that $\bar{\hat{\varepsilon}}_i = 0$ exactly, such that

$$\hat{\varepsilon}_{it(RE)} = q_{it} - x_{it}\hat{\beta}_{RE} - \left(\bar{q}_i - \bar{x}_i\hat{\beta}_{RE} - \bar{\hat{\varepsilon}}_{i(RE)}\right). \quad \text{(A16)}$$

Inserting the CRE estimate of equation (13) into (A16) and re-arranging terms yields

$$\begin{aligned}\hat{\varepsilon}_{it(RE)} &= (x_{it} - \bar{x}_i)\hat{\beta}_{1CR} + \bar{x}_i\hat{\beta}_{2CR} + \left(\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)}\right) \\ &\quad - x_{it}\hat{\beta}_{RE} - \left(\bar{q}_i - \bar{x}_i\hat{\beta}_{RE} - \bar{\hat{\varepsilon}}_{i(RE)}\right)\end{aligned} \quad \text{(A17)}$$

$$\begin{aligned}&\hat{\varepsilon}_{it(RE)} - \left(\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)}\right) \\ &\quad = -\left(\bar{q}_i - \bar{x}_i\hat{\beta}_{2CR}\right) + (x_{it} - \bar{x}_i)\left(\hat{\beta}_{1CR} - \hat{\beta}_{RE}\right) + \bar{\hat{\varepsilon}}_{i(RE)}\end{aligned} \quad \text{(A18)}$$

Combining this with (20) yields the second equation of (22d).

In each of (23a) – (23d), the first two equations are definitions. For the derivation of the last equation of (23a), note that, according to (1) and (14),

$$\begin{aligned}&\left(\hat{\mu}_{i(FE)} + \hat{\varepsilon}_{it(FE)}\right) - \left(\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)}\right) \\ &\quad = q_{it} - x_{it}\hat{\beta}_{FE} - \left[q_{it} - x_{it}\hat{\beta}_{1CR} - \bar{x}_i\left(\hat{\beta}_{2CR} - \hat{\beta}_{1CR}\right)\right].\end{aligned} \quad \text{(A19)}$$

This is equivalent to the third equation of (23a) since $\hat{\beta}_{FE} = \hat{\beta}_{1CR}$.

The third equation of (23b) results analogously using (1), (14) and $\hat{\beta}_{BE} = \hat{\beta}_{2CR}$:

$$\begin{aligned}&\left(\hat{\mu}_{i(BE)} + \hat{\varepsilon}_{it(BE)}\right) - \left(\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)}\right) \\ &\quad = q_{it} - x_{it}\hat{\beta}_{BE} - \left[q_{it} - x_{it}\hat{\beta}_{1CR} - \bar{x}_i\left(\hat{\beta}_{2CR} - \hat{\beta}_{1CR}\right)\right] \\ &\quad = q_{it} - x_{it}\hat{\beta}_{2CR} - \left[q_{it} - x_{it}\hat{\beta}_{1CR} - \bar{x}_i\left(\hat{\beta}_{2CR} - \hat{\beta}_{1CR}\right)\right] \\ &\quad = (x_{it} - \bar{x}_i)\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right).\end{aligned} \quad \text{(A20)}$$

To derive the third equation of (23c), use first (1) and (14) again, before applying equation (20):

$$\left(\hat{\mu}_{i(RE)} + \hat{\varepsilon}_{it(RE)}\right) - \left(\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)}\right)$$

$$= q_{it} - x_{it}\hat{\beta}_{RE} - [q_{it} - x_{it}\hat{\beta}_{1CR} - \bar{x}_i(\hat{\beta}_{2CR} - \hat{\beta}_{1CR})]$$

$$= x_{it}(\hat{\beta}_{1CR} - \hat{\beta}_{RE}) - \bar{x}_i(\hat{\beta}_{1CR} - \hat{\beta}_{2CR})$$

$$= x_{it}[\hat{\beta}_{1CR} - \hat{\beta}_{1CR} - \hat{\rho}_{RE}(\hat{\beta}_{2CR} - \hat{\beta}_{1CR})] - \bar{x}_i(\hat{\beta}_{1CR} - \hat{\beta}_{2CR})$$

$$= (x_{it}\hat{\rho}_{RE} - \bar{x}_i)(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}). \quad \text{(A21)}$$

In the same vain, the third equation of (23d) can be derived from (1), (14) and (18):

$$\hat{\omega}_{it(LS)} - (\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)})$$

$$= q_{it} - x_{it}\hat{\beta}_{LS} - [q_{it} - x_{it}\hat{\beta}_{1CR} - \bar{x}_i(\hat{\beta}_{2CR} - \hat{\beta}_{1CR})]$$

$$= x_{it}(\hat{\beta}_{1CR} - \hat{\beta}_{LS}) - \bar{x}_i(\hat{\beta}_{1CR} - \hat{\beta}_{2CR})$$

$$= x_{it}[\hat{\beta}_{1CR} - \hat{\beta}_{1CR} - \hat{\rho}_{LS}(\hat{\beta}_{2CR} - \hat{\beta}_{1CR})] - \bar{x}_i(\hat{\beta}_{1CR} - \hat{\beta}_{2CR})$$

$$= (x_{it}\hat{\rho}_{LS} - \bar{x}_i)(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}). \quad \text{(A22)}$$

Since strategy 2 estimates the same equations as strategy 1 with the sole difference of using effective instead of bilateral variables, all the derivations apply also to strategy 2; just replace each variable z by $\breve{z}$. To give an example, equation (22d) applied to strategy 2 becomes

$$\hat{\breve{m}}_{it}(RE,\varepsilon) - \hat{\breve{m}}_{it}(CR,\omega) = \hat{\breve{\varepsilon}}_{it(RE)} - (\hat{\breve{\mu}}_{i(CR)} + \hat{\breve{\varepsilon}}_{it(CR)})$$

$$= -(\bar{\breve{q}}_i - \bar{\breve{x}}_i\hat{\beta}_{2CR}) + (\breve{x}_{it} - \bar{\breve{x}}_i)\hat{\rho}_{RE}(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}) + \hat{\bar{\breve{\varepsilon}}}_{i(RE)} \quad \text{(A23)}$$

where the parameters $\hat{\beta}_{1CR}$, $\hat{\beta}_{2CR}$ and $\hat{\rho}_{RE}$ are obtained in an estimation procedure that uses effective variables.

Derivation of equations (25a)-(25d) and (26a)-(26d) in Table 3

For the derivation of equations (25a)-(25d) and (26a)-(26d), note first that, according to equations (3), (4) and (5), each effective misalignment $\breve{M}_{it}(\Gamma\varepsilon,\varepsilon)$ and $\breve{M}_{it}(\Gamma\omega,\omega)$ is related to the corresponding estimated bilateral log misalignment $\hat{m}_{it}(\Gamma\varepsilon,\varepsilon)$ and $\hat{m}_{it}(\Gamma\omega,\omega)$ in the following way:

$$\breve{M}_{it}(\Gamma\varepsilon,\varepsilon) = exp[\hat{m}_{it}(\Gamma\varepsilon,\varepsilon) - \textstyle\sum_{j=1}^{N} w_{ij}\hat{m}_{jt}(\Gamma\varepsilon,\varepsilon)]$$

$$= exp(\hat{\varepsilon}_{it(\Gamma\varepsilon)} - \textstyle\sum_{j=1}^{N} w_{ij}\hat{\varepsilon}_{jt(\Gamma\varepsilon)}), \quad \text{(A24)}$$

$$\breve{M}_{it}(\Gamma\omega,\omega) = exp[\hat{m}_{it}(\Gamma\omega,\omega) - \textstyle\sum_{j=1}^{N} w_{ij}\hat{m}_{jt}(\Gamma\omega,\omega)]$$

$$= exp(\hat{\omega}_{it(\Gamma\omega)} - \textstyle\sum_{j=1}^{N} w_{ij}\hat{\omega}_{jt(\Gamma\omega)}) \quad \text{(A25)}$$

where $exp(z) = e^z$, the set of estimation methods $\Gamma\varepsilon = \{CR, FE, BE, RE\}$ and $\Gamma\omega = \{CR, FE, BE, RE, LS\}$.

For the derivation of (25a), set $\Gamma\varepsilon = CR$ in (A24) and $\Gamma\omega = CR$ in (A25) to obtain

$$\breve{M}_{it}(CR,\varepsilon) - \breve{M}_{it}(CR,\omega)$$

$$\begin{aligned}
&= exp\left(\hat{\varepsilon}_{it(CR)} - \textstyle\sum_{j=1}^{N} w_{ij}\hat{\varepsilon}_{jt(CR)}\right) \\
&\quad -exp\left[\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)} - \textstyle\sum_{j=1}^{N} w_{ij}\left(\hat{\mu}_{j(CR)} + \hat{\varepsilon}_{jt(CR)}\right)\right] \\
&= exp\left(\hat{\varepsilon}_{it(CR)} - \textstyle\sum_{j=1}^{N} w_{ij}\hat{\varepsilon}_{jt(CR)}\right) \\
&\quad -exp\left(\hat{\varepsilon}_{it(CR)} - \textstyle\sum_{j=1}^{N} w_{ij}\hat{\varepsilon}_{jt(CR)} + \hat{\mu}_{i(CR)} - \textstyle\sum_{j=1}^{N} w_{ij}\hat{\mu}_{j(CR)}\right) \\
&= exp\left(\hat{\varepsilon}_{it(CR)} - \textstyle\sum_{j=1}^{N} w_{ij}\hat{\varepsilon}_{jt(CR)}\right) \\
&\quad -exp\left(\hat{\varepsilon}_{it(CR)} - \textstyle\sum_{j=1}^{N} w_{ij}\hat{\varepsilon}_{jt(CR)}\right) exp\left(\hat{\mu}_{i(CR)} - \textstyle\sum_{j=1}^{N} w_{ij}\hat{\mu}_{j(CR)}\right) \\
&= exp\left(\hat{\varepsilon}_{it(CR)} - \textstyle\sum_{j=1}^{N} w_{ij}\hat{\varepsilon}_{jt(CR)}\right)\left[1 - exp\left(\hat{\mu}_{i(CR)} - \textstyle\sum_{j=1}^{N} w_{ij}\hat{\mu}_{j(CR)}\right)\right] \\
&= exp\left(\breve{\varepsilon}_{it(CR)}\right)\left[1 - exp\left(\breve{\mu}_{i(CR)}\right)\right] \qquad \text{(A26)}
\end{aligned}$$

where the last equation makes use of the notation for an effective variable in strategy 1

$$\breve{z}_{it} = z_{it} - \textstyle\sum_{j=1}^{N} w_{ij} z_{jt}. \qquad \text{(A27)}$$

For the derivation of (25b), set $\Gamma\varepsilon = FE$ in (A24) and $\Gamma\omega = CR$ in (A25) to obtain

$$\breve{M}_{it}(FE,\varepsilon) - \breve{M}_{it}(CR,\omega)$$

$$\begin{aligned}
&= exp\left(\hat{\varepsilon}_{it(FE)} - \textstyle\sum_{j=1}^{N} w_{ij}\hat{\varepsilon}_{jt(FE)}\right) \\
&\quad -exp\left[\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)} - \textstyle\sum_{j=1}^{N} w_{ij}\left(\hat{\mu}_{j(CR)} + \hat{\varepsilon}_{jt(CR)}\right)\right] \\
&= exp\left[\hat{\varepsilon}_{it(CR)} - \hat{\bar{\varepsilon}}_{i(CR)} - \textstyle\sum_{j=1}^{N} w_{ij}\left(\hat{\varepsilon}_{jt(CR)} - \hat{\bar{\varepsilon}}_{j(CR)}\right)\right] \\
&\quad -exp\left[\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)} - \textstyle\sum_{j=1}^{N} w_{ij}\left(\hat{\mu}_{j(CR)} + \hat{\varepsilon}_{jt(CR)}\right)\right] \\
&= exp\left[\hat{\varepsilon}_{it(CR)} - \textstyle\sum_{j=1}^{N} w_{ij}\hat{\varepsilon}_{jt(CR)} - \left(\hat{\bar{\varepsilon}}_{i(CR)} - \textstyle\sum_{j=1}^{N} w_{ij}\hat{\bar{\varepsilon}}_{j(CR)}\right)\right] \\
&\quad -exp\left(\hat{\varepsilon}_{it(CR)} - \textstyle\sum_{j=1}^{N} w_{ij}\hat{\varepsilon}_{jt(CR)} + \hat{\mu}_{i(CR)} - \textstyle\sum_{j=1}^{N} w_{ij}\hat{\mu}_{j(CR)}\right) \\
&= exp\left(\hat{\varepsilon}_{it(CR)} - \textstyle\sum_{j=1}^{N} w_{ij}\hat{\varepsilon}_{jt(CR)}\right) \\
&\quad \cdot\left\{exp\left[-\left(\hat{\bar{\varepsilon}}_{i(CR)} - \textstyle\sum_{j=1}^{N} w_{ij}\hat{\bar{\varepsilon}}_{j(CR)}\right)\right] - exp\left(\hat{\mu}_{i(CR)} - \textstyle\sum_{j=1}^{N} w_{ij}\hat{\mu}_{j(CR)}\right)\right\} \\
&= exp\left(\breve{\varepsilon}_{it(CR)}\right)\left[exp\left(-\breve{\bar{\varepsilon}}_{i(CR)}\right) - exp\left(\breve{\mu}_{i(CR)}\right)\right] \qquad \text{(A28)}
\end{aligned}$$

where the second equation makes use of (A7) and the last one of (A27).

In order to derive (25c), set $\Gamma\varepsilon = BE$ in (A24) and $\Gamma\omega = CR$ in (A25). This yields

$\breve{M}_{it}(BE, \varepsilon) - \breve{M}_{it}(CR, \omega)$

$$= exp\left(\hat{\varepsilon}_{it(BE)} - \sum_{j=1}^{N} w_{ij}\hat{\varepsilon}_{jt(BE)}\right)$$
$$-exp\left[\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)} - \sum_{j=1}^{N} w_{ij}\left(\hat{\mu}_{j(CR)} + \hat{\varepsilon}_{jt(CR)}\right)\right]$$
$$= exp\{\hat{\varepsilon}_{it(CR)} - \hat{\bar{\varepsilon}}_{i(CR)} + (x_{it} - \bar{x}_i)\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right) - \sum_{j=1}^{N} w_{ij}\left[\hat{\varepsilon}_{jt(CR)} - \hat{\bar{\varepsilon}}_{j(CR)}\right.$$
$$\left.+\left(x_{jt} - \bar{x}_j\right)\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right)\right]\} - exp\left(\hat{\varepsilon}_{it(CR)} - \sum_{j=1}^{N} w_{ij}\hat{\varepsilon}_{jt(CR)} + \hat{\mu}_{i(CR)}\right.$$
$$\left.- \sum_{j=1}^{N} w_{ij}\hat{\mu}_{j(CR)}\right)$$
$$= exp\left(\hat{\varepsilon}_{it(CR)} - \sum_{j=1}^{N} w_{ij}\hat{\varepsilon}_{jt(CR)}\right)\{exp\left[(x_{it} - \bar{x}_i)\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right)\right.$$
$$\left.- \sum_{j=1}^{N} w_{ij}\left(x_{jt} - \bar{x}_j\right)\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right) - \left(\hat{\bar{\varepsilon}}_{i(CR)} - \sum_{j=1}^{N} w_{ij}\hat{\bar{\varepsilon}}_{j(CR)}\right)\right]$$
$$-exp\left(\hat{\mu}_{i(CR)} - \sum_{j=1}^{N} w_{ij}\hat{\mu}_{j(CR)}\right)\}$$
$$= exp\left(\breve{\varepsilon}_{it(CR)}\right)\{exp\left[\left(\overbrace{x_{it} - \bar{x}_i}\right)\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right) - \breve{\bar{\varepsilon}}_{i(CR)}\right] - exp\left(\breve{\mu}_{i(CR)}\right)\}$$
$$= exp\left(\breve{\varepsilon}_{it(CR)}\right)\{exp\left[(\breve{x}_{it} - \breve{\bar{x}}_i)\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right) - \breve{\bar{\varepsilon}}_{i(CR)}\right] - exp\left(\breve{\mu}_{i(CR)}\right)\} \quad \text{(A29)}$$

where the fourth equation makes use of (A27) and the second of

$$\hat{\varepsilon}_{it(BE)} = \hat{\varepsilon}_{it(CR)} - \hat{\bar{\varepsilon}}_{i(CR)} + (x_{it} - \bar{x}_i)\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right), \quad \text{(A30)}$$

which results from combining (22c) and (24).

Setting $\Gamma\varepsilon = RE$ in (A24) and $\Gamma\omega = CR$ in (A25) allows the derivation of (25d):

$\breve{M}_{it}(RE, \varepsilon) - \breve{M}_{it}(CR, \omega)$

$$= exp\left(\hat{\varepsilon}_{it(RE)} - \sum_{j=1}^{N} w_{ij}\hat{\varepsilon}_{jt(RE)}\right)$$
$$-exp\left[\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)} - \sum_{j=1}^{N} w_{ij}\left(\hat{\mu}_{j(CR)} + \hat{\varepsilon}_{jt(CR)}\right)\right]$$
$$= exp\{\hat{\varepsilon}_{it(CR)} - \hat{\bar{\varepsilon}}_{i(CR)} + \hat{\bar{\varepsilon}}_{i(RE)} + (x_{it} - \bar{x}_i)\hat{\rho}_{RE}\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right)$$
$$- \sum_{j=1}^{N} w_{ij}\left[\hat{\varepsilon}_{jt(CR)} - \hat{\bar{\varepsilon}}_{j(CR)} + \hat{\bar{\varepsilon}}_{j(RE)} + \left(x_{jt} - \bar{x}_j\right)\hat{\rho}_{RE}\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right)\right]\}$$
$$-exp\left(\hat{\varepsilon}_{it(CR)} - \sum_{j=1}^{N} w_{ij}\hat{\varepsilon}_{jt(CR)} + \hat{\mu}_{i(CR)} - \sum_{j=1}^{N} w_{ij}\hat{\mu}_{j(CR)}\right)$$
$$= exp\left(\hat{\varepsilon}_{it(CR)} - \sum_{j=1}^{N} w_{ij}\hat{\varepsilon}_{jt(CR)}\right)\{exp\left[(x_{it} - \bar{x}_i)\hat{\rho}_{RE}\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right)\right.$$
$$- \sum_{j=1}^{N} w_{ij}\left(x_{jt} - \bar{x}_j\right)\hat{\rho}_{RE}\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right) - \left(\hat{\bar{\varepsilon}}_{i(CR)} - \sum_{j=1}^{N} w_{ij}\hat{\bar{\varepsilon}}_{j(CR)}\right)$$
$$\left.+\left(\hat{\bar{\varepsilon}}_{i(RE)} - \sum_{j=1}^{N} w_{ij}\hat{\bar{\varepsilon}}_{j(RE)}\right)\right] - exp\left(\hat{\mu}_{i(CR)} - \sum_{j=1}^{N} w_{ij}\hat{\mu}_{j(CR)}\right)\}$$
$$= exp\left(\breve{\varepsilon}_{it(CR)}\right)\{exp\left[\left(\overbrace{x_{it} - \bar{x}_i}\right)\hat{\rho}_{RE}\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right) - \breve{\bar{\varepsilon}}_{i(CR)} + \breve{\bar{\varepsilon}}_{i(RE)}\right]$$
$$-exp\left(\breve{\mu}_{i(CR)}\right)\}$$

$$= \; exp\left(\breve{\varepsilon}_{it(CR)}\right)\left\{exp\left[(\breve{x}_{it} - \breve{\bar{x}}_i)\hat{\rho}_{RE}\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right) - \breve{\bar{\varepsilon}}_{i(CR)} + \breve{\bar{\varepsilon}}_{i(RE)}\right]\right.$$

$$\left. - exp\left(\breve{\mu}_{i(CR)}\right)\right\} \quad \text{(A31)}$$

where the fourth equation makes use of (A27) and the second of

$$\hat{\varepsilon}_{it(RE)} = \hat{\varepsilon}_{it(CR)} - \hat{\bar{\varepsilon}}_{i(CR)} + \hat{\bar{\varepsilon}}_{i(RE)} + (x_{it} - \bar{x}_i)\hat{\rho}_{RE}\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right), \quad \text{(A32)}$$

which results from combining (22d) and (24).

Equation (26a) can be derived if (A25) is used first with $\Gamma\omega = FE$ and then with $\Gamma\omega = CR$:

$$\breve{M}_{it}(FE, \omega) - \breve{M}_{it}(CR, \omega)$$

$$= \; exp\left[\hat{\mu}_{i(FE)} + \hat{\varepsilon}_{it(FE)} - \sum_{j=1}^{N} w_{ij}\left(\hat{\mu}_{j(FE)} + \hat{\varepsilon}_{jt(FE)}\right)\right]$$

$$- exp\left[\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)} - \sum_{j=1}^{N} w_{ij}\left(\hat{\mu}_{j(CR)} + \hat{\varepsilon}_{jt(CR)}\right)\right]$$

$$= exp\left\{\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)} - \bar{x}_i\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right) - \sum_{j=1}^{N} w_{ij}\left[\hat{\mu}_{j(CR)} + \hat{\varepsilon}_{jt(CR)}\right.\right.$$

$$\left.\left. - \bar{x}_j\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right)\right]\right\} - exp\left[\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)} - \sum_{j=1}^{N} w_{ij}\left(\hat{\mu}_{j(CR)} + \hat{\varepsilon}_{jt(CR)}\right)\right]$$

$$= exp\left\{\widehat{\omega}_{it(CR)} - \sum_{j=1}^{N} w_{ij}\widehat{\omega}_{jt(CR)} - \left[\bar{x}_i\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right)\right.\right.$$

$$\left.\left. - \sum_{j=1}^{N} w_{ij}\bar{x}_j\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right)\right]\right\} - exp\left(\widehat{\omega}_{it(CR)} - \sum_{j=1}^{N} w_{ij}\widehat{\omega}_{jt(CR)}\right)$$

$$= exp\left(\widehat{\omega}_{it(CR)} - \sum_{j=1}^{N} w_{ij}\widehat{\omega}_{jt(CR)}\right)$$

$$\cdot \left\{ exp\left\{-\left[\bar{x}_i\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right) - \sum_{j=1}^{N} w_{ij}\bar{x}_j\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right)\right]\right\} - 1\right\}$$

$$= exp\left(\breve{\omega}_{it(CR)}\right) \cdot \left\{ exp\left[-\breve{\bar{x}}_i\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right)\right] - 1\right\} \quad \text{(A33)}$$

where the second equation uses (23a) and the last one (A27).

For the derivation of (26b), use (A25) by first setting $\Gamma\omega = BE$ and then $\Gamma\omega = CR$:

$$\breve{M}_{it}(BE, \omega) - \breve{M}_{it}(CR, \omega)$$

$$= \; exp\left[\hat{\mu}_{i(BE)} + \hat{\varepsilon}_{it(BE)} - \sum_{j=1}^{N} w_{ij}\left(\hat{\mu}_{j(BE)} + \hat{\varepsilon}_{jt(BE)}\right)\right]$$

$$- exp\left[\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)} - \sum_{j=1}^{N} w_{ij}\left(\hat{\mu}_{j(CR)} + \hat{\varepsilon}_{jt(CR)}\right)\right]$$

$$= exp\left\{\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)} + (x_{it} - \bar{x}_i)\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right) - \sum_{j=1}^{N} w_{ij}\left[\hat{\mu}_{j(CR)} + \hat{\varepsilon}_{jt(CR)}\right.\right.$$

$$\left.\left. + \left(x_{jt} - \bar{x}_j\right)\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right)\right]\right\} - exp\left[\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)} - \sum_{j=1}^{N} w_{ij}\left(\hat{\mu}_{j(CR)} + \hat{\varepsilon}_{jt(CR)}\right)\right]$$

$$= exp\left[\widehat{\omega}_{it(CR)} - \sum_{j=1}^{N} w_{ij}\widehat{\omega}_{jt(CR)} + (x_{it} - \bar{x}_i)\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right)\right.$$

$$\left. - \sum_{j=1}^{N} w_{ij}\left(x_{jt} - \bar{x}_j\right)\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right)\right] - exp\left(\widehat{\omega}_{it(CR)} - \sum_{j=1}^{N} w_{ij}\widehat{\omega}_{jt(CR)}\right)$$

$$= exp\left(\widehat{\omega}_{it(CR)} - \sum_{j=1}^{N} w_{ij}\widehat{\omega}_{jt(CR)}\right)$$

$$\cdot \left\{ exp\left[(x_{it} - \bar{x}_i)\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right) - \sum_{j=1}^{N} w_{ij}\left(x_{jt} - \bar{x}_j\right)\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right)\right] - 1\right\}$$

$$= exp\left(\breve{\omega}_{it(CR)}\right) \cdot \left\{ exp\left[\left(\breve{x_{it} - \bar{x}_i}\right)\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right)\right] - 1\right\}$$

$$= exp\left(\breve{\omega}_{it(CR)}\right) \cdot \left\{ exp\left[(\breve{x}_{it} - \breve{\bar{x}}_i)\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right)\right] - 1\right\} \quad \text{(A34)}$$

where the second equation uses (23b) and the fifth one (A27).

Equation (26c) is derived by using (A25) where first $\Gamma\omega = RE$ and then $\Gamma\omega = CR$:

$$\breve{M}_{it}(RE, \omega) - \breve{M}_{it}(CR, \omega)$$

$$= exp\left[\hat{\mu}_{i(RE)} + \hat{\varepsilon}_{it(RE)} - \sum_{j=1}^{N} w_{ij}\left(\hat{\mu}_{j(RE)} + \hat{\varepsilon}_{jt(RE)}\right)\right]$$

$$-exp\left[\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)} - \sum_{j=1}^{N} w_{ij}\left(\hat{\mu}_{j(CR)} + \hat{\varepsilon}_{jt(CR)}\right)\right]$$

$$= exp\left\{\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)} + (x_{it}\hat{\rho}_{RE} - \bar{x}_i)\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right)\right.$$

$$\left. - \sum_{j=1}^{N} w_{ij}\left[\hat{\mu}_{j(CR)} + \hat{\varepsilon}_{jt(CR)} + \left(x_{jt}\hat{\rho}_{RE} - \bar{x}_j\right)\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right)\right]\right\}$$

$$-exp\left[\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)} - \sum_{j=1}^{N} w_{ij}\left(\hat{\mu}_{j(CR)} + \hat{\varepsilon}_{jt(CR)}\right)\right]$$

$$= exp\left[\widehat{\omega}_{it(CR)} - \sum_{j=1}^{N} w_{ij}\widehat{\omega}_{jt(CR)} + (x_{it}\hat{\rho}_{RE} - \bar{x}_i)\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right)\right.$$

$$\left. - \sum_{j=1}^{N} w_{ij}\left(x_{jt}\hat{\rho}_{RE} - \bar{x}_j\right)\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right)\right] - exp\left(\widehat{\omega}_{it(CR)} - \sum_{j=1}^{N} w_{ij}\widehat{\omega}_{jt(CR)}\right)$$

$$= exp\left(\widehat{\omega}_{it(CR)} - \sum_{j=1}^{N} w_{ij}\widehat{\omega}_{jt(CR)}\right) \cdot \left\{exp\left[(x_{it}\hat{\rho}_{RE} - \bar{x}_i)\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right)\right.\right.$$

$$\left.\left. - \sum_{j=1}^{N} w_{ij}\left(x_{jt}\hat{\rho}_{RE} - \bar{x}_j\right)\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right)\right] - 1\right\}$$

$$= exp\left(\breve{\omega}_{it(CR)}\right) \cdot \left\{ exp\left[\left(\breve{x_{it}\hat{\rho}_{RE} - \bar{x}_i}\right)\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right)\right] - 1\right\}$$

$$= exp\left(\breve{\omega}_{it(CR)}\right) \cdot \left\{ exp\left[(\breve{x}_{it}\hat{\rho}_{RE} - \breve{\bar{x}}_i)\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right)\right] - 1\right\} \quad \text{(A35)}$$

where the second equation uses (23c) and the fifth one (A27).

For the derivation of (26d), use (A25) by first setting $\Gamma\omega = LS$ and then $\Gamma\omega = CR$:

$$\breve{M}_{it}(LS, \omega) - \breve{M}_{it}(CR, \omega)$$

$$= exp\left(\widehat{\omega}_{it(LS)} - \sum_{j=1}^{N} w_{ij}\widehat{\omega}_{jt(LS)}\right)$$

$$-exp\left[\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)} - \sum_{j=1}^{N} w_{ij}\left(\hat{\mu}_{j(CR)} + \hat{\varepsilon}_{jt(CR)}\right)\right]$$

$$= exp\left\{\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)} + (x_{it}\hat{\rho}_{LS} - \bar{x}_i)\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right)\right.$$

$$\left. - \sum_{j=1}^{N} w_{ij}\left[\hat{\mu}_{j(CR)} + \hat{\varepsilon}_{jt(CR)} + \left(x_{jt}\hat{\rho}_{LS} - \bar{x}_j\right)\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right)\right]\right\}$$

$$-exp\left[\hat{\mu}_{i(CR)} + \hat{\varepsilon}_{it(CR)} - \sum_{j=1}^{N} w_{ij}\left(\hat{\mu}_{j(CR)} + \hat{\varepsilon}_{jt(CR)}\right)\right]$$

$$= exp\left(\breve{\omega}_{it(CR)}\right) \cdot \left\{ exp\left[(\breve{x}_{it}\hat{\rho}_{LS} - \breve{\bar{x}}_i)\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right)\right] - 1\right\} \quad \text{(A36)}$$

where the second equation uses (23d) and the third (A35).

If strategy 2 is used instead of strategy 1, the (adjusted) effective misalignments $\widetilde{M}_{it}(\Gamma\varepsilon, \varepsilon)$ and $\widetilde{M}_{it}(\Gamma\omega, \omega)$ are related to the corresponding estimated unadjusted effective log misalignment $\widehat{\widetilde{m}}_{it}(\Gamma\varepsilon, \varepsilon)$ and $\widehat{\widetilde{m}}_{it}(\Gamma\omega, \omega)$ through equations (8)-(10) as

$$\widetilde{M}_{it}(\Gamma\varepsilon, \varepsilon) = exp\left[\widehat{\widetilde{m}}_{it}(\Gamma\varepsilon, \varepsilon) - \sum_{j=1}^{N} \widetilde{w}_{jt}\widehat{\widetilde{m}}_{jt}(\Gamma\varepsilon, \varepsilon)\right]$$

$$= exp\left(\hat{\tilde{\varepsilon}}_{it(\Gamma\varepsilon)} - \sum_{j=1}^{N} \widetilde{w}_{jt}\hat{\tilde{\varepsilon}}_{jt(\Gamma\varepsilon)}\right), \quad \text{(A37)}$$

$$\widetilde{M}_{it}(\Gamma\omega, \omega) = exp\left[\widehat{\widetilde{m}}_{it}(\Gamma\omega, \omega) - \sum_{j=1}^{N} \widetilde{w}_{jt}\widehat{\widetilde{m}}_{jt}(\Gamma\omega, \omega)\right]$$

$$= exp\left(\widehat{\widetilde{\omega}}_{it(\Gamma\omega)} - \sum_{j=1}^{N} \widetilde{w}_{jt}\widehat{\widetilde{\omega}}_{jt(\Gamma\omega)}\right) \quad \text{(A38)}$$

A comparison of (A37) and (A38) with (A24) and (A25), respectively, shows that the mathematical structure of the equations is practically identical although the variables and parameters have a different meaning. Accordingly, equations (25a)-(25d) and (26a)-(26d) are valid for strategy 2, too, if each variable ž is replaced by z̃. As an additionally necessary adjustment, the time dependency of the weights $\widetilde{w}_{jt}$ implies that $\tilde{\mu}_{it}$ and $\tilde{\bar{\varepsilon}}_{it}$ are time dependent as well. To give an example, equation (25d) applied to strategy 2 becomes

$$\widetilde{M}_{it}(RE, \varepsilon) - \widetilde{M}_{it}(CR, \omega) = exp\left(\tilde{\varepsilon}_{it(CR)}\right)$$

$$\cdot \left\{exp\left[(\tilde{x}_{it} - \tilde{\bar{x}}_i)\hat{\rho}_{RE}\left(\hat{\beta}_{1CR} - \hat{\beta}_{2CR}\right) + \tilde{\bar{\varepsilon}}_{it(RE)} - \tilde{\bar{\varepsilon}}_{it(CR)}\right] - exp\left(\tilde{\mu}_{it(CR)}\right)\right\} \quad \text{(A39)}$$

where the parameters $\hat{\beta}_{1CR}$, $\hat{\beta}_{2CR}$ and $\hat{\rho}_{RE}$ are obtained in an estimation procedure that uses effective variables.

2. Data Appendix

Application 1

Productivity data are taken from the Conference Board's Total Economy Database; relative price level data are based on the IMF's World Economic Outlook "implied PPP exchange rates". Fischer and Hossfeld (2014) use the exogenously given group of 57 countries for which the European Central Bank and the Deutsche Bundesbank compute real effective exchange rates (cf. Schmitz et al., 2013). For data availability reasons, Fischer and Hossfeld's (2014) estimates based on labour productivity per hour worked, however, included only 46 of these countries. Since then, data availability has improved so that, in the present study, the panel contains 53 of the 57 exogenously given economies. These are Argentina, Australia, Austria, Belgium, Brazil, Bulgaria, Canada, Chile, China, Cyprus, the Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hong Kong, Hungary, Iceland, India, Indonesia, Ireland, Israel, Italy, Japan, Latvia, Lithuania, Luxembourg, Malaysia, Malta, Mexico, the Netherlands, New Zealand, Norway, the Philippines, Poland, Portugal, Romania, Russia, Singapore, the Slovak Republic, Slovenia, South Korea, Spain, Sweden, Switzerland, Taiwan, Thailand, Turkey, the United Kingdom, the United States, and Venezuela. Similar results to those shown here emerge if, instead of 53 countries, the original set of 46 countries is used. The countries whose observation period starts in 1995 instead of 1980 are the former communist transition economies and those that experienced a hyperinflation in the years before 1995.

Application 2

List of countries included: Australia, Austria, Belgium, Brazil, Canada, Chile, China, Colombia, the Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, India, Indonesia, Ireland, Italy, Japan, Malaysia, Mexico, the Netherlands, New Zealand, Norway, Pakistan, Peru, the Philippines, Poland, Portugal, Russia, South Africa, South Korea, Spain, Sweden, Switzerland, Thailand, Turkey, the United Kingdom, and the United States.

Per capita GDP measured in PPP terms: Source: Total Economy Database, The Conference Board. Comparable variable in IMF (2017) and Phillips et al. (2013): Output in PPP terms per working-age population relative to economies at the frontier of highest productivity.

Old age dependency ratio: Source: WDI (“Age dependency ratio, old (% of working-age population)”). Comparable variable in IMF (2017): IMF (2017) also considers the old age dependency ratio as explanatory variable, but the final specification used aging speed and the dependency ratio as explanatory variables that are supposed to capture demographics.

Government consumption per GDP: Source: WDI (“General government final consumption expenditure (% of GDP)”). Comparable variable in Phillips et al. (2013): An instrumented variable of a cyclically-adjusted fiscal balance was included in the regression. However, it was found to be not statistically significant.

Net foreign assets per GDP: Source: Update on the External Wealth of Nations Mark II database, see http://www.imf.org/~/media/Files/Publications/WP/2017/datasets/wp115.ashx and Lane and Milesi-Ferretti (2018). Comparable variable in IMF (2017): Net foreign asset to GDP ratio. In Phillips et al. (2013), this variable is considered but not included in the regression.

Trade openness: Source: WDI. Trade openness is computed as the sum of the two series "Exports of goods and services (% of GDP)" and "Imports of goods and services (% of GDP)". Comparable variable in IMF (2017) and Phillips et al. (2013): Trade openness.

Change in reserves per GDP (capturing foreign exchange intervention): Source: WDI. Change in reserves per GDP is computed from the two series "Total reserves (includes gold, current US$)" and "GDP (current US$)" as $(\text{Reserves/GDP})_t$ – $(\text{Reserves/GDP})_{t\text{-}1}$. Comparable variable in IMF (2017) and Phillips et al. (2013): A foreign exchange intervention measure which is an instrumented variable of the change in reserves to GDP interacted with a capital controls index.

Terms of trade: Source: WEO ("Terms of trade, total, US Dollars"). Comparable variable in IMF (2017) and Phillips et al. (2013): Commodity terms of trade.

Relative price levels: Source: WEO. Relative price levels are obtained by dividing the purchasing power parity series of a country by its nominal exchange rate against the US dollar. IMF (2017) uses a similar variable, in which the relative effective price level is computed for 2011, and REER indices are used to extend the data to other years of the observation period.

Variables which are not percentage shares are expressed in logs (as, for instance, in Fidora et al., 2017). These are the relative price levels, per capita GDP and the terms of trade. Since strategy 2 is used for the computation of the equilibrium exchange rates, the variables of each country need to be related to the weighted average of its trade partners. Therefore, equations (6a) and (6b) have been used to produce effective variables. As an exception, the terms of trade are not transformed into effective variables because they are already measured relative to the rest of the world (see Adler and Grisse, 2017). This is in line with IMF (2017), where the IMF abstains from a transformation of the commodity terms of trade, too.

The terms of trade variable is an index. This implies that, in contrast with the rest of the variables, it does not contain any meaningful cross-section variation. Therefore, the log terms of trade series have been normalized so that the country-specific mean equals zero. Otherwise, the BE and the CRE estimates would erroneously produce an effect of the terms of trade time-averages on the relative price levels, although the time-averages of an index are meaningless.

According to IMF (2017) and Phillips et al. (2013), some of the explanatory variables in the IMF regressions are lagged while others are included contemporarily. The regression results shown in the present paper are obtained exclusively with contemporary variables. As a robustness check, the same regressions have been performed with all the explanatory variables being lagged by one period. These regressions yield results that differ only slightly from the corresponding ones, which use contemporary variables instead.