Lifecycle Wages and Human Capital Investments: Selection and Missing Data

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First version: December 12, 2017
This version: January 2019

Abstract

We use rich administrative panel data on earnings to estimate individual-specific wage dynamics equations derived from a Ben Porath model. This structural model of human capital investments over the life-cycle allows for interruptions in labour market participation and explicitly deals with missing data and attrition. Selection is modelled using unobserved factors and factor loadings. Results evince the strong explanatory power of interruptions for wage processes even for males.

JEL Codes: C38, D91, I24, J24, J31

Keywords: Human capital investment, earnings dynamics, post-schooling earnings growth, factor models

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1 Introduction

Recent increases in earnings inequality across OECD countries have spurred researchers to investigate the dynamics of earnings or incomes and the insurance mechanisms that households use to protect themselves against earnings shocks when markets are incomplete (see Blundell, 2014 for a review). Most analyse interactions between labour earnings processes along the life-cycle and consumption dynamics (Meghir and Pistaferri, 2010), or household labour supply dynamics (Imai and Keane, 2004). Certain analyses more narrowly focus on the specification of earnings dynamics that can be studied using long panel survey or administrative data (Guvenen, Karahan, Ozkan and Song, 2015). In particular, there has been a few recent attempts to estimate specification of earnings or wage processes à la Mincer (1974) while including a lot of heterogeneity as in Browning, Ejrnaes and Alvarez (2012), Polachek, Das and Thamma-Apiroam (2015) or Magnac, Pistolesi and Roux (2014). These authors pay a particular attention to individualize as much as possible the earnings processes by estimating sets of individual-specific parameters beyond the permanent effects that appear in equations commonly estimated with panel data of earnings (Heckman, Lochner and Todd , 2006). In addition, Polachek et al., (2015) or Magnac et al. (2014) emphasize the importance of the economic underpinnings of the wage equations that are considered. In particular, they set up Ben Porath (1967) human capital model of earnings over the life-cycle, although in different guises, to justify their specifications. By doing so, parameters governing the earnings equation acquire a structural economic interpretation that can be related to individual characteristics if those are available. These parameters are related to ability to learn and to earn of individuals (Browning, Hechman and Hansen, 1999, Rubinstein and Weiss, 2006)

Yet, survey or administrative panel data on earnings are plagued with missing data and attrition issues. The most common attitude among researchers is to select earnings histories which are sufficiently long and to treat the missing observations in the earnings histories as random. This issue is particularly important when parameters are individual-specific since their estimation derives from individual time-series and therefore relies for consistency of those estimates on the number of periods being large. These time-series of wages are irregularly spaced over time because of interruptions in labour market participation. This procedure might induce sizeable small-sample biases in those estimates if the degree of attachment to the labor market, as measured by the reciprocal of the number of interruptions in individuals' careers, is somehow associated to individual parameters such as the ones which describe abilities, returns to human capital investments or their costs. In a nutshell, irregular wage histories are likely to be selected out by these procedures and the distribution of individual-specific parameters truncated out. Correction for missing data however is difficult in the absence of good instruments which would affect entry in and exit from the panel without affecting earnings or in the absence of credible alternative exclusion restrictions (Davezies and D'Haultfoeuille, 2011).

In this paper, we set out structural restrictions to identify the effect of selection. We build upon the structural model proposed by Magnac, Pistolesi and Roux (2018) who proposed a linear model for the logarithm of earnings over the life-cycle as a function of four individual-specific parameters: the initial level of human capital at entry in the labour market, the returns to human capital, their costs and the terminal value of human capital stocks. We show that this model extends to the case of two or more sectors in which these individual parameters are further differentiated. This setting fits the particular case in which wages in one sector of the labour market are observed while wages, if any, in an alternative sector are not. This provides us with a way of modelling temporary or permanent attrition in the life-cycle histories of wages in the private sector that we observe. The differential structure of rewards and costs of human capital investments across sectors creates a wedge between the accumulation processes in human capital in the two sectors (for instance Blundell, Costa-Dias, Meghir and Shaw, 2016, for part-time/full-time evidence) . In particular, we expect that rates of return and costs are different and that interruptions in the career have a sizeable effect on human capital investments. That leads us to introduce additional terms in wage equations reflecting the number of periods spent out of the labour market. These ideas borrow from the literature on the impact of interruptions during the life cycle in Mincer log-wage equations and in particular that interruptions have explanatory power for the gender wage gap (as reviewed for example in Polachek and Das, 2017).

Furthermore, to achieve conditional independence betwen the life-cycle wage equation and the selection process of labour market participation, we posit a factor structure for the residual process in wages and make the assumption that transitions between sectors are independent of wage processes in the two sectors conditionally on these unobserved factors. A set-up with factor structures was introduced in this literature by Aakvik, Heckman and Vytlacil (2005) and squares well with the fact that lots of heterogeneity affect wage histories over the life-cycle. The generalization of selection-on-unobservable-factors present in difference-in-differences methods was also explored by Gobillon and Magnac (2016).

In the empirical analysis, we estimate parameters using a long administrative panel dataset collected in France for social security purposes and which is typical of administrative datasets that can be found in many countries. We use wage observations of male individuals having entered the labour market between 1985 and 1992 and are followed until 2012 – if they do not leave the panel before. We estimate those factor models using the method of Bai (2009) with observed variables and observed factors that derives from the structural specification of the human capital model as well as unobserved factors that control for selection.

Some results are worth hightlighting. First, when looking at returns to experience after 20 years that approximately double wages in our preferred specification, a specification excluding the effect of interruptions and unobserved factors have a bias of 7.8%. Second, a large part of this bias comes from the influence of interruptions on human capital accumulation and not from unobserved factors so that selectivity seems mainly captured by interruptions and not by additional heterogeneity factors. Unsurprisibly, the higher the level of interruptions, the higher the bias for omitting interruptions. yet, unobserved factors and interruptions are strongly correlated which prove that unobserved factors play out in labor market selection.

The outline of the paper is the following. Section 2 briefly describes empirical evidence about the panel data on earnings that we use. Section 3 sets up the structural model and Section 4 the identifying restrictions of the econometric model. Section 5 presents our estimation strategy and results are reported in Section 6.

2 Evidence on Wage Dynamics

2.1 The data

In the empirical analysis, we use the *2011 DADS Grand Format-EDP* panel dataset in which we follow all individuals born in the first four days of October of an even year. It is constructed from two different sources (*Déclaration Annuelles des Données Sociales* i.e. *DADS* and *Echantillon Démographique Permanent*, i.e. *EDP*). The data record all jobs in the private sector since 1976 and certain jobs in the public sector since 1988.[1] We restrict our attention to jobs in the private sector because the information on public sector employees cover them partially only.

The DADS are collected for social security and tax purposes and contain details on job characteristics and in particular, labor earnings and days of work. Those are used to determine the status (full-time or part-time) and they record labor earnings and days of work. For a given individual and year, we aggregate earnings and days of work for all full-time jobs. We

[1] Three years are missing (1981, 1983 and 1990) because resources of the French Institute of Statistics were devoted to the collection of 1982 and 1990 censuses rather than to the management of the earnings data.

use these quantities to construct the daily wage.[2] The dataset contains $2,089,753$ observations. Information on the education level as measured by diploma is recovered from EDP which links DADS with data from the 1975, 1982, 1990, 1999 and 2006-2011 censuses. We consider the highest education levels obtained by individuals and regroup them into four categories: high-school drop outs, high-school graduates, two years, or short-track, college and college graduates with more than two years in college or from top engineering schools.

We selected observations in the following way. For coherency reasons, we recoded as missing person-year observations for which the daily wage is lower than 80% of the minimum wage or for which the number of days of work is lower than 180 (6 months). Part-time individual -year observations are coded as missing. We further restricted the sample to individuals for whom the information on the education level is not missing. Those observations define the sector denoted e for employment and we consider that at all other dates individuals are in the alternative sector, somewhat abusively denoted n for non-employment since it can be any other occupation not in the private sector. The year of entry into the panel is defined as the first year an individual works in sector e. We focus on males who enter the market over the 1985-1992 period and who are $16-30$ years old at the entry date. We obtain a sample of $178,098$ observations corresponding to $12,212$ males. We retain individuals whose wages in sector e are observed in at least 15 years to get reliable estimates and we end up with a sample of $137,315$ observations corresponding to $7,004$ males. Details on the change in sample size when making successive selections are given in Data Appendix A.

2.2 Descriptive statistics

We start by describing the profile of the logarithm of wages as a function of potential experience in the whole sample and by education level. Wages are deflated by the the World Bank consumer price index.[3] We restrict profiles to 20 years of potential experience only because the younger cohort enters in 1992 and the panel ends in 2011. We also assess to what extent our restriction that individual wages must be observed at least 15 times alter these profiles.

Figure 2 represents the mean log-wage as a function of potential experience for the unrestricted and restricted samples. Mean log-wages are increasing and slightly concave for low values of potential experience and they are not affected by our sample restriction. Figure 3 represents the mean log-wage by education level. As expected, the level of log-wages increases

[2] We ignore overlaps of job spans because they are exceptional for full-time jobs.
[3] https://data.worldbank.org/indicator/FP.CPI.TOTL?locations=FR

with education level and their slope as a function of potential experience is steeper for higher education levels so that mean log-wage differences across education levels increase with potential experience. As before, our sample restriction does not affect mean log-wage profiles.

Figure 2 also reports the variance of log-wages as a function of potential experience for the unrestricted and restricted samples. This variance is increasing and differences between samples are negligible. Variances of log-wages by education level are graphed in Figure 4 and as before, the sample restriction does not affect much the profile of variances by education level.

Interestingly, both levels and slopes differ in a sizable way across education levels. Higher education levels are characterized by a larger variance and a steeper slope although variance profiles for high-school graduates and short-track college graduates are very close. Moreover, there is some evidence of a Mincer dip (Mincer, 1974) in Figure 4 and it is probably still masked by the fact that we consider different cohorts (see Magnac, Pistolesi and Roux, 2014 for a single cohort analysis). Our complete specification estimated below shows more pronounced Mincer dips than this descriptive analysis.

As these statistics are very similar for the unrestricted and restricted samples, we focus from now on on the restricted sample and first analyze how mean log-wages reflect human capital investments. Indeed, mean log-wage profiles reported above are the composition of increasing investments with potential experience and increasing human capital prices due to other factors of production such as capital or technical progress. We derive the evolving price of human capital from a "flat spot" approach (Heckman, Lochner and Taber, 1998, Bowlus and Robinson, 2012). We consider the subsample of individuals aged $50-55$ in each year because those individuals presumably stop investing in human capital, at least as a first-order approximation. We define their median wage by year and education level as the price of human capital for a given education level in a given year. The resulting prices over the calendar years are reported in Figure 5. It shows that the price of human capital for high school dropouts increases over the period in France by roughly 50%, mainly because of increases in the minimum wage which have been larger than average wage increases since the 1970s (Cette et al., 2012). For other groups, prices increased until year 6 and became flatter afterwards. Increases sum up to around 20% over the whole period except for short-track college graduates whose price increases by more than 30% over the whole period. It corresponds to an increase in the supply of these education levels at the beginning of the 1990s (Albouy and Tavan, 2008).

Our resulting outcome variable, which stands for the effect of human capital investments on wage profiles over time and potential experience is the logarithm of wage from which we subtract the logarithm of the price of human capital corresponding to the education level of the individual in a given year. As shown in Figure 6, considering log-wages deflated with human capital prices does not affect much our descriptive results on the shape of profiles of mean and variance of log-wages although it does affect the scale of the mean log-wage profiles.

Serial correlations between log wages at different periods are reported in Table 1 and entail important stylized facts. The one-year lag correlation starts at .83 and grows until 0.94 at the end of the period of observation. Wages are getting more and more persistent when potential experience increases and this indicates that the variance of idiosyncratic shocks on wages tends to decrease over time (Magnac et al., 2014). Non stationarity is an important element to model in wage dynamics and this will be captured by factors in our empirical analysis. Second, the correlation decreases at longer lags although much less than at a geometric rate. At a 20-year lag, the correlation is equal to 0.28 far above $(.83)^{20}$. This likely denotes the presence of unobserved permanent heterogeneity which will be captured by factor loadings of observed and unobserved factors in our empirical analysis.

Table 2 reports statistics on interruptions in individual participation to sector e which sever the observation process of wages. The two parts in the Table, under the headings of No censorship and Censorship differ in the way the last periods of observation are dealt with. Under No censorship, the periods after the last period in sector e are treated as interruptions while under Censorship they are not. For instance, we would treat as an interruption, for an individual exiting in 2007 and being absent until the end of the panel in 2011, the periods between 2007 and 2011. Concentrating on the No censorship case, we can see that the cumulated duration in interruptions is 3.7 years ith respect with an average length of stay of about 21 years. The number of interruptions is low and equal to 1.44. The distribution of these interruptions is quite disperse however since 523 male individuals have more than 4 interruptions and their cumulative durations reach between 7 and 9 years. Those have a lesser attachment to employment in the private sector than others.

3 The economic model

In this section, we set up the model, analyze its structural predictions and derive the reduced form to be brought to the data.

3.1 Set up

We start from the description of human capital accumulation in two distinct sectors. We then present the timing of decisions and define value functions. We end up with the description of terminal conditions.

3.1.1 Human capital accumulation in two sectors

We assume that an individual at period t chooses a state in the labour market, s_t, among two denoted $s = n$ (non employment) and $s = e$ (employment) at each period $t \in \{t_0, ..., t_0 + d\}$ in which t_0 is an arbitrary initial date. All variables and parameters are individual-specific (although an individual subscript is omitted for readability). Individual earnings when the individual is in sector s are assumed to be given by:

$$y_t^s = \exp(\delta_t^s) H_t \exp(-\tau_t), \tag{1}$$

in which H_t is the stock of human capital at the beginning of period t, δ_t^s is the rental rate or "price" of human capital in sector s at time t and τ_t is a decision variable such that the term, $1-\exp(-\tau_t)$, can be interpreted as the fraction of non-leisure time, or alternatively the intensity of effort, devoted to investing in human capital. This fraction is increasing in τ_t, equal to zero when $\tau_t = 0$ and equal to one when $\tau_t = +\infty$. This is why we call, $\tau_t \geq 0$, the individual decision of investment in human capital at time t.

The technology of production of human capital in sector s is described by

$$H_{t+1}^s = H_t \exp[\rho^s \tau_t - \lambda_t^s], \tag{2}$$

in which ρ^s is the rate of return of human capital investments in sector s (fixed over time but individual specific) and λ_t^s is the depreciation of human capital in sector s at period t. Depreciation λ_t^s embeds individual-specific or aggregate shocks that depreciate previous vintages of human capital. Shocks are state-specific if human capital depreciation is larger when non-employed than when employed as on-the-job learning is more likely. Furthermore, on-the-job learning and learning when non-employed vary across individuals. The price of human capital, δ_t^s, and the depreciation, λ_t^s, are treated as stochastic processes whose properties are made precise below.

Period-t utility in sector s is written as:

$$\ln y_t^s - c\frac{(\tau_t)^2}{2} + \omega_t \mathbf{1}\{s = e\}, \tag{3}$$

in which the shock, ω_t, expresses a relative preference for working in sector e with respect to sector n. Furthermore, the cost of investment is quadratic in utility terms and indexed by an individual specific parameter, c. We assume that this cost parameter is not sector dependent as it is a parameter of the utility function. We neglect the linear component of the cost in terms of τ_t because it cannot be identified as log-earnings in sector s are:

$$\ln y_t^s = \delta_t^s + \ln H_t - \tau_t, \tag{4}$$

and the unit in which τ_t is expressed, is unobserved. Increasing marginal costs fits well with the interpretation of τ_t as an exerted effort which decreases current earnings and provides future returns. This is what makes unique the solution in the dynamic programming decision problem.

3.1.2 Timing and value functions

The timing of revelation of shocks, state variables and decisions about sectors and human capital investments is plotted in Figure 1. Our key assumption is that the revelation of sector preference shocks, ω_t, and the choice of sector, s_t, are made before shocks on prices and depreciations of human capital are revealed and decisions about human capital investments are made. This is a specific version of the Roy model which is known to lead to the absence of selectivity of sector choice on earnings (Heckman and Robb, 1985). However, in the current paper, this assumption is made conditional on a certain number of factors unobserved by the econometrician which act as controls for selectivity. We develop this framework further below.

About here Figure 1: Timing of the model

The first row in this figure reports the timing of the revelation of shocks on sector preferences, ω_t, and on price and depreciation of human capital, δ_t^s and λ_t^s. The second row reports the history of the time processes, δ^s, λ^s and ω up to the time described by the first row. In particular Z_t contains the history of ω up to period t and the history of δ^s, λ^s up to period $t-1$. History $Z_{t+1/2}$ completes Z_t with period t information on δ_t^s and λ_t^s. The third line reports the timing of decisions: the choice of sector is made after sector preference shocks are revealed and human capital investments after the revelation of shocks on prices and depreciation. The state variable H_t is inherited from the past according to equation (2) at the very beginning of period t. Below the timeline, the potential earnings, y_t, is a function of shocks on prices and depreciation.

Value functions at each stage of this timeline can now be constructed. If V_{t+1} is the value function at the beginning of period $t+1$, its arguments are the state variables, H_{t+1} and Z_{t+1}. At the previous interim stage $t+1/2$, these state variables are H_t, $Z_{t+1/2}$ and for each sector decision, $s \in \{n, e\}$, at time t, we model human capital investments as resulting from the following decision program:

$$W_t^s(H_t, Z_{t+1/2}) = \max_{\tau_t} \left\{ \delta_t^s + \ln H_t - \left(\tau_t + c\frac{(\tau_t)^2}{2} \right) + \beta \mathbb{E}_{t+1/2} \left[V_{t+1}(H_{t+1}^s, Z_{t+1}) \right] \right\}$$
subject to the human capital accumulation equation (2),

because of equations (3) and (4). In this expression, $E_{t+1/2}(.) = E(. \mid H_t, Z_{t+1/2})$ and β is the discount rate. This means in particular that the delay between t and $t+1/2$ is infinitely smaller than the delay between $t+1/2$ and $t+1$ despite our abusive notation.

At the beginning of period t, we model sector choice as resulting from:

$$s_t = e \text{ iff } \mathbb{E}_t W_t^e(H_t, Z_{t+1/2}) + \omega_t > \mathbb{E}_t W_t^n(H_t, Z_{t+1/2})), \tag{5}$$

where $E_t(.) = E(. \mid H_t, Z_t)$, which allows us to complete the definition of the recursive equation in V_t:

$$V_t(H_t, Z_t) = \max(\mathbb{E}_t W_t^e(H_t, Z_{t+1/2}) + \omega_t, \mathbb{E}_t W_t^n(H_t, Z_{t+1/2})).$$

As sector choice, denoted by s_t, affects the accumulation of human capital, the optimal level of investment is $\tau_t^{s_t}$. The level of human capital at date $t+1$ is then given by the simplified notation, $H_{t+1} \equiv H_{t+1}^{s_t}$.

3.1.3 Individual and terminal conditions

The initial level of human capital is supposed to be H_{t_0} at period t_0. The terminal condition of this decision program could be written by specifying an individual specific date at which investing in human capital stops as in Ben Porath (1967). We proceed differently by using the dual formulation that the value of human capital stocks at an arbitrary date $t_0 + d$ in the future is individual specific.[4] Specifically, we write that at the future date $t_0 + d + 1$ the value function or the discounted value of utility stream from $t_0 + d + 1$ onwards is given by:

$$V_{t_0+d+1}(H_{t_0+d+1}, Z_{t_0+d+1}) = a_{t_0+d+1}(Z_{t_0+d+1}) + \kappa \ln H_{t_0+d+1}, \tag{6}$$

[4] This will be the last date of observation in our empirical analysis further on.

in which the level a_{t_0+d+1} generically depends on Z_{t_0+d+1} and in which parameter κ, which is not indexed by t_0+d+1 for simplicity, is assumed to be independent of Z_{t_0+d+1} but is individual-specific.

To complete the description of the economic model, we further assume that the distribution of future shocks $(\omega_l, \delta_l^s, \lambda_l^s)_{l\geq t}$ conditionally on $Z_{t-1/2}$ does not depend on the state variable history $H_t, H_{t-1}, ., H_1$.

3.2 Analysis

3.2.1 Value functions and life cycle profile of investments

The sequence of investments between $t = t_0$ and $t = t_0 + d$ is called a life cycle profile of investments.

Proposition 1 *The sequence of value functions writes:*

$$W_t^s(H_t, Z_{t+1/2}) = a_t^s(Z_{t+1/2}) + \kappa_t \log H_t \text{ for } s = e, n$$

and:

$$V_t(H_t, Z_t) = a_t(Z_t) + \kappa_t \log H_t$$

in which

$$\kappa_t = \frac{1}{1-\beta} + \beta^{t_0+d-t}(\kappa - \frac{1}{1-\beta}).$$

and $a_t^{s_t}(Z_{t+1/2})$ *and* $a_t(Z_t)$ *are defined further below.*

Proof. Consider an individual who evaluates the consequences of working in sector s and choosing human capital investments, τ_t^s, whether it is positive or equal to zero.

The marginal value of human capital can be expressed as the derivative of the interim value function with respect to the level of human capital. By the envelope theorem or replacing by $\tau_t^s = 0$, we have that for any H_t:

$$\begin{aligned}\frac{\partial W_t^s}{\partial H_t} &= \frac{1}{H_t} + \beta \left\{ \exp\left[\rho^s \tau_t^s - \lambda_t^s\right] \mathbb{E}_{t+1/2}\left[\frac{\partial V_{t+1}}{\partial H_{t+1}}\right]\right\} \\ &= \frac{1}{H_t} + \frac{H_{t+1}}{H_t}\beta\mathbb{E}_{t+1/2}\left[\frac{\partial V_{t+1}}{\partial H_{t+1}}\right]\end{aligned}$$

since we have $\frac{H_{t+1}}{H_t} = \exp\left[\rho^s \tau_t^s - \lambda_t^s\right]$. This expression is equivalent to:

$$H_t \frac{\partial W_t^s}{\partial H_t} = 1 + \beta\mathbb{E}_{t+1/2}\left[H_{t+1}\frac{\partial V_{t+1}}{\partial H_{t+1}}\right],$$

and implies that:

$$H_t \mathbb{E}_t \frac{\partial W_t^s}{\partial H_t} = 1 + \beta \mathbb{E}_t \left[H_{t+1} \frac{\partial V_{t+1}}{\partial H_{t+1}} \right]. \tag{7}$$

This shows that derivatives do not depend on s i.e. $E_t \frac{\partial W_t^e}{\partial H_t} = E_t \frac{\partial W_t^n}{\partial H_t}$ and this proves that:

$$H_t \frac{\partial V_t}{\partial H_t} = H_t \frac{\partial}{\partial H_t} (\max(\mathbb{E}_t W_t^e + \omega_t, \mathbb{E}_t W_t^n)) = 1 + \beta \mathbb{E}_t \left[H_{t+1} \frac{\partial V_{t+1}}{\partial H_{t+1}} \right] \tag{8}$$

For $t = t_0 + d + 1$, specification (6) writes:

$$\frac{\partial V_{t_0+d+1}}{\partial H_{t_0+d+1}} = \frac{\kappa}{H_{t_0+d+1}} \Longrightarrow H_{t_0+d+1} \frac{\partial V_{t_0+d+1}}{\partial H_{t_0+d+1}} = \kappa. \tag{9}$$

Denote:

$$\kappa_t = H_t \frac{\partial V_t}{\partial H_t} \tag{10}$$

By backward induction, using (8) and the initial condition (9), all values κ_t are deterministic, that is, independent of Z_t. We obtain that:

$$\kappa_t = 1 + \beta \kappa_{t+1} \Longrightarrow \kappa_t - \frac{1}{1-\beta} = \beta(\kappa_{t+1} - \frac{1}{1-\beta}) \tag{11}$$

so that by backward induction:

$$\kappa_t = \frac{1}{1-\beta} + \beta^{t_0+d+1-t}(\kappa - \frac{1}{1-\beta}).$$

By integration of equations (7) and (10), we obtain the value functions of the Proposition in which the arbitrary constants of integration, $a_t^s(Z_{t+1/2})$ and $a_t(Z_t)$ are further defined below. ■

Proposition 2 *The sequence of potential investments between $t = t_0$ and $t = t_0 + d$ in each sector s is:*

$$\tau_t^s = \max\{0, \frac{1}{c}(\rho^s \beta \kappa_{t+1} - 1)\} \tag{12}$$

Proof. The first order condition of the maximization problem for $t \in [t_0, t_0 + d]$ with respect to the level of investment τ_t is

$$-(1 + c\tau_t) + \beta \rho^s \mathbb{E}_{t+1/2} \left[H_{t+1} \frac{\partial V_{t+1}}{\partial H_{t+1}} \right] = 0, \tag{13}$$

in which H_{t+1} is determined by equation (2). This first order condition delivers a positive optimal human capital investment, $\tau_t^s > 0$, if the following condition holds:

$$\beta \rho^s \mathbb{E}_{t+1/2} \left[H_{t+1} \frac{\partial V_{t+1}}{\partial H_{t+1}} \right] > 1. \tag{14}$$

Using equation (10), this condition is equivalent to $\beta\rho^s\kappa_{t+1} > 1$ and equation (13) yields the optimal investment which verifies:

$$(1 + c\tau_t^s) = \beta\rho^s\kappa_{t+1}, \tag{15}$$

and the second term in equation (12) follows. When $\beta\rho^s\kappa_{t+1} \leq 1$, we obtain that $\tau_t^s = 0$. Furthermore, as the second left hand side term in (13) is constant, the second order condition is satisfied if and only if $c > 0$. ■

Proposition 3 *The values of the sector-specific constant terms in Proposition 1 are:*

$$a_t^s(Z_{t+1/2}) = \delta_t^s - \beta\kappa_{t+1}\lambda_t^s + c\frac{(\tau_t^s)^2}{2} + \beta\mathbb{E}_{t+1/2}\left[a_{t+1}(Z_{t+1})\right].$$

in which τ_t^s is the optimal value of human capital investment when being in sector s as defined in equation (12).

Proof. Using proposition 1:

$$\begin{aligned} W_t^s(H_t, Z_{t+1/2}) &= \delta_t^s + \ln H_t - \left(\tau_t^s + c\frac{(\tau_t^s)^2}{2}\right) + \beta\mathbb{E}_{t+1/2}\left[V_{t+1}\right] \\ &= \delta_t^s + \ln H_t - \left(\tau_t^s + c\frac{(\tau_t^s)^2}{2}\right) + \beta\mathbb{E}_{t+1/2}\left[a_{t+1}(Z_{t+1}) + \kappa_{t+1}\log H_{t+1}\right] \\ &= \delta_t^s + \ln H_t - \left(\tau_t^s + c\frac{(\tau_t^s)^2}{2}\right) + \beta\mathbb{E}_{t+1/2}\left[a_{t+1}(Z_{t+1}) + \kappa_{t+1}(\ln H_t + \rho^s\tau_t^s - \lambda_t^s)\right]. \end{aligned}$$

By identifying constant terms and using (15), we get:

$$\begin{aligned} a_t^s(Z_{t+1/2}) &= \delta_t^s + \left(\beta\kappa_{t+1}\rho^s\tau_t^s - \tau_t^s - c\frac{(\tau_t^s)^2}{2}\right) - \beta\kappa_{t+1}\lambda_t^s + \beta\mathbb{E}_{t+1/2}\left[a_{t+1}(Z_{t+1})\right], \\ &= \delta_t^s + c\frac{(\tau_t^s)^2}{2} - \beta\kappa_{t+1}\lambda_t^s + \beta\mathbb{E}_{t+1/2}\left[a_{t+1}(Z_{t+1})\right]. \end{aligned}$$

by using Proposition 2. ■

Proposition 4 *The sector choice is determined by:*

$$\begin{aligned} s_t &= e \textit{ iff} \\ \omega_t + \mathbb{E}_t\left(\delta_t^e - \beta\kappa_{t+1}\lambda_t^e + c\frac{(\tau_t^e)^2}{2}\right) &\geq \mathbb{E}_t\left(\delta_t^n - \beta\kappa_{t+1}\lambda_t^n + c\frac{(\tau_t^n)^2}{2}\right). \end{aligned} \tag{16}$$

Proof. By equation (5) we have:

$$\begin{aligned} &\omega_t + \mathbb{E}_t\left[\delta_t^e + c\frac{(\tau_t^e)^2}{2} - \beta\kappa_{t+1}\lambda_t^e\right] + \beta\mathbb{E}_t\left[a_{t+1}(Z_{t+1})\right] + \kappa_t\log(H_t) \\ \geq\ &\mathbb{E}_t\left[\delta_t^n + c\frac{(\tau_t^n)^2}{2} - \beta\kappa_{t+1}\lambda_t^n\right] + \beta\mathbb{E}_t\left[a_{t+1}(Z_{t+1})\right] + \kappa_t\log(H_t). \end{aligned}$$

and we note that neither initial conditions H_t nor terminal conditions $E_t\left[a_{t+1}(Z_{t+1})\right]$ depend on current sector choice (absent any transition costs) and we obtain condition (16). It also yields:

$$a_t(Z_t) = \max(\omega_t + \mathbb{E}_t(\delta_t^s - \beta\kappa_{t+1}\lambda_t^s + c\frac{(\tau_t^s)^2}{2}), \mathbb{E}_t(\delta_t^n - \beta\kappa_{t+1}\lambda_t^n + c\frac{(\tau_t^n)^2}{2})) + \beta\mathbb{E}_t\left[a_{t+1}(Z_{t+1})\right].$$

■

3.3 The reduced form

Consider a worker who is in sector e at period t. As observations consist in earnings histories starting in the private sector beginning in t_0, $s_{t_0} = e$. Denote $t_1 = \min\{l; l \geq t_0, s_l = n\} \geq t_0$ the first period in sector n, t_2 the first return in sector e i.e. $t_2 = \min\{l; l > t_1, s_l = e\} > t_1+1$ and so forth by induction, and K_t the overall number of spells in sector n before period t. The sequence $(t_0, t_1, t_2, ., t_{2K_t})$ is then the sequence of transition dates into sector e (even index values) and into sector n (odd index values). We deduce from this setting that the mapping between date $l \leq t$ and the sector is given by:

$$\begin{aligned} s_l &= e \text{ for } t_{2k} \leq l \leq t_{2k+1} - 1 \\ &= n \text{ for } t_{2k+1} \leq l \leq t_{2k+2} - 1 \end{aligned}$$

where $k \leqslant K_t$.

Proposition 5 *Consider a worker who entered sector e for the first time at date t_0, is in sector e at date t, has experienced K_t spells in the alternate sector n before date t and whose trajectory among sectors e and n is described by the sequence $(t_0, ..., t_{2K_t})$, where t_{2k-1} is the date of entry into n at the k^{th} spell and t_{2k} is the date of return into sector e after this date. We also assume that $\tau_l^{s_l} > 0$ for any $t_0 \leq l < t_0 + d + 1$.*

Log earnings are:

$$\ln y_t = \eta_0 + \eta_1 t + \eta_2\beta^{-t} + \eta_3 x_t^{(3)} + \eta_4 x_t^{(4)} + v_t \tag{17}$$

in which:

$$\eta_0 = \ln H_{t_0} - \frac{\rho^e t_0 + 1}{c}\left(\frac{\rho^e\beta}{1-\beta} - 1\right) - \frac{(\rho^e)^2\beta}{c}\frac{\beta^{d+1}}{1-\beta}\left(\kappa - \frac{1}{1-\beta}\right) \quad (18)$$

$$\eta_1 = \frac{\rho^e}{c}\left(\rho^e\frac{\beta}{1-\beta} - 1\right) \quad (19)$$

$$\eta_2 = \beta^{t_0+d+1}\frac{\rho^e}{c}\left(\kappa - \frac{1}{1-\beta}\right)\left(\frac{\rho^e\beta}{1-\beta} - 1\right) \quad (20)$$

$$\eta_3 = \left(\frac{\rho^n}{c}\left(\frac{\rho^n\beta}{1-\beta} - 1\right) - \frac{\rho^e}{c}\left(\frac{\rho^e\beta}{1-\beta} - 1\right)\right) \quad (21)$$

$$\eta_4 = \frac{1}{c}\left((\rho^n)^2 - (\rho^e)^2\right)\left(\kappa - \frac{1}{1-\beta}\right)\frac{\beta^{t_0+d+1}}{1-\beta} \quad (22)$$

and $x_t^{(3)}$, $x_t^{(4)}$ and υ_t are defined by:

$$x_t^{(3)} = \sum_{k=0}^{K_t-1}\left(t_{2k+2} - t_{2k+1}\right)$$

$$x_t^{(4)} = \sum_{k=0}^{K_t-1}\left(\beta^{-t_{2k+2}+1} - \beta^{-t_{2k+1}+1}\right)$$

$$\upsilon_t = \delta^{s(t)}(t) - \sum_{l=t_0}^{t-1}\lambda^{s(l)}(l)$$

Proof. First, the stock of human capital in period t depends on previous investment choices and past depreciation that is

$$\begin{aligned}
H_t &= H_{t_{2K_t}}\exp\left[\sum_{l=t_{2K_t}}^{t-1}\rho^e\tau_l^e - \sum_{l=t_{2K_t}}^{t-1}\lambda_l^e\right] \\
&= H_{t_{2K_t-1}}\exp\left[\sum_{l=t_{2K_t}}^{t-1}\rho^e\tau_l^e - \sum_{l=t_{2K_t}}^{t-1}\lambda_l^e + \sum_{l=t_{2K_t-1}}^{t_{2K_t}-1}\rho^n\tau_l^n - \sum_{l=t_{2K_t-1}}^{t_{2K_t}-1}\lambda_l^n\right] \\
&\dots \\
&= H_{t_0}\exp\left[\sum_{l=t_0}^{t-1}\rho^{s_l}\tau_l^{s_l} - \sum_{l=t_0}^{t-1}\lambda_l^{s_l}\right]
\end{aligned}$$

At each date, we have that

$$\tau^{s_l} = \max\left\{0, \frac{1}{c}\left(\rho^{s_l}\beta\kappa_{l+1} - 1\right)\right\}$$

As long as investments remain strictly positive in both sectors we have that:

$$\begin{aligned}
\ln H_t &= \ln H_{t_0} - \sum_{l=t_0}^{t-1}\lambda_l^{s_l} + \sum_{l=t_0}^{t-1}\rho^{s_l}\tau_l^{s_l} \\
&= \ln H_{t_0} - \sum_{l=t_0}^{t-1}\lambda_l^{s_l} + \sum_{l=t_0}^{t-1}\frac{\rho^{s_l}}{c}\left(\rho^{s_l}\beta\kappa_{l+1} - 1\right)
\end{aligned}$$

Using the sequence of periods in every sector and replacing κ_{l+1} by its expression $\kappa_{l+1} = \frac{1}{1-\beta} + \beta^{t_0+d-l}(\kappa - \frac{1}{1-\beta})$ (see Proposition 1), the term $\sum_{l=t_0}^{t-1} \rho^{s_l} \frac{1}{c} \left(\rho^{s_l}\beta\kappa_{l+1} - 1\right)$ can be decomposed into:

$$\begin{aligned}
\sum_{l=t_0}^{t-1} \frac{\rho^{s(l)}}{c} \left(\rho^{s(l)}\beta\kappa_{l+1} - 1\right) \quad = \quad & \sum_{k=0}^{K_t-1} \sum_{l=t_{2k}}^{t_{2k+1}-1} \frac{\rho^e}{c} \left(\rho^e \frac{\beta}{1-\beta} - 1 + \rho^e \beta^{t_0+d+1-l} \left(\kappa - \frac{1}{1-\beta}\right)\right) \\
& + \sum_{l=t_{2K_t}}^{t-1} \frac{\rho^e}{c} \left(\rho^e \frac{\beta}{1-\beta} - 1 + \rho^e \beta^{t_0+d+1-l} \left(\kappa - \frac{1}{1-\beta}\right)\right) \\
& + \sum_{k=0}^{K_t-1} \sum_{l=t_{2k+1}}^{t_{2k+2}-1} \frac{\rho^n}{c} \left(\rho^n \frac{\beta}{1-\beta} - 1 + \rho^n \beta^{t_0+d+1-l} \left(\kappa - \frac{1}{1-\beta}\right)\right)
\end{aligned}$$

where the first two right-hand-side terms correspond to the accumulation of human capital when she is in sector e and the last one when she is in sector n. Since

$$\begin{aligned}
& \sum_{l=s_0}^{s_1-1} \frac{\rho^e}{c} \left(\rho^e \frac{\beta}{1-\beta} - 1 + \rho^e \beta^{t_0+d+1-l} \left(\kappa - \frac{1}{1-\beta}\right)\right) \\
= \quad & \frac{\rho^e}{c} \left(\rho^e \frac{\beta}{1-\beta} - 1\right)(s_1 - s_0) + \frac{(\rho^e)^2}{c} \beta^{t_0+d+1-s_0} \left(\kappa - \frac{1}{1-\beta}\right) \sum_{l=0}^{s_1-s_0-1} \beta^{-l} \\
= \quad & \frac{\rho^e}{c} \left(\rho^e \frac{\beta}{1-\beta} - 1\right)(s_1 - s_0) + \frac{(\rho^e)^2}{c} \frac{\beta^{t_0+d+2}}{1-\beta} \left(\kappa - \frac{1}{1-\beta}\right) \left(\beta^{-s_1} - \beta^{-s_0}\right)
\end{aligned}$$

the term $\sum_{l=t_0}^{t-1} \frac{\rho^{s_l}}{c} \left(\rho^{s_l}\beta\kappa_{l+1} - 1\right)$ simplifies into:

$$\begin{aligned}
& \frac{\rho^e}{c} \left(\rho^e \frac{\beta}{1-\beta} - 1\right) \sum_{k=0}^{K_t-1} (t_{2k+1} - t_{2k}) + \frac{(\rho^e)^2}{c} \left(\kappa - \frac{1}{1-\beta}\right) \frac{\beta^{t_0+d+2}}{1-\beta} \sum_{k=0}^{K_t-1} \left(\beta^{-t_{2k+1}} - \beta^{-t_{2k}}\right) \\
& + \frac{\rho^e}{c} \left(\rho^e \frac{\beta}{1-\beta} - 1\right)(t - t_{2K_t}) + \frac{(\rho^e)^2}{c} \left(\kappa - \frac{1}{1-\beta}\right) \frac{\beta^{t_0+d+2}}{1-\beta} \left(\beta^{-t} - \beta^{-t_{2K_t}}\right) \\
& + \frac{\rho^n}{c} \left(\rho^n \frac{\beta}{1-\beta} - 1\right) \sum_{k=0}^{K_t-1} (t_{2k+2} - t_{2k+1}) + \frac{(\rho^n)^2}{c} \left(\kappa - \frac{1}{1-\beta}\right) \frac{\beta^{t_0+d+2}}{1-\beta} \sum_{k=0}^{K_t-1} \left(\beta^{-t_{2k+2}} - \beta^{-t_{2k+1}}\right)
\end{aligned}$$

This term can be rearranged considering the differential accumulation of human capital between sectors e and n when the worker is in sector n. This leads to introducing the accumulation of human capital if the individual had been employed in sector e during the whole period:

$$
\begin{aligned}
& \sum_{l=t_0}^{t-1} \frac{\rho^{s_l}}{c} \left(\rho^{s_l} \beta \kappa_{l+1} - 1\right) \\
= \; & \frac{\rho^e}{c} \left(\rho^e \frac{\beta}{1-\beta} - 1\right) \left\{ \sum_{k=0}^{K_t - 1} \left[(t_{2k+1} - t_{2k}) + (t_{2k+2} - t_{2k+1}) \right] + (t - t_{2K_t}) \right\} \\
& + \left[\frac{\rho^n}{c} \left(\rho^n \frac{\beta}{1-\beta} - 1\right) - \frac{\rho^e}{c} \left(\rho^e \frac{\beta}{1-\beta} - 1\right) \right] \sum_{k=0}^{K_t - 1} (t_{2k+2} - t_{2k+1}) \\
& + \frac{(\rho^e)^2}{c} \frac{\beta^{t_0+d+2}}{1-\beta} \left(\kappa - \frac{1}{1-\beta}\right) \left\{ \sum_{k=0}^{K_t - 1} \left[\beta^{-t_{2k+1}} - \beta^{-t_{2k}} + \beta^{-t_{2k+2}} - \beta^{-t_{2k+1}} \right] + \beta^{-t} - \beta^{-t_{2K_t}} \right\} \\
& + \frac{(\rho^n)^2 - (\rho^e)^2}{c} \frac{\beta^{t_0+d+2}}{1-\beta} \left(\kappa - \frac{1}{1-\beta}\right) \sum_{k=0}^{K_t - 1} \left(\beta^{-t_{2k+2}} - \beta^{-t_{2k+1}}\right) \\
= \; & \frac{\rho^e}{c} \left(\rho^e \frac{\beta}{1-\beta} - 1\right) (t - t_0) + \frac{(\rho^e)^2}{c} \frac{\beta^{t_0+d+2}}{1-\beta} \left(\kappa - \frac{1}{1-\beta}\right) \left(\beta^{-t} - \beta^{-t_0}\right) \\
& + \left[\frac{\rho^n}{c} \left(\rho^n \frac{\beta}{1-\beta} - 1\right) - \frac{\rho^e}{c} \left(\rho^e \frac{\beta}{1-\beta} - 1\right) \right] \sum_{k=0}^{K_t - 1} (t_{2k+2} - t_{2k+1}) \\
& + \frac{(\rho^n)^2 - (\rho^e)^2}{c} \frac{\beta^{t_0+d+2}}{1-\beta} \left(\kappa - \frac{1}{1-\beta}\right) \sum_{k=0}^{K_t - 1} \left(\beta^{-t_{2k+2}} - \beta^{-t_{2k+1}}\right)
\end{aligned}
$$

Defining

$$
\begin{aligned}
x_t^{(3)} &= \sum_{k=0}^{K_t - 1} (t_{2k+2} - t_{2k+1}) \\
x_t^{(4)} &= \sum_{k=0}^{K_t - 1} \left(\beta^{-t_{2k+2}} - \beta^{-t_{2k+1}}\right)
\end{aligned} \tag{23}
$$

and

$$
\begin{aligned}
\eta_3 &= \frac{\rho^n}{c} \left(\rho^n \frac{\beta}{1-\beta} - 1\right) - \frac{\rho^e}{c} \left(\rho^e \frac{\beta}{1-\beta} - 1\right) \\
\eta_4 &= \frac{1}{c} \frac{\beta^{T+2}}{1-\beta} \left(\kappa - \frac{1}{1-\beta}\right) \left((\rho^n)^2 - (\rho^e)^2\right)
\end{aligned}
$$

Human capital at date t has the following expression:

$$
\begin{aligned}
\ln H_t = \; & \ln H_{t_0} - \sum_{l=t_0}^{t-1} \lambda_l^{s_l} + \eta_3 x_t^{(3)} + \eta_4 x_t^{(4)} \\
& + \frac{\rho^e}{c} \left(\rho^e \frac{\beta}{1-\beta} - 1\right) (t - t_0) \\
& + \frac{(\rho^e)^2}{c(1-\beta)} \beta^{t_0+d+2} \left(\kappa - \frac{1}{1-\beta}\right) \left(\beta^{-t} - \beta^{-t_0}\right)
\end{aligned}
$$

This expression can then be plugged into the earnings equation which becomes:

$$
\begin{aligned}
\ln y_t &= \delta_t + \ln H_t - \tau_t \\
&= \delta_t + \ln H_{t_0} - \sum_{l=t_0}^{t-1} \lambda_l^{s_l} + \eta_3 x_t^{(3)} + \eta_4 x_t^{(4)} \\
&\quad + \frac{\rho^e}{c}\left(\rho^e \frac{\beta}{1-\beta} - 1\right)(t - t_0) + \frac{(\rho^e)^2}{c\,(1-\beta)} \beta^{t_0+d+2}\left(\kappa - \frac{1}{1-\beta}\right)\left(\beta^{-t} - \beta^{-t_0}\right) \\
&\quad - \frac{1}{c}\left(\frac{\rho^e \beta}{1-\beta} + \rho^e \beta^{t_0+d+1-t}(\kappa - \frac{1}{1-\beta}) - 1\right) \\
&= \ln H_{t_0} - \frac{\rho^e t_0 + 1}{c}\left(\frac{\rho^e \beta}{1-\beta} - 1\right) - \frac{(\rho^e)^2 \beta}{c} \frac{\beta^{d+1}}{1-\beta}\left(\kappa - \frac{1}{1-\beta}\right) \\
&\quad + \frac{\rho^e}{c}\left(\rho^e \frac{\beta}{1-\beta} - 1\right) t \\
&\quad + \frac{\rho^e}{c}\left(\frac{\rho^e \beta}{1-\beta} - 1\right)\beta^{t_0+d+1}\left(\kappa - \frac{1}{1-\beta}\right)\beta^{-t} \\
&\quad + \delta_t - \sum_{l=t_0}^{t-1} \lambda_l^{s_l} + \eta_3 x_t^{(3)} + \eta_4 x_t^{(4)}
\end{aligned}
$$

■

4 Econometric model: Identifying restrictions

In the data, we observe wages in case of employment only during periods $1, ..., T$ (i.e. the dates covered by our panel data) and we use this information to estimate equation (17). If the individual is employed, the wage involved in this equation is the one observed in the data. If the individual is not employed, it is the counterfactual wage if the individual were employed. In particular, we allow structural parameters implicit in this equation, ρ_i^e, ρ_i^n, c_i, κ_i and $\log(H_{i1})$, i.e. returns in both sectors, the cost of effort, the terminal value of human capital and the initial value of human capital, to be individual-specific while we restrict heterogeneity by assuming that the discount factor β is homogeneous. The log earnings equation can thus be written as

$$
\begin{aligned}
\ln y_{it} &= \eta_{i0} + \eta_{i1} t + \eta_{i2} \beta^{-t} + \eta_{i3} x_{it}^{(3)} + \eta_{i4} x_{it}^{(4)} + v_{it}, \\
&= \eta_{i0} + x_{it} \eta_i + v_{it},
\end{aligned}
$$

where $v_{it} = \delta_{it} - \sum_{l=t_0}^{t-1} \lambda_l^{s_{il}}$ in which s_{il} is the sector chosen by individual i at period l and $x_{it} = (t, \beta^{-t}, x_{it}^{(3)}, x_{it}^{(4)})$ and $\eta_i = (\eta_{ij})_{j=1,.,4}$.[5]

[5] From now on, when there is no superscript, parameters are those in the employment sector; otherwise the superscript gives the sector.

We start by looking at the identification of parameters η in the case selection in employment is exogenous. We then turn to stating the conditions under which selection is conditionally exogenous.

4.1 Identification under exogenous selection

We begin with elements about identification when selection in employment is exogenous. Note that the number of structural parameters and the number of reduced form parameters are both equal to 5 for each individual. Yet, a necessary condition for point identification is that there is enough individual mobility across sectors. Indeed, consider an individual i who is employed during the whole period in sector e, or who moves only once out of sector e to sector n, so that $x_{it}^{(3)} = x_{it}^{(4)} = 0$ for all dates t during which this individual is working in sector e. Parameters η_{i3} and η_{i4} are not identified. Turn now to an individual making two transitions, one from e to n first, and then a return from n to e later. In this case, $x_{it}^{(3)} = (t_{2i} - t_{1i})\, 1_{t \geqslant t_{2i}}$ and $x_{it}^{(4)} = \left(\beta^{-t_{2i}} - \beta^{-t_{1i}}\right) 1_{t \geqslant t_{2i}}$, and the two variables $x_{it}^{(3)}$ and $x_{it}^{(4)}$ are proportional to $1_{t \geqslant t_{2i}}$. Parameters η_{i3} and η_{i4} are not separately identified since the linear combination $\eta_{i3}\,(t_{2i} - t_{1i}) + \eta_{i4}\left(\beta^{-t_{2i}} - \beta^{-t_{1i}}\right)$ only is. Furthermore, an additional final exit from employment would not have any additional identifying power. It is only if an individual makes four transitions (two from e to n and two from n to e) that parameters η_{i3} and η_{i4} are identified separately. Remark that underidentification of parameters η_{i3} and η_{i4} does not affect the identification of the other parameters η_{i0}, η_{i1} and η_{i2}.

4.2 Missing conditionally at random assumption

We now discuss the identifying assumptions that we adopt and that make selection exogeneous when we impose the structural model and in particular the participation equation (16). First, stochastic processes ω_{it} (i.e. sector preference), δ_{it} (i.e. human capital price) and λ_{it} (i.e. depreciation), are specified as linear factors:

$$\omega_{it} = \varphi_t^{(\omega)}\theta_i^{(\omega)} + \widetilde{\omega}_{it}, \tag{24}$$

$$\delta_{it}^s = \varphi_t^{(\delta),s}\theta_i^{(\delta),s} + \widetilde{\delta}_{it}^s, \tag{25}$$

$$\lambda_{it}^s = \varphi_t^{(\lambda),s}\theta_i^{(\lambda),s} + \widetilde{\lambda}_{it}^s. \tag{26}$$

in which residual random shocks satisfy the following orthogonality restrictions:

$$\begin{aligned} \left(\widetilde{\omega}_{it}, \widetilde{\delta}_{it}^s, \widetilde{\lambda}_{it}^s\right)_{t\geq 1, s\in\{n,e\}} &\perp \left(\varphi_t^{(\omega)}\theta_i^{(\omega)}, \varphi_t^{(\delta),s}\theta_i^{(\delta),s}, \varphi_t^{(\lambda),s}\theta_i^{(\lambda),s}\right)_{t\geq 1, s\in\{n,e\}}, \\ E\left(\widetilde{\omega}_{it}\,|\Phi_t, \theta_i\right) &= E\left(\widetilde{\delta}_{it}^s\,|\Phi_{t+1/2}, \theta_i\right) = E\left(\widetilde{\lambda}_{it}^s\,|\Phi_{t+1/2}, \theta_i\right) = 0. \end{aligned} \tag{27}$$

where we denote $\Phi_t = \left\{\varphi_t^{(\omega)}, \Phi_{t-1}\right\}$ to mimic for factors the construction of history Z_t and we define $\Phi_{t+1/2}$ accordingly that is $\Phi_{t+1/2} = \left\{\varphi_t^{(\delta),e}, \varphi_t^{(\delta),s}, \varphi_t^{(\lambda),e}, \varphi_t^{(\lambda),n}, \Phi_t\right\}$, consistently with the timing of Figure 1. We also denote $\theta_i = \left\{\theta_i^{(\omega)}, \theta_i^{(\delta),e}, \theta_i^{(\delta),n}, \theta_i^{(\lambda),e}, \theta_i^{(\lambda),n}\right\}$ and extend the notation Z_t and $Z_{t+1/2}$ in a natural way to $\tilde{Z}_t$ and $\tilde{Z}_{t+1/2}$ which now refer to the histories of residual random shocks $\widetilde{\omega}_{it}$, $\widetilde{\delta}_{it}^s$ and $\widetilde{\lambda}_{it}^s$.

Second, we assume that:

Assumption M(issing)C(onditionally on)F(actors)A(t)R(andom):

$$\widetilde{\omega}_{it} \perp \tilde{Z}_{t-1/2} \Big| \Phi_t, \theta_i \tag{28}$$

$$(\widetilde{\delta}_{it}^s, \widetilde{\lambda}_{it}^s) \perp \tilde{Z}_t \Big| \Phi_{t+1/2}, \theta_i \tag{29}$$

Note that it implies that $\{\widetilde{\omega}_{it}\}_{t\geq 1}$ and $\{(\widetilde{\delta}_{it}^s, \widetilde{\lambda}_{it}^s)\}_{t\geq 1}$ are independent and that they are both independently distributed over time (although they could be heteroscedastic).

We now prove that these assumptions in a linear factor setting imply that selection is exogenous and that experience variables $x_{it}^{(3)}$ and $x_{it}^{(4)}$ are exogenous. First rewrite the wage equation under assumptions (24)-(26):

$$\ln y_{it} = \eta_{i0} + x_{it}\eta_i + \varphi_t^{(\delta)}\theta_i^{(\delta)} - \left[\sum_{l=t_0}^{t-1} \varphi_l^{(\lambda),n} 1_{\{s_{il}=n\}}\right] \theta_i^{(\lambda),n} - \left[\sum_{l=t_0}^{t-1} \varphi_l^{(\lambda),e} 1_{\{s_{il}=e\}}\right] \theta_i^{(\lambda),e} + \widetilde{v}_{it} \tag{30}$$

where $\widetilde{v}_{it} = \widetilde{\delta}_{it} - \sum_{l=t_0}^{t-1} \widetilde{\lambda}_l^{s_{il}}$ can be rewritten as:

$$\widetilde{v}_{it} = \widetilde{\delta}_{it} - \left(\sum_{l=t_0}^{t-1} \widetilde{\lambda}_{il}^n 1_{\{s_{il}=n\}}\right) - \left(\sum_{l=t_0}^{t-1} \widetilde{\lambda}_{il}^e 1_{\{s_{il}=e\}}\right). \tag{31}$$

Second, the sector choice equation (16) can be rewritten as:

$$\tilde{\omega}_{it} + \mathbb{E}_t\left(\tilde{\delta}_{it}^e - \beta\kappa_{it+1}\tilde{\lambda}_{it}^e\right) - \mathbb{E}_t\left(\tilde{\delta}_{it}^n - \beta\kappa_{it+1}\tilde{\lambda}_{it}^n\right) \geq f(\varphi_t, \theta_i)$$

in which $f(\varphi_t, \theta_i)$ is a function of factors and factor loadings which in particular subsumes investment terms like $c_i\frac{(\tau_{it}^s)^2}{2}$.[6] Because of definition (27) and condition (29) we have that $E_t\left(\tilde{\delta}_{it}^s - \beta\kappa_{it+1}\tilde{\lambda}_{it}^s\right) = 0$ for $s = e, n$ and the selection equation rewrites as:

$$\tilde{\omega}_{it} \geq f(\varphi_t, \theta_i). \tag{32}$$

[6] Indeed, the sector choice depends on the terms $\tau_i^s(t)$ as shown by equation (16), which depend themselves on ρ_i^s through equation (12).

Furthermore, conditions (28) and (29) imply that:

- $\tilde{\omega}_{it}$ is independent of $\widetilde{\delta}_{it}$ given factors and factor loadings $(\Phi_{t+1/2}, \theta_i)$,
- $\tilde{\omega}_{it}$ is independent of the history of depreciation shocks, $\widetilde{\lambda}_i^s$, $s \in \{n, e\}$, up to date $t-1$, given factors and factor loadings $(\Phi_{t+1/2}, \theta_i)$,
- $\tilde{\omega}_{it}$ is independent of the history of sector preferences, $\widetilde{\omega}_i$, up to date $t-1$, given factors and factor loadings (Φ_t, θ_i).

and this in turn implies that $\tilde{\omega}_{it}$ and $\widetilde{v}_{it}$ are independent given factors and factor loadings $(\Phi_{t+1/2}, \theta_i)$. This proves that under conditions (28) and (29), selection is exogenous.[7]

Furthermore, explanatory variables $x_{it}^{(3)}$ and $x_{it}^{(4)}$ are exogenous under the same conditions. Indeed, these two variables can be written as functions of past sector choices as stated in equation (23). For instance:

$$x_{it}^{(3)} = \sum_{l=1}^{t-1} 1_{\{s_{il}=n\}}.$$

We evaluate $E\left(\widetilde{v}_{it} \middle| x_{it}^{(3)}, x_{it}^{(4)}, \Phi_{t+1/2}, \theta_i\right)$ as given by equation (31). First, $(x_{it}^{(3)}, x_{it}^{(4)})$ and $\widetilde{\delta}_{it}$ are independent because of condition (29). Moreover the second and third terms of $\widetilde{v}_{it}$ are such that:

$$\begin{aligned}
& E\left[\left(\sum_{l=t_0}^{t-1} \widetilde{\lambda}_{il}^s 1_{\{s_{il}=s\}}\right) \mid x_{it}^{(3)}, x_{it}^{(4)}, \varphi_t, \theta_i\right] \\
= \; & E\left[E\left(\sum_{l=t_0}^{t-1} \widetilde{\lambda}_{il}^s 1_{\{s_{il}=s\}} \mid \omega_{it-1,.,}\omega_{i1}, \varphi_t, \theta_i\right) \mid x_{it}^{(3)}, x_{it}^{(4)}, \Phi_{t+1/2}, \theta_i\right] \\
= \; & E\left[\left(\sum_{l=t_0}^{t-1} E(\widetilde{\lambda}_{il}^s \mid \omega_{it-1,.,}\omega_{i1},, \varphi_t, \theta_i) 1_{\{s_{il}=s\}}\right) \mid x_{it}^{(3)}, x_{it}^{(4)}, \Phi_{t+1/2}, \theta_i\right] \\
= \; & 0
\end{aligned}$$

because the processes $\{\widetilde{\omega}_{it}\}_{t\geq 1}$ and $\{(\widetilde{\delta}_{it}^s, \widetilde{\lambda}_{it}^s)\}_{t\geq 1}$ are independent over time conditional on factors and factor loadings. This is why we obtain that covariates are exogenous since:

$$E\left(\widetilde{v}_{it} \middle| x_{it}^{(3)}, x_{it}^{(4)}, \Phi_{t+1/2}, \theta_i\right) = 0.$$

[7] These conditions are sufficient and far from necessary and are stated this way for the sake of simplicity. In particular, the independence assumption between preference shocks, ω_{it}, and price shocks, δ_{it}, might be substantially weakened.

5 Estimation strategy

This strategy is dictated by our available data which consist in employment status and wage histories when employed for cohorts of individuals entering the labour market between 1985 and 1992 in France. No information is available when individuals are not employed.

5.1 Estimation procedure

We estimate the model pooling all cohorts together and making the simplifying assumption that factors and factor loadings associated with the depreciation rate of human capital are the same for the two sectors: $\varphi_t^{(\lambda),e} = \varphi_t^{(\lambda),n}$ and $\theta_i^{(\lambda),e} = \theta_i^{(\lambda),n}$. In that case, interactive terms associated with the rental price and depreciation rate of human capital enter additively in a similar way in the wage equation and they are thus undistinguishable. Without loss of generality, we thus drop terms $\varphi_t^{(\lambda),s}\theta_i^{(\lambda),s}$ from the wage equation and we relabel $\varphi_t^{(\delta)}\theta_i^{(\delta)}$ as $\varphi_t\theta_i$. The wage equation becomes:

$$y_{it} = \eta_{i0} + x_{it}\eta_i + \varphi_t\theta_i + \widetilde{v}_{it} \tag{33}$$

where:

$$\begin{aligned} x_{it} &= \left(t, \beta^{-t}, x_{it}^{(3)}, x_{it}^{(4)}\right) \\ \eta_i &= \{\eta_{i1}, \eta_{i2}, \eta_{i3}, \eta_{i4}\}' \end{aligned}$$

Because of the presence of additive individual fixed effects, η_{i0}, and individual effects, η_{i1} and η_{i2}, interacting with individual-invariant explanatory variables, t and β^{-t}, a normalisation restriction on factors is needed and we adopt the following one:

$$\varphi \perp \left(e_T, x^{(1)}, x^{(2)}\right) \tag{34}$$

with $\varphi = (\varphi_1, ..., \varphi_T)'$, $x^{(1)} = (1, ..., T)'$ and $x^{(2)} = (1, ..., \beta^{-T})'$. Note in particular that this restriction yields that the average of interactive time effects is zero: $\sum_{t=1}^{T}\varphi_t = 0$. There is no such normalisation on interactive individual effects, θ_i, because the structural model does not impose any additive time fixed effects.

Our approach consists in maximizing a pseudo-likelihood for observations for which wages are observed. Minimizing the sum of squares is equivalent to maximizing the log-likelihood of identically and independently distributed normals. As the model involves interactive effects and the panel is not balanced, we use an EM algorithm. In the expectation step, we replace wages with their linear predictions at dates at which workers have not yet entered the labor market – a form of left-censoring – or are not employed (sector n).[8] In the maximization step, we maximize the quasi-likelihood for observations corresponding to all individuals and dates. Heyden and Morton (1996) show that this EM algorithm is valid and allows to recover the pseudo-score estimators of parameters. Note that this procedure is merely a statistical device to recover consistent estimators of parameters, and it does not matter that wages are imputed using our model specification at dates where the model is not verified.

We now describe our following iteration algorithm in which we use (k) as a superscript for parameters at step k. Our initial conditions, $\varphi^{(0)}$, are such that $\frac{\varphi^{(0)}(\varphi^{(0)})'}{T} = I$, the identity matrix. When y_{it} is not observed, it is replaced with the prediction obtained from individual-specific regressions of observed values of y_{it} on x_{it}.

The updating from step $k-1$ to step k is the following:

1. We regress y_{it} on x_{it} and $\varphi_t^{(k-1)}$ for each individual considering only periods at which they are observed, and we recover the estimators $\eta_i^{(k)}$, $\eta_{0i}^{(k)}$ and $\theta_i^{(k)}$.[9]

2. We predict the values of y_{it} when not observed or censored using the formula: $\widehat{y}_{it} = \eta_{0i}^{(k)} + x_{it}\eta_i^{(k)} + \varphi_t^{(k-1)}\theta_i^{(k)}$.

3. We estimate the factor model: $y_{it} - \eta_{0i}^{(k)} - x_{it}\eta_i^{(k)} = \varphi_t\theta_i + \widetilde{\widetilde{v}}_{it}$ and recover the estimator $\varphi_t^{(k)}$ using Bai (2009)'s approach. Accordingly, we impose the identification restrictions: $\varphi\varphi'/T = I$, $\theta\theta'/N$ is diagonal and the first element of each row of φ_1 is positive.[10] For the additional identification restrictions (34) to be verified, we project $\varphi_t^{(k)}$ on the space orthogonal to e_T, $x^{(1)}$ and $x^{(2)}$. We then re-normalize the projection within this space such that the identification restriction $\varphi_t^{(k)}\varphi_t^{(k)\prime}/T = I$ is still verified and such that $\varphi_1 > 0$

We need a criterium to stop the iterative procedure after a meaningful number of steps, such that there is convergence when the criterium takes values that are small enough. We choose

[8] Some workers are more than 50 years old and according to a flat-spot approach we assume that they no longer accumulate human capital. We also replace their wages by their linear prediction after 50 as a mere statistical device to balance the panel. This is akin to right-censoring.

[9] Note that we retain the estimator of θ_i at this step rather than the one from Bai's procedure at step 3 of previous iteration to avoid using imputed values of y_{it} to estimate θ_i. This makes the algorithm converge faster. Note also that even if $\theta_i^{(k)}\theta_i^{(k)\prime}/N$ is not diagonal by construction at each iteration of our algorithm, it becomes diagonal as the algorithm converges since estimated parameters converge to the least square solution as shown in Appendix C.

[10] Alternatively, regressing $y_{it} - \eta_{0i}^{(k)} - x_{it}\eta_i^{(k)}$ on $\theta_i^{(k)}$ under the constraint $\varphi\varphi'/T = I$ would deliver another estimate of φ.

this criterium to be the minimum of two subcriteria for factors and factor loadings. According to Bai's approach, factors can be recovered as the K eigenvectors (multiplied by $\sqrt{T}$ because of the restriction $\varphi^{(k)}\varphi^{(k)\prime}/T = I$) corresponding to the K largest eigenvalues of matrix $\sum_{i=1}^{N}\left(y_i - \eta_{0i}^{(k)} - x_i\eta_i^{(k)}\right)\left(y_i - \eta_{0i}^{(k)} - x_i\eta_i^{(k)}\right)'$ where $y_i = (y_{i1}, ..., y_{iT})'$ and $x_i = (x_{i1}', ..., x_{iT}')'$. It is then problematic to consider the convergence of estimated factors one by one, since their order can change between two iterations when their eigenvalues are close. Consequently, we rather consider that there is convergence when the space generated by estimated factors converges. Our first subcriterium is thus: $C_1 \equiv \left\| M_{\varphi^{(k-1)}}\varphi^{(k)} \right\| /RT$.

Furthermore, it is too demanding to have each factor loading converge and we rather want to achieve convergence for their averages and covariance matrices. Our second subcriterium is thus: $C_2 \equiv \min(c_1, c_2)$ where:

$$c_1 = N\left(\overline{\theta}^{(k)} - \overline{\theta}^{(k-1)}\right)' V\left(\overline{\theta}^{(k-1)}\right)^{-1}\left(\overline{\theta}^{(k)} - \overline{\theta}^{(k-1)}\right)$$

with $\overline{\theta}^{(k-1)} = \sum_{i=1}^{N}\theta_i^{(k-1)}/N$ (the inverse of variance $V\left(\overline{\theta}^{(k-1)}\right)$ being used to give less weight to averages of factor loadings estimated with more uncertainty), and :

$$c_2 = tr\left[\left(V\left(\overline{\theta}^{(k)}\right) - V\left(\overline{\theta}^{(k-1)}\right)\right)\left(V\left(\overline{\theta}^{(k)}\right) - V\left(\overline{\theta}^{(k-1)}\right)\right)'\right]/tr\left[V\left(\overline{\theta}^{(k-1)}\right)\right]$$

using the fact that $tr\left[(A-B)'(A-B)\right]$ is a distance between matrices A and B, and dividing by $tr\left[V\left(\overline{\theta}^{(\delta)}\right)\right]$ as a normalization. Our overall criterium is $C = \min(C_1, C_2)$ and we consider that there is convergence when $C < tol$ where $tol = 1e-4$.

6 Results

We present estimation results of our main specification described in equation (33) and that includes two factors. We also comment how estimates vary when changing the number of factors.

6.1 Estimated coefficients and explanatory power

We first decompose the variance into: (1) the overall effect of *observed factors,* $(1, t, \beta^{-t})$,whose interactions with factor loadings describe potential log-wages in sector e due to human capital investments and include the additive individual effect $(\eta_{i0} + t\eta_{i1} + \beta^{-t}\eta_{i1})$, (2) the overall effect of absences or *interruptions* from the panel $(x_{it}^{(3)}\eta_{i3} + x_{it}^{(4)}\eta_{i4})$ (3) the overall effect of *unobserved factors* $(\varphi_t\theta_i)$ and(4) the residual $(\widetilde{v}_{it})$. We distinguish neither linear, η_{i1}, from non-linear, η_{i2}, effects of potential experience, nor between linear and non linear effects of interruptions, η_{i3} and η_{i4} because both elements within those pairs of factor loadings are highly correlated.

Table 3 reports the estimated correlations between these groups of effects, their correlations with log-wages (y_{it}), as well as their own estimated variances. It shows that the correlation between the overall effect of observed factors and the overall effect of absences is negative and sizable (-0.85). There might be a mechanical effect due to the fact that the number of years during which a male individual is absent from the panel is positively related to the length of the observation period and therefore to the level of human capital investments. More substantially, this denotes that larger human capital investments are associated with much fewer interruptions or with much smaller duration of those interruptions. Unobserved factors and observed factors effects are also negatively correlated (-0.23). One possible interpretation is that temporary shocks to human capital prices or depreciation might be inversely related to the level of human capital stocks. As the correlation between the overall effect of factors and the effects of absences is small (-0.05), further selection terms are unlikely to be governing unobserved factors.

Unsurprisingly, observed factors have the largest explanatory power since its overall effect has the largest variance and the largest correlation with log-wages (0.43). Variables denoting the number of periods outside the panel also have a large explanatory power but the correlation of their overall effect with log-wages is close to zero (0.004). Recall however that the effect of interruptions are not identified for observations which are continuously or quasi-continuously observed over the whole period. In this case, their estimates are implicitly set to zero.

Besides, unobserved factors do not have a large explanatory power but at the same time it is far from being negligible, since the variance of their overall effect is more than one-third that of log-wages (although the correlation between their overall effect and log-wages is only 0.05). If instead of the correlation matrix, we return to the covariance matrix, the variance of log-wages (0.147) can be decomposed into a covariance between log-wages and observed factors equal to 0.133, between log wages and interruptions of 0.001, between log wages and unobserved factors of 0.005 and finally the covariance of residual terms is equal to 0.008. Heterogeneity terms on top of potential experience are not that important at the level of the whole population. Note that the variance of residuals is close to zero, suggesting that the numerous sources of unobserved heterogeneity introduced in the model are enough to get a excellent fit for log-wages. Furthermore, Figure 7 shows that the time profile of the two factors introduced in the specification have cycles and, without surprise, those are countercyclical since our identification restriction imposes that factors are orthogonal.

6.2 Wage profiles

We begin with analyzing the evolution of average log wages as a function of potential experience and turn to the analysis of the profiles of their variances along the life-cycle. We contrast these evolutions across education levels and across the number of interruptions in private sector individual careers.

6.2.1 Evolution of mean wages with potential experience

We predict individual log-wages in teh counterfactual situation in which individuals would have been employed continuously in the private sector since their entry on the labour market. More precisely, the counterfactual log-wage of an individual i at any given date t is predicted to be:

$$y_{it}^c = \widehat{\eta}_{i0}^c + \widehat{\eta}_{i1}^c t + \widehat{\eta}_{i2}^c \beta^{-t} \tag{35}$$

when $(\widehat{\eta}_{i0}^c, \widehat{\eta}_{i1}^c, \widehat{\eta}_{i2}^c)$ are estimates of individual parameters obtained. Those predictions can be interpreted as potential log-wages without any interruption. As observed factors used to construct these counterfactuals, x_{i1} and x_{i2}, are written as a function of calendar time, it is simpler to abstract from calendar effects and consider a counterfactual situation in which all individuals enter the labor market during the same year – chosen to be 1985. Counterfactual individual parameters corresponding to that situation can be computed from structural formulas (18)-(20) and they verify $(\eta_{i0}^c, \eta_{i1}^c, \eta_{i2}^c) = (\eta_{i0} + \eta_{i1}(T - t_{0i}), \eta_{i1}, \eta_{i2} \exp\left[\beta^{-(T-t_{0i})}\right])$. This presentation allows us to focus on the effects of potential experience on log-wages in the private sector for all cohorts over the first twenty years of potential experience during which all cohorts are present in the data.

We present results on how the population mean and variance of counterfactual log-wages evolve with potential experience using sets of estimates derived from different specifications. Our main and preferred specification includes the observed factors, the variables describing interruptions and two unobserved factors. Alternative specifications either omit interruption variables or vary the number of unobserved factors. Discrepancies between counterfactual predicted profiles indicate how results are biased by the omission of these components. We also conduct the same exercise distinguishing counterfactual log-wage profiles by education level and the number of interruptions.

Figure 8 displays the average counterfactual log-wage profiles. As log-wages are deflated with the price of human capital, their level is not meaningful and the mean counterfactual log-wage in the main specification is normalized to zero at the initial date. Results show that the mean counterfactual log-wage is increasing and slightly concave in the main specification. The mean yearly increase in log-wages over the whole period is 3.7% – wages approximately doubling over 20 years – but 17 years after the entry on the labor market, this average return decreases to 1.5% only because of lower human capital investments (concave profiles).

When the estimated specification retains either none, one or three unobserved factors, the profile of the mean counterfactual log-wage is hardly affected although the curvature of the profile is less pronounced with no or one factor only. When unobserved factors and variables related to non-employment (x_{i3} and x_{i4}) are both omitted, the mean counterfactual log-wage profile is flatter and less concave in a sizeable way. This profile starts above the main specification one but crosses it after five years and ends up below. After twenty years of potential experience, the difference of mean counterfactual log-wages is around 7.8 percentage points between the specifications – out of a total of about 100 points as mentionned above. Overall, this highlights, not so much the importance of including unobserved factors, but the importance of allowing for individual variables describing private sector interruptions.

We repeat the same exercise for subsamples stratified by education level and the number of interruptions. As above, education levels are grouped into high school drop-outs, high-school graduates, some college or college graduates. As the price of human capital varies across education levels, the comparison of mean counterfactual log-wages across education levels is not meaningful and we normalize profiles in such a way that their starting values at the initial date, in the main specification, are zero for every education level. Figure 9 plots profiles for two specifications, the main one and when there are no unobserved factors. First, the slope of the profiles increases with the level of education and profiles are slightly concave at all education levels. For the main specification, the mean yearly increase in log-wage is twice larger for college graduates than for high-school drop-outs (5.9% vs. 2.9%). For every education level, the predicted profile in the specification with no unobserved factor is close to the one derived using the main specification. This suggests that ignoring selection hardly biases the estimated returns to experience whatever the education level.

Figure 10 plots log-wage profiles for the main specification and when both unobserved factors and interruption variables are omitted. Interestingly, for every education level, the profile in the main specification is above the one using estimates omitting unobserved factors and interruption variables. Returns to experience are biased downward when ignoring interruptions during which human capital is likely to deteriorate. The length of interruption periods and deterioration rates vary across education levels, and the discrepancy between the two profiles is the largest for high-school drop-outs and workers with some college experience. This is not surprising because high-school drop-outs experience more frequent and longer periods of interruptions which affects human cpital stocks (see below). Moreover short-track college individuals are less likely to have attractive options outside private-sector jobs that can prevent the deterioration of their human capital. In contrast, college and high-school graduates may engage into work activities outside the private sector in which the deterioration of human capital might be less likely. In particular, these groups could go through spells of white collar self-employment as professionals or blue collar self-employment when holding a technical high-school diploma.

We repeat the same exercise for subgroups of workers depending on the number of interruptions (0, 1, 2, 3 or $\geqslant 4$). The comparison of wage levels is not meaningful because the composition in terms of education levels varies across subgroups and wages are deflated with education level-specific prices of human capital. For instance, high-school drop-outs are over-represented in the subgroup with four or more interruptions. Consequently, we normalize profiles as before in such a way that their starting values at the initial date in the main specification are zero for each subgroup.

Figure 11 reports for every subgroup the mean counterfactual log-wage in the main specification and for the alternative specification in which there are no unobserved factors. Interestingly, it shows that the return to experience is biased downward when the specification has no factor and the discrepancy between profiles using different specifications tends to increase with the number of interruptions. This Figure shows that in the specification without unobserved factors, the prediction of potential wages for permanent workers and workers with interruptions are different while they are closing in when two unobserved factors are estimated. This evinces that variables describing interruptions, x^3 and x^4, and unobserved factors are correlated.

Finally, Figure 12 displays, for every subgroup, the mean counterfactual log-wage in the main specification and in an alternative specification which omits unobserved factors and interruptions. For a given subgroup, the difference between the two profiles is even larger than previously and it increases with the number of interruptions. This points out again that returns to human capital are underestimated when interruption periods are not taken into account. The downward bias is very large when interruption periods are frequent. After 20 years, the difference between the two profiles is as large as 28.6 percentage points for individuals experiencing four interruptions or more.

6.2.2 Evolution of wage dispersion with experience

We now assess how the variance of counterfactual log-wages evolves with experience for alternative specifications and subgroups. An important issue is the sampling error on the estimated individual parameters – of an order of magnitude equal to the reciprocal of the number of observed periods – that biases the empirical estimates of population variances (for instance, Arellano and Bonhomme, 2012). We propose a simple approach to deal with the sampling error in which the sampling error on estimated factors is ignored. This is a rather unconsequential approximation since factors are period-specific effects estimated using a large number of individuals. This approach is detailed in Appendix D and we comment results on bias-corrected variances and compare them with uncorrected variances to assess the effect of our small-sample correction.

Figures 13 displays the profile of variances of log-wages as a function of potential experience. For our main specification, the variance decreases until four years of potential experience before increasing until the end of our twenty-year period. This is consistent with a Mincer dip in wage profiles due to the crossing of heterogenous log-wage profiles (Mincer, 1974). This makes a big difference for the variance of potential wages. At the dip, the variance estimate is around .25 while it approaches 1 after 20 years of potential experience

This is true for corrected and uncorrected estimates. This correction is however very significant for all specifications and in particular in the main specification, decreases estimated variances of log-wages after 20 years of potential experience from a raw value of about 1 to a corrected value of .5. When groups of variables are omitted, this also systematically biases variances downwards. Specifically, allowing for more unobserved factors in the specification increases estimated variances. Increasing the number of factors might play out on the precision of estimates of individual parameters and mechanically increase variance estimates. Our small-sample bias-correction does not seem to succeed in balancing away this additional source of small sample-bias.

Figures 14 and 15 report estimates of the profiles of variance for different education levels. We find again that omitting factors biases downwards variance estimates for all education levels. More originally, we find that college and high school graduates have the highest predicted variance of log-wages after 20 years, the variance estimates of short-track college and high school drop-outs being slightly lower. This is to be related to the ordering between these groups obtained when average log-wages across education levels are compared.

Finally, Figures 16 and 17 display variance estimates in subsamples stratified by the number of interruptions. the profiles of variance of log-wages for the subsample of individuals without any interruption in their careers is much flatter than in the other subsamples. The order of magnitude of variances is much higher in subsamples in which the number of interruptions is above 3 and the profiles of these variance estimates are much steeper.

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Figure 1: Timing of the model

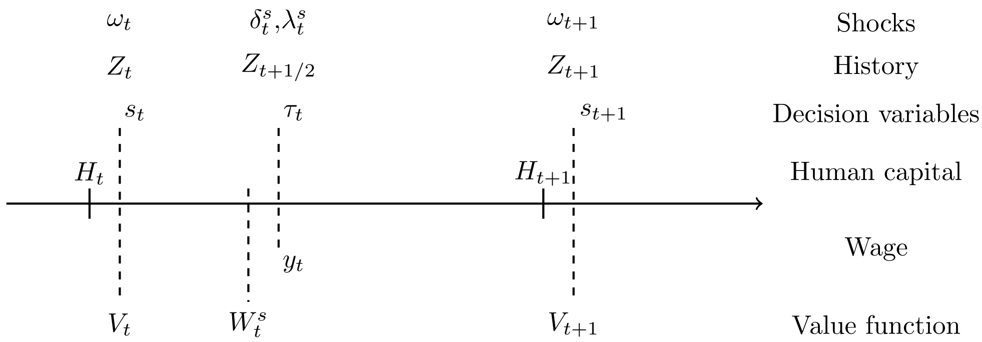
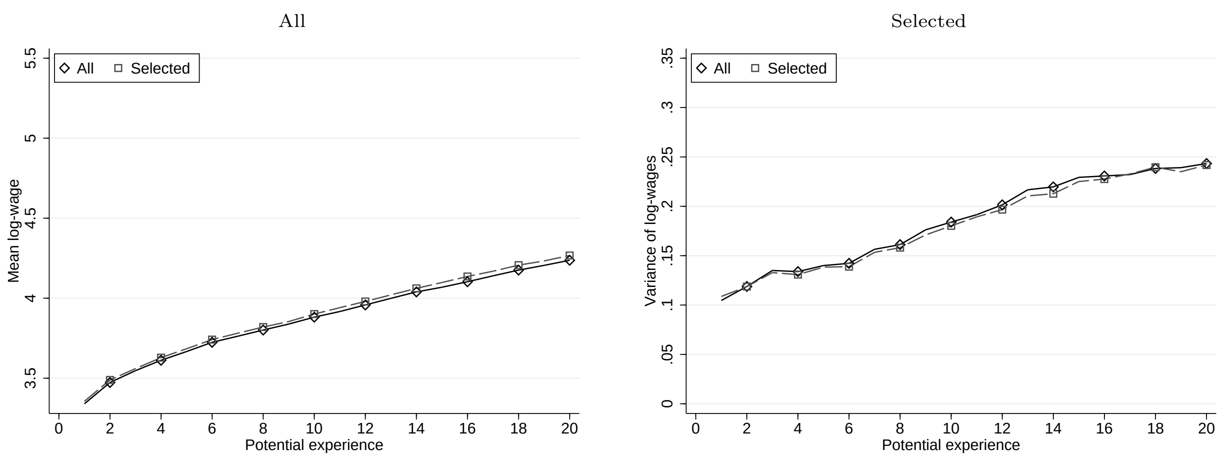
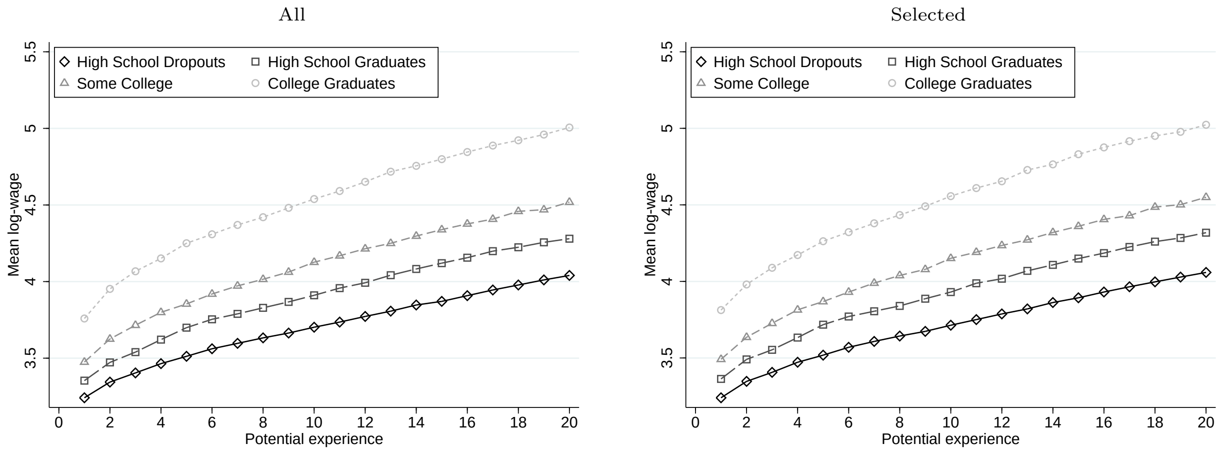


Figure 2: Mean log-wage as a function of potential experience



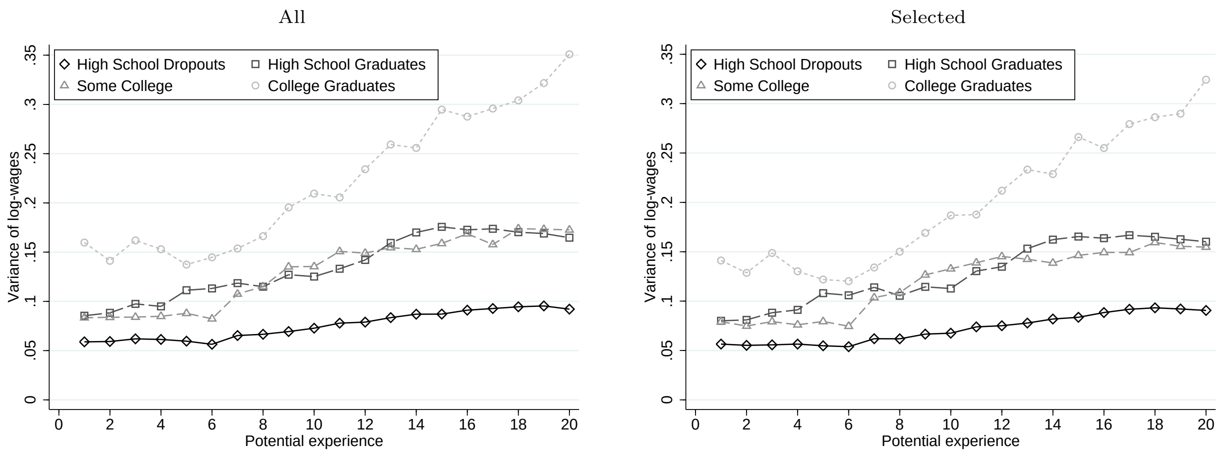
Note: "All": all individuals of our seven cohorts; "Selected": individuals of our seven cohorts who are employed at least 15 years during their timespan.

Figure 3: Mean log-wage as a function of potential experience, by diploma



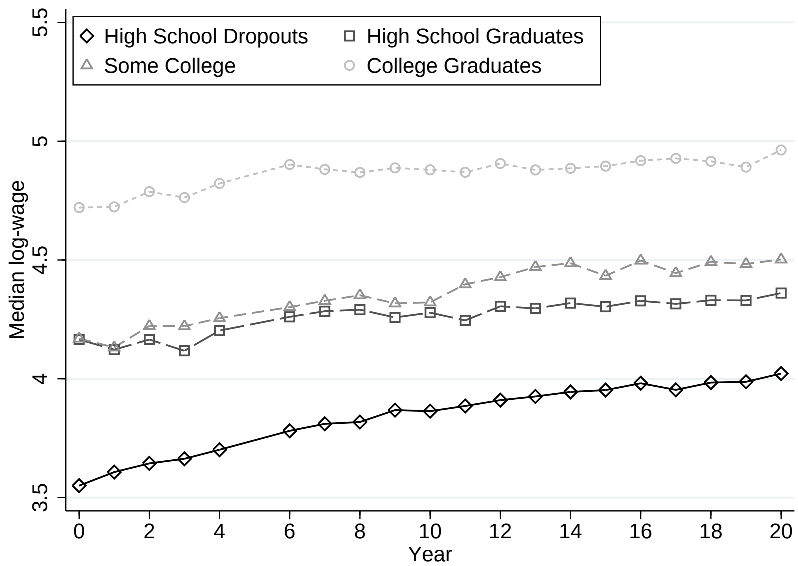
Note: "All": all individuals entering the labour market between 1985 and 1992; "Selected": individuals entering the labour market between 1985 and 1992 who are employed at least 15 years in our panel data.

Figure 4: Variance of log-wages as a function of potential experience, by diploma



Note: "All": all individuals entering the labour market between 1985 and 1992; "Selected": individuals entering the labour market between 1985 and 1992 who are employed at least 15 years in our panel data.

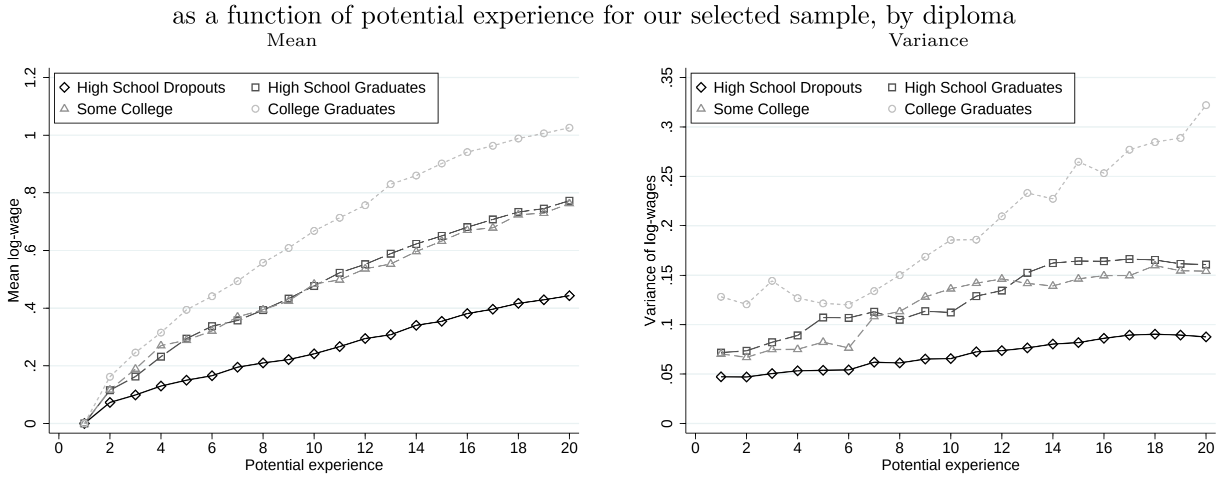
Figure 5: Price of human capital by diploma



Note:Mean log-wage computed on the subsample of observations such that individuals are 50-55 years old.

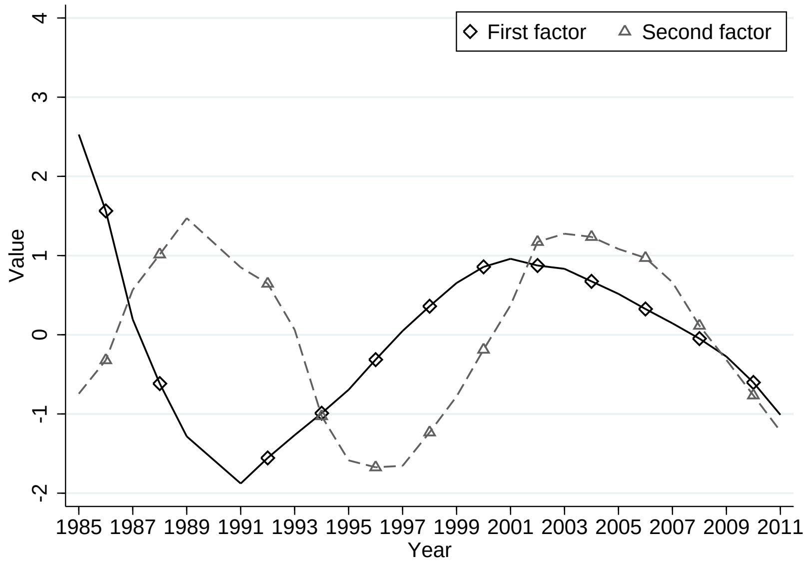
Figure 6: Mean and variance of log-wages deflated with prices of human capital

as a function of potential experience for our selected sample, by diploma



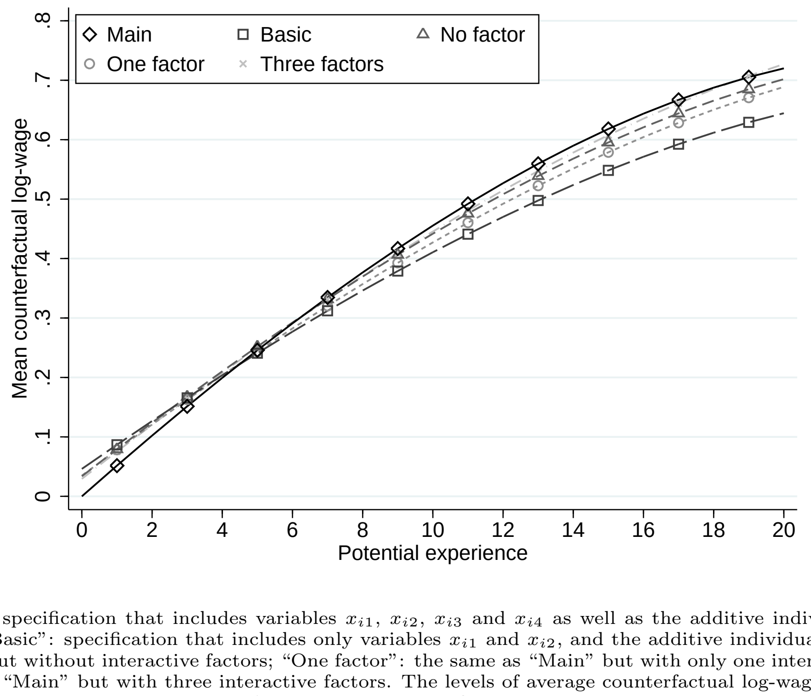
Note: “All”: all individuals entering the labour market between 1985 and 1992; “Selected”: individuals entering the labour market between 1985 and 1992 who are employed at least 15 years in our panel data.

Figure 7: Value of factors as a function of time, main specification



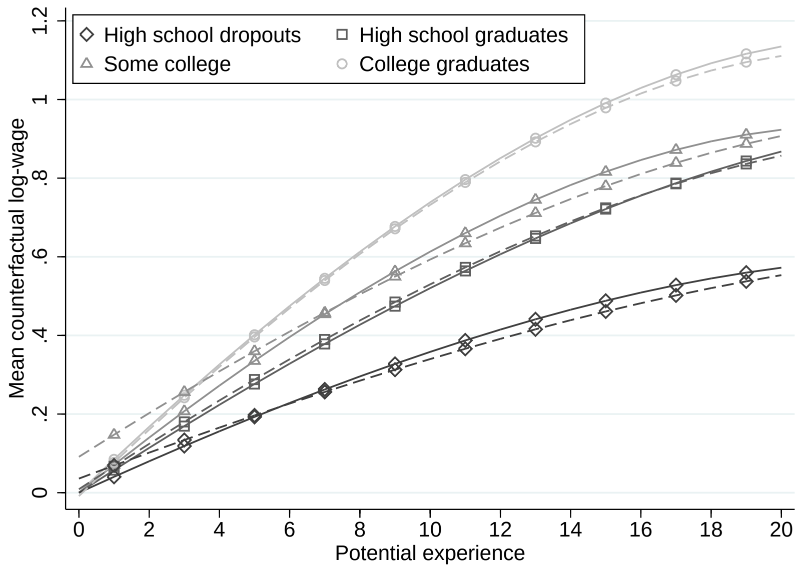
Note: The main specification includes two factors, and the sign of factors was initially normalized such that their value is positive in 1985. However, the opposite is represented for the second factor for its curve to be consistent with that of factors obtained with specifications that include one or three factors (see Figure A.1).

Figure 8: Mean counterfactual log-wage as a function of potential experience



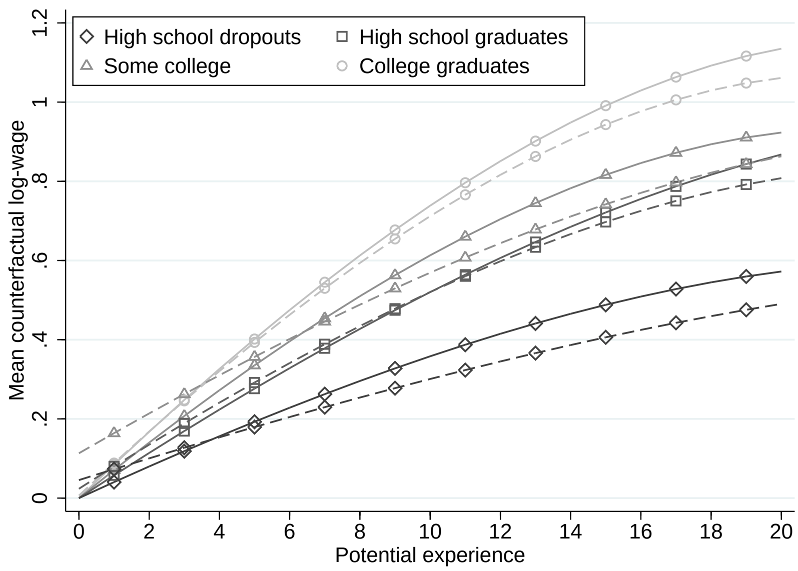
Note: "Main": main specification that includes variables x_{i1}, x_{i2}, x_{i3} and x_{i4} as well as the additive individual effect and two interactive factors; "Basic": specification that includes only variables x_{i1} and x_{i2}, and the additive individual effect; "No factor": the same as "Main" but without interactive factors; "One factor": the same as "Main" but with only one interactive factor; "Three factors": the same as "Main" but with three interactive factors. The levels of average counterfactual log-wages are normalized for all specifications using the value at period zero of the benchmark specification.

Figure 9: Mean counterfactual log-wage as a function of potential experience by education level, main specification and specification with no factor



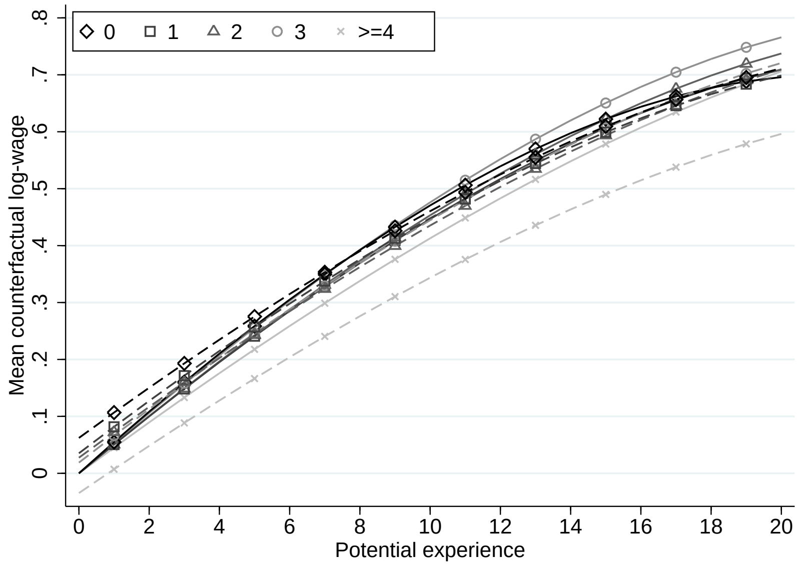
Note: In solid line: main specification that includes variables x_{i1}, x_{i2}, x_{i3} and x_{i4} as well as the additive individual effect and two interactive factors; in dashed line: same specification as the main one, but without interactive factor. For each diploma, the levels of average counterfactual log-wages are normalized for the two specifications using the value at period zero of the benchmark specification.

Figure 10: Mean counterfactual log-wage as a function of potential experience by education level, main and basic specifications



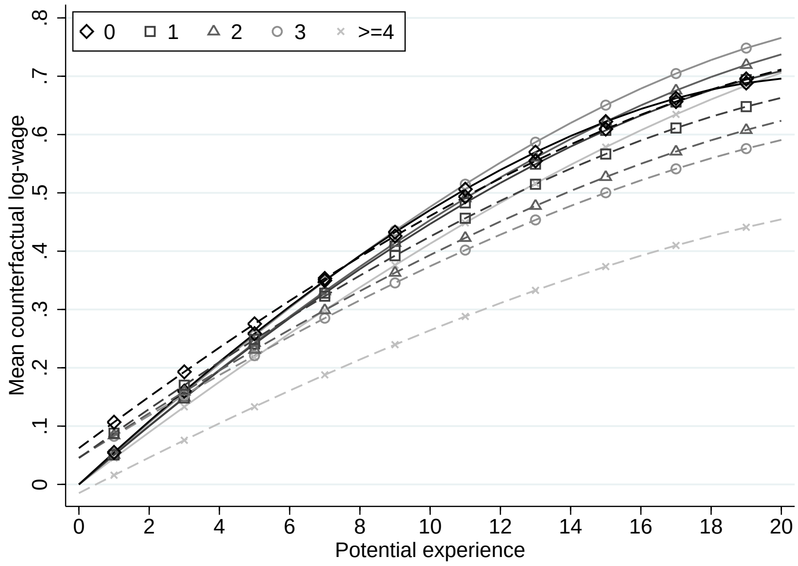
Note: In solid line: main specification that includes variables x_{i1}, x_{i2}, x_{i3} and x_{i4} as well as the additive individual effect and two interactive factors; in dashed line: basic specification that includes only variables x_{i1} and x_{i2}, and the additive individual effect. For each diploma, the levels of average counterfactual log-wages are normalized for the two specifications using the value at period zero of the benchmark specification.

Figure 11: Mean counterfactual log-wage as a function of potential experience by number of interruptions, main specification and specification with no factor



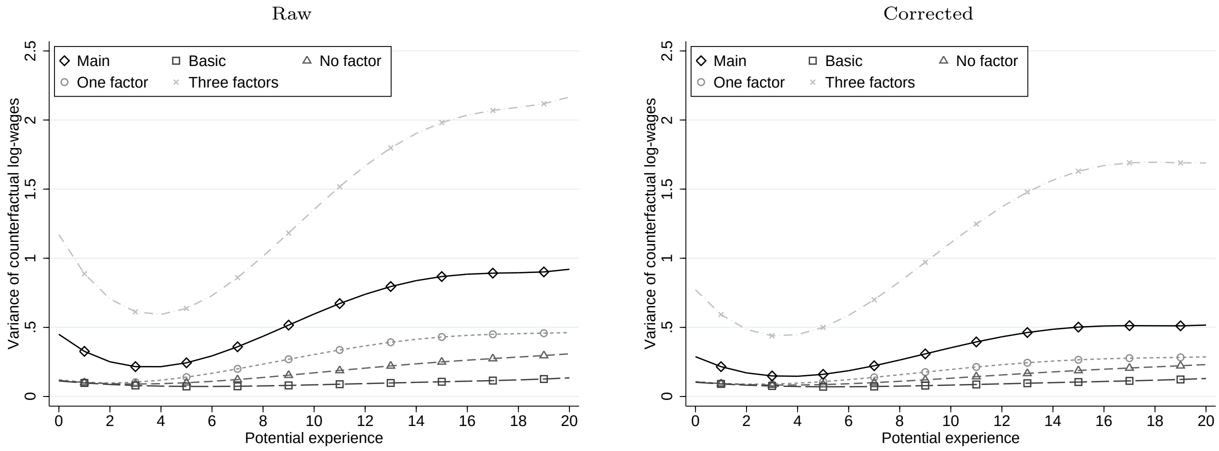
Note: In solid line: main specification that includes variables x_{i1}, x_{i2}, x_{i3} and x_{i4} as well as the additive individual effect and two interactive factors; in dashed line: same specification as the main one, but without interactive factor. For each number of interruptions, the levels of average counterfactual log-wages are normalized for the two specifications using the value at period zero of the benchmark specification.

Figure 12: Mean counterfactual log-wage as a function of potential experience by number of interruptions, main and basic specifications



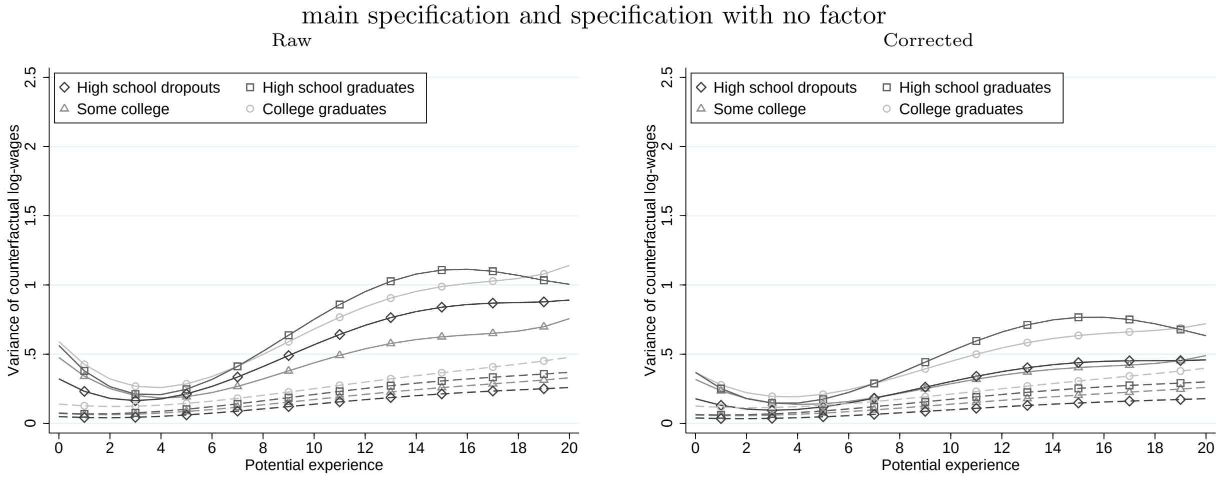
Note: In solid line: main specification that includes variables x_{i1}, x_{i2}, x_{i3} and x_{i4} as well as the additive individual effect and two interactive factors; in dashed line: basic specification that includes only variables x_{i1} and x_{i2}, and the additive individual effect. For each number of interruptions, the levels of average counterfactual log-wages are normalized for the two specifications using the value at period zero of the benchmark specification.

Figure 13: Variance of counterfactual log-wages as a function of potential experience



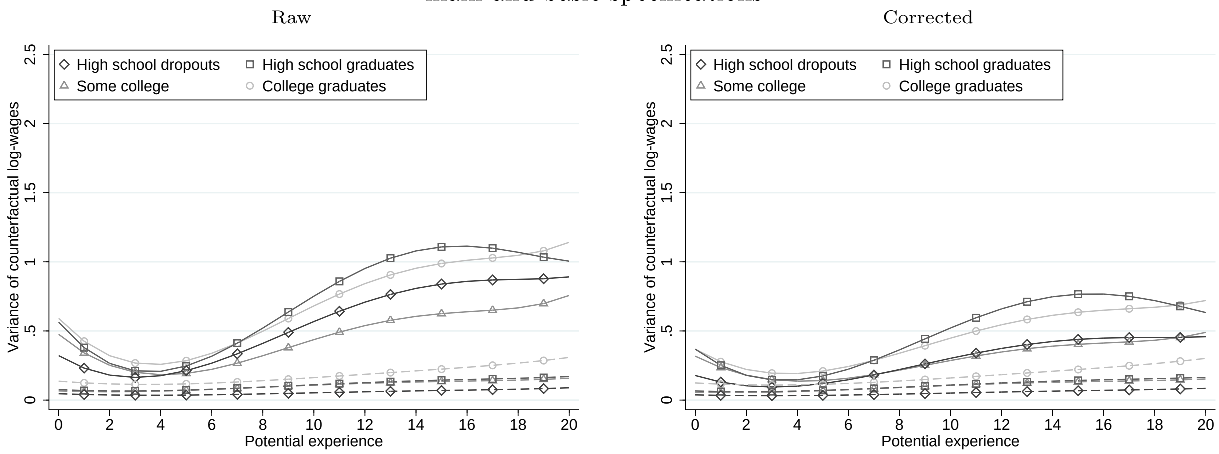
Note: "Main": main specification that includes variables x_{i1}, x_{i2}, x_{i3} and x_{i4} as well as the additive individual effect and two interactive factors; "Basic": specification that includes only variables x_{i1} and x_{i2}, and the additive individual effect; "No factor": the same as "Main" but without interactive factors; "One factor": the same as "Main" but with only one interactive factor; "Three factors": the same as "Main" but with three interactive factors. "Raw": variance without any correction for sampling error; "Corrected": variance with correction for sampling error.

Figure 14: Variance of counterfactual log-wages as a function of potential experience by education level, main specification and specification with no factor



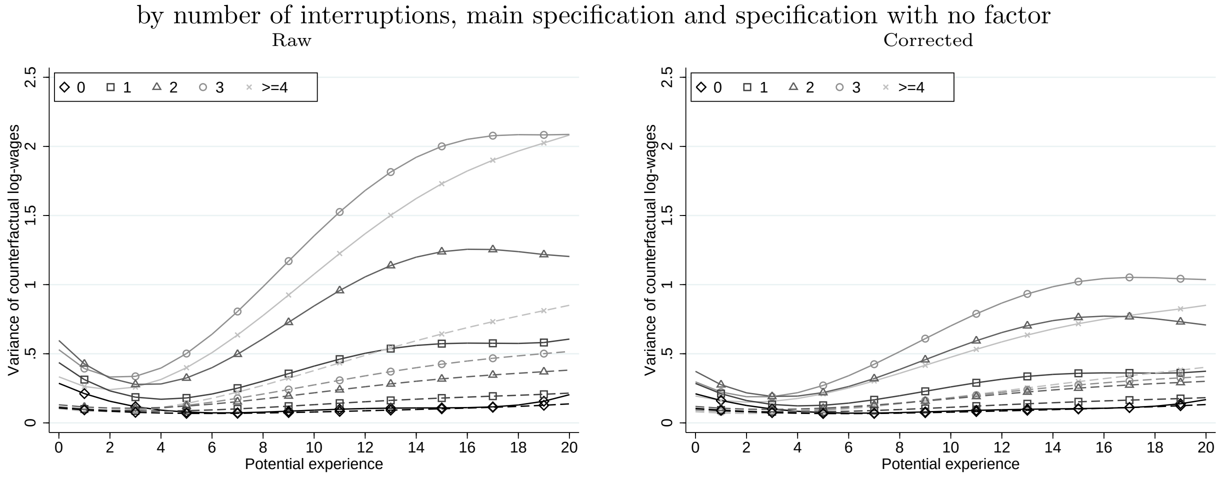
Note: In solid line: main specification that includes variables x_{i1}, x_{i2}, x_{i3} and x_{i4} as well as the additive individual effect and two interactive factors; in dashed line: same specification as the main one, but without interactive factor. "Raw": variance without any correction for sampling error; "Corrected": variance with correction for sampling error.

Figure 15: Variance of counterfactual log-wages as a function of potential experience by education level, main and basic specifications



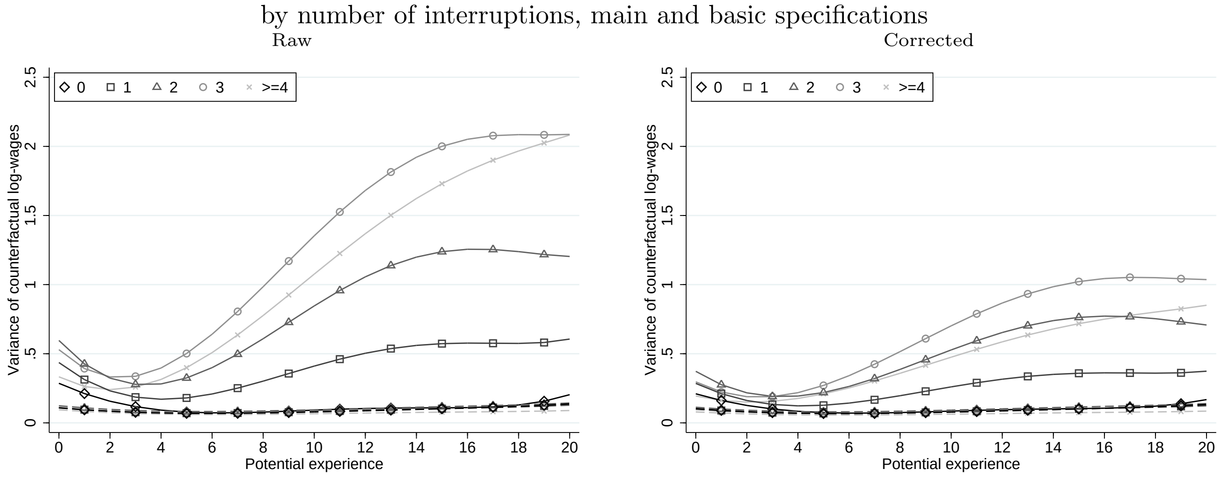
Note: In solid line: main specification that includes variables x_{i1}, x_{i2}, x_{i3} and x_{i4} as well as the additive individual effect and two interactive factors; in dashed line: basic specification that includes only variables x_{i1} and x_{i2}, and the additive individual effect. "Raw": variance without any correction for sampling error; "Corrected": variance with correction for sampling error.

Figure 16: Variance of counterfactual log-wages as a function of potential experience by number of interruptions, main specification and specification with no factor



Note: In solid line: main specification that includes variables x_{i1}, x_{i2}, x_{i3} and x_{i4} as well as the additive individual effect and two interactive factors; in dashed line: same specification as the main one, but without interactive factor. "Raw": variance without any correction for sampling error; "Corrected": variance with correction for sampling error.

Figure 17: Variance of counterfactual log-wages as a function of potential experience by number of interruptions, main and basic specifications



Note: In solid line: main specification that includes variables x_{i1}, x_{i2}, x_{i3} and x_{i4} as well as the additive individual effect and two interactive factors; in dashed line: basic specification that includes only variables x_{i1} and x_{i2}, and the additive individual effect. "Raw": variance without any correction for sampling error; "Corrected": variance with correction for sampling error.

Table 1: Correlation of log-wages deflated with prices of human capital at two values of potential experience

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1.00																			
2	0.83	1.00																		
3	0.75	0.85	1.00																	
4	0.70	0.79	0.87	1.00																
5	0.66	0.74	0.79	0.86	1.00															
6	0.63	0.70	0.75	0.81	0.87	1.00														
7	0.59	0.66	0.73	0.78	0.83	0.89	1.00													
8	0.55	0.62	0.68	0.73	0.79	0.84	0.90	1.00												
9	0.50	0.58	0.64	0.69	0.75	0.79	0.84	0.89	1.00											
10	0.46	0.54	0.59	0.65	0.70	0.75	0.78	0.83	0.88	1.00										
11	0.43	0.51	0.56	0.63	0.67	0.72	0.75	0.81	0.86	0.90	1.00									
12	0.42	0.50	0.55	0.60	0.65	0.69	0.73	0.78	0.82	0.86	0.90	1.00								
13	0.37	0.45	0.51	0.57	0.61	0.64	0.70	0.75	0.79	0.82	0.86	0.90	1.00							
14	0.37	0.43	0.50	0.56	0.60	0.65	0.69	0.73	0.77	0.80	0.83	0.87	0.91	1.00						
15	0.32	0.40	0.46	0.51	0.56	0.61	0.65	0.70	0.74	0.76	0.81	0.84	0.88	0.91	1.00					
16	0.31	0.39	0.45	0.50	0.55	0.60	0.63	0.68	0.72	0.75	0.78	0.82	0.85	0.89	0.91	1.00				
17	0.31	0.38	0.45	0.51	0.54	0.60	0.63	0.67	0.71	0.73	0.77	0.81	0.84	0.86	0.89	0.92	1.00			
18	0.29	0.36	0.43	0.50	0.53	0.57	0.62	0.66	0.69	0.71	0.74	0.79	0.81	0.84	0.87	0.90	0.93	1.00		
19	0.28	0.36	0.42	0.48	0.51	0.57	0.60	0.64	0.68	0.70	0.73	0.77	0.80	0.83	0.86	0.88	0.91	0.93	1.00	
20	0.28	0.35	0.40	0.48	0.50	0.56	0.59	0.64	0.67	0.69	0.72	0.76	0.79	0.81	0.83	0.85	0.89	0.91	0.94	1.00

Table 2: Descriptive statistics on interruptions

		No censorship					Censorship		
Number of interruptions	Number of individuals	Proportion in interruption	Cumulated duration in interruption	Average number of interruptions	Number of interruptions	Number of individuals	Proportion in interruption	Cumulated duration in interruption	Average number of interruptions
All	7004	0.154	3.7	1.44	All	7004	0.154	2.5	1.44
0	1219	0.000	0.0	0.00	0	1560	0.000	0.0	0.00
1	2279	0.110	2.6	1.00	1	2463	0.110	1.9	1.00
2	1933	0.196	4.7	2.00	2	1795	0.196	3.5	2.00
3	1050	0.261	6.3	3.00	3	830	0.261	4.8	3.00
4	383	0.321	7.7	4.00	4	279	0.321	6.2	4.00
5	118	0.355	8.5	5.00	5	69	0.355	7.2	5.00
6	22	0.378	9.3	6.00	6	8	0.378	8.6	6.00

Table 3: Variance analysis of the effects, main specification

	Variance	Correlation				
		Log-wage	Potential experience effect	Non-employment effect	Effect of factors	Residual
Log-wage	0.147	1.000	0.425	0.004	0.051	0.229
Pot. exp. effect	0.671	0.425	1.000	-0.848	-0.233	0.000
Non-empl. effect	0.501	0.004	-0.848	1.000	-0.046	-0.000
Effect of factors	0.059	0.051	-0.233	-0.046	1.000	0.000
Residual	0.008	0.229	0.000	-0.000	0.000	1.000

Note: "Potential experience effect": sum of all effects related to potential experience and the individual additive effect: $\eta_{i0} + \eta_{i1} t + \eta_{i2} \beta^{-t}$; "Non-employment effect": sum of all effects related to being absent from the panel: $\eta_{i3} x_{it}^{(3)} + \eta_{i4} x_{it}^{(4)}$; residual: $\widetilde{v}_{it}$.

APPENDIX

A Additional information on the data

In the raw data, there are $4,884,767$ person-job-year observations in the public and private sector over the 1976-2012 period corresponding to individuals born in the first four days of October. For individuals born an odd year, there is no information before 2002. When restricting the sample to males, we are left with $2,658,470$ observations. For consistency across time, we restrict our attention to individuals born on even years over the whole period, and this makes the sample size drop to $2,017,624$ observations. When considering only jobs in the private sector, we are left with $1,772,511$ observations, and when considering only full-time positions, the sample size decreases to $1,520,615$ observations. We also delete jobs for workers on a training period and apprentices, and this leaves us with $1,492,091$ observations. Once jobs are aggregated per individual-year, we end up with $1,365,837$ observations.

We then restrict the sample to jobs such that the wage is lower than 80% of the minimum wage. To compute the minimum wage, we use a national time series of gross hourly values. Over the 1976-1998 period, we transform them into monthly values by multiplying them with the number of working hours fixed legally to 169 (ie. 39 hours per week). After 1998, some firms change their number of working hours to 151.67 (ie. 35 hours per week) and this becomes the legal number in 2001. Therefore, from 1999 onwards, we compute two monthly values depending on whether the number of working hours is 169 or 151.67, and we consider that there is a transition over the 1999-2006 period between the two values consistently with the evolution of the proportion of individuals working 35 hours per week.[A.1] From 2007 onwards, we consider that the number of working hours is 151.67. We then decrease monthly values by 20% to remove payroll taxes and obtain net monthly values. The deletion of observations such that the wage is lower than 80% of the minimum wage makes the sample decrease to $1,354,104$ observations.

We keep only individual-year observations such that the total amount of working days is larger than 6 months, and the sample then includes $1,192,377$ observations corresponding to $102,425$ males. We keep only observations for individuals entering the labor market over the 1985-1992 period (ie. individuals observed for the first time in the panel during that period), and we are left with $200,756$ observations corresponding to $15,039$ are males. After restricting the sample to individuals aged $16-30$, our sample includes $178,111$ observations corresponding to $12,216$ males. We delete individuals for whom the education level is missing (4 individuals) and this leaves us with $178,098$ observations corresponding to $12,212$ male individuals. Finally, we keep individuals who were present at least 15 years, which leaves us with 7,004 individuals with 137,315 observations.

The education level is defined as the diploma obtained. Using French diploma names, high-school drop-outs includes no diploma, CAP, BEPC and CEP; high-school graduates includes baccalauréat and low-level technical diplomas; short-track college graduates gather BTS, DUT and DEUG diploma holders; college graduates include 3-year and more college diplomas and Grandes Ecoles.

[A.1] We use as proportions for every years over the 1999-2006 period: 10%, 20%, 30%, 40%, 60%, 70%, 80% and 90%.

B Structural restrictions

We start from the structural restrictions in which we omit subscript i:

$$\kappa \in (0, \frac{1}{1-\beta}), c > 0, \rho^s > 0$$

and we impose that investments in both sectors are positive until T so that (since investments are decreasing over time):

$$\rho^s \beta \kappa_{T+1} \geq 1. \tag{B.1}$$

Note that it implies that $\rho^s > 0$ and the structural restrictions write:

$$\kappa \in (0, \frac{1}{1-\beta}), c > 0, \rho^s \beta \kappa_{T+1} \geq 1. \tag{B.2}$$

We assume that β is fixed.

B.1 Necessary conditions

We start from the system of equations (18)-(22) relating reduced form and structural parameters:

$$\begin{aligned}
\eta_0 &= \ln H(t_0) - \frac{\rho^e t_0 + 1}{c}\left(\frac{\rho_i^e \beta}{1-\beta} - 1\right) - \frac{(\rho^e)^2 \beta}{c}\frac{\beta^{T+1-t_0}}{1-\beta}\left(\kappa - \frac{1}{1-\beta}\right) \\
\eta_1 &= \frac{\rho^e}{c}\left(\rho^e \frac{\beta}{1-\beta} - 1\right) \\
\eta_2 &= \beta^{T+1}\frac{\rho^e}{c}\left(\kappa - \frac{1}{1-\beta}\right)\left(\frac{\rho^e \beta}{1-\beta} - 1\right) \\
\eta_3 &= \left(\frac{\rho^n}{c}\left(\frac{\rho^n \beta}{1-\beta} - 1\right) - \frac{\rho^e}{c}\left(\frac{\rho^e \beta}{1-\beta} - 1\right)\right) \\
\eta_4 &= \frac{1}{c}\left((\rho^n)^2 - (\rho^e)^2\right)\left(\kappa - \frac{1}{1-\beta}\right)\frac{\beta^{T+1}}{1-\beta}
\end{aligned} \tag{B.3}$$

and we first note that the first equation is the only equation including η_0 and $\ln H(t_0)$ and as no constraint binds on the latter, parameter η_0 is unconstrained.

Considering now the ratio of the third and second equations in system (B.3), we get:

$$\kappa - \frac{1}{1-\beta} = \frac{\eta_2}{\eta_1}\beta^{-(T+1)},$$

and therefore as a consequence of the first restriction in system (B.2):

$$\frac{\eta_2}{\eta_1} \in (-\frac{\beta^{T+1}}{1-\beta}, 0). \tag{B.4}$$

Using the expression for κ_{t+1} given by equation (11), we have that:

$$\kappa_{t+1} - \frac{1}{1-\beta} = \beta^{T-t}(\kappa - \frac{1}{1-\beta}) = \frac{\eta_2}{\eta_1}\beta^{-(t+1)},$$

Furthermore as $\kappa_{T+1} = \kappa$, restriction (B.1) writes:

$$\rho^s \beta \kappa_{T+1} = \rho^s \frac{\beta}{1-\beta} + \frac{\eta_2}{\eta_1} \rho^s \beta^{-T} \geq 1,$$

and we have that:

$$\rho^s \geq \frac{1}{\frac{\beta}{1-\beta} + \frac{\eta_2}{\eta_1}\beta^{-T}}, \tag{B.5}$$

since the denominator is equal to $\beta\kappa > 0$ This implies in particular that because $\frac{\eta_2}{\eta_1} \leq 0$, we have:

$$\rho^s \geq \frac{1-\beta}{\beta}. \tag{B.6}$$

Consider now the remaining equations in system (B.3). Denote:

$$\varphi(\rho) = \rho(\rho \frac{\beta}{1-\beta} - 1)$$

and note that because $\rho^s \geq \frac{1-\beta}{\beta}$ we have that:

$$\varphi(\rho^s) > 0, \varphi'(\rho^s) > 0. \tag{B.7}$$

Rewrite lines 2, 4 and 5 in system (B.3) as:

$$\begin{aligned} c\eta_1 &= \varphi(\rho^e), \\ c(\eta_1 + \eta_3) &= \varphi(\rho^n), \\ c\eta_4 &= ((\rho^n)^2 - (\rho^e)^2) \frac{\eta_2}{\eta_1} \frac{1}{1-\beta}. \end{aligned} \tag{B.8}$$

It is now immediate that $c > 0$ and restriction (B.7) implies that

$$\eta_1 > 0, \eta_1 + \eta_3 > 0, \tag{B.9}$$

and that as $c\eta_3 = \varphi(\rho^n) - \varphi(\rho^e)$, $\varphi(.)$ is increasing and $\frac{\eta_2}{\eta_1} < 0$ that:

$$\eta_3 \eta_4 < 0. \tag{B.10}$$

Write also:

$$c\eta_3 = \varphi(\rho^n) - \varphi(\rho^e) = ((\rho^n)^2 - (\rho^e)^2) \frac{\beta}{1-\beta} - (\rho^n - \rho^e),$$

so that because of the third equation in system (B.8):

$$c\eta_3 = c\eta_4 \frac{\eta_1}{\eta_2} \beta - (\rho^n - \rho^e)$$

and denoting $\xi = \frac{\eta_4}{\eta_3} \frac{\eta_1}{\eta_2} > 0$ (because of restrictions (B.4) and (B.10)):

$$\frac{\rho^n - \rho^e}{c} = \eta_3(\beta\xi - 1).$$

As η_3 has the same sign as $\varphi(\rho^n) - \varphi(\rho^e)$ and thus as $\rho^n - \rho^e$ from the above, this leads to the fourth restriction:

$$\beta\xi > 1. \tag{B.11}$$

Furthermore, the third equation in system (B.8) can also be written as:

$$\frac{(\rho^n)^2 - (\rho^e)^2}{c} = \eta_4 \frac{\eta_1}{\eta_2}(1-\beta) = \eta_3 \xi (1-\beta)$$

so that we arrive at:

$$\rho^n + \rho^e = \frac{\xi(1-\beta)}{(\xi\beta - 1)}.$$

Equation (B.5) yields:

$$\begin{aligned}
\frac{\rho^n + \rho^e}{2} &= \frac{1}{2}\frac{\xi(1-\beta)}{(\xi\beta-1)} > \frac{1}{\frac{\beta}{1-\beta} + \frac{\eta_2}{\eta_1}\beta^{-T}}, \\
&\iff \xi(\beta + (1-\beta)\frac{\eta_2}{\eta_1}\beta^{-T}) > 2(\xi\beta - 1), \\
&\iff \xi(\beta - (1-\beta)\frac{\eta_2}{\eta_1}\beta^{-T}) < 2
\end{aligned}$$

and therefore (since $\frac{\eta_2}{\eta_1} < 0$):

$$\xi < \frac{2}{\beta - (1-\beta)\frac{\eta_2}{\eta_1}\beta^{-T}}. \tag{B.12}$$

It is easy to check that since $\frac{\eta_2}{\eta_1} > -\frac{\beta^{T+1}}{1-\beta}$, then restrictions (B.11) and (B.12) do not define an empty set for ξ.

Nonetheless, it does not ensure that both ρ^e and ρ^n satisfy (B.5).

We know that:

$$\rho^e + \rho^n = \varphi^{-1}(c\eta_1) + \varphi^{-1}(c(\eta_3 + \eta_1))$$

is strictly increasing with c when $c > 0$ because of constraints (B.9). Under the constraint (B.12) there does exist a unique $c > 0$ such that:

$$\varphi^{-1}(c\eta_1) + \varphi^{-1}(c(\eta_3 + \eta_1)) = \frac{\xi(1-\beta)}{(\xi\beta - 1)}$$

and such that $\frac{\rho^n+\rho^e}{2} = \frac{1}{2}\frac{\xi(1-\beta)}{(\xi\beta-1)} > \frac{1}{\frac{\beta}{1-\beta}+\frac{\eta_2}{\eta_1}\beta^{-T}}$. Define c^* such a solution. We then have the condition:

$$\varphi^{-1}(\min(c^*\eta_1, c^*(\eta_3 + \eta_1))) > \frac{1}{\frac{\beta}{1-\beta} + \frac{\eta_2}{\eta_1}\beta^{-T}}. \tag{B.13}$$

B.2 Sufficient conditions

Suppose that :

$$\begin{aligned}
\frac{\eta_2}{\eta_1} &\in [-\frac{\beta^{T+1}}{1-\beta}, 0], \\
\eta_1 &> 0, \eta_1 + \eta_3 > 0, \\
\eta_3\eta_4 &< 0, \\
\frac{1}{\beta} &< \xi < \frac{2}{\beta - (1-\beta)\frac{\eta_2}{\eta_1}\beta^{-T}}, \\
\varphi^{-1}(\min(c^*\eta_1, c^*(\eta_3 + \eta_1))) &> \frac{1}{\frac{\beta}{1-\beta} + \frac{\eta_2}{\eta_1}\beta^{-T}}.
\end{aligned}$$

in which c^* is defined above.

We can first derive that:

$$\kappa - \frac{1}{1-\beta} = \frac{\eta_2}{\eta_1}\beta^{-(T+1)} \in [0, \frac{1}{1-\beta}].$$

Furthermore, we know that:

$$\rho^e + \rho^n = \varphi^{-1}(c\eta_1) + \varphi^{-1}(c(\eta_3 + \eta_1))$$

is strictly increasing with c when $c > 0$ because of constraints (B.9). Under the constraint (B.12) there does exist a unique $c > 0$ such that:

$$\varphi^{-1}(c\eta_1) + \varphi^{-1}(c(\eta_3 + \eta_1)) = \frac{\xi(1-\beta)}{(\xi\beta - 1)}$$

and such that $\frac{\rho^n+\rho^e}{2} = \frac{1}{2}\frac{\xi(1-\beta)}{(\xi\beta-1)} > \frac{1}{\frac{\beta}{1-\beta}+\frac{\eta_2}{\eta_1}\beta^{-T}}$. This is the definition of c^*.

Moreover, because of restriction (B.13) we also have that

$$\rho^e = \varphi^{-1}(c^*\eta_1) > \frac{1}{\frac{\beta}{1-\beta} + \frac{\eta_2}{\eta_1}\beta^{-T}}, \rho^n = \varphi^{-1}(c^*(\eta_3 + \eta_1)) > \frac{1}{\frac{\beta}{1-\beta} + \frac{\eta_2}{\eta_1}\beta^{-T}},$$

and therefore the structural constraints $\rho^s\beta\kappa_{T+1} \geq 1$.

C Convergence of the iterative estimation procedure

We use a specific iterative procedure to find the solution of the sum-of-squares minimization program. We show in this section that our iteractive procedure converges to the solution of this program as the number of iterations tends to infinity.

The sum osf squares we consider is given by:

$$C\left(\theta,\varphi,\eta\right)=\sum_{i,t|s(i,t)=1}\left(y_{it}-\eta_{i0}-x_{it}\eta_i-\varphi_t\theta_i\right)^2 \tag{C.14}$$

For a given set of parameters, say for instance η_i, we denote by $\eta_i^{(k)}$ the value of the estimates at the k^{th} iteration.

As explained in the text, the first stage of our algorithm consists in minimizing $C\left(\theta,\varphi^{(k-1)},\eta\right)$ with respect to θ and η – maintaining $\varphi^{(k-1)}$ constant. We denote the values of the arguments of the minimizer as $\eta^{(k)}=(\eta_i^{(k)})_{i=1,.,n}$ and $\theta^{(k)}=(\theta_i^{(k)})_{i=1,.,n}$.

At the second stage, we impute log-wages that are not observed using the formula:

$$y_{it}^{(k)}=\eta_{i0}^{(k)}+x_{it}\eta_i^{(k)}+\varphi_t^{(k-1)}\theta_i^{(k)} \tag{C.15}$$

At the third stage, we recover values of θ and φ – fixing the values of $y_{it}^{(k)}$, $\eta_{i0}^{(k)}$ and $\eta_i^{(k)}$ – that minimize the sum of squares:

$$\widetilde{C}\left(\theta,\varphi,\eta^{(k)}\right)=C\left(\theta,\varphi,\eta^{(k)}\right)+\sum_{i,t|s(i,t)=0}\left(y_{it}^{(k)}-\eta_{i0}^{(k)}-x_{it}\eta_i^{(k)}-\varphi_t\theta_i\right)^2 \tag{C.16}$$

using Bai's algorithm and we denote these values, $\widetilde{\theta}^{(k)}$ and $\varphi^{(k)}$.

We now show that the sum of squares decreases at each iteration of our algorithm.

Lemma 6

$$C\left(\widetilde{\theta}^{(k)},\varphi^{(k)},\eta^{(k)}\right)\leqslant C\left(\widetilde{\theta}^{(k-1)},\varphi^{(k-1)},\eta^{(k-1)}\right). \tag{C.17}$$

Proof. From the first stage of our algorithm, we have that:

$$C\left(\theta^{(k)},\varphi^{(k-1)},\eta^{(k)}\right)\leqslant C\left(\theta^{(k-1)},\varphi^{(k-1)},\eta^{(k-1)}\right), \tag{C.18}$$

since $\theta^{(k)},\eta^{(k)}$ are minimizers of the left-hand side. Using the definition of $y_{it}^{(k)}$, we also have that

$$\widetilde{C}\left(\theta^{(k)},\varphi^{(k-1)},\eta^{(k)}\right)=C\left(\theta^{(k)},\varphi^{(k-1)},\eta^{(k)}\right), \tag{C.19}$$

since the sum of squares on the right hand side of equation (C.16) is equal to zero. The third stage of our algorithm yields, by minimization :

$$\widetilde{C}\left(\widetilde{\theta}^{(k)},\varphi^{(k)},\eta^{(k)}\right)\leqslant \widetilde{C}\left(\theta^{(k)},\varphi^{(k-1)},\eta^{(k)}\right). \tag{C.20}$$

From equation (C.16), we have

$$C\left(\widetilde{\theta}^{(k)},\varphi^{(k)},\eta^{(k)}\right)\leqslant \widetilde{C}\left(\widetilde{\theta}^{(k)},\varphi^{(k)},\eta^{(k)}\right)$$

and get, using equations (C.20), (C.19) and (C.18) successively:

$$\begin{aligned}C\left(\widetilde{\theta}^{(k)},\varphi^{(k)},\eta^{(k)}\right) &\leqslant \widetilde{C}\left(\theta^{(k)},\varphi^{(k-1)},\eta^{(k)}\right)=C\left(\theta^{(k)},\varphi^{(k-1)},\eta^{(k)}\right)\\ &\leqslant C\left(\widetilde{\theta}^{(k-1)},\varphi^{(k-1)},\eta^{(k-1)}\right). \end{aligned} \tag{C.21}$$

■

This shows that the sum of squares is decreasing at each iteration. In fact, it is strictly decreasing as shown by the following lemma:

Lemma 7

$$C\left(\widetilde{\theta}^{(k)},\varphi^{(k)},\eta^{(k)}\right)=C\left(\widetilde{\theta}^{(k-1)},\varphi^{(k-1)},\eta^{(k-1)}\right)\Longrightarrow\left(\widetilde{\theta}^{(k)},\varphi^{(k)},\eta^{(k)}\right)=\left(\widetilde{\theta}^{(k-1)},\varphi^{(k-1)},\eta^{(k-1)}\right).$$

Proof. The left-hand side equality implies that:

$$C\left(\widetilde{\theta}^{(k)},\varphi^{(k)},\eta^{(k)}\right)=\widetilde{C}\left(\widetilde{\theta}^{(k)},\varphi^{(k)},\eta^{(k)}\right)$$

according to equation (**??**). Using (C.16), this yields

$$\sum_{i,t|s(i,t)=0}\left(\varphi_t^{(k-1)}\theta_i^{(k)}-\varphi_t^{(k)}\widetilde{\theta}_i^{(k)}\right)^2=0$$

and thus $\varphi_t^{(k-1)}\theta_i^{(k)}=\varphi_t^{(k)}\widetilde{\theta}_i^{(k)}$ for all i,t such that $s(i,t)=0$. Considering also that there are identification restrictions on parameters, we then have generically $\varphi_t^{(k-1)}=\varphi_t^{(k)}$ and $\widetilde{\theta}_i^{(k)}=\theta_i^{(k)}$ for all i, t. From (**??**), we also have that

$$C\left(\theta^{(k)},\varphi^{(k-1)},\eta^{(k)}\right)=C\left(\widetilde{\theta}^{(k-1)},\varphi^{(k-1)},\eta^{(k-1)}\right).$$

As C is strictly concave, the solution in the first step is unique for a given $\varphi^{(k-1)}$, and we get that $\theta^{(k)}=\widetilde{\theta}^{(k-1)}$ and $\eta^{(k)}=\eta^{(k-1)}$. Putting all the equality on parameters together, we obtain $\left(\widetilde{\theta}^{(k)},\varphi^{(k)},\eta^{(k)}\right)=\left(\widetilde{\theta}^{(k-1)},\varphi^{(k-1)},\eta^{(k-1)}\right)$. ■

Using the contraposition of the lemma and equation (C.17), we have that

$$\left(\widetilde{\theta}^{(k)},\varphi^{(k)},\eta^{(k)}\right)\neq\left(\widetilde{\theta}^{(k-1)},\varphi^{(k-1)},\eta^{(k-1)}\right)\Longrightarrow C\left(\widetilde{\theta}^{(k)},\varphi^{(k)},\eta^{(k)}\right)<C\left(\widetilde{\theta}^{(k-1)},\varphi^{(k-1)},\eta^{(k-1)}\right),$$

which shows that the sum of squares is strictly decreasing at each iteration. As it is bounded below by zero, it converges to a value $\overline{C}$ and parameters converge to the value of its minimizers $\left(\widehat{\widehat{\theta}},\widehat{\varphi},\widehat{\eta}\right)$. Hence, $\theta^{(k)}$ converges to the value of θ denoted $\widehat{\theta}$ that minimizes $C\left(\theta,\widehat{\varphi},\widehat{\eta}\right)$. We also have that $\widetilde{\theta}^{(k)}$ is the value that minimizes:

$$\widetilde{C}\left(\theta,\widehat{\varphi},\widehat{\eta}\right)=C\left(\theta,\widehat{\varphi},\widehat{\eta}\right)+\sum_{i,t|s(i,t)=0}\left(\widehat{\varphi}\left(\widehat{\theta}_i-\theta_i\right)\right)^2$$

As $C\left(\theta,\widehat{\varphi},\widehat{\eta}\right)$ is minimum in $\widehat{\theta}$, and the second (positive) right-hand side term is positive but zero for $\theta=\widehat{\theta}$, then $\widetilde{C}\left(\theta,\widehat{\varphi},\widehat{\eta}\right)$ is minimized at $\widehat{\theta}$ and we have $\widehat{\widehat{\theta}}=\widehat{\theta}$. Overall, step 1 yields that $\widehat{\theta}$ and $\widehat{\eta}$ verify the least squares first-order conditions, and step 3 makes $\widehat{\varphi}$ verify the least squares first-order conditions. Hence, $\left(\widehat{\theta},\widehat{\eta},\widehat{\varphi}\right)$ is the least squares solution.

D Bias-corrected variances of counterfactual log-wages

Denote $\phi_i = (\eta'_{i0}, \eta'_{i1}, \theta'_i)'$ the true value of individual parameters for individual i. The unfeasible solution in the first step of our algorithm when factors φ_t are known is given by:

$$\phi_i^u = \phi_i + (Z_i'Z_i)^{-1} Z_i'\widetilde{v}_i \tag{D.22}$$

in which $Z_i = \left(1'_{T_i}, X'_i, \varphi'_i\right)'$ and 1_{T_i} is a $T_i \times 1$ vector full of ones. T_i the number of periods individual i is observed in the panel, $X_i = \left(x_{it_1}, ..., x_{it_{T_i}}\right)'$ the set of observed characteristics at those periods and $\varphi_i = \left(\varphi_{t_1}, ..., \varphi_{T_i}\right)'$. The conditional variance of the unfeasible solution is given by:

$$V\left(\phi_i^u \left| Z_i\right.\right) = V\left(\phi_i \left| Z_i\right.\right) + E(\sigma_i^2 \mid Z_i)\left(Z_i'Z_i\right)^{-1} \tag{D.23}$$

in which $\sigma_i^2 I$ is the variance of ε_i when we allow for **individual** heteroskedasticity but no serial autocorrelation.

Decomposing the variance, we have:

$$\begin{aligned} V\left(\phi_i\right) &= EV\left(\phi_i \left| Z_i\right.\right) + V\left(E(\phi_i \mid Z_i)\right), \\ V\left(\phi_i^u\right) &= EV\left(\phi_i^u \left| Z_i\right.\right) + V\left(E(\phi_i^u \mid Z_i)\right). \end{aligned}$$

By equation (D.22), we also have since $E(\widetilde{v}_i \mid Z_i) = 0$:

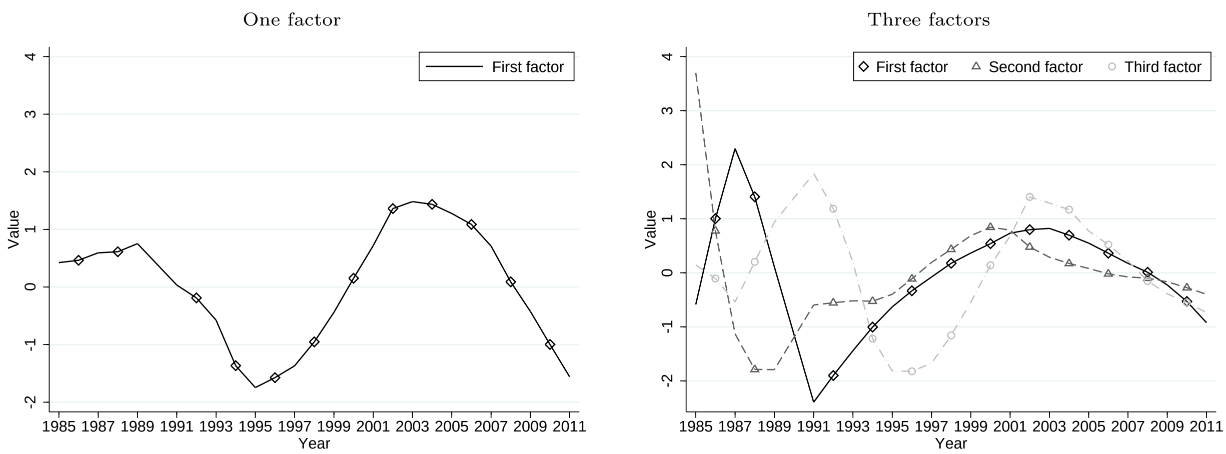
$$E(\phi_i^u \mid Z_i) = E(\phi_i \mid Z_i)$$

and thus, we get

$$V\left(\phi_i\right) = V\left(\phi_i^u\right) - E\left[\sigma_i^2\left(Z_i'Z_i\right)^{-1}\right] \tag{D.24}$$

and the two right-hand side terms can be computed after replacing φ_i by its least-square estimator $\widehat{\varphi}_i = \left(\widehat{\varphi}_{t_1}, ..., \widehat{\varphi}_{T_i}\right)'$ in the expression of Z_i and using instead $\widehat{Z}_i = \left(1'_{T_i}, X'_i, \widehat{\varphi}'_i\right)'$. Indeed, the first term can then be approximated with the empirical variance of estimated individual parameters and the second one can be estimated as the average of ordinary least square variances obtained in the first step of our iterative estimation method for every individuals. We can then easily recover an estimator of $V\left(y_{it}^c\right)$ from (35) using (D.24), since the counterfactual log-wage is a linear combination of estimated individual parameters.

Figure A.1: Value of factors as a function of time, main specification



Note: The sign of factors was initially normalized such that their value is positive in 1985. However, for the specification with three factors, the opposite is represented for the first factor for its curve to be consistent with that of factors obtained with specifications that include one or two factors (see this Figure and Figure 7).

Table A.1: Proportion of individuals occupying a job at both potential experiences s and t

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1.00	0.97	0.87	0.90	0.95	0.96	0.85	0.86	0.88	0.88	0.91	0.91	0.90	0.91	0.91	0.91	0.90	0.90	0.89	0.88
2	0.97	0.97	0.95	0.93	0.96	0.95	0.83	0.84	0.86	0.86	0.88	0.89	0.88	0.88	0.87	0.87	0.87	0.87	0.86	0.85
3	0.87	0.95	0.87	0.88	0.88	0.87	0.75	0.76	0.78	0.78	0.80	0.80	0.79	0.79	0.79	0.79	0.78	0.79	0.77	0.77
4	0.90	0.93	0.88	0.90	0.94	0.91	0.78	0.78	0.79	0.80	0.82	0.82	0.81	0.82	0.81	0.81	0.80	0.80	0.79	0.79
5	0.95	0.96	0.88	0.94	0.95	1.01	0.84	0.84	0.85	0.85	0.87	0.87	0.86	0.87	0.86	0.86	0.86	0.86	0.84	0.84
6	0.96	0.95	0.87	0.91	1.01	0.96	0.86	0.85	0.86	0.86	0.88	0.88	0.87	0.87	0.87	0.87	0.86	0.86	0.85	0.84
7	0.85	0.83	0.75	0.78	0.84	0.86	0.85	0.77	0.77	0.77	0.78	0.78	0.77	0.78	0.77	0.77	0.77	0.77	0.75	0.75
8	0.86	0.84	0.76	0.78	0.84	0.85	0.77	0.86	0.79	0.78	0.80	0.79	0.78	0.79	0.78	0.78	0.78	0.78	0.76	0.76
9	0.88	0.86	0.78	0.79	0.85	0.86	0.77	0.79	0.88	0.81	0.82	0.81	0.80	0.80	0.80	0.80	0.79	0.79	0.78	0.78
10	0.88	0.86	0.78	0.80	0.85	0.86	0.77	0.78	0.81	0.88	0.84	0.82	0.81	0.81	0.81	0.80	0.80	0.80	0.79	0.78
11	0.91	0.88	0.80	0.82	0.87	0.88	0.78	0.80	0.82	0.84	0.91	0.86	0.84	0.84	0.84	0.83	0.82	0.82	0.81	0.80
12	0.91	0.89	0.80	0.82	0.87	0.88	0.78	0.79	0.81	0.82	0.86	0.91	0.85	0.85	0.84	0.83	0.82	0.82	0.81	0.81
13	0.90	0.88	0.79	0.81	0.86	0.87	0.77	0.78	0.80	0.81	0.84	0.85	0.90	0.86	0.84	0.83	0.82	0.82	0.80	0.80
14	0.91	0.88	0.79	0.82	0.87	0.87	0.78	0.79	0.80	0.81	0.84	0.85	0.86	0.91	0.85	0.84	0.82	0.82	0.81	0.80
15	0.91	0.87	0.79	0.81	0.86	0.87	0.77	0.78	0.80	0.81	0.84	0.84	0.84	0.85	0.91	0.86	0.83	0.83	0.81	0.81
16	0.91	0.87	0.79	0.81	0.86	0.87	0.77	0.78	0.80	0.80	0.83	0.83	0.83	0.84	0.86	0.91	0.85	0.83	0.82	0.81
17	0.90	0.87	0.78	0.80	0.86	0.86	0.77	0.78	0.79	0.80	0.82	0.82	0.82	0.82	0.83	0.85	0.90	0.85	0.82	0.81
18	0.90	0.87	0.79	0.80	0.86	0.86	0.77	0.78	0.79	0.80	0.82	0.82	0.82	0.82	0.83	0.83	0.85	0.90	0.84	0.82
19	0.89	0.86	0.77	0.79	0.84	0.85	0.75	0.76	0.78	0.79	0.81	0.81	0.80	0.81	0.81	0.82	0.82	0.84	0.89	0.84
20	0.88	0.85	0.77	0.79	0.84	0.84	0.75	0.76	0.78	0.78	0.80	0.81	0.80	0.80	0.81	0.81	0.81	0.82	0.84	0.88

Table A.2: Proportion of individuals in a given cohort occupying a job at potential experience t

Cohort	N	Potential experience																										
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1985	897	1.00	0.73	0.70	0.76	0.83	0.83	0.86	0.88	0.87	0.69	0.87	0.88	0.89	0.91	0.91	0.91	0.88	0.89	0.81	0.84	0.86	0.84	0.85	0.79	0.77	0.63	0.63
1986	818	1.00	0.73	0.75	0.80	0.80	0.86	0.85	0.84	0.68	0.87	0.89	0.90	0.91	0.91	0.91	0.91	0.89	0.84	0.86	0.88	0.87	0.87	0.83	0.80	0.71	0.69	0.00
1987	896	1.00	0.70	0.77	0.77	0.82	0.85	0.85	0.69	0.89	0.89	0.90	0.92	0.92	0.91	0.91	0.91	0.83	0.86	0.87	0.89	0.88	0.86	0.84	0.77	0.77	0.00	0.00
1988	905	1.00	0.79	0.79	0.81	0.86	0.83	0.66	0.87	0.89	0.90	0.94	0.93	0.94	0.92	0.92	0.85	0.87	0.89	0.90	0.90	0.88	0.86	0.80	0.77	0.00	0.00	0.00
1989	1123	1.00	1.00	0.80	0.85	0.82	0.65	0.85	0.87	0.90	0.92	0.90	0.93	0.91	0.92	0.87	0.90	0.92	0.93	0.94	0.93	0.90	0.86	0.84	0.00	0.00	0.00	0.00
1991	1469	1.00	0.86	0.82	0.68	0.87	0.90	0.91	0.91	0.92	0.93	0.93	0.93	0.88	0.90	0.91	0.91	0.93	0.93	0.90	0.87	0.86	0.00	0.00	0.00	0.00	0.00	0.00
1992	896	1.00	0.79	0.64	0.87	0.86	0.91	0.93	0.93	0.94	0.94	0.95	0.87	0.89	0.92	0.95	0.96	0.95	0.93	0.91	0.88	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table A.3: Proportion of individuals with a given number of job periods when non-employed at potential experience t

Potential experience	N	Number of job periods 0	1	2	3	4	≥ 5	pdefinitif
2	1321	0.540	0.079	0.046	0.017	0.027	0.273	0.017
3	1677	0.470	0.052	0.057	0.035	0.027	0.341	0.018
4	1531	0.415	0.062	0.066	0.046	0.021	0.376	0.014
5	1121	0.531	0.082	0.054	0.034	0.018	0.282	.
6	1161	0.387	0.088	0.050	0.042	0.030	0.392	0.011
7	1049	0.411	0.071	0.069	0.027	0.025	0.387	0.011
8	974	0.387	0.080	0.035	0.021	0.034	0.433	0.010
9	854	0.389	0.056	0.043	0.039	0.023	0.443	0.007
10	820	0.343	0.073	0.055	0.032	0.040	0.384	0.073
11	615	0.390	0.046	0.037	0.046	0.029	0.410	0.042
12	622	0.429	0.058	0.035	0.050	0.035	0.349	0.043
13	666	0.437	0.059	0.029	0.030	0.017	0.407	0.023
14	617	0.365	0.057	0.037	0.029	0.041	0.446	0.026
15	631	0.431	0.049	0.033	0.040	0.032	0.401	0.014
16	644	0.422	0.067	0.037	0.037	0.047	0.376	0.014
17	702	0.462	0.078	0.046	0.050	0.067	0.279	0.019
18	690	0.501	0.058	0.070	0.061	0.028	0.258	0.025
19	775	0.582	0.072	0.070	0.026	0.048	0.159	0.044
20	810	0.647	0.105	0.019	0.038	0.049	0.090	0.052
21	1670	0.886	0.016	0.030	0.016	0.023	0.019	0.010
22	3031	0.946	0.020	0.010	0.009	0.006	.	0.009
23	3143	0.969	0.010	0.010	0.006	.	.	0.004
24	4251	0.984	0.009	0.004	.	.	.	0.003
25	5053	0.992	0.005	.	.	.	.	0.004
26	5873	0.996	.	.	.	.	.	0.004
27	6442	.	.	.	.	.	.	1.000

Table A.4: Proportion of individuals with a given number of non-employment periods when occupying a job at potential experience t

Potential Experience	N	Number of non-employment periods 0	1	2	3	4	≥ 5	pdefinitif
1	7004	0.811	0.087	0.048	0.028	0.013	0.012	.
2	5683	0.831	0.097	0.039	0.014	0.010	0.008	.
3	5327	0.861	0.089	0.025	0.014	0.005	0.007	.
4	5473	0.911	0.040	0.029	0.008	0.007	0.005	.
5	5883	0.904	0.064	0.017	0.008	0.003	0.004	.
6	5843	0.897	0.069	0.017	0.008	0.004	0.005	.
7	5955	0.909	0.062	0.017	0.006	0.003	0.003	.
8	6030	0.921	0.054	0.013	0.006	0.002	0.004	.
9	6150	0.921	0.060	0.011	0.005	0.001	0.003	.
10	6184	0.946	0.036	0.008	0.003	0.003	0.003	.
11	6389	0.940	0.037	0.010	0.008	0.003	0.002	.
12	6382	0.937	0.040	0.012	0.004	0.003	0.003	.
13	6338	0.949	0.036	0.007	0.005	0.002	0.001	.
14	6387	0.936	0.040	0.011	0.008	0.003	0.002	.
15	6373	0.942	0.036	0.009	0.005	0.002	0.003	0.003
16	6360	0.932	0.038	0.012	0.006	0.002	0.002	0.008
17	6302	0.942	0.032	0.010	0.005	0.001	0.001	0.009
18	6314	0.932	0.032	0.014	0.003	0.002	0.000	0.017
19	6229	0.942	0.024	0.007	0.002	0.000	0.001	0.024
20	6194	0.815	0.017	0.007	0.000	0.001	0.000	0.160
21	5334	0.709	0.015	0.005	0.001	0.000	0.000	0.269
22	3973	0.931	0.015	0.003	0.001	.	.	0.051
23	3861	0.688	0.010	0.004	0.000	.	.	0.298
24	2753	0.683	0.008	0.003	.	.	.	0.306
25	1951	0.558	0.006	.	.	.	.	0.436
26	1131	0.476	.	.	.	.	.	0.524
27	562	.	.	.	.	.	.	1.000