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Trend Inflation and Inflation Compensation

by Juan Angel Garcia and Aubrey Poon

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Abstract

This paper incorporates market-based inflation expectations to the growing literature on trend inflation estimation, and finds that there has been a significant decline in euro area trend inflation since 2013. This finding is robust to using different measures of long-term inflation expectations in the estimation, both market-based and surveys. That evidence: (i) supports the expansion of ECB's UMP measures since 2015; (ii) provides a metric to monitor long-term inflation expectations following their introduction, and the likelihood of a sustained return of inflation towards levels below but close to 2% over the medium term.

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1 Introduction

The analysis of inflation expectations is a crucial element of modern monetary policy. Long-term inflation expectations shed light on the credibility of monetary policy, and it is widely agreed that the monetary transmission mechanism is most effective when long-term inflation expectations are strongly anchored. It is therefore not surprising that central bankers in their public statements (see, e.g., Bernanke 2007, Draghi, 2014, 2015; Yellen, 2015) and specialized press and market commentary (e.g. The Economist, 2014, 2017; Financial Times, 2016) nowadays discuss in some detail developments in long-term inflation expectations.

At the same time, analysis of trend inflation has become increasingly relevant for researchers and central banks. The estimated level and variability of the trend inflation provide direct information on the degree of anchoring of inflation expectations and its evolution over time. Indeed, an important literature combining unobserved components models with stochastic volatility (UCSV) to estimate long-term trend inflation has emerged in recent years (e.g. Stock and Watson, 2015, Chan, Koop and Potter, 2013, Bednar and Clark, 2014, Garnier, Mertens, and Nelson, 2015, and Mertens, 2015).

This paper makes two contributions to the existing literature. First, we incorporate market-based inflation expectations into the estimation of trend inflation in UCSV models. Building on Kozicki and Tinsley (2012) and others, Chan, Clark and Koop (2018) incorporated long-term survey expectations into the modeling of trend inflation. A key motivation for their work was to assess whether survey measures of long-term inflation expectations have become disconnected with actual inflation in the low inflation environment experienced in recent years.

Market-based measures of inflation expectations are particularly relevant for trend inflation analysis because, in contrast to survey measures, they have declined significantly to historical low levels in recent years. In our benchmark specification, we adapt the empirical framework of Chan, Clark and Koop (2018) to the use of market-based inflation expectations. Specifically, our UCSV model jointly provides an estimation of trend inflation consistent with market-based inflation expectations along the lines of recent time-series literature. We then use the level and variation of trend inflation to assess the extent to which the protracted period of below-target inflation since 2013 has affected euro area trend inflation.

The second contribution of this paper is to provide a decomposition of the observed long-term inflation compensation into inflation expectations and inflation risk premium (and other premia) along the lines of the term structure literature. Our model specification provides an implicit estimation of long-term inflation risk premia without the need to model (and the risk of mispecification of) linkages along the whole term structure. Specifically, in our specification the observed inflation compensation reflects the sum of trend inflation, an inflation risk premia component and an additional error term associated to other premia.

Our main findings are as follows. We show that trend inflation had been relatively well anchored around the 2% level between 2004-2012. Importantly, the anchoring of inflation expectations was broadly unchanged during most of the Global Financial Crisis period, including the intensification of the financial turbulences following the collapse of Lehman Brothers and the onset of the European debt crisis in 2010. Yet, there has been a significant decline in trend inflation estimates since 2013, pointing to a weakening in the anchoring of long-term inflation expectations in the euro area. The protracted decline in actual inflation, which has remained below the 2% level since January 2013, led to a significant decline in benchmark long-term inflation compensation measures (five-year forward IL swap rate in five years), which in second-half of August 2014 starting being priced in below 2% for the first time in the history of the euro area and reached a historical minimum below 1.3% in the summer of 2016. We show that such a decline has reflected a gradual but persistent decline in trend inflation levels over the following two years towards levels around 1.5% by early 2015.

We show that our findings are robust to using different measures of long-term inflation compensation. Moreover, alternative trend inflation estimates using survey data on long-term inflation expectations display very similar dynamics, and, importantly, also corroborate the point in time at which long-term inflation expectations started declining in late 2012. Our robustness checks therefore provide substantial support for the decline in euro area trend inflation in recent years reflecting a protracted weakness of inflation dynamics in the euro area and not a mispricing in the IL market. In contrast, we find evidence of a sizable and statistically significant bias in euro area survey inflation expectations, in line with the evidence on the U.S. reported in Chan Clark and Koop (2018).

Our findings have important implications for the analysis of long-term inflation expectations in the euro area. First, the evidence on the protracted decline in trend inflation since 2013 provides substantial support for the expansion of the ECB's unconventional monetary policy UMP measures to direct purchases of sovereign bonds (QE), among other assets, since early 2015. Furthermore, as ECB's QE seems to have just managed to attenuate the decline in long-term inflation compensation measures and trend inflation, but both have remained significantly below their historical averages since then, this paper provides a useful metric to help monitor the likelihood of a sustained return of inflation towards (levels below but close to) 2% over the medium term (Draghi, 2018).

Second, the framework introduced in this paper also provides an implicit decomposition of long-term inflation compensation into the level of inflation expectations, given by trend inflation priced in in the IL instruments, and the additional premia requested by investors. In particular, we show that the rebound in long-term inflation compensation between November 2016 and January 2017 reflects a slight recovery of long-term inflation expectations (about 15 basis points) but mainly an increase in inflation risk premia (about 25 basis points).

The remainder of the paper is organized as follows. Section 2 provides an overview of recent developments in long-term inflation expectations and the euro area IL swap market compensation. Section 3 describes our empirical model, and we report the main results from our benchmark specification in Section 4. Section 5 provides several robustness checks to our benchmark specification, and discusses the estimates of inflation risk premia implicit in our framework. Section 6 finally concludes.

2 Market-based inflation compensation and expectations

The main goal in this paper is to provide reliable estimates of trend inflation in the euro area. Over the last two decades, the analysis of inflation expectations has been further reinvigorated by the issuance of bonds and derivatives (mainly swaps but also other instruments) whose payments are indexed to inflation developments in many advanced and emerging economies. The yield spread between comparable conventional bonds and inflation-linked (IL) bonds is often referred to as the *break-even inflation rate* (BEIR) because it provides an estimate of the level of expected inflation at which a (risk-neutral) investor would be indifferent between holding either type of bond. BEIRs often provide more timely and comprehensive information (across time horizons) on investors' inflation expectations compared to survey-based expectations, and have by now become important and closely-monitored indicators.

In addition to the expected inflation, however, BEIRs and IL swap rates incorporate other factors, notably inflation risk premia, and should better be interpreted as the overall inflation *compensation* requested by investors to hold nominal assets, rather than a pure measure of expected inflation. The inflation risk premium captures markets' pricing of risks surrounding inflation expectations. Abstracting from liquidity premium—which may arise from trading frictions or insufficient market activity and is therefore unrelated to inflation expectations—inflation compensation measures should then be interpreted as an indicator of market participants' inflation expectations in a broader sense rather than a single point estimate, comprising information on inflation risks. Changes in inflation compensation measures over time could reflect either changes in the level of expected inflation, changes in the perceived risks and uncertainty about future inflation or a combination of both. From a central bank's perspective, both components are of relevance. A credible commitment to price stability should anchor the level of expected inflation to its policy objective, with the degree of perceived uncertainty about future inflation developments providing information about how firmly inflation expectations may be anchored.

We measure inflation compensation using data from the inflation-linked (IL) swap market in the euro area. Using BEIRs requires the estimation of nominal and real term structures from conventional and IL government bonds issued by euro area governments. The issuance of IL bonds in the euro area has remained relatively limited so far,[1] at least compared to the TIPS issuance in the United States for example. As a result, there is significant market segmentation in the euro area IL bond market, and the onset of the Global Financial Crisis and the subsequent euro area debt crisis has led to the presence of significant differences in sovereign and liquidity risk embodied in the prices of those bonds. Since a key goal of this paper is to assess the anchoring of inflation expectations after the onset of the global financial crisis and particularly since 2013 we instead use data from the euro area IL swap market. IL swaps—a derivative though which one party commits to pay a fixed rate of inflation in exchange for the actual inflation over the length of the contract— provide inflation compensation measures that, being solely based on net exchanges of flows at the end of the contract, should not incorporate a liquidity premium, and can therefore provide a cleaner measure of inflation compensation than bond-based BEIRs.[2] The estimation of the inflation risk premium usually takes place in the context of term structure models. Estimates available from central banks vary significantly across specifications even for a single country, but they generally point to significant variation in inflation risk premium over time and across maturities, which often makes the interpretation of changes in inflation compensation far from straightforward.

[1] Upto now, only France, Italy, Germany, Spain and Greece have issued some IL bonds.

[2] There exist some approaches to correct for liquidity premium, based on relative traded volumes or asset-swap spreads for example, but the adjustment necessarily involves some assumptions on some model specification or its presence across maturities.

2.1 The euro area IL swap market

The euro-area IL swaps market is the most mature and largest IL swaps market in the world in terms of trading volumes, a very liquid market for actively hedging exposures to euro area-wide HICP (excluding tobacco).[3] Market participants typically include pension funds willing to hedge their inflation exposures but also a substantial arbitrage activity between the sovereign market for inflation-linked debt and the IL swap market. Given the limitations of the euro area sovereign IL bond market, the IL swaps as the main market in which to hedge inflation outcomes.

The largest traded volumes are typically zero-coupon contracts, through which, net payments of a fixed rate of inflation for a floating rate reflecting actual inflation in euro-area consumer prices are exchanged at maturity. In a zero-coupon IL swap, the fixed inflation rate leg of the swap reflects the compensation requested by the holder of the contract for expected inflation over the life of the contract plus a premium for bearing the uncertainty associated to future inflation, the inflation risk premium. Such an inflation compensation measure can be obtained directly from the market quotes, without the need to estimate the nominal and real zero-coupon term structures from traded bonds and therefore minimizing the impact of potential model mispecification in our analysis.

Euro area IL swap contracts have been very actively traded since 2004 over a wide range of maturities from 1 to 30 years, although market intelligence suggests that the five and ten year maturities have tended to concentrate a significant amount of liquidity. In the light of those considerations, and despite the fact that the ECB has repeatedly stated that the analysis of inflation expectations comprises a wide range of indicators, the five-year IL forward swap rate five years ahead capturing inflation compensation between 5 and 10 years ahead has become the most widely used measure to assess developments in euro area long-term inflation expectations (e.g. Draghi, 2014).

Forward IL swap rates provide a direct measure of market's inflation compensation at medium-to-long term horizons. The price of a spot zero-coupon swap with a 10-year maturity, s_t^{10y}, reflects the average inflation compensation over the next ten years. Similarly for the

[3]The overwhelming majority of euro area financial instruments linked to inflation are index to the HICP *excluding* tobacco. This convention follows the issuance of French IL bonds that were indexed to the euro area HICP exTobacco to comply with French regulation aimed at keeping tobacco taxation decisions independent from other considerations. Other national treasuries joining the market at a later stage, as well as derivative products have followed that convention. Given the limited weight of tobacco in the overall HICP index this convention has no material influence.

five-year spot rate, s_t^{5y}. In contrast, by construction, a forward IL swap rate five-year forward in five years, f_t^{5y5y}, reflects the inflation compensation priced in between five and ten years ahead, a medium-to-long term period that captures well the movements in inflation compensation we are interested. Formally, the long-term forward IL swap rates implicit in the term structure of IL swap rates can be calculated from the five and ten year sport rates as follows

$$(1 + f_t^{\,5y5y}) = \frac{(1 + s_t^{10y})^{10}}{(1 + s_t^{5y})^5} \tag{1}$$

2.2 An overview of euro area long-term inflation expectations

Figure 1 provides some graphical evidence on the behavior of private sector's inflation expectations over recent years. Specifically it depicts the long-term forward IL swap rate (five-year forward in five years) together with two survey measures of long-term inflation expectations, from Consensus Economics (6 to 10 years ahead) and from the ECB's Survey of professional Forecasters (five-years ahead, ECB's SPF, see Garcia, 2003).

[Figure 1 around here]

There are two important features of the behavior of euro area inflation expectations over the last decade that are worth noting. First, long-term forward inflation compensation tended to be significantly above survey measures of long-term inflation expectations. Assuming the level of long-term inflation expectations was relatively similar in both market and survey-based measures, the discrepancy between the two measures of long-term inflation expectations has been generally attributed to the presence of the inflation risk premia in inflation compensation measures (see Hordalh and Tristani, 2010, Garcia and Werner, 2014). Whether market- and survey-based measures reflect the same level of long-term inflation expectations has however become less clear in recent years. Long-term forward inflation compensation declined significantly in the euro area since actual inflation rates drifted downwards since 2013. Such a decline was not exclusive to the euro area, and have attracted substantial attention among policymakers (Draghi, 2014, 2015; Yellen, 2015), as well as in specialized press and market commentary (e.g. The Economist, 2014, 2017; Financial Times, 2016). In contrast to long-term inflation compensation, survey measures of inflation expectations have remained relatively more stable in the euro area, although a significant decline away from the 2% reference level for price stability can also be appreciated since mid-2013.

The striking differences in the behavior of long-term inflation compensation and survey measures of inflation expectations have strong implications for the assessment of their information content. On the one hand, markets may have overpriced the risks of severe deflation in the euro area. On the other hand, surveys may have become disconnected with actual inflation dynamics in a low inflation environment. A crucial goal of this paper is to shed light on the those questions by means of the empirical estimation of trend inflation.

3 A model for trend inflation

This paper uses a statistical model for the decomposition of observed inflation into a trend level and a temporary/cyclical component in the sense of Beveridge and Nelson (1981). Specifically, the model will decompose inflation π_t into a trend component π_t^*, and a deviation from trend, or inflation gap c_t, component, that is $\pi_t = \pi_t^* + c_t$, using an unobserved components framework allowing for stochastic volatility (as specified below). The permanent component, or trend inflation π_t^*, reflects the most likely inflation rate to be observed once the transitory influences on inflation die away. Trend inflation can therefore be interpreted as the optimal conditional long-term inflation forecast.

Our model specification for trend inflation estimation draws from Chan et al. (2018). Our contribution is to extend the analysis and interpretation to the market-based inflation expectations. Formally comprises the following set of equations:

$$\pi_t - \pi_t^* = b_t(\pi_{t-1} - \pi_{t-1}^*) + v_t, \quad v_t \sim N(0, e^{h_{v,t}}), \tag{2}$$

$$ILS_t = d_{0,t} + d_{1,t}\pi_t^* + \epsilon_{z,t} + \psi\epsilon_{z,t-1}, \quad \epsilon_{z,t} \sim N(0, \sigma_w^2), \tag{3}$$

$$\pi_t^* = \pi_{t-1}^* + n_t, \quad n_t \sim N(0, e^{h_{n,t}}), \tag{4}$$

$$b_t = b_{t-1} + \epsilon_{b,t}, \quad \epsilon_{b,t} \sim TN(0, \sigma_b^2), \tag{5}$$

$$d_{i,t} - \mu_{i,t} = \rho_{d_i}(d_{i,t-1} - \mu_{d_i}) + \epsilon_{d_i,t}, \quad \epsilon_{d_i,t} \sim N(0, \sigma_{d_i}^2), \quad i = 0, 1, \tag{6}$$

$$h_{i,t} = h_{i,t-1} + \eta_{h_i} \ , \quad \eta_{h_i} \sim N(0, \sigma_{h_i}^2), \quad i = v, n. \tag{7}$$

Equation (2) is a commonly used standard measurement equation that relates current inflation π_t and trend inflation π_t^* to past inflation and past trend inflation respectively. The b_t is a time-varying parameter that measures the evolution of the degree of persistence in the inflation gap. Note that in equation (5) a truncated normal is assumed on the variance of the b_t to ensure that $0 < b_t < 1$ is satisfy at each point of time, so that the inflation gap in (2) is stationary at each point of time and the conditional expectation of this process converges to zero as the forecast horizon increases.

Equation (3) is fundamental for the main goal of this paper, so we provide some additional information on its interpretation here. In our model specification long-run inflation compensation ILS_t depends on the long-term inflation trend, with a slope coefficient $d_{1,t}$ that is allowed to vary over time, a time-varying intercept $d_{0,t}$ that captures risk premia, and an MA(1) error term to allow for persistence in a long-term inflation compensation that may not be fully captured by persistence in trend inflation. Historical values of long-term inflation compensation measures show that they have hovered about 2% level which suggests that trend inflation is likely to be, at least in quantitative terms, the most important component of inflation compensation (Figure 1). The motivation for adding this additional relationship between long-term inflation expectations (in our case ILS_t) and π_t^* is to improve the precision of the trend inflation estimates. Once the level of long-term inflation expectations and its market pricing onto inflation compensation is pinned down, the remaining part of the observed inflation compensation reflects the premia requested by investors. Such premia may include a premium related to the perceived inflation risks, as well as a potential liquidity premium. Both of those premia components are unobservable, and while their quantitative values are to a large extent dependent on the model specification available evidence suggests they exhibit significant variation over time. We show that our specification of long-term inflation compensation, with a time-varying intercept $d_{0,t}$,and an MA(1) error term can capture well those characteristics of the premia and provide additional quantitative evidence on their size and variation over time, conditional on a reliable estimation of the level of long-term inflation expectations.

Equation (4) and (5) are the transition or state equations for the trend inflation and the time-varying parameters $d_{i,t}$ respectively. The model also allows for stochastic volatility in the inflation gap (1) and trend inflation (4) equations, which in the related literature has often found to be very useful in the estimation of trend inflation models like ours. Lastly, all the errors stated above are assumed to be independent over time and with each other.

We use Bayesian methods to estimate the model and implement a standard Markov Chain Monte Carlo (MCMC) algorithm. Details on the estimation approach as well as some sensitivity analysis on some key priors used in the estimation are provided in the Appendix.

4 Inflation trends: level and uncertainty

Our estimates show that trend inflation had been relatively stable around the 2% level for most for the sample (see Figure 2). The low level at the beginning of our sample most likely just reflects the fact that the euro area IL swap market was still under development in early 2004, but by the end of that year a long-term inflation level of 2% level was already priced in, and it remained close to that level until 2012. Importantly, the anchoring of inflation expectations was broadly unchanged during most of the Global Financial Crisis (GFC) period, including the intensification of the financial turbulences following the collapse of Lehman Brothers and the onset of the European debt crisis since early 2010.

[Figure 2 around here]

Despite its resilience in the aftermath of the global financial crisis, our trend inflation estimates declined significantly from 2013. The protracted decline in actual inflation, which went below the 2% mark in January 2013 and has remained below target since then, in turn triggered a gradual but persistent decline in trend inflation estimates. From a level close to 2% at the beginning of 2013, trend inflation declined by around 70 basis points over the following two years, and reached levels slightly above 1.3% by early 2015. This downward re-pricing of long-term trend inflation led to a significant decline in benchmark long-term inflation compensation measures (five-year forward IL swap rate in five years), which in second-half of August 2014 were priced in below 2% for the first time in the history of the euro area, and reached a historical minima below 1.3% in the summer of 2016.

The announcement of an expansion of the ECB's unconventional monetary policy measures to direct purchases of sovereign bonds (QE) among other assets in January 2015 seems to have just managed to attenuate the decline in long-term inflation compensation measures and trend inflation so far, but both remained significantly below their historical averages since then. Only almost two years after the introduction of bond purchases and with the temporary rebound in actual inflation in late 2016 due to based effects on energy prices, trend inflation and long-term inflation compensation have partially rebounded but still remain far from their historical average.

Interestingly, our estimates suggest that uncertainty surrounding trend inflation has not increased despite the substantial increase in volatility in actual inflation. On the contrary the decline in trend inflation since 2013 has been accompanied by a slight reduction in the uncertainty surrounding the posterior estimates, which clearly indicate that the decline in trend inflation is clearly significant from a statistical point of view.[4]

5 Robustness checks and discussion

This section provides some sensitivity analysis on our benchmark specification to show that the main conclusions for the analysis described above hold in different settings. Specifically, we investigate the robustness of our trend inflation estimates to: (i) different measures of long-term inflation compensation; (ii) using survey measures as information of long-term inflation expectations. In addition, as a by-product of trend inflation estimation, the framework introduced in this paper provides an implicit decomposition of long-term inflation compensation into the level of inflation expectations, given by trend inflation interpreted as the optimal long-term conditional forecast that is priced in in the IL instruments, and the additional premia requested by investors. As an illustration of the usefulness of that decomposition for policy analysis we address an specific question that has become particularly relevant for the ECB policy debate (Draghi, 2017) since late 2016: to what extent the rebound in long-term inflation compensation experienced between November 2016 and January 2017 was related to a change in inflation expectations and which part reflected a repricing of premia in IL markets?

5.1 Different inflation compensation measures

One of the key advantages of financial indicators of inflation expectations is that, in very well developed IL markets like the euro area, inflation compensation measures are traded over a large number of maturities. Even at the long-end of the term structure of inflation compensation the market deepening allows for constructing several measures of long-term inflation compensation to cross-check the market pricing of trend inflation at different maturities.

To assess the robustness of our key findings we carry out the trend inflation estimation using two other long-term forward rates at the long-end of the IL swap curve, namely the one-

[4] Moreover, Garcia and Werner (2018) also finds a significant deterioration in the anchoring of inflation expectations in the euro area when looking at the response of long-term forward inflation compensation to macroeconomic news.

year forward in nine years and the five-year forward in ten years. The former helps assess the pricing of inflation compensation at the most liquid segment of the curve, the ten-year maturity. The latter covers the pricing of long-term inflation expectations in a curve segment populated by institutional investors like pension funds whose main interest is hedging long-term inflation exposures and are therefore most interested in an accurate pricing of expectations.

[Figure 3 around here]

The key empirical findings from our benchmark specification using the five-year forward inflation compensation are robust to other measures of long-term inflation compensation. Importantly, all measures confirm the protracted decline in trend inflation since 2013. Quantitative differences are relatively small, and remain within standard confidence bands around our benchmark specification over the whole sample.

5.2 Evidence from survey data

We have documented the discrepancies between inflation compensation and survey measures of inflation expectations since 2013, with the reaction of survey measures to the low inflation in the euro area being rather muted compared to that of inflation compensation. Given the robustness of the decline of trend inflation shown in different measures of inflation compensation it is important to assess the extent to which the relatively high level of survey measures questions the decline in trend inflation, or whether they have just become more disconnected with actual inflation in recent years in the euro area as they have been in the U.S.

We estimate trend inflation using long-term (6-10 years) Consensus Economics forecasts as measure of inflation expectations. A comparison of the estimated trend inflation using survey data and our benchmark estimation based on the five-year forward IL in five years is provided in Figure 4.

[Figure 4 around here]

Results corroborate that the decline in trend inflation we documented is not related to the use of inflation compensation measures. Indeed, there is a substantial consistency between the trend inflation estimates using survey and inflation compensation data. First, the survey-based trend inflation lies within standard statistical confidence intervals from our benchmark estimates over the whole sample. Second, both inflation compensation and survey-based estimates also show very similar dynamics, and, importantly, also corroborate the point in time at which long-term inflation expectations started declining in late 2012. Despite the significant differences in the observed behavior of market and survey-based indicators of inflation expectations in recent years, the similar information they provide in terms of underlying trend inflation points to a very strong weakening of inflation dynamics in the euro area in recent years.

This evidence is also broadly in line with the findings of the most recent literature looking at euro area survey data. For example, using a very different framework Lyziac and Paloviita (2017) finds that long-term inflation expectations in the ECB's Survey of Professional Forecasters display some signs of de-anchoring in the financial crisis period (see also Figure 1), as measured by increased sensitivity to shorter-term inflation expectations and actual HICP inflation, as well as less weight to the ECB's inflation target (or a lower one) in the formation of those expectations.

5.3 The role of long-term information

Our results provide strong evidence of a significant decline in trend inflation since 2013. We have shown that our main conclusion is robust to the use of different inflation compensation and survey measures of long-term inflation expectations. This section provides additional evidence on the importance of adding long-term inflation expectations into the specification for the correct estimation of trend inflation.

As illustration, we consider an alternative model specification that does not incorporate equation (3). From a modeling perspective, the benchmark specification used in this paper then collapses to the UCSV model used in Chan et al. (2013) without imposing bounds for trend inflation. Intuitively, this alternative specification does not incorporate long-term information in the form of a time-varying *"end point" a la* Kozicki and Tinsley (2012). Intuitively, the estimation of trend inflation would then be based solely on the observed inflation dynamics.

Figure 5 depicts the estimates of trend inflation from a model without information on long-term inflation expectations. To ease comparison, the resulting trend inflation is shown together with our benchmark specification using long-term inflation compensation (five-year forward inflation compensation rate in five years) and long-term survey expectations. Results suggest that using long-term information is crucial for trend inflation estimation, particularly in a disinflationary environment.

[Figure 5 around here]

Trend inflation estimates before 2012 were not significantly different from those incorporating long-term inflation expectations. However since 2012, with disinflation pressures mounting and leading to a protracted period of low inflation and eventually deflation, the alternative trend inflation estimates decline sharply, moving significantly below our benchmark estimates

That evidence further stresses that accounting for the anchoring of inflation expectations, and its variation over time, may be crucial for understanding actual inflation dynamics. When estimating trend inflation, long-term inflation expectations do provide information on the level of inflation expected to prevail over the medium-to-long term. There is ample evidence that such forward looking information is quantitatively important for understanding inflation dynamics, and the comparison of results with and without incorporating long-term information helps highlight the forces at work in our framework.

In the history of the euro area, there are two episodes of particular interest from the perspective of inflation dynamics and the role of the anchoring of inflation expectations. First, in the aftermath of Lehman Brothers collapse the intensification of global financial turbulences triggered a recession and a brief spell of deflation in 2009. Inflation however rebounded sharply over 2010 (see Figure 1). We argue that the strong anchoring of inflation expectations, with trend inflation broadly insensitive to actual inflation, was fundamental in that regard. Indeed, downward pressures on inflation dynamics at the time appeared to be mainly of temporary nature and that was the way in which economic agents perceived them. Both inflation compensation and survey measures remained broadly constant over that period. Moreover, before 2012 the largest discrepancy between our benchmark estimates for trend inflation and those without incorporating long-term information is observed around 2009, with the latter pointing to a stronger decline.

The other episode of interest, and the main focus of this paper, is the protracted period of below-target inflation since 2013. Our results point to two important forces shaping euro area inflation since late 2012. First, strong disinflationary pressures led to weak inflation dynamics. The sharp decline in trend inflation estimates without incorporating long-term expectations provides evidence on how weak actual inflation dynamics has been.

The role of long-term inflation expectations in this second disinflationary episode may be even more important. Our trend inflation estimates point to a significant weakening in the anchoring of euro area inflation expectations, and strongly suggest that such weakening may have played a fundamental role to explain the subdued inflation dynamics and the protracted period of below-target inflation since 2013. Yet, our results are consistent with a significantly weaker, but nonetheless only partial weakening of long-term inflation expectations. The fact that despite trend inflation declining below the 2% mark in our benchmark specification, it nonetheless remains significantly above the alternative specification without incorporating long-term expectations supports that interpretation.

6 Trend inflation and inflation expectations

Long-term inflation expectations are a crucial element of the empirical framework employed in this paper to estimate trend inflation. Focusing on the euro area evidence, we advocate for the use of financial indicators of inflation expectations as measures of long-term inflation expectations. Our choice is mainly motivated by the fact that, over the protracted disinflation period experienced in the euro area since late 2012, the behavior of survey measures of inflation expectations was rather muted, and suggested that they were somewhat disconnected with actual inflation developments. Such a muted reaction has been even more surprising when compared to the unprecedented decline in long-term forward inflation compensation measures since 2013. Our estimates point to significant decline in euro area trend inflation over the last few years.

This section first provides additional empirical evidence on the relationship between trend inflation and survey measures as well as between trend inflation and inflation compensation in the euro area. We then discuss how our findings may shed new light on the on-going research analyzing the discrepancies between US inflation expectations and trend inflation.

6.1 Survey expectations and trend inflation

Beyond the consistency in terms of trend inflation estimates, our estimates using euro area survey data allows for a quantification of the reasons behind the discrepancy of the observed, or reported, surveys and the underlying trend inflation. Indeed, one of the key motivations behind the framework developed by Chan et al. (2018) was to investigate whether US survey inflation expectations had become disconnected from actual inflation developments and trend inflation since the GFC.

Specifically, if survey expectations are employed instead of ILS_t in equation (3) $SUR_t = d_{0,t}^{sur} + d_{1,t}^{sur}\pi_t^* + \epsilon_{z,t} + \psi\epsilon_{z,t-1}$, the above framework allows to quantify the extent to which survey measures provide *unbiased* estimates of an econometric estimate of trend inflation. conceptually, the disconnection of survey expectations from trend inflation can reflect two distinct but complementary dimensions, namely $d_{0,t}^{sur} \neq 0$— a systematic *"level"* bias— and $d_{1,t}^{sur} \neq 1$, a lack of *"efficiency"* that would suggest a more relevant departure from rational expectations in the sense that long-term expectations may become less related to the long-term trend in inflation. Given that our estimates of d_{1t} are not statistically significant from 1 (see Section 7 for details) the main reason behind the discrepancy comes from a significant bias, captured by $d_{0,t}^{sur}$ in our framework.

Figure 6 depicts our estimates of the level bias in survey measures of inflation expectations, relative to the estimated trend inflation, as well as standard uncertainty bands around it. The results confirm the limited reliability of survey measures in the low inflation environment. While statistically insignificant before 2011-12, the bias in survey measures increased significantly since then, reaching almost 40 basis points during most of the below-target period. Despite their recent decrease, the survey bias remains sizable, which warns about the reliability of survey measures as indicators of inflation expectations, at least if taken as face value.

[Figure 6 around here]

The discrepancy between euro area survey inflation expectations and trend inflation in recent years is not qualitative and quantitatively very different from the findings of Chan et al (2018) for US survey expectations. This evidence has some important implications for the monitoring of indicators of inflation expectations particularly in central banks over in recent years. Analyzing the reasons behind that discrepancy in the case of the euro area is beyond the scope of this paper, but we offer some potential explanations recently put forward in some recent research on the U.S. case.

Coibion and Gorodnichenko (2015) provided regression evidence that survey forecasts departure from full rationality may be related to information rigidities leading to a sluggish adjustment of (U.S.) survey expectations. Such an interpretation is consistent with our findings for euro area surveys. In equation (3) the time varying parameters (and the MA error term) in equation (3) can be interpreted as capturing the effects of information rigidities in a flexible way, without imposing additional parameter restrictions arising from a particular theoretical model of expectations formation (see also Chan et al., 2018). A fundamental characteristic of our findings is that the discrepancy between survey expectations and trend inflation has been particularly strong during the protracted disinflation in the euro area from late 2012. The extend to which there may be important asymmetries in the adjustment of inflation expectations when observing inflation realizations below-target for a protracted period of time is an important issue to investigate. The approach introduced in Mertens and Nason (2015), where inflation and survey inflation expectations are modeled jointly and allowing for the strength of the information rigidities to vary over time using an additional latent state—incorporating autoregressive dynamics and trend inflation—on which the observed survey expectations depend, may be quite promising in that regard.

6.2 Inflation compensation, trend inflation and risk premia

A fundamental contribution of this paper to the growing literature on trend inflation estimation is the use of inflation compensation in the estimation. At least in the case of the euro area, we have argued this is quite a natural choice to investigate a potential weakening in the anchoring of long-term inflation expectations since 2013, given the sluggish adjustment of survey measures. Indeed, we have shown that, although survey data broadly provides the same message in terms of trend inflation, observed survey results exhibit significant biases, which may undermine their information content. In turn, the relationship between inflation compensation and the (unobserved) trend inflation is often clouded by the presence of risk premia in the latter. This section shows how our empirical framework can be used to shed light on that relationship.

A by-product of our benchmark trend inflation estimation is an implicit decomposition of long-term inflation compensation into the level of inflation expectations, given by trend inflation as the optimal long-term conditional forecast, and the additional premia requested by investors. Specifically, in our empirical framework, the equation $ILS_t = d_{0,t} + d_{1,t}\pi_t^* + \epsilon_{z,t} + \psi\epsilon_{z,t-1}$ can be used to gauge the decomposition of long-term inflation compensation. Once the part explained by trend inflation $d_{1,t}\pi_t^*$ is estimated, the remaining part of the observed inflation compensation is by definition the premia. In general terms, $d_{0,t}$ estimates can be used to gauge the inflation risk premium (IRP), while the MA(1) error terms can be attributed to liquidity premia or other temporary market disturbances. Figure 7 provides a decomposition of our benchmark measure of long-term forward inflation compensation (the five-year forward IL swap in five years).

[Figure 7 around here]

Information on the movements of long-term expectations and inflation risk premia is very useful for monetary policymaking. We have argued that both concepts are very relevant for monetary policy and the recent experience of the euro area is a clear example of the usefulness of getting information on their movements. To help focus the discussion, Figure 8 provides additional evidence on the dynamics of euro area inflation risk premia. As before, together with our benchmark specification using the 5-year forward IL swap rate in five years we also consider long-term inflation compensation at different maturities, namely the 1-year forward in nine years and the 5-year forward in ten years. There are a few important insights worth highlighting. We will first focus on the pre-2013 period. We then discuss the most recent period of below-target inflation, and how our estimates shed new light on the policy discussions during the most recent period.

Before 2013, our model results are broadly in line with previous findings on euro area IRP. First, IRP exhibits substantial time variation, in particular when compared to the trend inflation estimates (see Figure 3). Second, the estimated average level of IRP for our benchmark specification—around 30 basis points on average—is consistent with existing research evidence using term structure models (e.g. Hordalh and Tristani, 2010, Garcia and Werner, 2014). Moreover, they imply a relatively low level of IRP ahead of the global financial crisis —around 20 basis points before 2008—and a significant increase during the spring of that year following the surge of oil prices and actual inflation. The crisis period triggered significant volatility in the premia, with highs around late 2010 amid inflationary concerns about the expansionary monetary policy measures implemented in the post-Lehman collapse period, followed by severe declines showing recurrent deflationary fears as the debt crisis in periphery countries evolve, before stabilizing at around 30 basis points from early 2012.

[Figure 8 around here]

The protracted disinflation from early 2013 however triggered significant declines in IRP. Interestingly, our model estimates suggest that the decline in inflation compensation measures since late 2012 was initially mainly driven by lower trend inflation. Yet, the IRP however fell by around 30 basis points over 2014, hovering around zero for most of 2015 and even turning negative over most of 2016. This evidence on long-term inflation risk premium in euro area complements recent findings of a negative IRP at shorter maturities during the financial crisis period (e.g. Camba-Mendez and Werner, 2017).

While our estimates suggest that IRP at long maturities have mainly remained in positive territory, our finding of a temporary negative IRP is consistent with the recent literature that emphasizes not only the time variation of risk premia, but also the fact that it may change sign. Standard finance theory suggests that the inflation risk premium reflects the correlation between inflation and the marginal inter-temporal rate of substitution of consumption of the representative investor. As useful metric to gauge that relationship, U.S. stock and nominal bond returns are reported to be positively correlated before the 1990s (e.g. Campbell et al., 2018), suggesting that higher inflation was bad news for stocks and bonds. Over the last two decades however negative correlations have been reported not only for the U.S. but also in the euro area markets (e.g. Fleckenstein et al , 2016).[5] Campbell et al. (2018) have argued that the role of nominal bonds may have changed from inflation bets to "deflation hedges", as, since nominal bonds will perform well under deflation, they will be a good investment when deflation fears intensify. Investors may then be willing to forgo some return for such a hedge, thereby leading to a negative IRP priced in. We believe that, given potential difficulties of standard term structure models to accommodate such changes in the sign of the IRP the framework introduced in this paper can offer a flexible alternative.

The dynamics of our IRP estimates are fully consistent with the evolution of implicit forward Risk Neutral Densities (RNDs) of Gimeno and Ibañez (2017) using euro area IL swap and inflation options (caps and floors) data. Specifically, the few periods of negative inflation risk premia in our sample (around mid-2010, mid-2011, early 2015 and second-half of 2016) coincide with a rise in the implied probability of inflation being below zero in the long-term forward RND and increase in the balance of risk towards lower inflation outcomes.[6]

The decomposition of IL swap rates also sheds light on a specific question that has become particularly relevant for the ECB policy debate (Draghi, 2017) since late 2016: to what extent has the rebound in long-term inflation compensation been related to a change in inflation expectations and which part has reflected a repricing of premia in IL markets? Our estimates suggest that, while the rebound of inflation compensation of about 40 basis points between November 2016 and January 2017 was encouraging, the bulk of the increase is related to higher premia (around 25 basis points) while trend inflation has merely risen by about 15 basis points, and at around 1.6% it is still far form ECB's price stability objective. [7]

[5] Fleckenstein et al (2016) report an average correlation between daily stock and bond returns on all outstanding U.S. Treasuries, measured using the Barclays U.S. Treasury Index of -0.18 between August 1998 and October 2014, and of -0.22 in the euro area.

[6] Calculated as the probability mass associated to inflation higher than 2% over the probability mass associated to inflation lower than 1.5%.

[7] The rebound of inflation risk premia into positive territory since late 2016 is also consistent with the sharp improvement in those two dimensions of the long-term forward RND reported in Gimeno and Ibañez (2016).

7 Additional model results

In the previous sections we have focused on our benchmark model results, their robustness regarding trend inflation, and the additional dimensions necessary to understand the dynamics of inflation expectations. To our knowledge this is however the first paper focusing on euro area trend inflation, and particularly over the recent sample of below-target inflation. We therefore report here other model results and compare them among different specifications to provide a broader overview of the results for the euro area economy.

Figure 9 reports the estimates for the degree of inflation persistence (Panel (a)), the coefficient of trend inflation in long-term inflation expectations (Panel (b)), and the stochastic volatility governing the dynamics of the inflation gap (Panel (c)) and the trend inflation (Panel (d)). Beyond some specific details, a key message from all this model dimensions is the robustness across different model specifications: as for the trend inflation estimates, in all cases the model parameters are very similar and, even if small differences arise, quantitatively lie within the standard uncertainty bands surrounding our benchmark estimates.

[Figure 9 around here]

Our results point to the presence of a sustained upward trend in inflation persistence in the euro area over the sample as a whole. While even at the end of the sample the b_t parameter remains around 0.4, this upward trend is quite relevant, for, first, it is consistent with the idea of higher inflation persistence in the later part of the sample and, second, it may contribute to explain why the rebound of inflation in the euro area following the disinflation since late 2012 has been so slow.

The impact of trend inflation on long-term inflation expectations has been quite stable in our sample. There is however a noticeable difference: while d_{1t} estimates for inflation compensation measures tend to be slightly higher than, for survey measures they are below 1 during all the sample. While this may explain the stronger decline in inflation compensation than in survey measures, the estimates remain quite close and statistically not different from 1. Indeed, we have also run restricted versions of the model fixing the trend inflation parameter to 1 and the results were qualitatively similar, and always within the standard uncertainty bands around the benchmark estimates.

We also find evidence of significant stochastic volatility in the trend inflation but particularly in the inflation gap equation. In the latter case, there was an important surge in the volatility of the shocks around in the aftermath of Lehman Brothers collapse, followed by a partial decline and stabilization until late 2013, when a second surge in volatility was observed. Inflation gap shock volatility has remained high since then. The volatility of trend inflation shocks has however remained relatively constant through the global financial crisis and the low inflation period. When longer maturity inflation compensation (5-year forward in ten years) is used there is some evidence of higher shock volatility than in our benchmark specification, but differences lie well within standard confidence bands.

8 Concluding remarks

This paper incorporates the use of market-based inflation expectations to the growing literature on trend inflation estimation. The growth of markets for inflation-linked products in many countries since 2004 has been remarkable, and it is not surprising that market-based expectations nowadays play a fundamental role in the monitoring of inflation expectations in many central banks and specialized media. We show that inflation compensation measures can play an important role in the analysis of trend inflation as well.

We provide strong evidence of a significant deterioration in long-term inflation expectations in the euro area since 2013. Our trend inflation estimates declined significantly below 2% to historically low levels around 1.3% by mid-2016. These findings, which are robust to different measures of long-term inflation compensation and the use of survey measures, provide support to the expansion of UMP measures by the ECB since early 2015. Moreover, they also provide a useful metric to assess the evolution of long-term inflation expectations and the likelihood of a sustained return of inflation towards levels below, but close to, 2% over the medium term.

Extending the estimation of trend inflation to market-based inflation expectations can in turn allow for gathering further international evidence on global inflation trends, not only in advance economies but also in many emerging markets with already well develop markets for inflation-linked products. It also provides estimates of inflation risk premia without the need to impose restrictions for the pricing of inflation across maturities that are common in the macro-finance literature on the term structure of interest rates. These two research avenues can help expand the toolkit for monitoring inflation expectations in many central banks.

Appendix: Estimation approach

Priors of the Model

We implement the same priors as Chan et al. (2018) for the Model given in equation (1) to (6). Firstly, we initialize the state equations (3), (4), (5) and (6) by

$$\pi_1^* \sim N(\pi_0^*, V_{\pi^*} e^{h_{n,1}}), \tag{8}$$

$$b_1 \sim N(b_0, V_b), \tag{9}$$

$$d_{i,1} \sim N(\mu_{d,i}, \tfrac{\sigma_{d,i}^2}{(1-\rho_{d,i}^2)}), \quad i = 0, 1, \tag{10}$$

$$h_{i,1} \sim N(h_{i,0}, V_{h_i}), \quad i = v, n, \tag{11}$$

where $\pi_0^* = b_0 = h_{i,0} = 0$ and $V_{\pi^*} = V_b = V_{h_i} = 100$. For all the model parameters, we implement independent priors for each of them. Thus,

$$\mu_{d,0} \sim (a_0, V_\mu), \tag{12}$$

$$\mu_{d,1} \sim (a_1, V_\mu), \tag{13}$$

$$\rho_{d,i} \sim TN_{(0,1)}(a_2, V_\rho), \quad i = 0, 1. \tag{14}$$

where $TN_{(0,1)}(\mu, \sigma)$ denotes the $N(\mu, \sigma)$ distribution truncated to the interval $(0, 1)$ and we set $a_0 = 0$, $a_1 = 1$, $a_2 = 0.95$ and $V_\mu = V_\rho = 0.1^2$. These choices of prior imply relatively informative priors centered at the values which imply trend inflation is equal to long-run forecast (apart from a mean zero error). The prior for MA(1) coefficient is

$$\psi \sim TN_{(-1,1)}(0, V_\psi), \tag{15}$$

where $V_\psi = 0.25^2$. Lastly, we assume independent inverse gamma priors for the all variance parameters where

$$\sigma^2_{d,0}, \sigma^2_w, \sigma^2_{h_v}, \sigma^2_{h_n} \sim IG(\nu_j, S_j), \quad j = \sigma^2_{d,0}, \sigma^2_w, \sigma^2_{h_v}, \sigma^2_{h_n}, \tag{16}$$

$$\sigma^2_{d,1}, \sigma^2_b, \sim IG(\nu_g, S_g), \quad g = \sigma^2_{d,1}, \sigma^2_b, \tag{17}$$

where $\nu_{\sigma^2_{d,0}} = \nu_{\sigma^2_{d,1}} = \nu_{\sigma^2_w} = \nu_{\sigma^2_{hv}} = \nu_{\sigma^2_{hn}} = \nu_{\sigma^2_b} = 5$, $S_{\sigma^2_{d,0}} = S_{\sigma^2_w} = S_{\sigma^2_{hv}} = S_{\sigma^2_{hn}} = 0.04$ and $S_{\sigma^2_{d,1}} = S_{\sigma^2_b} = 0.004$. Chan et al. (2017) notes that these prior choices are relatively non-informative and they also found that these priors are fairly robust in terms of a prior sensitive analysis.

Gibbs Sampler

To simulate the posterior distributions, we follow Chan et al. (2018) and implement a nine block Gibbs Sampler that sequentially draws from each conditional posterior distribution. First, let's denote $\theta = (\psi, \mu_{d,0}, \mu_{d,1}, \rho_{d,0}, \rho_{d,1}, \sigma^2_{d,0}, \sigma^2_{d,1}, \sigma^2_b, \sigma^2_z, \sigma^2_{h_v}, \sigma^2_{h_n})'$, $\pi = (\pi_1, \ldots, \pi_T)'$, $\boldsymbol{b} = (b_1, \ldots, b_T)'$, $\boldsymbol{d} = (d_{0,1}, d_{1,1}, \ldots, d_{0,T}, d_{1,T})'$ and $\boldsymbol{h}_i = (h_{i,1}, \ldots, h_{i,T})'$. The outline of the steps are:

1. Draw $p(\pi^*|Data, \boldsymbol{b}, \boldsymbol{d}, \boldsymbol{h}_v, \boldsymbol{h}_n, \theta)$,
2. Draw $p(\boldsymbol{b}|Data, \pi^*, \boldsymbol{d}, \boldsymbol{h}_v, \boldsymbol{h}_n, \theta)$,
3. Draw $p(\boldsymbol{d}|Data, \pi^*, \boldsymbol{b}, \boldsymbol{h}_v, \boldsymbol{h}_n, \theta)$,
4. Draw $p(\boldsymbol{h}_v, \boldsymbol{h}_n|Data, \pi^*, \boldsymbol{b}, \boldsymbol{d}, \theta)$,
5. Draw $p(\mu_{d,0}, \mu_{d,1}|Data, \pi^*, \boldsymbol{b}, \boldsymbol{d}, \boldsymbol{h}_v, \boldsymbol{h}_n, \theta_{-\{\mu_{d,0}, \mu_{d,1}\}})$,
6. Draw $p(\sigma^2_{d,0}, \sigma^2_{d,1}|Data, \pi^*, \boldsymbol{b}, \boldsymbol{d}, \boldsymbol{h}_v, \boldsymbol{h}_n, \theta_{-\{\sigma^2_{d,0}, \sigma^2_{d,1}\}})$,
7. Draw $p(\rho_{d,0}, \rho_{d,1}|Data, \pi^*, \boldsymbol{b}, \boldsymbol{d}, \boldsymbol{h}_v, \boldsymbol{h}_n, \theta_{-\{\rho_{d,0}, \rho_{d,1}\}})$,
8. Draw $p(\psi|Data, \pi^*, \boldsymbol{b}, \boldsymbol{d}, \boldsymbol{h}_v, \boldsymbol{h}_n, \theta_{-\{\psi\}})$,
9. Draw $p(\sigma^2_b, \sigma^2_w, \sigma^2_{h_v}, \sigma^2_{h_n}|Data, \pi^*, \boldsymbol{b}, \boldsymbol{d}, \boldsymbol{h}_v, \boldsymbol{h}_n, \theta_{-\{\sigma^2_b, \sigma^2_w, \sigma^2_{h_v}, \sigma^2_{h_n}\}})$,

Draw $p(\pi^*|Data, \boldsymbol{b}, \boldsymbol{d}, \boldsymbol{h}_v, \boldsymbol{h}_n, \theta)$

Firstly, we can rewrite the measurement equation of (1) into

$$\mathbf{H}_b\pi = \mathbf{H}_b\pi^* + \tilde{\alpha}_{\pi^*} + v, \quad v \sim N(0, \Lambda_v) \ , \tag{18}$$

where $\tilde{\alpha}_{\pi^*} = (b_1(\pi_0 - \pi_0^*), 0, \ldots, 0)'$, $\Lambda_v = diag(e^{h_{v,1}}, \ldots, e^{h_{v,T}})'$, $v = (v_1, \ldots, v_T)'$ and

$$\mathbf{H}_b = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -b_2 & 1 & 0 & \cdots & 0 \\ 0 & -b_3 & 1 & \ddots & 0 \\ \vdots & & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -b_T & 1 \end{bmatrix}. \tag{19}$$

Since $|\mathbf{H}_b| = 1$ for any b, $\mathbf{H}_b$ is invertible. Therefore, we have

$$(\pi|\pi^*, \boldsymbol{b}, \boldsymbol{h}_v) \sim N(\pi^* + \alpha_{\pi^*}, (\mathbf{H}_b^{'}\Lambda_v^{-1}\mathbf{H}_b)^{-1}), \tag{20}$$

where $\alpha_{\pi^*} = \mathbf{H}_b^{-1}\tilde{\alpha}_{\pi^*}$. Next, we can also rewrite equation (2) into

$$\boldsymbol{z} = d_0 + \mathbf{X}_{\pi^*}\pi^* + \mathbf{H}_\psi\epsilon_z, \quad \epsilon_z \sim N(0, \sigma_w^2\mathbf{I}_T), \tag{21}$$

where $d_0 = (d_{0,1}, \ldots, d_{0,T})'$, $\mathbf{X}_{\pi^*} = diag(d_{1,1}, \ldots, d_{1,T})$, $\epsilon_z = (\epsilon_{z,1}, \ldots, \epsilon_{z,T})'$, $\boldsymbol{z} = (z_1, \ldots, z_T)'$ and

$$\mathbf{H}_\psi = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ \psi & 1 & 0 & \cdots & 0 \\ 0 & \psi & 1 & \ddots & 0 \\ \vdots & & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \psi & 1 \end{bmatrix}. \tag{22}$$

Therefore, we have

$$(z|d_0, \pi^*, \psi, \sigma_w^2) \sim N(d_0 + \mathbf{X}_{\pi^*}\pi^*, \sigma_w^2\mathbf{H}_\psi\mathbf{H}_\psi^{'}). \tag{23}$$

Lastly, we can rewrite the state equation of (3)

$$\mathbf{H}\pi^* = \delta_{\pi^*} + \boldsymbol{n}_t, \quad \boldsymbol{n}_t \sim N(0, \Lambda_n), \tag{24}$$

where $\delta_{\pi^*} = (\pi_0^*, 0, \ldots, 0)'$, $\Lambda_n = diag(e^{h_{n,1}}V_{\pi^*}, e^{h_{n,2}}, \ldots, e^{h_{n,T}})'$ and

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & \ddots & 0 \\ \vdots & & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -1 & 1 \end{bmatrix}. \tag{25}$$

Therefore we have

$$(\pi^*|\boldsymbol{h}_n) \sim N(\delta_{\pi^*}, (\mathbf{H}'\Lambda_n^{-1}\mathbf{H})^{-1}). \tag{26}$$

To find the conditional posterior of $p(\pi^*|Data, \boldsymbol{b}, \boldsymbol{d}, \boldsymbol{h}_v, \boldsymbol{h}_n, \theta)$, we combine (19), (22) and (25) to obtain

$$\log p(\pi^*|Data, \boldsymbol{b}, \boldsymbol{d}, \boldsymbol{h}_v, \boldsymbol{h}_n, \theta) \propto -\frac{1}{2}(\pi - \pi^* - \alpha_{\pi^*})'(\mathbf{H}_b^{'}\Lambda_v^{-1}\mathbf{H}_b)\frac{1}{2}(\pi - \pi^* - \alpha_{\pi^*}),$$

$$-\frac{1}{2\sigma_w^2}(z - d_0 - \mathbf{X}_{\pi^*}\pi^*)'(\mathbf{H}_\psi\mathbf{H}_\psi^{'})^{-1}(z - d_0 - \mathbf{X}_{\pi^*}\pi^*) - \frac{1}{2}(\pi^* - \delta_{\pi^*})'(\mathbf{H}'\Lambda_n^{-1}\mathbf{H})(\pi^* - \delta_{\pi^*}), \tag{27}$$

$$\propto -\frac{1}{2}(\pi - \hat{\pi}^*)'\mathbf{K}_{\pi^*}(\pi - \hat{\pi}^*), \tag{28}$$

where the conditional posterior is

$$(\pi^*|Data, \boldsymbol{b}, \boldsymbol{d}, \boldsymbol{h}_v, \boldsymbol{h}_n, \theta) \sim (\hat{\pi}^*, \mathbf{K}_{\pi^*}^{-1}), \tag{29}$$

where

$$\mathbf{K}_{\pi^*} = (\mathbf{H}_b^{'}\Lambda_v^{-1}\mathbf{H}_b + \frac{1}{\sigma_w^2}\hat{\mathbf{X}}_{\pi^*}^{'}\hat{\mathbf{X}}_{\pi^*} + \mathbf{H}'\Lambda_n^{-1}\mathbf{H})^{-1}, \tag{30}$$

$$\hat{\pi}^* = \mathbf{K}_{\pi^*}^{-1}(\mathbf{H}_b' \Lambda_v^{-1} \mathbf{H}_b(\pi - \alpha_{\pi^*}) + \frac{1}{\sigma_w^2} \hat{\mathbf{X}}_{\pi^*}' \tilde{\boldsymbol{z}} + \mathbf{H}' \Lambda_n^{-1} \mathbf{H} \boldsymbol{\delta}_{\boldsymbol{\pi}^*}), \tag{31}$$

where $\tilde{\boldsymbol{z}} = \mathbf{H}_\psi^{-1}(z - d_0)$ and $\hat{\mathbf{X}}_{\pi^*} = \mathbf{H}_\psi^{-1}\mathbf{X}_{\pi^*}$.Notice that the precision matrix $\mathbf{K}_{\pi^*}^{-1}$ is a band matrix, which means we can apply the precision sampler technique of Chan and Jeliazkov (2009) to draw $\hat{\pi}^*$. As discussed in Chan et al. (2018) most of the elements of $\hat{\mathbf{X}}_{\pi^*}$that are away from the diagonal band are close to zero. Therefore, they construct a band approximation by replacing all elements below the absolute value of 10^{-6} with zero.

Draw $p(\boldsymbol{b}|Data, \pi^*, \boldsymbol{d}, \boldsymbol{h}_v, \boldsymbol{h}_n, \theta)$

To derive this conditional posterior, the inequality $0 < b_t < 1$ must be satisfied. As a result of this inequality, this conditional posterior is non-normal, which means a Metropolis-Hasting step has to be undertaken to simulate the posterior draws. First, we can rewrite the measurement equation of (1) as:

$$\tilde{\pi} = \mathbf{X}_b \boldsymbol{b} + v, \quad v \sim N(0, \Lambda_v), \tag{32}$$

where$\tilde{\pi} = (\pi_1 - \pi_1^*, \ldots, \pi_T - \pi_T^*)'$ and $\mathbf{X}_b = diag(\pi_0 - \pi_0^*, \ldots, \pi_{T-1} - \pi_{T-1}^*)'$. Next, we can rewrite the state equation of b_t (4) into

$$\mathbf{H}\boldsymbol{b} = \tilde{\delta}_b + \epsilon_b, \quad \epsilon_b \sim N(0, \sigma_b^2 \mathbf{I}_T), \tag{33}$$

where $\tilde{\delta}_b = (b_0, 0, \ldots, 0)'$and the elements of $\epsilon_b = (\epsilon_{b,1}, \ldots, \epsilon_{b,T})'$ are independent truncated normal variables. Note that $\Pr(0 < b_1 < 1) = \Phi(\frac{1-b_0}{\sqrt{V_b}}) - \Phi(\frac{b_0}{\sqrt{V_b}})$ and

$$\Pr(0 < b_t < 1) = \Phi(\frac{1 - b_{t-1}}{\sigma_b}) - \Phi(\frac{-b_{t-1}}{\sigma_b}), \tag{34}$$

where $\Phi(.)$ is the cumulative distribution function of the standard normal distribution. Thus, the prior density for b is

$$\log p(\boldsymbol{b}|\sigma_b^2) \propto -\frac{1}{2}(\boldsymbol{b} - \delta_b)' \mathbf{H}' \Sigma_b^{-1} \mathbf{H}(\boldsymbol{b} - \delta_b) + g(\boldsymbol{b}, \sigma_b^2), \tag{35}$$

where $\Sigma_b = diag(V_b, \sigma_b^2, \ldots, \sigma_b^2)$, $\delta_b = \mathbf{H}^{-1}\tilde{\delta}_b$ and

$$g(\boldsymbol{b}, \sigma_b^2) = -\sum_{t=2}^{T} \log(\Phi(\frac{1 - b_{t-1}}{\sigma_b}) - \Phi(\frac{-b_{t-1}}{\sigma_b})). \tag{36}$$

To get the conditional posterior, we combine (32) and (35) to obtain

$$\log p(\boldsymbol{b}|Data, \pi^*, \boldsymbol{d}, \boldsymbol{h}_v, \boldsymbol{h}_n, \theta) \propto -\frac{1}{2}(\boldsymbol{b} - \hat{\boldsymbol{b}})'\mathbf{K}_b^{-1}(\boldsymbol{b} - \hat{\boldsymbol{b}}) + g(\boldsymbol{b}, \sigma_b^2), \tag{37}$$

Thus,

$$(\boldsymbol{b}|Data, \pi^*, \boldsymbol{d}, \boldsymbol{h}_v, \boldsymbol{h}_n, \theta) \sim N(\hat{\boldsymbol{b}}, \mathbf{K}_b^{-1}) \tag{38}$$

where

$$\mathbf{K}_b = (\mathbf{H}^{'}\Sigma_b^{-1}\mathbf{H} + \mathbf{X}_b^{'}\Lambda_v^{-1}\mathbf{X}_b), \tag{39}$$

$$\hat{\boldsymbol{b}} = \mathbf{K}_b^{-1}(\mathbf{H}^{'}\Sigma_b^{-1}\mathbf{H}\delta_b + \mathbf{X}_b^{'}\Lambda_v^{-1}\tilde{\pi}). \tag{40}$$

As mentioned above, a Metropolis-Hasting step is taken to draw $\boldsymbol{b}$. First, candidate draws are obtain from distribution of equation (37) and then they are accepted or reject via the Metropolis-Hasting step.

Draw $p(\boldsymbol{d}|Data, \pi^*, \boldsymbol{b}, \boldsymbol{h}_v, \boldsymbol{h}_n, \theta)$,

To sample from this conditional posterior, we first need to rewrite (2) and (5) into

$$z = \mathbf{X}_d\boldsymbol{d} + \boldsymbol{H}_\psi\epsilon_z, \quad \epsilon_z \sim N(0, \sigma_w^2\mathbf{I}_T), \tag{41}$$

$$\mathbf{H}_\rho\boldsymbol{d} = \tilde{\delta}_d + \epsilon_d, \quad \epsilon_d \sim N(0, \Sigma_d), \tag{42}$$

where $\tilde{\delta}_d = (\mu_{d,0}, \mu_{d,1}, (1 - \rho_{d,0})\mu_{d,0}, (1 - \rho_{d,1})\mu_{d,1}, \ldots, (1 - \rho_{d,0})\mu_{d,0}, (1 - \rho_{d,1})\mu_{d,1})'$, $\Sigma_d = diag(\frac{\sigma_{d,0}^2}{(1-\rho_{d,0}^2)}, \frac{\sigma_{d,1}^2}{(1-\rho_{d,1}^2)}, \sigma_{d,0}^2, \sigma_{d,1}^2, \ldots, \sigma_{d,0}^2, \sigma_{d,1}^2)'$,

$$\mathbf{X}_d = \begin{bmatrix} 1 & \pi_1^* & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \pi_2^* & 0 & \cdots & 0 \\ \vdots & & & \ddots & \ddots & & \vdots \\ 0 & 0 & 0 & 0 & 0 & 1 & \pi_T^* \end{bmatrix}, \tag{43}$$

and

$$\mathbf{H}_\rho = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & 0 & \cdots & 0 \\ -\rho_{d,0} & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & -\rho_{d,1} & 0 & 1 & \ddots & & 0 \\ 0 & 0 & \ddots & 0 & \ddots & \ddots & \\ \vdots & & \ddots & -\rho_{d,0} & \ddots & \ddots & 0 \\ 0 & 0 & 0 & 0 & -\rho_{d,1} & 0 & 1 \end{bmatrix}. \tag{44}$$

Combining (40) and (41), we can derive the conditional posterior

$$\log p(\boldsymbol{d}|Data, \pi^*, \boldsymbol{b}, \boldsymbol{h}_v, \boldsymbol{h}_n, \theta) \propto -\frac{1}{2\sigma_w^2}(z-\mathbf{X}_d\boldsymbol{d})'(\mathbf{H}_\psi\mathbf{H}_\psi^{'})^{-1}(z-\mathbf{X}_d\boldsymbol{d}) - \frac{1}{2}(\boldsymbol{d}-\delta_d)'\mathbf{H}_\rho^{'}\Sigma_d^{-1}\mathbf{H}_\rho(\boldsymbol{d}-\delta_d), \tag{45}$$

where $\delta_d = \mathbf{H}_\rho^{-1}\tilde{\delta}_d$. Thus from (44), the conditional posterior is

$$(\boldsymbol{d}|Data, \pi^*, \boldsymbol{b}, \boldsymbol{h}_v, \boldsymbol{h}_n, \theta) \sim N(\hat{\boldsymbol{d}}, \mathbf{K}_d^{-1}), \tag{46}$$

where

$$\mathbf{K}_d = (\mathbf{H}_\rho^{'}\Sigma_d^{-1}\mathbf{H}_\rho + \frac{1}{\sigma_w^2}\tilde{\mathbf{X}}_d^{'}\tilde{\mathbf{X}}_d), \tag{47}$$

$$\hat{\boldsymbol{d}} = \mathbf{K}_d^{-1}(\mathbf{H}_\rho^{'}\Sigma_d^{-1}\tilde{\delta}_d + \frac{1}{\sigma_w^2}\tilde{\mathbf{X}}_d^{'}\mathbf{H}_\psi^{-1}z), \tag{48}$$

where $\tilde{\mathbf{X}}_d = \mathbf{H}_\psi^{-1}\mathbf{X}_d$. Again, we construct a band approximation of $\tilde{\mathbf{X}}_d$ by replacing all elements less than 10^{-6} with zero. Similar to step 1, the precision sampler approach of Chan and Jeliazkov (2009) is used to sample $\hat{\boldsymbol{d}}$.

Draw $p(\boldsymbol{h}_v, \boldsymbol{h}_n|Data, \pi^*, \boldsymbol{b}, \boldsymbol{d}, \theta)$

To draw the stochastic volatilizes of $\boldsymbol{h}_v, \boldsymbol{h}_n$, we implement the precision sampler technique by Chan and Hsiao (2014) and follow their procedure whereby they implement the Kim, Shepherd and Chib (1998) auxiliary mixture sampler in approximating the $\log-\chi^2_1$ distribution using a seven component Gaussian mixture density with fixed parameters. For more information, please see Chan and Hsiao (2014).

Draw $p(\mu_{d,0}, \mu_{d,1}|Data, \pi^*, \boldsymbol{b}, \boldsymbol{d}, \boldsymbol{h}_v, \boldsymbol{h}_n, \theta_{-\{\mu_{d,0},\mu_{d,1}\}})$ and $p(\sigma^2_{d,0}, \sigma^2_{d,1}|Data, \pi^*, \boldsymbol{b}, \boldsymbol{d}, \boldsymbol{h}_v, \boldsymbol{h}_n, \theta_{-\{\sigma^2_{d,0},\sigma^2_{d,1}\}})$

Both these conditional posteriors are standard:

$$(\mu_{d,i}|Data, \pi^*, \boldsymbol{b}, \boldsymbol{d}, \boldsymbol{h}_v, \boldsymbol{h}_n, \theta_{-\{\mu_{d,0},\mu_{d,1}\}}) \sim N(\hat{\mu}_{d,i}, \mathbf{K}^{-1}_{d,i}), \tag{49}$$

$$(\sigma^2_{d,i}|Data, \pi^*, \boldsymbol{b}, \boldsymbol{d}, \boldsymbol{h}_v, \boldsymbol{h}_n, \theta_{-\{\sigma^2_{d,0},\sigma^2_{d,1}\}}) \sim IG(\nu_{d,i} + \frac{T}{2}, \tilde{S}_{d,i}), \tag{50}$$

where $\mathbf{K}_{d,i} = \frac{1}{V_\mu} + \frac{(1-\rho^2_{d,i})}{\sigma^2_{d,i}} + (T-1)\frac{(1-\rho_{d,i})^2}{\sigma^2_{d,i}}$, $\hat{\mu}_{d,i} = \mathbf{K}^{-1}_{d,i}(\frac{a_i}{V_\mu} + \frac{(1-\rho^2_{d,i})d_{i,1}}{\sigma^2_{d,i}} + \sum_{t=2}^{T} \frac{(1-\rho_{d,i})(d_{i,t}-\rho_{d,i}d_{i,t-1})}{\sigma^2_{d,i}})$ and $\tilde{S}_{d,i} = S_{d,i} + \frac{((1-\rho^2_{d,i})(d_{i,1}-\mu_{d,i})^2 + \sum_{t=2}^{T}(d_{i,t}-\mu_{d,i}(1-\rho_{d,i})-\rho_{d,i}d_{i,t-1})^2}{2}$.

Draw $p(\rho_{d,0}, \rho_{d,1}|Data, \pi^*, \boldsymbol{b}, \boldsymbol{d}, \boldsymbol{h}_v, \boldsymbol{h}_n, \theta_{-\{\rho_{d,0},\rho_{d,1}\}})$

$$p(\rho_{d,i}|Data, \pi^*, \boldsymbol{b}, \boldsymbol{d}, \boldsymbol{h}_v, \boldsymbol{h}_n, \theta_{-\{\rho_{d,0},\rho_{d,1}\}}) \propto p(\rho_{d,i}) g_{\rho_{d,i}}(\rho_{d,i}) e^{-\frac{1}{2}\sum_{t=2}^{T}(d_{i,t}-\mu_{d,i}-\rho_{d,i}(d_{i,t-1}-\mu_{d,i}))^2}, \tag{51}$$

where $p(\rho_{d,i})$ is the truncated normal prior for $\rho_{d,i}$ and $g_{\rho_{d,i}}(\rho_{d,i}) = (1-\rho^2_{d,i})^{\frac{1}{2}} \exp(-\frac{1}{2\sigma^2_{d,i}}(1-\rho^2_{d,i})(d_{i,1}-\mu_{d,i})^2)$. This conditional density is non-standard, which means a Metropolis-Hasting step must be undertaken to draw $\rho_{d,i}$. We follow Chan et al. (2017) where they implement an independence chain Metropolis-Hasting step with a proposal distribution $N(\hat{\rho}_{d,i}, K^{-1}_{\rho_{d,i}})$, where $K_{\rho_{d,i}} = (\frac{1}{V_\rho} + \frac{X'_{\rho_{d,i}} X_{\rho_{d,i}}}{\sigma^2_{d,i}})$ and $\hat{\rho}_{d,i} = K^{-1}_{\rho_{d,i}}(\frac{a_2}{V_\rho} + \frac{X'_{\rho_{d,i}} y_{\rho_{d,i}}}{\sigma^2_{d,i}})$, with $X_{\rho_{d,i}} = (d_{i,1}-\mu_{d,i}, \ldots, d_{i,t-1}-\mu_{d,i})'$ and $y_{\rho_{d,i}} = (d_{i,2}-\mu_{d,i}, \ldots, d_{i,T}-\mu_{d,i})'$.

Draw $p(\psi|Data, \pi^*, \boldsymbol{b}, \boldsymbol{d}, \boldsymbol{h}_v, \boldsymbol{h}_n, \theta_{-\{\psi\}})$

To draw ψ, we follow Chan (2013) by implementing an independence chain Metropolis-Hasting step. Specifically, we evaluate the log-density below using band matrix routines, where we maximize it numerically to obtain the mode and negative Hessian, denoted as $\hat{\psi}$ and K_ψ. Then, we generate candidate draws from the $N(\hat{\psi}, K_{\psi}^{-1})$ distribution.

$$\log p(\psi|Data, \pi^*, \boldsymbol{b}, \boldsymbol{d}, \boldsymbol{h}_v, \boldsymbol{h}_n, \theta_{-\{\psi\}}) \propto \log p(z|\pi^*, \boldsymbol{d}, \sigma_w^2) + \log p(\psi), \tag{52}$$

$$\propto -\frac{1}{2\sigma_w^2}(z - d_0 - \mathbf{X}_{\pi^*}\pi^*)'(\mathbf{H}_{\psi}\mathbf{H}_{\psi}^{'})^{-1}(z - d_0 - \mathbf{X}_{\pi^*}\pi^*) + \log p(\psi), \tag{53}$$

where $\log p(\psi)$ is the prior density of ψ.

Draw $p(\sigma_b^2, \sigma_w^2, \sigma_{h_v}^2, \sigma_{h_n}^2|Data, \pi^*, \boldsymbol{b}, \boldsymbol{d}, \boldsymbol{h}_v, \boldsymbol{h}_n, \theta_{-\{\sigma_b^2, \sigma_z^2, \sigma_{h_v}^2, \sigma_{h_n}^2\}})$

All these variance parameters are conditionally independent given the data and states. $\sigma_z^2, \sigma_{h_v}^2, \sigma_{h_n}^2$ all follow standard inverse-Gamma distributions

$$(\sigma_w^2|Data, \pi^*, \boldsymbol{b}, \boldsymbol{d}, \boldsymbol{h}_v, \boldsymbol{h}_n, \theta_{-\{\sigma_b^2, \sigma_z^2, \sigma_{h_v}^2, \sigma_{h_n}^2\}}) \sim IG(\nu_{\sigma_w^2} + \frac{T}{2}, S_{\sigma_w^2} + \frac{1}{2}\sum_{t=1}^{T}\tilde{\epsilon}_{z,t}^2), \tag{54}$$

$$(\ \sigma_{h_i}^2|Data, \pi^*, \boldsymbol{b}, \boldsymbol{d}, \boldsymbol{h}_v, \boldsymbol{h}_n, \theta_{-\{\sigma_b^2, \sigma_z^2, \sigma_{h_v}^2, \sigma_{h_n}^2\}}) \sim IG(\nu_{\sigma_{h_i}^2} + \tfrac{T-1}{2}, S_{\sigma_{h_i}^2} + \tfrac{1}{2}\textstyle\sum_{t=2}^{T}(h_{i,t} - h_{i,t-1})^2), \quad i = v, n, \tag{55}$$

where the elements of $\tilde{\epsilon}_z$ can be computed as $\tilde{\epsilon}_z = \mathbf{H}_{\psi}^{-1}(z - \mathbf{X}_d\boldsymbol{d})$. To draw σ_b^2, an Metropolis-Hasting step has to be undertaken since the conditional density is non-standard given

$$\log(\sigma_b^2|Data, \pi^*, \boldsymbol{b}, \boldsymbol{d}, \boldsymbol{h}_v, \boldsymbol{h}_n, \theta_{-\{\sigma_b^2, \sigma_z^2, \sigma_{h_v}^2, \sigma_{h_n}^2\}}) \propto -(\nu_{\sigma_b^2} + 1)\log - \frac{S_{\sigma_b^2}}{\sigma_b^2} - \frac{T-1}{2}\log\sigma_b^2 \ldots$$

$$\ldots - \frac{1}{2\sigma_b^2}\sum_{t=2}^{T}(b_t - b_{t-1})^2 + g_b(b, \sigma_b^2). \tag{56}$$

To implement the Metropolis-Hasting step, we first draw from a proposal density

$$IG(\nu_{\sigma_b^2} + \frac{T-1}{2}, S_{\sigma_b^2} + \frac{1}{2}\sum_{t=2}^{T}(b_t - b_{t-1})^2). \tag{57}$$

Prior sensitivity analysis

As mentioned above, majority of the priors specified in the model are relatively non-informative. However, we undertake a prior sensitivity analysis for the hyperparameter V_μ, which is a key parameter in the state equation for $d_{i,t}$. Moreover, since a multiplicative term involving two latent series $d_{1,t}\pi_t^*$ appears in the equation $ILS_t = d_{0,t} + d_{1,t}\pi_t^* + \epsilon_{z,t} + \psi\epsilon_{z,t-1}$, this Appendix will also ease concerns about their independent identification by reporting some results for different choices of V_μ. In the paper, we set $V_\mu = 0.1^2$, which implies an informative, but not too tight, prior attaching appreciable weight to intervals of ± 0.2 around the theoretically-justified prior mean values. Here we compare our benchmark results with those from a more informative choice $V_\mu = 0.025^2$, and those from the case of a very non-informative choice $V_\mu = 1$. Intuitively, for the more informative choice $V_\mu = 0.025^2$, we are implying a tighter prior, and suggesting that the prior mean of $d_{0,t}$ is closer to 0, and that of $d_{1,t}$ is very close to 1. With this in mind, this tighter prior suggests that the estimated trend inflation tracks closely IL swaps, or equivalently that the difference between the trend inflation and the IL swap rates—that is the premia in our interpretation— would be much smaller. As regards to the very non-informative prior $V_\mu = 1$, this choice would imply a loose prior, through which a larger weight is placed on the data when determining the trend inflation estimates, which, in turn, would tend to have the opposite effect compared to the tighter prior.

[Figure A1 around here]

As expected, Figure A1 shows that a tighter prior would move trend inflation estimates closer to the observed level of the IL swap rates. Indeed, while remaining within the uncertainty bands of our benchmark specification, prior to the financial crisis the level of trend inflation looks relatively high, for there is widespread agreement that the euro area long-term inflation expectations were well anchored then. Importantly, however, the estimates, although declining from higher levels also display a significant decline since 2013, thereby corroborating our main findings. A non-informative (looser) prior instead leads to lower trend inflation estimates over the whole sample. While those lower trend inflation levels remain below but relatively close to the 2% level prior to the financial crisis, they collapse sharply to implausibly low levels from 2013.

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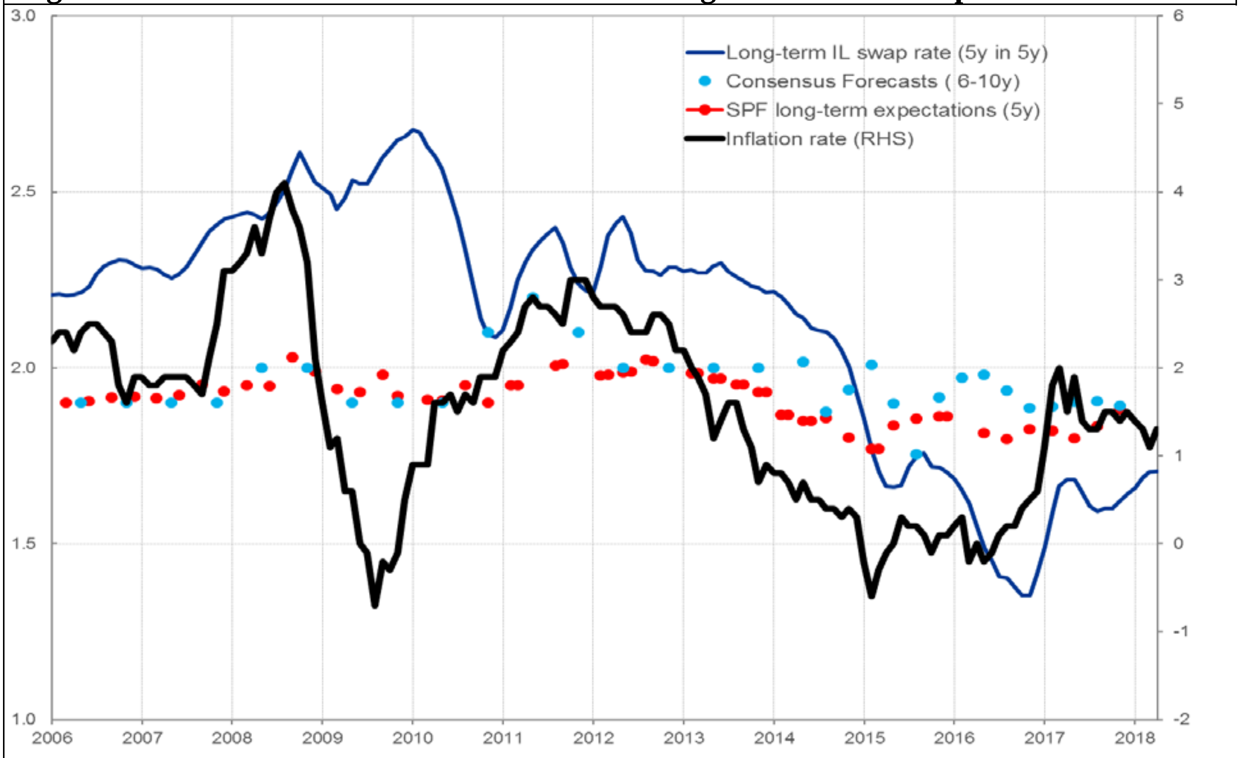
Tables and Figures

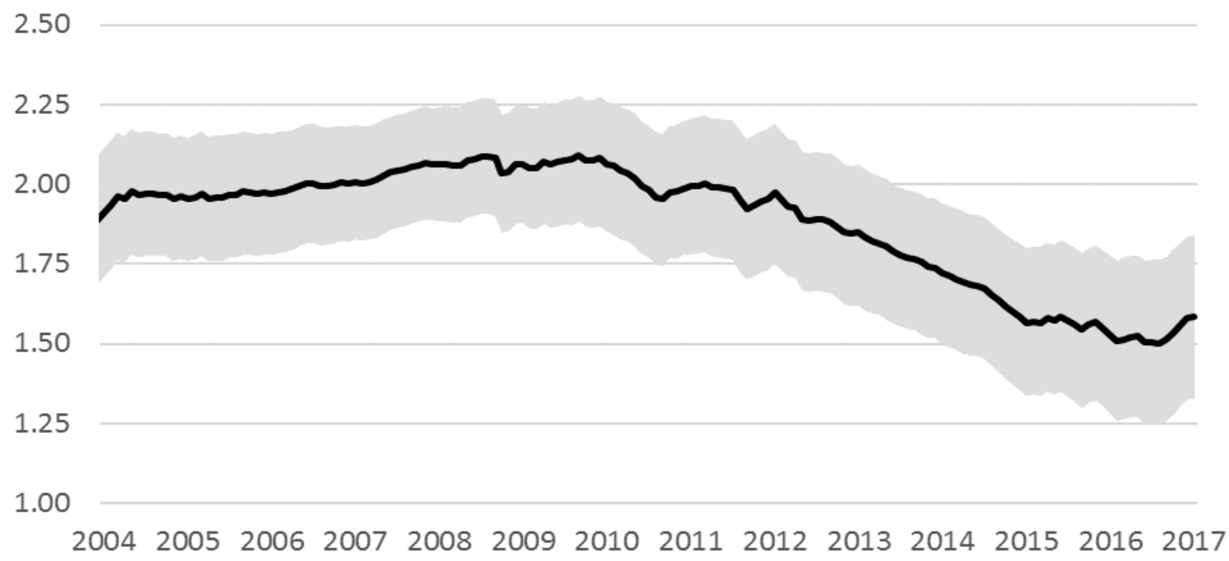
Figure 1. Euro area inflation and indicators of long-term inflation expectations



Note: The Figure depicts realized inflation (year-on -year rate of growth of HICP) and some indicators of inflation long-term inflation expectation in the euro area. First, our benchmark measure of long-term inflation compensation, the five-year forward inflation-linked swap rate five years ahead (blue line, calculation details can be found in Section 2 of the paper). The chart also includes two measures of survey expectations: Consensus Economics forecasts 6-to-10 years ahead (blue dots) and the 5-years ahead expectations from ECB's Survey of Professional Forecasters (red dots. For background information on the ECB's SPF, see Garcia, 2003). The chart illustrates two important features of euro area inflation expectations over the last decade First, long-term forward inflation compensation tended to be significantly above survey measures of long-term inflation expectations. with the discrepancy widely attributed to the presence of the inflation risk premia in inflation compensation. In recent years, however, long-term forward inflation compensation declined significantly as actual inflation rates drifted downwards since 2013. Survey measures of inflation expectations have in contrast remained relatively more stable in the euro area, but a significant decline away from the 2% target level of the ECB can also be appreciated since mid-2013.

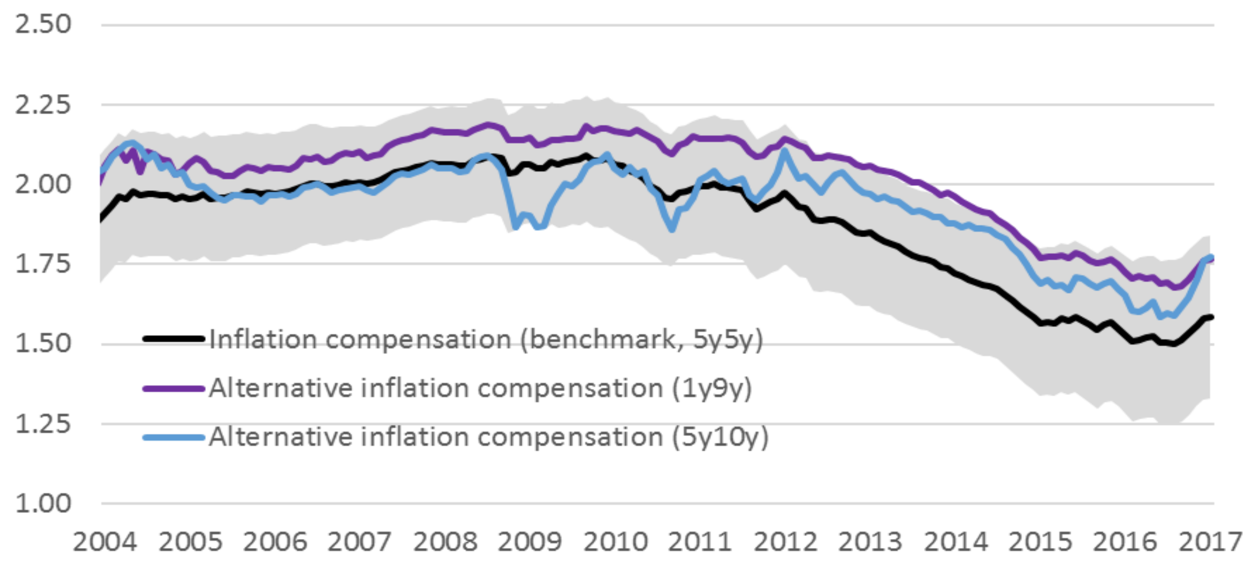
Sources: Eurostat, ICAP, Consensus Economics, ECB's SPF, and author's calculations.

Figure 2: Euro area trend inflation estimates



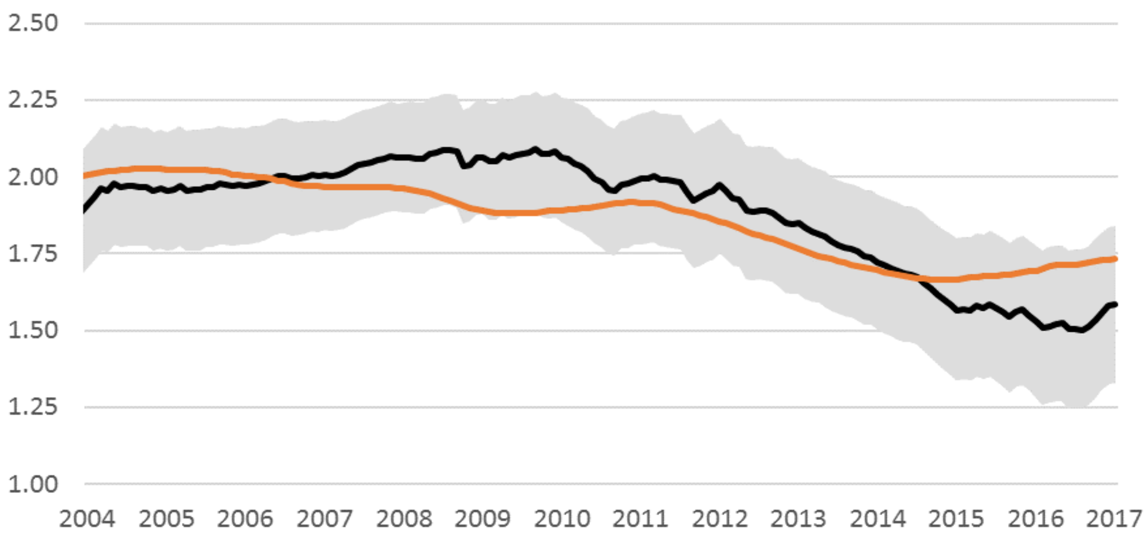
Note: Trend inflation estimates are based on from our benchmark model specification (see Sections 3 and 4). Shadow area denotes 16th and 84th percentiles around our benchmark model estimates. The chart illustrates the weaker anchoring of euro area inflation expectations since 2013. While trend inflation had been around 2% between 2004-2012, including following the collapse of Lehman Brothers and the European debt crisis in 2010, there has been a significant deterioration in the anchoring of long-term inflation expectations in the euro area from mid-2012. This downward re-pricing of long-term trend inflation led to a significant decline in benchmark long-term inflation compensation measures (five-year forward IL swap rate in five years), which in second-half of August 2014 starting being priced in below 2% for the first time in the history of the euro area, and reached a historical minimum below 1.3% in the summer of 2016 (see Figure 1). The launching of ECB's QE measures in early 2015 just managed to attenuate the decline in long-term inflation compensation measures and trend inflation, but both remained significantly below their historical averages since then.

Figure 3: Trend inflation estimates using different measures of long-term inflation compensation



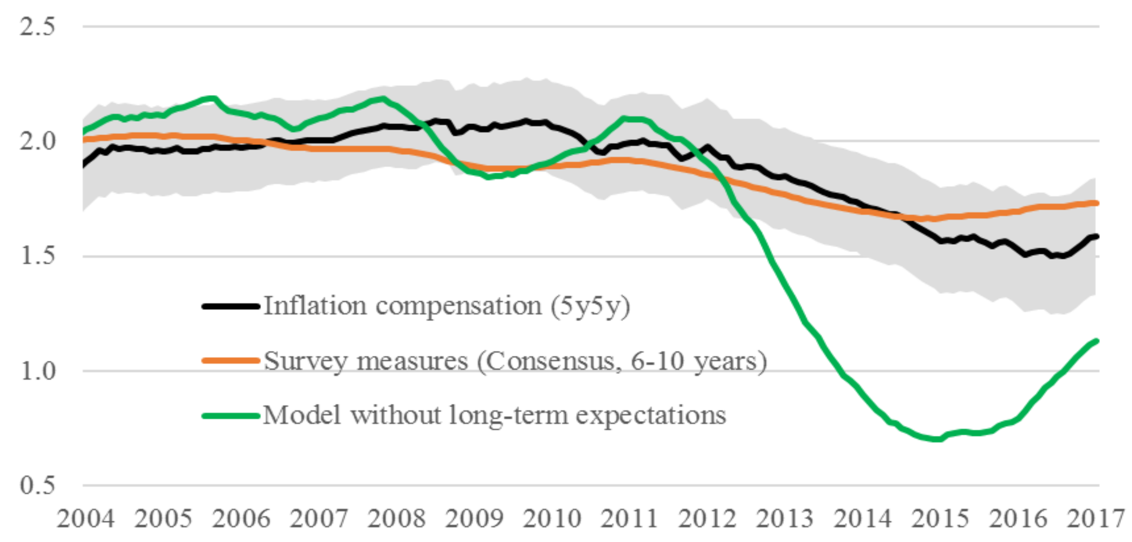
Note: Trend inflation estimates based on five-year forward IL swap rates in five years (black line, benchmark model specification, see Section 3), the one-year forward in nine years (grey line) and the five-year forward in ten years (blue line). Shadow area represents the 16th and 84th percentiles around our benchmark model estimates. Trend inflation estimates are very consistent across different measures of long-term inflation compensation.

Figure 4. Trend inflation estimates using long-term inflation compensation and survey inflation expectations (five-year forward IL swap rates in five years and Consensus Economics forecasts 6-to-10 years ahead)



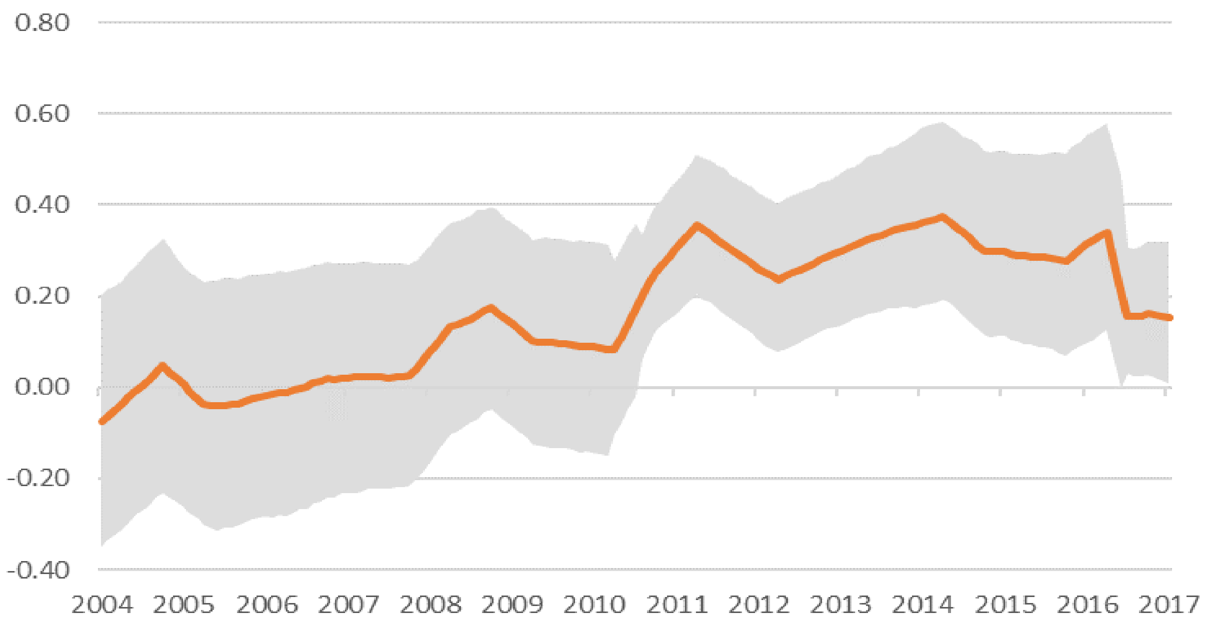
Note: Trend inflation estimates based on five-year forward IL swap rates in five years (black line, benchmark model specification, see Section 3), and survey data (blue line, Consensus Economics forecasts 6-to-10 years ahead). Shadow area represents the 16th and 84th percentiles around our benchmark model estimates. Trend inflation estimates are very consistent across different measures of long-term inflation expectations.

Figure 5. Trend inflation estimates using model specifications with and without long-term information (inflation compensation and survey inflation expectations (five-year forward IL swap rates in five years and Consensus Economics forecasts 6-to-10 years ahead)



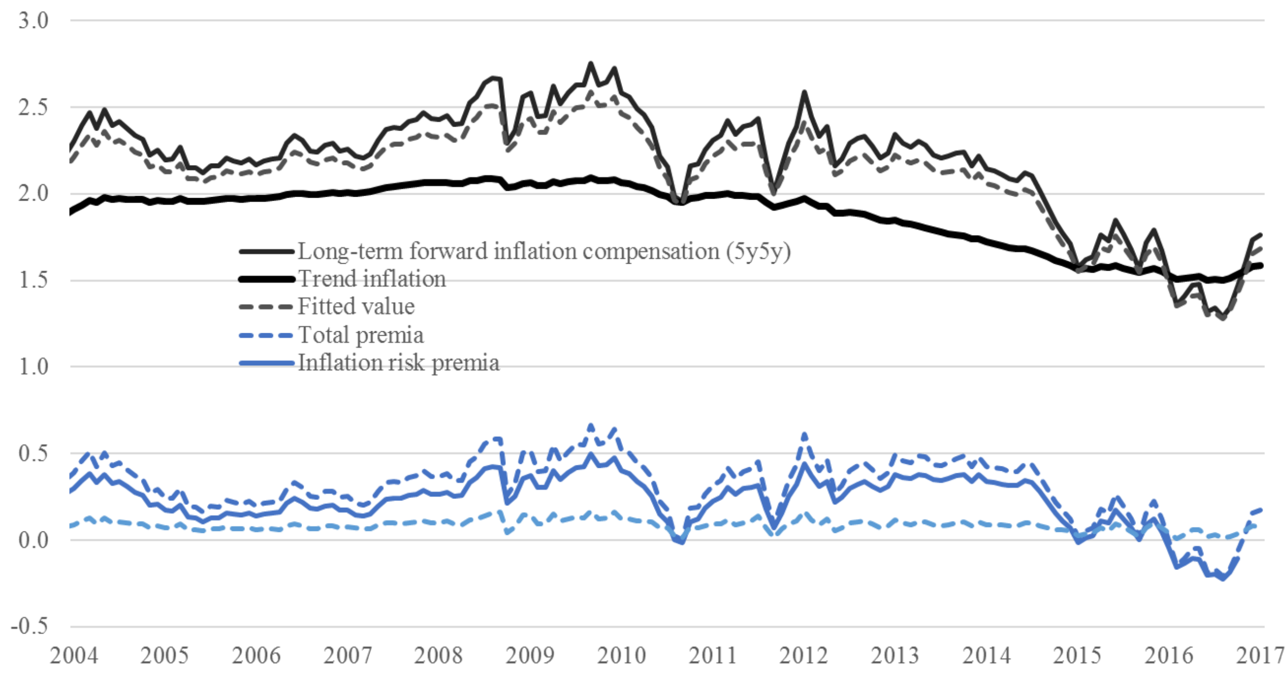
Note: Trend inflation estimates using long-term inflation expectations: five-year forward inflation compensation in five years (black line, benchmark model specification, see Section 3), survey data (orange line, Consensus Economics forecasts 6-to-10 years ahead); and estimates without long-term expectations (green line). In the latter case the model specification is just consistent with Chan et al. (2013) without imposing bounds in the estimation. Shadow area represents the 16th and 84th percentiles around our benchmark model estimates.

Figure 6: Bias in the estimation of trend inflation using survey inflation expectations (Consensus Economics forecasts 6-to-10 years ahead)



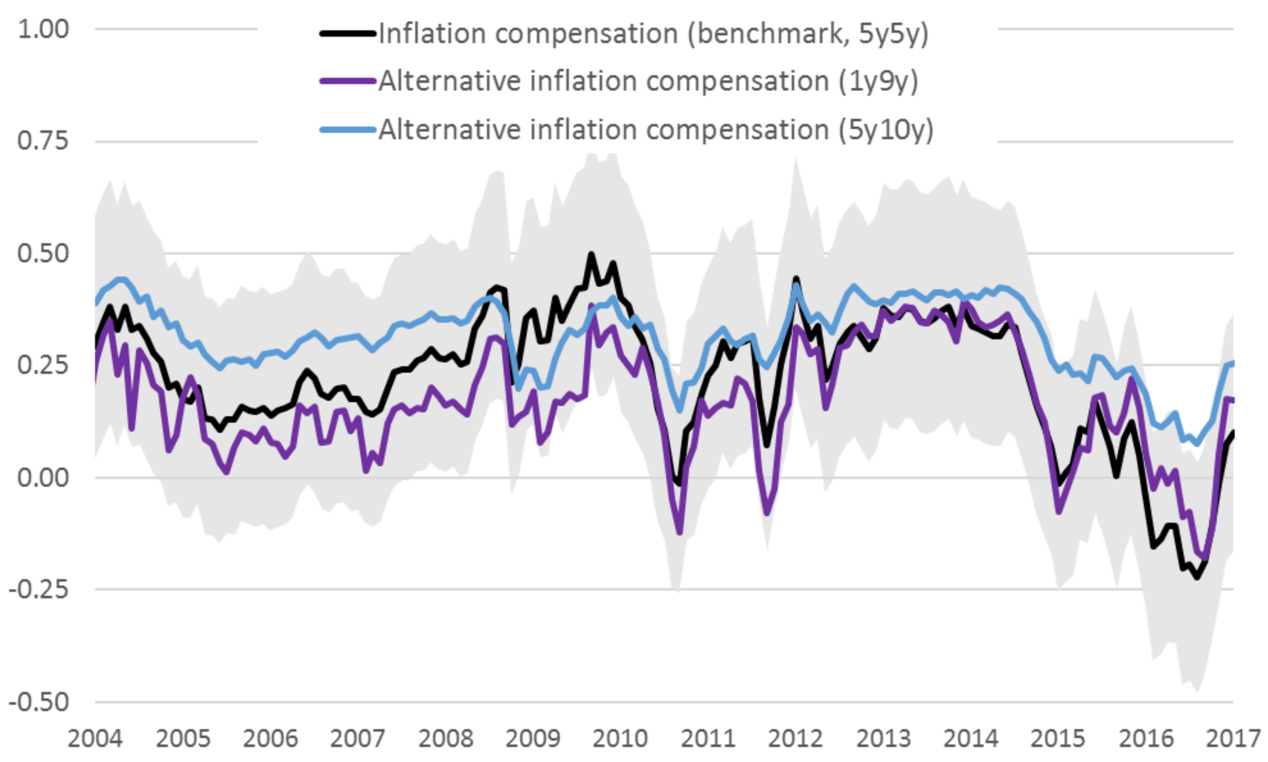
Note: The Figure reports the presence of a quantitatively important bias in the trend estimates using survey data (see equation (2) in Section 3 of the paper). Shadow area represents the 16th and 84th percentiles around the bias estimates.

Figure 7: Decomposition of long-term inflation compensation based on our trend inflation model (benchmark model specification using five-year forward IL swap rates in five years)



Note: A by-product of our benchmark trend inflation estimation is an implicit decomposition of long-term inflation compensation into the level of inflation expectations, given by trend inflation as the optimal long-term conditional forecast, and the additional premia requested by investors. Without loss of generality we can assume that our model specification (see equation (2) in Section 3) provides direct estimates of the inflation risk premia (d0t), while the MA(1) error terms can be attributed to liquidity premia or other temporary market disturbances. The Figure reports the decomposition of our benchmark measure of long-term forward inflation compensation (the five-year forward IL swap in five years).

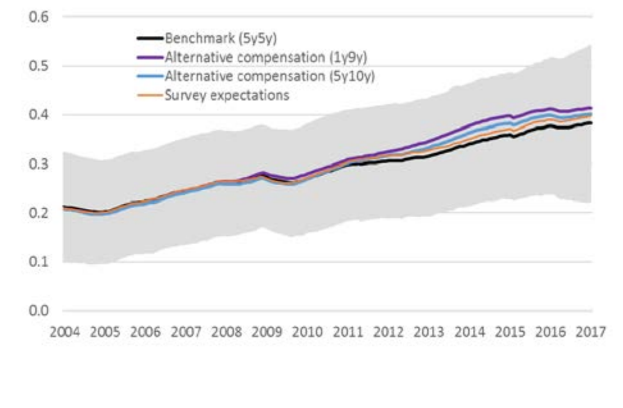
Figure 8: Estimates of inflation risk premium using different measures of long-term inflation compensation



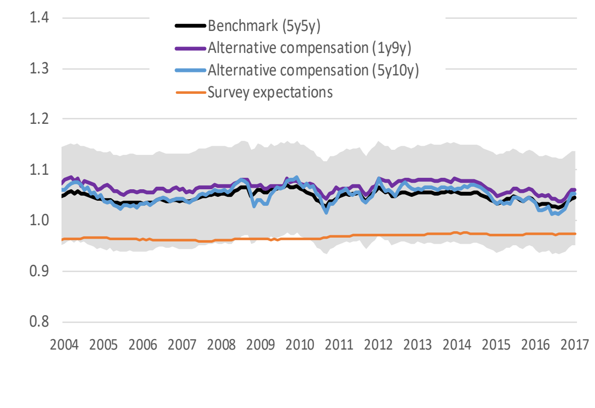
Note: Inflation risk premium estimates based on five-year forward IL swap rates in five years (black line, benchmark model specification, see Section 3), the one-year forward in nine years (grey line) and the five-year forward in ten years (blue line). Shadow area represents the 16th and 84th percentiles around our benchmark model estimates.

Figure 9: Additional model results using different measures of long-term inflation compensation and survey inflation expectations

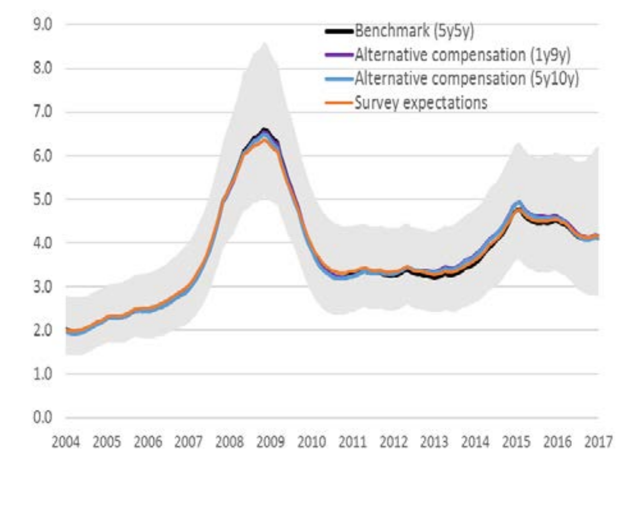
Panel (a): bt



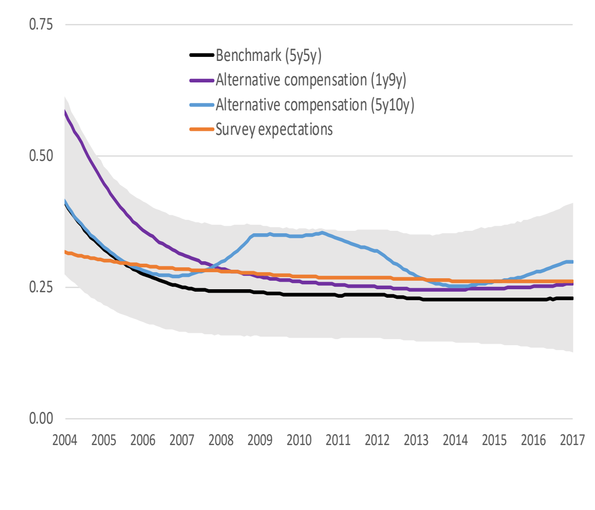
Panel (b): d1t



Panel (c): Volatility of shocks to inflation gap

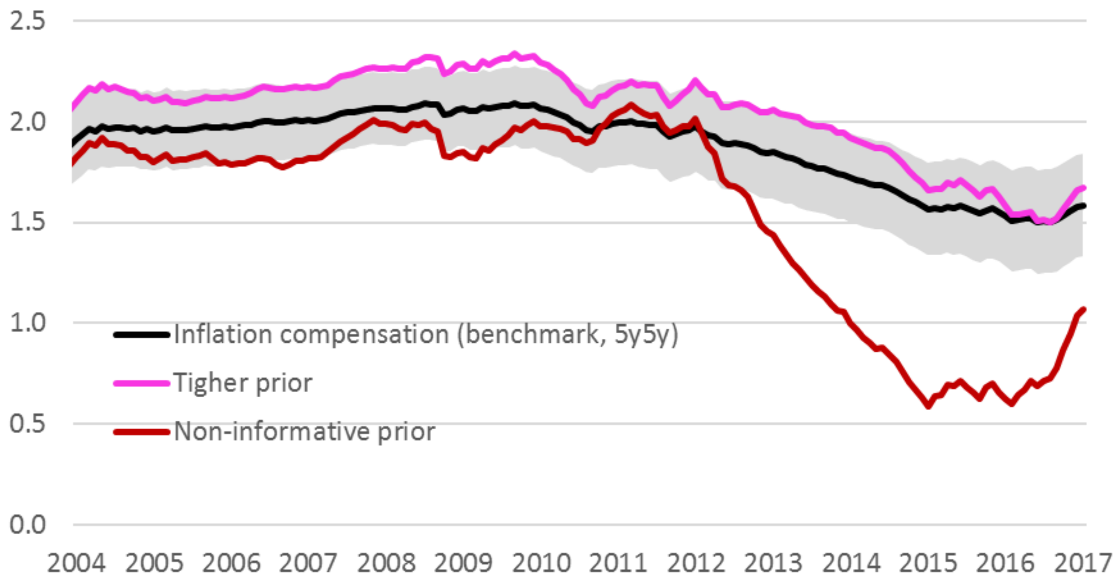


Panel (d): Volatility of shocks to trend inflation



Note: Posterior means from model specifications using different measures of long-term inflation compensation and survey inflation expectations. Shadow area represents the 16th and 84th percentiles for our benchmark specification based on the 5-year forward inflation compensation in five years.

Figure A1. Trend inflation estimates using different priors
(benchmark specification, a tighter prior and a non-informative prior)



Note: Trend inflation estimates using three alternative priors for the Vμ hyperparameter using five-year forward in five years inflation compensation in the estimation (see Appendix for details). Specifically, results for three choices are shown (i) our benchmark choice $V\mu=0.1^2$ (black line); (ii) a tighter informative $V\mu=0.025^2$ (pink line) and a non-informative prior $V\mu=1^2$ (brown line). Shadow area represents the 16th and 84th percentiles around our benchmark model estimates.