The (un)demand for money in Canada

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Abstract

A novel dataset is used to estimate the deposit functions for banknotes in Canada for three denominations, \$1,000, \$100 and \$50. The broad conclusion is that denominations are different monies. There are large and significant deposit costs for the \$1,000 banknote, but insignificant costs for the \$100 and \$50 denominations. The interest rate elasticity of deposit is positive for the \$1,000 but negative for the \$100 and \$50. 5% of the \$1,000, 30% of the \$100 and 22% of the \$50 banknotes ever issued by the Bank of Canada do not circulate through financial institutions (in Canada).

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1 Introduction

One difficulty with the empirical analysis of models of money demand is that the researcher typically requires individual- or household-level survey data. Survey responses are generally received from a subset of individuals or households invited to participate in the survey, and those responses are, in most cases, self-reported. Since one salient feature of money is anonymity, it is unclear whether individuals who respond to surveys are representative of typical money users and/or whether the responses they give are accurate or truthful. Using aggregate money data is no less problematic since one cannot easily identify money demand from flow data on withdrawals and deposits and stock data on notes in circulation, which is the data commonly reported by central banks. In this paper, we use unique deposit data from the Bank of Canada to study the complement of money demand: given outstanding monetary balances, what factors lead to their deposit at financial institutions? Succinctly, we study the undemand for money.

To study the undemand for money we exploit episodes of active currency management by the Bank of Canada that allow us to measure cash deposits at financial institutions in Canada (in this paper we will interchangeably use money, currency and cash to refer to banknotes issued by the Bank of Canada). On several occasions since 2000, the Bank of Canada has, with the voluntary participation of the financial institutions in the Banknote Distribution System (BNDS), accelerated the withdrawal of certain series of banknotes from circulation in the economy.[1] Accelerated withdrawal programs require financial institutions to pass any banknotes of a targeted series deposited or held at their branches directly to the Bank of Canada. Banknotes targeted by such accelerated withdrawal programs are labelled unfit.[2] The Bank of Canada redeems these unfit notes and may replace them with a new series of banknotes of the same denomination. Unfit banknote deposits are observed by the Bank of Canada since no unfit banknote is reissued by a financial institution. Thus, the Bank of Canada observes the *deposit* of every banknote targeted in an accelerated withdrawal program.

As one example of such an accelerated withdrawal program, when the Bank of Canada introduced polymer $100 banknotes, it simultaneously declared all remaining, ever-issued, paper $100 banknotes unfit. At the time of the accelerated withdrawal program for the $100, there were

[1] There are nine financial institutions in the BNDS. A brief discussion of the BNDS is presented in Section 2.

[2] As we discuss in Section 2, banknotes may also be labelled unfit if they fail to pass a quality screening by banknote sorting machines at Bank of Canada processing centres.

roughly 312 million $100 banknotes held outside the Bank of Canada. Using data from the Bank of Canada's currency processing centres, we construct a novel dataset of cash deposits for three episodes of accelerated withdrawal programs: from 2000-2015 for the $1,000 banknote, from 2011-2015 for the $100 banknote and from 2012-2015 for the $50 banknote. These three accelerated withdrawal programs are similar to the extent that all existing paper banknote series of these denominations were declared unfit. One notable difference is that the $1,000 banknote was declared unfit and no replacement series of this denomination was issued, whereas polymer banknote series were issued for the $100 and $50 denominations. The rationales for an accelerated withdrawal program are varied, but these programs are generally undertaken to ensure that public confidence in banknotes remains high. For instance, at the time of the $1,000 accelerated withdrawal program, there were concerns raised by law enforcement that the $1,000 denomination was overwhelmingly used for criminal transactions.

We show how to use a standard inventory-theoretic choice framework to exploit our dataset to recover estimates of the fractions of outstanding, ever-issued banknotes of a denomination that circulate, and their probability of being deposited as functions of the opportunity costs. We define circulation broadly to include any banknote that, *ex ante*, has a non-zero probability of being deposited. Thus, our definition of circulation does not include banknotes that are hoarded, have been lost, are held outside of Canada or that may exclusively circulate outside of the legitimate (formal) economy. One nice feature of our identification scheme is that we do not require individual data from depositors. We show that the choice, for an individual or merchant, of whether to deposit or hold a banknote of a given denomination depends only on a wedge driven by the opportunity costs incurred from not depositing and that this wedge is the same for all individuals and merchants. Our choice framework yields a standard logit specification for banknote deposits.

The dataset we construct, using Bank of Canada currency processing data, contains all banknote deposits geographically distributed across Canada (roughly according to provincial boundaries) for three denominations: $1,000, $100 and $50. Our estimates suggest that, for these denominations, between 5 and 30% of ever-issued notes that are unredeemed by the Bank of Canada do not actively circulate through financial institutions (in Canada). As a fraction of the $70 billion currency liabilities of the Bank of Canada as of 2015, this represents approximately 17% of the total value. Perhaps importantly, we find that the factors that affect the probability of deposit for a $1,000 banknote are very different from those for a \$100 or \$50 banknote. Thus, while monetary value is divisible by construction, the functions of money appear denomination-specific. We also find that the interest rate elasticity of deposit is denomination-specific in *sign* – it is positive for the \$1,000 and negative for the \$100 and \$50. Changes in the price level matter for the \$1,000 in half of the regions (accounting, however, for a supermajority of the Canadian population) but are mostly insignificant for the \$100 and \$50. The broad flavour of our empirical findings is that denominations are different monies, and we find evidence in favour of the non-neutrality of monetary denominations.

One concern with the comparison between denominations is that the \$1,000 denomination may suffer from survivorship bias as, once deposited, these notes cannot be re-obtained from a financial institution. In contrast, both the \$100 and \$50 denominations continue to be issued by the Bank of Canada, albeit as polymer notes. We conduct a robustness exercise that accounts for survivorship bias in the distribution of note holdings for the \$1,000 and find no difference to our conclusion that denominations are different monies.

We also find that the probability of deposit is inversely related to the face value of the note: \$50 notes have a deposit probability of roughly 10% per month, \$100 notes of roughly 7% and \$1,000 of roughly 2%. We argue that these deposit probabilities pose something of a puzzle, since the opportunity cost in terms of consumption of holding notes is rising in proportion to the denomination. Thus, deposit probabilities are inversely related to their opportunity costs, which appears to stand in contrast to typical predictions from most theoretical monetary models.

The sample period for the \$1,000 denomination encompasses a period of legislative changes to financial regulations governing reporting requirements for large-value cash transactions and also the financial crisis of 2008/09. The financial tracking changes required regulated institutions (e.g., banks, investment brokers, accountants, money services businesses) to report the identities of individuals making large-value cash deposits to the authorities. Thus while depositing large cash sums remained a legal activity, such transactions were no longer anonymous. We show that our choice framework can identify the costs associated with the heightened reporting requirements and loss of anonymity. We find that, for the \$1,000 note, changes to financial tracking laws for large-value cash deposits had no significant effect for the probability of deposit at a financial institution for any of the 10 regions. In contrast, our results for the financial crisis, which we proxy by the Lehman

Brothers bankruptcy, suggest that the crisis led to a 2-3 times increase in the deposit probabilities in 5 of the 10 regions. These regions are also, in almost all cases, the regions in which the inflation elasticity of deposit for the $1,000 is significantly different from zero.

The demand for money has a long and voluminous history in economics. We contribute to three strands of this research. First, our results appear to be informative for the recently revived debate regarding measurement of monetary aggregates. Lucas and Nicolini (2015), Belongia and Ireland (2015), Hendrickson (2014) and Barnett *et al.* (2013) examine the empirical content of US monetary aggregates for economic activity and propose alternative measures of money supply to the simple-sum aggregates typically reported (e.g., M1, M2, etc.). Lucas and Nicolini (2015) extend the theoretical model of Prescott (1987) and Freeman and Kydland (2000) to define a new monetary aggregate, NewM1, using currency, reserves and commercial bank deposits as distinct elements. Belongia and Ireland (2015), Hendrickson (2014) and Barnett *et al.* (2013) examine the recent empirical performance of Divisia monetary aggregates introduced in Barnett (1980). While the results in this paper are silent as to the relative merits of Divisia monetary aggregates or NewM1, our findings suggest that even simple sums for currency are misleading measures of currency liquidity, at least for Canada.

Our paper also provides estimates of regional patterns of money circulation in Canada. The estimation of regional circulation is typically challenging because econometricians rarely observe both withdrawals and deposits for a given banknote. Thus, an econometrician is unable to determine where a note circulates between these two events. Our model, as a by-product of the choice-theoretical framework, provides an estimate of the stock of existing notes in circulation by the regional distribution of Bank of Canada processing centres when an accelerated withdrawal program is implemented. We find some evidence of regional differences in per capita money holdings. There appears to be no previous research measuring banknote circulation in domestic regions in Canada (or indeed any other nation). The lack of research does not appear to be due to a lack of interest. Dow (1982) argues that understanding regional money multipliers is useful for (New) Keynesian models because such models are often very applicable to regional government spending and because aggregate dynamics may depend on the aggregation of regional multipliers. Mulligan and Sala-i-Martin (1992) emphasize the importance of regional (disaggregated) data for the estimation of money demand. Both of these papers focus on non-cash measures of money such as demand deposits.

In a partly related, empirical literature, Bartzsch and Seitz (2016) and Bartzsch *et al.* (2011a,b) provide estimates for the shares of euros circulating outside of the Eurozone and Doyle (2000) and Porter and Judson (1996) provide estimates of the foreign demand for US dollars. Our identification approach, based upon a choice-theoretic framework rather than time-series regressions using monetary aggregates, is fundamentally different from these existing approaches. One potential limitation of our approach is that we cannot separately identify foreign demand from other non-circulating notes, e.g., hoarded notes, although in our theoretical model such a distinction is immaterial.[3] However, the time-series approach is also limited in that it typically assumes that all domestic demand in the proxy countries used for identification is in fact domestic, and circulating. At least for one proxy country often used in this literature, Canada, our results suggest this is untrue. The time-series approach is also, typically, atheoretic in terms of the specification of money demand, whereas our model is structural.

A final literature to which our results may apply is the study of the denomination structure of money. Most attention has focused on the optimal divisibility of monetary instruments, typically in models in which money is principally introduced as a means of payment. For example, Telser (1995), Van Hove and Heyndels (1996), Lee *et al.* (2005), Lee (2010) and Bouhdaoui *et al.* (2011) consider the optimal denomination structures of monetary instruments as payment instruments. Our results would appear to cast some doubt on some conclusions from this literature, particularly for the higher denomination notes that we study, because these conclusions are based on environments in which monetary objects circulate for transactional purposes. Our results suggest that monetary units are not equally likely to circulate and that monetary objects with different nominal face values are unique monies. An alternative theoretical view from Wallace and Zhu (2004) is that money is a commodity good. The results we present in this paper are not a formal test of the models proposed in Wallace and Zhu; however, the non-neutrality results we find for banknotes appear consistent with the random-matching model of non-divisible money proposed in that paper.

Our estimates are informative for both policy analysis and operational planning by central banks. Calza and Zaghini (2011) stress the importance of circulating banknotes for the calculation of

[3]In our model, notes either have a zero probability of deposit in the initial period or a positive probability. Foreign-held notes and hoarded notes should, almost certainly, have a zero probability of deposit since both are held, presumably, for future consumption. In this sense, hoarded notes and foreign-held notes are one and the same.

the welfare effects of inflation, and Judson (2011) stresses the importance of measuring the quantity of US banknotes circulating abroad for seigniorage. Rogoff (2014) emphasizes the importance of circulating currency for the zero lower bound on the nominal interest rate. Our results also suggest that the zero lower bound on the nominal interest rate may be particularly important in practical terms, given the proportions of high-denomination notes that appear to be non-circulating.

Our results are also informative for the question of whether electronic currencies, or other alternative payment instruments, may replace physical specie. It would appear that the answers to such questions are almost certainly denomination-specific. One may also wonder how feasible it would be, or how long it would take, to replace circulating physical specie with digital currencies without some measure of demonetization, given the large proportion of notes that do not circulate into financial institutions and the relatively low deposit rates for those that do. Indeed, in terms of operational planning, estimates of the deposit probabilities of actively circulating notes are useful for planning future accelerated withdrawal programs.

2 The Banknote Distribution System

Together with nine private financial institutions, the Bank of Canada distributes banknotes across Canada via the BNDS. The BNDS was reformed in 1996 as part of a large-scale overhaul of the payments system in Canada.[4] The BNDS owns two currency processing centres, in Toronto and Montréal, from which it distributes banknotes to one of 44 regional distribution centres. Each regional distribution centre is owned by one of the nine financial institutions that are part of the BNDS.[5] The regional distribution centres receive (distribute) banknotes from (to) bank branches. The regional distribution centres are associated with 10 regional distribution points (roughly mapped to the provinces of Canada) and at these points the financial institutions exchange fit banknotes from their distribution centres using the BNDS. Fit banknotes are notes that pass a quality-index threshold for redistribution to the economy. The quality score of a note is determined by factors such as tears, graffiti, folds, stains, colour degradation and other machine-readable quality measures. Stock and flow data is transmitted to the Bank of Canada through a digital inventory management program, which records note counts by denomination.

[4] Bilkes (1997) provides a comprehensive overview of the changes.

[5] The nine financial institutions that are members of the BNDS are Banque Nationale, Desjardins, Banque Laurentienne, Scotia Bank, the Royal Bank of Canada, Toronto Dominion, Canadian Imperial Bank of Commerce, Alberta Treasury Board and the Credit Union Central of Alberta.

Financial institutions deposit unfit banknotes received by their regional distribution centres to one of the 10 regional distribution points, where these deposits are processed by the Bank of Canada and sent to one of the two processing centres. The regional distribution points are: Newfoundland, Halifax, Quebec City, Montréal, Ottawa, Toronto, Winnipeg, Regina, Calgary and Vancouver. During accelerated withdrawal programs, all notes of the chosen denomination received by a regional distribution centre are deposited though the BNDS system to one of the processing centres. The data we report in this paper are the deposit data at the processing centres (which are dated to their arrival at the regional distribution centre). Figures 2, 3 and 4 in the Appendix show the deposits by region for the $50, $100 and $1,000 denominations, respectively. One feature of the data, which is expected, is the decay process in banknote deposits. This pattern is largely true for all denominations in all regions. However, for the $50 denomination, there is a spike in each region for the initial period after the introduction of the $50 polymer note. This is due to the start date of the accelerated withdrawal process, March 26th, which means that the initial period is roughly 1/6 of a month, unlike for the other denominations. One complication from this start date is that the initial unfit notes were unlikely to be processed in the month of March, for two reasons. The first is the distance from the regional distribution centres to the processing centres. The second is the delays due to normal operating queues in the processing of notes. As a result, for our estimations for the $50 denominations we chose April as the start of the accelerated withdrawal program.

3 A Simple Choice Model of Deposits

In this section we propose a simple model that starts *in media res* – agents in the economy hold notes that they have already withdrawn from the central bank. Each agent faces the same options, per period, with respect to the banknotes in his, her or its possession. Banknotes can be (1) deposited at a Bank; (2) held for future consumption (saved); or (3) passed to a different agent.[6] Figure 1 illustrates the possible paths that a banknote can take in a given period (the ... indicate that these may continue for an unobserved number of agents).[7] Crucially for our empirical identification, each agent holding a banknote of a given denomination faces the same set of choices. The data we

[6] Technically, a note could be destroyed; however, we assume that agents do not wilfully do so. In addition, any damaged (mutilated) notes can be deposited at the Bank of Canada offices for face-value redemption in electronic funds (which effectively deposits them).

[7] While we assume that agents do meet and (potentially) exchange notes, we are agnostic about the type of matching between agents that is implicit in path (3).

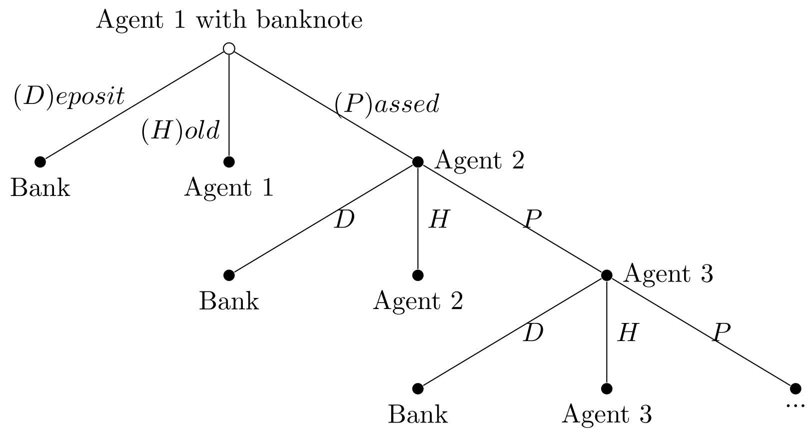

Figure 1: Banknote Circulation Paths Per Period

observe are the number of banknotes deposited at the Bank in each period (monthly in our data). At the end of every period, any note not deposited at the Bank is assumed to be held by an agent in the economy who has made the choice to not deposit the note.

We assume that depositing bank notes is idiosyncratically costly for agents and that these costs may depend on the denomination being deposited. There are a number of reasons why different denominations may have different deposit costs. First, the physical costs of cash holdings are denomination-specific. All else equal, the weight and volume required to store cash in \$1,000 notes is 1/10th that for \$100 notes and 1/20th that for \$50 notes. Thus the opportunity cost of deposit may be proportional to the denomination. Second, individuals may be less likely to carry higher denomination notes in personal wallets, which implies greater transportation costs to banks for these notes. Third, high-denomination notes are less likely to be passed in trade – particularly the \$1,000 – so the chance that a high-denomination note is traded to an agent with idiosyncratically lower deposit costs is lower for high denominations. Since our focus is on the deposit cost faced by a given *banknote* in a given period, the third point is germane, as we shall argue.

One benefit of depositing bank notes is that agents can earn the market rate of interest. Consider a banknote of denomination d. An agent n depositing d earns an expected real return:

$$\frac{\mathsf{d}(1+i_t)(1-\tau^n_{\mathsf{d}})}{(1+\mathbb{E}_t[\pi_{t+1}])},$$

where i_t is the market nominal interest rate, $\tau^n_{\mathsf{d},t}$ is the deposit cost for denomination d in period

t for an agent n, and $\mathbb{E}_t[\pi_{t+1}]$ is the expected inflation rate. Similarly, if an agent decides to not deposit a bill (hold) then the agent earns an expected return:

$$\frac{\mathsf{d}(1+m_t)}{(1+\mathbb{E}_t[\pi_{t+1}])},$$

where m_t is the nominal rate of return when not deposited. m_t is often assumed to equal 0 in theoretical work, but there is no reason to assume so here. It is plausible that either informal finance or informal economies may provide nominal returns for denominations and that these returns may also be denomination-specific. For instance, money laundering or some avenues of tax avoidance may provide positive rates of return for participants (and thus for the banknote).

Embedding these rates of return into a simple intertemporal choice problem for an agent with felicity function $u(c)$, where c is consumption and u is differentiable, and intertemporal discount factor β implies that for an agent to deposit a note it must be that:

$$\mathbb{E}_t[\frac{\beta\mathsf{d}(1+i_t)(1-\tau^n_{\mathsf{d},t})}{1+\pi_{t+1}}u'(c_{t+1})] \geq u'(c_t).$$

Similarly, for an agent to hold a note and not deposit it, it must be that:

$$\mathbb{E}_t[\frac{\beta\mathsf{d}(1+m_t)}{1+\pi_{t+1}}u'(c_{t+1})] \geq u'(c_t).$$

Inductive reasoning implies that an agent n who deposits a note must prefer depositing to holding the note; thus,

$$\mathbb{E}_t[\frac{\beta\mathsf{d}(1+i_t)(1-\tau^n_{\mathsf{d},t})}{1+\pi_{t+1}}u'(c_{t+1})] \geq \mathbb{E}_t[\frac{\beta\mathsf{d}(1+m_t)}{1+\pi_{t+1}}u'(c_{t+1})]. \tag{1}$$

Rearranging the expectations, we note:

$$\beta\mathsf{d}(1+i_t)(1-\tau^n_{\mathsf{d},t})\mathbb{E}_t[\frac{u'(c_{t+1})}{1+\pi_{t+1}}] \geq \beta\mathsf{d}(1+m_t)\mathbb{E}_t[\frac{u'(c_{t+1})}{1+\pi_{t+1}}]. \tag{2}$$

Since the terms inside the expectation are identical on both the left-hand side (LHS) and the right-hand side (RHS), for an agent n to deposit a note it must be that:

$$(1+i_t)(1-\tau^n_{\mathsf{d},t}) \geq (1+m_t). \tag{3}$$

We assume that the non-market return, m_t, is proportional to the market return so that $(1+m_t) = (1+i_t)^{\sigma}$. This formulation is convenient since $\sigma < 1$ implies that the non-market return is less than the market return, $\sigma = 0$ implies $m_t = 0$ and $\sigma > 1$ implies that the non-market return exceeds the market return. Taking logarithms, assuming i_t is small and using the logarithmic approximation and substituting for $1 + m_t$ yields:

$$\ln(1 - \tau^n_{\mathsf{d},t}) + (1 - \sigma)i_t \geq 0. \tag{4}$$

Equation (4) is the condition for a note to be deposited by the agent holding the note. We assume that agents face idiosyncratic deposit costs (such as weather shocks, traffic delays, illness) for notes held in their possession, although Equation (4) also implies that the threshold $\tau^n_{\mathsf{d},t}$ for a note to be deposited is identical across all agents in a given period t. Thus, for a note to be deposited an agent holding the note must have a deposit cost lower than $(1 - \sigma)i_t$. Given technological progress in the financial sector and general increases in the price level, transaction costs likely change over time and so the threshold for depositing a note in period t is unlikely to be constant for all periods $t + k$, $k > 1$. We allow for change over time in the transaction fees such that the deposit cost in period t is scaled by the change in the price level

$$1 - \tau_{\mathsf{d},t} = (1 - \tau^n_{\mathsf{d}})(1 + \Pi_t)^{\delta},$$

where Π_t is the change in the price level at time t compared with $t = 0$ and δ is a scaling parameter. This formulation also allows for different trend changes across time in the different regions we examine empirically because the price level is observed regionally. Replacing $1 - \tau^n_{\mathsf{d},t}$ in Equation (4) yields

$$\ln(1 - \tau^n_{\mathsf{d}}) + (1 - \sigma)i_t + \delta \ln(1 + \Pi_t) \geq 0. \tag{5}$$

Finally, notes can pass between agents and so, at the margin, whether an individual note is deposited depends on the minimum value of the deposit cost for agents holding that note in a given period t. Although we do not observe the individual deposit costs, we do observe whether a note is deposited or not. If a note is deposited, then the (last) agent holding that note must have had a deposit cost below the threshold for deposit. If a note is not deposited, then the deposit cost for all agents holding that note in the period must have been higher than the threshold for deposit. Thus, for each note, the relevant deposit cost is the minimum deposit cost for all agents holding that note in the given period. We assume that for each note in circulation a deposit cost shock, e, is drawn from a Type 1 extreme value distribution, with mean x_{d} (the average of the individual depositors' $1-\tau^n_{\mathsf{d}}$), independently in each period. The Type 1 extreme value, or Gumbel, distribution is well-suited to this problem. Extreme value distributions are the limiting distributions for the minimum (maximum) of random observations from the same arbitrary distribution, which is consistent with notes (potentially) passing amongst many agents in a given period and only being deposited by the agent with a deposit cost, $\ln(1-\tau^n_{\mathsf{d}})$, below the threshold. Given e and assuming that i_t is small, we approximate the threshold decision to deposit a bill or not as:

$$U^D = x_{\mathsf{d}} + (1-\sigma)i_t + \delta\ln(1+\Pi_t) + e.$$

In our empirical analysis, we assume that agents have rational expectations over i_t and that transaction fees are known.[8] The first assumption might appear innocuous because the nominal interest offered at the time of deposit is typically advertised (although one concern is that agents face interest rates different from those we assume). The second assumption regarding transaction fees may appear innocuous but our assumption that fees are indexed to the general price level may imply a potential mismeasurement. In our empirical analysis, we address the concerns over the potential endogeneity of both i_t and Π_t using a control function approach.

Under the assumption that e is distributed as a Type 1 extreme value random variable, we can write the probability that a note is deposited at a financial institution in region j in period t, $p_{j,t}$, as:

$$p_{j,t} = \frac{\exp^{x_{j,\mathsf{d}}+(1-\sigma)i_t+\delta\ln(1+\Pi_{j,t})}}{1+\exp^{x_{j,\mathsf{d}}+(1-\sigma)i_t+\delta\ln(1+\Pi_{j,t})}}, \tag{6}$$

where we allow the deposit probability to vary across regions and over time.

3.1 Determining the Stock of Notes in Circulation

We label the outstanding stock of unfit notes that circulate in region j in period t as $M_{j,t}$. Since we have defined the probability of deposit, $p_{j,t}$, then it must be that the number of notes deposited per period per region is $p_{j,t}M_{j,t}$. We define unfit note deposits at the Bank of Canada in region j

[8] We note that these are contemporaneous expectations over the current period realizations that an agent faces once arrived at a financial institution and not expectations over the future evolution of these prices.

in period t as $UF_{j,t}$ and define $M_{j,0}$ as the stock of initial currency circulating in region j at the time of the unfit policy change. Thus, in the initial period $t = 0$,

$$UF_{j,0} = M_{j,0} p_{j,0}, \tag{7}$$

which allows us to pin down the initial circulation of notes in region j. In subsequent periods, $t \geq 1$, assuming no new notes of the denomination were added in period 0, expected unfit note deposits are given by:

$$UF_{j,t} = p_{j,t}(M_{j,0} - \sum_{m=0}^{t-1} UF_{j,m}), \tag{8}$$

where the term in brackets is the cumulative stock of banknotes remaining in circulation. In theory, we are free to impose the restriction that $M_{j,0} = UF_{j,0}/p_{j,0}$; however, in our empirical application below, we estimate $M_{j,0}$ as a fixed effect for each region j and sum the total across the regions j because imposing the restriction directly is problematic when we control for the possible endogeneity of $\imath_t$ and $\Pi_{j,t}$. (However, we can report that there is no significant difference (and typically only a very small quantitative difference) when we impose the restriction in our models without an endogeneity correction.) We then compare this sum with the Bank of Canada's record of the number of notes in circulation in Canada. As we document, our estimates of the total stock of notes in circulation are typically lower than the number of notes in circulation. Given our definition of circulation, we conclude that these missing notes have been destroyed or lost; are hoarded; are held internationally; or circulate in a sector of the economy that does not deposit notes at financial institutions.

Our first application is to estimate $p_{j,t}$ and $M_{j,0}$ using data for the \$1,000 banknote that was declared unfit on May 12, 2000. The BNDS data from the Bank of Canada provides the number of unfit deposits for these notes for each region in Canada. The use of the \$1,000 banknote is particularly useful because the period of the policy change also includes a period when the financial tracking rule for large-value cash deposits changed and also encompasses the financial crisis. The change in the financial tracking rule arguably imposed a higher cost for depositing notes by requiring large-value cash depositors to provide personal information. We are able to quantify how the change in policy affected the probability of deposit for the \$1,000 note by region and find that it had no statistically significant effect. However, as we show below, the financial crisis affected deposit probabilities in roughly half of the regions.

As a second application, we estimate the deposit probabilities and the stocks of \$50 and \$100 notes in circulation using data from the introduction of the polymer notes for these denominations. The Bank of Canada introduced polymer \$100 bills on Nov 14, 2011 and \$50 bills on March 26, 2012 and simultaneously declared existing paper notes of these denominations to be unfit on the same day (though, unlike the \$1,000 note, the Bank of Canada issued polymer notes of these denominations as replacements).[9] Unfortunately, for the \$50 and \$100 notes, the Bank of Canada's unfit data do not distinguish between unfit paper notes and polymer notes that have become unfit because of some type of physical deformation (e.g., creased through use, defaced, torn, etc.). Thus, the unfit series for these notes are a mixture of paper note deposits and polymer notes and so, by extension, our empirical identification is less clean.

The unfit \$50 and \$100 notes deposited at the Bank of Canada in the initial period of polymer introduction will be *all* notes currently deposited at a financial institution in region j, so we can continue to define the initial stock of paper notes in circulation as:

$$UF_{j,0} = p_{j,0} M_{j,0}.$$

However, unlike for the \$1,000 note, the number of unfit \$50 and \$100 notes received by the Bank of Canada in period 1 is equal to the number of paper notes received by financial institutions plus the number of new polymer notes that were introduced in period $t = 0$ that became unfit. Thus,

$$UF_{j,1} = p_{j,1}(M_{j,0} - UF_{j,0} + \ddot{q}_{j,0}),$$

where the first element of the RHS is the number of newly deposited paper notes from the total pool of such notes and the second term, $\ddot{q}_{j,0}$, is the number of unfit polymer notes from the number of polymer notes introduced in period $t = 0$. We define $\ddot{q}_{j,t-1}$:

$$\ddot{q}_{j,t-1} = \begin{cases} 0 & \text{if } t = 0 \\ \sum_{n=1}^{t-1} \gamma^n (1-\gamma^n)^{t-(n+1)} Q_{j,n-1} & \text{if } t \geq 1; \end{cases} \tag{9}$$

where γ is the probability that a polymer note has become unfit in a given period (we ignore differences across regions in unfitness rates) and $Q_{j,t}$ is the number of polymer notes withdrawn in

[9] Although the Bank of Canada subsequently introduced polymer notes of other denominations, they did not simultaneously declare existing paper notes unfit for those denominations. Accelerated withdrawal programs require the voluntary, active involvement and participation of all financial institutions that operate regional distribution centres and can be logistically challenging, particularly for financial institutions.

region j in period t. This specification assigns the same per-period probability that a polymer note becomes unfit to the population of polymer notes in circulation. Thus $\ddot{q}_{j,t}$ is the expected number of unfit polymer notes deposited with the Bank of Canada given the existing age distribution of polymer notes in circulation.

We choose to estimate γ using auxiliary data from circulation trials conducted by the Currency Department of the Bank of Canada, which are designed precisely for this purpose. The circulation trials follow roughly 20 million polymer notes (for both the \$50 and \$100 denominations) and use optical scanners to assess the fitness of these notes as they circulate through the Bank of Canada's Agency Operation Centres. Our estimates of γ for the \$50 and \$100 denominations are from these trials. The circulation trials report cumulative unfit percentages for each denomination. We assume a constant per-month unfitness rate, γ, and a continuous-time environment to estimate α from the cumulative unfit percentages. We estimate $\gamma = 0.00009$ for the \$100 denomination and $\gamma = 0.00022$ for the \$50. These estimates are precise as there appears to be little time-series variation in the unfitness rates for the polymer notes.[10] We experiment with a range of α around our estimated values for both the \$100 and \$50 and do not qualitatively or quantitatively find different results from those we report in the paper.[11] Nevertheless, it is probable that our estimates of $\ddot{q}_{j,t}$ are mismeasured, which could imply that our estimates of $p_{j,t}$ are biased if the measurement errors are correlated with the deposit probabilities. We again use a control function approach to account for such endogeneity.

4 Results

In this section we present our main results for the \$1,000 denomination, as well as our results for the \$50 and \$100 denominations. One advantage of our data is that we are able to use regional-level unfit deposit data from the 10 Bank of Canada regional distribution points: Newfoundland, Halifax, Quebec City, Montréal, Ottawa, Toronto, Winnipeg, Regina, Calgary and Vancouver. Moreover, given the parsimony of our model, we estimate for these regions separately (we do not impose any cross-equation restrictions). Our sample sizes are balanced across regions and are 180, 42 and 38

[10] We do not report the full estimation results but these are available upon request to the authors, subject to the data availability policies of the Bank of Canada.

[11] An alternative approach would be to estimate α directly in our main regression; however, we do not choose to do this because identification of γ in our main equation, while theoretically possible, relies on exploiting a difference between a polynomial $\ddot{q}$ and the sum of unfit notes. Given our relatively short time series, our estimates of γ in this approach are generally not very robust.

for each of the 10 regions for the \$1,000, \$100 and \$50, respectively.[12]

We adopt the following baseline econometric specification of the deposit probability for each denomination,

$$p_{j,t} = \frac{\exp^{\alpha_{1,j}+\alpha_{2,j}i_t-\alpha_{3,j}\ln(1+\Pi_{j,t})+\omega_j S_t}}{1+\exp^{\alpha_{1,j}+\alpha_{2,j}i_t-\alpha_{3,j}\ln(1+\Pi_{j,t})+\omega_j S_t}}, \tag{10}$$

with $\alpha_{1,j} \equiv x_{\mathsf{d}}$ for region j and $\alpha_{2,j} \equiv (1-\sigma)$ and $\alpha_{3,j} \equiv \delta$ being the relative contributions of the nominal interest rate (converted to monthly returns to be consistent with the deposit data) and the non-seasonally adjusted price level for deposits in region j, respectively, and $\omega_j S_t$ being a vector of quarterly dummy variables to account for possible seasonal patterns in deposits.[13] Thus, for the \$1,000:

$$UF_{j,t} = p_{j,t}(M_{j,0} - \sum_{m=0}^{t-1} UF_{j,m}) + u_{j,t}, \tag{11}$$

and for the \$100 and \$50:

$$UF_{j,t} = p_{j,t}(M_{j,0} - \sum_{m=0}^{t-1} UF_{j,m} + \ddot{q}_{j,t-1}) + \tilde{u}_{j,t}, \tag{12}$$

where $u_{j,t}$ and $\tilde{u}_{j,t}$ are the regression residuals. We estimate the model for each region using non-linear least squares. In our regressions where we use control functions to adjust for the possible endogeneity of the nominal interest rate and CPI inflation, we augment our specification of the deposit probability to include the residuals from the first-stage regressions for these variables. Given our use of control functions, we use a bootstrap procedure to estimate the standard errors. We bootstrap with 199 resamples of our data for every model we present. Arguably, 199 is on the low side but we do not observe much difference in our estimates for trials with 999 resamples.

Our model identifies two variables of interest, the stock of notes in circulation at $t = 0$, $M_{j,0}$ and the deposit probabilities, $p_{j,t}$, which allow us to estimate the interest elasticity of banknote deposits. First, our model identifies the stock of notes in circulation in each region (recall that we

[12] Clearly imposing cross-equation restrictions and estimating a non-linear seeming-unrelated regression, for example, would increase our apparent sample sizes by a factor of 10. We chose not to present our results in this way so that differences in the estimated parameters and standard errors act roughly as a cross-validation exercise for our model specification. Since we find reasonably stable parameter estimates across all specifications and our method is a conservative approach in terms of standard errors, we choose to present our results from separate regressions. Results from NLSUR specifications are available upon request.

[13] We do not include quarterly dummy variables for the \$50 denomination in the results we report. We found no evidence of significant quarterly differences for the \$50, and the inclusion of the dummy variables in a few regions made identification of $\alpha_{1,j}$ separately from the dummies in the short sample for the \$50 difficult. However, apart from weaker identification of $\alpha_{i,j}$ in a few regions, there is no substantive difference in the results for the \$50. The results with the quarterly dummies are available upon request.

define circulation as having a non-zero probability of being deposited at $t = 0$) and so summing across regions provides an estimate of the number of notes in circulation at the time such notes were declared unfit. Comparing these estimates with the number of notes ever issued of that denomination at that time provides an estimate of the number of destroyed, irretrievably lost notes or notes that circulate outside of the financial system. Our model also isolates the effects of the price level and nominal interest rates separately for the deposits of banknotes. While this is not direct evidence of the effect of these factors for new money demand (i.e., withdrawals from financial institutions) it is evidence of the effect of these factors for the demand to hold money.

Because the Bank of Canada regions do not exactly correspond to the Canadian provinces, we match the regions with the nearest provincial-level data on prices, which is obtained from Statistics Canada. We use the 30-day treasury bill rate as the nominal interest rate to correspond with the one-month periodicity of our data. We are naturally concerned that the price level and/or the treasury bill rate is (are) either mismeasured (perhaps because of errors in expectations by agents, which we do not observe or because our geographies do not exactly overlap) or determined by an omitted variable contemporaneously with unfit deposit levels. We control for endogeneity using a control function approach where we follow Ng and Bai (2009) and use two-period lagged values of the first four principal components (two components for the \$100 and \$50 denominations because of the shorter sample period) from interest rate data, asset market data and foreign exchange rate data (we present this data in more detail in the Appendix). We also include two-period lagged values of gasoline and food prices from the CPI subindices as control variables. These instruments will be valid as long as innovations to our treasury bill and CPI inflation data are not predictable at more than a one-month frequency. In most regions and denominations, our control function approach does not suggest any evidence of endogeneity.

4.1 Notes in Circulation: Estimates

We first present a summary of our estimates of notes in circulation for the \$1,000, \$100 and \$50 denominations in the regions we observe.[14] Although we estimate both, including and excluding control functions, in the interests of brevity, we summarize our results below using the estimates from the specification that appears most suitable. Thus, if our regression estimates show evidence of endogeneity we present our estimates in italics from the control function specification. If we

[14] The detailed regression results for each region are reported in the Appendix.

find no evidence of endogeneity we present our baseline results without italics. We define evidence of endogeneity to be either a p-value less than 0.05 on an individual control function parameter estimate or a p-value on joint significance of the control function parameter estimates less than 0.05. We acknowledge that these criteria are arbitrary but our mean estimates of notes in circulation are not sensitive to any conventional threshold we might choose (the standard errors do change meaningfully in a few cases). In the Appendix, we report the individual regression results by region.[15]

Perhaps not surprisingly, there is a strong correlation between market size and the location of notes, with the three largest regions (Toronto, Montréal and Vancouver) accounting for a supermajority of outstanding notes. We use our 'mean' estimates to calculate the total outstanding stock of unfit notes of each denomination that can be accounted for by our model. We include our estimates of the proportion of notes not in circulation and label this region "Unknown" as its location is uncertain.[16] To put these estimates into context, we calculate the value of notes not in circulation using our baseline estimates as $11.5 billion, which, to put it in perspective, is roughly 17% of the currency liabilities of the Bank of Canada as of November 2015. One possibility is that these notes have been destroyed or irretrievably lost, although, given the large-value denominations involved, this would seem unlikely to explain the entire discrepancy. Nevertheless, despite being unobserved, the Unknown Region is the largest region in terms of the circulation of $100 notes, fully 5 percentage points larger than Toronto. It is the second-largest region in terms of notes for the $50 denomination, but rather unremarkable for the $1,000 denomination. The geographic concentrations of banknotes largely follow the regional populations, with Vancouver having slightly more $100 notes per capita than the other regions overall, and Toronto and Montréal having much more $1,000 notes per capita. The concentration of $1,000 notes in Toronto and Montréal accounts for roughly two-thirds of the initial outstanding circulation of $1,000 notes. One possibility is that this concentration of notes is a legacy from the initial introduction of such notes as a settlement vehicle for financial institutions. However, our estimates of the initial stock of circulating notes is imprecise in one region for both the $50 and $1,000 denominations, although it is precise for all

[15] We consider only the two specifications of our regression equations, i.e., with and without the control functions, and do not test down our models to more parsimonious specifications, as doing so would require arbitrary decisions over sequences and p-value thresholds.

[16] Our estimates are compatible with the findings of Hogg, Liang and Stuber (2012) who examined the deposits to date of notes in accelerated withdrawal programs. Our estimates of the circulating specie are higher than the actual receipts observed by those authors.

regions for the \$100. Thus, we cannot statistically reject that, for the \$50 and \$1,000, the Unknown Region is Winnipeg and Winnipeg and Vancouver, respectively.

A final observation is that the proportion of \$1,000 notes in the Unknown Region is significantly smaller than for either the \$100 or \$50 denominations. On the one hand, this result appears intuitive as the opportunity cost of this denomination is, ignoring deposit costs, 10 and 20 times larger than for the \$100 and \$50 denominations, respectively. On the other hand, these results suggest that the reason for cash holding in the Unknown Region is unlikely to be for savings since the physical storage costs for the \$1,000 note is, for a given nominal amount, the lowest of the three denominations studied. To the extent that the Unknown Region is similar across denominations, we conclude that these results are indicative that the Unknown Region either represents an (sub) economy with circulating physical specie or else irrational hoarding. Further support for this conclusion is that our estimates of σ for the \$100 and \$50 denominations strongly suggest that the expected returns from holding these denominations are far greater than the returns from depositing it.

Table 1: Circulation by Region

	\$1,000			\$100			\$50		
Region	Estimate	Std Err	Share	Estimate	Std Err	Share	Estimate	Std Err	Share
Calgary	**138235**	22292.19	4%	**1.72E+07**	3.15E+05	6%	**1.09E+07**	1.12E+06	6%
Halifax	**145334**	7224.873	4%	**9552655**	2607087	3%	**4943648**	553376.8	3%
Montréal	**1094830**	40580.76	31%	**5.83E+07**	3.87E+06	19%	**3.09E+07**	2.56E+06	16%
Newfoundland	**18881**	1081.797	1%	**2327261**	215534.8	1%	**1542480**	153238.9	1%
Ottawa	**69329**	7449.631	2%	**3705896**	136397.3	1%	**3432704**	320158.4	2%
Quebec City	**226085**	12682.06	6%	**7630105**	580253.2	2%	**3575339**	515178	2%
Regina	**44941**	7182.756	1%	**3382492**	464734.5	1%	**3199168**	126662	2%
Winnipeg	89657	3304120	3%	**5144663**	291795.9	2%	5316793	692155.8	3%
Toronto	**1238588**	**93263**	35%	**7.93E+07**	9.01E+06	25%	**5.65E+07**	3.34E+06	29%
Vancouver	*299497*	*8400207*	8%	***3.33E+07***	*4.20E+05*	11%	**1.81E+07**	9.33E+05	9%
Unknown	182803		5%	9.20E+07		30%	4.21E+07		22%

Notes: Italicized estimates are from the control function specification. Bold indicates the estimates have a p-value of less than 0.01. Share refers to the estimate as a proportion of ever-issued notes outstanding from the Bank of Canada at the time of the unfit announcement.

4.2 Deposit Probabilities

We next present our estimates of the average deposit probabilities of the unfit banknotes. To clarify, these deposit probabilities are the average conditional probability that an undeposited and unfit note is deposited in a financial institution.[17] There are at least two reasons why a note will not be deposited: (1) the note is held for anticipated exchange, or (2) the note is being held as savings

[17] These deposit probabilities are conditional on the nominal interest rate and the price level for the sample period.

Table 2: Circulation by Region

Region	$1,000 Estimate	$1,000 Std Err	$1,000 Share	$100 Estimate	$100 Std Err	$100 Share	$50 Estimate	$50 Std Err	$50 Share
Calgary	**138235**	22292.19	4%	**1.72E+07**	3.15E+05	6%	**1.39E+07**	5.84E+05	7%
Halifax	**145334**	7224.873	4%	**9.55E+06**	2607087	3%	**5860054**	244323.7	3%
Montreal	**1094830**	40580.76	31%	**5.83E+07**	3.87E+06	19%	**3.28E+07**	2.05E+06	17%
Newfoundland	**18881**	1081.797	1%	**2327261**	215534.8	1%	**1782229**	111964.6	1%
Ottawa	**69329**	7449.631	2%	**3705896**	136397.3	1%	**3386818**	218132.1	2%
Quebec City	**226085**	12682.06	6%	**7630105**	580253.2	2%	4189492	5.69E+08	2%
Regina	**44941**	7182.756	1%	**3382492**	464734.5	1%	3199168	126662	2%
Winnipeg	***64390***	*14204.23*	2%	**5144663**	291795.9	2%	5316793	692155.8	3%
Toronto	***1288871***	*147449.9*	36%	**7.93E+07**	9.01E+06	25%	**6.10E+07**	2.04E+06	32%
Vancouver	*299497*	*8400207*	8%	***3.33E+07***	*4.20E+05*	11%	**1.93E+07**	2.58E+05	10%
Unknown	157788		4%	9.20E+07		30%	4.21E+07		22%

Notes: Italicized estimates are from the control function specification. Estimates for the $50 denomination for Regina and Winnipeg are from the specification without quarterly dummy variables. Bold indicates the estimates have a p-value of less than 0.01. Share refers to the estimate as a proportion of ever-issued notes outstanding from the Bank of Canada at the time of the unfit announcement.

because the cost of deposit to a financial institution is greater than the potential gain.[18] If (1) is correct we might expect deposit rates to rise in the face value of the denomination since these notes are less typically used in transactions; see Henry, Huynh and Shen (2015). If (2) is correct, we should observe deposit probabilities that are increasing in the value of the denomination, assuming that the gain from depositing is simply interest earnings. As Table 3 suggests, the pattern we observe is inconsistent with our expectations from (1) and (2). Of course, one possibility is that the deposit probabilities differ only because the sample periods do not coincide so that the conditioning variables are of different magnitudes. We explore and reject this conjecture.

The average estimated deposit probabilities imply that the probability of deposit is inversely related to the value of the denomination. In all regions, the $50 denomination has the highest average estimated deposit probability, followed by the $100, while the $1,000 denomination has the lowest deposit probability. The number of observations for each denomination is different because the unfit episodes have different calendar timing. Nevertheless, the overall picture of the deposit probabilities for the $1,000 denomination does not appear to depend on changes in the underlying interest rates for the sample period, as the standard errors of the estimate are tight. The estimates we have chosen to present in Table 3 summarize the estimates across the various specifications that we consider. In the Table, the estimated deposit probabilities for Vancouver for the $1,000 and the $100 are those calculated from the control function regressions, but these estimates are not sensitive to the endogeneity correction. In the Appendix, we present detailed estimates from

[18] We acknowledge that these reasons, while exhaustive in terms of motivation, do not separately capture notions of hoarding which, we believe, could plausibly be interpreted as either reason.

additional specifications that confirm that our main estimates are essentially unchanged across specifications. We also note that the deposit probabilities are, by and large, fairly homogeneous across regions.

Table 3: Average Estimated Deposit Probabilities by Region (in %)

	$1,000		$100		$50	
Region	Average	Std Dev	Average	Std Dev	Average	Std Dev
Calgary	1.21%	0.71%	9.59%	6.04%	6.24%	2.72%
Halifax	0.92%	0.33%	4.58%	1.40%	11.93%	9.59%
Montreal	1.05%	0.69%	4.34%	0.93%	9.02%	6.41%
Newfoundland	1.40%	0.39%	4.76%	1.25%	9.82%	5.10%
Ottawa	0.75%	0.75%	6.43%	2.81%	7.91%	6.08%
Quebec City	0.79%	0.58%	3.87%	0.76%	7.57%	5.35%
Regina	0.66%	0.64%	6.03%	2.18%	18.98%	12.46%
Winnipeg	*0.86%*	*0.38%*	6.33%	2.65%	13.12%	8.91%
Toronto	*0.76%*	*0.65%*	7.06%	2.43%	10.41%	5.96%
Vancouver	*1.51%*	*0.62%*	*7.81%*	*2.60%*	12.95%	10.17%

Notes: Italicized estimates are from the control function specification. Estimates for the $50 denomination for Regina and Winnipeg are from the specification without quarterly dummy variables.

Our individual regression results, reported in the Appendix, suggest that the key driver of the deposit probability is the nominal interest rate while the price level changes are generally only significantly different from zero for the $1,000 denomination and are not otherwise statistically significant. The deposit costs also are a driver of deposits for the $1,000 note. Generally speaking, we do not observe seasonal effects for the deposit probabilities for the $1,000 and $100 denominations but do observe some seasonal patterns for the $50. We calculate the average interest elasticity of unfit note deposits for our data to gauge how responsive unfit note deposits are to interest rate changes. The measure of the sample average interest rate elasticity for region j is:

$$\varepsilon_i(\iota, UF) = \frac{1}{T}\sum_{n=1}^{t} \alpha_2 p_{j,n}(1 - p_{j,n})\frac{i_n}{UF_{j,n}}(M_{j,0} - \sum_{m=0}^{n-1} UF_{j,m}).$$

The estimates are presented in Table 4. For the $1,000 the sample interest rate elasticity of deposit is roughly 0.35, for the $100 it is roughly -2 and for the $50 it is roughly -4. The estimates suggest that the elasticities are declining in the face value of the note, which is consistent with an opportunity cost of money interpretation. However, the negative interest rate elasticities for the $100 and $50 denominations imply that increasing interest rates lower deposits which is not consistent with an interpretation of interest rates as the opportunity cost of holding notes. The negative elasticities for the \$100 and \$50 denominations are largely driven by the estimates of σ. The mean estimates of σ that are statistically different from zero range from roughly -2200 to -6300 for the \$100 and \$50 denominations. These estimates imply that m_t ranges from roughly 1.6 to 4.5, which suggests that the \$100 and \$50 denominations may earn significant returns at the monthly frequency. To put these extraordinary returns in context, they imply a compounded daily return of up to 5% (i.e., if held by the same individual) or an average daily return of 15% if not compounded. These rates of return, while high, appear to be roughly within the range of the sales tax rates across regions.

Since Canadian treasury bill rates were unusually low by historical standards during the \$100 and \$50 sample periods (ranging between 0.5 and 1% at annual rates), one possibility is that the sample interest rate elasticity is non-linear in the interest rate even if the model does not suggest such a channel. It is also possible that the introduction of the \$100 polymer bill led to a change in usage of all high-denomination notes, which implied that all such notes changed their interest elasticity.

To investigate this hypothesis, we introduced a dummy variable for the \$100 sample period and also interacted this dummy variable with the interest rate in the regressions for the \$1,000 note.[19] The coefficient estimates reject non-linearity in the interest rate elasticity in all regions, but provide some support for the hypothesis that the introduction of the polymer \$100 changed the fixed cost of depositing the \$1,000 note in two regions (Ottawa and Vancouver). In these regions, the introduction of the \$100 polymer lowered the fixed cost associated with the deposit of the \$1,000. One narrative consistent with this finding is that the increased security provided by the new polymer security features and durability led to a switch by non-depositers from paper \$1,000 to polymer \$100.[20] However, in general, the introduction of the \$100 polymer note had little impact on the deposits of the \$1,000 unfit paper notes.

Thus, our estimates suggest that deposits of the \$1,000 denomination increase with rises in the nominal interest rate. In contrast, deposits of the circulating \$100 and \$50 decrease by a factor of 2

[19] The sample period for the \$50 is almost identical to that of the \$100 and, in terms of the level of interest rates, there is very little difference.

[20] The detailed results of these regressions are available upon request. One might additionally suspect that deposit rate variation would depend on criminal activity in the regions. Unfortunately, we are unable to explore this conjecture as the crime data is annual and is also likely to be subject to under-reporting (measurement errors) for the type of property crime most likely to be associated with cash. Finding a suitable instrument for such measurement errors is beyond the scope of this project.

and 4, respectively, with rises in the nominal interest rate. The results for the $1,000 are suggestive of an interest rate opportunity cost of holding money as in standard monetary theory. Surprisingly, those for the $100 and $50 are suggestive of the opposite; indeed, our estimates suggest that the return from holding notes (and perhaps using them in an informal economy) are significantly larger than the market interest rate – although we remind the reader that this estimate is only for notes in circulation.

While these numbers are not directly comparable with the interest rate elasticity of money *demand* because they measure the *undemand* of money balances, they do suggest that the distribution of banknote denominations plays a crucial role in the evaluation of such elasticities. We conclude that the composition of money balances should not be overlooked.

In a similar manner, we calculate the (log) price-level elasticity of deposit since the change in the log price level is significant for some regions for the $1,000 note. We find an elasticity of deposit of roughly -1.5 for the regions for which log price level is a significant covariate. We interpret this finding as evidence that rising nominal prices reduce the benefits of deposit for the high-denomination bills in these regions.

Table 4: Average Sample Interest Rate Elasticity

	Interest Rate Elasticity			Price Level Elasticity		
Region	$1,000	$100	$50	$1000	$100	$50
Calgary	**0.43**	**-2.65**	-2.84	-1.24	0.02	-15.33
Halifax	**0.30**	-1.25	**-5.22**	**-0.67**	-0.38	4.42
Montreal	**0.27**	**-1.61**	**-3.81**	**-1.17**	-0.14	10.01
Newfoundland	**0.47**	-1.46	**-3.44**	-0.26	**-0.28**	-0.19
Ottawa	**0.25**	**-2.40**	**-4.26**	**-1.74**	0.08	**11.73**
Quebec City	**0.27**	**-1.55**	**-4.57**	**-1.40**	-0.19	1.73
Regina	**0.14**	**-2.17**	-3.91	-2.16	-0.34	19.75
Winnipeg	**0.44**	**-2.80**	-4.70	**-1.87**	-0.19	11.04
Toronto	**0.21**	**-1.84**	**-3.45**	**-1.78**	-0.04	3.79
Vancouver	***0.47***	***-1.75***	**-4.14**	-0.45	**0.11**	33.27

Notes: Italicized estimates are from the control function specification. Estimates for the $50 denomination for Regina and Winnipeg are from the specification without quarterly dummy variables. Bold indicates the estimates have a p-value of less than 0.01.

4.3 Survivorship

Our estimates of the deposit probabilities and elasticities reported thus far rely on a stationary distribution for the unobserved deposit costs, e, at least conditioning on the price level. This restriction seems palatable for the \$100 and \$50 denominations since the introduction of polymer notes at the time the existing notes were declared unfit means that the ability of an agent to obtain such a denomination was unaffected. However, for the \$1,000 note, the declaration of unfit status was not accompanied by the introduction of a new series. Thus, \$1,000 notes became harder to obtain. This suggests that there may be a survivorship bias evident in the distribution of e over time.

To examine the robustness of our results to survivorship, we adapt our model to reflect increasing (mean) costs over time. We write the deposit decision for an agent as:

$$U_t^D = (\ln(x_{j,\mathsf{d}}) + \ln(1+\bar{x}^t)) + (1-\sigma)i_t + \delta \ln(1+\Pi_{j,t}) + e_t,$$

where $\ln(1+\bar{x}^t)$ is the increasing cost to the surviving agents for depositing a note ($\bar{x}$ is the growth rate in mean costs). This specification implies that notes may be increasingly held by agents who find depositing the note to be more costly than agents in a previous period, if $\bar{x} > 0$, which reflects the survivorship bias. We note that the inclusion of $\bar{x}$ may also help to explain the deposit elasticity for the \$1,000 denomination since interest rates were mostly declining over our sample period. If the survivorship bias leads to fewer deposits, then this may weaken the observed relationship between the interest rate and the deposits.

We estimate this version of our model using a non-linear seeming-unrelated regression, imposing the constraint that $\alpha_{k,j} = \alpha_k$. We impose the constraint to aid identification of $\bar{x}$ different from the time path of i_t because i_t is common across regions. However, given that the earlier results for the baseline specification without $\bar{x}$ suggest some differences in $\alpha_{k,j}$ across regions, this restriction may increase the variance in our estimates of α_k while aiding identification of $\bar{x}$ (survivorship bias is likely to be more common across regions). Our results suggest that the survivorship bias does not play an important role (see Table 15). Our estimates for $\bar{x}$ are insignificantly different from zero in the baseline specification and we find no evidence that the control function specification is preferable.[21] Our remaining point estimates are likewise essentially unaffected and we observe that the variance of α_k is actually smaller than in the individual regressions, which suggests that this restriction does not impose undue structure on the data. We conclude that our evidence for the deposit rates and interest elasticities reported above is unchanged by survivorship.

[21] For these regressions we report jackknife standard errors since bootstrapping our data is complicated by the inclusion of $\ln(1+\bar{x}^t)$.

Comparing all the results reported thus far for the $1,000, $100 and $50 denominations, and the detailed estimates presented in Tables 9 to 15, there appears to be little similarity in the deposit probabilities and the fractions of notes in circulation. We find evidence that the $1,000 denomination has statistically significant fixed costs and that its deposit rates rise as the nominal interest rate rises. We find no evidence of fixed costs for the $100 or $50 denominations and find statistically significant evidence that their deposit rates fall as the nominal interest rate rises.

We emphasize that our results are robust across regions, without imposing any cross-equation restrictions beyond the model structure. Indeed, we interpret the similarity of our results, for each denomination, across regions as validation for our theoretical model. If our theoretical choice model were grossly mis-specified, we would expect to see results vary across regions for each denomination, which we do not.

4.4 Regulatory Change and the Financial Crisis

The sample period from May 2000 until April 2015 for the $1,000 unfit notes encompasses a financial regulatory change for cash deposits that was ushered in after September 11th, 2001. In Canada, the government created Fintrac, an agency with oversight over financial transactions deemed suspicious (thresholds for such are legally defined). One regulatory change, enforced as of November 30, 2002, required financial institutions to report large cash deposits in excess of $10,000 and to require depositors to identify themselves and the source of the funds being deposited. This reporting requirement may have imposed a cost on deposits of bank notes, likely particularly so for large denominations such as the $1,000. Individuals processing large-value cash transactions regularly would be most likely to trigger the reporting requirements. To examine the impact of the regulatory change, we include a dummy variable in our deposit probability specification.[22]

A second, arguably unexpected, change that may have affected banknote deposits was the financial crisis arising from the Lehman Brothers bankruptcy.[23] One plausible mechanism is that bank deposits became less desirable because of depositor concerns over the soundness of the financial system, although we note that no Canadian financial insititution did fail during this period. We introduce a second dummy variable dated from September 2009 to account for the effect of the Lehman bankruptcy for the deposit probability of the $1,000 notes.

[22]Unfortunately, the sample periods for the $100 and $50 are too short (or too late) for a similar analysis.

[23]The exact timing of when the financial crisis became a salient effect is, of course, unclear. However, we do not find much difference due to alternative datings, such as the Bear Stearns bankruptcy.

We present the estimated coefficients for the Fintrac dummy variable and the Lehman bankruptcy dummy variable in Table 5. Our results suggest no evidence that the Fintrac legislation affected the costs of deposit of the $1,000 banknote. In contrast, the Lehman bankruptcy had a significant effect and reduced the costs of deposit in roughly half of the regions. In the final column of the table, we calculate the Lehman effect as a fraction of the expected fixed effect for deposit costs on deposits, $e^{-\ln(x_\mathsf{d})+q}/e^{-\ln(x_\mathsf{d})}$, where q is the estimated coefficient on the Lehman bankruptcy dummy. To a first-order approximation, this fraction represents the unconditional increase in deposit rates due to the Lehman effect. For the regions in which the Lehman bankruptcy dummy variable is significant, the estimated fraction of the fixed effect is roughly 200-300%. Of particular interest is that the regions in which the Lehman bankruptcy dummy is significant are also the regions in which the price level coefficient is estimated to be significant. One narrative is that depositors may have expected that the response of monetary authorities to the financial crisis was likely to lead to inflation and so their response was to deposit notes. We also note that the regions in which the Lehman bankruptcy dummy was significant constitute roughly four-fifths of the Canadian population.

Table 5: Marginal Effect on Deposits of Fintrac Legislation

	Fintrac Legislation			Lehman Bankruptcy		
Region	Estimate	Std Err	FE Share	Estimate	Std Err	FE Share
Calgary	0.26	0.32	129%	1.29	0.51	363%
Halifax	-0.09	0.12	91%	**0.57**	**0.14**	176%
Montréal	0.09	0.11	109%	**0.66**	**0.14**	193%
Newfoundland	0.03	0.21	103%	0.62	15.07	185%
Ottawa	0.27	0.24	130%	**1.20**	**0.43**	331%
Quebec City	0.11	0.14	112%	**0.81**	**0.21**	225%
Regina	0.40	0.53	150%	1.83	1.29	624%
Winnipeg	0.32	0.19	138%	0.89	0.39	243%
Toronto	*0.34*	*0.18*	140%	**1.25**	**0.32**	349%
Vancouver	*-0.01*	*0.16*	144%	***0.72***	***0.18***	297%

Notes: Italicized estimates are from the control function specification. Bold estimates are statistically significantly different from 0 at the 5% level. FE Share refers to the percentage of the deposit costs of the estimated dummy parameter.

5 Discussion

The estimates of the deposit elasticities and the proportion of notes held in the Unknown Region are evidence that banknote denominations circulate as different monies. In particular, the estimates suggest that $1,000 circulating as a single banknote or as ten $100 banknotes has (i) a different probability of deposit; (ii) an interest elasticity of deposit of opposite sign and (iii) a different fixed cost of deposit. These facts suggest that money balances exhibit non-neutrality, at least for Canadian money.

The estimates do not, however, identify the optimal divisibility of money. In most theoretical models of denomination structure, buyers and sellers should maintain sufficiently divisible cash balances to facilitate trade, though as Lee *et al.* (2005) note, too small a denomination is not welfare-improving either. Indeed, Telser (1995), Van Hove and Heyndels (1996) and Lee (2010) all consider the question of optimal denomination structures and, in general, their results suggest that the $1,000 has very limited value as a means to facilitate exchange. Our results regarding the non-neutrality of denominations appear consistent with the commodity-money, random-matching, environment proposed in Wallace and Zhu (2004). Our results are, however, not a formal test of that environment and we are sanguine about whether alternative model environments might also be consistent with our empirical results.

One issue that would appear relevant to the optimal divisibility literature, and does not appear to have been previously considered, is the different costs of depositing notes. The estimates imply no statistically significant fixed costs for deposits of the $50 and $100 denominations but do imply a statistically significant fixed cost for the $1,000. For the $1,000, our estimates of α_1 in all regions are roughly -4.5, which implies $1 - \tau_{\mathsf{d}} \approx 0.01$ for depositors. This implies that current period costs for depositing and then withdrawing the $1,000 note are much larger (approximately 100 times larger) than simply holding the note for the future period. This suggests that the costs of deposit for the $1,000 note are larger than simply the foregone interest from our discussion above and also suggests that the costs of deposit may reflect more than simply a monetary calculation. Although our model cannot distinguish between the costs of acceptance at merchants and the costs of depositing directly at financial institutions, it does suggest that the $1,000 note possesses characteristics that may make it costly as a medium of exchange. Returning to the $50 and $100 denominations, our estimates do not reject that $1 - \tau_{\mathsf{d}} = 1$ for these denominations.

A final observation is that our estimates for the Unknown Region may illuminate at least some of the differences in usage across denominations. The estimates suggest that the $1,000 note has the lowest deposit probability, the highest cost of deposit of the denominations we study and a positive interest rate elasticity of deposit. These observations are consistent with the $1,000 note functioning as a method of saving. This narrative is also consistent with our finding that the Lehman Brothers bankruptcy increased the deposit probability by 200-300% for the $1,000 note. In contrast, the estimated deposit costs for the $50 and $100 are insignificantly different from zero, so the lack of deposits is not due to deposit transaction costs. For the $50 and $100 notes, the size of the Unknown Regions appears consistent with usage in unmeasured parts of the economy. The deposit probabilities for the $100 and $50 denominations are negatively related to the nominal interest rate, which suggests that their intrinsic value as a medium of exchange increases in economic expansions (given that the short-term nominal interest rate is generally procyclical).

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A Appendix

Figure 2: $50 Unfit Note Deposits by Region

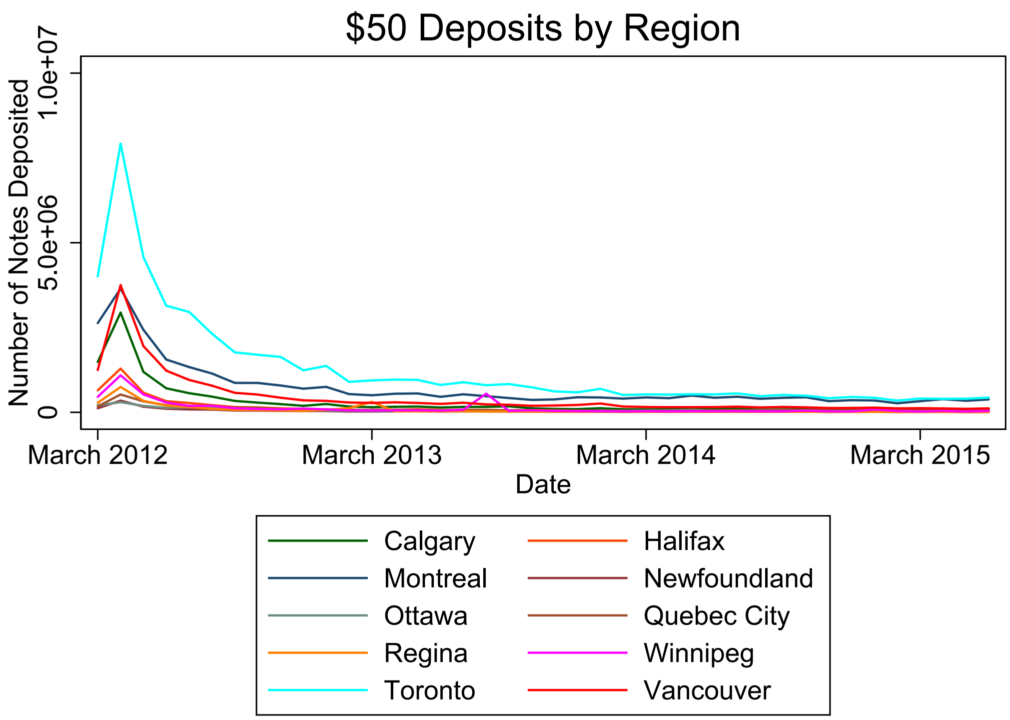

Figure 3: $100 Unfit Note Deposits by Region

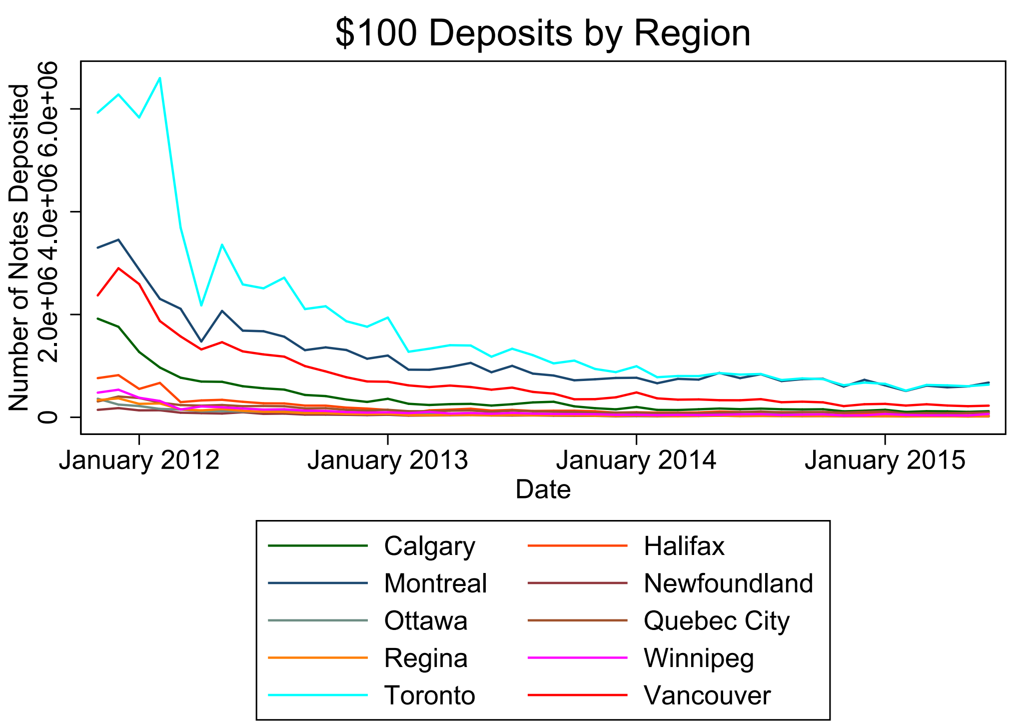

Figure 4: $1,000 Unfit Note Deposits by Region

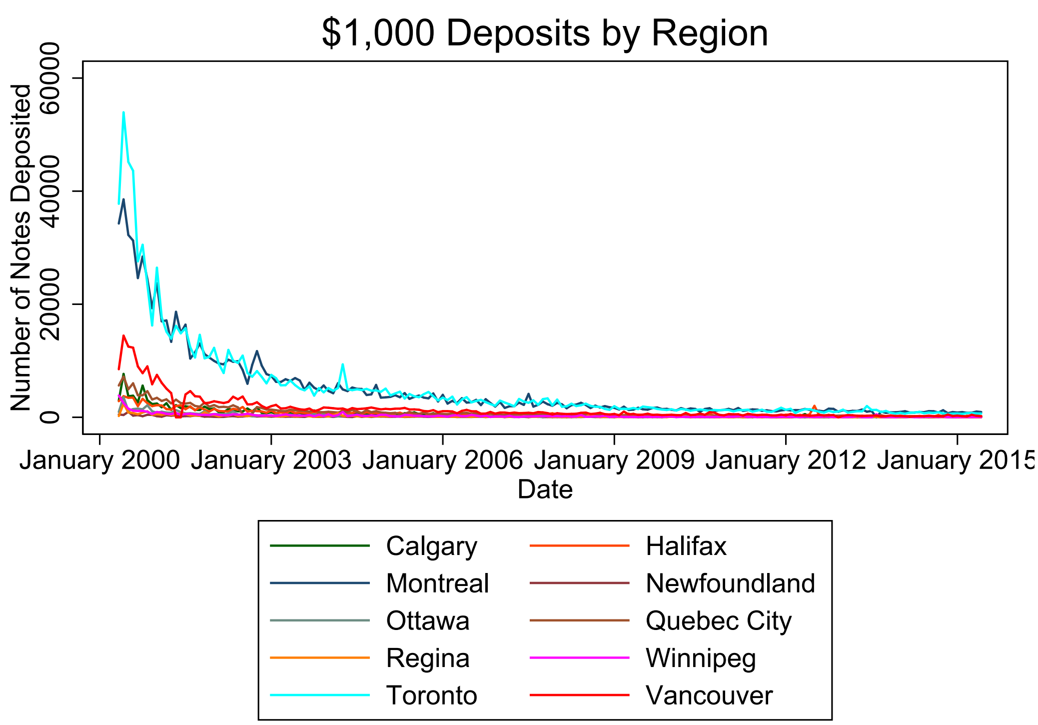

Table 6: Control Function Interest Rate Variable List

Bank rate, last Tuesday or last Thursday
Bank rate
Chartered bank administered interest rates -
prime business
Chartered bank - consumer loan rate
Forward premium or discount (-), United
States dollar in Canada: 1 month
Forward premium or discount (-), United
States dollar in Canada: 3 month
Prime corporate paper rate: 1 month
Prime corporate paper rate: 2 month
Prime corporate paper rate: 3 month
Bankers' acceptances: 1 month
Bankers' acceptances: 2 month
Bankers' acceptances: 3 month
Overnight money market financing, 7 day average
Selected Government of Canada benchmark bond yields: 2 year
Selected Government of Canada benchmark bond yields: 3 year
Selected Government of Canada benchmark bond yields: 5 year
Selected Government of Canada benchmark bond yields: 7 year
Selected Government of Canada benchmark bond yields: 10 years
Selected Government of Canada benchmark bond yields: long term
Government of Canada marketable bonds, average yield: 1-3 year
Government of Canada marketable bonds, average yield: 3-5 year
Government of Canada marketable bonds, average yield: 5-10 year
Government of Canada marketable bonds, average yield: over 10 years
Chartered bank - 90 day term deposits
Chartered bank - conventional mortgage: 1 year
Chartered bank - conventional mortgage: 3 year
Chartered bank - conventional mortgage: 5 year
Chartered bank - 5 year personal fixed term
Chartered bank - daily interest savings (balance over $100,000)
Chartered bank - non-chequable savings deposits
Chartered bank - Guaranteed Investment Certificates: 1 year
Chartered bank - Guaranteed Investment Certificates: 3 year
Chartered bank - Guaranteed Investment Certificates: 5 year
Treasury bill auction - average yields: 3 month
Treasury bill auction - average yields: 3 month, average at values
Treasury bill auction - average yields: 6 month
Treasury bill auction - average yields: 1 year
Treasury bill auction - amount auctioned: 3 month (dollars x 1,000,000)
Treasury bill auction - amount auctioned: 6 month (dollars x 1,000,000)
Treasury bill auction - amount auctioned: 1 year (dollars x 1,000,000)
Treasury bill auction - amount maturing (dollars x 1,000,000)
Treasury bills: 1 month
Treasury bills: 2 month
Treasury bills: 3 month
Treasury bills: 6 month
Treasury bills: 1 year
Real return bonds, long-term
Government of Canada marketable bonds, average yield, average of Wednesdays: 1-3
Government of Canada marketable bonds, average yield, average of Wednesdays: 3-5
Government of Canada marketable bonds, average yield, average of Wednesdays: 5-10
Government of Canada marketable bonds, average yield, average of Wednesdays: over 10
Average residential mortgage lending rate: 5 year
Chartered bank - chequable personal savings deposit rate
First coupon of Canada Savings Bonds

Table 7: Control Function Asset Market Variable List

Standard and Poor's/Toronto Stock Exchange Composite Index, high
Standard and Poor's/Toronto Stock Exchange Composite Index, low
Standard and Poor's/Toronto Stock Exchange Composite Index, close
Toronto Stock Exchange, stock dividend yields (composite), closing quotations
Standard and Poor's/Toronto Stock Exchange 60 Index
Standard and Poor's/Toronto Stock Exchange Canadian Consumer Discretionary Index
Standard and Poor's/Toronto Stock Exchange Canadian Consumer Staples Index
Standard and Poor's/Toronto Stock Exchange Canadian Energy Index
Standard and Poor's/Toronto Stock Exchange Canadian Financial Index
Standard and Poor's/Toronto Stock Exchange Canadian Gold Index
Standard and Poor's/Toronto Stock Exchange Canadian Industrial Index
Standard and Poor's/Toronto Stock Exchange Canadian Information Technology Index
Standard and Poor's/Toronto Stock Exchange Canadian Materials Index
Standard and Poor's/Toronto Stock Exchange Canadian Diversified Metals and Mining
Standard and Poor's/Toronto Stock Exchange Canadian Telecommunication Service
Standard and Poor's/Toronto Stock Exchange Canadian Utilities Index

Table 8: Control Function Foreign Exchange Market Variable List

United States dollar, noon spot rate, average
United States dollar, 90-day forward noon rate
Danish krone, noon spot rate, average
Japanese yen, noon spot rate, average
Norwegian krone, noon spot rate, average
Swedish krona, noon spot rate, average
Swiss franc, noon spot rate, average
United Kingdom pound sterling, noon spot rate, average
United Kingdom pound sterling, 90-day forward noon rate
United States dollar, closing spot rate
United States dollar, highest spot rate
United States dollar, lowest spot rate
United States dollar, 90-day forward closing rate
United States dollar, 30-day forward closing rate
United States dollar, 60-day forward closingrate
United States dollar, 180-day forward closing rate
United States dollar, 1-year forward closing rate
Australian dollar, noon spot rate, average
Hong Kong dollar, noon spot rate, average
New Zealand dollar, noon spot rate, average
Mexican pesos, noon spot rate, average
Canadian dollar effective exchange rate index (CERI) (1992=100)
European euro, noon spot rate, average

Table 9: Estimates by Region, $1,000 Denomination

Region	1		2		3		4		5	
	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.
T Bill	255.36	75.74	157.96	33.21	158.02	22.56	235.31	81.53	154.40	72.61
CPI	6.83	3.51	3.51	1.27	9.09	1.90	1.47	2.22	14.01	4.94
x_d	-4.12	0.28	-4.51	0.14	-3.92	0.10	-4.81	0.26	-3.60	0.37
Q3	-0.34	0.26	-0.02	0.11	-0.10	0.09	0.32	0.19	-0.65	0.22
Q4	-0.16	0.28	-0.08	0.10	-0.12	0.10	0.02	0.15	-0.37	0.23
Q1	-0.50	0.29	-0.19	0.10	-0.16	0.09	-0.17	0.19	-0.62	0.19
Fintrac1	0.26	0.32	-0.09	0.12	0.09	0.11	0.03	0.21	0.27	0.24
Lehman	1.29	5.12E-01	0.57	0.14	0.66	0.14	0.62	15.07	1.20	0.43
M_0	138235.20	22292.19	155406.30	9724.87	1094830.00	40580.76	18880.57	1081.80	69329.22	7449.63
R^2	0.92		0.94		0.98		0.86		0.93	
Nobs	181		181		181		181		181	
Region	6		7		8		9		10	
	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.
T Bill	18.38	4.09	24.69	11.08	24.82	7.18	26.83	6.11	29.88	7.29
CPI	-0.21	1.13	2.83	1.80	0.57	0.86	0.72	1.27	0.16	1.33
x_d	-4.29	0.15	-4.22	0.40	-4.61	0.23	-4.38	0.24	-4.80	0.41
Q3	-0.14	0.13	-0.67	0.35	-0.22	0.23	-0.06	0.18	0.18	0.37
Q4	-0.15	0.12	-0.49	0.34	-0.42	0.25	-0.32	0.21	-0.04	0.42
Q1	-0.18	0.12	-0.78	0.42	-0.34	0.21	-0.24	0.19	0.03	0.37
Fintrac1	-0.23	0.08	-0.45	2.49E+05	0.00	0.11	-0.23	0.12	-0.17	0.18
Lehman	0.44	13.23	0.87	46054.75	0.69	0.26	0.69	1.02E+08	0.83	3.74E+09
M_0	184579	2766	35808	2038	53483	1132	1004240	19981	285887	8118
R^2	0.95		0.84		0.89		0.95		0.92	
Nobs	181		181		181		181		181	

Table 10: Estimates by Region Control Function Correction, $1,000 Denomination

Region	1		2		3		4		5	
	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.
Treasury Bill	182.83	64.04	140.69	39.65	148.12	25.60	146.59	49.08	148.82	53.64
CPI	4.27	1.92	2.77	1.36	7.54	1.70	-0.15	1.58	7.29	2.60
x_d	-4.43	1.03	-4.54	0.16	-4.04	0.11	-4.61	0.20	-3.95	0.24
Q3	0.02	0.14	0.01	0.11	-0.02	0.09	0.14	0.13	-0.25	0.12
Q4	0.15	0.15	-0.05	0.09	-0.03	0.08	0.05	0.15	-0.07	0.17
Q1	-0.03	0.12	-0.12	0.09	-0.05	0.07	-0.12	0.18	-0.34	0.13
CF1	-329.94	411.60	-295.84	180.07	-187.77	140.19	-206.85	274.77	-118.10	273.31
CF2	7.50	5.72	3.78	4.69	4.84	5.41	3.37	6.08	0.96	8.65
Fintrac1	-0.05	0.15	-0.14	0.12	-0.01	0.09	-0.09	0.14	0.02	0.14
Lehman	0.52	0.24	0.42	0.14	0.45	0.11	0.26	0.15	0.57	1.62E-01
M_0	150402	11700000	156804	12483	1107789	72123	18991	4011	64471	10686.07
R^2	0.94		0.93		0.98		0.89		0.94	
Nobs	181		181		181		181		181	
Region	6		7		8		9		10	
	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.
Treasury Bill	15.82	4.43	8.20	5.83	16.33	3.87	21.26	5.46	24.79	5.81
CPI	0.32	2.06	1.70	3.16	-1.09	2.10	-1.38	2.90	1.03	3.63
x_d	-4.31	0.16	-4.12	0.35	-4.65	0.16	-4.43	0.20	-4.99	0.44
Q3	-0.03	0.11	-0.15	0.18	0.05	0.12	0.06	0.11	0.55	0.34
Q4	-0.03	0.09	0.01	0.21	-0.07	0.15	-0.05	0.14	0.36	0.38
Q1	-0.05	0.11	-0.10	0.17	-0.06	0.13	-0.11	0.16	0.49	0.35
CF1	-18.03	18.05	-6.78	27.95	-37.97	23.67	-41.17	17.84	-36.27	17.57
CF2	-0.90	2.15	1.17	3.54	-1.47	2.55	-3.77	2.95	1.47	4.40
Fintrac1	-0.24	0.07	-0.38	0.16	0.04	0.12	-0.23	0.09	-0.21	0.10
Lehman	-0.14	0.13	-0.67	0.35	-0.22	0.23	-0.06	0.18	0.18	0.37
M_0	186417	1727	36702	982	54607	1823	1020626	10521	290462	3871
R^2	0.96		0.87		0.91		0.97		0.94	
Nobs	181		181		181		181		181	

CF1 and CF2 refer to control functions for the nominal interest rate and inflation respectively.

Table 11: Estimates by Region, \$100 Denomination

Region	1		2		3		4		5
	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.
Treasury Bill	-5164.68	1217.93	-1850.88	1679.86	-2333.53	916.95	-2164.44	1142.56	-3523.74
CPI	-3.99	4.18	19.78	12.29	6.87	9.24	12.72	6.35	-4.13
x_{d}	1.35	0.98	-1.22	1.30	-1.23	0.70	-1.04	0.96	-0.30
Q3	0.10	0.13	-0.08	0.13	0.03	0.09	-0.08	0.12	0.16
Q4	-0.11	0.13	-0.12	0.16	-0.06	0.08	-0.16	0.12	-0.02
Q1	-0.06	0.13	-0.05	0.22	-0.05	0.09	-0.11	0.12	0.03
M_0	1.67E+07	3.15E+05	9.55E+06	2.61E+06	5.83E+07	3.87E+06	2.33E+06	2.16E+05	3.71E+06
R^2	0.99		0.96		0.99		0.98		0.98
Nobs	42		42		42		42		42

Region	6		7		8		9		10
	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.
Treasury Bill	-2199.04	600.64	-3417.71	1186.15	-4272.23	1635.00	-2790.59	1086.81	-3265.00
CPI	9.13	7.50	19.41	11.21	8.57	6.28	2.52	8.85	0.12
x_{d}	-1.38	0.50	0.25	0.95	0.64	1.32	-0.62	0.85	-0.09
Q3	0.07	0.08	-0.08	0.12	0.04	0.15	0.11	0.13	0.07
Q4	-0.13	0.09	-0.24	0.15	-0.08	0.16	0.00	0.14	-0.23
Q1	-0.07	0.08	-0.13	0.14	-0.10	0.18	0.24	0.18	-0.08
M_0	7630105	580253	3382492	464735	5144663	291796	7.93E+07	9009425	3.36E+07
R^2	0.98		0.99		0.97		0.97		0.99
Nobs	42		42		42		42		42

Table 12: Estimates by Region Control Function Correction, \$100 Denomination

Region	1		2		3		4		5	
	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.
Treasury Bill	-4022.48	1585.27	-5456.78	3169.10	-2911.57	1357.59	-1710.04	1388.75	-2502.37	1085.71
CPI	-0.83	6.81	-0.33	17.32	-13.07	9.96	10.23	8.73	-9.42	5.72
x_{d}	0.47	1.34	1.59	2.60	-0.96	1.03	-1.42	1.25	-1.18	0.88
Q3	0.07	0.09	0.02	0.13	0.10	0.07	-0.08	0.10	0.16	0.07
Q4	-0.26	0.12	-0.27	0.18	0.00	0.09	-0.26	0.13	-0.07	0.10
Q1	-0.25	0.15	-0.81	0.27	-0.09	0.12	-0.14	0.15	-0.03	0.12
CF1	3917.31	1870.07	8011.50	4695.74	2097.47	1603.21	2519.62	2719.76	2231.44	1671.82
CF2	-5.18	13.23	-52.51	34.30	-20.21	17.01	-12.82	15.76	-3.24	15.22
CF3	-3.67E+03	4.56E+03	-9.52E+02	1.76E+05	-4.93E+03	9.99E+03	-5.16E+03	1.86E+04	-4.78E+03	5.13E+03
M_0	1.72E+07	4.34E+06	8.62E+06	9.43E+06	5.33E+07	3.56E+06	2.35E+06	2.12E+06	3.75E+06	1.86E+05
R^2	0.99		0.97		0.99		0.98		0.98	
Nobs	42		42		42		42		42	

Region	6		7		8		9		10	
	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.
Treasury Bill	-2336.23	1029.18	-4135.39	2256.84	-1392.62	1216.50	-3397.57	2635.08	-2613.23	875.10
CPI	-4.26	9.39	13.88	17.22	9.98	7.06	1.79	15.48	-21.93	10.14
x_{d}	-1.39	0.80	0.82	1.97	-1.74	1.31	-0.13	2.08	-0.75	0.72
Q3	0.11	0.06	-0.03	0.12	-0.03	0.10	0.13	0.15	0.10	0.05
Q4	-0.05	0.09	-0.22	0.20	-0.18	0.14	0.06	0.20	0.02	0.11
Q1	-0.14	0.09	-0.22	0.23	-0.13	0.18	0.21	0.24	-0.01	0.11
CF1	1739.19	1246.52	2202.62	3634.54	846.71	2563.97	2091.57	4047.25	857.42	1287.94
CF2	-6.65	14.71	-17.77	25.15	-12.24	29.18	-23.56	33.61	-23.56	12.27
CF3	-8217.09	6216.39	-6249.62	4332111.00	-8502.15	4826084.00	-6985.95	48004.14	-5066.81	3526.53
M_0	7.11E+06	6.22E+05	3.32E+06	1.63E+08	5.64E+06	3.09E+08	7.85E+07	2.77E+07	3.33E+07	4.20E+05
R^2	0.99		0.98		0.97		0.97		1.00	
Nobs	42		42		42		42		42	

CF1, CF2 and CF3 refer to control functions for the nominal interest rate, inflation and polymer unfit notes, respectively.

Table 13: Estimates by Region, $50 Denomination

Region	1		2		3		4		5	
	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.
Treasury Bill	-4455.28	3524.81	-7770.01	3688.24	-5671.47	2173.27	-5104.17	2345.79	-5982.17	1723.83
CPI	17.36	13.85	-4.89	13.96	-10.95	10.84	0.21	10.10	-12.06	5.36
x_d	18.66	12.12	-0.93	13.13	-9.04	9.89	1.98	8.99	-10.31	4.95
Q3	-0.20	0.24	-0.14	0.26	-0.11	0.15	-0.12	0.18	0.28	0.14
Q4	-0.75	0.28	-0.61	0.26	-0.41	0.19	-0.49	0.20	-0.31	0.19
Q1	-0.78	0.29	-1.17	4.70E+09	-0.65	0.26	-0.70	0.27	-0.60	0.20
M_0	13900000.00	583563.50	5860054.00	244323.70	32800000.00	2045529.00	1782229.00	111964.60	3386818.00	218132.10
R^2	0.96		0.95		0.97		0.97		0.95	
Nobs	38		38		38		38		38	
	6		7		8		9		10	
	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.
Treasury Bill	-6295.69	2019.85	-4740.57	6334.79	-5173.80	11762.56	-5192.78	1603.60	-6222.52	1995.37
CPI	-1.75	11.06	-18.04	28.69	-8.98	80.01	-4.18	8.07	-37.48	19.32
x_d	0.47	10.30	-16.50	31.00	-7.43	83.58	-2.46	7.47	-34.77	19.65
Q3	0.02	0.19					-0.01	0.13	-0.07	0.17
Q4	-0.36	0.24					-0.32	0.15	-0.35	1.84E-01
Q1	-0.67	0.29					-0.52	0.21	-0.63	0.23
M_0	4.19E+06	5.69E+08	3.20E+06	1.27E+05	5.32E+06	6.92E+05	6.10E+07	2.04E+06	1.93E+07	2.58E+05
R^2	0.94		0.76		0.64		0.98		0.98	
Nobs	38		38		38		38		38	

Table 14: Estimates by Region Control Function Correction, $50 Denomination

Region	1		2		3		4		5	
	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.
Treasury Bill	-4194.053	2305.624	-4435.94	2794.19	-2911.55	2710.23	-805.51	1155.17	-2895.38	3236.16
CPI	7.306262	6.960092	7.88	8.64	-6.67	10.98	16.46	6.63	-2.36	8.35
x_d	7.868348	6.43754	8.67	7.75	-7.38	10.21	14.06	6.10	-3.65	7.90
Q3	0.2927157	0.117428	0.21	0.14	0.17	1.51	0.22	0.11	0.49	0.94
Q4	-0.2346496	0.1595193	-0.17	0.17	-0.07	1.51	-0.12	0.12	-0.09	0.97
Q1	-0.3997752	0.2520622	-0.64	0.31	-0.26	1.49	-0.17	0.14	-0.26	0.98
CF1	1782.42	3410.031	2536.24	4282.47	948.77	2834.00	-2837.51	2970.29	-1199.73	4659.48
CF2	5.942883	17.95248	-6.69	21.56	-18.19	22.32	0.48	16.63	6.29	37.90
CF3	2.83E+02	7.67E+03	0.04	0.97	-271.85	1484.58	-1326.09	4107.36	-303.56	38090.16
M_0	1.39E+07	1.09E+07	6.11E+06	3.05E+08	3.48E+07	3.11E+06	2.05E+06	2.31E+05	4.12E+06	3.47E+08
R^2	0.98	0.00	0.98		0.98		0.99		0.95	
Nobs	38		38		38		38		38	
Region	6		7		8		9		10	
	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.	Est.	Std Err.
Treasury Bill	-4571.06	3620.60	14509.59	36583.84	77090.26	46778.18	-3182.16	1725.48	-4791.04	1948.42
CPI	7.85	14.32	-27.99	94.14	-55.57	61.82	4.31	6.09	-22.63	12.11
x_d	8.18	13.13	-42.77	95.97	-124.62	77.80	4.12	5.73	-21.37	11.75
Q3	0.35	0.19	0.73	31.25	4.88	2154.76	0.19	0.10	0.20	0.11
Q4	0.02	0.27	0.61	31.38	4.91	4.50	-0.09	0.14	-0.07	0.13
Q1	-0.34	0.37	1.77	31.37	6.92	5.34	-0.25	0.20	-0.33	0.20
CF1	3619.89	4511.32	16001.62	49427.38	-61371.65	69199.88	330.25	2853.92	1304.46	2728.31
CF2	13.78	40.65	57.13	166.97	21.54	319.91	-4.01	19.21	-19.34	22.78
CF3	-2.25E+03	3.19E+05	4.49E+02	1.06E+07	1.75E+04	1.17E+07	-2.54E+01	1.16E+03	-1.10E+03	1.07E+03
M_0	4663979	5.31E+06	3160480	5.34E+06	5261994	11E+08	6.32E+05	3100789	1.95E+05	408738
R^2	0.95		0.85		0.81		0.99		0.99	
Nobs	38		38		38		38		38	

CF1, CF2 and CF3 refer to control functions for the nominal interest rate, inflation and polymer unfit notes, respectively.

Table 15: Depositor Heterogeneity $1,000 Denomination: NLSUR

	NLSUR		CF NLSUR	
	Est.	Std Err.	Est.	Std Err.
Treasury Bill	197.375	32.66484	114.7382	393.8377
CPI	9.518971	3.331117	5.96606	12.10477
x_{d}	-0.065	0.182885	-4.13632	980946.9
$\bar{x}$	0.027497	0.128873	0.819291	0.372474
Q3	-0.065	0.182885	-0.03113	0.250461
Q4	-0.14904	0.189218	-0.06698	0.374522
Q1	-0.1602	0.157171	-0.04228	0.283553
Fintrac1	7.24E-05	0.112453	-0.08597	980945
Lehman	0.386423	0.158838	0.068536	12.99774
CF1			-246.622	377.9773
CF2			1.957839	17.54737
$M_{0,\text{Calgary}}$	151790.4	28141.99	147109.6	172292.1
$M_{0,\text{Halifax}}$	125146.4	17361.2	129626.6	500937.1
$M_{0,\text{Montréal}}$	1094248	185514.3	1066351	572176.5
$M_{0,\text{Newfoundland}}$	17476.43	1869.902	18397.43	73276.98
$M_{0,\text{Ottawa}}$	63306.99	12457.12	61078.53	60700.49
$M_{0,\text{Quebec City}}$	190094.7	31571.51	186433.6	199776.4
$M_{0,\text{Regina}}$	36257.92	5958.578	36421.12	144342.7
$M_{0,\text{Winnipeg}}$	58190.18	9174.65	53073.56	174896.2
$M_{0,\text{Toronto}}$	1256367	236323.6	1191436	1311358
$M_{0,\text{Vancouver}}$	326647.7	51520.16	325302.6	176088.2

CF1 and CF2 refer to control functions for the nominal interest rate and inflation, respectively.