Identification of Structural Vector Autoregressions Through Higher Unconditional Moments*

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Abstract

This paper pursues two objectives. First, we determine the identification conditions of SVAR processes through the third and fourth unconditional moments of the statistical innovations. Our findings provide novel insights when the entire system is not identified, as they highlight which subset of structural parameters is identified and which is not. Second, we elaborate a tractable testing procedure to verify whether the identification conditions hold, prior to the estimation of the structural parameters of the SVAR process. To do so, we design a new bootstrap procedure that improves the small-sample properties of rank tests for the symmetry and kurtosis of the structural shocks.
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1. Introduction

Econometric methods for simultaneous equation models highlight the importance of verifying the identification conditions before proceeding to the (often challenging) estimation exercise of the model parameters. Namely, it is only if the identification conditions hold that it becomes worthwhile to perform the estimation of all the model parameters, since otherwise the estimation is doomed to fail. In this vein, this paper pursues two objectives. First, we derive the identification conditions for Structural Vector Autoregressive (SVAR) processes through higher unconditional moments. Second, we develop a tractable method to verify whether a SVAR process is identified, prior to the estimation of the structural parameters.

SVAR processes represent systems of simultaneous, dynamic, linear equations, in which the structural parameters reflect the contemporaneous interactions across the current variables of interest and the dynamic feedbacks between these current variables and their lagged values. Such processes are frequently used in macroeconomics to assess the dynamic responses of the variables of interest to various structural, or economic, shocks.

Conventional SVAR analyses assume, either explicitly or implicitly, that the structural shocks are generated from unconditional normal distributions. In this context, the unconditional covariances of the statistical innovations is the only information that can be used to identify the structural parameters. As is well known, this information is insufficient to identify all the parameters, so that short-run, long-run, or sign restrictions, which may be dubious, need to be placed on certain parameters. Unfortunately, the imposed restrictions are not innocuous as they often alter the sign, magnitude, and shape of dynamic responses. Moreover, the validity of the restrictions cannot be verified by applying formal statistical tests.

Recent developments in independent component analysis consider the possibility that some structural shocks follow unconditional non-normal distributions. In this environment, all the structural parameters invloved in the SVAR are locally identified, without placing any restrictions, when at least all, but one, structural shocks are non-normally distributed (see Comon, 1994; Eriksson and Koivunen, 2004; Gouriéroux, Monfort, and Renne, 2017a). Then, the dynamic response matrices are unique, up to changes in sign and permutations of columns. It also becomes possible to perform statistical tests to verify the relevance of commonly imposed restrictions.[1]

A key goal of this paper is to determine the identification conditions of SVAR processes through the third and fourth unconditional moments of the statistical innovations. Intuitively, not only the covariances of the statistical innovations, as in conventional analyses, but also the coskewnesses and excess cokurtoses can be exploited to identify extra structural parameters, and, hence, to relax some of the debatable restrictions. We derive the order (necessary) and rank (sufficient) conditions for local identification, where the latter are obtained by extending the developments of Lütkepohl (2007) — rather than relying on the properties of multivariate normal distributions as in independent component analysis. We further express these conditions in terms of simple formulas, which exclusively involve the numbers of statistical innovations and structural shocks displaying skewness and excess kurtosis. Given this information, it is most easy for empirical researchers to determine whether or not the structural system is identified.

Our findings regarding the identification of the entire structural system parallel the existing results. That is, all the structural parameters are identified when at least all, but one, structural shocks exhibit skewness and/or excess kurtosis.[2] Our findings further provide novel insights when the entire SVAR process is not identified, as they highlight which subset of structural parameters is identified and which is not. This leads to two important implications. The first implication is that one can establish which structural subsystem is identified. Note that this subsystem documents the dynamic responses of all the variables included in the SVAR process to the structural shocks which are asymmetric and/or non-mesokurtic. Hence, these responses can be traced without imposing any restrictions on the structural parameters. The second implication is that one can determine the structural parameters for which some restrictions must be placed on in order to achieve the identification of the entire system. Such restrictions are required to evaluate the dynamic responses to the structural shocks which are symmetric and mesokurtic.

[1] In contrast to local identification, global identification often requires the imposition of ad hoc restrictions (see Gouriéroux, Monfort, and Renne, 2017b; Lanne, Meitz, and Saikkonen, 2017). These restrictions insure that the dynamic response matrices are unique, although the structural shocks usually have no economic meaning.

[2] The number of asymmetric and/or non-mesokurtic statistical innovations must also be, at least, as large as the number of structural shocks featuring skewness and/or excess kurtosis.

Another prime aim of this paper is to elaborate a tractable testing procedure to verify whether the identification conditions hold, prior to the estimation of the structural parameters involved in the SVAR process. As stated above, our identification conditions require the knowledge of the numbers of statistical innovations displaying skewness and excess kurtosis. In principle, such information can be inferred by applying Jarque-Bera tests for symmetry and kurtosis, given that the statistical innovations are readily measured from the reduced-form residuals. The identification conditions also necessitate the determination of the numbers of asymmetric and non-mesokurtic structural shocks. At first glance, this may seem problematic for practitioners, as the structural shocks become measurable only once the structural system is estimated.[3] However, we demonstrate that the numbers of structural shocks displaying skewness and excess kurtosis correspond to the ranks of the coskewness and excess cokurtosis matrices of the statistical innovations, where these matrices are easily constructed from the sample estimates of the moments of the reduced-form residuals — without having to proceed to the estimation of the structural system.

In practice, it is recommended to perform Kilian and Demiroglu's (2000) bootstrap procedure to approximate the finite-sample distributions of Jarque-Bera tests for the symmetry and kurtosis of the statistical innovations. It has been documented that this procedure substantially improves the finite-sample properties of the test for kurtosis, given that the finite-sample critical values converge extremely slowly to their asymptotic counterparts. In this paper, we design a new bootstrap procedure to approximate the finite-sample distributions of rank tests for the symmetry and kurtosis of the structural shocks. We show that this procedure allows to overcome size distortions. Specifically, for symmetry both the Wald and likelihood-ratio versions of the rank test with bootstrap critical values feature empirical sizes that are almost identical to the nominal sizes, regardless of the number of observations in the sample. In comparison, the Wald test with asymptotic distribution has empirical sizes that slightly deviate from the nominal ones, and the likelihood-ratio test with limiting distributions has empirical sizes that are substantially smaller than the nominal counterparts. For kurtosis, the bootstrap version of the Wald and likelihood-ratio test statistics are essentially

[3] As a result, existing studies do not verify whether the structural shocks are asymmetric or non-mesokurtic before proceeding to the estimation of the structural system; see for example Moneta, Entner, Hoyer, and Coad (2013), Lanne, Meitz, and Saikkonen (2017), and Gouriéroux, Monfort, and Renne (2017b).

free of size distortions for all sample sizes. In sharp contrast, the Wald and likelihood-ratio tests with asymptotic distributions imply empirical sizes that are systematically close to zero, even for large samples.

Finally, we illustrate our developments by identifying the effects of fiscal policies on economic activity; a topic that has received renewed interest in light of the recent great recession. For this purpose, we perform the analysis on a trivariate SVAR process which include taxes, public spending, and output for quarterly U.S. data over the post-1980 period. The empirical results for Jarque-Bera tests with finite-sample distributions indicate that the hypothesis of symmetry is not rejected for all statistical innovations, while the hypothesis of zero excess kurtosis is rejected only for the statistical innovation associated with taxes. Furthermore, the Wald and likelihood-ratio bootstrap versions for the rank tests imply that all the structural shocks are symmetric and only the tax shock is non-mesokurtic. Based on these informations, both the order and rank conditions reveal that the subsytem relating all the variables to the tax shock is identified, whereas the subsytem relating the variables to the spending shock is under-identified. Accordingly, the dynamic response of output following a tax shock can be assessed without imposing any restrictions on the structural parameters, whereas the dynamic response to a spending shock can only be evaluated under certain restrictions. Furthermore, we show that the restrictions invoked in the seminal study of Blanchard and Perotti (2002) imply that the subsytem relating the variables to the spending shock becomes over-identified, which can alter the dynamic response relative to that obtained under exact identification. Instead, we explore alternative sets of restrictions that lead to exact identification. Our findings suggest that the dynamic tax multiplier is uniquely defined, but is always less than one as it reaches a peak of 0.61. In contrast, the dynamic spending multiplier depend on which of our set of restrictions is used, but it always attains values larger than one at impact and at the peak.

This paper is organized as follows. Section 2 motivates, from an economic perspective, the identification through higher unconditional moments. Section 3 derives the order and rank conditions for the identification of the structural parameters involved in SVAR processes. Section 4 develops a tractable testing procedure to test whether the identification conditions hold, before the estimation of the structural parameters. Section 5 presents an application related to the identification of the structural parameters determining the dynamic responses of output to fiscal shocks. Section 6 concludes.

2. Motivation

This section motivates the strategy of identifying SVAR processes through higher unconditional moments. To do so, we provide a simple example to gain some economic intuition about how the information related to asymmetric and non-mesokurtic distributions can be exploited to achieve identification. Specifically, we consider the following bivariate SVAR process (in innovation form):

$$\nu_{y,t} = -\alpha_d \nu_{p,t} + \omega_d \epsilon_{d,t}, \tag{1}$$

$$\nu_{p,t} = \alpha_s \nu_{y,t} + \omega_s \epsilon_{s,t}. \tag{2}$$

This system expresses a downward-sloping demand curve (1) and an upward-sloping (inverse) supply curve (2) of a good. The terms $\nu_{y,t}$ and $\nu_{p,t}$ represent the statistical innovations associated with the quantity and price of the good, while $\epsilon_{d,t}$ and $\epsilon_{s,t}$ are structural shocks capturing the demand and supply shocks with the following unconditional scedastic structure: $\sigma_{\epsilon,dd} = E[\epsilon_{d,t}^2] = 1$, $\sigma_{\epsilon,ss} = E[\epsilon_{s,t}^2] = 1$, and $\sigma_{\epsilon,ds} = E[\epsilon_{d,t}\epsilon_{s,t}] = 0$. The positive parameters α_d and α_s are related to the slopes of the demand and supply curves, whereas the positive parameters ω_d and ω_s are related to the shifts of the curves following demand and supply shocks.

System (1)–(2) involves four parameters that have to be identified: α_d, α_s, ω_d, and ω_s. As usual, three of these parameters, say for illustration purposes, α_d, ω_d, and ω_s, can potentially be identified through the distinct elements of the unconditional covariance matrix of the statistical innovations: $\sigma_{\nu,yy} = E[\nu_{y,t}^2]$, $\sigma_{\nu,pp} = E[\nu_{p,t}^2]$, and $\sigma_{\nu,yp} = E[\nu_{y,t}\nu_{p,t}]$. Importantly, the remaining parameter, α_s, can potentially be identified through higher unconditional moments, reflecting, for example, asymmetric and non-mesokurtic distributions.

As a starting point, Figures 1 and 2 depict the densities and the scatter plot of simulated shocks for the case where $\epsilon_{d,t}$ and $\epsilon_{s,t}$ are normally distributed. The simulations are generated for the following parametrization of equations (1)–(2): $\alpha_d = \alpha_s = 0.5$, $\omega_d = \omega_s = 1$, $\epsilon_{d,t} \sim N(0,1)$, and $\epsilon_{s,t} \sim N(0,1)$.[4] As expected, $\nu_{y,t}$ and $\nu_{p,t}$ are also normally distributed and the realizations of these innovations form a spherical cloud in the ($\nu_{y,t}$, $\nu_{p,t}$) plan. In this context, shifts in the demand and supply curves are as likely to generate the realizations of $\nu_{y,t}$ and $\nu_{p,t}$. Consequently, these realizations are not informative about the slope of either of the two curves, so that α_s cannot be identified. In this context, possible identification strategies are to impose one short-run restriction (e.g. Sims, 1980), long-run restriction (e.g. Blanchard and Quah, 1989), or sign restriction (e.g. Uhlig, 2005) in order to identify α_s.

Figures 1 and 2 also show the case where $\epsilon_{d,t}$ follows a mixture of normal distributions, while $\epsilon_{s,t}$ is normally distributed: $2.1755 \times \epsilon_{d,t} \sim N(1,1)$ with probability 0.7887 and $2.1755 \times \epsilon_{d,t} \sim N(-3.7326,1)$ with probability 0.2113, whereas $\epsilon_{s,t} \sim N(0,1)$. The resulting density of $\epsilon_{d,t}$ is mesokurtic as it is characterized by a zero excess kurtosis, as for the normal distribution, but it displays an asymmetry given that it is left skewed.[5] This third moment of $\epsilon_{d,t}$ implies a negative skewness that is more pronounced for $\nu_{y,t}$ than for $\nu_{p,t}$. As a result, the scatter plot of $\nu_{y,t}$ and $\nu_{p,t}$ exhibits an elliptical shape along the supply curve. This occurs because large negative values are more often observed for $\epsilon_{d,t}$ (than for $\epsilon_{s,t}$), and this induces substantial leftward shifts of the demand curve (relative to those associated with the supply curve). These shifts of the demand curve imply movements along the supply curve, so that it becomes possible to identify the slope of the supply curve, α_s.

Finally, Figures 3 and 4 display the case where $\epsilon_{d,t}$ follows a Student's t-distribution, while $\epsilon_{s,t}$ is normally distributed: $1.291 \times \epsilon_{d,t} \sim t(5)$ and $\epsilon_{s,t} \sim N(0,1)$. For this parametrization, the density of $\epsilon_{d,t}$ is symmetric, similarly to the normal distribution, but it is leptokurtic given that it has fat tails.[6] The fourth moment of $\epsilon_{d,t}$ translates into a large positive excess kurtosis for $\nu_{y,t}$ and a small excess kurtosis for $\nu_{p,t}$. This leads to an elliptical shape along the supply curve, where the extreme

[4] As is well known, this parametrization implies the following unconditional moments for the demand and supply shocks: the expectations are $E[\epsilon_{d,t}] = 0$ and $E[\epsilon_{s,t}] = 0$, the variances are $E[\epsilon_{d,t}^2] = 1$ and $E[\epsilon_{s,t}^2] = 1$, the skewnesses are $s_{\epsilon,d,dd} = E[\epsilon_{d,t}^3] = 0$ and $s_{\epsilon,s,ss} = E[\epsilon_{s,t}^3] = 0$, and the excess kurtoses are $\kappa^e_{\epsilon,dd,dd} = E[\epsilon_{d,t}^4] - 3 = 0$ and $\kappa^e_{\epsilon,ss,ss} = E[\epsilon_{s,t}^4] - 3 = 0$.

[5] The unconditional moments of the demand shock are as follows: the expectation is $E[\epsilon_{d,t}] = 0$, the variance is $E[\epsilon_{d,t}^2] = 1$, the skewness is $s_{\epsilon,d,dd} = E[\epsilon_{d,t}^3] = -0.9907$, and the excess kurtosis is $\kappa^e_{\epsilon,dd,dd} = E[\epsilon_{d,t}^4] - 3 = 0$.

[6] Specifically, the unconditional moments of the demand shock are the following: the expectation is $E[\epsilon_{d,t}] = 0$, the variance is $E[\epsilon_{d,t}^2] = 1$, the skewness is $s_{\epsilon,d,dd} = E[\epsilon_{d,t}^3] = 0$, and the excess kurtosis is $\kappa^e_{\epsilon,dd,dd} = E[\epsilon_{d,t}^4] - 3 = 6$.

realizations of $\epsilon_{d,t}$ (compared to those of $\epsilon_{s,t}$) generate pronounced leftward and rightward shifts of the demand curve (relative to those associated with the supply curve). Again, these shifts of the demand curve imply movements along the supply curve, so the slope parameter α_s is identified.

The examples presented so far highlight that the parameter α_s is identified when the unconditional distribution of the demand shock $\epsilon_{d,t}$ is either asymmetric or non-mesokurtic. Note that α_s is also identified when $\epsilon_{d,t}$ exhibits conditional heteroskedasticity. This is because conditional heteroskedasticity typically implies unconditional leptokurtic (even for the case of conditional mesokurtic distributions), and, as discussed above, it is precisely the presence of unconditional non-mesokurtic demand shock that leads to the identification of α_s. However, using the information provided in the unconditional non-mesokurtic distribution of the demand shock offers the important advantage of avoiding to take a stand on whether the time-varying conditional volatility of $\epsilon_{d,t}$ is determined by fixing a priori the dates of the structural breaks (e.g. Rigobon, 2003), is specified via GARCH processes (Normandin and Phaneuf, 2004), or is modeled by regime switching processes with Markov chains (Lanne, Lütkepohl, and Maciejowska, 2010) or smooth transitions (Lütkepohl and Netšunajev, 2014).

Taken altogether, these examples suggest that exploiting the information of the structural shocks related to higher unconditional moments, such as the third and fourth moments, help to identify additional parameters of a SVAR process (relative to the case where only the second unconditional moments are considered). In constrast, ignoring this information leads to the important consequence that more identifying restrictions must be imposed on the structural parameters.

3. Identification

In this section, we first present the SVAR specification. We then derive the order and rank conditions of identification through higher unconditional moments.

3.1 Specification

We consider a structural system that takes the form of the following p-order SVAR process:

$$\Phi x_t = \Phi_0 + \sum_{\tau=1}^{p} \Phi_\tau x_{t-\tau} + \epsilon_t. \tag{3}$$

The $(n \times 1)$ vector x_t includes the variables of interest. The $(n \times 1)$ vector ϵ_t contains the structural shocks. These shocks are assumed to be cross-sectionally independent and serially uncorrelated.[7] The $(n \times 1)$ vector Φ_0 incorporates n unrestricted intercepts. The non-singular $(n \times n)$ matrix Φ captures up to n^2 unrestricted contemporaneous interactions among the variables.[8] The $(n \times n)$ matrix Φ_τ contains n^2 unrestricted dynamic feedbacks between the variables.

The first four unconditional moments of the structural shocks of system (3) are obtained from the following expressions:

$$
\begin{aligned}
M_\epsilon &= E[\epsilon_t], & (4)\\
\Sigma_\epsilon &= E[\epsilon_t \epsilon_t'], & (5)\\
S_\epsilon &= E[\epsilon_t \epsilon_t' \otimes \epsilon_t'], & (6)\\
K_\epsilon^e &= K_\epsilon - K_{\tilde{\epsilon}} = E[\epsilon_t \epsilon_t' \otimes \epsilon_t' \otimes \epsilon_t'] - E[\tilde{\epsilon}_t \tilde{\epsilon}_t' \otimes \tilde{\epsilon}_t' \otimes \tilde{\epsilon}_t'], & (7)
\end{aligned}
$$

where E is the unconditional expectation operator and $\otimes$ denotes the Kronecker product. As is common practice, the $(n \times 1)$ vector of expectations is fixed to $M_\epsilon = [\mu_{\epsilon,i}] = 0$ and the $(n \times n)$ covariance matrix is normalized to $\Sigma_\epsilon = [\sigma_{\epsilon,ij}] = I$ (for $i, j = 1, \ldots, n$).[9] Also, the $(n \times n^2)$ coskewness matrix concatenates n symmetric $(n \times n)$ submatrices: $S_\epsilon = [S_{\epsilon,1}, \ldots, S_{\epsilon,n}]$, where $S_{\epsilon,k} = [s_{\epsilon,k,ij}] = E[\epsilon_{k,t}\epsilon_{i,t}\epsilon_{j,t}]$. The n unconstrained skewnesses of the structural shocks may be non-zero, $s_{\epsilon,k,kk} \neq 0$, whereas all coskewnesses are null, $s_{\epsilon,k,ii} = s_{\epsilon,k,ij} = 0$ (for $i, j \neq k$), given that the structural shocks are independent. Finally, the $(n \times n^3)$ excess cokurtosis matrix, K_ϵ^e, is the difference between the cokurtosis matrix, K_ϵ, of the true (potentially) non-normal structural shocks, ϵ_t, and the cokurtosis matrix, $K_{\tilde{\epsilon}}$, associated with hypothetical normal structural shocks, $\tilde{\epsilon}_t$. These matrices of cokurtoses stack n^2 symmetric $(n \times n)$ submatrices: $K_\xi = [K_{\xi,11}, \ldots, K_{\xi,1n}, \ldots, K_{\xi,n1}, \ldots, K_{\xi,nn}]$, where $K_{\xi,k\ell} = [\kappa_{\xi,k\ell,ij}] = E[\xi_{k,t}\xi_{\ell,t}\xi_{i,t}\xi_{j,t}]$ and $\xi_t = \epsilon_t$, $\tilde{\epsilon}_t$. The n kurtoses are unconstrained for the true structural shocks so that they may differ from 3, $\kappa_{\xi,kk,kk} \neq 3$ when $\xi_t = \epsilon_t$, and are constrained

[7] Note that the stronger assumption that the structural shocks are also serially independent is often invoked in independent component analysis for the derivations of the conditions of identification and/or the statistical properties of estimators (e.g. Gouriéroux, Monfort, and Renne 2017a,b; Lanne, Meitz, and Saikkonen, 2017). Assuming instead that the shocks are serially uncorrelated offers the advantage of allowing for the existence of conditional heteroskedasticity, which is a feature that characterizes many macroeconomic and financial time series.

[8] The assumption of non singularity ensures that there is no redundant variables included in the SVAR process.

[9] Note that the inclusion of the n unrestricted intercepts in Φ_0 of system (3) ensures that $M_\epsilon = 0$. Also, the inclusion of n unrestricted diagonal elements in Φ allows for the normalization $\Sigma_\epsilon = I$ without loss of generality.

for the hypothetical structural shocks such that $\kappa_{\xi,kk,kk} = 3$ when $\xi_t = \tilde{\epsilon}_t$, whereas the cokurtoses are either unity, $\kappa_{\xi,kk,ii} = \sigma_{\xi,kk}\sigma_{\xi,ii} = 1$ (for $i \neq k$), or null, $\kappa_{\xi,kk,ki} = \kappa_{\xi,kk,ij} = \kappa_{\xi,k\ell,ij} = 0$ (for $i, j, \ell \neq k$) when $\xi_t = \epsilon_t, \tilde{\epsilon}_t$, given that the true and hypothetical structural shocks are independent.[10]

Next, the reduced form associated with system (3) corresponds to the following p-order VAR process:

$$x_t = \Gamma_0 + \sum_{\tau=1}^{p} \Gamma_\tau x_{t-\tau} + \nu_t, \tag{8}$$

where $\Gamma_0 = \Theta\Phi_0$, $\Gamma_\tau = \Theta\Phi_\tau$, and the non-singular matrix $\Theta = \Phi^{-1}$ captures the impact responses of the variables of interest to the various structural shocks, whereas ν_t includes the statistical innovations. These innovations are related to the structural shocks as follows:

$$\nu_t = \Theta\epsilon_t. \tag{9}$$

Also, the first four unconditional moments of the statistical innovations are:

$$M_\nu = E[\nu_t], = \Theta M_\epsilon, \tag{10}$$

$$\Sigma_\nu = E[\nu_t\nu_t'] = \Theta\Sigma_\epsilon\Theta', \tag{11}$$

$$S_\nu = E[\nu_t\nu_t' \otimes \nu_t'] = \Theta S_\epsilon(\Theta' \otimes \Theta'), \tag{12}$$

$$K_\nu^e = K_\nu - K_{\tilde{\nu}} = E[\nu_t\nu_t' \otimes \nu_t' \otimes \nu_t'] - E[\tilde{\nu}_t\tilde{\nu}_t' \otimes \tilde{\nu}_t' \otimes \tilde{\nu}_t'] = \Theta K_\epsilon^e(\Theta' \otimes \Theta' \otimes \Theta'). \tag{13}$$

Here, $M_\nu = [\mu_{\nu,i}] = 0$ given that $M_\epsilon = 0$ and $\Sigma_\nu = [\sigma_{\nu,ij}] = \Theta\Theta'$ since $\Sigma_\epsilon = I$. Moreover, $S_\nu = [S_{\nu,1}, \ldots, S_{\nu,n}]$ with $S_{\nu,k} = [s_{\nu,k,ij}]$ and $K_\upsilon = [K_{\upsilon,11}, \ldots, K_{\upsilon,1n}, \ldots, K_{\upsilon,n1}, \ldots, K_{\upsilon,nn}]$ with $K_{\upsilon,k\ell} = [\kappa_{\upsilon,k\ell,ij}]$, while $\upsilon_t = \nu_t, \tilde{\nu}_t$, where ν_t captures the true (potentially) non-normal statistical innovations and $\tilde{\nu}_t$ contains hypothetical normal statistical innovations. As is well known, the symmetric matrix Σ_ν contains $\frac{n(n+1)}{2}$ distinct elements. Furthermore, the matrices S_ν and K_ν^e include up to $\frac{n(n+1)(n+2)}{6}$ and $\frac{n(n+1)(n+2)(n+3)}{24}$ distinct elements.[11]

[10] As an example, the bivariate system (1)–(2) implies that all the elements of S_ϵ are null, except potentially the (1,1) and (2,4) elements which correspond to the skewnesses of the demand shock, $s_{\epsilon,d,dd}$, and supply shock, $s_{\epsilon,s,ss}$. Also, all the elements of K_ϵ^e are null, with the possible exceptions of the (1,1) and (2,8) elements which capture the excess kurtoses of the demand shock, $\kappa_{\epsilon,dd,dd}^e = (\kappa_{\epsilon,dd,dd} - 3)$, and supply shock, $\kappa_{\epsilon,ss,ss}^e = (\kappa_{\epsilon,ss,ss} - 3)$, and the (1,4), (1,6), (1,7), (2,2), (2,3), and (2,5) elements which correspond to the product $\sigma_{\epsilon,dd}\sigma_{\epsilon,ss} = 1$.

[11] For example, the bivariate system (1)–(2) implies that Σ_ν incorporates 3 distinct elements, namely $\sigma_{\nu,yy}$, $\sigma_{\nu,yp}$, and $\sigma_{\nu,pp}$. Also, S_ν includes up to 4 distinct elements: $s_{\nu,y,yy}$, $s_{\nu,y,yp}$, $s_{\nu,y,pp}$, and $s_{\nu,p,pp}$. Finally, K_ν^e involves up to 5 distinct elements: $\kappa_{\nu,yy,yy}^e$, $\kappa_{\nu,yy,yp}^e$, $\kappa_{\nu,yy,pp}^e$, $\kappa_{\nu,yp,pp}^e$, and $\kappa_{\nu,pp,pp}^e$.

3.2 Identification Conditions

We now determine the order and rank conditions of local identification (rather than global identification), which establish the conditions for identifying the parameters associated with the structural form (3) from the distinct elements and the rank associated with the reduced form (8).[12] For expositional purposes, these conditions are mainly derived from two cases; the first case exploits the skewness of the structural shocks, whereas the second case focuses on the excess kurtosis of the structural shocks. This presentation accords with the economic illustrations highlighting that identification can be achieved when the unconditional distributions of the structural shocks are either asymmetric or non-mesokurtic (see Section 2). For each case, we elaborate the conditions required to identify the impact responses involved in Θ and the skewnesses or excess kurtoses of the structural shocks included in S_ϵ or K_ϵ^e from the unconditional moments of the statistical innovations contained in Σ_ν and S_ν or K_ν^e. For completeness, note that, once these parameters are identified, it is trivial to identify the other structural parameters included in Φ_0 and Φ_τ (where $\tau = 1, \ldots, p$) through the relations $\Phi_0 = \Theta^{-1}\Gamma_0$ and $\Phi_\tau = \Theta^{-1}\Gamma_\tau$.

3.2.1 Order Conditions

Let us denote by η and ρ the number of parameters involved in the structural form and the number of distinct elements in the reduced form. The order conditions are given by $\rho = \eta$ and $\rho > \eta$, which represent necessary conditions for the exact- and over-identification of the entire structural system. To the best of our knowledge, these conditions have never been established for SVAR with non-normal structural shocks.

We begin by examining our first case, which exploits the skewness of the structural shocks. On

[12] Note that, under local identification, the matrix incorporating the impact responses, Θ, is unique up to sign changes and permutations of columns. Changing the signs of columns means that negative instead of positive structural shocks (and vice versa) are considered. Permuting the columns implies alternative orderings of the structural shocks. Global identification, which implies that Θ is unique, is usually achieved by imposing certain sets of restrictions, such as $\Theta'\Theta = I$ or $\theta_i'\theta_i = 1$ (for $i = 1, \ldots, n$) where θ_i denotes the i^{th} column of Θ. These restrictions are not innocuous from both statistical and economic perspectives. From a statistical viewpoint, some sets of restrictions, such as $\Theta'\Theta = I$, may imply that the pseudo maximum likelihood (PML) approach provides estimators which are numerically cumbersome, while other sets of restrictions, such as $\theta_i'\theta_i = 1$ (for $i = 1, \ldots, n$), may lead to PML estimators that are statistically inconsistent (see Gouriéroux, Monfort, and Renne, 2017b). From an economic standpoint, the various sets of restrictions, such as $\Theta'\Theta = I$ and $\theta_i'\theta_i = 1$ (for $i = 1, \ldots, n$), may not ensure the intepretation of the structural shocks in terms of economic phenomena.

the one hand, the number of parameters in the structural form is $\eta = [n^2 - (n - n_s)m_s] + m_s$ given that there are $n^2 - (n - n_s)m_s$ and m_s non-zero structural parameters to identify in the impact response and skewness matrices, Θ and S_ϵ, where n_s represents the number of skewed statistical innovations and m_s corresponds to the the number of asymmetric structural shocks. To clarify the number of non-zero parameters involved in Θ, we consider two configurations of the partitioned representation of relation (9):

$$\begin{pmatrix} \nu_{s,t} \\ \nu_{ns,t} \end{pmatrix} = \begin{pmatrix} \Theta_{s,s} & \Theta_{s,ns} \\ \Theta_{ns,s} & \Theta_{ns,ns} \end{pmatrix} \begin{pmatrix} \epsilon_{s,t} \\ \epsilon_{ns,t} \end{pmatrix}, \tag{14}$$

where $\nu_{s,t}$ and $\nu_{ns,t}$ are subvectors that collect, respectively, the n_s and $(n - n_s)$ skewed and non-skewed statistical innovations, $\epsilon_{s,t}$ and $\epsilon_{ns,t}$ contain the m_s and $(n-m_s)$ asymmetric and symmetric structural shocks, and $\Theta_{i,j}$ are conformable submatrices of impact responses (for $i, j = s, ns$).[13] The first configuration assumes that $n = n_s$ and $n_s \geq m_s$.[14] In this context, all the statistical innovations display asymmetries as soon as one structural shock is skewed. This holds even when all the impact responses are non zero, so that the n^2 parameters involved in the matrix Θ have to be identified. For example, when $n = n_s = 2$ and $m_s = 1$ (where, say, $\epsilon_{s,t} = \epsilon_{1,t}$ is the only asymmetric structural shock so that $s_{\epsilon,1,11} \neq 0$), then the skewness and all coskewnesses obtained form (12) are different from zero for each statistical innovation: $s_{\nu,k,ij} = (\theta_{k1}\theta_{i1}\theta_{j1})s_{\epsilon,1,11} \neq 0$ (for $k, i, j = 1, 2$), where θ_{ij} corresponds to the (i, j) element of the matrix Θ involved in relation (9). The second configuration postulates that $n > n_s$ and $n_s \geq m_s$. In this environment, the $(n - n_s)$ statistical innovations contained in $\nu_{ns,t}$ are symmetric, as long as these innovations are not related to the m_s skewed structural shocks included in $\epsilon_{s,t}$. This absence of relation occurs when $\Theta_{ns,s} = 0$, which implies that $(n - n_s)m_s$ impact reponses are zero such that the remaining $n^2 - (n - n_s)m_s$ parameters of the matrix Θ have to be identified. For instance, when $n = 2$ and $n_s = m_s = 1$ (where, say, $\epsilon_{s,t} = \epsilon_{1,t}$ is skewed), then $s_{\nu,2,ij} = (\theta_{21}\theta_{i1}\theta_{j1})s_{\epsilon,1,11} = 0$ (for $i, j = 1, 2$) only if a single impact response is zero: $\Theta_{ns,s} = \theta_{21} = 0$.

[13] Note that all the elements included in $\nu_{s,t}$ and $\epsilon_{s,t}$ are asymmetric, but some of these elements may also be non-mesokurtic. Moreover, the elements in $\nu_{ns,t}$ and $\epsilon_{ns,t}$ are symmetric, but possibly non-mesokurtic.

[14] For the configurations, we analyze the non-trivial cases where there exists at least one structural shock that is skewed ($m_s \geq 1$). Also, the configurations stipulate that $n \geq n_s \geq m_s$ to ensure that the impact response matrix Θ is non-singular.

On the other hand, the number of distinct elements from the reduced from is $\rho = \left[\frac{n(n+1)}{2}\right] + \left[\frac{n_s(n_s+1)(n_s+2)}{6}\right]$. As already mentioned, there are $\frac{n(n+1)}{2}$ distinct elements in Σ_ν. Moreover, there are $\left[\frac{n_s(n_s+1)(n_s+2)}{6}\right]$ distinct elements in S_ν. This can be seen from the examples given for the two configurations discussed above. For the example of the first configuration, $n = n_s = 2$ and $m_s = 1$ (where $\epsilon_{s,t} = \epsilon_{1,t}$ is skewed), recall that all the impact responses are non zero. In this case, $s_{\nu,k,ij} = (\theta_{k1}\theta_{i1}\theta_{j1})s_{\epsilon,1,11} \neq 0$ (for $k, i, j = 1, 2$), so that $\left[\frac{n_s(n_s+1)(n_s+2)}{6}\right] = 4$ reaches the maximal number of distinct elements in S_ν. For the example of the second configuration, $n = 2$ and $n_s = m_s = 1$, remind that $\Theta_{ns,s} = \theta_{21} = 0$. In this context, $s_{\nu,1,11} = \theta_{11}^3 s_{\epsilon,1,11}$ is the only non-zero element in S_ν, so that $\left[\frac{n_s(n_s+1)(n_s+2)}{6}\right] = 1$.

Table 1 illustrates the order conditions for a structural system involving four variables ($n = 4$). The necessary condition for identification is violated ($\rho < \eta$) when there are one or two skewed statistical innovations ($n_s = 1$ or $n_s = 2$) and a single asymmetric structural shock ($m_s = 1$). In constrast, the order condition for exact identification ($\rho = \eta$) holds when there are two skewed statistical innovations and structural shocks ($n_s = 2$ and $m_s = 2$). Finally, the necessary condition for over-identification ($\rho > \eta$) is verified for all the other specifications.

We now turn to the second case which focuses on the excess kurtosis of the structural shocks. In this context, the partitioned representation of relation (9) becomes:

$$\begin{pmatrix} \nu_{\kappa,t} \\ \nu_{n\kappa,t} \end{pmatrix} = \begin{pmatrix} \Theta_{\kappa,\kappa} & \Theta_{\kappa,n\kappa} \\ 0 & \Theta_{n\kappa,n\kappa} \end{pmatrix} \begin{pmatrix} \epsilon_{\kappa,t} \\ \epsilon_{n\kappa,t} \end{pmatrix}, \tag{15}$$

where $\nu_{\kappa,t}$ and $\nu_{n\kappa,t}$ are subvectors that collect, respectively, the n_κ and $(n - n_\kappa)$ non-mesokurtic and mesokurtic statistical innovations, while $\epsilon_{\kappa,t}$ and $\epsilon_{n\kappa,t}$ contain the m_κ and $(n - m_\kappa)$ non-mesokurtic and mesokurtic structural shocks.[15]

Invoking analogous arguments as those elaborated above implies that $\eta = [n^2 - (n - n_\kappa)m_\kappa] + m_\kappa$ — there are $n^2 - (n - n_\kappa)m_\kappa$ and m_κ non-zero structural parameters in Θ and K_ϵ^e to identify, where n_κ and m_κ are the numbers of non-mesokurtic statistical innovations and structural shocks. Also, $\rho = \left[\frac{n(n+1)}{2}\right] + \left[\frac{n_\kappa(n_\kappa+1)(n_\kappa+2)(n_\kappa+3)}{24}\right]$, because there are $\frac{n(n+1)}{2}$ and $\frac{n_\kappa(n_\kappa+1)(n_\kappa+2)(n_\kappa+3)}{24}$ distinct

[15] All the terms incorporated in $\nu_{\kappa,t}$ and $\epsilon_{\kappa,t}$ are non-mesokurtic, but some of these terms may also be asymmetric. Furthermeore, the terms in $\nu_{n\kappa,t}$ and $\epsilon_{n\kappa,t}$ are mesokurtic, but possibly asymmetric.

elements in Σ_ν and K^e_ν. Coming back to the example of a structural system with four variables, the necessary condition for identification is now violated only when $n_\kappa = m_\kappa = 1$, whereas the order condition for exact identification holds when $n_\kappa = 2$ and $m_\kappa = 1$, and that for over-identification holds for all the other specifications.

Finally, we present the general case which takes into account both the skewness and excess kurtosis of the structural shocks. To do so, the relation (9) is partitioned according to the characteristics of the statistical innovations and structural shocks as following:

$$\begin{pmatrix} \nu_{ss,t} \\ \nu_{\kappa\kappa,t} \\ \nu_{s\kappa,t} \\ \nu_{ns\kappa,t} \end{pmatrix} = \begin{pmatrix} \Theta_{ss,ss} & 0 & 0 & \Theta_{ss,ns\kappa} \\ 0 & \Theta_{\kappa\kappa,\kappa\kappa} & 0 & \Theta_{\kappa\kappa,ns\kappa} \\ \Theta_{s\kappa,ss} & \Theta_{s\kappa,\kappa\kappa} & \Theta_{s\kappa,s\kappa} & \Theta_{s\kappa,ns\kappa} \\ 0 & 0 & 0 & \Theta_{ns\kappa,ns\kappa} \end{pmatrix} \begin{pmatrix} \epsilon_{ss,t} \\ \epsilon_{\kappa\kappa,t} \\ \epsilon_{s\kappa,t} \\ \epsilon_{ns\kappa,t} \end{pmatrix}. \tag{16}$$

Here, $\nu_{ss,t}$, $\nu_{\kappa\kappa,t}$, $\nu_{s\kappa,t}$, and $\nu_{ns\kappa,t}$ are subvectors that collect, respectively, the n_{ss}, $n_{\kappa\kappa}$, $n_{s\kappa}$, and $(n - n_{ss} - n_{\kappa\kappa} - n_{s\kappa})$ statistical innovations that are exclusively skewed, only non-mesokurtic, both asymmetric and non-mesokurtic, and both symmetric and mesokurtic. Likewise, the subvectors $\epsilon_{ss,t}$, $\epsilon_{\kappa\kappa,t}$, $\epsilon_{s\kappa,t}$, and $\epsilon_{ns\kappa,t}$ contain, respectively, the m_{ss}, $m_{\kappa\kappa}$, $m_{s\kappa}$, and $(m - m_{ss} - m_{\kappa\kappa} - m_{s\kappa})$ structural shocks that are exclusively skewed, only non-mesokurtic, both asymmetric and non-mesokurtic, and both symmetric and mesokurtic.[16] In this environment, $\eta = [n^2 - (n - n_{ss} - n_{\kappa\kappa} - n_{s\kappa})(m_{ss} + m_{\kappa\kappa} + m_{s\kappa}) - n_{ss}m_\kappa - n_{\kappa\kappa}m_s] + [m_s + m_\kappa]$, where $n_s = n_{ss} + n_{s\kappa}$, $n_\kappa = n_{\kappa\kappa} + n_{s\kappa}$, $m_s = m_{ss} + m_{s\kappa}$, and $m_\kappa = m_{\kappa\kappa} + m_{s\kappa}$. In the expression for η, the term in the first set of brackets corresponds to the number of non-zero parameters in Θ, whereas the term in the second set of brackets represents the number of non-zero skewnesses and excess kurtoses in S_ϵ and K^e_ϵ.[17] Moreover, $\rho = \left[\frac{n(n+1)}{2}\right] + \left[\frac{n(n_s+1)(n_s+2)}{6}\right] + \left[\frac{n_\kappa(n_\kappa+1)(n_\kappa+2)(n_\kappa+3)}{24}\right]$, given the number of distinct

[16] System (16) is related to (14) as $\nu_{s,t} = \begin{pmatrix} \nu_{ss,t} & \nu_{s\kappa,t} \end{pmatrix}'$, $\nu_{ns,t} = \begin{pmatrix} \nu_{\kappa\kappa,t} & \nu_{ns\kappa,t} \end{pmatrix}'$, $\epsilon_{s,t} = \begin{pmatrix} \epsilon_{ss,t} & \epsilon_{s\kappa,t} \end{pmatrix}'$, and $\epsilon_{ns,t} = \begin{pmatrix} \epsilon_{\kappa\kappa,t} & \epsilon_{ns\kappa,t} \end{pmatrix}'$. Likewise, system (16) is related to (15) as $\nu_{\kappa,t} = \begin{pmatrix} \nu_{\kappa\kappa,t} & \nu_{s\kappa,t} \end{pmatrix}'$, $\nu_{n\kappa,t} = \begin{pmatrix} \nu_{ss,t} & \nu_{ns\kappa,t} \end{pmatrix}'$, $\epsilon_{\kappa,t} = \begin{pmatrix} \epsilon_{\kappa\kappa,t} & \epsilon_{s\kappa,t} \end{pmatrix}'$, and $\epsilon_{n\kappa,t} = \begin{pmatrix} \epsilon_{ss,t} & \epsilon_{ns\kappa,t} \end{pmatrix}'$. For system (16), note further that the statistical innovations involved in $\nu_{ss,t}$ are mesokurtic. It can be shown that this occurs because $\nu_{ss,t}$ is a linear combination of the structural shocks contained in $\epsilon_{ss,t}$ and $\epsilon_{ns\kappa,t}$, where these shocks are mutually independent and are characterized by unconditional expectations and excess kurtoses that are equal to zero. Similarly, the statistical innovations collected in $\nu_{\kappa\kappa,t}$ are symmetric.

[17] Note that there are $(n - n_{ss} - n_{\kappa\kappa} - n_{s\kappa})(m_{ss} + m_{\kappa\kappa} + m_{s\kappa})$ zero parameters in Θ to ensure that the statistical innovations that do not exhibit skewness and excess kurtosis are unrelated to the asymmetric and/or non-mesokurtic structural shocks. Also, there are $n_{ss}m_\kappa$ additional zero parameters in Θ, such that the statistical innovations displaying only skewness are disconnected from the structural shocks featuring excess kurtosis. Finally, there are $n_{\kappa\kappa}m_s$ zero parameters in Θ, so that the statistical innovations exhibiting exclusively excess kurtosis are not linked to the structural shocks which are asymmetric.

elements in Σ_ν, S_ν, and K_ν^e.

3.2.2 Rank Condition

In this Section, we formally derive the rank condition and simple formulas which allow practitioners to evaluate easily the rank condition. The rank condition $r = \eta$ represents the sufficient condition for the identification of the entire structural system, where r corresponds to the rank associated with the unconditional moment matrices of the statistical innovations. Extending the developments of Lütkepohl (2007), we derive this condition from the ranks of the Jacobian matrices associated with the structural parameters to identify. In doing so, our approach leads to similar results regarding the identification of all the structural parameters than those highlighted in the literature on independent component analysis, which relies on the properties of multivariate normal distributions.[18]

If it turns out that the entire structural system is not identified, then our approach further allows to establish which structural parameters are identified and which are not. This gives rise to two important implications. First, it permits to assess which structural subsystem is identified. This subsystem documents the effects induced by the asymmetric and/or non-mesokurtic structural shocks. Second, it enables to determine the structural parameters for which some restrictions must be placed on in order to acheive the identification of the entire system. This is required to document the effects of the symmetric and mesokurtic structural shocks. As far as we know, these key implications have never been examined in previous studies.

Again, we first consider the case which exploits the asymmetry of the structural shocks. As explained above, the number of parameters involved in the structural form is $\eta = [n^2 - (n-n_s)m_s] +$

[18] In independent component analysis, the identification condition of the entire structural system is derived by confronting the two following relations:

$$\begin{aligned} \nu_t &= \Theta \epsilon_t, \\ \nu_t &= \Theta^* \epsilon_t^*, \end{aligned}$$

where $\Theta^* = \Theta Q'$, $\epsilon_t^* = Q\epsilon_t$, and $Q'Q = I$. Given that the elements of ϵ_t^* and those of ϵ_t are respectively independent, then all elements of ϵ_t^* and ϵ_t are normally distributed as long as the matrix Q has no zero elements (e.g., Darmois, 1953; Ghurye and Olkin, 1961). Accordingly, the system is not identified as $\Theta^* \neq \Theta$. In contrast, at least $(n-1)$ elements of ϵ_t^* and of ϵ_t are non-normally distributed when Q corresponds to a permuation matrix (see Comon, 1994; Eriksson and Koivunen, 2004; Gouriéroux, Monfort, and Renne, 2017a). In this context, the structural parameters involved in the impact response matrix Θ are locally identified, as Θ is unique up to sign changes and permutations of columns.

m_s. Also, the rank associated with the reduced form is equal to the rank of the following Jacobian matrix:

$$J = \begin{bmatrix} J_{\theta_s} & J_{\theta_{ns}} & J_{s_\epsilon} \end{bmatrix} = \begin{bmatrix} J_{\sigma_\nu,\theta_s} & J_{\sigma_\nu,\theta_{ns}} & J_{\sigma_\nu,s_\epsilon} \\ J_{s_\nu,\theta_s} & J_{s_\nu,\theta_{ns}} & J_{s_\nu,s_\epsilon} \end{bmatrix}. \tag{17}$$

Here, $J_{\theta_s} = \begin{bmatrix} J'_{\sigma_\nu,\theta_s} & J'_{s_\nu,\theta_s} \end{bmatrix}'$, $J_{\theta_{ns}} = \begin{bmatrix} J'_{\sigma_\nu,\theta_{ns}} & J'_{s_\nu,\theta_{ns}} \end{bmatrix}'$, $J_{s_\epsilon} = \begin{bmatrix} J'_{\sigma_\nu,s_\epsilon} & J'_{s_\nu,s_\epsilon} \end{bmatrix}'$, and $J_{y,x} = \frac{\partial y}{\partial x'}$. Moreover, the vector σ_ν vectorizes the lower triangular part of the symmetric covariance matrix Σ_ν, and the vector s_ν collects the distinct elements of the coskewness matrix S_ν. Finally, the vector θ_s stacks the columns of the matrix $\Theta_{s,s}$ in system (14), the vector θ_{ns} contains the elements of the matrices $\Theta_{s,ns}$ and $\Theta_{ns,ns}$, and the vector s_ϵ includes the non-zero elements of the skewness matrix S_ϵ. The analytical derivatives involved in (17) are detailed in the Appendix.

The rank of the Jacobian matrix (17), $r = rk[J]$, can be evaluated from the analytical derivatives. Simple formulas can be deduced from the analytical derivation of the rank condition. As a result, the rank r can be easily assessed from the number of variables involved in the system, n, the number of asymmetric statistical innovations, n_s, and the number of skewed structural shocks, m_s. Specifically, the rank corresponds to the sum of three components: $r = r_s + r_{ns} + r_{s_\epsilon}$, with $r_s = rk[J_{\theta_s}] = n_s \times m_s$, $r_{ns} = rk[J_{\theta_{ns}}] = \sum_{i=0}^{n-m_s}(n-i) - m_s$, and $r_{s_\epsilon} = rk\left[J_{s_\epsilon}\right] = m_s$.

The components $r_s = n_s \times m_s$ and $r_{s_\epsilon} = m_s$ reveal that the information contained in the second and third moments of the statistical innovations, Σ_ν and S_ν, allows to identify all the $n_s \times m_s$ elements of the matrix $\Theta_{s,s}$ relating the aymmetric statistical innovations to the skewed structural shocks, as well as all the m_s non-zero elements of the skewness matrix S_ϵ. The intuition for this result can be gained from the two following features. First, $rk[J_{s_\nu,\theta_s}] = n_s \times m_s$ and $rk[J_{s_\nu,s_\epsilon}] = m_s$, but $rk\begin{bmatrix} J_{s_\nu,\theta_s} & J_{s_\nu,s_\epsilon} \end{bmatrix} = n_s \times m_s$. This implies that the coskewness matrix S_ν identifies the elements of $\Theta_{s,s}$ and S_ϵ jointly, but not separately. To illustrate this, consider an example where $n = 2$ and $n_s = m_s = 1$ (where $\epsilon_{s,t} = \epsilon_{1,t}$ is skewed), so that $\Theta_{s,s} = \theta_{11}$ and $\Theta_{ns,s} = \theta_{21} = 0$. In this context, the skewness of the statistical innovation $\nu_{1,t}$, $s_{\nu,1,11} = \theta_{11}^3 s_{\epsilon,1,11}$, identifies the parameters θ_{11} and $s_{\epsilon,1,11}$ jointly, but not individually. Second, $J_{\sigma_\nu,\theta_s} \neq 0$ whereas $J_{\sigma_\nu,s_\epsilon} = 0$. This implies that the covariance matrix Σ_ν disentangles the parameters involved in $\Theta_{s,s}$ from those contained in S_ϵ, so that it becomes possible to identify individually each parameter in $\Theta_{s,s}$ and S_ϵ. Coming back to the previous example, the variance of $\nu_{1,t}$, $\sigma_{\nu,11} = (\theta_{11}^2 + \theta_{12}^2)$, disentangles the parameter θ_{11} from $s_{\epsilon,1,11}$, given that this variance is related to θ_{11} but not to $s_{\epsilon,1,11}$.

The component $r_{ns} = \sum_{i=0}^{n-m_s}(n-i)-m_s$ indicates whether the remaining information contained in the second moments of the statistical innovations, Σ_ν, allows to identify all the $n \times (n-m_s)$ elements of the matrices $\Theta_{s,ns}$ and $\Theta_{ns,ns}$ relating the statistical innovations to the symmetric structural shocks. The intuition for this result is obtained from the following features: $J_{s_\nu,\theta_{ns}} = 0$ and $J_{\sigma_\nu,\theta_{ns}} \neq 0$. This implies that only the information captured in Σ_ν, which is independent of that already used to identify $\Theta_{s,s}$, can be exploited to identify the parameters included in $\Theta_{s,ns}$ and $\Theta_{ns,ns}$.

Our findings relying on the rank of the Jacobian matrix (17) parallel the existing results of the literature on independent component analysis. This literature highlights that all the structural parameters are identified when at least all, but one, structural shocks are non-normally distributed (see Comon, 1994; Eriksson and Koivunen, 2004; Gouriéroux, Monfort, and Renne, 2017a). Our findings state that the entire structural system is identified when at least all, but one, structural shocks are skewed. Specifically, when all structural shocks are asymmetric, $m_s = n$, then all the structural parameters are identified as $\eta = r = n^2+n$ — where $\eta = [n^2-(n-n_s)m_s]+m_s = n^2+n$ and $r = r_s + r_{ns} + r_{s_\epsilon}$, with $n_s = n$, $r_s = n^2$, $r_{ns} = 0$, and $r_{s_\epsilon} = n$.[19] When all, but one, structural shocks are skewed, $m_s = n-1$, then all the structural parameters are identified as either $\eta = r = n^2+n-1$ with $n_s = n$, $r_s = n(n-1)$, $r_{ns} = n$, and $r_{s_\epsilon} = n-1$, or $\eta = r = n^2$ with $n_s = n-1$, $r_s = (n-1)^2$, $r_{ns} = n$, and $r_{s_\epsilon} = n-1$.

Table 2 illustrates the rank condition for a structural system involving four variables ($n = 4$). This condition ($r = \eta$) holds for three specifications, namely *i*) $n_s = m_s = 4$, *ii*) $n_s = 4$ and $m_s = 3$, and *iii*) $n_s = m_s = 3$. Also, comparing Tables 1 and 2 indicates that the order conditions are verified for more specifications than the rank condition. In this sense, the order conditions are milder than the rank condition. This is expected because the order conditions represent necessary conditions for the identification of the entire structural system, whereas the rank condition is a sufficient condition.

[19] Recall that the conditions $n \geq n_s \geq m_s$ are required to ensure that the impact response matrix Θ is non-singular.

Importantly, our approach relying on the rank of the Jacobian matrix (17) further provides novel insights when the entire structural system is not identified. In particular, as already explained above, the moments Σ_ν and S_ν allow to identify the $n_s \times m_s$ structural parameters included in $\Theta_{s,s}$ and the m_s distinct elements involved in S_ϵ. Hence, the subsystem relating all the statistical innovations to the skewed structural shocks is always identified. This subsystem traces the effects generated by the structural shocks displaying skewness. For example, the impact responses of the variables associated with the skewed statistical innovations are given by $\Theta_{s,s}$, whereas those of the variables related to the symmetric statistical innovations are equal to zero. Hence, if the structural shocks of interest are skewed, then their effects can be assessed without imposing any restrictions on the structural parameters. This contrasts with the common practice which consists in evaluating the effects of a subset of structural shocks under certain identifying restrictions, whether or not these structural shocks are asymmetric. Examples include studies which document exclusively the effects of monetary policy shocks (Sims, 1992; Bernanke and Mihov, 1998; Bjørnland and Leitemo, 2009), fiscal policy shocks (e.g. Blanchard and Perotti, 2002; Galí, López-Salido, and Vallés, 2007; Monacelli and Perotti, 2010), oil shocks (e.g. Kilian, 2009; Kilian and Park, 2009), or technology shocks (e.g. Galí, 1999; Francis and Ramey, 2009; Fève and Guay, 2010) by imposing short-run and/or long-run restrictions.

Moreover, the under-identification of the entire structural system occurs when the moments Σ_ν does not permit to identify all the $n \times (n - m_s)$ elements contained in $\Theta_{s,ns}$ and $\Theta_{ns,ns}$. As a result, certain restrictions on the structural parameters involved in θ_{ns} must be imposed. For illustration purposes, consider the following (linear) short-run restrictions $R\theta_{ns} = q$. In this context, the rank condition holds when:

$$rk[J^+] = rk\begin{bmatrix} J^+_{\theta_s} & J^+_{\theta_{ns}} & J^+_{s_\epsilon} \end{bmatrix} = rk\begin{bmatrix} J_{\sigma_\nu,\theta_s} & J_{\sigma_\nu,\theta_{ns}} & J_{\sigma_\nu,s_\epsilon} \\ J_{s_\nu,\theta_s} & J_{s_\nu,\theta_{ns}} & J_{s_\nu,s_\epsilon} \\ 0 & R & 0 \end{bmatrix} = \eta, \tag{18}$$

where J^+ is the augmented Jacobian matrix, $J^+_{\theta_s} = \begin{bmatrix} J'_{\sigma_\nu,\theta_s} & J'_{s_\nu,\theta_s} & 0' \end{bmatrix}'$, $J^+_{\theta_{ns}} = \begin{bmatrix} J'_{\sigma_\nu,\theta_{ns}} & J'_{s_\nu,\theta_{ns}} & R' \end{bmatrix}'$, and $J^+_{s_\epsilon} = \begin{bmatrix} J'_{\sigma_\nu,s_\epsilon} & J'_{s_\nu,s_\epsilon} & 0' \end{bmatrix}'$. The rank condition (18) states that $(\eta - r)$ linearly independent restrictions on θ_{ns} are needed to identfify the entire structural system. Hence, if the structural shocks of interest are symmetric, then their effects can only be gauged when $(\eta - r)$ restrictions are placed on θ_{ns}. In expression (18), the relevant short-run restrictions imply $(\eta - r)$ zero impact responses of some variables to certain symmetric structural shocks. It is straightforward to show that relevant long-run restrictions imply $(\eta - r)$ zero dynamic responses (evaluated over an infinite horizon) of some variables to certain symmetric shocks.

We next analyze the case which focuses on the excess kurtosis of the structural shocks. Under the short-run restrictions $R\theta_{n\kappa} = q$, the rank condition is verified if:

$$rk[J^+] = rk\begin{bmatrix} J^+_{\theta_\kappa} & J^+_{\theta_{n\kappa}} & J^+_{\kappa^e_\epsilon} \end{bmatrix} = rk\begin{bmatrix} J_{\sigma_\nu,\theta_\kappa} & J_{\sigma_\nu,\theta_{n\kappa}} & J_{\sigma_\nu,\kappa^e_\epsilon} \\ J_{\kappa^e_\nu,\theta_\kappa} & J_{\kappa^e_\nu,\theta_{n\kappa}} & J_{\kappa^e_\nu,\kappa^e_\epsilon} \\ 0 & R & 0 \end{bmatrix} = \eta. \tag{19}$$

Here, the vectors κ^e_ν and κ^e_ϵ incorporate the distinct elements of the cokurtosis matrices K^e_ν and K^e_ϵ. Also, the vector θ_κ collects the parameters of the matrix $\Theta_{\kappa,\kappa}$ in system (15), while the vector $\theta_{n\kappa}$ includes the elements of the matrices $\Theta_{\kappa,n\kappa}$ and $\Theta_{n\kappa,n\kappa}$. Again, the analytical derivatives involved in (19) are relegated in the Appendix.

Recall that the number of structural parameters to identify in this case is $\eta = [n^2 - (n - n_\kappa)m_\kappa] + m_\kappa$. Now, let's first consider the eventuality that no restrictions are placed on the structural parameters, that is $R = 0$. Then, the rank of J^+ corresponds to $r = r_\kappa + r_{n\kappa} + r_{\kappa^e_\epsilon}$ with $r_\kappa = rk[J^+_{\theta_\kappa}] = n_\kappa \times m_\kappa$, $r_{n\kappa} = rk[J^+_{\theta_{n\kappa}}] = \sum_{i=0}^{n-m_\kappa}(n-i) - m_\kappa$, and $r_{\kappa^e_\epsilon} = rk\begin{bmatrix} J^+_{\kappa^e_\epsilon} \end{bmatrix} = m_\kappa$. Consequently, the entire structural system is identified, that is $\eta = r$, when at least all, but one, structural shocks display excess kurtosis. Also, invoking analogous arguments as those developed above reveals that, whether or not $\eta = r$, the subsystem relating all the statistical innovations to the non-mesokurtic structural shocks is identified, as the information contained in Σ_ν and K^e_ν always allows to recover the structural parameters involved in $\Theta_{\kappa,\kappa}$ and K^e_ϵ. Hence, if the structural shocks of interest display excess kurtosis, then their effects can be documented without imposing any restrictions on the structural parameters.

Let's now contemplate the eventuality that some restrictions are imposed on the structural parameters ($R \neq 0$). These restrictions are required when the remaining information captured in Σ_ν does not allow to identify all the structural parameters in $\Theta_{\kappa,n\kappa}$ and $\Theta_{n\kappa,n\kappa}$. In this context, the entire structural system is identified when $(\eta - r)$ linearly independent restrictions are imposed on $\theta_{n\kappa}$, where these restrictions can take the form of the short-run restrictions $R\theta_{n\kappa} = q$. Thus, if the structural shocks of interest do not exhibit excess kurtosis, then their effects can only be determined when $(\eta - r)$ restrictions are placed on $\theta_{n\kappa}$.

Finally, we establish a proposition providing the rank condition for the identification of the structural parameters, under short-run restrictions, for the general case where the structural shocks display skewness and/or excess kurtosis.

Proposition 1 *Given the unconditional moments of the statistical innovations, Σ_ν, S_ν, and K_ν^e, the system of equations (11)–(13) has a locally unique solution if and only if*

$$\begin{aligned} rk[J^+] &= rk\left[J^+_{\theta_{ss}} \quad J^+_{\theta_{\kappa\kappa}} \quad J^+_{\theta_{s\kappa}} \quad J^+_{\theta_{ns\kappa}} \quad J^+_{s_\epsilon} \quad J^+_{\kappa^e_\epsilon}\right] \\ &= rk\begin{bmatrix} J_{\sigma_\nu,\theta_{ss}} & J_{\sigma_\nu,\theta_{kk}} & J_{\sigma_\nu,\theta_{sk}} & J_{\sigma_\nu,\theta_{ns\kappa}} & J_{\sigma_\nu,s_\epsilon} & J_{\sigma_\nu,\kappa^e_\epsilon} \\ J_{s_\nu,\theta_{ss}} & J_{s_\nu,\theta_{kk}} & J_{s_\nu,\theta_{sk}} & J_{s_\nu,\theta_{ns\kappa}} & J_{s_\nu,s_\epsilon} & J_{s_\nu,\kappa^e_\epsilon} \\ J_{\kappa^e_\nu,\theta_{ss}} & J_{\kappa^e_\nu,\theta_{kk}} & J_{\kappa^e_\nu,\theta_{sk}} & J_{\kappa^e_\nu,\theta_{ns\kappa}} & J_{\kappa^e_\nu,s_\epsilon} & J_{\kappa^e_\nu,\kappa^e_\epsilon} \\ 0 & 0 & 0 & R & 0 & 0 \end{bmatrix} = \eta, \end{aligned} \tag{20}$$

where the vector θ_{ss} stacks by columns the $n_s \times m_{ss}$ parameters involved in the matrices $\Theta_{ss,ss}$ and $\Theta_{s\kappa,ss}$ relating the asymmetric (and possibly non-mesokurtic) statistical innovations to the structural shocks displaying only skewness in system (16), the vector $\theta_{\kappa\kappa}$ contains the $n_\kappa \times m_{\kappa\kappa}$ parameters of the matrices $\Theta_{\kappa\kappa,\kappa\kappa}$ and $\Theta_{s\kappa,\kappa\kappa}$ linking the non-mesokurtic (and possibly asymmetric) statistical innovations to the structural shocks exhibiting exclusively excess kurtosis, the vector $\theta_{s\kappa}$ includes the $n_{s\kappa} \times m_{s\kappa}$ parameters of the matrix $\Theta_{s\kappa,s\kappa}$ associating the asymmetric and non-mesokurtic statistical innovations to the structural shocks featuring both skewness and excess kurtosis, $\theta_{ns\kappa}$ incorporates the $n \times [n - (m_{ss} + m_{\kappa\kappa} + m_{s\kappa})]$ parameters of the matrices $\Theta_{ss,ns\kappa}$, $\Theta_{\kappa\kappa,ns\kappa}$, $\Theta_{s\kappa,ns\kappa}$, and $\Theta_{ns\kappa,ns\kappa}$ relating all statistical innovations to the structural shocks which are both symmetric and mesokurtic, and the matrix R forms the short-run restrictions $R\theta_{ns\kappa} = q$.

The analytical derivatives involved in (20) are reported in the Appendix. As explained above, the number of structural parameters to identify is $\eta = [n^2 - (n - n_{ss} - n_{\kappa\kappa} - n_{s\kappa})(m_{ss} + m_{\kappa\kappa} + m_{s\kappa}) - n_{ss}m_\kappa - n_{\kappa\kappa}m_s] + [m_s + m_\kappa]$. When no restrictions are imposed on the structural parameters ($R = 0$), then $rk\,[J^+] = r$ with $r = r_{ss} + r_{\kappa\kappa} + r_{s\kappa} + r_{ns\kappa} + r_{s_\epsilon} + r_{\kappa^e_\epsilon}$, $r_{ss} = rk[J^+_{\theta_{ss}}] = n_s \times m_{ss}$, $r_{\kappa\kappa} = rk[J^+_{\theta_{\kappa\kappa}}] = n_\kappa \times m_{\kappa\kappa}$, $r_{s\kappa} = rk[J^+_{\theta_{\kappa\kappa}}] = n_{s\kappa} \times m_{s\kappa}$, $r_{ns\kappa} = rk[J^+_{\theta_{ns\kappa}}] = \sum_{i=0}^{n-(m_{ss}+m_{\kappa\kappa}+m_{s\kappa})}(n - i) - (m_{ss}+m_{\kappa\kappa}+m_{s\kappa})$, $r_{s_\epsilon} = rk\left[J^+_{s_\epsilon}\right] = m_s$, and $r_{\kappa^e_\epsilon} = rk\left[J^+_{\kappa^e_\epsilon}\right] = m_\kappa$. In this context, **Proposition**

1 has two implications. First, the entire structural system is identified, that is $\eta = r$, when at least all, but one, structural shocks exhibit skewness and/or excess kurtosis. Second, whether or not $\eta = r$, the subsystem relating all the statistical innovations to the asymmetric and/or non-mesokurtic structural shocks is identified, given that the information contained in Σ_ν, S_ν, and K^e_ν always allows to recover the structural parameters involved in θ_{ss}, $\theta_{\kappa\kappa}$, $\theta_{s\kappa}, S_\epsilon$, and K^e_ϵ — that is $[r_{ss} + r_{\kappa\kappa} + r_{s\kappa}] + [r_{s_\epsilon} + r_{\kappa^e_\epsilon}] = [n_s m_{ss} + n_\kappa m_{\kappa\kappa} + n_{s\kappa} \times m_{s\kappa}] + [m_s + m_\kappa]$. When some restrictions are placed on the structural parameters ($R \neq 0$), these restrictions are required if the remaining information captured in Σ_ν does not allow to identify all the structural parameters contained in $\theta_{ns\kappa}$ — that is $r_{ns\kappa} < n \times [n - (m_{ss} + m_{\kappa\kappa} + m_{s\kappa})]$. In this environment, **Proposition 1** states that the entire structural system becomes identified only if $(\eta - r)$ linearly independent restrictions are imposed on $\theta_{ns\kappa}$.

4. Testing Procedure

In this section, we elaborate, for the first time in the literature, a testing procedure to verify the symmetry and excess kurtosis of the structural shocks, prior to the estimation of the SVAR process. Specifically, we develop a tractable procedure to verify whether the order and rank conditions hold by assessing the numbers of asymmetric and/or non-mesokurtic statistical innovations and structural shocks. We then outline a bootstrap procedure to improve the small-sample properties of rank tests designed to verify the numbers of statistical innovations and structural shocks displaying skewness and/or excess kurtosis.

4.1 Verification of the Order and Rank Conditions

The order and rank conditions for identification are useful as long as they are verified before proceeding to the estimation of the SVAR (3); it is only if they hold that it becomes worthwhile to perform the estimation of all the structural parameters involved in the system. As explained above, the order conditions, $\rho \geq \eta$, and the rank condition, $r = \eta$, can be verified from the numbers of asymmetric and/or non-mesokurtic statistical innovations and structural shocks. Although the statistical innovations are readily measured from the reduced-form residuals, the structural shocks become measurable only once the SVAR is estimated.[20]

We start by presenting two ways to test the number of statistical innovations exhibiting skewness and/or excess kurtosis. The first one consists in applying Jarque-Bera tests for symmetry and kurtosis. That is, the number of statistical innovations displaying skewness, n_s, is equal to the number of variables for which the null hypothesis $s_{\nu,i,ii} = 0$ is rejected. Likewise, the number of statistical innovations exhibiting excess kurtosis, n_κ, corresponds to the number of variables for which the hypothesis $\kappa_{\nu,ii,ii} - 3\sigma_{\nu,i}^4 = 0$ is refuted — where $\sigma_{\nu,i}$ is the standard deviation of the statistical innovation $\nu_{i,t}$. Also, the number of statistical innovations featuring skewness and/or excess kurtosis, $n_{ss} + n_{\kappa\kappa} + n_{s\kappa}$, is the number of variables for which the joint hypothesis $s_{\nu,i,ii} = 0$ and $\kappa_{\nu,ii,ii} - 3\sigma_{\nu,i}^4 = 0$ is rejected. Using these informations, the numbers of statistical innovations displaying exclusively skewness, n_{ss}, excess kurtosis, $n_{\kappa\kappa}$, or both, $n_{s\kappa}$, are easily deduced — given that $n_s = n_{ss} + n_{s\kappa}$ and $n_\kappa = n_{\kappa\kappa} + n_{s\kappa}$.[21]

The second way relies on rank tests. That is, n_s is equal to the rank of the following matrix $\tilde{S}_\nu = diag(s_{\nu,i,ii})$, where $i = 1, \ldots, n$. Similarly, n_κ is the rank of $\tilde{K}_\nu^e = diag[(\kappa_{\nu,ii,ii} - 3\sigma_{\nu,i}^4)]$, where $i = 1, \ldots, n$. In addition, $n_{ss} + n_{\kappa\kappa} + n_{s\kappa} = rk[\tilde{\Psi}_\nu]$, where $\tilde{\Psi}_\nu = \begin{pmatrix} \tilde{S}_\nu & \tilde{K}_\nu^e \end{pmatrix}$. Again, these informations allow to recover n_{ss}, $n_{\kappa\kappa}$, and $n_{s\kappa}$.

We next develop a new method to test the number of asymmetric and/or non-mesokurtic structural shocks, which relies exclusively on the statistical innovations — where the latter can be evaluated from the reduced form (8) before the estimation of the structural form (3). Specifically, the number of skewed structural shocks, m_s, corresponds to the rank of the coskewness matrix of

[20] Empirically, asymmetric (either postive or negative skewness) and leptokurtic behaviors have been extensively documented for stock returns (see for example, Fama, 1965; Harris, 1986; Richardson and Smith, 1993; Bekaert and Harvey, 1997), government and corporate bond returns (e.g. Roll, 1970; Fujiwara, Körber, and Nagakura, 2013; Bai, Bali, and Wen, 2016), spot and forward foreign exchange rates (e.g. Westerfield, 1977; McFarland, Pettit, and Sung, 1982; Boothe and Glassman, 1987; Corlu and Corlu, 2015), and commodity prices (e.g. Mandelbrot, 1963; Clark, 1973; Dusak, 1973). Likewise, positive excess kurtosis have been detected for several macroeconomic series, including indicators related to the economic activity — e.g. real GDP, the components of the real aggregate expenditure, industrial production, and unemployment — as well as a variety of indices of the cost of living — e.g. GDP deflator and CPI (see for example, Blanchard and Watson, 1986; Balke and Fomby, 1994; Bai and Ng, 2005; Lanne, Meitz, and Saikkonen, 2016; Gouriéroux, Montford, and Renne, 2017a,b). Note that the studies just reported highlight the existence of skewness and/or excess kurtosis for the variables of interest or for the statistical innovations related to these variables, but never for the structural shocks.

[21] Specifically, $n_{s\kappa}$ is determined from $n_{ss} + n_{\kappa\kappa} + n_{s\kappa} = (n_s - n_{s\kappa}) + (n_\kappa - n_{s\kappa}) + n_{s\kappa}$, where the value for $n_{ss} + n_{\kappa\kappa} + n_{s\kappa}$ is obtained by testing the joint hypotheses $s_{\nu,i,ii} = 0$ and $\kappa_{\nu,ii,ii} - 3\sigma_{\nu,i}^4 = 0$, the value for n_s is find by testing the hypothesis $s_{\nu,i,ii} = 0$, and the value for n_κ is obtained by testing the hypothesis $\kappa_{\nu,ii,ii} - 3\sigma_{\nu,i}^4 = 0$. Then, n_{ss} and $n_{\kappa\kappa}$ are determined from $n_s = n_{ss} + n_{s\kappa}$ and $n_\kappa = n_{\kappa\kappa} + n_{s\kappa}$.

the statistical innovations, S_ν. To see this, it useful to rewrite the relation (14) as follows:

$$\begin{pmatrix} \nu_{s,t} \\ \nu_{ns,t} \end{pmatrix} = \begin{pmatrix} \Theta_{s,s} \\ 0 \end{pmatrix} \epsilon_{s,t} + \begin{pmatrix} \Theta_{s,ns} \\ \Theta_{ns,ns} \end{pmatrix} \epsilon_{ns,t},$$

or more compactly,

$$\nu_t = \Theta_s \epsilon_{s,t} + \Theta_{ns} \epsilon_{ns,t}, \tag{21}$$

where Θ_s is a $(n \times m_s)$ matrix of rank m_s and Θ_{ns} is a $(n \times (n - m_s))$ matrix of rank $n - m_s$. Given that all elements contained in $\epsilon_{s,t}$ and $\epsilon_{ns,t}$ are independent and that all terms involved in $\epsilon_{ns,t}$ are symmetric, equation (12) can be expressed as:

$$S_\nu = \Theta_s S_s (\Theta_s' \otimes \Theta_s'), \tag{22}$$

where $S_s = E[\epsilon_{s,t} \epsilon_{s,t}' \otimes \epsilon_{s,t}']$ is a $(m_s \times m_s^2)$ matrix of rank m_s. As a result, equation (22) implies that $m_s = rk[S_\nu]$.

Analogously, the number of non-mesokurtic structural shocks, m_κ, is given by the rank of the excess cokurtosis matrix of the statistical innovations, K_ν^e. This holds because equation (13) can be rewritten as:

$$K_\nu^e = \Theta_\kappa K_\kappa^e (\Theta_\kappa' \otimes \Theta_\kappa' \otimes \Theta_\kappa'), \tag{23}$$

where $\Theta_\kappa = \begin{pmatrix} \Theta_{\kappa,\kappa}' & 0' \end{pmatrix}'$ is a $(n \times m_\kappa)$ matrix of rank m_κ formed from the relation (15), $K_\kappa^e = (E[\epsilon_{\kappa,t} \epsilon_{\kappa,t}' \otimes \epsilon_{\kappa,t}' \otimes \epsilon_{\kappa,t}'] - E[\tilde{\epsilon}_{\kappa,t} \tilde{\epsilon}_{\kappa,t}' \otimes \tilde{\epsilon}_{\kappa,t}' \otimes \tilde{\epsilon}_{\kappa,t}'])$ is a $(m_\kappa \times m_\kappa^3)$ matrix of rank m_κ that involves the fourth unconditional moments of the true non-meskurtic and hypothetical normal structural shocks, $\epsilon_{\kappa,t}$ and $\tilde{\epsilon}_{\kappa,t}$. Equation (23) implies that $m_\kappa = rk[K_\nu^e]$.

Also, the number of skewed and/or non-mesokurtic structural shocks, $m_{ss} + m_{\kappa\kappa} + m_{s\kappa}$, is the rank of $\Psi_\nu = \begin{pmatrix} S_\nu & K_\nu^e \end{pmatrix}$. To see this, note that equations (22) and (23) imply that $rk\left[\begin{pmatrix} S_\nu & K_\nu^e \end{pmatrix}\right] = rk\left[\begin{pmatrix} \Theta_s & \Theta_\kappa \end{pmatrix}\right]$. Using the relation (16), the latter matrices are partitioned as $\Theta_s = \begin{pmatrix} \Theta_{ss} & \Theta_{s\kappa} \end{pmatrix}$ and $\Theta_\kappa = \begin{pmatrix} \Theta_{\kappa\kappa} & \Theta_{s\kappa} \end{pmatrix}$, where $\Theta_{ss} = \begin{pmatrix} \Theta_{ss,ss}' & 0' & \Theta_{s\kappa,ss}' & 0' \end{pmatrix}'$ is a $(n \times m_{ss})$ matrix of rank m_{ss} relating the statistical innovations to the structural shocks displaying exclusively skewness, $\Theta_{\kappa\kappa} = \begin{pmatrix} 0' & \Theta_{\kappa\kappa,\kappa\kappa}' & \Theta_{s\kappa,\kappa\kappa}' & 0' \end{pmatrix}'$ is a $(n \times m_{\kappa\kappa})$ matrix of rank $m_{\kappa\kappa}$ linking the statistical innovations to the structural shocks exhibiting only excess kurtosis, and $\Theta_{s\kappa} = \begin{pmatrix} 0' & 0' & \Theta'_{s\kappa,s\kappa} & 0' \end{pmatrix}'$ is a $(n \times m_{s\kappa})$ matrix of rank $m_{s\kappa}$ relating the statistical innovations to the structural shocks featuring both skewness and excess kurtosis. Accordingly, $rk\left[\begin{pmatrix} \Theta_s & \Theta_\kappa \end{pmatrix}\right] = rk[\Theta_{ss}] + rk[\Theta_{\kappa\kappa}] + rk[\Theta_{s\kappa}] = m_{ss} + m_{\kappa\kappa} + m_{s\kappa}$. From these results, the numbers of structural shocks displaying exclusively skewness, m_{ss}, excess kurtosis, $m_{\kappa\kappa}$, or both, $m_{s\kappa}$, are readily deduced — given that $m_s = m_{ss} + m_{s\kappa}$ and $m_\kappa = m_{\kappa\kappa} + m_{s\kappa}$.

4.2 Bootstrap Procedures

As is common practice, the Jarque-Bera tests for symmetry and kurtosis, allowing to determine n_s, n_κ, and $n_{ss} + n_{\kappa\kappa} + n_{s\kappa}$, rely on the following statistics:

$$\hat{S} = \frac{(T-p)}{6}\hat{s}^2_{u,i,ii}, \tag{24}$$

$$\hat{K} = \frac{(T-p)}{24}(\hat{\kappa}_{u,ii,ii} - 3)^2, \tag{25}$$

$$\hat{J} = \hat{S} + \hat{K}. \tag{26}$$

These statistics are computed from the sample estimates of the skewness $\hat{s}_{u,i,ii} = \frac{1}{T-p}\sum_{t=p+1}^{T} \hat{u}^3_{i,t}$ and kurtosis $\hat{\kappa}_{u,ii,ii} = \frac{1}{T-p}\sum_{t=p+1}^{T} \hat{u}^4_{i,t}$ of the estimated standardized statistical innovations $\hat{u}_{i,t} = \frac{\hat{\nu}_{i,t}}{\hat{\sigma}_{\nu,i}}$, where $\hat{\nu}_{i,t}$ is the OLS residual of the reduced form (8), $\hat{\sigma}_{\nu,i}$ is the estimate of the standard deviation of the residual, and T is the sample size. The statistics (24), (25), and (26) follow asymptotically chi-squared distributions, given that $\hat{s}_{u,i,ii}$ and $\hat{\kappa}_{u,ii,ii}$ have limiting normal distributions. From analytical approximations of the first four moments, it can be shown that $\hat{s}_{u,i,ii}$ has a symmetric leptokurtic distribution which fairly rapidly tends to a normal distribution as the sample size increases, but $\hat{\kappa}_{u,ii,ii}$ has a very skewed distribution that hardly converges to a normal distribution (see Mardia, 1980). This implies that the finite-sample critical values to test the null hypothesis $K = 0$ converge extremely slowly to their asymptotic counterparts. Numerical simulations further suggest that the use of these asymptotic critical values leads to severe size distortions, as the empirical size often substantially deviates from the nominal size even for samples as large as $T = 5,000$ (see Kilian and Demiroglu, 2000; Bai and Ng, 2005).

To circumvent this problem, Kilian and Demiroglu (2000) develop a bootstrap procedure to compute finite-sample critical values to test the hypotheses $S = 0$, $K = 0$, and $J = 0$. Monte Carlo analyses highlight that the Jarque-Bera tests for symmetry and kurtosis are virtually free of size distortions when the critical values are computed from the bootstrap procedure, even for samples as small as $T = 125$.

For the rank tests, we use the following likelihood-ratio (LR) and Wald (W) statistics:[22]

$$\widehat{CRT}^{LR}_{r^*} = (T-p)\sum_{i=r^*+1}^{n} \ln(1+\hat{\lambda}_i), \tag{27}$$

$$\widehat{CRT}^{W}_{r^*} = (T-p)\sum_{i=r^*+1}^{n} \hat{\lambda}_i, \tag{28}$$

where $\hat{\lambda}_i$ are the estimates of the eigenvalues of the quadratic form of the matrix of interest (with $\hat{\lambda}_1 \geq \ldots \geq \hat{\lambda}_n \geq 0$) and r^* is the rank of this matrix under the null hypothesis. To determine n_s, n_κ, and $n_{ss} + n_{\kappa\kappa} + n_{s\kappa}$, the matrices of interest are $\tilde{S}_u$, $\tilde{K}^e_u$, and $\tilde{\Psi}_u$, where $\tilde{S}_u = diag(s_{u,i,ii})$, $\tilde{K}^e_u = diag[(\kappa_{u,ii,ii}-3)]$ (with $i = 1, \ldots, n$), and $\tilde{\Psi}_u = \begin{pmatrix}\tilde{S}_u & \tilde{K}^e_u\end{pmatrix}$. These matrices are computed from the sample estimates $\hat{s}_{u,i,ii}$ and $\hat{\kappa}_{u,ii,ii}$ presented above.[23] To find m_s, m_κ, and $m_{ss}+m_{\kappa\kappa}+m_{s\kappa}$, the matrices of interest are S_u, K^e_u, and Ψ_u. These matrices are constructed from the sample estimates of the coskweness $\hat{s}_{u,k,ij} = \frac{1}{T-p}\sum_{t=p+1}^{T} \hat{u}_{k,t}\hat{u}_{i,t}\hat{u}_{j,t}$ and cokurtosis $\hat{\kappa}_{u,k\ell,ij} = \frac{1}{T-p}\sum_{t=p+1}^{T} \hat{u}_{k,t}\hat{u}_{\ell,t}\hat{u}_{i,t}\hat{u}_{j,t}$ of the estimated normalized statistical innovations, as well as the cokurtoses $\kappa_{\tilde{u},kk,kk} = 3$, $\kappa_{\tilde{u},kk,ii} = \sigma_{\tilde{u},kk}\sigma_{\tilde{u},ii} = 1$ (for $i \neq k$), and $\kappa_{\tilde{u},kk,ki} = \kappa_{\tilde{u},kk,ij} = \kappa_{\tilde{u},k\ell,ij} = 0$ (for $\ell, i, j \neq k$) of hypothetical normal statistical innovations. Moreover, the estimate of the normalized statistical innovations coresponds to $\hat{u}_t = \hat{\Omega}^{-1}\hat{\nu}_t$, where $\hat{\nu}_t$ contains the OLS residuals of the reduced form (8) and $\hat{\Omega}$ is a lower triangular matrix obtained from the Cholesky decompostion of the estimated covariance matrix of the OLS residuals; i.e. $\hat{\Sigma}_\nu = \hat{\Omega}\hat{\Omega}'$.[24] Robin and Smith (2000) show that, under some regularity conditions, the statistics (27) and (28) have limiting distributions that are weighted sums of independent chi-squared variables, despite that the estimators of $vec(S_u)$, $vec(K^e_u)$, and

[22] See Anderson (1951) and Robin and Smith (2000).

[23] Note that $rk[\tilde{S}_u] = rk[\tilde{S}_\nu] = n_s$, $rk[\tilde{K}^e_u] = rk[\tilde{K}^e_\nu] = n_\kappa$, and $rk[\tilde{\Psi}_u] = rk[\tilde{\Psi}_\nu] = n_{ss} + n_{\kappa\kappa} + n_{s\kappa}$ given that $\tilde{S}_u = \tilde{\Omega}^{-3}\tilde{S}_\nu$ and $\tilde{K}^e_u = \tilde{\Omega}^{-4}\tilde{K}^e_\nu$, where $\nu_t = \tilde{\Omega}u_t$ and $\tilde{\Omega} = diag\left(\sigma_{\nu,i}\right)$ with $i = 1, \ldots, n$.

[24] Here, $rk[S_u] = rk[S_\nu] = m_s$, $rk[K^e_u] = rk[K^e_\nu] = m_\kappa$, and $rk[\Psi_u] = rk[\Psi_\nu] = m_{ss} + m_{\kappa\kappa} + m_{s\kappa}$ given that $S_u = \Omega^{-1}S_\nu(\Omega^{-1\prime} \otimes \Omega^{-1\prime})$ and $K^e_u = \Omega^{-1}K^e_\nu(\Omega^{-1\prime} \otimes \Omega^{-1\prime} \otimes \Omega^{-1\prime})$, where $\nu_t = \Omega u_t$.

$vec(\Psi_u)$ have not full rank asymptotic covariance matrices.[25] These distributions are used to find the asymptotic critical values to test that $CRT^{LR}_{r^*} = 0$ and $CRT^{W}_{r^*} = 0$ under the null hypothesis that the rank is r^*.

Alternatively, we design a bootstrap procedure for computing finite-sample critical values to test the hypotheses $CRT^{LR}_{r^*} = 0$ and $CRT^{W}_{r^*} = 0$. We illustrate the various steps of the procedure by focusing on the rank of S_u in order to determine m_s.

Step 1. Under the null hypothesis $rk[S_u] = r^*$ (i.e. $r^* = m_s$ is the assumed number of asymmetric structural shocks), the vector $u^*_t = \begin{pmatrix} u^{*\prime}_{r^*,t} & u^{*\prime}_{n-r^*,t} \end{pmatrix}'$ is generated as follows. The elements contained in the $(r^* \times 1)$ subvector $u^*_{r^*,t}$ are obtained by bootstraping those included in the vector $w_{r^*,t} = C'_{r^*}\hat{u}_t$ for $t = (p+1), \ldots, T$, where C_{r^*} is a $(n \times r^*)$ matrix stacking the eigenvectors associated with the r^* largest eigenvalues of the quadratic form $S_u\ S'_u$ and $\hat{u}_t$ is the $(n \times 1)$ vector collecting the estimated normalized statistical innovations.[26] The elements contained in the $[(n - r^*) \times 1]$ subvector $u^*_{n-r^*,t}$ are drawned from $u^*_{n-r^*,t} \sim N(0, I)$ for $t = (p + 1), \ldots, T$.

Step 2. The bootstrap sample is generated recursively from the VAR process (8) as:

$$x^*_t = \hat{\Gamma}_0 + \sum_{\tau=1}^{p} \hat{\Gamma}_\tau x^*_{t-\tau} + \hat{\Omega} u^*_t, \tag{29}$$

for $t = (p+1), \ldots, T$. To do so, the starting values of x^*_t for $t = 1, \ldots, p$ are generated by randomly drawing a block of the actual data of length p, while $\hat{\Gamma}_0$, $\hat{\Gamma}_\tau$, and $\hat{\Omega}$ are the estimates of the reduced-form parameters obtained by applying OLS on the actual sample.

Step 3. The VAR process is estimated to yield:

$$x^*_t = \hat{\Gamma}^*_0 + \sum_{\tau=1}^{p} \hat{\Gamma}^*_\tau x^*_{t-\tau} + \hat{\Omega}^* \hat{u}^*_t, \tag{30}$$

where $\hat{\Gamma}^*_0$, $\hat{\Gamma}^*_\tau$, and $\hat{\Omega}^*$ are the estimates obtained by performing OLS on the bootstrap sample, whereas $\hat{u}^*_t$ corresponds to the normalized residuals.

Step 4. The normalized residuals $\hat{u}^*_t$ are used to compute the bootstrap analogues of the statistics (27) and (28).

[25] Note that most rank tests require non-singular asymptotic covariance matrices (see Camba-Méndez and Kapetanios, 2008).

[26] This implies that the elements contained in $w_{r^*,t}$ correspond to linear combinations of the normalized statistical innovations which are the most asymmetric.

Step 5. Steps 1 to 4 are repeated 2,000 times to compute the empirical distributions of the statistics (27) and (28). Selecting the appropriate quantiles of these empirical distibutions yield the finite-sample critical values to test the hypotheses $CRT_{r^*}^{LR} = 0$ and $CRT_{r^*}^{W} = 0$.

A similar procedure can be done to compute the finite-sample critical values for the ranks of K_u^e and Ψ_u to determine m_κ and $m_{ss} + m_{\kappa\kappa} + m_{s\kappa}$. Likewise, the procedure can be used to obtain the finite-sample critical values for the ranks of $\tilde{S}_u$, $\tilde{K}_u^e$, and $\tilde{\Psi}_u$ to find n_s, n_κ, and $n_{ss} + n_{\kappa\kappa} + n_{s\kappa}$.

To document the possible size distortions of rank tests with asymptotic and finite-sample distributions, we focus on the ranks of S_u and K_u^e to deduce m_s and m_κ. The empirical sizes of these rank tests are evaluated by simulating 10,000 samples of size T from the bivariate system (1)–(2), where the supply shock follows a normal distribution whereas the demand shock is either generated by a normal distribution under the null hypothesis $r^* = 0$ or by non-normal distributions (i.e. a mixture of normal distributions when the shock is asymmetric and a Student's t-distribution when the shock is non-mesokurtic) under $r^* = 1$. Table 3 presents the empirical sizes of the Wald and likelihood-ratio versions of the rank tests for symmetry with asymptotic distributions, where the limiting critical values are computed as in Robin and Smith (2000). For the Wald test, the results indicate the existence of a mild size distortion when $T = 100$, but this distortion quickly vanishes as T increases. For example, the empirical sizes of 3.92 percent under $r^* = 0$ and 5.79 percent under $r^* = 1$ reported for $T = 100$ become almost equal to the nominal size of 5 percent when $T \geq 200$. For the likelihood-ratio test, however, the size distortion documented for small samples is more important than that reported for the Wald test, and such distortion is still observed for samples as large as $T = 1,000$. Table 4 shows the empirical sizes of the Wald and likelihood-ratio versions of the rank tests for kurtosis with asymptotic distributions. For both the Wald and LR tests, the size distortions are severe under the hypotheses $r^* = 0$ and $r^* = 1$. Specifically, the empirical sizes are systematically close to zero, and, as such, they are substantially smaller than the nominal sizes even for samples as large as $T = 5,000$.

Finally, Tables 5 and 6 display the empirical sizes related to the rank tests for symmetry and kurtosis with finite-sample distributions, where the critical values are constructed from the bootstrap procedure developed above. For symmetry, both the Wald and likelihood-ratio tests are essentially free of size distortions; the empirical sizes are very close to the nominal sizes for all T. Note that, although the empirical sizes of the Wald tests with finite-sample and asymptotic distributions are similar, the empirical sizes of the likelihood-ratio test with finite-sample distributions deviate markedly from those obtained from the asymptotic counterpart. For kurtosis, the empirical sizes of the Wald and likelihood-ratio tests with finite-sample distributions are almost identical to the nominal sizes, regardless of the sample size T. Importantly, these findings are strikingly different than those reported for tests with asymptotic distributions.

Overall, the Kilian-Demiroglu bootstrap procedure for Jarque-Bera tests for symmetry and kurtosis as well as our bootstrap procedure for rank tests allow to overcome size distortions. Consequently, these bootstrap procedures are most useful to determine the numbers of asymmetric and/or non-mesokurtic statistical innovations and structural shocks, in order to assess whether the order and rank conditions hold.

5. Application

We now apply the developments presented above to identify the effects of fiscal policies on economic activity. This represents a classical question in macroeconomics. Also, it has received renewed interest in light of the recent great recession and the ongoing debate about which type of government interventions stimulate the most the economy.

To identify the effects of fiscal policies from a global perspective, we rely on the specification invoked in the seminal paper of Blanchard and Perotti (2002):

$$\nu_{\tau,t} = \alpha_1 \nu_{y,t} + \alpha_2 \omega_g \epsilon_{g,t} + \omega_\tau \epsilon_{\tau,t}, \tag{31}$$

$$\nu_{g,t} = \beta_1 \nu_{y,t} + \beta_2 \omega_\tau \epsilon_{\tau,t} + \omega_g \epsilon_{g,t}, \tag{32}$$

$$\nu_{y,t} = \gamma_1 \nu_{\tau,t} + \gamma_2 \nu_{g,t} + \omega_y \epsilon_{y,t}. \tag{33}$$

The statistical innovations $\nu_{\tau,t}$, $\nu_{g,t}$, and $\nu_{y,t}$ capture the unanticipated movements in taxes, government spending, and output. The structural shocks $\epsilon_{\tau,t}$ and $\epsilon_{g,t}$ represent the tax and spending shocks that reflect unexpected, exogenous, discretionary changes in taxes and government expenditures, whereas $\epsilon_{y,t}$ captures the non-fiscal shocks that affect output. Equations (31) and (32) describe the government's tax and spending rules. Specifically, the rule (31) highlights that taxes may vary in response to changes in output or to spending shocks. The rule (32) has an analogous interpretation for public spending. In these rules, the parameters α_1 and β_1 potentially measure the automatic and government's systematic responses of taxes and government spending to changes in output, whereas α_2 and β_2 allow for interactions between tax and spending policies. Equation (33) relates changes in output to changes in taxes and government expenditures, and to non-fiscal shocks. Finally, the terms ω_τ, ω_g, and ω_y are scaling parameters.

The specification (31)–(33) can be expressed in the form of relation (9) as:

$$\begin{pmatrix} \nu_{\tau,t} \\ \nu_{g,t} \\ \nu_{y,t} \end{pmatrix} = \frac{1}{\Delta} \begin{pmatrix} (1+\alpha_1\beta_2\gamma_2-\beta_1\gamma_2)\omega_\tau & (\alpha_2+\alpha_1\gamma_2-\alpha_2\beta_1\gamma_2)\omega_g & \alpha_1\omega_y \\ (\beta_2+\beta_1\gamma_1-\alpha_1\beta_2\gamma_1)\omega_\tau & (1+\alpha_2\beta_1\gamma_1-\alpha_1\gamma_1)\omega_g & \beta_1\omega_y \\ (\gamma_1+\beta_2\gamma_2)\omega_\tau & (\alpha_2\gamma_1+\gamma_2)\omega_g & \omega_y \end{pmatrix} \begin{pmatrix} \epsilon_{\tau,t} \\ \epsilon_{g,t} \\ \epsilon_{y,t} \end{pmatrix}, \tag{34}$$

where $\Delta = (1-\alpha_1\gamma_1-\beta_1\gamma_2)$. For short, we refer to θ_{ij} as the (i,j) element of the matrix in (34).

Empirically, the relation (34) is evaluated for quarterly U.S. data from 1980-I to 2015-III.[27] Output corresponds to the logarithm of real GDP per capita, taxes are defined as the logarithm of real total government receipts net of transfer payments per capita, and government spending is the logarithm of the sum of real consumption and gross investment expenditures per capita. The series are expressed in real terms using the GDP deflator and in per capita terms using total population. Also, taxes and government spending are measured for the general government, i.e. the sum of federal (defense and non-defense), state, and local governments.[28]

As explained previously, it is crucial to verify whether the identification conditions hold before proceeding to the estimation of the structural parameters. To do so, we apply Jarque-Bera and rank tests for the statistical innovations, where the finite-sample critical values are computed by the bootstrap procedures discussed in Section 4.2.[29] The results for Jarque-Bera tests reveal that the hypothesis of symmetry is not rejected at the 5 percent level for all (standardized) statistical

[27] A similar starting date of the sample is selected by Perotti (2004), Favero and Giavazzi (2009), and Bouakez, Chihi, and Normandin (2014) to avoid structural breaks.

[28] The data are seasonally adjusted at the source and are taken from the National Income and Products Accounts (NIPA), except for total population which is obtained from the Federal Reserve Bank of Saint-Louis' FRED database.

[29] The statistical innovations are measured by the OLS residuals of the reduced form (8). The reduced form includes a linear deterministic trend and eight lags, which correspond to the most parsimonious lag structure for which all reduced-form residuals are serially uncorrelated.

innovations, whereas the hypothesis of zero excess kurtosis is rejected only for the statistical innovation associated with taxes, $\nu_{\tau,t}$. The findings for rank tests confirm that $n_{ss} = n_{s\kappa} = 0$ and $n_{\kappa\kappa} = 1$, as both the likelihood-ratio and Wald versions of the tests lead to the conclusions that $rk[\tilde{S}_u] = n_s = 0$, $rk[\tilde{K}^e_u] = n_\kappa = 1$, and $rk[\tilde{\Psi}_u] = n_{ss} + n_{\kappa\kappa} + n_{s\kappa} = 1$. Furthermore, the results for rank tests indicate that all the structural shocks are symmetric and only one shock is non-mesokurtic (i.e. $m_{ss} = m_{s\kappa} = 0$ and $m_{\kappa\kappa} = 1$), given that the likelihood-ratio and Wald versions of the tests imply that $rk[S_u] = m_s = 0$, $rk[K^e_u] = m_\kappa = 1$, and $rk[\Psi_u] = m_{ss} + m_{\kappa\kappa} + m_{s\kappa} = 1$. Overall, these findings suggest that the tax shock, $\epsilon_{\tau,t}$, displays excess kurtosis.

These test results are central for assessing the effects of fiscal policies. To see this, first note that these findings imply that $\theta_{21} = \theta_{31} = 0$, where these true values ensure that the mesokurtic statistical innovations $\nu_{y,t}$ and $\nu_{g,t}$ are not related to the non-mesokurtic structural shock $\epsilon_{\tau,t}$. In this context, the number of structural parameters is $\eta = [n^2 - (n - n_\kappa)m_\kappa] + m_\kappa = 8$ (i.e. the parameters θ_{11}, θ_{ij} with $i = 1, 2, 3$ and $j = 2, 3$, and the excess kurtosis of tax shock, $\kappa^e_{\epsilon,\tau\tau,\tau\tau}$), whereas the number of distinct elements in the reduced form is $\rho = \left[\frac{n(n+1)}{2}\right] + \left[\frac{n_\kappa(n_\kappa+1)(n_\kappa+2)(n_\kappa+3)}{24}\right] = 7$, and the rank associated with the reduced form is $r = r_\kappa + r_{n\kappa} + r_{\kappa^e_\epsilon} = 7$ — with $r_\kappa = n_\kappa \times m_\kappa = 1$, $r_{n\kappa} = \sum_{i=0}^{n-m_k}(n-i) - m_\kappa = 5$, and $r_{\kappa^e_\epsilon} = m_\kappa = 1$. Hence, the order and rank conditions, $\rho \geq \eta$ and $r = \eta$, are violated, so that the entire system is not identified.

However, the subsytem relating all statistical innovations, $\nu_{\tau,t}$, $\nu_{g,t}$, and $\nu_{y,t}$, to the tax shock, $\epsilon_{\tau,t}$, is identified. This leads to the important implication that the responses of output, taxes, and government spending following a tax shock can be evaluated without imposing any restrictions on the structural parameters. In contrast, the subsytem relating the statistical innovations to the structural shocks $\epsilon_{g,t}$ and $\epsilon_{y,t}$ is under-identified. To achieve the idenfication of this subsystem, $(\eta - r) = 1$ restriction must be imposed. Of course, the nature of the restriction selected affects the magnitudes and shapes of the responses of the variables to a spending shock.

At this point it is useful to analyze the consequences of the commonly used identifying restrictions elaborated by Blanchard and Perotti (2002) on the evaluation of the effects of a spending shock. The first set of restrictions fixes $\alpha_2 = 0$ such that taxes do not vary following a spending shock. It also calibrates $\alpha_1 = 2.08$ and $\beta_1 = 0$ using institutional information about tax and transfer systems, where such information allows to measure automatic adjustments of taxes and public spending, rather than the government's systematic responses to fluctuations in output (see Blanchard and Perotti, 2002). In principle, this identification strategy may allow for $\theta_{21} \neq 0$ and $\theta_{31} \neq 0$. In practice, however, this apparent flexibility is illusive in the sense that the estimates of β_2 and γ_1 should tend to zero to recover the true values $\theta_{21} = \theta_{31} = 0$. Table 7 shows the estimates of the structural parameters of system (34); it confirms that the estimates of β_2 and γ_1 are numerically small.[30] Moreover, the misleading case $\theta_{21} \neq 0$ and $\theta_{31} \neq 0$ forces to impose three restrictions: two restrictions are required to compensate for the 'pseudo' deviations of θ_{21} and θ_{31} from zero (despite the fact that the true values are $\theta_{21} = \theta_{31} = 0$) and, as stated above, one restrictrion is needed to identify the subsystem linking the statistical innovations to the structural shocks $\epsilon_{g,t}$ and $\epsilon_{y,t}$. Here, the three restrictions, implying that $\theta_{12} = \alpha_1\theta_{32}$, $\theta_{13} = \alpha_1\theta_{33}$, and $\theta_{23} = 0$, are placed on the subsystem relating the statistical innovations to the shock $\epsilon_{g,t}$ and $\epsilon_{y,t}$, so that it becomes over-identified. Such an over-identification alters the responses of the variables to a spending shock, relative to the those obtained under exact identification.

The second set of identifying restrictions invoked by Blanchard and Perotti (2002) imposes $\beta_2 = 0$ so that government spending is not affected by tax shocks, as well as $\alpha_1 = 2.08$ and $\beta_1 = 0$ to capture only automatic adjustments. Again, this identification strategy allows for $\theta_{31} \neq 0$. However, the estimate of γ_1 is negligible to yield the true value $\theta_{31} = 0$, as illustrated in Table 7. Moreover, two restrictions, namely $\theta_{13} = \alpha_1\theta_{33}$ and $\theta_{23} = 0$, lead to the over-identification of the subsystem allowing to trace the responses of the variables to a spending shock.

We now rely on the test results of the identification conditions to design two alternative sets of restrictions than those just presented. The first set imposes $\beta_2 = \gamma_1 = 0$ to recover the true values $\theta_{21} = \theta_{31} = 0$. It also fixes $\beta_1 = 0$, so that the single restriction $\theta_{23} = 0$ leads to the exact identification of the subsytem determining the responses of the variables to a spending shock. Note that the true value $\theta_{21} = 0$ and the restriction $\theta_{23} = 0$ imply that public spending is predetermined,

[30] In general, the estimation of the structural parameters requires to use the information contained in the covariance, coskewness, and excess cokurtosis matrices of the statistical innovations, Σ_ν, S_ν, and K^e_ν. In our application, however, the different sets of restrictions that are invoked always imply that the relation (34) involves six unrestricted parameters. Hence, we estimate these parameters by exploiting exclusively the decomposition $\Sigma_\nu = \Theta\Theta'$, where Σ_ν is measured by the sample estimator of the covariance matrix of the reduced-form residuals.

which is an assumption frequently invoked in practice (e.g., Fatás and Mihov, 2001; Galí, López-Salido, and Vallés, 2007; Monacelli and Perotti, 2010). This time, however, the parameter α_1 is estimated from the data, rather than calibrated from institutional information. As such, the estimated value of α_1 may measure not only the automatic adjustments, but also the government's systematic responses of taxes to fluctuations in output. In particular, the value of α_1 is smaller (larger), than the one capturing exclusively automatic adjustments, when the systematic component is a concave (convex) function of output (Bouakez, Chihi, and Normandin, 2014).

The second set of restrictions that we explore also fixes $\beta_2 = \gamma_1 = 0$ to yield the true values $\theta_{21} = \theta_{31} = 0$. It further sets $\alpha_2 = 0$, such that the single restriction $\theta_{12}\theta_{33} = \theta_{13}\theta_{32}$ enables one to exactly identify the effects of a spending shock. Note that the parameters α_1 and β_1 are estimated from the data (rather than calibrated from institutional information) to measure the automatic and systematic responses of taxes and public spending to changes in output. In particular, $\beta_1 < 0$ ($\beta_1 > 0$) suggests that public spending is countercyclical (procyclical). Finally, observe that $\beta_1 \neq 0$ implies that $\theta_{23} \neq 0$, so that government expenditures are no longer predetermined.

Table 8 reports the estimates of the structural parameters obtained under the alternative sets of restrictions. Note that the estimate of α_1 obtained under the restrictions $\beta_2 = \gamma_1 = \beta_1 = 0$ is less than the calibrated value of 2.08, and is even smaller under the restrictions $\beta_2 = \gamma_1 = \alpha_2 = 0$. Also, the estimate of β_1 computed under the restrictions $\beta_2 = \gamma_1 = \alpha_2 = 0$ is close to zero, which suggests that public spending is predetermined. Moreover, the estimate of γ_2 is slightly larger under the restrictions $\beta_2 = \gamma_1 = \beta_1 = 0$, which implies that the impact response of output to a spending shock (measured by θ_{32}) is larger.[31]

To complete the analysis, Tables 9 and 10 show the effects of fiscal shocks obtained under the

[31] Note that, for each alternative set of restrictions, the estimated tax shocks are not significantly related to the first four lags of a dummy variable that represents the dates identified by Romer and Romer (2010). These dates, documented from presidential speeches and Congressional reports, mark the announcements of exogenous changes in U.S. tax policy (i.e. legislated tax policy actions that are not taken for the purpose of offsetting factors that could affect output growth). Likewise, the estimated spending shocks are not significantly related to the first four lags of i) the dates corresponding to the expected changes in U.S. military spending identified by Ramey (2011), and ii) the forecasts of government spending provided in the Federal Reserve Bank of Philadelphia's Survey of Professional Forecasters. Overall, these findings imply that the fiscal shocks extracted from the SVAR are not Granger-caused by "news" about future taxes and public spending, identified using the narrative approach or based on forecasts. This evidence, albeit suggestive, indicates that the fiscal foresight problem is not sufficiently severe to undermine the SVAR approach.

restrictions $\beta_2 = \gamma_1 = \beta_1 = 0$ and $\beta_2 = \gamma_1 = \alpha_2 = 0$. To ease the interpretation, these effects are measured by the dynamic multipliers defined as the dollar change in output occuring in quarter $t+i$ that results from a dollar cut (increase) in the exogenous component of taxes (public spending) in period t. For the tax multipliers, several observations are worth stressing. First, the multipliers are identical across the two sets of restrictions. This arises because the subsystem relating the statistical innovations to the tax shock is identified, even when no restrictions are imposed. As a result, the effects of tax shocks are uniquely defined, and, hence, are not affected by the selection of the restrictions placed on the subsystem linking the statistical innovations to the structural shocks $\epsilon_{g,t}$ and $\epsilon_{y,t}$. Second, this uniqueness of the multipliers holds despite that the estimates of α_1 differ across the two sets of restrictions. Consequently, the response of output to a tax shock is not entirely pinned down by the value of α_1 (for a given covariance matrix of the statistical innovations Σ_ν), unlike what is suggested in Caldara and Kamps (2017). Third, the impact multiplier is zero, given that the output does not instantanously respond to a tax cut (i.e. $\theta_{31} = 0$). Fourth, the dynamic multipliers are always less than one, as they reached a peak of 0.61 after 14 quarters.

In constrast, the spending multipliers differ across the two sets of restrictions. This occurs because the effects of a spending shock are influenced by the nature of the restriction placed on the subsystem linking the statistical innovations to the shock $\epsilon_{g,t}$ and $\epsilon_{y,t}$. In particular, the impact multiplier is larger under the restrictions $\beta_2 = \gamma_1 = \beta_1 = 0$, given that the estimate of γ_2 (and thus the impact response of output to the spending shock θ_{32}) is larger under this set of restrictions. Likewise, the dynamic multipliers are larger under the restrictions $\beta_2 = \gamma_1 = \beta_1 = 0$. Despite the fact that the spending multipliers depend on the set of restrictions used, it is interesting to note that they always reach values larger than one at impact and at the peak, and as such they substantially exceed the impact and maximal tax multipliers.

6. Conclusion

In this paper, we first derived the identification conditions of SVAR processes through higher unconditional moments. These conditions are solely related to the numbers of statistical innovations and structural shocks that display skewness and/or excess kurtosis. Furthermore, these conditions establish which structural parameters are identified and which are not. For practitioners, this yields useful guidances about which structural parameters need to be restricted to achieve the identification of the entire system.

We then developed a tractable procedure to verify whether a SVAR process is identified, prior to the estimation of the structural parameters. In particular, the numbers of asymmetric and non-mesokurtic statistical innovations are inferred by applying Jarque-Bera tests. Also, the numbers of structural shocks exhibiting skewness and excess kurtosis correspond to the ranks of the third and fourth unconditional moment matrices of the statistical innovations. A bootstrap procedure is designed to improve the small-sample properties of these rank tests. The bootstrap version of the tests are virtually free of size distortions, whereas existing tests with asymptotic distributions suffer from severe size distortions even for large samples.

7. Appendix

This Appendix details the analytical partial derivatives involved in the Jacobians matrices (18), (19), and (20). First, the partial derivatives of the second unconditional moments of the statistical innovations with respect to the structural parameters are:

$$
\begin{aligned}
J_{\sigma_\nu,\theta_i} &= 2D_\sigma^+(\Theta \otimes I_n)\Upsilon_{\theta_i}, \\
J_{\sigma_\nu,s_\epsilon} &= 0, \\
J_{\sigma_\nu,\kappa_\epsilon^e} &= 0,
\end{aligned}
$$

where $i = s, ns$ in (18), $i = \kappa, n\kappa$ in (19), and $i = ss, \kappa\kappa, s\kappa, n$ in (20). The vectorization of the distinct elements of the second moments yields $\sigma_\nu = D_\sigma^+ vec(\Sigma_\nu)$, where $\sigma_\nu = vech(\Sigma_\nu)$, $D_\sigma^+ = (D_\sigma' D_\sigma)^{-1} D_\sigma'$, and D_σ is the $\left(n^2 \times \frac{n(n+1)}{2}\right)$ duplication matrix such that $D_\sigma \sigma_\nu = vec(\Sigma_\nu)$. Using this vectorization, we obtain $\frac{\partial \sigma_\nu}{\partial \theta_i'} = D_\sigma^+ \frac{\partial vec(\Sigma_\nu)}{\partial vec(\Theta)'} \frac{\partial vec(\Theta)}{\partial \theta_i'}$. Equation (11) leads to $vec(\Sigma_\nu) = (\Theta \otimes \Theta) vec(I_n)$, so that $\frac{\partial vec(\Sigma_\nu)}{\partial vec(\Theta)'} = 2(\Theta \otimes I_n)$ (see Lütkepohl, 2007, p. 363). Also, $\frac{\partial vec(\Theta)}{\partial \theta_i'} = \Upsilon_{\theta_i}$ is a matrix containing the values one and zero such that only the partial derivatives with respect to the elements of the vector θ_i are selected. As an example, consider the relation (14) with $n = 2$ and $n_s = m_s = 1$ (where the asymmetric statistical innovation and structural shock are ordered first), then the $(n^2 \times n_s m_s)$ selection matrix corresponds to $\Upsilon_{\theta_s} = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}'$ and $\theta_s = vec(\Theta_{s,s})$. Moreover, $\frac{\partial \sigma_\nu}{\partial s_\epsilon'} = D_\sigma^+ \frac{\partial vec(\Sigma_\nu)}{\partial vec(S_\epsilon)'} \frac{\partial vec(S_\epsilon)}{\partial s_\epsilon'}$, where $\frac{\partial vec(\Sigma_\nu)}{\partial vec(S_\epsilon)'} = 0$ given that Σ_ν is not a function of the skewnesses of the structural shocks. Likewise, $\frac{\partial \sigma_\nu}{\partial \kappa_\epsilon^{e\prime}} = D_\sigma^+ \frac{\partial vec(\Sigma_\nu)}{\partial vec(K_\epsilon^e)'} \frac{\partial vec(K_\epsilon^e)}{\partial \kappa_\epsilon^{e\prime}}$ with $\frac{\partial vec(\Sigma_\nu)}{\partial vec(K_\epsilon^e)'} = 0$.

Next, the partial derivatives of the third unconditional moments of the statistical innovations with respect to the structural parameters are:

$$
\begin{aligned}
J_{s_\nu,\theta_i} &= D_s^+\{(I_{n^2} \otimes \Theta S_\epsilon)[(I_n \otimes C_{n,n} \otimes I_n)[(I_{n^2} \otimes vec(\Theta')) + (vec(\Theta') \otimes I_{n^2})]C_{n,n}] + [(\Theta \otimes \Theta)S_\epsilon' \otimes I_n]\}\Upsilon_{\theta_i}, \\
J_{s_\nu,s_\epsilon} &= D_s^+(\Theta \otimes \Theta \otimes \Theta)\Upsilon_{s_\epsilon}, \\
J_{s_\nu,\kappa_\epsilon^e} &= 0,
\end{aligned}
$$

where $i = s, ns$ in (18) and $i = ss, \kappa\kappa, s\kappa, n$ in (20). The vectorization of the non-zero distinct elements of the third moments corresponds to $s_\nu = D_s^+ vec(S_\nu)$, where $D_s^+ = (D_s' D_s)^{-1} D_s'$, and

D_s is the $\left(n^3 \times \frac{n_s(n_s+1)(n_s+2)}{6}\right)$ matrix such that $D_s s_\nu = vec(S_\nu)$. As an example, for a bivariate system with $n = n_s = 2$, then:

$$D_s = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

For the case with $n = 2$ and $n_s = 1$ (where the asymmetric statistical innovation is ordered first), then $D_s = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}'$.

Using the above vectorization, we have $\frac{\partial s_\nu}{\partial \theta_i'} = D_s^+ \frac{\partial vec(S_\nu)}{\partial vec(\Theta)'} \frac{\partial vec(\Theta)}{\partial \theta_i'}$ with $\frac{\partial vec(\Theta)}{\partial \theta_i'} = \Upsilon_{\theta_i}$. Rewriting equation (12) as $vec(S_\nu) = [(\Theta \otimes \Theta) \otimes \Theta] vec(S_\epsilon)$, then $\frac{\partial vec(S_\nu)}{\partial vec(\Theta)'} = (I_{n^2} \otimes \Theta S_\epsilon) \frac{\partial vec(\Theta' \otimes \Theta')}{\partial vec(\Theta)'} + [(\Theta \otimes \Theta) S_\epsilon' \otimes I_n]$, where $\frac{\partial vec(\Theta' \otimes \Theta')}{\partial vec(\Theta)'} = (I_n \otimes C_{n,n} \otimes I_n)[(I_{n^2} \otimes vec(\Theta')) + (vec(\Theta') \otimes I_{n^2})] \frac{\partial vec(\Theta')}{\partial vec(\Theta)'}$ with $\frac{\partial vec(\Theta')}{\partial vec(\Theta)'} = C_{n,n}$ (see Magnus and Neudecker, 2007, pp. 208–209), and $C_{n,m}$ is a $(nm \times nm)$ commutation matrix implying that $C_{n,m} vec(A) = vec(A')$ for the arbitrary $(n \times m)$ matrix A. Note that $\frac{\partial s_\nu}{\partial \theta_i'} = 0$ for $i = ns$ in (18) and for $i = \kappa\kappa, n$ in (20), since S_ν is not a function of the structural parameters relating the statistical innovations to the symmetric structural shocks. Furthermore, $\frac{\partial s_\nu}{\partial s_\epsilon} = D_s^+ \frac{\partial vec(S_\nu)}{\partial vec(S_\epsilon)'} \frac{\partial vec(S_\epsilon)}{\partial s_\epsilon{}'}$, where $\frac{\partial vec(S_\nu)}{\partial vec(S_\epsilon)'} = (\Theta \otimes \Theta \otimes \Theta)$ and $\frac{\partial vec(S_\epsilon)}{\partial s_\epsilon{}'} = \Upsilon_{s_\epsilon}$ is a $(n^3 \times m_s)$ matrix selecting the partial derivatives with respect to the elements of s_ϵ. In particular, for a system with $n = m_s = 2$, then Υ_{s_ϵ} has values one for the (1,1) and (8,2) elements, and zero elsewhere. For the system with $n = 2$ and $m_s = 1$, then Υ_{s_ϵ} has values one for the (1,1) element, and zero elsewhere. Moreover, $\frac{\partial s_\nu}{\partial \kappa_\epsilon^e} = D_s^+ \frac{\partial vec(S_\nu)}{\partial vec(K_\epsilon^e)'} \frac{\partial vec(K_\epsilon^e)}{\partial \kappa_\epsilon^{e\prime}}$, where $\frac{\partial vec(S_\nu)}{\partial vec(K_\epsilon^e)'} = 0$ given that S_ν is not a function of the excess kurtoses of the structural shocks.

Finally, the partial derivatives of the fourth unconditional moments of the statistical innovations with respect to the structural parameters are:

$$\begin{aligned} J_{\kappa_\nu^e, \theta_i} &= D_\kappa^+ \{(I_{n^2} \otimes \Theta K_\epsilon^e)(I_{n^2} \otimes C_{n,n^2} \otimes I_n)[(I_{n^4} \otimes vec(\Theta'))(I_n \otimes C_{n,n} \otimes I_n) \\ &\quad \times [(I_{n^2} \otimes vec(\Theta') + (vec(\Theta') \otimes I_{n^2})]C_{n,n} + (vec(\Theta' \otimes \Theta') \otimes I_{n^2})C_{n,n}] + [(\Theta \otimes \Theta \otimes \Theta) K_\epsilon^{e\prime} \otimes I_n]\} \Upsilon_{\theta_i}, \\ J_{\kappa_\nu^e, s_\epsilon} &= 0, \\ J_{\kappa_\nu^e, \kappa_\epsilon^e} &= D_\kappa^+ (\Theta \otimes \Theta \otimes \Theta \otimes \Theta) \Upsilon_{\kappa_\epsilon^e}, \end{aligned}$$

where $i = \kappa, n\kappa$ in (19) and $i = ss, \kappa\kappa, s\kappa, n$ in (20). The vectorization of the non-zero distinct elements of the fourth moments is $\kappa_\nu^e = D_\kappa^+ vec(K_\nu^e)$, where $D_\kappa^+ = (D_\kappa' D_\kappa)^{-1} D_\kappa'$, and D_κ is the $\left(n^4 \times \frac{n_\kappa(n_\kappa+1)(n_\kappa+2)(n_\kappa+3)}{24}\right)$ matrix such that $D_\kappa \kappa_\nu^e = vec(K_\nu^e)$. For example, when $n = n_\kappa = 2$, then:

$$D_\kappa = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Instead, when $n = 2$ and $n_\kappa = 1$ (where the non-mesokurtic statistical innovation is ordered first), then $D_\kappa = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}'$.

Using the above vectorization, we have $\frac{\partial \kappa_\nu^e}{\partial \theta_i'} = D_\kappa^+ \frac{\partial vec(K_\nu^e)}{\partial vec(\Theta)'} \frac{\partial vec(\Theta)}{\partial \theta_i'}$ with $\frac{\partial vec(\Theta)}{\partial \theta_i'} = \Upsilon_{\theta_i}$. Given that equation (13) implies $vec(K_\nu^e) = [(\Theta \otimes \Theta \otimes \Theta) \otimes \Theta] vec(K_\epsilon^e)$, then $\frac{\partial vec(K_\nu^e)}{\partial vec(\Theta)'} = (I_{n^2} \otimes \Theta K_\epsilon^e) \frac{\partial vec(\Theta' \otimes \Theta' \otimes \Theta')}{\partial vec(\Theta)'} + [(\Theta' \otimes \Theta' \otimes \Theta') K_\epsilon^{e\prime} \otimes I_n]$, where $\frac{\partial vec(\Theta' \otimes \Theta' \otimes \Theta')}{\partial vec(\Theta)'} = (I_{n^2} \otimes C_{n,n^2} \otimes I_n) \left[(I_{n^4} \otimes vec(\Theta')) \frac{\partial vec(\Theta' \otimes \Theta')}{\partial vec(\Theta)'} + [vec(\Theta' \otimes \Theta') \otimes I_{n^2}]\right] \frac{\partial vec(\Theta')}{\partial vec(\Theta)'}$, and, as shown above, $\frac{\partial vec(\Theta' \otimes \Theta')}{\partial vec(\Theta)'} = (I_n \otimes C_{n,n} \otimes I_n)[(I_{n^2} \otimes vec(\Theta')) + (vec(\Theta') \otimes I_{n^2})] \frac{\partial vec(\Theta')}{\partial vec(\Theta)'}$ and $\frac{\partial vec(\Theta')}{\partial vec(\Theta)'} = C_{n,n}$. Note that $\frac{\partial \kappa_\nu^e}{\partial \theta_i'} = 0$ for $i = n\kappa$ in (19) and for $i = ss, n$ in (20), since K_ν^e is not a function of the structural parameters relating the statistical innovations to the mesokurtic structural shocks. Moreover, $\frac{\partial \kappa_\nu^e}{\partial s_\epsilon} = D_\kappa^+ \frac{\partial vec(K_\nu^e)}{\partial vec(S_\epsilon)'} \frac{\partial vec(S_\epsilon)}{\partial s_\epsilon{}'}$, where $\frac{\partial vec(K_\nu^e)}{\partial vec(S_\epsilon)'} = 0$ given that K_ν^e is not a function of the skewnesses of the structural shocks. In addition, $\frac{\partial \kappa_\nu^e}{\partial \kappa_\epsilon^e} = D_\kappa^+ \frac{\partial vec(K_\nu^e)}{\partial vec(K_\epsilon^e)'} \frac{\partial vec(K_\epsilon^e)}{\partial \kappa_\epsilon^{e\prime}}$, where $\frac{\partial vec(K_\nu^e)}{\partial vec(K_\epsilon^e)'} = (\Theta \otimes \Theta \otimes \Theta \otimes \Theta)$ and $\frac{\partial vec(K_\epsilon^e)}{\partial \kappa_\epsilon^{e\prime}} = \Upsilon_{\kappa_\epsilon^e}$ is a $(n^4 \times m_\kappa)$ matrix selecting the partial derivatives with respect to the elements of κ_ϵ^e. For example, when $n = m_\kappa = 2$, then $\Upsilon_{\kappa_\epsilon^e}$ has values one for the (1,1) and (16,2) elements, and zero elsewhere. For the system with $n = 2$ and $m_\kappa = 1$, then $\Upsilon_{\kappa_\epsilon^e}$ has values one for the (1,1) element, and zero elsewhere.

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Table 1. Order Conditions for a System with Four Variables ($n = 4$)

		m_s = 1	m_s = 2	m_s = 3	m_s = 4
n_s	1	$\eta = 14$ $\rho = 11$	—	—	—
	2	$\eta = 15$ $\rho = 14$	$\boldsymbol{\eta} = \mathbf{14}$ $\boldsymbol{\rho} = \mathbf{14}$	—	—
	3	$\boldsymbol{\eta} = \mathbf{16}$ $\boldsymbol{\rho} = \mathbf{20}$	$\boldsymbol{\eta} = \mathbf{16}$ $\boldsymbol{\rho} = \mathbf{20}$	$\boldsymbol{\eta} = \mathbf{16}$ $\boldsymbol{\rho} = \mathbf{20}$	—
	4	$\boldsymbol{\eta} = \mathbf{17}$ $\boldsymbol{\rho} = \mathbf{30}$	$\boldsymbol{\eta} = \mathbf{18}$ $\boldsymbol{\rho} = \mathbf{30}$	$\boldsymbol{\eta} = \mathbf{19}$ $\boldsymbol{\rho} = \mathbf{30}$	$\boldsymbol{\eta} = \mathbf{20}$ $\boldsymbol{\rho} = \mathbf{30}$

Notes. The terms n_s and m_s refer to the numbers of skewed statistical innovations and structural shocks, whereas $\eta = [n^2 - (n - n_s)m_s] + m_s$ corresponds to the number of parameters in the structural form, and $\rho = \left[\frac{n(n+1)}{2}\right] + \left[\frac{n_s(n_s+1)(n_s+2)}{6}\right]$ is the number of distinct elements in the reduced form. — indicates the configurations where the impact response matrix Θ is singular. Bold characters indicate the cases where the order conditions, $\rho \geq \eta$, hold.

Table 2. Rank Condition for a System with Four Variables ($n = 4$)

n_s	m_s = 1	2	3	4
1	$\eta = 14$ $r_s = 1$ $r_{ns} = 9$ $r_{s_\epsilon} = 1$ $r = 11$	—	—	—
2	$\eta = 15$ $r_s = 2$ $r_{ns} = 9$ $r_{s_\epsilon} = 1$ $r = 12$	$\eta = 14$ $r_s = 4$ $r_{ns} = 7$ $r_{s_\epsilon} = 2$ $r = 13$	—	—
3	$\eta = 16$ $r_s = 3$ $r_{ns} = 9$ $r_{s_\epsilon} = 1$ $r = 13$	$\eta = 16$ $r_s = 6$ $r_{ns} = 7$ $r_{s_\epsilon} = 2$ $r = 15$	$\boldsymbol{\eta} = \mathbf{16}$ $\mathbf{r}_s = \mathbf{9}$ $\mathbf{r}_{ns} = \mathbf{4}$ $\mathbf{r}_{s_\epsilon} = \mathbf{3}$ $\mathbf{r} = \mathbf{16}$	—
4	$\eta = 17$ $r_s = 4$ $r_{ns} = 9$ $r_{s_\epsilon} = 1$ $r = 14$	$\eta = 18$ $r_s = 8$ $r_{ns} = 7$ $r_{s_\epsilon} = 2$ $r = 17$	$\boldsymbol{\eta} = \mathbf{19}$ $\mathbf{r}_s = \mathbf{12}$ $\mathbf{r}_{ns} = \mathbf{4}$ $\mathbf{r}_{s_\epsilon} = \mathbf{3}$ $\mathbf{r} = \mathbf{19}$	$\boldsymbol{\eta} = \mathbf{20}$ $\mathbf{r}_s = \mathbf{16}$ $\mathbf{r}_{ns} = \mathbf{0}$ $\mathbf{r}_{s_\epsilon} = \mathbf{4}$ $\mathbf{r} = \mathbf{20}$

Notes. The terms n_s and m_s refer to the numbers of skewed statistical innovations and structural shocks, whereas $\eta = [n^2 - (n - n_s)m_s] + m_s$ corresponds to the number of parameters in the structural form, and $r = r_s + r_{ns} + r_{s_\epsilon}$ is the rank associated with the reduced form, with $r_s = n_s \times m_s$, $r_{ns} = \sum_{i=0}^{n-m_s} (n-i) - m_s$, and $r_{s_\epsilon} = m_s$. — indicates the configurations where the impact response matrix Θ is singular. Bold characters indicate the cases where the rank condition, $r = \eta$, holds.

Table 3. Empirical Sizes of Rank Tests with Asymptotic Distributions: Symmetry

		$r^* = 0$				
	Wald			LR		
T	10 %	5%	1%	10 %	5%	1%
100	8.72	3.92	0.53	2.68	0.63	0.01
200	9.99	4.66	0.80	5.81	1.91	0.12
500	9.93	4.69	0.81	7.97	3.36	0.41
$1,000$	9.73	4.63	0.70	8.65	3.94	0.52
$5,000$	10.03	5.22	1.09	9.90	4.97	1.02
		$r^* = 1$				
	Wald			LR		
T	10 %	5%	1%	10 %	5%	1%
100	11.83	5.79	1.52	7.86	3.22	0.51
200	10.87	5.30	1.18	8.60	3.66	0.53
500	10.89	5.20	1.06	9.74	4.42	0.63
$1,000$	9.97	4.82	1.03	9.45	4.36	0.86
$5,000$	10.61	5.59	1.02	10.05	5.47	0.99

Notes. Entries are the empirical sizes (in percentage) of the rank tests with asymptotic distributions under the null hypothesis that $rk[S_u] = r^*$. The empirical sizes are evaluated for the bivariate specification (1)–(2), where the parameters are set as follows: $\alpha_d = \alpha_s = 0.5$ and $\omega_d = \omega_s = 1$. Also, the distributions are $\epsilon_{s,t} \sim N(0,1)$, and i) $\epsilon_{d,t} \sim N(0,1)$ under $r^* = 0$ or ii) $2.1755 \times \epsilon_{d,t} \sim N(1,1)$ with probability 0.7887 and $2.1755 \times \epsilon_{d,t} \sim N(-3.7326, 1)$ with probability 0.2113 under $r^* = 1$. For each parametrization, 10,000 simulated samples of size T are generated to compute the proportions of time that the Wald statistic $\widehat{CRT}^{W}_{r^*}$ and the likelihood-ratio (LR) statistic $\widehat{CRT}^{LR}_{r^*}$ associated with S_u exceed the asymptotic critical values, where the latters are computed as in Robin and Smith (2000).

Table 4. Empirical Sizes of Rank Tests with Asymptotic Distributions: Kurtosis

$r^* = 0$						
	Wald			LR		
T	10 %	5%	1%	10 %	5%	1%
100	0.70	0.10	0.00	0.00	0.00	0.00
200	0.00	0.00	0.00	0.00	0.00	0.00
500	0.00	0.00	0.00	0.00	0.00	0.00
1,000	0.00	0.00	0.00	0.00	0.00	0.00
5,000	0.00	0.00	0.00	0.00	0.00	0.00
$r^* = 1$						
	Wald			LR		
T	10 %	5%	1%	10 %	5%	1%
100	1.19	0.53	0.08	0.37	0.07	0.00
200	1.05	0.38	0.04	0.33	0.06	0.00
500	0.68	0.21	0.02	0.36	0.12	0.00
1,000	0.54	0.21	0.05	0.32	0.09	0.00
5,000	0.36	0.10	0.02	0.32	0.08	0.02

Notes. Entries are the empirical sizes (in percentage) of the rank tests with asymptotic distributions under the null hypothesis that $rk[K_u^e] = r^*$. The empirical sizes are evaluated for the bivariate specification (1)–(2), where the parameters are set as follows: $\alpha_d = \alpha_s = 0.5$ and $\omega_d = \omega_s = 1$. Also, the distributions are $\epsilon_{s,t} \sim N(0,1)$, and i) $\epsilon_{d,t} \sim N(0,1)$ under $r^* = 0$ or ii) $1.291 \times \epsilon_{d,t} \sim t(5)$ under $r^* = 1$. For each parametrization, 10,000 simulated samples of size T are generated to compute the proportions of time that the Wald statistic $\widehat{CRT}_{r^*}^{W}$ and the likelihood-ratio (LR) statistic $\widehat{CRT}_{r^*}^{LR}$ associated with K_u^e exceed the asymptotic critical values, where the latters are computed as in Robin and Smith (2000).

Table 5. Empirical Sizes of Rank Tests with Finite-Sample Distributions: Symmetry

$r^* = 0$						
	Wald			LR		
T	10 %	5%	1%	10 %	5%	1%
100	9.42	4.65	0.98	9.56	4.85	1.01
200	10.17	5.25	0.98	10.19	5.20	1.00
500	10.14	5.04	1.10	10.29	4.99	1.12
$1,000$	9.82	4.91	0.92	9.87	4.90	0.92
$5,000$	10.02	5.10	1.12	9.98	5.11	1.11
$r^* = 1$						
	Wald			LR		
T	10 %	5%	1%	10 %	5%	1%
100	11.41	6.35	1.47	11.41	6.35	1.47
200	9.11	4.86	1.42	9.11	4.86	1.42
500	9.29	4.55	1.07	9.29	4.55	1.07
$1,000$	8.39	4.26	1.02	8.39	4.26	1.02
$5,000$	9.20	4.68	0.96	9.20	4.68	0.96

Notes. Entries are the empirical sizes (in percentage) of the rank tests with finite-sample distributions under the null hypothesis that $rk[S_u] = r^*$. The empirical sizes are evaluated for the bivariate specification (1)–(2), where the parameters are set as follows: $\alpha_d = \alpha_s = 0.5$ and $\omega_d = \omega_s = 1$. Also, the distributions are $\epsilon_{s,t} \sim N(0,1)$, and i) $\epsilon_{d,t} \sim N(0,1)$ under $r^* = 0$ or ii) $2.1755 \times \epsilon_{d,t} \sim N(1,1)$ with probability 0.7887 and $2.1755 \times \epsilon_{d,t} \sim N(-3.7326,1)$ with probability 0.2113 under $r^* = 1$. For each parametrization, 10,000 simulated samples of size T are generated to compute the proportions of time that the Wald statistic $\widehat{CRT}^{W}_{r^*}$ and the likelihood-ratio (LR) statistic $\widehat{CRT}^{LR}_{r^*}$ associated with S_u exceed the finite-sample critical values, where the latters are computed by the bootstrap procedure elaborated in Section 4.2.

Table 6. Empirical Sizes of Rank Tests with Finite-Sample Distributions: Kurtosis

$r^* = 0$						
	Wald			LR		
T	10 %	5%	1%	10 %	5%	1%
100	10.12	5.00	0.99	10.43	4.93	1.09
200	9.74	5.14	1.23	9.75	5.19	1.21
500	9.81	4.91	1.01	9.86	4.87	1.00
1,000	9.71	4.60	1.04	9.75	4.58	1.03
5,000	9.84	4.88	1.02	9.83	4.89	1.03
$r^* = 1$						
	Wald			LR		
T	10 %	5%	1%	10 %	5%	1%
100	9.90	4.98	0.93	9.90	4.98	0.93
200	10.65	5.67	1.19	10.65	5.67	1.19
500	9.88	5.08	1.15	9.88	5.08	1.15
1,000	10.10	4.95	0.95	10.10	4.95	0.95
5,000	9.71	4.76	0.95	9.71	4.76	0.95

Notes. Entries are the empirical sizes (in percentage) of the rank tests with finite-sample distributions under the null hypothesis that $rk[K_u^e] = r^*$. The empirical sizes are evaluated for the bivariate specification (1)–(2), where the parameters are set as follows: $\alpha_d = \alpha_s = 0.5$ and $\omega_d = \omega_s = 1$. Also, the distributions are $\epsilon_{s,t} \sim N(0,1)$, and i) $\epsilon_{d,t} \sim N(0,1)$ under $r^* = 0$, and ii) $1.291 \times \epsilon_{d,t} \sim t(5)$ under $r^* = 1$. For each parametrization, 10,000 simulated samples of size T are generated to compute the proportions of time that the Wald statistic $\widehat{CRT}_{r^*}^{W}$ and the likelihood-ratio (LR) statistic $\widehat{CRT}_{r^*}^{LR}$ associated with K_u^e exceed the finite-sample critical values, where the latters are computed by the bootstrap procedure elaborated in Section 4.2.

Table 7. Estimates of the Structural Parameters under the Blanchard-Perotti Sets of Restrictions

Parameter	$\alpha_1 = 2.08$, $\beta_1 = 0$, and $\alpha_2 = 0$	$\alpha_1 = 2.08$, $\beta_1 = 0$, and $\beta_2 = 0$
α_2	–	-0.011
		$(-1.283, 0.952)$
β_2	-0.003	–
	$(-0.026, 0.020)$	
γ_1	-0.002	-0.003
	$(-0.020, 0.015)$	$(-0.020, 0.015)$
γ_2	0.252	0.204
	$(0.135, 0.382)$	$(0.130, 0.388)$
ω_τ	0.047	0.047
	$(0.035, 0.049)$	$(0.035, 0.049)$
ω_g	0.007	0.007
	$(0.005, 0.007)$	$(0.005, 0.007)$
ω_y	0.005	0.005
	$(0.004, 0.005)$	$(0.004, 0.005)$

Notes. Entries correspond to the estimates of the structural parameters obtained under the two Blanchard-Perotti sets of restrictions, namely i) $\alpha_1 = 2.08$, $\beta_1 = 0$, and $\alpha_2 = 0$, and ii) $\alpha_1 = 2.08$, $\beta_1 = 0$, and $\beta_2 = 0$. Numbers in parentheses represent the 90 percent confidence intervals, where the confidence intervals are computed from 10,000 bootstrap samples.

Table 8. Estimates of the Structural Parameters under the Alternative Sets of Restrictions

Parameter	$\gamma_1 = 0$, $\beta_2 = 0$, and $\beta_1 = 0$	$\gamma_1 = 0$, $\beta_2 = 0$, and $\alpha_2 = 0$
α_1	1.854	1.777
	$(0.313, 3.778)$	$(0.182, 4.670)$
α_2	-0.078	–
	$(-1.281, 0.899)$	
β_1	–	0.006
		$(-0.834, 0.103)$
γ_2	0.252	0.224
	$(0.134, 0.389)$	$(0.002, 0.477)$
ω_τ	0.047	0.047
	$(0.035, 0.049)$	$(0.035, 0.049)$
ω_g	0.007	0.007
	$(0.005, 0.007)$	$(0.005, 0.007)$
ω_y	0.005	0.005
	$(0.004, 0.005)$	$(0.002, 0.005)$

Notes. Entries correspond to the estimates of the structural parameters obtained under two alternative sets of restrictions, namely i) $\gamma_1 = 0$, $\beta_1 = 0$, and $\beta_2 = 0$, and ii) $\gamma_1 = 0$, $\beta_2 = 0$, and $\alpha_2 = 0$. Numbers in parentheses represent the 90 percent confidence intervals, where the confidence intervals are computed from 10,000 bootstrap samples.

Table 9. Tax Multipliers

under the Alternative Sets of Restrictions

Quarter	$\gamma_1 = 0$, $\beta_2 = 0$, and $\beta_1 = 0$	$\gamma_1 = 0$, $\beta_2 = 0$, and $\alpha_2 = 0$
1	0.00	0.00
4	0.05	0.05
8	0.24	0.24
Peak	0.61	0.61
	[14]	[14]

Notes. Entries correspond to the tax multiplier: the dollar change in output at a given horizon that results from a dollar cut in the exogenous component of taxes. An asterisk indicates that the 90 percent confidence interval does not include zero, where the confidence intervals are computed from 10,000 bootstrap samples. Numbers between brackets indicate the quarters in which the maximum value of the multiplier is reached.

Table 10. Spending Multipliers

under the Alternative Sets of Restrictions

Quarter	$\gamma_1 = 0$, $\beta_2 = 0$, and $\beta_1 = 0$	$\gamma_1 = 0$, $\beta_2 = 0$, and $\alpha_2 = 0$
1	1.28*	1.14*
4	1.24	0.98
8	0.72	0.49
Peak	1.76*	1.53
	[3]	[3]

Notes. Entries correspond to the spending multiplier: the dollar change in output at a given horizon that results from a dollar increase in the exogenous component of government spending. An asterisk indicates that the 90 percent confidence interval does not include zero, where the confidence intervals are computed from 10,000 bootstrap samples. Numbers between brackets indicate the quarters in which the maximum value of the multiplier is reached.