Trends in the distribution of household income and changes in the interdependence between income sources*

Iryna Kyzyma
Luxembourg Institute of Socio-Economic Research, and IZA Bonn

Alessio Fusco
Luxembourg Institute of Socio-Economic Research

Philippe Van Kerm
Luxembourg Institute of Socio-Economic Research
and University of Luxembourg

Abstract
We develop a copula-based decomposition framework which allows identifying how changes in the dependence structure between various income sources underlie shifts in the distribution of household income over time. In contrast to the existing inequality decomposition techniques, the method isolates the contribution of the changing interdependence of all income sources from the contribution attributable to changes in their marginal distributions. These two components can be broken down further to inspect how changes in the dependence structure between particular income sources and their marginal distributions reflect on the distribution of household income. The proposed methodology is illustrated on data from Luxembourgish Socio-Economic Panel in order to understand the change in the distribution of household disposable income between 2003 and 2012.

Keywords: distribution of household income, changes over time, copula-based decomposition; income sources; dependence structure

JEL classification codes: C14 ; D31; D33.

* This research has been funded by the Luxembourg Fonds national de la Recherche (PhD grant agreement 2011-1, 2011-2015) and by core funding for LISER from the Ministry of Higher Education and Research of Luxembourg. Comments by Martin Biewen, Jean-François Carpantier, Robin Jessen, Marie Le Mouel, Jamel Trabelsi, Don Williams, participants of the 33rd IARIW conference in Rotterdam, Conference on Public Economics of Economic Inequality in Berlin, 2nd Luxembourg workshop on Household Finance and Consumption, Augustin Cournot doctoral days in Strasbourg, IMPALLA/ESPANET conference in Luxembourg, and research seminars at the Centre for European Economic Reesearch (ZEW), Karlsruhe Institute of Technology, Kent State University, Luxembourg Institute of Socio-Economic Research, and SEMILUX seminar in Luxembourg are gratefully acknowledged. Iryna Kyzyma also thanks the Multidisciplinary Program in Inequality and Social Policy at Harvard University, where a part of the work on this paper has been accomplished, for its hospitality. The usual disclaimers apply.

1. Introduction

Many rich countries have experienced sharp changes in the distribution of total household income over recent decades (OECD, 2015). Widening the gap between low-income and high-income families, they have resulted in the rise of income inequality, poverty and polarization indexes. Inspection of household income sources, such as earnings, capital income, social transfers or taxes, is useful to understand the nature of changes underlying the level and dispersion of total income over time. This approach has a long history starting with the seminal works of Shorrocks (1982) and Lerman and Yitzhaki (1985), who have proposed decomposition techniques to uncover the contribution of each income source to total income inequality at a given point in time. It has also been used recently, among others, by Fiorio (2011), Larrimore (2013), García-Peñalosa and Orgiazzi (2013), Brewer and Wren-Lewis (2016) to explain changes in income inequality over time in a number of OECD countries.

Most of the existing methods for the analysis of income sources are limited to particular indices of inequality (Jenkins and Van Kerm, 2009).[1] Given that household disposable income is the sum of income components (e.g. pre-tax income sources as well as social security transfers, taxes and social insurance contributions), one can assess the contribution of each income source to overall inequality. According to Sastre and Trannoy (2002), the decomposition of inequality by factors can take two forms. Within what they refer to as the 'global methods', the contribution of each source is computed for all pre-determined components and the sum of these contributions add up to the inequality index to be explained. Examples of such methods are Shorrocks (1982)'s decomposition rule or Lerman and Yitzhaki (1985)'s decomposition method. In the context of 'local methods', the contribution of one specific income source is computed using a before/after calculation: each source contribution is then equal to the difference between overall inequality and the inequality obtained when the income component is not taken into account (see Jenkins, 1995; Cancian and Reed, 1998; Fuest et al., 2010 for various applications and Lerman (1999) or Mussard (2007) for reviews of this literature).

'Global' and 'local' methods share some limits. First, these methods focus on specific distributional measures providing little evidence about changes that occur along the income distribution. In some cases, however, it might be important to know what happens where in the distribution. For example, the distribution might become more unequal over time because

[1] Note, however, that a method of decomposition by income source for the Sen index of poverty was proposed by Mussard and Pi Alperin (2011) and for the indexes of polarization by Araar (2008) and Deutsch et al. (2013).

of the increased dispersion in its upper tail, lower tail or both. Second, focus on specific summary measures makes these methods dependent on the index chosen: Shorrocks (1982) relies on the square of the coefficient of variation while Lerman and Yitzhaki (1985) rely on the Gini coefficient. Contribution of income components to the overall inequality may then be sensitive to the index used. Finally, both 'global' and 'local' methods aim to assess the contribution of different income sources to the overall inequality at a given point in time. In the context of rising inequalities, however, an inter-temporal analysis is also needed. It can help to identify the forces lying behind the observed trends in inequality and, hence, make policy interventions more informative.

As a response to these limitations, a number of scholars have proposed techniques which allow decomposing the change in the entire income distribution by income sources. The literature in this field builds on the seminal work of Burtless (1999) who use the rank-preserving income exchange to evaluate, among other factors, how changes in the marginal distributions of female and male earnings have contributed to the shift in the distribution of total income in the US between 1979 and 1996.[2] In a similar spirit, Fournier (2001), Daly and Valletta (2006), Fiorio (2011), and Larrimore (2013) assess the contributions of the changes in various income sources, to the trends in income inequality in Italy, Taiwan, and the US. Although all these studies provide extensive evidence on how shifts in marginal distributions of income sources underlie the distribution of household income, only a few of them address the changing interdependence structure between income sources. Fournier (2001), for example, investigates how changes in the rank-correlation between each income source and its complementary to total household income reflect on the distribution of household income. Larrimore (2013) goes further and investigates in a sequential manner how rank correlations between income sources contribute to the trends in total income inequality.

The main limitation of the studies mentioned above is their inability to clearly partition the overall shift in the distribution of household income into two components attributable to (1) changes in the marginal distributions of income sources and (2) their dependence structure. This, in turn, makes it difficult to draw conclusions about the relative portion of the distributional change which is associated with the change in the rank dependence of income sources as opposed to the change in their marginal distributions. In addition, the existing studies do not provide a formalized decomposition framework which

[2] The method foresees construction of the counterfactual income distribution by assigning to individuals with certain ranks in one period incomes to which their ranks would entitle them in another period.

would allow performing a detailed decomposition of the change in income distribution with respect to changes in the dependence structure between various income sources.

This paper aims to extend previous literature by developing a formalized semi-parametric decomposition procedure which makes it possible to partition the change in the distribution of total household income into two components attributable to changes in the marginal distributions of all income sources and their dependence structure. As a next step, it offers a detailed framework of how these two terms can be decomposed further into the contributions associated with separate income sources. The main advantage of the proposed methodology lies in its ability to explore the contribution of each income source to the change in the distribution of household income from two angles – the change in its marginal distribution and interdependence with other income sources. The latter is especially important in the era when spouses' incomes are becoming more and more correlated (Burtless, 1999; Larrimore, 2013) and wealth is getting highly concentrated among selected individuals (Piketty, 2014).

We apply the proposed methodology to decompose the change in the disposable household income in Luxembourg between 2003 and 2012. Over the past 25 years, Luxembourg has experienced a remarkable economic growth induced primarily by the expansion of the financial sector and other service activities, such as information and communication technologies (STATEC, 2003). The economic growth led to an improvement of the overall living standards of the resident population (Osier, 2012). However, following the trend of other rich countries, the levels of inequality and income poverty have been on the rise in Luxembourg, with an especially large increase being observed from 2000 onwards (Fusco et al., 2013). This evidence makes Luxembourg an interesting case to study, since alongside the fast rising living standards, the dispersion of income has increased substantially.

The paper is structured as follows. Section 2 presents the proposed decomposition methodology while Section 3 describes data used for the empirical application. Section 4 summarizes the changes in the distribution of total disposable income and its components in Luxembourg between 2003 and 2012. Section 5 provides the results of the decomposition exercise and Section 6 concludes.

2. A copula based decomposition approach

Specification of the model

Consider a population of N individuals consisting of two sub-groups where each sub-group is observed at one point in time $t \in \{0, 1\}$. Each individual i ($i=1, \ldots, N$) receives income from different income sources $k=1, \ldots, K$ (e.g, earnings, capital, public transfers etc.) which form a set of mutually exclusive income components, y_k, so that

$$y_i^t = \sum_{k=1}^{K} y_{ik}^t \tag{2.1}$$

where y_i^t is total income which individual i has in period t.

The cumulative distribution function (CDF) of total income, $F^t(y)$, in time period t can then be derived as a joint probability density function (PDF) of all income components $g(y_1, \ldots, y_k)$ in that period integrated over the region of their values:

$$F^t(y) = \Pr[Y_1 + \ldots + Y_k \le y \mid T = t] = \int_0^{y} \ldots \int_0^{y-y_1-\ldots-y_{k-1}} g^t(y_1, \ldots, y_k)\, dy_1 \ldots dy_k \tag{2.2}$$

According to Sklar's theorem (Sklar, 1959), the joint CDF of income components in period t, $G^t(y_1, \ldots, y_k)$, can be expressed as a function of their marginal CDFs, $F_{y_1}^t(y_1), \ldots, F_{y_k}^t(y_k)$, and the copula, $C_{y_1,\ldots,y_k}^t$:

$$G^t(y_1, \ldots, y_k) = C_{y_1,\ldots,y_k}^t(F_{y_1}^t(y_1), \ldots, F_{y_k}^t(y_k)). \tag{2.3}$$

The marginal distributions $F_{y_1}^t(y_1), \ldots, F_{y_k}^t(y_k)$ in Equation (2.3) represent the distributions of income values in each income source k. The copula links these marginal distributions together by capturing the dependence structure between them. Formally, copula can be seen as the joint CDF of k uniformly distributed variables ($r_{y1}, \ldots, r_{yk}$) where each variable contains information about the ordered positions of individuals in the marginal distributions of various income components.

By substituting Equation (2.3) into Equation (2.2) we can re-express the CDF of total household income as a function of copula and marginal distributions of income sources:

$$F^t(y) = \Pr[Y_1 + ... + Y_k \le y \mid T = t] = \int_0^y ... \int_0^{y-y_1-...-y_{k-1}} dC^t_{y_1,...,y_k}(F^t_{y_1}(y_1),...,F^t_{y_k}(y_k)) \qquad (2.4)$$

The copula-based specification of $F^t(y)$ in Equation (2.4) allows partitioning the CDF of total household income into two components – one attributable to the dependence structure between all income sources (the copula), and another one attributable to their marginal CDFs. This, in turn, sets the ground for a decomposition procedure which enables us to isolate the contribution of the changing interdependence between income sources from other factors while analyzing the change in the income distribution over time.

Aggregate decomposition

Building on the decomposability property of cumulative distribution functions and Equation (2.4), the change in the distribution of household disposable income between two points in time, $\Delta F(y)$, can be defined as the difference in the CDFs of total household income between a final period *t = 1* and a base period *t = 0*:

$$\Delta F(y) = F^{(1)}(y) - F^{(0)}(y) =$$

$$\int_0^y ... \int_0^{y-y_1-...-y_{k-1}} dC^{(1)}_{y_1,...,y_k}(F^{(1)}_{y_1}(y_1),...,F^{(1)}_{y_k}(y_k)) - \int_0^y ... \int_0^{y-y_1-...-y_{k-1}} dC^{(0)}_{y_1,...,y_k}(F^{(0)}_{y_1}(y_1),...,F^{(0)}_{y_k}(y_k)) \qquad (2.5)$$

Specification of Equation (2.5) implies that the change in the CDF of total household income over time can be potentially generated by two factors: (i) changes in the copula linking the marginal CDFs of all income sources together, $C^t_{y_1,...,y_k}$, and (ii) changes in the marginal CDFs themselves, $F^t_{y_1}(y_1),...,F^t_{y_k}(y_k)$:

$$\Delta F(y) = \Theta(\Delta F^{(0,1)}_C(y), \Delta F^{(0,1)}_M(y)) \qquad (2.6)$$

where $\Delta F^{(0,1)}_C(y)$ stands for the contribution of the copula and $\Delta F^{(0,1)}_M(y)$ stands for the contribution of the marginal CDFs of all income sources.

In order to quantify $\Delta F^{(0,1)}_C(y)$, one needs to construct a counterfactual distribution of household income which would have prevailed in the final period if the interdependence between all income sources had remained the same as in the base period. The difference

between the actual distribution in the final period and the constructed counterfactual yields the contribution of the copula to the observed change in the CDF of total household income (Equation 2.7). Similarly, to identify $\Delta F_M^{(0,1)}(y)$, one needs to deduct from the actual distribution of household income in the final period the counterfactual CDF which would have prevailed in this period if marginal distributions of all income sources had remained at their levels in the base period (Equation 2.8):

$$\Delta F_C^{(0,1)}(y) = \int_0^y \ldots \int_0^{y-y_1-\ldots-y_{k-1}} dC^{(1)}_{y_1,\ldots,y_k}(F^{(1)}_{y_1}(y_1),\ldots,F^{(1)}_{y_k}(y_k)) - \int_0^y \ldots \int_0^{y-y_1-\ldots-y_{k-1}} dC^{(0)}_{y_1,\ldots,y_k}(F^{(1)}_{y_1}(y_1),\ldots,F^{(1)}_{y_k}(y_k)) \tag{2.7}$$

$$\Delta F_M^{(0,1)}(y) = \int_0^y \ldots \int_0^{y-y_1-\ldots-y_{k-1}} dC^{(1)}_{y_1,\ldots,y_k}(F^{(1)}_{y_1}(y_1),\ldots,F^{(1)}_{y_k}(y_k)) - \int_0^y \ldots \int_0^{y-y_1-\ldots-y_{k-1}} dC^{(1)}_{y_1,\ldots,y_k}(F^{(0)}_{y_1}(y_1),\ldots,F^{(0)}_{y_k}(y_k)) \tag{2.8}$$

For linear decomposition, the sum of $\Delta F_C^{(0,1)}(y)$ and $\Delta F_M^{(0,1)}(y)$ would result into $\Delta F(y)$. However, if decomposition is non-linear, as in our case, the sum of these two components will not necessarily be equal to the total change in the CDF of household disposable income. The exact decomposition will be possible only if the copula and marginal distributions of income sources are independent from each other. This is unlikely to hold in the situation when the effect of copula is contingent on the effect of marginal CDFs of income sources. The exact decomposition in such a case can still be achieved in a sequential manner, where the contributions of various components are defined one by one. A typical limitation of this approach is that the results of decomposition might depend on the sequence chosen (Fortin et al., 2011). Depending on the research purposes, one might prefer to identify the contribution of the copula before the contribution of the marginal CDFs of income components, or the other way around. Since the primary goal of this paper is to propose a formalized decomposition framework which would allow separating contribution of the copula from the contribution of the marginal CDFs of income sources, estimating $\Delta F_C^{(0,1)}(y)$ first is viewed as more appropriate:

$$\Delta F(y) = [\int_0^y \dots \int_0^{y-y_1-\dots-y_{k-1}} dC^{(1)}_{y_1,\dots,y_k}(F^{(1)}_{y_1}(y_1),\dots,F^{(1)}_{y_k}(y_k)) - \int_0^y \dots \int_0^{y-y_1-\dots-y_{k-1}} dC^{(0)}_{y_1,\dots,y_k}(F^{(1)}_{y_1}(y_1),\dots,F^{(1)}_{y_k}(y_k))] +$$

$$[\int_0^y \dots \int_0^{y-y_1-\dots-y_{k-1}} dC^{(0)}_{y_1,\dots,y_k}(F^{(1)}_{y_1}(y_1),\dots,F^{(1)}_{y_k}(y_k)) - \int_0^y \dots \int_0^{y-y_1-\dots-y_{k-1}} dC^{(0)}_{y_1,\dots,y_k}(F^{(0)}_{y_1}(y_1),\dots,F^{(0)}_{y_k}(y_k))] =$$

$$= \Delta F_C^{(0,1)}(y) + \Delta F_M^{(0,1)}(y) \qquad (2.9)$$

In Equation (2.9), $\Delta F_C^{(0,1)}(y)$ captures the direct contribution of the change in the interdependence structure between all income sources (copula) to the change in the distribution of household income. The second decomposition component, $\Delta F_M^{(0,1)}(y)$, reflects the change in the distribution of household income induced by the shifts in the marginal CDFs of income sources either directly or via their interactions with the copula.

Further detailed decomposition

The aggregate decomposition provides only a general picture of the factors standing behind the change in the distribution of household disposable income over time. In order to understand changes in the dependence structure and marginal CDFs of which income sources drive the observed shift in the income distribution, a further detailed decomposition within each component of the aggregate decomposition is needed.

To disentangle the total copula effect, $\Delta F_C^{(0,1)}(y)$, in several sub-components attributable to changes in the rank interdependence between separate income sources, one needs to move the rank structure between these income sources back to the base year while keeping other factors unchanged. Let us assume that we are interested in how the change in the rank correlation between income components y_1 and y_2 (e.g. earnings of households heads and spouses) has reflected on the distribution of total disposable income between year *0* and year *1*. To answer this question, we can build on copula properties which allow re-expressing the CDF of the sum of these two income sources as a function of their copula and marginal distributions:

$$F^t_{y_1+y_2}(y_1+y_2) = \int_0^y \int_0^{(y_1+y_2)-y_1} dC^t_{y_1,y_2}(F^t_{y_1}(y_1), F^t_{y_2}(y_2)) \qquad (2.10)$$

where $F^t_{y_1+y_2}(y_1+y_2)$ is the marginal CDF of the income component derived by pooling income values from sources y_1 and y_2.

Since $F^t_{y_1+y_2}(y_1+y_2)$ is one of the income sources, we can re-express the CDF of total household income as follows:

$$F^t(y) = \int_0^y \ldots \int_0^{y-(y_1+y_2)-y_3\ldots-y_{k-1}} dC^t_{y_1+y_2,y_3,\ldots,y_k}(F^t_{y_1+y_2}(y_1+y_2), F^t_{y_3}(y_3),\ldots,F^t_{y_k}(y_k)) \qquad (2.11)$$

Equation (2.11) can be used to construct a counterfactual CDF of total household income, $F^C(y)$ which would have prevailed in the final year if the CDF of the sum of two income sources, y_1 and y_2, had remained the same as in the base year:

$$F^C(y) = \int_0^{+\infty} \ldots \int_0^{y-(y_1+y_2)-y_3\ldots-y_{k-1}} dC^{(0,1)}_{y_1+y_2,y_3,\ldots,y_k}(F^0_{y_1+y_2}(y_1+y_2), F^1_{y_3}(y_3),\ldots,F^1_{y_k}(y_k)) \qquad (2.12)$$

From Equation (2.10) it follows that moving $F^t_{y_1+y_2}(y_1+y_2)$ back to the base period is identical to moving both marginal CDFs of y_1 and y_2 and the dependence structure between them back to the base period. To disentangle the contribution of the dependence structure between these two income sources from the contribution induced by changes in their marginal CDFs, one needs to take the difference between the counterfactual distribution in Equation (2.12) and the counterfactual distribution which would have prevailed in the final year if only marginal distributions of both income sources (but not their dependence structure) had remained the same as in the base year:

$$\Delta F^{(0,1)}_{C_{y1,y2}}(y) = \int_0^{+\infty} \ldots \int_0^{y-(y_1+y_2)-y_3\ldots-y_{k-1}} dC^{(0,1)}_{y_1+y_2,y_3,\ldots,y_k}(F^0_{y_1+y_2}(y_1+y_2), F^1_{y_3}(y_3),\ldots,F^1_{y_k}(y_k)) -$$

$$- \int_0^{+\infty} \ldots \int_0^{y-(y_1+y_2)-y_3\ldots-y_{k-1}} dC^{(1)}_{y_1,\ldots,y_k}(F^0_{y_1}(y_1), F^0_{y_1}(y_1), F^1_{y_3}(y_3),\ldots,F^1_{y_k}(y_k)) \qquad (2.13)$$

where $\Delta F^{(0,1)}_{C_{y1,y2}}(y)$ is the contribution of the change in the rank interdependence between y_1 and y_2 to the total change in the CDF of household income.

The procedure described for income sources y_1 and y_2 can be applied for any other pairs of income sources and also extended to capture the interrelationship between three and more income components.

To partition the contribution of the shift in the marginal CDFs of all income sources to the distribution of total household income, $\Delta F_M^{(0,1)}(y)$, into a set of components attributable to various income sources, one needs to construct a set of counterfactual distributions of household income which would have prevailed in the final period if the marginal distributions of given income sources had remained as in the base period. For example, to isolate the contribution of the change in the marginal CDF of income source y_1 from the contributions induced by changes in other components (factors), one needs to construct a counterfactual distribution of household income which would have prevailed only if the marginal distribution of this income component had remained unchanged over time, $F_{M_{y1}}^{(C)}(y)$:

$$F_{M_{y1}}^{C}(y) = \int_0^{y} \ldots \int_0^{y-y_1-\ldots-y_{k-1}} dC_{y_1,\ldots,y_k}^{(1)}(F_{y_1}^{(0)}(y_1), F_{y_2}^{(1)}(y_1), \ldots, F_{y_k}^{(1)}(y_k)) \qquad (2.14)$$

Subtraction of this counterfactual distribution from the actual distribution in the final period yields the contribution of the change in the marginal CDF of y_1 to the shift in the distribution of total household income. In a similar way, one can derive the ceteris paribus contributions associated with changes in the marginal distributions of other income sources.

Similarly to the case of aggregate decomposition, due to non-linearity of the procedure, the sum of all copula or marginal-related decomposition components will not necessarily result into the total of $\Delta F_C^{(0,1)}(y)$ or $\Delta F_M^{(0,1)}(y)$. The exact decomposition, again, can be achieved via a sequential procedure, where the contributions of different income sources are derived in a sequence.

3. Data

The analysis is performed with data from the *Socio-Economic Panel "Liewen zu Lëtzebuerg"* (PSELL). This is an annual representative longitudinal survey which collects data on income and living conditions of individuals and private households residing in Grand

Duchy of Luxembourg.[3] Apart from questions about demographic and socio-economic characteristics of individuals and their households, it also includes information on total household income and its components which is important for studying the contribution of different income sources to the change in the entire distribution of household disposable income over time. The PSELL consists of three independent but consecutive panels: PSELL I (1985-1994), PSELL II (1995-2002) and PSELL III (2003-onwards). In this paper we use data from PSELL III for the years 2004 and 2013 which contain income information for 2003 and 2012 accordingly.

To perform a decomposition exercise, we partition total net household income into seven components:

$$\text{Total net household income} = E_h + E_s + E_o + P + CI + PT - ITC \qquad (3.1)$$

where E_h is gross earnings of household head, E_s gross earnings of spouse, E_o gross earnings of other household members, P pensions, CI capital income, PT public transfers, and ITC income taxes and social security contributions.[4] Household head is defined according to the rules of the tax-benefit system in Luxembourg: in couple-headed households the man is the head while in single-headed households the head of the household can be either a man or a woman. All income components are equalized with the OECD-modified equivalence scale and adjusted for the prices of 2005.[5] The unit of analysis is the individual, with the final sample comprising 8994 individuals for 2004 and 9963 individuals for 2013.

The implementation of the decomposition procedure described above requires identification of unique ranks within each income source (so that $r_{y1,1} \leq r_{y1,2} \ldots \leq r_{y1,n}$). This is straightforward for continuously distributed variables without ties, but less so when several individuals in the sample score the same on some variables, which is usually the case in income data. In such a situation, identification of unique ranks requires additional assumptions. One of the possible solutions is to assign ranks to the tied observations randomly. It might slightly change the dependence structure between income components but the overall bias will be kept at minimum. As an alternative, one might rank individuals with

[3] The survey does not cover individuals living in institutional establishments and international civil servants.

[4] The gross earnings components include salary income and income from self-employment. The pension component includes old-age and survivor pensions. Capital income includes income from rent/land, interests and dividends from capital.

[5] The OECD-modified equivalence scale gives the value 1 to the first adult member in the household, the value 0.5 to each additional adult member, and the value 0.3 to each child below 14 years old.

tied observations in one income component according to their scores in other income components. For example, one might want to assign a higher rank to the individual who also scores higher in other income components (the assumption of maximum inequality). Similarly, one might consider assigning higher ranks in one income component to individuals who score lower in other income components (the assumption of minimum inequality). All three possibilities have been explored in the paper but the results proved to be consistent regardless of the approach taken. Hence, for the sake of simplicity, in what follows we present the decomposition results based on the random assignment of ranks to the tied observations.

4. Changes in the distribution of household disposable income and its summary measures in Luxembourg between 2004 and 2013

Figure 4.1 plots the change in the distribution of household disposable income in Luxembourg in 2013, as compared with 2004. The differences are taken between the values of log income at each percentile of the income distribution. [6]

< Insert Figure 4.1 here >

Figure 4.1 shows that incomes have evolved differently in the lower and upper parts of the distribution. Whereas the income values of the individuals located at the bottom of the distribution have decreased over time, the income values of those at the top have increased. Remarkably, that the size of the income growth at the top mirrors the size of the income decline at the bottom. All in all, these trends signify that the distribution of household disposable income has become more dispersed and unequal in Luxembourg between 2004 and 2013.

Table 4.1 below characterizes changes in the distribution of household disposable income observed in Figure 4.1 with various summary measures. We focus on the standard indexes – percentile income ratios (P90/P10, P90/P50, P50/P10), the Gini coefficient, the percentage of individuals living below the poverty line, and the percentage of individuals living above the richness line. To shed light on how the standards of living have changed over time, the table also describes trends in the mean and median income.

[6] For example, the value of household disposable income at the 25th percentile in 2013 was 19682 Euros (equivalent to 14.26 on the base-2 logarithmic scale) whereas in 2004 it was 21107 Euros (equivalent to 14.37 on the base-2 logarithmic scale). We plot the difference between the two which is equal to -0.11 log points.

< Insert Table 4.1 here >

A larger increase in the mean than in the median income confirms numerically that the distribution of household disposable income has become more dispersed between 2004 and 2013. The evolution of the percentile income ratios gives a notion in what parts of the distribution this dispersion has occurred. During the period studied, the income ratios between the 90^{th} and the 50^{th} percentiles and between the 50^{th} and the 10^{th} percentiles of the distribution increased almost identically implying that incomes have become more dispersed along the entire distribution of household income. This is also confirmed by the trends in the Gini coefficient and the P90/P10 ratio which have increased significantly over time. The relative poverty and richness rates also rose between 2004 and 2013, although the estimates are less precise than for inequality indexes.

5. Decomposition results

In this section, we investigate which factors can be hold responsible for the change in the distribution of household disposable income in Luxembourg documented above. We start with the aggregate decomposition (Sub-section 5.1), where we partition the overall shift in the distribution of household income into two components attributable to (1) changes in the dependence structure (copula) of all income sources taken together, and (2) changes in their marginal distributions. As a next step (Sub-section 5.2), we look inside the two components and analyse how shifts in the rank interdependence of separate income sources and their marginal distributions have reflected on the distribution of household disposable income in Luxembourg over time. As a final step (Sub-section 5.3), we perform a set of sensitivity checks to test the reliability of our results.

5.1. Aggregate decomposition

Figure 5.1 illustrates how the change in the rank interdependence of income sources (Panel A) and their marginal distributions (Panel B) has reflected on the shift in the distribution of household disposable income in Luxembourg. The respectful contributions are defined in line with Equation 2.9 and show the absolute decline / increase in incomes in certain areas of the distribution attributed to the factors of interest.

< Insert Figure 5.1 here >

Figure 4.1 shows that the decline in income values at the bottom of the distribution (between the 10th to 50th percentiles) is mainly driven by the changes in the copula, as compared to the changes in the marginal distributions of income sources. This finding implies that individuals who are located at the bottom of the household income distribution and rank low on one income source not only tend to rank low on other income sources, but this relationship has also intensified over time. Had the dependence structure of different income sources remained the same as in 2004, the decline in incomes we actually observe would not have taken place, other things being equal.

In contrast to the copula, whose contribution is mainly concentrated at the lower tail of the income distribution, the shifts in the marginal distributions of the income sources have reflected predominantly on its upper tail. Although the increase in incomes is documented from the 30th percentile of the income distribution onwards, the results are significant only in the area between the 75th and 90th percentiles. In general, it allows us to conclude that if the marginal distributions of all income sources had remained at their 2004 level, the distribution of household disposable income would have been less unequal in 2013.

Table 5.1 provides the results of the aggregate decomposition for the distributional summary measures. It shows that both, changes in the marginal distributions of the income sources and their dependence structure, are associated with the increase in all inequality and poverty measures. The associations, however, are more or less precise only for the contributions of the marginal CDFs of income components, and only for the Gini and P90/P10 indexes.[7] According to the estimates, more than 2/3 of the observed increase in the Gini coefficient and the P90/P10 ratio can be attributed to the shifts in the marginal distributions of income sources.

While looking at the relative size of the contributions, Table 5.1 shows that the shifts in the marginal distributions of the income sources explain most of the increase in all distributional summary measures except of poverty, whereas the contribution of the copula to this increase is relatively small. In contrast, the change in the copula accounts for a substantial portion of the increase in the relative poverty rate. According to the estimates, had the rank interrelationship between income sources remained the same as in 2004, we would have observed 63.4% smaller increase in the relative poverty rate in 2013. These findings are also consistent with the graphical evidence from Figure 5.1 which shows that the change in

[7] The inability to obtain significant results for other distributional measures might be related to the relatively small sample size since Luxembourg is a country with a small population.

the copula has induced a significant downward shift in incomes at the bottom of the income distribution.

5.2. Detailed decomposition

The decomposition performed in Section 5.1 provides only a general picture of the factors standing behind the change in the distribution of household disposable income in Luxembourg. In what follows, we look inside the two major components of the aggregate decomposition in order to identify how changes in the rank interdependence between certain income sources and changes in their marginal CDFs underlie the change in the distribution of household disposable income over time. While doing it, we follow the natural process of household income formation and consider the contributions of the earnings related income sources first, with the subsequent introduction of capital income, public transfers, and taxes to the decomposition sequence.

Figure 5.2 illustrates the contributions of the changes in the rank interdependence between various income sources to the shift in the distribution of household disposable income. Table 5.2 quantifies these contributions via a set of distributional summary measures. We first focus on the change in the rank correlation between earnings of household heads and spouses and then, by adding one component at a time, investigate how the change in its rank correlation with the previously included income sources has reflected on the distribution of total household income.

< Insert Figure 5.2 here >

< Insert Table 5.2 here >

Figure 5.2 shows that the change in the rank dependence between earnings of household heads and spouses as well as in the rank dependence of capital income with all earnings related income sources is significantly associated with an increase in income dispersion in Luxembourg between 2004 and 2013. In contrast, changes in the rank dependence of public transfers and taxes with other income sources demonstrate an equalizing effect on the distribution of total household income over time. The shifts in other factors of the copula decomposition have not produced a significant influence on the distribution of household disposable income.

The graphical evidence from Figure 5.2 is also confirmed numerically. Panel A in Table 5.2 shows that, where statistically significant, changes in the rank dependence between market income sources have contributed to the increase in inequality and poverty indexes. This increase has been substantially offset by the changes in the rank interdependence of market income sources with public transfers and taxes. For example, the change in the rank correlation between public transfers and market income sources has almost cancelled out the increase in the Gini coefficient induced by the increased rank correlation between earnings of household heads and spouses.

While looking at the contributions of particular income sources, which were found significant on Figure 5.2, one can see that the change in the rank correlation between earnings of household heads and spouses is associated with a decline in incomes at the bottom and increase in incomes at the top of the household income distribution. This evidence is in line with the findings for other countries which indicate that earnings of household heads and spouses have become more correlated over time resulting in an increase in income dispersion across households. Indeed, the estimates in Table 5.2 (Panel A) show that the change in the rank correlation between earnings of household heads and spouses is associated with the largest increase in all inequality and poverty measures attributed to the decomposition components within the copula block. For example, around 1/3 of the total increase in the Gini coefficient and almost 2/3 of the total increase in all polarization ratios can be assigned to the changed interdependence between earnings of household heads and spouses. The contribution is even more profound for the poverty rate – had the copula between earnings of household heads and spouses remained unchanged over time, the poverty rate would have been even lower in 2013 than it was in 2004.

Another factor which has contributed significantly to the increase in inequality in Luxembourg is the change in the dependence structure between capital income and other earnings related income sources. Figure 5.2 shows that the change in this factor has produced a statistically significant decline in incomes at the bottom and in the middle of the household income distribution. As a consequence the distribution has become more unequal in 2013 as compared to 2004, shifting the Gini coefficient and percentile ratios up. According to Table 5.2, almost 30% of the increase in the Gini can be attributed to the change in the rank interdependence between capital income and other earnings related income sources.

As mentioned above, the increase in inequality and poverty indexes induced by the changes in the dependence structure between the market income sources has been partially

eliminated by the changes in the rank interdependence of these sources with transfers. Figure 5.2 shows that the change in the dependence structure between public transfers and other income sources (with the exception of taxes) has generated an increase in incomes at the bottom 20th percentile of the income distribution. This implies that individuals with relatively low earnings and capital income were receiving relatively more public transfers in 2013 as compared to 2004. Due to this fact, inequality and poverty measures decreased, although the decrease is found statistically significant only for the Gini coefficient (see Panel A in Table 5.2). Had the rank interdependence between public transfers and all other market income sources remained at its 2004 level, the Gini coefficient would have been 0.007 points higher in 2013.

Similarly to public transfers, the change in the rank interdependence of taxes with other income sources (including public transfers) contributed to equalization of the distribution of household disposable income. The graphical evidence shows that it has resulted in a significant increase in incomes at the very bottom of the income distribution and a decline in incomes at the very top. While looking at the changes in distributional measures in Table 5.2, one can see that all of them are negatively influenced by the change in the rank interdependence between taxes and other income sources. The estimates, however, are significant only for the richness rate, and somewhat significant for the P90/P50 ratio. Had the rank interdependence of taxes with other income sources remained at its 2004 level, the richness rate would have been 1.63 percentage points higher in 2013. This evidence implies that rich households were paying relatively more taxes in 2013 as compared to 2004.

Figure 5.3 provides the first insight in the contributions of marginal CDFs of various income sources to the shift in the distribution of household disposable income. While deriving these source-specific contributions we followed the same sequence as in the detailed copula decomposition and defined the contributions of earnings related income sources first.

< Insert Figure 5.3 here >

Figure 5.3 indicates that changes in the marginal distributions of most income sources (with the exception of earnings of other household members and capital income) have influenced significantly the distribution of household disposable income in Luxembourg. Looking at earnings of household head first, one can see that the change in their marginal distribution has resulted in the decline of income values along the entire distribution of

household income (except of its top 10 percentile), with an especially profound decline documented at the very bottom of the distribution. The change in the marginal CDF of earnings of spouses, in turn, is associated with an increase in total household income at all points of the distribution. Moreover, the size of the increase is relatively constant along the distribution of household income. This evidence is very much in line with the findings for other rich countries (see, among others, Burtless, 1999, Daly and Valletta, 2006, and Larrimore, 2013) which show the increased importance of spouse's (typically female) earnings for the distribution of total household income.

Figure 5.3 also reveals a substantial contribution of pensions to the income growth along the entire distribution of household income. Nevertheless, not all households have benefited equally from this growth – the graphical evidence shows that the change in the marginal distribution of pensions was more favourable for households located at the top of the household income distribution as compared to those located at the bottom. Moreover, households at the very bottom of the total income distribution were not affected at all by the change in the marginal distribution of this income source.

In contrast to the market income sources, changes in the marginal distributions of public transfers and taxes have contributed to equalization of incomes across households. Figure 5.2 shows that the change in the marginal distribution of public transfers is associated with the increase in incomes of all households, but especially those located at the bottom of the income distribution. In contrast, changes in the marginal distribution of taxes are accountable for the decline in incomes. The observed decline, however, increases proportionally as we move along the distribution of household income which signifies that rich individuals tend to pay relatively more taxes nowadays than they used to ten years ago.

Panel B in Table 5.2 compliments the graphical evidence from Figure 5.3 by quantifying the contributions of the changes in the marginal distribution of each income source to the distributional summary measures. It shows that the change in the marginal distribution of household heads' earnings was the major driver of the increase in all inequality and poverty measures in Luxembourg between 2003 and 2012. The change in the marginal distribution of pensions was second the most important contributor to the increase in the above mentioned measures. The unfavourable changes in the marginal distributions of these two income sources were partially offset by the shifts in the marginal distributions of public transfers and taxes. For example, around 50% of the increase in the Gini coefficient

induced by the shift in the marginal distribution of earnings of household heads was offset by the changing distributions of taxes and transfers.

5.3. Sensitivity checks

In order to investigate to what extent the results of our decomposition exercise are sensitive to the methodological choices made, we have performed two sensitivity checks. First, we performed reverse order decomposition and identified the contributions of the marginal CDFs of income sources to the change in the entire distribution of household income before the contributions of the copula-related components. Secondly, we repeated the entire decomposition procedure by taking 2013 as the reference year and 2004 as the final year.

The reverse order decomposition has yielded similar results. The size of the copula contributions to the shifts in the distributional summary measures, however, is a bit larger when the contributions of the marginal CDFs of income sources are identified before the contributions of the copula in the process of aggregate decomposition. This is not surprising because the copula component in this case also picks up the interaction effect between the copula and the marginal CDFs of income sources. The estimates of the copula contributions have also gained precision and become significant for the Gini coefficient, p90/p10, and p90/p50 measures. In contrast, the vast majority of the contributions of the changes in the marginal CDFs of income sources have become smaller in size and largely insignificant.

In the detailed reverse order decomposition, the contributions of the copula components to the change in the distribution of household income were found very similar to the ones from the main order decomposition. Similarly to the case of reverse order aggregate decomposition, the estimates of the copula contributions became larger in size and also gained significance for a number of distributional summary measures. In particular, the contributions of the changing rank dependence of pensions with earnings related income components, as well as public transfers with market income components, became highly significant for p90/p10, p50/p10 indexes and for the poverty rate. The nature of the contributions, however, remained the same: the change in the rank dependence of pensions with other earnings related market incomes is associated with the increases in inequality and poverty measures whereas the change in the rank dependence of public transfers with other market related income sources is associated with a decrease in inequality and poverty

measures. The contributions of the marginal CDFs of income sources in reverse order decomposition, where significant, are also similar to the ones from the main decomposition.

Taking 2004 as the final year and 2013 as the base year has also yielded the results similar to the ones obtained in Sections 5.1 and 5.2. In particular, the results of the aggregate decomposition reveal that the change in the dependence structure of income sources is responsible for higher inequality and poverty measures in 2013 as compared to 2004. The contributions of the copula to inequality indexes are also found more significant when we move from 2004 to 2013 rather than the other way around in the decomposition exercise. Similarly Section 5.2, the contributions of the change in the marginal CDFs of income components are found insignificant when 2013 is taken as the reference year.

The results of the detailed decomposition in Appendix B are to a large extent in line with the results in Table 5.2. The major difference lies in the contribution of the changing dependence structure of taxes with other income components. In the main decomposition exercise, this component is found as such that contributes to the decrease in the distributional summary measures. The decomposition with 2004 as the final year, however, yields the opposite results, but these results are found insignificant.

6. Conclusions

This paper proposes a formalized decomposition framework which makes it possible to decompose the change in the entire distribution of household income by income sources. Building on copula theory, it allows partitioning the overall change in the distribution of household disposable income into two components attributable to: (i) changes in the marginal distributions of different income sources, and (ii) changes in their dependence structure (copula). It also offers a flexible way how these two components can be decomposed further in order to identify the contributions of the changes in particular income sources and the dependence structure between them to the shift in the distribution of household income. The proposed methodology is then used to explain the change in the distribution of total disposable income in Luxembourg between 2003 and 2012.

The results indicate that the distribution of household disposable income has become more unequal in Luxembourg between 2004 and 2013 resulting in the increase in all inequality and poverty measures. The observed shift in the income distribution has been induced by both – changes in the marginal distributions of income sources and their interdependence with each other (the copula). While changes in the marginal distributions of

income sources are hold responsible predominantly for the growth of incomes in the upper part of the income distribution, the change in the copula account for the decline in incomes in its lower tail. As a result, the shifts in the marginal distributions of income sources explain most of the increase in the inequality measures and the richness rate whereas the change in the copula induced a substantial portion of the increase in the relative poverty rate.

These findings imply that it is predominantly well-off households that have benefited from the growth of incomes coming from various sources over time. The households at the bottom of the distribution not only have not enjoyed this growth, they also became more likely to have low incomes in all income sources. For example, those who score low in the earnings of household heads also have become more prone to score low in the earnings of spouses, capital income etc.

A detailed decomposition of the copula contribution to the change in the distribution of household income in Luxembourg has shown that it is the rank interdependence between earnings of household heads and spouses that has generated the largest unequalizing effect. The rank correlation of capital income with other market income sources is the second largest factor accounting for the more unequal distribution of household disposable income in 2013 as compared to 2004. In contrast, changes in the interdependence of public transfers and taxes with other income sources have produced an equalizing effect on the distribution of household income over time. This evidence suggests that individuals with relatively low market incomes were more likely to receive high public transfers and pay low taxes in 2013 as compared to 2004.

While looking at the contributions of the marginal distributions of various income sources to the shift in the distribution of household income in Luxembourg, we have found that it is mainly the marginal distributions of earnings of household heads and capital income that have made the distribution significantly more unequal over time. The unequalizing contributions of these two types of income sources have been partially cancelled out by the changes in the marginal distributions of public transfers and taxes. The changes in the public transfers were more favourable for the households located at the bottom of the total income distribution whereas the changes in taxes induced a decline in incomes of those at the top.

References

Araar, A. (2008), *On the decomposition of polarization indices: Illustrations with chinese and nigerian household surveys*, Cahiers de recherche 0806, CIRPEE.

Biewen, M. and Juhasz, A. (2012), 'Understanding rising income inequality in Germany', *Review of Income and Wealth* **58**, 622–647.

Brewer, M. and Wren-Lewis, L. (2016). Accounting for changes in income inequality: Decomposition analyses for the UK, 1978-2008. *Oxford Bulletin of Economics and Statistics*, **78**(3), 289-322.

Burtless, G. (1999), 'Effects of growing wage disparities and changing family composition on the u.s. income distribution', *European Economic Review* **43**(4-6), 853–865.

Cancian, M. and Reed, D. (1998), 'Assessing the effects of wives' earnings on family income inequality', *The Review of Economics and Statistics* **80**(1), 73–79.

Cowell, F. and Fiorio, C. (2011), 'Inequality decomposition reconciliation', *Journal of Economic Inequality* **9**(4), 509–528.

Daly, M. C. and Valletta, R. G. (2006), 'Inequality and poverty in United States: The effects of rising dispersion of men's earnings and changing family behaviour', *Economica* **73**(289), 75–98.

Deutsch, J., Fusco, A. and Silber, J. (2013), 'The big trilogy (bipolarization, inequality and polarization): One saga but three different stories', *Economics: The Open-Access, Open-Assessment E-Journal* **7**(2013-22).

DiNardo, J., Fortin, N. M. and Lemieux, T. (1996), 'Labor market institutions and the distribution of wages, 1973-1992: A semiparametric approach', *Econometrica* **64**(5), 1001–1044.

Fiorio, C. V. (2011), 'Understanding Italian inequality trends', *Oxford Bulletin of Economics and Statistics* **73**(2), 255–275.

Fortin, N., Lemieux, T., and Firpo, S. (2011). Decomposition methods in Economics, *in* Ashenfelter, O. and Card, D. *Handbook of Labor Economics*, vol. 4, part A, pp. 1-102.

Fournier, M. (2001), 'Inequality decomposition by factor component: an approach illustrated on the Taiwanese case', *Recherches économiques de Louvain* **67**(4), 381–403.

Friedline, T., Masa, R.D., Chowa, G.A.N. (2015). Transforming wealth: Using the inverse hyperbolic sine and splines to predict youth's math achievement. *Social Science Research* **49**, 264-287.

Fuest, C., Niehues, J. and Peichl, A. (2010), 'The redistributive effects of tax benefit systems in the enlarged EU', *Public Finance Review* **38**(4), 473–500.

Fusco, A., Van Kerm, P., Alieva, A., Bellani, L., Etienne-Robert, F., Guio, A.-C., Kyzyma, I., Leduc, K., Liégeois, P., Pi Alperin, M.-N., Reinstadler, A., Sierminska, E., Sologon, D., Thill, P., Valentova, M. and Voicu, B. (2013), Luxembourg: has inequality grown enough to matter?, *in* B. Nolan, N. B. Salverda, Wiemer, D. Checchi, I. Marx, A. McKnight, I. Tóth and H. Van de Werfhorst, eds, '*Changing Inequalities and Societal Impacts in Rich Countries: Thirty Countries Experiences*', Oxford University Press.

García-Peñalosa, C. and Orgiazzi, E. (2013), 'Factor components of inequality: A cross-country study', *Review of Income and Wealth* pp. n/a–n/a.

Hyslop, D. R. and Maré, D. C. (2005), 'Understanding New Zealand's changing income distribution, 1983-1998: A semi-parametric analysis', *Economica* **72**(3), 469–495.

Jenkins, S. P. (1995), 'Accounting for inequality trends: Decomposition analyses for the UK, 1971-86', *Economica* **62**(245), 29–63.

Jenkins, S. P. and Van Kerm, P. (2005), 'Accounting for income distribution trends: A density function decomposition approach', *The Journal of Economic Inequality* **3**(1), 43–61.

Jenkins, S. P. and Van Kerm, P. (2009), The measurement of economic inequality, *in* W. Salverda, B. Nolan and S. Tim, eds, 'Oxford Handbook on Economic Inequality', Oxford University Press, chapter 3, pp. 40–67.

Larrimore, J. (2013), 'Accounting for United States household income inequality trends: The changing importance of household structure and male and female labor earnings inequality', *Review of Income and Wealth* pp. n/a–n/a.

Lerman, R. (1999), How do income sources affect income inequality?, *in* J. Silber, ed., 'Handbook on Income Inequality Measurement', Kluwer Academic Publishers.

Lerman, R. and Yitzhaki, S. (1985), 'Income inequality effects by income source: A new approach and applications to the United States', *Review of Economics and Statistics* **67**, 151–156.

Mussard, S. (2007), 'La décomposition des mesures dinégalité en sources de revenu: méthodes et applications', *L'Actualité Economique* **83**(3), 415–445.

Mussard, S. and Pi Alperin, M. N. (2011), 'Poverty growth in Scandinavian countries: A Sen multi-decomposition', *Economic Modelling* **28**(6), 2842–2853.

OECD (2015), '*In it together: why less inequality benefits all?*' Paris: OECD Publishing.

Osier G. (2012), "L'évolution du revenu des ménages depuis le milieu des années 80", STATEC, Collection le Luxembourg 1960-2010, Luxembourg.

Peichl, A., Pestel, N. and Schneider, H. (2012), Does the size matter? The impact of changes in household structure on income distribution in Germany. *The Review of Income and Wealth*, **58** (1), pp. 118-141.

Peichl, A., Schaefer, T. and Scheicher, C. (2010), Measuring richness and poverty: A micro data application to Europe and Germany. Review of Income and Wealth, 56 (3), 597-619.

Piketty, T. (2014). Capital in the twenty first century. Cambridge: Harvard University Press.

Rothe, C. (2012), 'Partial distributional policy effects', *Econometrica* **80**(5), 2269–2301.

Sastre, M. and Trannoy, A. (2002), 'Shapley inequality decomposition by factor components: Some methodological issues', *Journal of Economics* **9**(1), 51–89.

Shorrocks, A. (1982), 'Inequality decomposition by factor components', *Econometrica* **50**, 193–211.

Sklar, A. (1959), Fonctions de repartition à n dimensions et leurs marges. Publications de l'Institut de Statistique de l'Université de Paris, **8**, 229-231.

STATEC (2003), Economic and social portrait of Luxembourg, Technical report.

Van Kerm, P. (2003). Adaptive kernel density estimation. *The Stata Journal*, **3** (2), pp. 148-156.

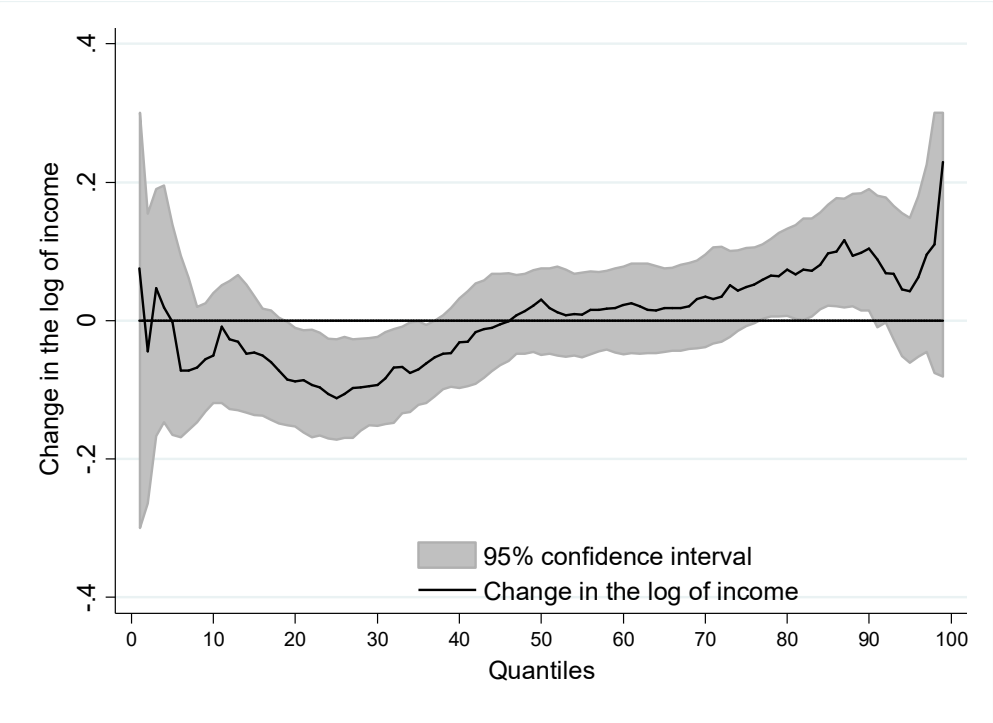

Figure 4.1. The change in the distribution of household disposable income in Luxembourg between 2004 and 2013

Source: Authors' calculations based on the PSELL III, cross-sectionally weighted data. 95% confidence intervals are based on 500 bootstrap replications. For presentation purposes, we truncated the low bound confidence interval at the 1st percentile of the income distribution and the upper bound confidence interval at the 98-99th percentiles of the distribution.

Note: The Figure depicts the differences in the base-2 logarithms of income values between 2012 and 2003, at 99 equally spaced percentiles of the income distribution.

Table 4.1. Changes in the distributional summary measures in Luxembourg between 2004 and 2013

Indexes	2004	2013	Change (2004 to 2013)	
			Estimate	(Standard error)
Mean income	31345.53	32586.49	+1241	(675.39)
Median income	27680.81	28221.04	+ 540	(655.93)
P90/P10	3.274	3.642	+0.368	(0.140)
P90/P50	1.801	1.902	+0.101	(0.056)
P50/P10	1.818	1.914	+0.096	(0.058)
Gini	0.266	0.303	+0.037	(0.010)
Poverty rate (%)	13.54	15.72	+2.18	(1.36)
Richness rate (%)	6.92	8.18	+1.26	(0.94)

Source: Authors' calculations based on the PSELL III, cross-sectionally weighted data. Standard errors are based on 500 bootstrap replications.

Note: All measures are obtained for household equivalized disposable income. The poverty line is defined as 60% of the median household disposable income. The richness threshold is twice the median household disposable income in a given year.

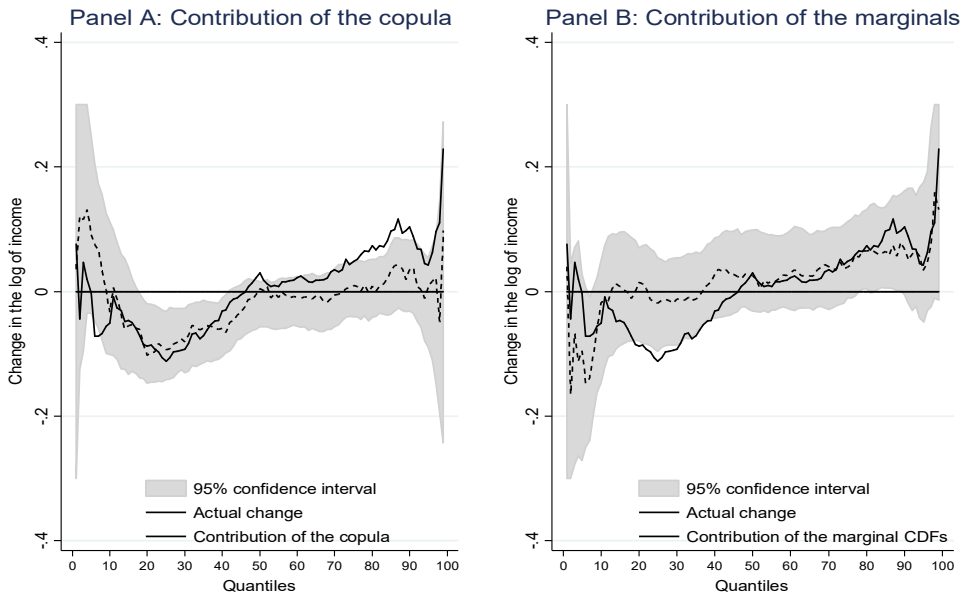

Figure 5.1. Aggregate decomposition of the change in the household disposable income in Luxembourg between 2004 and 2013

Source: PSELL 3, cross-sectionally weighted data.

Note: The Figure depicts the differences in the base-2 logarithms of real income values at 99 equally spaced percentiles of the income distribution. The line 'actual change' captures the difference in actual income values between 2013 and 2004. The contributions of the decomposition components (changes in the copula and marginal CDFs of income sources) are identified according to Equation 2.9 in the text.

Table 5.1. Aggregate decomposition of the change in the distribution of household equivalized disposable income in Luxembourg between 2004 and 2013

Distributional summary measure	Overall change	Change due to the copula		Change due to the marginal CDFs of income sources	
		Estimate (SE)	In %	Estimate (SE)	In %
Gini	+0.029	+0.008	+26.15	*+0.021*	+73.85
	(0.011)	(0.008)		*(0.013*)	
P90/P10	+0.362	+0.103	+28.51	*+0.259*	+71.49
	(0.139)	(0.173)		*(0.156)*	
P90/P50	+0.099	+0.045	+45.99	+0.053	+54.01
	(0.054)	(0.052)		(0.048)	
P50/P10	+0.095	+0.009	+9.11	+0.087	+90.89
	(0.057)	(0.086)		(0.073)	
Poverty rate, %	+2.029	+1.286	+63.41	+0.742	+36.59
	(1.30)	(1.31)		(1.07)	
Richness rate, %	+0.861	+0.058	+6.82	+0.802	+93.18
	(0.89)	(0.78)		(0.81)	

Source: Authors' calculations based on the PSELL III, cross-sectionally weighted data.

Note: Overall changes in the distributional summary measures between 2004 and 2013 are expressed in absolute terms and calculated using predicted values of incomes. The standard errors in the parentheses are based on 500 bootstrap replications.

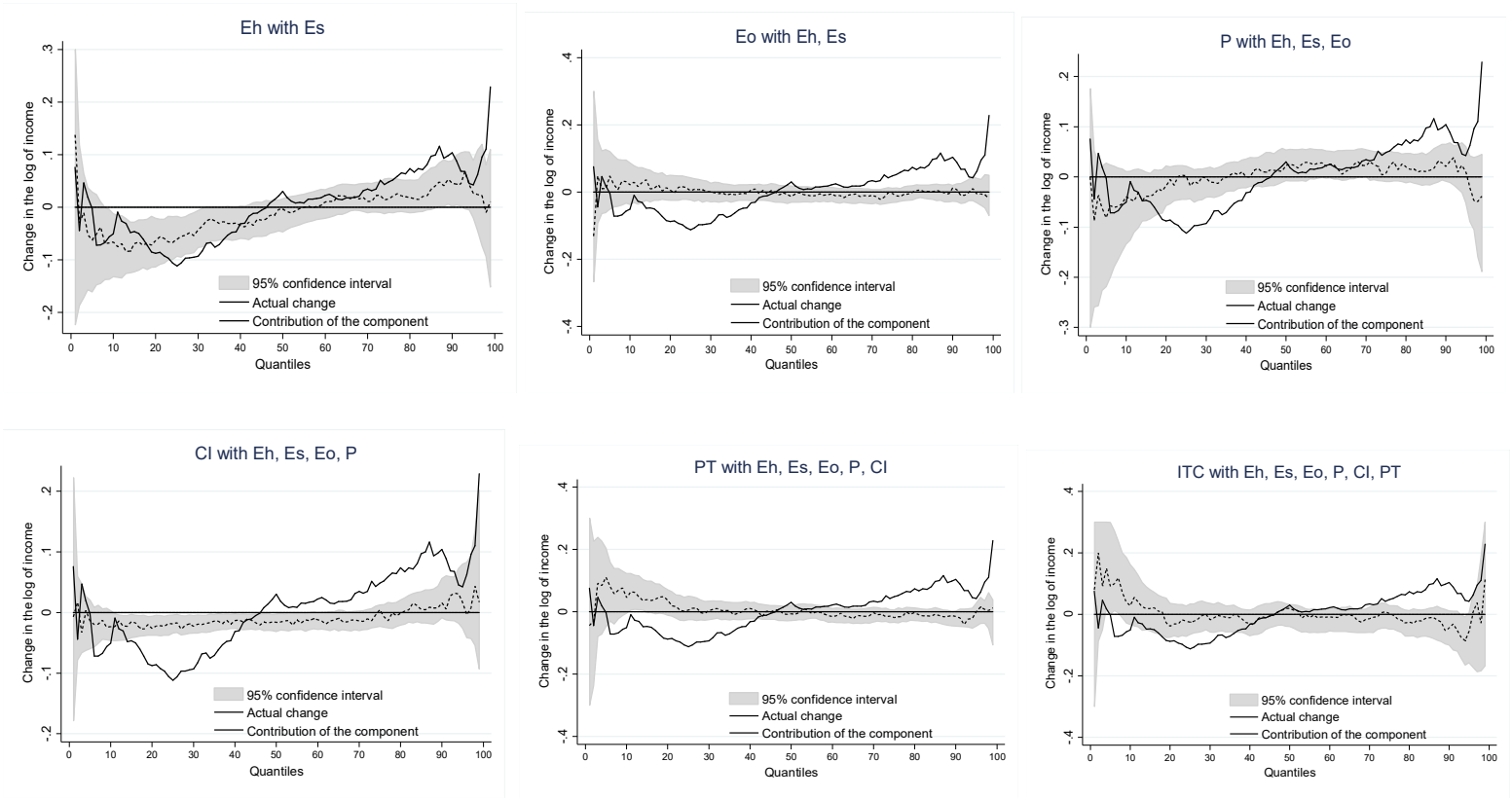

Figure 5.2. Detailed sequential decomposition of the copula contribution to the change in the distribution of household income

Source: Authors' calculations based on the PSELL III, cross-sectionally weighted data.

Note: The line 'actual change' captures the actual difference in the base-2 logarithm of real income values between 2012 and 2003 at 99 equally spaced percentiles of the income distribution. The contributions of the decomposition components are identified in a sequence following the algorithm described in Section 5.

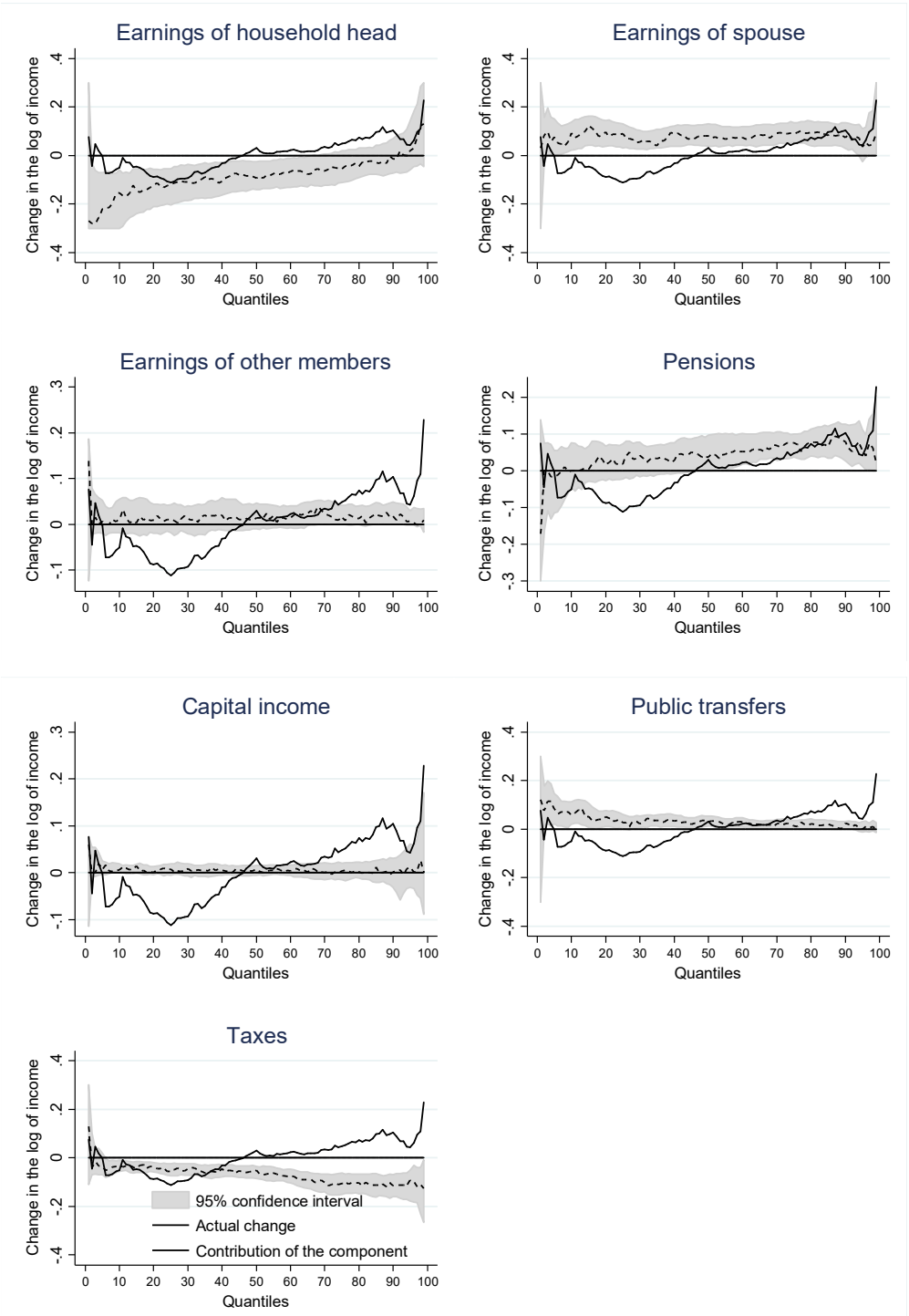

Figure 5.3. Detailed sequential decomposition of the contribution of marginal distributions to the change in the distribution of household income

Source: Authors' calculations based on the PSELL III, cross-sectionally weighted data.

Note: The line 'actual change' captures the actual difference in the base-2 logarithm of real income values between 2012 and 2003 at 99 equally spaced percentiles of the income distribution. The contributions of the decomposition components are identified in a sequence following the algorithm described in Section 5.

Table 5.2. Detailed decomposition of the change in the distribution of household disposable income in Luxembourg between 2004 and 2013

Decomposition component	Contribution to the change in the following measures					
	Gini	P90/P10	P90/P50	P50/P10	Poverty rate	Richness
Overall change	**+0.029**	**+0.362**	**+0.099**	**+0.096**	**+2.03**	**+0.86**
	(0.011)	(0.139)	(0.054)	(0.057)	(1.30)	(0.90)
Eh with Es	+0.010	+0.252	+0.063	+0.079	+2.44	+0.66
	(0.004)	**(0.095)**	*(0.036)*	*(0.047)*	**(1.01)**	(0.55)
Eo with Eh, Es	-0.003	-0.050	+0.026	-0.056	-0.99	+0.36
	(0.002)	(0.078)	(0.020)	(0.047)	(1.00)	(0.30)
P with Eh, Es, Eo	+0.003	+0.117	-0.006	+0.078	+1.33	+0.10
	(0.003)	(0.096)	(0.030)	(0.059)	(1.28)	(0.51)
CI with Eh, Es, Eo, P	+0.007	+0.079	+0.044	+0.003	+0.24	+0.55
	(0.001)	*(0.042)*	**(0.022)**	(0.016)	(0.42)	(0.35)
PT with Eh, Es, Eo, P, CI	-0.007	-0.115	-0.008	-0.065	-1.59	+0.02
	(0.002)	(0.073)	(0.022)	(0.043)	(1.06)	(0.36)
ITC with Eh, Es, Eo, P, CI, PT	-0.002	-0.180	*-0.073*	-0.030	-0.14	-1.63
	(0.006)	(0.140)	*(0.044)*	(0.078)	(1.16)	**(0.74)**
Total due to the copula	+0.008	+0.103	+0.046	+0.009	+1.29	+0.06
	(0.008)	(0.173)	(0.052)	(0.086)	(1.31)	(0.78)
Eh	+0.030	+0.353	+0.100	+0.093	+1.58	+1.80
	(0.011)	**(0.146)**	**(0.036)**	(0.065)	*(0.96)*	**(6.94)**
Es	-0.001	-0.019	-0.017	+0.006	-0.16	-0.07
	(0.003)	(0.084)	(0.032)	(0.041)	(0.008)	(0.004)
Eo	-0.000	+0.019	+0.011	-0.001	+0.05	-0.13
	(0.001)	(0.043)	(0.019)	(0.019)	(0.52)	(0.24)
P	+0.007	+0.169	+0.049	+0.047	+0.73	+0.49
	(0.003)	**(0.068)**	(0.033)	*(0.024)*	(0.68)	(0.40)
CI	-0.000	-0.006	-0.007	+0.004	-0.00	+0.03
	(0.002)	(0.030)	(0.014)	(0.008)	(0.21)	(0.26)
PT	-0.006	-0.079	-0.008	-0.037	-1.03	-0.46
	(0.001)	*(0.047)*	(0.012)	(0.025)	*(0.006)*	**(0.002)**
ITC	-0.009	-0.178	-0.075	-0.025	-0.43	-0.86
	(0.005)	**(0.054)**	**(0.025)**	(0.018)	(0.42)	*(0.47)*
Total due to marginals	+0.021	*+0.259*	+0.053	+0.087	+0.74	+0.80
	(0.013)	*(0.156)*	(0.048)	(0.073)	(1.07)	(0.81)

Source: Authors' calculations based on the PSELL III, cross-sectionally weighted data.

Note: E_h is gross earnings of household head, E_s - gross earnings of spouse, E_o - gross earnings of other household members, *P* - pensions, *CI* - capital income, *PT* - public transfers, and *ITC* - income taxes and social security contributions. All income components are adjusted for the household size using the OECD-modified equivalized scale.

Appendix A
A reverse order decomposition of the change in the distribution of household disposable income in Luxembourg between 2004 and 2013

Decomposition component	Contribution to the change in the following measures					
	Gini	P90/P10	P90/P50	P50/P10	Poverty rate	Richness
Overall change	**+0.029**	**+0.362**	*+0.099*	+0.096	+2.03	+0.86
	(0.011)	(0.139)	(0.054)	(0.057)	(1.30)	(0.90)
Copula components						
Eh with Es	**+0.01**	**+0.303**	***+0.115***	+0.050	+1.77	+2.12
	(0.004)	**(0.111)**	**(0.042)**	(0.050)	(1.03)	(0.77)
Eo with Eh, Es	-0.003	-0.096	+0.012	-0.066	-1.14	-0.01
	(0.002)	(0.076)	(0.021)	(0.044)	(0.89)	(0.31)
P with Eh, Es, Eo	+0.002	**+0.219**	-0.023	**+0.145**	**+2.69**	-0.49
	(0.003)	**(0.103)**	(0.022)	**(0.059)**	**(1.06)**	(0.35)
CI with Eh, Es, Eo, P	**+0.007**	*+0.082*	*+0.039*	+0.008	+0.29	**+0.88**
	(0.002)	*(0.047)*	*(0.023)*	(0.015)	(0.34)	**(0.39)**
PT with Eh, Es, Eo, P, CI	**-0.007**	**-0.213**	-0.028	**-0.092**	**-2.22**	-0.42
	(0.002)	**(0.079)**	(0.019)	**(0.045)**	**(0.91)**	(0.33)
ITC with Eh, Es, Eo, P, CI, PT	+0.003	-0.022	-0.006	-0.006	-0.41	-0.70
	(0.002)	(0.061)	(0.028)	(0.027)	(0.50)	(0.49)
Total due to the copula	**+0.013**	**+0.273**	**+0.109**	+0.039	+0.98	+1.38
	(0.005)	**(0.136)**	**(0.051)**	(0.064)	(1.35)	(0.91)
Eh	**+0.029**	**+0.401**	**+0.101**	**+0.116**	**+2.51**	**+1.30**
	(0.011)	**(0.125)**	**(0.037)**	**(0.050)**	**(1.08)**	**(0.64)**
Es	-0.002	-0.059	+0.006	-0.039	-1.13	+0.31
	(0.004)	(0.081)	(0.027)	(0.036)	(0.84)	(0.52)
Eo	-0.001	-0.022	-0.006	-0.006	+0.05	+0.05
	(0.002)	(0.054)	(0.018)	(0.024)	(0.51)	(0.33)
P	+0.005	**+0.136**	-0.017	**+0.092**	**+2.01**	-0.47
	(0.003)	**(0.070)**	(0.032)	**(0.024)**	**(0.67)**	(0.47)
CI	+0.000	-0.009	-0.003	-0.002	+0.000	-0.19
	(0.002)	(0.031)	(0.016)	(0.008)	(0.19)	(0.27)
PT	**-0.007**	**-0.188**	**-0.034**	**-0.069**	**-1.78**	-0.24
	(0.002)	**(0.056)**	**(0.012)**	**(0.027)**	**(0.67)**	(0.23)
ITC	-0.008	**-0.170**	**-0.057**	**-0.035**	-0.61	**-1.28**
	(0.005)	**(0.059)**	**(0.027)**	**(0.018)**	(0.49)	**(0.51)**
Total due to marginals	+0.016	+0.089	-0.010	+0.057	+1.05	-0.52
	(0.013)	(0.146)	(0.046)	(0.062)	(1.23)	(0.85)

Note: In this decomposition procedure, the contributions of the changes in the marginal CDFs of the income components are identified first and the copula contributions second. E_h is gross earnings of household head, E_s - gross earnings of spouse, E_o - gross earnings of other household members, *P* - pensions, *CI* - capital income, *PT* - public transfers, and *ITC* - income taxes and social security contributions. All income components are adjusted for the household size using the OECD-modified equivalized scale.

Appendix B

Decomposition of the change in the distribution of household disposable income in Luxembourg (taking 2013 as the base year)

Decomposition component	Contribution to the change in the following measures					
	Gini	P90/P10	P90/P50	P50/P10	Poverty rate	Richness
Overall change	**-0.029**	**-0.360**	*-0.095*	*-0.099*	**-2.03**	**-0.85**
	(0.012)	**(0.139)**	*(0.055)*	*(0.057)*	(1.39)	(0.96)
Eh with Es	**-0.011**	-0.244	**-0.103**	-0.020	-1.45	**-1.72**
	(0.005)	(0.171)	**(0.057)**	(0.079)	(1.05)	**(0.83)**
Eo with Eh, Es	+0.003	+0.068	-0.036	+0.069	-0.17	-0.27
	(0.003)	(0.126)	(0.029)	(0.061)	(0.90)	(0.42)
P with Eh, Es, Eo	-0.001	-0.151	+0.057	-0.129	-1.50	+0.78
	(0.004)	(0.158)	(0.039)	(0.078)	(1.24)	(0.63)
CI with Eh, Es, Eo, P	*-0.009*	-0.123	-0.015	-0.147	*-0.88*	-0.14
	(0.005)	(0.072)	(0.026)	(0.028)	*(0.51)*	(0.46)
PT with Eh, Es, Eo, P, CI	+0.007	*+0.316*	+0.025	+0.130	+1.46	+0.19
	(0.002)	*(0.150)*	(0.031)	(0.079)	(1.13)	(0.48)
ITC with Eh, Es, Eo, P, CI, PT	-0.002	-0.141	-0.041	-0.040	+1.53	-0.29
	(0.006)	(0.165)	(0.054)	(0.079)	(1.28)	(0.91)
Total due to the copula	**-0.013**	**-0.275**	**-0.113**	-0.037	-1.01	-1.45
	(0.006)	**(0.138)**	**(0.053)**	(0.067)	(1.45)	(0.93)
Eh	**-0.031**	**-0.407**	-0.060	**-0.150**	**-2.82**	-1.21
	(0.013)	**(0.158)**	(0.052)	**(0.056)**	**(1.09)**	(0.83)
Es	+0.003	+0.060	-0.045	+0.074	+1.21	-0.53
	(0.003)	(0.118)	(0.032)	(0.049)	(0.80)	(0.51)
Eo	+0.001	+0.032	+0.007	+0.010	+0.38	-0.07
	(0.002)	(0.058)	(0.023)	(0.022)	(0.53)	(0.36)
P	-0.003	**-0.180**	+0.020	**-0.110**	-2.26	+0.68
	(0.003)	**(0.074)**	(0.030)	**(0.029)**	(0.75)	(0.55)
CI	-0.000	*+0.048*	+0.009	+0.015	-0.11	+0.25
	(0.002)	*(0.029)*	(0.012)	(0.010)	(0.22)	(0.25)
PT	**+0.007**	**+0.190**	**+0.032**	*+0.065*	**+1.71**	+0.32
	(0.002)	**(0.083)**	**(0.013)**	*(0.035)*	**(0.75)**	(0.29)
ITC	**+0.007**	**+0.172**	**+0.055**	*+0.034*	+0.87	**+1.16**
	(0.003)	**(0.054)**	**(0.024)**	*(0.017)*	(0.47)	**(0.45)**
Total due to marginals	-0.016	-0.085	+0.018	-0.062	-1.02	+0.60
	(0.011)	(0.153)	(0.049)	(0.064)	(1.21)	(0.87)

Note: In this decomposition procedure, the contributions of the changes in the marginal CDFs of the income components are identified first and the copula contributions second. E_h is gross earnings of household head, E_s - gross earnings of spouse, E_o - gross earnings of other household members, *P* - pensions, *CI* - capital income, *PT* - public transfers, and *ITC* - income taxes and social security contributions. All income components are adjusted for the household size using the OECD-modified equivalized scale.