Conditional Choice Probability Estimation of Continuous-Time Job Search Models

Preliminary and Incomplete

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Abstract

Non-stationary job search models arise in many applications where workers can anticipate a policy change. The most prominent example of such policies is the expiration of unemployment benefits, extensively studied by van den Berg (1990). However, estimating these models poses a considerable computational challenge because of the need to solve a differential equation numerically at each step of the optimization routine. In this paper, we propose a novel, computationally feasible method of estimating non-stationary job search models. We adapt conditional choice probability methods, widely used in the dynamic discrete choice literature, to job search models and show how the hazard rate out of unemployment and the distribution of the accepted wages, which can be estimated in many datasets, can be used to infer the value of unemployment. We demonstrate how to apply our method by analyzing the effect of the unemployment benefit expiration on duration of unemployment using data from the Survey of Income and Program Participation (SIPP) in 1996-2007.

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1 Introduction

Non-stationary job search models arise in many applications when workers can anticipate a policy switch. The most common example of this situation is expiration of unemployment insurance (hereafter, UI) after the certain fixed period of unemployment. As a result of this policy, workers' reservation wages change with the duration of unemployment, going down as a worker approaches UI expiration date.

The question of the effect of UI expiration on unemployment is extremely policy relevant. For instance, during the Great Recession, the US economy experienced a massive extension of UI duration: workers could receive UI for up to 99 weeks, which was a more than threefold increase from the usual UI duration of 26 weeks. Although multiple papers have studied the effect of this policy on unemployment[1], they have used a reduced-form approach which does not allow to predict the effect of alternative policies, such as a smaller UI extension.

Non-stationary job search models in continuous time were extensive studied by van den Berg (1990) but his framework has not been widely used, despite the large interest in the effect of UI expiration on unemployment. The reason why non-stationary models have not gained bigger popularity lies in the computational burden of estimating them. In particular, the model outlined in van den Berg (1990) does not have a closed-form solution for the reservation wages. Instead, reservation wages need to be solved for numerically at each step of an optimization routine. This is time consuming despite increasing computational capacities of modern computers.

In this paper, we propose a novel, computationally feasible method of estimating non-stationary job search models which overcomes this challenge. We introduce preference shocks into a continuous-time job search model, which allows us to apply conditional choice probability methods, widely used in the dynamic discrete choice literature, to the estimation of continuous-time job search models. We show that in this extended model, a differential equation for the value of unemployment has a closed-form solution in terms of probability to accept a job. Based on this result, we propose a two-stage estimation procedure. At the first stage, we estimate the hazard rate out of unemployment and distribution of wages for newly created jobs. At the second stage, we use these two objects to estimate the structural parameters of the model using maximum likelihood.

[1] See, for example, Hagedorn et al. (2015), Farber and Valletta (2015), and Rothstein (2011).

Continuous-time search models were first estimated using conditional choice probability approach in the context of an entry model in Arcidiacono et al. (2016). This paper uses the same idea to estimate continuous-time job search models. Furthermore, to our knowledge, this is the first paper which applies conditional choice probability methods to estimate not only stationary, but non-stationary models as well.

Conditional choice probability methods are based on the result of Hotz and Miller (1993), that a value of a discrete choice can be expressed in terms of the conditional probability of making this choice. In this paper, we apply this approach to a job search setting. Indeed, the hazard rate out of unemployment is a product of job offer arrival rate and probability to accept an offer when it arrives. Thus, the hazard rate out of unemployment can be used to infer the probability to accept a job offer. However, the hazard rate contains information about the average probability to accept a job offer. We demonstrate that the hazard rates can be combined with the estimated distribution of wages for newly created jobs to recover the probability to accept a job offer at all levels of offered wages.

The rest of the paper is structured as follows. In Section 2, we extend a stationary continuous-time job search model to allow for preference shocks, and explain how the model changes when UI can expire. In Section 3, we show how the value of unemployment can be expressed in terms of probability to accept a job offer, and outline the two-stage estimation procedure. In Section 4 we apply this procedure to analyze the effect of UI expiration on duration of unemployment and wages at newly created jobs in the US using the data from the Survey of Income and Program Participation (SIPP) in 1996-2007. Finally, Section 5 concludes.

2 Model

We start by building a non-stationary job-search model with wage posting. We proceed in two steps. First, we incorporate preference shocks into the basic stationary continuous-time job search model. This allows us to obtain closed-form expressions for the value functions in terms of observed choice probabilities and structural parameters, similarly to Arcidiacono et al. (2016). Next, we introduce non-stationarity into the model by allowing unemployment insurance (hereafter, UI) to expire, as in van den Berg (1990). UI expiration makes the value function of unemployment and the hazard rate out of unemployment duration dependent. It also creates a computational challenge, since the econometrician needs to solve a differential equation to derive the value function of unemployment. In Section 3, we show how the observed hazard rates out of unemployment can be used to obtain a closed-form expression for the value function of unemployment and thus lift the computational challenge of estimating non-stationary job search models.

2.1 Stationary Job Search

Consider an economy in continuous time with infinitely lived workers, who discount the future at rate ρ. Workers are looking for jobs and can observe only one job offer at a time. Furthermore, search is costly and a worker has to wait to receive a job offer. Following standard job search models, we abstract from the labor force participation decision but we model both search while unemployed and on-the-job search.

In our model, both employed and unemployed workers are looking for jobs and they can receive job offers which are characterized by wages drawn from a known wage distribution with the p.d.f. $f(w)$[2]. Each time a worker receives an offer, she has to decide whether to accept it or turn it down based on the expected value which she can get if she continues to search. We model the arrival of job offers as a Poisson process, and allow employed and unemployed workers to sample jobs at different frequencies.

We allow workers to value the job offers they receive differently. We model these differences through the preference shock ε which is drawn independently whenever a new job offer arrives. The preference shock ε represents the relative attractiveness of a new job compared to the current state of the individual (employment at the current job for the employed or unemployment for the unemployed). Thus, a job offer with wage w can be accepted or rejected depending on the realization of the preference shock ε. We will denote the *ex ante* probability of accepting a job offer with wage w by $p_1(w, w_0)$ for the employed, where w_0 is the wage at the current job, and by $p_0(w)$ for the unemployed.

Finally, we abstract from the wage premium for tenure on the job and assume that workers are paid the same wage w while the job lasts. However, the job can end if the worker gets laid off, which occurs at the exogenous rate δ.

Now we can formally write the workers' problems. Consider first the unemployed. Denoting by λ_0 the rate of job offer arrival to the unemployed and by b the value of

[2]We assume that employed and unemployed individuals face the same offered wage distribution, though this assumption can be relaxed at the cost of the clarity of exposition.

leisure, the value of unemployment V_0 writes as:

$$(\lambda_0 + \rho)V_0 = b + \lambda_0 \mathbb{E}_{w,\varepsilon}\left[\max\left\{V_1(w) - c_0 - V_0 + \varepsilon, 0\right\}\right], \tag{2.1}$$

where $V_1(w)$ is the value of employment with the wage w and c_0 is the switching cost an unemployed worker has to pay once he becomes employed. The Emax term is the expected value resulting from the optimal choice conditional on the job offer arrival. The expectation is taken both with the respect to the possible offered wages w and realizations of the preference shock ε. Assuming that ε has a logistic distribution, we can write a closed-form expression for the *ex ante* probability to accept a job offer with wage w as:

$$p_0(w) = \frac{1}{1 + \exp\left\{V_0 - V_1(w) + c_0\right\}}. \tag{2.2}$$

In our model, the probability to accept a job offer (2.2) plays the same role as the reservation wage in classic job-search models; i.e., it maps the offered wage into the choice which brings the highest expected value to the worker in the future. However, unlike the classic model where the reservation wage is unobservable and has to be recovered from a non-linear equation, we will be able to obtain an estimate of the probability to accept an offer directly from the data. This will guide our estimation procedure, explained in detail in Section 3.2.

Using the properties of the logistic distribution (McFadden, 1974), we can rewrite the value function of unemployment in terms of the job acceptance probability:

$$(\lambda_0 + \rho)V_0 = b + \lambda_0 \mathbb{E}_w\left[V_1(w)\right] - \lambda_0 c_0 - \lambda_0 \mathbb{E}_w\left[\log\left(p_0(w)\right)\right] + \lambda_0\gamma. \tag{2.3}$$

Similarly, denoting by λ_1 the rate of job offer arrival to the employed and denoting the offered wage by w, the value of employment V_1 can be written as:

$$\begin{aligned}(\lambda_1 + \rho + \delta)V_1(w) = {} & w + \delta V_0 \\ & + \lambda_1 \mathbb{E}_{w'}\Big[V_1(w')\Big] - \lambda_1 c_1 - \lambda_1 \mathbb{E}_{w'}\Big[\log\Big(p_1(w, w')\Big)\Big] + \lambda_1\gamma.\end{aligned} \tag{2.4}$$

Using equations (2.3) and (2.4), we can write the expressions for the value functions solely in terms of the (unknown) structural parameters λ_0, λ_1, c_0, c_1 and $f(w)$, and the ex ante probabilities to accept an offer:

$$V_0 = \beta_0 + \beta_1\lambda_0\mathbb{E}_w[\log(p_0(w))] + \beta_2\lambda_1\mathbb{E}_w[\mathbb{E}_{w'}[\log(p_1(w, w')]] \tag{2.5}$$

$$V_1(w) = \alpha_0 + \alpha_1 w + \alpha_2\lambda_0\mathbb{E}_w[\log(p_0(w))] + \alpha_3\lambda_1\mathbb{E}_w[\mathbb{E}_{w'}[\log(p_1(w, w')]] \tag{2.6}$$

where α-s and β-s are known functions of the structural parameters of the model.[3]

2.2 Non-stationary Job Search

Having derived the expression for the value of unemployment V_0 and the value of employment V_1 in the stationary case, we can now extend the model to allow for the unemployment insurance to expire. UI expiration implies that the flow utility of unemployment is $b(t)$ before the UI expiration date T, and constant and equal to $\bar{b}$ afterwards.[4] This makes the value function of unemployment non-stationary, since the value of continued search now varies depending on the time left till the UI expiration date. However, UI expiration has no effect on the value of employment, which remains stationary as before. Thus, we now turn to the derivation of the value of unemployment $V_0(t)$.

Since the value of search diminishes as unemployment lasts longer and the worker gets closer to UI expiration, the value of unemployment is now duration dependent and should be indexed by the duration of unemployment t. Then, the value of unemployment writes as:

$$(\rho + \lambda_0)V_0(t) = b(t) + \dot{V}_0(t) + \lambda_0 \mathbb{E}_{w,\varepsilon}\left[\max\left\{V_1(w) - c_0 - V_0(t) + \varepsilon, 0\right\}\right], \tag{2.7}$$

where $\dot{V}_0(t) = \mathrm{d}V_0(t)\big/\mathrm{d}t$. The only difference between the value of unemployment in the stationary case (2.1) and non-stationary case (2.7) is the derivative of the value of unemployment with respect to duration of unemployment $\dot{V}_0(t)$. It represents the change in the option value of search due to approaching the UI expiration date.

Using the properties of the logistic distribution, the value of unemployment can be expressed in terms of the ex ante probability to accept a job offer $p_0(w,t)$:

$$(\lambda_0 + \rho)V_0(t) = b(t) + \dot{V}_0(t) + \lambda_0 \mathbb{E}_w\left[V_1(w)\right] - \lambda_0 c_0 - \lambda_0 \mathbb{E}_w\left[\log\Big(p_0(w,t)\Big)\right] + \lambda_0 \gamma. \tag{2.8}$$

Thus, after allowing UI to expire we no longer have a closed form expression for the value of unemployment $V_0(t)$ as a function of the job acceptance probabilities and the structural parameters. Instead, we have a differential equation for the value function.

[3] The explicit expressions are: $\beta_0 = \frac{\rho+\delta}{\rho^2+\lambda_0\rho+\rho\delta}\big(b + \lambda_0(\gamma - c_0)\big) + \frac{\lambda_0}{\rho^2+\lambda_0\rho+\rho\delta}\big(\mu + \lambda_1(\gamma - c_1)\big)$, $\beta_1 = -\frac{\rho+\delta}{\rho^2+\lambda_0\rho+\rho\delta}$, $\beta_2 = -\frac{\lambda_0}{\rho^2+\lambda_0\rho+\rho\delta}$, $\alpha_0 =$, $\alpha_1 = \frac{1}{\rho+\delta+\lambda_1}$, $\alpha_2 = -\frac{\delta}{\rho^2+\lambda_0\rho+\rho\delta}$, and $\alpha_3 = \frac{\lambda_0+\rho}{\rho^2+\lambda_0\rho+\rho\delta}$.

[4] The assumption that the flow utility is constant after the UI expiration date T is not required and is made for the clarity of the exposition

However, unlike in the classic paper of van den Berg (1990), this equation is an ordinary differential equation, which admits an exact solution. Indeed, equation (2.8) can be rewritten as:

$$\dot{V}_0(t) = (\lambda_0 + \rho)V_0(t) + \phi(t) + b(t) - \nu - C \tag{2.9}$$

$$\phi(t) = \lambda_0 \mathbb{E}_w \left[\log\left(p_0(w,t)\right)\right] \tag{2.10}$$

$$\nu = \lambda_0 \mathbb{E}_w \left[V_1(w)\right] \tag{2.11}$$

$$C = \lambda_0(\gamma - c_0). \tag{2.12}$$

Equation (2.9) is a first-order linear differential equation in $V_0(t)$; ν and C are constants, and $\phi(t)$ and $b(t)$ are known functions of t. The solution to equation (2.9) writes as:

$$V_0(t) = e^{(\lambda_0+\rho)t}\left(V_0(0) + \int_0^t e^{-(\lambda_0+\rho)s}\Big(\phi(s) + b(t)\Big)\,\mathrm{d}s\right) + \frac{1}{\rho+\lambda_0}\left(1 - e^{(\rho+\lambda_0)t}\right)\Big(\nu + C\Big). \tag{2.13}$$

Thus, we have expressed the value of unemployment at each duration of the unemployment spell in terms of the structural parameters, offer acceptance probabilities $p_0(w,t)$, expected value of a job with wages $\mathbb{E}_w\left[V_1(w)\right]$, and the value of unemployment at the start of the unemployment spell $V_0(0)$. The expected value of a job with the wages $\mathbb{E}_w\left[V_1(w)\right]$ is itself a function of the offer acceptance probability and the structural parameters (see equation 2.4). The value of unemployment at the start of the unemployment spell can be obtained from the terminal condition at the moment when

UI expires $V_0(T) = \bar{V}_0$[5]:

$$V_0(0) = \frac{\lambda_0}{\kappa_1}\Upsilon_2 + \left(1 + \frac{\lambda_0\delta}{\kappa_1}\right)\Upsilon_1, \tag{2.14}$$

$$\begin{aligned}\Upsilon_1 = & -\Phi(T) - \frac{1}{\rho+\lambda_0}e^{-(\rho+\lambda_0)T}\Psi(T) \\ & + \frac{1}{\rho+\lambda_0}e^{-(\rho+\lambda_0)T}(b_s - b_{ns}) \\ & + \frac{1}{\rho+\lambda_0}\Big(\lambda_0(\gamma - c_1) + b_{ns}\Big),\end{aligned} \tag{2.15}$$

$$\Upsilon_2 = \mu + \lambda_2(\gamma - c_2) - \lambda_2\mathbb{E}_w\left[\mathbb{E}_{w'}\Big[\log\Big(p_1(w,w')\Big) \mid w\Big]\right], \tag{2.16}$$

where $\Phi(t) = \lambda_0\int_0^t e^{-(\rho+\lambda_0)s}\,\mathbb{E}_w\Big[\log\Big(p_0(w,s)\Big)\Big]\,\mathrm{d}s$ and $\Psi(t) = \lambda_0\mathbb{E}_w\Big[\log\Big(p_0(w,t)\Big)\Big]$.

The key difference between our model and the classic model of van den Berg (1990) lays in the treatment of the reservation wage. In the classic model, the differential equation for the value of unemployment takes the form:

$$(\lambda_0 + \rho)V_0(t) = b(t) + \dot{V}_0(t) + \lambda_0\int_{w^*(t)}^{\infty} V_1(w)f(w)\,\mathrm{d}w, \tag{2.17}$$

where $w^*(t)$ is the reservation wage at the duration of unemployment t. Since both the offered wage distribution $f(w)$ and the reservation wage $w^*(t)$ are unobserved, the differential equation (2.17) does not have a closed-form solution even if the value of employment $V_1(w)$ is known.

In our model, the reservation wage itself is not observed as well. However, since the offer acceptance is no longer deterministic, we can observe the offer acceptance probability, albeit indirectly (see the next section for details). The offer acceptance probability $p_0(w,t)$ is duration dependent for the same reasons that the reservation wage $w^*(t)$ is duration dependent in the classic model. Thus, the offer acceptance probability is informative about the shape of the reservation wage $w^*(t|\varepsilon)$ with respect to duration of unemployment. And the fact that we can indirectly observe these probabilities allows us to treat the term $\phi(t)$ in equation (2.9) as a known function.

Similarly to van den Berg (1990), we do not observe the offered wage distribution $f(w)$ directly so we will have to calculate the integral $\int_0^t e^{-(\lambda_0+\rho)s}\phi(s)\,\mathrm{d}s$ numerically at each step of the optimization routine given the parametrization of the distribution

[5]Another option to obtain $V_0(0)$ is to introduce voluntary quits to the model and express $V_0(0)$ as a function of $V_1(w)$ and the probability to quit the job with a wage level w.

$f(w)$, but we will *not* have to solve the differential equation (2.9) numerically. Thus, adopting the conditional choice probability approach to the non-stationary job search model allows us to overcome the problem of numerical differentiation, though it still leaves the problem of numerical integration.

3 Identification

In the previous section, we have expressed the value of employment and unemployment as a function of conditional choice probabilities and the structural parameters of the model. In this section, we demonstrate how to take the model to the data. First, we show how to obtain the conditional choice probabilities, since in our case they are not directly observed. Second, we parametrize the distributions of the offered wage, which can not be recovered from the data either. Finally, we propose a two-stage estimation procedure which allows us to recover the parameters of interest. At the first stage, we recover the functions related to the conditional choice probabilities, and at the second stage we estimate the structural parameters of the model via MLE.

3.1 Conditional Choice Probabilities

Unlike in traditional discrete choice models, the probability to accept a job offer in job search models is not observed directly. This is typically the case for job search models with wage posting, since we normally do not observe the rejected job offers. However, the model makes predictions about the frequency of transitions from unemployment to employment, and about the frequency of job switches. Denoting by $h_0(t)$ the hazard rate out of unemployment at the duration of unemployment t, we can express this hazard as:

$$h_0(t) = \lambda_0 \mathbb{E}_w \Big[p_0(w,t) \Big], \tag{3.1}$$

and, similarly, we can express the hazard rate of job switching $h_1(w_0)$ as:

$$h_1(w_0) = \lambda_1 \mathbb{E}_w \Big[p_1(w,w_0) \Big], \tag{3.2}$$

where w_0 is the wage rate at the old job. Both the hazard rate out of unemployment and the hazard rate of job switching represent the frequency of accepting a job offer conditional on having received one. Unemployed workers receive job offers at the rate

λ_0 and employed workers receive job offers at the rate λ_1. Conditional on the job offer, an unemployed (employed) worker accepts an offer with wage level w with the probability $p_0(w,t)$ ($p_1(w,w_0)$, respectively). Since we do not observe the offered wages, we integrate them out and get the expressions for the hazard rates. Note that both $h_0(t)$ and $h_1(w_0)$ can be estimated from the data on durations of unemployment and employment spells, as long as the wage level on the old job is observed as well.

Furthermore, we observe the offered wage if it was accepted – i.e. when an unemployed worker gets a new job or an employed worker makes a job switch. Denoting by $g_0(w,t)$ the p.d.f. of accepted wages for the new jobs out of unemployment and by $g_1(w,w_0)$ the p.d.f. of accepted wages for the new jobs as a result of job switches, these densities can be written as:

$$g_0(w,t) = f(w)\,\frac{p_0(w,t)}{\mathbb{E}_w\left[p_0(w,t)\right]}, \tag{3.3}$$

$$g_1(w,w_0) = f(w)\,\frac{p_1(w,w_0)}{\mathbb{E}_w\left[p_1(w,w_0)\right]}. \tag{3.4}$$

That is, the likelihood of observing a new job with wage w is the likelihood of the wage w being offered, conditional on this offer being accepted. Applying Bayes' rule immediately leads to equations (3.3) and (3.4).

Unlike the expressions for the hazard rates out of unemployment, which reveal an average acceptance probability for each spell type, the density of accepted wages provides information on the actual acceptance probabilities at each level of the accepted wages w observed in the data. Note that if the sample is large enough, the support of the offered wage distribution is the same as the support of the accepted wage distribution, since the probability of the offer acceptance is non-zero for all wage levels (though it can be very-very small by magnitude).

Combining equations (3.1) and (3.3), we can obtain the expression for the probability to accept a job offer with wage level w while unemployed:

$$p_0(w,t) = \frac{h_0(t)}{\lambda_0}\frac{g_0(w,t)}{f(w)}. \tag{3.5}$$

Similarly, the probability to accept an job offer with wage level w while employed is:

$$p_1(w,w_0) = \frac{h_1(w_0)}{\lambda_1}\frac{g_1(w,w_0)}{f(w)}. \tag{3.6}$$

In equations (3.5) and (3.6) the hazard rates $h_0(t)$ and $h_1(w_0)$, as well as the accepted wage densities $g_0(w,t)$ and $g_1(w,w_0)$, can be estimated from the data. Thus, we have identified the conditional choice probabilities up to the constants λ_0, λ_1 and the p.d.f. of the offered wage density. We will treat λ_0 and λ_1 as parameters to be estimated at the second stage. Similarly, we will parametrize the offered wage distribution and estimate its moments at the second stage.

To be able to take the expressions for the acceptance probabilities (3.5) and (3.6) to the data, we need to make assumptions about the p.d.f. of the offered wage distribution $f(w)$. The offered wage distribution cannot be recovered non-parametrically, because we only observe accepted wages and do not impose parametric restrictions on the choice probabilities $p_0(w,t)$ and $p_1(w,w_0)$. This lack of parametric restrictions on the acceptance probabilities is crucial since we will use them to obtain the shape of the value functions at the second stage. Thus, we assume that the offered wage is distributed log-normally with parameters μ and σ, which we will be estimating together with the other structural parameters of the model. Parametrization of $f(w)$ allows us to treat $p_0(w,t)$ and $p_1(w,w_0)$ as identified up to the parameters λ_0, λ_1, μ and σ. Furthermore, since the shape of the offered wage distribution is now fixed, we can also express the integrals $\mathbb{E}_w\left[\mathbb{E}_{w'}\Big[\log\Big(p_1(w,w')\Big) \mid w\Big]\right]$ and $\mathbb{E}_{w'}\left[\log\Big(p_0(w',t)\Big)\right]$ required for the value functions up to parameters to be estimated.

3.2 Two-stage Estimation Procedure

Based on the results in the previous subsection, we propose a two-stage estimation procedure. At the first stage, we estimate the hazard rate out of unemployment $\hat{h}_0(t)$, the hazard rate of job switching $\hat{h}_1(w_0)$, and the p.d.f. of the accepted wage for the new jobs out of unemployment $\hat{g}_0(w,t)$ and for the job switches $\hat{g}_1(w,w_0)$. All four objects can be estimated non-parametrically.

At the second stage, we estimate the structural parameters using the maximum likelihood estimator. First, we use the first stage estimates to form the observed acceptance probabilities in equations (3.5) and (3.6). Next, we express the value of unemployment $V_0(t)$ and the value of employment $V_1(w)$ as functions of the choice probabilities and structural parameters which we would like to estimate.

Using the results of Section 2, the value of a job with wage w can be written as a function of structural parameters and conditional choice probabilities:

$$V_1(w) = \frac{1}{\lambda_2+\rho+\delta}\Big[w + \lambda_2(\gamma - c_2) - \lambda_2 \mathbb{E}_{w'}\Big[\log\Big(p_1(w,w')\Big) \mid w\Big] + \frac{\kappa_2}{\kappa_1}\Upsilon_2 + \Big(1+\frac{\kappa_2}{\kappa_1}\Big)\delta\Upsilon_1\Big], \quad (3.7)$$

and the value of being unemployed after t months of unemployment can be written as follows:

$$V_0(t) = -\frac{1}{\rho+\lambda_0}e^{-(\rho+\lambda_0)(T-t)}\Psi(T) + e^{(\lambda_0+\rho)t}\Big[\Phi(t)-\Phi(T)\Big] + \frac{\lambda_0}{\kappa_1}\Upsilon_2 + \frac{\lambda_0\delta}{\kappa_1}\Upsilon_1 + \frac{1}{\rho+\lambda_0}e^{-(\rho+\lambda_0)(T-t)}(b_s - b_{ns}) + \frac{1}{\rho+\lambda_0}\Big(\lambda_0(\gamma-c_1)+b_{ns}\Big), \quad (3.8)$$

where Υ_1 and Υ_2 are functions of conditional choice probabilities as well, and are given by equations (2.15) and (2.16) on p. 8.

After obtaining the value functions, we can construct acceptance probabilities implied by the structural parameters and the estimated conditional choice probabilities, since the Extreme Value Type I distribution gives us closed-form expressions for these probabilities:

$$p_0(w,t) = \frac{\exp\{V_1(w)-c_0\}}{\exp\{V_0(t)\}+\exp\{V_1(w)-c_0\}}, \quad (3.9)$$

$$p_1(w,w_0) = \frac{\exp\{V_1(w)-c_1\}}{\exp\{V_1(w_0)\}+\exp\{V_1(w)-c_1\}}, \quad (3.10)$$

where c_0 is the switching cost out of unemployment and c_1 is the switching cost of changing a job.

Using these acceptance probabilities, we can now obtain the hazard rate out of unemployment and the hazard rate of job switching implied by the model:

$$h_0(t) = \lambda_0 \mathbb{E}_w p_0(w,t), \quad (3.11)$$

$$h_1(w_0) = \lambda_1 \mathbb{E}_w\Big[p_1(w,w_0)\Big], \quad (3.12)$$

as well as the p.d.f. of accepted wages:

$$g_0(w,t) = f(w)\,\frac{p_0(w,t)}{\mathbb{E}_w\left[p_0(w,t)\right]}, \quad (3.13)$$

$$g_1(w,w_0) = f(w)\,\frac{p_1(w,w_0)}{\mathbb{E}_w\left[p_1(w,w_0)\right]}. \quad (3.14)$$

Finally, we form the likelihood function. It has four components: the likelihood of the unemployment spell duration, which is based on $h_0(t)$, the density of the accepted wage for the new jobs out of unemployment, based on $g_0(w,t)$, the likelihood of the employment spell duration, based on $h_1(w_0)$, and the density of the accepted wage when changing a job, based on $g_1(w,w_0)$. The exact expressions for the likelihood contribution are given in Section 4.3 where we discuss the empirical implementation on the two-stage procedure.

4 Empirical Implementation

In this section, we apply the two-stage procedure described in Section 3.2 to study the effect of unemployment benefits expiration on duration of unemployment and wages at the newly created jobs. We estimate the model using the data on white males from the Survey of Income and Program Participation (SIPP) in 1996-2007. First, we describe how we construct the estimation sample. Then we introduce the necessary parametrization and discuss the first-stage results. Finally, we explain how the second stage is implemented and present the results.

4.1 Data

We estimate the model using data from the Survey of Program and Participation (SIPP). SIPP is a nationally representative panel dataset which renews its sample approximately every four years. We pool the data from the 1996 and 2004 panels.[6] SIPP provides details on workers labor force status, employment history, wages, and hours at each job at monthly frequency. We restrict our analysis to the white males aged 25–55 during the time of the survey, and estimate the model using the sample of high school graduates and college graduates separately.

Although SIPP does not collect information on the duration of unemployment explicitly, it provides such details as weekly labor force status and the starting date of

[6] Although SIPP is available from 1986 to 2013, we focus on these panels for three reasons. First, we consider the period before the Great Recession, and thus do no include the 2008 panel in the estimation. Second, related to the prior one, we exclude the 2001 panel as we do not consider the 2001 recession either. Third, SIPP underwent a substantial redesign in 1996, and contains more details on workers' employment starting from 1996.

a current job. Based on this information, we construct duration of each employment and unemployment spell and record labor force status at the end of each month. Since our analysis focuses on the role of unemployment benefits expiration in duration of unemployment, we use only unemployment spells with duration of less than 2 years. Finally, to be able to construct consistent employment histories, we exclude individuals who attrit once even if they come back to the sample.

The final sample contains information on 29,960 individuals with a high school degree and 13,804 individuals with a college degree (see Tables 1a and 1b). The average worker with a high school degree is 39 years old while the average worker with a college degree is 40 years old. Most workers have only 1 spell in the data, but the average number of spells per worker is 1.3–1.4 in both education groups.

Among the unemployment spells, the average spell has a duration of 2.25 months in both education groups, and about 70% of unemployment spells end with a new job. On the other hand, only 14–15% of employment spells end with a new job, and the average duration of an employment spell in the estimation sample is 6.5 years for both education groups. On average, college graduates get higher wages and receive a larger wage increase when switching jobs than high school graduates.[7]

4.2 First Stage

At the first stage, we estimate four key objects: 1) hazard rate out of unemployment to a new job, 2) hazard rate of job switching, 3) accepted wage density at new jobs of previously unemployed workers, and 4) accepted wage density at new jobs of previously employed workers. We then use the estimates of these four objects to construct conditional choice probabilities as described in Section 3, and to estimate the structural parameters of the model.

Both hazard rates and accepted wage distributions are estimated semi-parametrically. We model hazard rates as a proportional hazard, and we model accepted wage distributions as mixtures of two normal distributions. Furthermore, we allow hazard rates and distributions of accepted wages to vary with unobserved characteristics of the workers. We model these unobserved characteristics as a mixture distribution (Heckman and Singer, 1984). We assume that there are K types of workers, with population probability π_k each, and recover these parameters in the estimation.

[7]For comparability, we deflate wages using the national CPI.

Each worker i can have multiple spells of employment and unemployment in the data. Denoting by J_i^E and $J_i^{\bar{E}}$ the number of completed and incomplete employment spells for worker i, and by J_i^U and $J_i^{\bar{U}}$ the number of completed and incomplete unemployment spells for worker i, respectively, the full likelihood contribution for this worker can be written as follows:

$$L_i = \sum_k \pi_k \left(\prod_{j=1}^{J_i^E} L_{ijk}^{h^E}\left(\theta^{h^E}, X_i\right) L_{ijk}^{w^E}\left(\theta^{w^E}, X_i\right) \prod_{j=1}^{J_i^{\bar{E}}} L_{ijk}^{h^E}\left(\theta^{h^E}, X_i\right) \right.$$
$$\left. \prod_{j=1}^{J_i^U} L_{ijk}^{h^U}\left(\theta^{h^U}, X_i\right) L_{ijk}^{w^U}\left(\theta^{w^U}, X_i\right) \prod_{j=1}^{J_i^{\bar{U}}} L_{ijk}^{h^U}\left(\theta^{h^U}, X_i\right) \right), \tag{4.1}$$

where $L_{ijk}^{h^E}$ is the likelihood contribution of an employment spell, $L_{ijk}^{w^E}$ is the likelihood contribution of accepted wages for a job switcher, $L_{ijk}^{h^U}$ is the likelihood contribution of an unemployment spell, $L_{ijk}^{w^U}$ is the likelihood contribution of accepted wages for a newly employed worker, θ^{h^E}, θ^{w^E}, θ^{h^U} and θ^{w^U} are parameters to be estimated, and X_i are observable characteristics of a worker i.

We estimate these parameters using an EM algorithm. Using the properties of EM algorithms, the full likelihood contribution (4.1) of worker i can be expressed as follows:

$$\log L_i = \sum_k q_{ik} \left(\sum_{j=1}^{J_i^E} \left(\log L_{ijk}^{h^E}\left(\theta^{h^E}, X_i\right) + \log L_{ijk}^{w^E}\left(\theta^{w^E}, X_i\right)\right) + \sum_{j=1}^{J_i^{\bar{E}}} \log L_{ijk}^{h^E}\left(\theta^{h^E}, X_i\right) \right.$$
$$\left. + \sum_{j=1}^{J_i^U} \left(\log L_{ijk}^{h^U}\left(\theta^{h^U}, X_i\right) + \log L_{ijk}^{w^U}\left(\theta^{w^U}, X_i\right)\right) + \sum_{j=1}^{J_i^{\bar{U}}} \log L_{ijk}^{h^U}\left(\theta^{h^U}, X_i\right) \right). \tag{4.2}$$

Because the parameters of the hazard rates θ^{h^E} and θ^{h^U} do not have any common terms with the parameters of the accepted wage distribution θ^{w^E} and θ^{w^U}, these four sets of parameters can be estimated separately via partial likelihood. However, contribution of all four components is required to obtain the posterior probability of being each type $q_{ik}(Z_i)$. Thus, we implement the following EM-algorithm:

1. Initialize a guess for type probabilities $\pi_k^{(0)}$ and posterior probabilities $q_{ik}^{(0)}$.

2. **M-step.** Taking posterior probabilities $q_{ik}^{(m-1)}$ as given, estimate parameters of:

(a) hazard out of unemployment;

(b) hazard of job switching;

(c) accepted wage out of unemployment;

(d) accepted wage of job switchers.

3. **E-step.** Taking type probabilities $\pi_k^{(m-1)}$ as given, renew posterior probabilities using the estimates of parameters from the M-step:

$$q_{ik}^{(m)} = \frac{\pi_k^{(m-1)} \prod_{j=1}^{J_i^E} L_{ijk}^{h^E} L_{ijk}^{w^E} \prod_{j=1}^{J_i^U} L_{ijk}^{h^U} L_{ijk}^{w^U}}{\sum_k \pi_k^{(m-1)} \left(\prod_{j=1}^{J_i^E} L_{ijk}^{h^E} L_{ijk}^{w^E} \prod_{j=1}^{J_i^U} L_{ijk}^{h^U} L_{ijk}^{w^U} \right)}, \quad (4.3)$$

where $L_{ijk}^{h^E}$, $L_{ijk}^{w^E}$, $L_{ijk}^{h^U}$, and $L_{ijk}^{w^U}$ are evaluated at the parameter values obtained at the M-step of the algorithm.

Renew type probabilities using the new guess for posterior probabilities:

$$\pi_k^{(m)} = \frac{1}{N} \sum_i q_{ik}^{(m)}. \quad (4.4)$$

4. Repeat steps 2-3 until convergence.

4.2.1 Hazard rates.

We model the hazard rate out of unemployment and the hazard rate of job switching as proportional hazards, allowing the baseline hazard to depend on the duration of the spell. Thus, the hazard rate out of unemployment of a type-k individual who has been unemployed for t months and is experiencing her jth unemployment spell, denoted by $h_{ijk}^U(t)$, can be written as follows:

$$h_{ijk}^U(t) = h_0^U(t) \exp\left\{\mu_k^U + \gamma^U X_{ijt}\right\}, \quad (4.5)$$

where $h_0^U(t)$ is the baseline hazard, μ_k^U is the k-type specific shifter, and X_{ijt} are the characteristics of an individual, which we allow to vary with the duration of the spell. In particular, we allow the hazard rate out of unemployment to vary with the reference month in the data. SIPP questionnaires are administered every four months, and ask retrospective questions about the previous four months. This design leads to a well-known problem, seam-bias, which creates measurement error in transition rates between labor force statuses. By estimating how different the transition rate out of unemployment is during the fourth reference month, when the seam bias can be expected, we ensure that our results are not driven by this feature of the data.

We model baseline hazard $h_0^U(t)$ as a piecewise-constant function, allowing it to be different at each month of unemployment before UI expire, and holding it constant the UI expiration (7 months of unemployment and longer). Denote by t_j an observation-month for a spell j, and by T_i the set of unemployment durations which are observed in the spell j. Finally, denote by $d^U(t_j)$ an indicator function for whether the spell was completed at the month t_j. Then the log likelihood contribution of a spell j is:

$$\log L_{ijk}^{h^U} = \sum_{t_j \in T_i} \left(-(1 - d^U(t_j)) h_{ijk}^U(t_j) + d^U(t_j) \log \left(1 - \exp\{-h_{ijk}^U(t_j)\} \right) \right). \tag{4.6}$$

We parametrize the hazard of job switching similarly to the hazard out of unemployment, except that we allow the hazard rate to vary with the wage rate of the current job, but assume that hazard rate of job switching does not vary with tenure on the job. As a result, the hazard rate of job switching of a type-k individual who is currently in her jth employment spell and whose wage at the current job is w_0, denoted by $h_{ijk}^E(t)$, can be written as follows:

$$h_{ijk}^E(w_0) = h_0^E(w_0) \exp\left\{ \mu_k^E + \gamma^E X_{ijt} \right\}, \tag{4.7}$$

where $h_0^E(w_0)$ is baseline hazard and μ_k^U is the k-type specific shifter. Just as in the case with the hazard out of unemployment, we include the indicator for the fourth reference month in X_{ijt} to account for the measurement error caused by the seam bias.

We split all current job spells into 5 percentile groups based on the current log wages. As a result, the log likelihood contribution of an employment spell j is:

$$\log L_{ijk}^{h^E} = \sum_{t_j \in T_i} \left(-(1 - d^E(t_j)) h_{ijk}^E(w_0) + d^E(t_j) \log \left(1 - \exp\{-h_{ijk}^E(w_0)\} \right) \right). \tag{4.8}$$

4.2.2 Accepted wage distributions.

We estimate the distribution of accepted wages out of unemployment and the distribution of accepted wages of job switchers separately. We model each of these distributions as a mixture of two normal distributions. We allow the distribution of accepted wages out of unemployment to vary with duration of unemployment in a following way:

$$g_0(w,t) = \omega^U(t)\phi(\mu_1^U, \sigma_1^U) + \Big(1-\omega^U(t)\Big)\phi(\mu_2^U, \sigma_2^U), \tag{4.9}$$

where $\phi(\mu, \sigma)$ is the p.d.f. of a normal distribution with mean μ and standard deviation σ. We model $\omega^U(t)$ as a second degree polynomial of duration of unemployment t:

$$\omega^U(t) = \frac{1}{1+\exp\{a_{10}^U + a_{11}^U t + a_{12}^U t^2\}}. \tag{4.10}$$

Coefficients a_{10}, a_{11} and a_{12} are estimated together with the mixture parameters μ_1^U, μ_2^U, σ_1^U , and σ_2^U.

The distribution of accepted wages for job switchers is modeled in a similar way:

$$g_1(w,w_0) = \omega^E(w_0)\phi(\mu_1^E, \sigma_1^E) + \Big(1-\omega^E(w_0)\Big)\phi(\mu_2^E, \sigma_2^E), \tag{4.11}$$

where $\omega^E(w_0)$ is a second degree polynomial of the current wage w_0:

$$\omega^E(w_0) = \frac{1}{1+\exp\{a_{10}^E + a_{11}^E w_0 + a_{12}^E w_0^2\}}. \tag{4.12}$$

4.3 Second Stage

At the second stage, we are interested in estimated four groups of parameters:

- arrival rates λ_0 and λ_1;
- switching costs;
- parameters of the offered wage distribution μ and σ;
- the value of leisure before and after UI expiration b_{ns} and b_s, and scale parameter of the flow utility function η.

Having recovered the first stage parameters for each of the K types of workers, and the posterior probability of being each type $\hat{q}_{ik}$, at the second stage we estimate the structural parameters for the first worker type.

Using the strategy described in Section 3.2, we construct the hazard rate out of unemployment $\tilde{h}_0$, hazard rate of job switching $\tilde{h}_1$, density of accepted wage for newly created jobs out of unemployment $\tilde{g}_0$, and density of accepted wage for job switchers $\tilde{g}_1$

implied by the values of the structural parameters. Then, the log likelihood contribution of an unemployment spell is:

$$\log L_i^{h^U} = \sum_{t_j \in T_i} \Big(-(1 - d^U(t_j))\tilde{h}_0(t_j)\exp\{\hat{\gamma}^U X_{it_j}\} + d^U(t_j)\log\Big(1 - \exp\{-\tilde{h}_0(t_j)\exp\{\hat{\gamma}^U X_{it_j}\}\}\Big)\Big), \quad (4.13)$$

where $\hat{\gamma}^U$ is the estimate from the first stage. The log likelihood contribution of an employment spell is:

$$\log L_i^{h^E} = \sum_{t_j \in T_i} \Big(-(1 - d^E(t_j))\tilde{h}_1(t_j)\exp\{\hat{\gamma}^E X_{it_j}\} + d^E(t_j)\log\Big(1 - \exp\{-\tilde{h}_1(t_j)\exp\{\hat{\gamma}^E X_{it_j}\}\}\Big)\Big). \quad (4.14)$$

Similarly, the log likelihood contribution of accepted wages for newly created jobs out of unemployment is:

$$\log L_i^{w^U} = \log \tilde{g}_0(w_{1,i}), \quad (4.15)$$

and the log likelihood contribution of accepted wages for job switchers is:

$$\log L_i^{w^E} = \log \tilde{g}_1(w_{1,i}). \quad (4.16)$$

The resulting log likelihood function used to estimate the structural parameters is:

$$\log L = \sum_i \hat{q}_{i1}\Big(\log L_i^{h^U} + \log L_i^{h^E} + \log L_i^{w^U} + \log L_i^{w^E}\Big). \quad (4.17)$$

4.4 Results

In progress.

5 Conclusion

In this paper, we propose a novel approach to estimating job search models. We extend traditional continuous-time job search model with on-the-job search to allow for preference shocks which, in turn, allows us to estimate the model using conditional choice probability methods, widely used in the discrete choice literature. We propose a simple two-stage procedure of estimating job search models. At the first stage, we estimate the hazard rate out of unemployment and the hazard rate of job switching, as well as the density of accepted wages at the newly created jobs. These objects inform us on the shape of the probability to accept a job offer as a function of offered wages and duration of unemployment for unemployed workers, and as a function of offered wages and current wages for employed workers. At the second stage, we estimate structural parameters of the model using the maximum likelihood estimator. Conditional choice probability methods allow us to obtain closed-form expressions for the value functions. As a result, we overcome the computational burden of estimating complicated job search models, such as non-stationary models. We illustrate our method by studying the impact of unemployment benefits expiration on the duration of unemployment and wages in the US, using data from the Survey and Program Participation (SIPP) in 1996-2007.

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Tables

Table 1. Summary statistics, estimation sample.

(a) High school graduates.

	Unemployment spells	Employment spells
Means		
Duration, months	2.26	76.44
Log real wage at the current job		2.35
Log real wage at the new job	2.39	2.39
Difference in log real wage		0.09
Age, years	38.57	38.97
No. spells per worker	1.29	1.41
Share of complete spells	0.67	0.15
No. observations		
No. complete spells	4837	6086
No. spells	7170	41881
No. workers	29960	29960

Sample: White males, age 25-55. SIPP, 1996–2007.

(b) College graduates.

	Unemployment spells	Employment spells
Means		
Duration, months	2.25	80.03
Log real wage at the current job		2.73
Log real wage at the new job	2.77	2.75
Difference in log real wage		0.13
Age, years	40.05	40.1
No. spells per worker	1.3	1.34
Share of complete spells	0.71	0.14
No. observations		
No. complete spells	1627	2540
No. spells	2307	18517
No. workers	13804	13804

Sample: White males, age 25-55. SIPP, 1996–2007.