Shrinkage in Set-Identified SVARs

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- Since Faust (98), Canova & De Nicoló (02, *JME*), and Uhlig (05, *JME*), set-identified SVARs have become common.
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- This comes at a cost:
  1. Weak restrictions may prevent from recovering informative economic conclusions.
  2. Risk of retaining structural parameters with implausible implications.
- Challenge: come up with a small number of additional uncontentious restrictions that help shrink the set of admissible structural parameters and reach clear economic conclusions. ▸ Sign vs Ordering Restrictions

This Paper

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- FEV decomposition: how much of the FEV is explained by each structural shock.
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- In this talk: only monetary policy application.

Relevant Literature

- Arias, Caldara & Rubio-Ramirez (17) impose sign and zero restrictions on the structural component of monetary policy instead of restricting impulse responses.
- Narrative sign restrictions in Antolin-Diaz & Rubio-Ramirez (17, *AER*).
- Heterogeneity restrictions in Amir-Ahmadi & Drautzburg (17) for identification of productivity news and defense spending shocks.
- Kilian & Murphy (12, *JEEA*): on top of sign restrictions, they impose bounds on the magnitude of the short-run oil supply elasticity and on the impact response of real activity to oil shocks.

The Econometric Framework

- SVAR(p):

$$A_0 y_t = c + \sum_{j=1}^{p} A_j y_{t-j} + \epsilon_t, \;\; \epsilon_t | (y_{t-1}, \dots) \sim \mathcal{N}(0, I_n).$$

- Reduced-form VAR(p):

$$y_t = b + \sum_{j=1}^{p} B_j y_{t-j} + u_t, \;\; u_t | (y_{t-1}, \dots) \sim \mathcal{N}(0, \Sigma)$$

  where $\Sigma = A_0^{-1}(A_0^{-1})'$ and $\beta = vec(B)$.
- The prior for the reduced-form is a conjugate $\mathcal{NIW}$ specification: $\Sigma \sim \mathcal{IW}(\Psi, d), \;\; \beta | \Sigma \sim \mathcal{N}(b, \Sigma \otimes \Omega)$.

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- The prior for the reduced-form is a conjugate $\mathcal{NIW}$ specification: $\Sigma \sim \mathcal{IW}(\Psi, d), \ \ \beta | \Sigma \sim \mathcal{N}(b, \Sigma \otimes \Omega)$.
- For estimation and inference, I employ the algorithm in Arias, Rubio-Ramirez & Waggoner (18, *Ecta*); Giacomini & Kitagawa (15) is used as robustness check.

Bivariate Model

- Simple SVAR(0):

$$A \begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{pmatrix}, \quad A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}.$$

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- Structural shocks $(\epsilon_{1t}, \epsilon_{2t}) \sim \mathcal{N}(0, I)$.
- Structural parameters: $\theta = vec(A) \in \Theta$.
- Structural parameter of interest: impulse response of y_{1t} to ϵ_{1t}, $\alpha \equiv IR^0(y_1, \epsilon_1)$.
- Reduced-form parameters: Cholesky decomposition of the Var-cov matrix of (y_{1t}, y_{2t}), $\phi = (\sigma_{11}, \sigma_{21}, \sigma_{22}) \in \Phi$.

Bivariate Model (cont)

- Model M^1 (sign restrictions): $IR^0(y_{1t}, \epsilon_{2t}) \leq 0$ and $IR^0(y_{2t}, \epsilon_{1t}) \geq 0$. α is unrestricted. α is set-identified; the identified set is

$$IS_\alpha(\phi) \equiv \begin{cases} \left[\sigma_{11} \cos\left(\text{arctan}\left(\frac{\sigma_{22}}{\sigma_{21}}\right)\right), \sigma_{11}\right] \text{ for } \sigma_{21} > 0, \\ \left[0, \sigma_{11} \cos\left(\text{arctan}\left(\frac{-\sigma_{21}}{\sigma_{22}}\right)\right)\right] \text{ for } \sigma_{21} \leq 0. \end{cases}$$

Illustrative Example: Restrictions on the FEV

- Model M^2 (restrictions on the FEV): M^1 + shock ϵ_{2t} explains variable y_{2t} more than y_{1t}: $\frac{IR^0(y_{2t},\epsilon_{2t})^2}{var(y_{2t})} \geq \frac{IR^0(y_{1t},\epsilon_{2t})^2}{var(y_{1t})}$. α is unrestricted.

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- M^2 shrinks $IS_\alpha(\phi)$ with respect to M^1.
- The same holds in a trivariate setting; $IS_\alpha(\phi)$ of the variable that is not subject to restrictions on the FEV is shrunk.

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IRFs, $h=0$	-	-	-	-	-	-	$+$
FEV, $h=0$	$[0.05, 0.15]$	$[0.03, 0.13]$	$[0.02, 0.12]$	$[0.00, 0.02]$	$[0.02, 0.09]$	$[0.00, 0.03]$	$[0.37, 0.66]$
FEV, $h=1$	$[0.07, 0.17]$	$[0.06, 0.13]$	$[0.03, 0.12]$	$[0.01, 0.03]$	$[0.04, 0.11]$	$[0.03, 0.10]$	$[0.30, 0.60]$

Monte-Carlo Simulation

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- Let us evaluate the performance of restrictions on the FEV vis-a-vis standard sign restrictions.
- Fix the DGP at posterior means in Smets & Wouters (07, *AER*).
- Run 1000 replications.
- For each replication, generate $T^* = 200$ and estimate
  1. Model 1 (Sign Restrictions): $IR^0_{\Delta ci} \leq 0$, $IR^0_{\Delta li} \leq 0$, $IR^0_{\Delta wi} \leq 0$, $IR^0_{li} \leq 0$, $IR^0_{\Delta \pi i} \leq 0$, $IR^0_{ii} \geq 0$. $IR_{\Delta yi}$ is left unrestricted.
  2. Model 2 (Sign and Zero Restrictions): Model 1 and $IR^{\infty}_{\Delta yi} = 0$ (Blanchard & Quah, 89 *AER*).
  3. Model 3 (Restrictions on the FEV): Model 1 and $FEV^h_{ki} \leq FEV^h_{ii}$ at $h = 0, 1$, where $k = \{\Delta c, \Delta I, \Delta w, l, \Delta \pi\}$.

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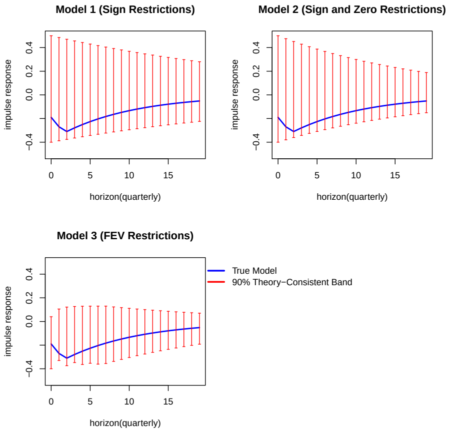

Empirical Application

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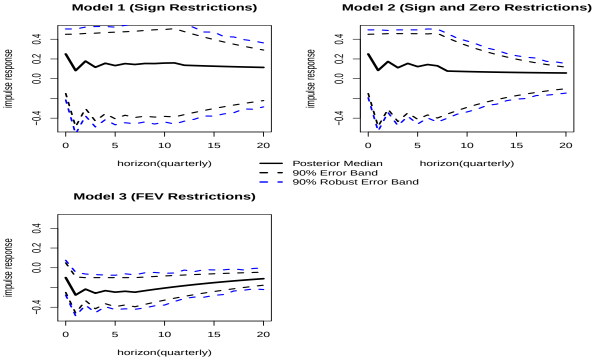
- Let us impose the robust restrictions on a 7-var model.
- Same variables as DGP.
- Dataset in Stock & Watson (2008), USA, 1959Q1-2008Q4.
- Same three models as before
  1. Model 1: sign restrictions.
  2. Model 2: sign and zero restrictions.
  3. Model 3: restrictions on the FEV.

Output Impulse Responses



Non-Emptiness and Convexity of the Identified Set

- Sufficient conditions to guarantee that restrictions on the FEV deliver a non-empty and convex identified set.
- Non-emptiness guarantees that restrictions are supported by the reduced-form.
- Convexity is useful. It greatly simplifies the computation/interpretation of the identified set.

Non-Emptiness and Convexity of the Identified Set (cont)

Lemma 1: Restrictions on the FEV only

If restrictions on the FEV restrict only the j-th structural shock and satisfy some rank conditions, $IS_\alpha(\phi)$ is non-empty and convex.

Lemma 2: FEV + Sign restrictions

Assume that sign restrictions restrict the j-th structural shock and are supported by ϕ. If restrictions on the FEV restrict only the j^*-th structural shock and satisfy some rank conditions, $IS_\alpha(\phi)$ is non-empty and convex.

Lemma 3: FEV + Sign and Zero restrictions

Assume that conditions in Lemma 2 hold. If zero restrictions satisfy some rank conditions, $IS_\alpha(\phi)$ is non-empty and convex.

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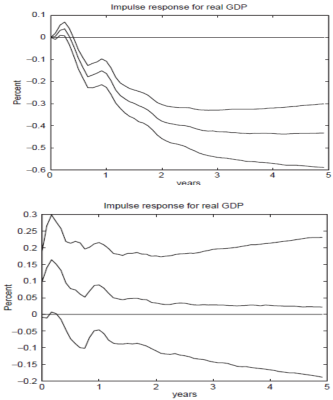
- Not in this talk: FEV restrictions to identify productivity shocks.

▸ Motivation


Impulse response for real GDP
Percent
years
0.1
0
−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
0
1
2
3
4
5
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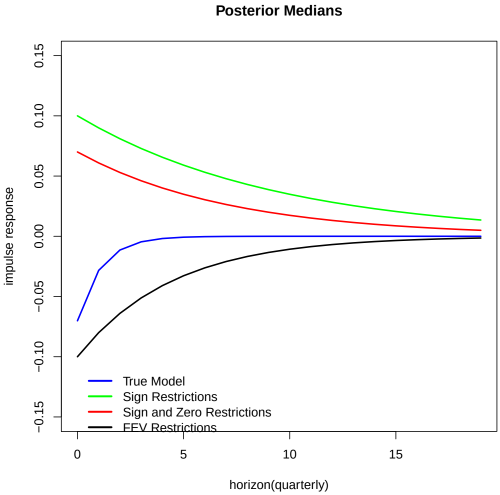
Sample Analysis (cont)



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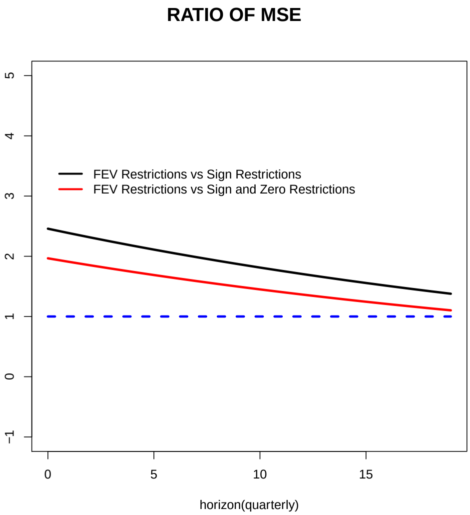

Estimation and Inference: Arias, Rubio-Ramirez & Waggoner, 18

Agnostic Prior: prior specification does not imply additional restrictions beyond those explicitly imposed.

1. Specify proper prior ($\mathcal{NIW}$) for the reduced-form $\tilde{\pi}_{\phi}$.
2. Run the Bayesian reduced-form to obtain posterior $\tilde{\pi}_{\phi|Y}$.
3. Draw $\phi \sim \tilde{\pi}_{\phi|Y}$.
4. Uhlig (05): $A = Q'\Sigma_{tr}^{-1} \Rightarrow$ Draw Q from flat distribution. Rotation Matrix
5. Retain $IR(\phi, Q)$ only if restrictions are satisfied. Otherwise, reject ϕ.

Let us go back to the Illustrative Example

The Current Practice Model M^1 (set-identified): $\frac{IR^0(y_{2t},\epsilon_{2t})^2}{var(y_{2t})} \geq \frac{IR^0(y_{1t},\epsilon_{2t})^2}{var(y_{1t})}$.

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Prior for $\phi \Rightarrow$ posterior for $\phi \not\Rightarrow$ posterior for α.

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- To have one posterior for α, we have to specify a conditional prior for α given ϕ!

$$\pi_{\alpha|Y} = \int_{\Phi} \pi_{\alpha|\phi} d\pi_{\phi|Y} \quad (1)$$

- Traditional approach: $\pi_{\alpha|\phi} \sim$ flat.
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- Baumeister & Hamilton (15, *Ecta*): flat specification on $\alpha|\phi$ can lead to unintentional informative inference and does not vanish even asymptotically.
- Interpret the "lack of knowledge within $IS_{\alpha}(\phi)$" as "the lack of ability to specify a credible prior $\pi_{\alpha|\phi}$".

Giacomini & Kitagawa (15) solution

- Specify a unique π_ϕ and accept any $\{\pi_{\alpha|\phi} : \phi \in \Phi\}$ that assigns prob one on $IS_\alpha(\phi)$.
- The set of priors: $\Pi_\alpha = \left\{\pi_\alpha = \int_\Phi \pi_{\alpha|\phi} d\pi_\phi : \pi_{\alpha|\phi}(IS_\alpha(\phi)) = 1, \forall\phi\right\}$
- Apply the Bayes rule prior by prior $\Rightarrow$ the set of posteriors:

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$$\left[\inf_{\pi_{\alpha|Y}\in\Pi_{\alpha|Y}} E_{\alpha|Y}(\alpha), \sup_{\pi_{\alpha|Y}\in\Pi_{\alpha|Y}} E_{\alpha|Y}(\alpha)\right] = \left[E_{\phi|Y}(l(\phi)), E_{\phi|Y}(u(\phi))\right]$$

consistent estimator of $IS_\alpha(\phi_{true})$, where $[l(\phi), u(\phi)]$ is the convex hull of $IS_\alpha(\phi)$.

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  consistent estimator of $IS_\alpha(\phi_{true})$, where $[l(\phi), u(\phi)]$ is the convex hull of $IS_\alpha(\phi)$.
- Application of linear programming provides "posterior mean bounds" and an associated credible region.

Posterior Median in Set-Identification

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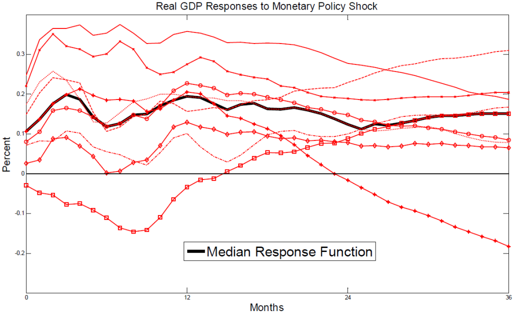
- Alternative measures:
    - Penalty function in Fry & Pagan (11, *JEL*).
    - Mode approach in Inoue & Kilian (13, *JoE*).

For example . . .

Source: Inoue & Kilian (13, *JoE*).

Figure 1: Randomly Selected Response Functions from a Sign-Identified VAR Model



NOTES: Based on nine randomly selected responses from the posterior of the model used as an empirical example in section 4.2. The median response function is constructed from the pointwise posterior medians. It coincides with responses from six different admissible structural models depending on the horizon.

DSGE and VAR

- Some references: Fernandez-Villaverde, Rubio-Ramirez, Sargent, & Watson (07, *AER*), Ravenna (07, *JME*), Fernandez-Villaverde, Rubio-Ramirez & Sargent (05).

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- Some references: Fernandez-Villaverde, Rubio-Ramirez, Sargent, & Watson (07, *AER*), Ravenna (07, *JME*), Fernandez-Villaverde, Rubio-Ramirez & Sargent (05).
- y includes all endogenous state variables of the DSGE model. Under an invertibility condition, y then possesses a finite order VAR representation.
- Otherwise, Under invertibility and stability conditions provided in Fernandez-Villaverde, Rubio-Ramirez, & Sargent (05), y then follows a VAR representation of infinite or finite order. Ravenna (07, *JME*) identifies conditions for the existence of a finite order VAR.