

Inference in Games without Nash Equilibrium: An Application to Restaurants Competition in Opening Hours

Erhao Xie*

January 16, 2018

Abstract

This paper relaxes the Bayesian Nash Equilibrium (BNE) assumption commonly imposed in empirical discrete choice games with incomplete information. Instead of assuming that players have rational expectations, my model treats a player's belief about the behavior of other players as an unrestricted unknown function. I study the joint identification of belief and payoff functions. I show that in games where one player has more actions than the other player, the payoff function is partially identified with neither equilibrium restrictions nor the usual exclusion restrictions. Furthermore, if the cardinality of players' action sets varies across games, then the payoff and belief functions are point identified up to scale normalizations, and the restriction of equilibrium beliefs is testable. For games without asymmetric action sets, I obtain very similar identification results without imposing restrictions on beliefs, as long as the payoff function satisfies a condition of multiplicative separability. I apply this model and its identification results to study store hours competition between McDonald's and KFC in China. In relatively rich markets, I reject the null hypothesis that KFC has rational beliefs. Furthermore, the estimation results of the payoff functions indicate that the store hours decision is a type of vertical differentiation. By operating through the night, a firm not only attracts night-time consumers but also can steal competitors' day-time customers. This result has implications on the optimal regulation of stores' opening hours.

Keywords: Empirical Games, Biased Beliefs, Identification, Store Hours, Incomplete Information

JEL Codes: C57, L13, L85

*E-mail: erhao.xie@mail.utoronto.ca. I am indebted to my supervisor Victor Aguirregabiria for his invaluable supervision and continuous encouragement. I am also grateful to my committee members, Yao Luo and Ryan Webb, for their generous support and helpful comments. I thank Ismael Mourifié, Eduardo Souza-Rodrigues, Yuanyuan Wan, Ping Xiao, and seminar participants at the University of Toronto, Tsinghua University, 2016 DWAE, 2017 CEA, Young Economist Symposium (Yale), 2017 EARIE, and 2017 JEI for insightful and constructive comments. Finally, I thank Qinggang Meng, a general manager at McDonald's, for providing detailed industry insights. All errors are my own.

1 Introduction

Over the last decade, game theoretic models with incomplete information have been actively applied to study oligopolistic competition and individuals' social interactive behaviors.¹ In this stream of literature, researchers commonly assume that players' observed choices are consistent with Bayesian Nash Equilibrium (BNE). Under this powerful solution concept, researchers estimate players' utility/payoff functions and predict their behaviors in counterfactual environments.

Despite its power and usefulness in applied empirical work, BNE places a strong restriction on players' expectations such that each player has equilibrium/unbiased beliefs about other players' behaviors (i.e. a player's beliefs are other players' *actual* choice probabilities given the available information). In reality, economic agents could have limited ability to process information and predict other players' strategies. In addition, many empirical games have multiple equilibria, which further complicates the construction of unbiased belief. In these games, a player could be uncertain about which equilibrium strategy is chosen by other players.² Furthermore, market conditions and government policies often vary dramatically. This poses difficulties in learning other players' behaviors through past experience. Finally, recent empirical work has shown the failure of Nash Equilibrium in different types of games using both field and experimental data. A partial list includes Goeree and Holt (2001), Goldfarb and Xiao (2011), Asker et al. (2016), Doraszelski, Lewis and Pakes (2016), Kashaev (2016), Aguirregabiria and Magesan (2017), Aguirregabiria and Xie (2017) and Jeon (2017).

If players have biased beliefs in games, falsely imposing the equilibrium condition would bias the estimates of payoff functions and counterfactual predictions. To address this issue, this paper relaxes the rational belief assumption. Specifically, each player maximizes her expected utility given her subjective belief, which can be any probability distribution over the other player's action set. This nests BNE as a special case when each player forms an equilibrium belief. It also allows players' behaviors to be off-

¹A partial list of work includes firms entry studied by Gowrisankaran and Krainer (2011) and Aradillas-Lopez and Gandhi (2016); product differentiation studied by Seim (2006), Augereau, Greenstein and Rysman (2006), and Sweeting (2009); social interactive effect analyzed by Brock and Durlauf (2001, 2007) and Bajari et al. (2010); network structure by Vitorino (2012) and methodological contributions by Aradillas-Lopez (2010, 2012).

²This type of strategic uncertainty is defined by Van Huyk, Battalio, and Beil (1990) and Crawford and Haller (1990), and studied by Morris and Shin (2002, 2004), and Heinemann, Nagel and Ockenfels (2009), among others. As argued by Besanko et al. (2010), it could be very common in oligopolistic competition. Moreover, the existence of multiple equilibria can facilitate the identification of players' payoffs as shown by Sweeting (2009), De Paula and Tang (2012), and Aradillas-Lopez and Gandhi (2016); also see Aguirregabiria and Mira (2017) and Xiao (2015).

equilibrium due to biased beliefs. In the estimation of the model, both utility/payoff and belief are treated as unknown unrestricted functions, in contrast to the standard approach in the literature of imposing equilibrium restrictions.

The principle of revealed preference implies that, under general conditions, researchers can infer the expected utility function using data on players' choices. However, since expected utility is a composite function of the utility and belief functions, it is challenging to separately identify these two functions. In this paper, I show that if the cardinality of the action set (i.e. the number of possible choices of a player) varies across players, the payoff function is partially identified without imposing equilibrium restrictions. I then characterize the identified set. To the best of my knowledge, this is the first result on nonparametric identification of players' payoff functions that imposes neither equilibrium restrictions nor exclusion restrictions. Furthermore, there are many examples and applications of games where players have a different number of possible choices. For instance, in models of competition in multi-product or multi-store markets (e.g. in price, quantity or quality choice), firms with different numbers of products or stores face a different number of actions when choice is at the product or store level.

Suppose further that the cardinality of each player's action set has variation across games (e.g. a firm has a different number of stores in different markets). Then, the base return (e.g. monopoly profit in an entry game) is identified, and each player's interactive effect is identified up to her belief at *only one* realization of the state variable. It consequently identifies the sign and a lower bound of the interactive effect. These results shed light on the nature of the game (e.g. strategic substitutes and strategic complements). Furthermore, researchers can infer how a player adjusts her beliefs across different games. It naturally yields a testable restriction of unbiased beliefs and provides information on how players form expectations in games. The proofs of all identification results are constructive and imply naturally two-step estimators.

In many empirical applications, every player has the same number of actions and the cardinality of action set remains constant across observations. In these applications, I show that when the payoff function satisfies a condition of multiplicative separability, researchers can establish a similar asymmetric feature as in the games with asymmetric action sets. As a result, the payoff function is also partially identified. Additionally, I show that the standard exclusion restrictions can further shrink the identified

set of payoff functions. Therefore, the identification results proposed in this paper apply to a broad class of games.

I apply these identification and estimation results to study the operating-hour decisions of Kentucky Fried Chicken (KFC) and McDonald's (McD) in China. Many countries have strong regulations on stores' opening hours in the retail industry and there is an ongoing debate on the deregulation of such restrictions, especially in Europe. The strategic nature of opening hours is a key element in the evaluation of these deregulation policies. However, there are mixed views on the strategic role of opening hours in the theoretical literature. Inderst and Irmen (2005), Shy and Stenbacka (2006, 2008), and Wenzel (2010, 2011) treat store hours as a dimension of horizontal differentiation since consumers may prefer to shop or consume at different times of the day. Under this view, firms' operating-hour decisions would be strategic substitutes. On the other hand, Western-style fast food industry experts indicate that 24-hour stores provide the staff with sufficient time to prepare for high-volume demand at breakfast. Such incentive could be strong, especially in busy markets. Moreover, longer operation times can enhance consumers' perception of the company and as a method of establishing brand value. Under this view, a store's operating-hour decision is more like a dimension of vertical differentiation (i.e. quality choice), as studied in Ferris (1990) and Klemperer and Padilla (1997). In this way, stores' operating hours can be strategic complements instead of substitutes. Unfortunately, there are few empirical studies on this question, partly due to a lack of appropriate data on store hours, and to limited variation in store-hour decisions caused by strict regulations.³ As a country without regulations on opening hours, China provides an ideal empirical setting to evaluate the strategic nature of store hours.

Although KFC and McD have competed in China for almost 30 years and are familiar with each other, drastic changes in the economic environment and market conditions could preclude perfect prediction of competitor's behaviors. At the macroeconomic level, China maintains its economic growth at a miracle rate. At the industrial level, a survey conducted by China Market Research Group suggests that Chinese consumers have started to lose their trust in KFC and McD after their peak in 2010. The decline has accelerated since 2012, when KFC encountered a series of food safety scandals.⁴ In addition, despite

³Tanguay, Vallge and Lanoie (1995) focus on how store hours affect firms' pricing decision. Skuterud (2005) studies the impact of deregulation policy on employment. In contrast, my paper focuses on competition in opening hours. To the best of my knowledge, there exists only one other paper (Kügler and Weiss, 2016) who investigate the same empirical question as this paper; they find the strategic effect of opening hours to be insignificant in the Austrian gasoline market.

⁴See news "Chinese consumers are losing trust in McDonald's and KFC", retrieved from

the rapid growth of China's catering industry, Euromonitor International reports that KFC's sales have dropped since 2012, while McD's sales remain constant.⁵ These dramatic changes in economic conditions and consumer preferences represent structural transformations and are potential sources of firms' biased beliefs. Falsely imposing the equilibrium condition could bias both the magnitude and the sign of the interactive effect. Consequently, the strategic nature of store hours can be incorrectly quantified. Finally, detecting biased beliefs and measuring how players adjust their expectations are important in and of themselves, especially in rapidly changing economic environments.

This paper models operating hours competition as an incomplete information game such that each chain simultaneously decides how many of its existing outlets to remain open at night. Different numbers of outlets owned by two chains in various markets naturally construct the asymmetry and variation of action sets required for identification of both payoff and belief functions. Without imposing the BNE assumption, I find that KFC tends to extend its store hours when it expects McD to do so. This is consistent with the interpretation of vertical differentiation. Moreover, the estimation results reject the null hypothesis that KFC has rational beliefs. Specifically, KFC under-evaluates the impact of McD's supply shipment costs on its operation-hour decision in markets where consumers have higher income.

Recently, Aradillas-Lopez and Tamer (2008), Goldfarb and Xiao (2011), Fershtman and Pakes (2012), Kline and Tamer (2012), Uetake and Watanabe (2013), An (2010), Gillen (2010), Asker et al. (2017), Doraszelski, Lewis and Pakes (2017), and Kashaev (2016), among others, study players' non-equilibrium behaviors in games. In this literature, my paper is closely related to Aguirregabiria and Magesan (2017) and Aguirregabiria and Xie (2017), who show that the equilibrium assumption is testable with an exclusion restriction that only affects a single player's payoffs. However, to identify the payoff, they need to assume players form equilibrium beliefs at several realizations of the exclusion restriction. In contrast, this paper exploits the asymmetry between the cardinality of players' action sets and potential variation in cardinality across observations. Through this, the identification of both payoff and belief functions can be established in absence of exclusion restrictions.

The rest of this paper is organized as follows. Section 2 describes the model. Section 3 presents the identification results. The empirical application is shown in section 4, and section 5 discusses further

<http://www.businessinsider.com/r-food-scares-strip-mcdonalds-kfc-of-treat-status-in-china-2015-8>

⁵See news "China Starts to Lose Its Taste for McDonald's and KFC", retrieved from <https://www.agweb.com/article/china-starts-to-loses-its-taste-for-mcdonalds-and-kfc-blmg/>

extensions. I conclude in Section 6. Some generalizations of identification results are left to the appendix.

2 Model

Consider a two-player static game. Players are indexed by $i \in \{1, 2\}$ and $-i$ represents the other player. Appendix D shows how to generalize the identification results to a game with multiple players and section 5 discusses an extension to dynamic games. Each player i simultaneously chooses an action denoted by a_i from her action set $A_i = \{0, 1, \dots, J_i\}$. Players are allowed to have different action sets and different numbers of actions. The Cartesian product $A = A_1 \times A_2$ represents the space of action profiles in this game. Let $\mathbf{a} = (a_1, a_2) \in A$ be an action profile or realized outcome of this game. Player i 's payoff for the action profile \mathbf{a} is

$$\Pi_i(\mathbf{x}, \boldsymbol{\varepsilon}_i, \mathbf{a}) = \tilde{\Pi}_i(\mathbf{x}, a_i, a_{-i}) + \varepsilon_i(a_i),$$

where $\mathbf{x} \in \mathbb{R}^{L_x}$ denotes a vector of state variables that affect players' payoffs and is public information. The term $\varepsilon_i(a_i)$ represents a variable that affects player i 's payoff of action a_i . It is private information observed only by player i and unobserved by player $-i$. Therefore, it is a game of incomplete information. The payoffs $\tilde{\Pi}_i(\mathbf{x}, a_i, a_{-i})$ are non-parametrically specified.

Define $\pi_i(\mathbf{x}, a_i) = \tilde{\Pi}_i(\mathbf{x}, a_i, a_{-i} = 0)$ and $\delta_i(\mathbf{x}, a_i, a_{-i}) = \tilde{\Pi}_i(\mathbf{x}, a_i, a_{-i}) - \tilde{\Pi}_i(\mathbf{x}, a_i, a_{-i} = 0)$. By construction, $\delta_i(\mathbf{x}, a_i, a_{-i} = 0) = 0$. Without loss of generality, the payoff function can be written as

$$\Pi_i(\mathbf{x}, \boldsymbol{\varepsilon}_i, \mathbf{a}) = \pi_i(\mathbf{x}, a_i) + \delta_i(\mathbf{x}, a_i, a_{-i}) \cdot \mathbb{1}(a_{-i} \neq 0) + \varepsilon_i(a_i). \quad (1)$$

Throughout the paper, I consider the payoff function specified by equation (1) for expositional purpose. Note that it is non-parametrically specified. Following the language of De Paula and Tang (2012), $\pi_i(\cdot)$ is referred to as the base return and it represents player i 's payoff when the other player chooses action 0. Term $\delta_i(\cdot)$ is referred to as the interactive effect/payoff and it measures how player i 's payoff is affected by player $-i$'s behavior.

Assumption 1 states an independence restriction imposed on player's private information.

Assumption 1. (a) for each $i = 1, 2$, $\boldsymbol{\varepsilon}_i = (\varepsilon_i(0), \varepsilon_i(1), \dots, \varepsilon_i(J_i))'$ follows a CDF $G_i(\cdot)$ that is absolutely

continuous with respect to Lebesgue measure in \mathbb{R}^{J_i+1} .

(b) ε_i is independent across players and independent of public information \mathbf{x} .

It is important to note that a relaxation of assumption 1 can also achieve identification. For more details, the model and identification results are generalizable to the case that $G_i(\cdot)$ depends on a vector of finite-dimensional unknown parameters. This extended model and its identification results are presented in Appendix B.⁶

Assumption 2. (a) Each player's belief about the other player's behavior depends only on public information \mathbf{x} .

(b) Each player chooses an action that maximizes her expected payoff given her belief.

Assumption 2 (a) assumes that player i 's belief about the other player's behavior does not depend on her private information. Given assumption 1 (b) such that players have independent private information, ε_i has no predictive power about player $-i$'s payoff and behavior; consequently, ε_i does not affect player i 's belief. Specifically, define $b_i^j(\mathbf{x})$ as player i 's belief about the probability that player $-i$ will choose action j . Moreover, let $\mathbf{b}_i(\mathbf{x}) = (b_i^0(\mathbf{x}), \dots, b_i^{J-i}(\mathbf{x}))'$ be a vector of belief functions that are non-parametrically specified. The only assumption I impose is that the belief functions must be valid probability distributions over the other player's action set (i.e. $0 \leq b_i^j(\mathbf{x}) \leq 1 \forall j$ and $\sum_{j=0}^{J-i} b_i^j(\mathbf{x}) = 1$).

Given payoff and belief functions, player i 's expected payoff of action a_i is

$$E[\Pi_i(\mathbf{x}, \varepsilon_i, a_i)] = \pi_i(\mathbf{x}, a_i) + \sum_{j=1}^{J-i} \delta_i(\mathbf{x}, a_i, a_{-i} = j) \cdot b_i^j(\mathbf{x}) + \varepsilon_i(a_i). \quad (2)$$

Assumption 2 (b) states that player i chooses an action that maximizes this expected payoff. Define $a_i^*(\mathbf{x}, \varepsilon_i)$ as player i 's strategy function, which can be characterized as

$$a_i^*(\mathbf{x}, \varepsilon_i) = \operatorname{argmax}_{a_i \in A_i} \left\{ \pi_i(\mathbf{x}, a_i) + \sum_{j=1}^{J-i} \delta_i(\mathbf{x}, a_i, a_{-i} = j) \cdot b_i^j(\mathbf{x}) + \varepsilon_i(a_i) \right\}. \quad (3)$$

As player $-i$ does not observe ε_i , her rational expectation of player i 's behavior is player i 's best response probability function or *conditional choice probability* (CCP). Let $\mathbf{p}_i(\mathbf{x}) = (p_i^0(\mathbf{x}), \dots, p_i^{J-i}(\mathbf{x}))'$

⁶Aradillas-Lopez (2010), Wan and Xu (2014), and Xu (2014) extend the independence assumption of ε_i and allow it to be correlated across players even conditional on observed state variables. Despite their power, their applicability is very limited in my framework. Note that player i 's belief will depend on ε_i if it is correlated with ε_{-i} . From an econometrician's perspective, the belief is an unknown function that depends on an unknown variable ε_i . With the BNE assumption, we can solve for this belief using a fixed-point algorithm; however, it will not work if BNE fails as is allowed in my framework.

denote a vector of player i 's CCPs where $p_i^j(\mathbf{x})$ is her choice probability of action j conditional on state variable \mathbf{x} . Given the best response function $a_i^*(\mathbf{x}, \boldsymbol{\varepsilon}_i)$ defined above, the conditional choice probability takes the following form

$$p_i^j(\mathbf{x}) = \int \mathbb{1}\{a_i^*(\mathbf{x}, \boldsymbol{\varepsilon}_i) = j\} dG_i(\boldsymbol{\varepsilon}_i). \quad (4)$$

For instance, if $\varepsilon_i(a_i)$ is type 1 extreme value distributed and independent across actions, the conditional choice probability $p_i^k(\mathbf{x})$ is

$$p_i^k(\mathbf{x}) = \frac{\exp\left\{\pi_i(\mathbf{x}, a_i = k) + \sum_{j=1}^{J-i} \delta_i(\mathbf{x}, a_i = k, a_{-i} = j) \cdot b_i^j(\mathbf{x})\right\}}{\sum_{l=0}^{J_i} \exp\left\{\pi_i(\mathbf{x}, a_i = l) + \sum_{j=1}^{J-i} \delta_i(\mathbf{x}, a_i = l, a_{-i} = j) \cdot b_i^j(\mathbf{x})\right\}}.$$

The conditional choice probability defined by equation (4) only assumes that a player maximizes expected payoff given her belief. Such belief is allowed to be any probability distribution over the other player's action set. In contrast, Bayesian Nash Equilibrium restricts players to be perfectly rational in the sense that a player's belief is the other player's true choice probability conditional on the available information. My framework therefore nests BNE as a special case and is summarized by definition 1.

Definition 1. Players' behaviors are consistent with Bayesian Nash Equilibrium if each player's belief is the other player's actual conditional choice probability, i.e. $b_i^j(\mathbf{x}) = p_{-i}^j(\mathbf{x}) \forall 0 \leq j \leq J_i$ and $i = 1, 2$.

Remark. Some identification results in this paper exploit variation in players' action sets. To understand those results, it is important to keep in mind that player i 's choice probabilities and beliefs depend on J_1 and J_2 as they affect the dimensions of vectors \mathbf{p}_i and \mathbf{b}_i . A notation for choice probabilities and beliefs that emphasizes this dependence is $\mathbf{p}_{i, J_i, J_{-i}}(\mathbf{x})$ and $\mathbf{b}_{i, J_i, J_{-i}}(\mathbf{x})$. For the sake of notation simplicity, I maintain the expression of $\mathbf{p}_i(\mathbf{x})$ and $\mathbf{b}_i(\mathbf{x})$ and only include J_i and J_{-i} as explicit arguments when necessary.

Example 1. *Operating hours game in the empirical application in section 4.* There are two fast food chains, KFC and McD, competing through decisions on business hours. Suppose chain i owns J_i number of outlets in a given market, and chooses some number of its existing stores to be nonstop service stores. Two chains could own different numbers of outlets and consequently have heterogeneous action sets. Term $\pi_i(\mathbf{x}, a_i = k)$ represents chain i 's profit of opening k nonstop service stores if the other chain closes

all stores at night (e.g. monopoly profit). Vector \mathbf{x} represents variables that affect each chain's profit such as income per-capita and population at the local market. When chain $-i$ decides to operate j stores at night, it will have some impact on chain i 's night profit and this is captured by $\delta_i(\mathbf{x}, a_i = k, a_{-i} = j)$. Finally, $\varepsilon_i(a_i)$ represents chain i 's private information of its own profitability such as managerial skill and staff's coordination efficiency.

3 Identification

In this section, I first present conditions on the data generating process. In subsection 3.2, I show how to exploit the asymmetry and variation of action sets to identify each player's payoff and belief functions, without requiring standard exclusion restrictions. Subsection 3.3 considers another type of games in which players have the same action set that remains constant across observations but the payoff function satisfies a multiplicative separability condition. I show that a similar asymmetric feature can be constructed in this type of games such that the payoff function is partially identified. Of course, the introduction of standard exclusion restrictions provides additional identification power. Therefore, my identification results hold in a broad class of games. All proofs are left to the appendix.

3.1 Conditions on the Data Generating Process

Suppose researchers have access to a data set about the same two players that play M independent games (e.g. one game in each of M isolated markets). In each game/observation, which is indexed by m , both players and the econometrician observe realizations of the state variables \mathbf{x}_m . Moreover, each player i observes her own payoff shock $\varepsilon_{i,m}$. Researchers cannot observe player i 's private information but know its probability distribution $G_i(\cdot)$. Given the observed state variables, each player forms a belief and chooses her optimal action.

Assumption 3. *A player forms the same beliefs for any two observations with the same public information. That is, for $m \neq m'$ but $\mathbf{x}_m = \mathbf{x}_{m'} = \mathbf{x}$, we have $\mathbf{b}_{i,m}(\mathbf{x}) = \mathbf{b}_{i,m'}(\mathbf{x}) = \mathbf{b}_i(\mathbf{x})$.*

Assumption 3 states that each player has a unique belief conditional on public information. In models that impose equilibrium restrictions, an analogous assumption would be assuming that each player em-

employs the same equilibrium strategy when multiple equilibria exist. Therefore, even though the model allows for multiple equilibria, there is a unique equilibrium observed in the data. Such an assumption is commonly used in the literature.⁷ In section 5, I discuss how to relax this assumption of belief uniqueness.

The asymptotic consistency comes from M going to infinity. In this situation, $\hat{\mathbf{p}}_i(\mathbf{x}_m)$ is consistently estimated.⁸ Consequently, for identification results, $\mathbf{p}_i(\mathbf{x})$ is assumed to be known by researchers for every realization of \mathbf{x} . Researcher's objective is to identify player i 's base return $\pi_i(\mathbf{x}, a_i)$, interactive effect $\delta_i(\mathbf{x}, a_i, a_{-i})$ and belief function $\mathbf{b}_i(\mathbf{x})$ using the data described above.

It is known in the discrete choice literature that only differences in payoffs are identified. Therefore, a normalization is required to achieve identification, as summarized in assumption 4.

Assumption 4. *For player $i = 1, 2$, the payoff for action 0 is normalized to zero. That is, $\pi_i(\mathbf{x}, a_i = 0) = 0$ and $\delta_i(\mathbf{x}, a_i = 0, a_{-i}) = 0 \forall \mathbf{x}, a_{-i}$.*

Even though normalizing the payoff of one action to zero is an innocuous assumption in single-agent discrete choice models, it imposes some restrictions on a player's payoff when players interact with each other. Specifically, this normalization restricts player i 's payoff of action 0 to be unaffected by the other player's action. Such normalization is plausible if action 0 is modeled as an outside option. For instance, it represents firm i does not enter a particular market in a standard entry game. Consequently, its profit is independent of other firm's behaviors in such market.⁹

I investigate identification *conditional* on any $\mathbf{x} \in \mathbb{R}^{L_x}$, though is suppressed for notation simplicity throughout this section. Following Hotz and Miller (1993), assumption 1 and 4 imply that there is a one-to-one mapping $F_i(\cdot) : \mathbb{R}^{J_i+1} \rightarrow \mathbb{R}^{J_i+1}$ from player i 's conditional choice probability to her expected payoff. Specifically, let $F_i(\cdot)$ be the inverse of the integral function defined by equation (4), we then have

⁷A partial list includes Aguirregabiria and Mira (2007), Bajari, Benkard and Levin (2007), Pakes, Ostrovsky and Berry (2007), Pesendorfer and Schmidt-Dengler (2008), Bajari et al., (2010), and Aradillas-Lopez (2012), etc.

⁸If \mathbf{x} were discrete variables with finite support, or continuous variables with a smooth choice probability function; then the choice probability can be consistently estimated by the standard kernel estimator. If \mathbf{x} were continuous variables and $\mathbf{p}_i(\mathbf{x})$ had some points of discontinuity that are not known by researchers ex-ante, then some variants of the standard kernel method developed by Müller (1992) and Delgado and Hidalgo (2000) can still establish the consistency and asymptotic normality for the choice probability estimator.

⁹In situation that it is implausible to assume player i 's payoff of $a_i = 0$ is invariant to the other player's choices, all the identification results still hold if researcher's interest is the difference of payoff rather than payoff. For instance, the interested parameters are $\tilde{\pi}_i(\mathbf{x}, a_i) = \pi_i(\mathbf{x}, a_i) - \pi_i(\mathbf{x}, a_i = 0)$ and $\tilde{\delta}_i(\mathbf{x}, a_i, a_{-i}) = \delta_i(\mathbf{x}, a_i, a_{-i}) - \delta(\mathbf{x}, a_i = 0, a_{-i})$.

$$\pi_i(a_i = k) + \sum_{j=1}^{J_{-i}} \delta_i(a_i = k, a_{-i} = j) \cdot b_i^j = F_i^k(\mathbf{p}_i) \quad \forall 0 \leq k \leq J_i, \quad (5)$$

where $F_i^k(\cdot)$ denotes the k^{th} element of the inverse function. Note that $F_i^0(\mathbf{p}_i) = 0$ based on the normalization stated in assumption 4. Given that $G_i(\cdot)$ is known by researchers, $F_i(\cdot)$ is also known. For instance, if $\varepsilon_i(a_i)$ is independently type 1 extreme value distributed, we have the mapping $F_i^k(\mathbf{p}_i) = \log\left(\frac{p_i^k}{p_i^0}\right)$.

In empirical games with incomplete information, researchers commonly assume that players play an equilibrium strategy and the conditions described in definition 1 hold. Consequently, we can replace b_i^j with its counterpart p_{-i}^j in equation (5). As p_{-i}^j can be consistently estimated, this equilibrium assumption provides identification power.¹⁰

For the rest of this paper, I drop the BNE conditions as described in definition 1. In the econometric model, each player is an expected utility maximizer and her belief is allowed to be any probability distribution over the other player's action set. Under this framework, I investigate the identification of payoffs and beliefs.

3.2 Identification with Asymmetric Action Spaces

This subsection exploits the asymmetry and variation in players' action sets to identify each player's payoffs and beliefs. Let $\mathcal{J} = \{0, 1, \dots, \bar{J}\}$ denote the support of J_i for each player i . In example 1, each chain could have a different number of actions that varies across markets.

Intuitively, observational data reveal player i 's expected payoff of every action since p_{-i}^j is consistently estimated. These J_i restrictions depend only on J_{-i} unknown belief parameters (i.e. the number of player $-i$'s actions minus one). When $J_i > J_{-i}$, I show that it is possible to obtain a transformation of equation (5) such that beliefs are differenced out and what remains is a relationship between unknown payoffs and known choice probabilities. To the best of my knowledge, this is the first result on non-parametric identification of payoffs in games that imposes neither equilibrium restrictions nor exclusion restrictions. Furthermore, if J_{-i} varies across the M markets and takes on a value of zero with positive probability, then we would have a player i 's single agent problem and her base return $\pi_i(\cdot)$ would be point identified. Finally, comparing between markets with $J_{-i} = 0$ and ones where $J_{-i} > 0$ provides information on player

¹⁰For estimation techniques, see Su (2014) for an excellent discussion and comparison of different methods.

i 's interactive payoffs and beliefs.

Proposition 1. (a) Suppose assumptions 1 to 4 hold and the data contain observations with $J_i > J_{-i} = 1$, then for any two choice alternatives j and k , the identified set of player i 's payoff parameters $\pi_i(\cdot)$ and $\delta_i(\cdot)$ is given by the set of values that satisfy the following restriction

$$\frac{F_i^j(\mathbf{p}_i) - \pi_i(a_i = j)}{F_i^k(\mathbf{p}_i) - \pi_i(a_i = k)} = \frac{\delta_i(a_i = j, a_{-i} = 1)}{\delta_i(a_i = k, a_{-i} = 1)}.$$

(b) Further, suppose that the data also contain observations with $J_{-i} = 0$, then player i 's base return $\pi_i(a_i = k)$ is point identified as $F_i^k(\mathbf{p}_{i, J_{-i}=0}) \forall k$. Furthermore, the identified set of player i 's interactive effect and belief is given by the set of values that satisfy the following restriction

$$\delta_i(a_i = k, a_{-i} = 1)b_i^1 = F_i^k(\mathbf{p}_{i, J_{-i}=1}) - F_i^k(\mathbf{p}_{i, J_{-i}=0}) \forall 0 \leq k \leq J_i.$$

The base return $\pi_i(a_i)$ is point identified. In example 1, this term represents chain i 's monopoly profit in the night market, since day-time profits are normalized to zero. This paper also characterizes the identified set of player i 's interactive effect and beliefs. In this subsection, I focus on the case where J_{-i} takes on values of zero or one. The results for $J_{-i} > 1$ are very similar and presented in appendix C. Since $0 \leq b_i^1 \leq 1$, the sign and lower bound of player i 's interactive payoff are identified. Furthermore, researchers can infer how player i adjusts her beliefs across different observations; it naturally implies a testable restriction of unbiased belief.

Proposition 2. Under the conditions met in proposition 1 such that $\delta_i(a_i, a_{-i} = 1)b_i^1$ is point identified, if such term is non-zero, it follows that

(a) the interactive effect ratio $\frac{\delta_i(a_i=j, a_{-i}=1)}{\delta_i(a_i=k, a_{-i}=1)}$ is point identified for every two actions j and k .

(b) the sign of $\delta_i(a_i, a_{-i} = 1)$ and lower bound of $|\delta_i(a_i, a_{-i} = 1)|$ are identified for every a_i .

(c) suppose the data contain observations with $J'_i, J''_i \geq 1$, then $\frac{b_{i, J'_i}^1}{b_{i, J''_i}^1}$ is identified and naturally implies

a testable restriction of unbiased belief: $\frac{b_{i, J'_i}^1}{b_{i, J''_i}^1} = \frac{p_{-i, J'_i}^1}{p_{-i, J''_i}^1}$.

The interactive effect ratio $\frac{\delta_i(a_i=j, a_{-i}=1)}{\delta_i(a_i=k, a_{-i}=1)}$ sheds light on a player's choice incentive. It concludes which of player i 's action is more sensitive to the other player's behavior. In competitive games, a player

has the incentive to choose an action that is insensitive to the other player's action. For instance, in an entry/expansion game, an incentive for a firm to open an additional store is to alleviate the negative impact of other firms; such incentive can be measured by the interactive effect ratio, as it represents how the negative impact is attenuated when a firm opens one additional store. Similarly, an incentive for cooperation is also quantified by the interactive effect ratio in coordination games. For instance, a player has an incentive to choose a sensitive action to exploit positive spillover effects. Finally, in the context of a product choice game, the interactive effect ratio provides information about which product of firm i is a close substitute for the other firm's product.

The sign and ratio of the interactive effect also determine the strategic nature of the game. Under conditions such that $\frac{\delta_i(a_i=j, a_{-i}=1)}{\delta_i(a_i=k, a_{-i}=1)} > 1$ for all $j > k$, players' actions are strategic substitutes if the sign of the interactive effect is negative and are strategic complements if it is positive.¹¹ Determining the strategic nature is one of the central questions in my empirical application; if firms competition in operating hours shows strategic complementarity, this would imply that this strategic choice is related to vertical product differentiation, and would have implications on the optimal (de)regulation of opening hours. Furthermore, inferring the sign of this interactive component is the main empirical question in many papers, such as Sweeting (2009) and De Paula and Tang (2012).

Suppose player i 's action space varies across observations; therefore, her optimal strategy may differ. In response, player $-i$ may also adjust her strategy accordingly. Proposition 2 (c) states that researchers can identify how player i adjusts her belief when her action space varies. When player i has rational expectations, such adjustment of belief should equal the actual adjustment of the other player's choice probability. This naturally constructs a testable restriction of unbiased belief.

Table 1 summarizes the identification results in this section and compares them with existing literature. The first row presents the result in Aguirregabiria and Magesan (2017), who show the identification power of the exclusion restriction and prove that a function of belief is identified. However, there is no identification results for payoff functions.¹² In contrast, this paper shows that the asymmetry between

¹¹To see this clearly, recall that $\delta_i(a_i = k, a_{-i} = 1) = \Pi_i(a_i = k, a_{-i} = 1) - \Pi_i(a_i = k, a_{-i} = 0)$, where $\Pi_i(a_i, a_{-i})$ represents player i 's payoff of outcome (a_i, a_{-i}) . Consequently, $\frac{\delta_i(a_i=j, a_{-i}=1)}{\delta_i(a_i=k, a_{-i}=1)} > 1 \forall j > k$ is equivalent to $|\Pi_i(a_i = k, a_{-i} = 1) - \Pi_i(a_i = k, a_{-i} = 0)| < |\Pi_i(a_i = j, a_{-i} = 1) - \Pi_i(a_i = j, a_{-i} = 0)|$, for all $j > k$. With $\delta_i(a_i, a_{-i} = 1) < 0$, it is precisely the condition of strategic substitutes since player i 's incentive to choose a higher action decreases as player $-i$ increases her action.

¹²To identify payoff, they need to assume that researchers know players' beliefs at several realizations of the exclusion restriction.

Table 1: Summary of Identification Results

Model Restrictions	Identified Set Payoff	$\frac{\delta_i(a_i=j, a_{-i}=1)}{\delta_i(a_i=k, a_{-i}=1)}$	$\pi_i(a_i, a_{-i})$	Sign, L.B. of $\delta_i(a_i, a_{-i})$	Identified Set Belief	Unbiased Belief Test
Exclusion Restriction z_i : Existing Literature					✓	✓
Asymmetric Action Sets	✓					
Variation in Action Sets	✓	✓	✓	✓	✓	✓

players' action sets provides additional identification power. Without an exclusion restriction, it partially identifies the payoff function. Additionally, variation of players' action sets across observations would allow researchers to almost point identify each player's payoff and belief. It naturally implies a testable restriction of unbiased belief.

The variation in cardinality of a player's action set is analogous to the empirical auction literature that exploits variation in the number of bidders to identify each bidder's payoff function. A partial list includes Paarsch (1992), Bajari and Hortacsu (2003, 2005), and Guerre, Perrigne and Vuong (2009). Similar to bidders endogenously participating in auctions, the variation in players' action sets could also be endogenous. The method proposed by Guerre, Perrigne and Vuong (2009) can be used to deal with such selection issue.

3.3 Identification with Multiplicative Separability in the Interactive Payoff

In many empirical applications, players have the same number of possible choices; moreover, the action space may remain constant across observations. In those cases, the identification results from previous subsections cannot be directly applied. However, in this subsection, I show that a similar asymmetric feature can be constructed through conventional restrictions on a player's payoff function. For instance, suppose the interactive effect $\delta_i(a_i, a_{-i})$ is multiplicative separable between a player's own action and her opponent's action; then each player's belief is summarized by a sufficient statistic that is interpreted as the player's subjective expectation. This result implies a reduction in the dimension of player i 's beliefs from J_{-i} to one (i.e. the dimension of the sufficient statistic). As a result, each player's identification problem

mimics the structure of the asymmetric game described in subsection 3.2 regardless of the number of actions available to the opponent. Therefore, the payoff function is partially identified. Moreover, when researchers can observe usual exclusion restrictions, we can sharpen the identified set and achieve similar results as when there is variation in players' action sets.

In this subsection, suppose $J_1 = J_2 = \bar{J} > 1$. Assumption 5 states a conventional payoff function such that the interactive effect is multiplicative separable between the two players' actions.

Assumption 5. $\delta_i(a_i, a_{-i}) = \delta_i(a_i, a_{-i} = 1) \cdot \eta_i(a_{-i})$ with $\eta_i(a_{-i} = 1) = 1$.

In the empirical application described by example 1, $\delta_i(a_i = j, a_{-i} = 1)$ measures the impact of player $-i$'s operating its first 24-hour store on player i 's profit of opening j nonstop service stores. Consequently, $\eta_i(a_{-i} = k)$ captures the proportional change of the interactive effect when player $-i$ opens k stores at night. Assumption 5 restricts this proportional rate of change to be constant across player i 's different actions.¹³ This restriction is commonly imposed in the estimation of games with multiple players and multiple actions. For instance, Aradillas-Lopez and Gandhi (2016) exploit the same restriction; they refer to $\eta_i(\cdot)$ as the strategic index and to $\delta_i(\cdot)$ as the overall scale of the strategic effect.¹⁴ Most current literature restricts $\eta_i(a_{-i})$ to be a particular functional form that only depends on the other player's action; some examples include $\eta_i(a_{-i}) = \log(1 + a_{-i})$, i.e. each additional store opened by player $-i$ affects player i 's profit at a diminishing rate as specified in Nishida (2014) or $\eta_i(a_{-i}) = a_{-i}$, i.e. each additional store opened by player $-i$ affects player i 's profit at a constant rate as specified in Augereau, Greenstein and Rysman (2006). In contrast, this paper does not impose any parametric restriction on the function $\eta_i(\cdot)$ and it is a non-parametric function of \mathbf{x} , which is suppressed in this section for notation simplicity. Under assumption 5, player i 's expected payoff for action a_i defined in equation (2) becomes

¹³For instance, $\frac{\delta_i(a_i=k, a_{-i}=j)}{\delta_i(a_i=k, a_{-i}=j')} = \frac{\delta_i(a_i=k', a_{-i}=j)}{\delta_i(a_i=k', a_{-i}=j')}$.

¹⁴Aradillas-Lopez and Gandhi (2016) allow ϵ_i to enter $\delta_i(a_i, a_{-i} = 1)$ in a non-linear fashion while it is additive separable in my framework. However, their interest is in the strategic index $\eta_i(a_{-i})$ and need to assume that $\delta_i(a_i, a_{-i} = 1)$ is non-decreasing in a_i to establish the identification result. In contrast, my interest is $\delta_i(a_i, a_{-i} = 1)$ and so do not require any more restrictions on payoff functions.

$$\begin{aligned}
E[\Pi_i(\boldsymbol{\varepsilon}_i, a_i)] &= \pi_i(a_i) + \sum_{j=1}^{J-i} \delta_i(a_i, a_{-i} = 1) \cdot \eta_i(a_{-i} = j) \cdot b_i^j + \varepsilon_i(a_i) \\
&= \pi_i(a_i) + \delta_i(a_i, a_{-i} = 1) \cdot \left[\sum_{j=1}^{J-i} \eta_i(a_{-i} = j) \cdot b_i^j \right] + \varepsilon_i(a_i) \\
&= \pi_i(a_i) + \delta_i(a_i, a_{-i} = 1) \cdot g_i + \varepsilon_i(a_i),
\end{aligned} \tag{6}$$

where $g_i = \sum_{j=1}^{J-i} \eta_i(a_{-i} = j) \cdot b_i^j$ represents player i 's subjective expectation of the value $\eta_i(a_{-i} = j)$. In the previous subsection, if $J-i = 1$, then player i 's expected payoff of action a_i is

$$E[\Pi_i(\boldsymbol{\varepsilon}_i, a_i)] = \pi_i(a_i) + \delta_i(a_i, a_{-i} = 1) \cdot b_i^1 + \varepsilon_i(a_i).$$

It is easy to see that if we treat g_i in equation (6) as being analogous to b_i^1 in the above equation, then these two equations would share the same structure. As a result, the identification result in proposition 1 (a) holds trivially. This is summarized in the following proposition.

Proposition 3. *Under assumption 1 to 5, for any two actions j and k , player i 's payoff parameters $\pi_i(\cdot)$ and $\delta_i(\cdot)$ are given by the set of values that satisfy the following restriction*

$$\frac{F_i^j(\mathbf{p}_i) - \pi_i(a_i = j)}{F_i^k(\mathbf{p}_i) - \pi_i(a_i = k)} = \frac{\delta_i(a_i = j, a_{-i} = 1)}{\delta_i(a_i = k, a_{-i} = 1)}.$$

Existing literature on the estimation of empirical games assumes the existence of an exclusion restriction, a variable that affects one player's payoff but not that of other players'. Without such restriction, the payoff function is non-identified, even with the BNE assumption (Bajari et al. (2010) and Aradillas-Lopez (2010)). In this subsection, I show that with the usual exclusion restrictions, researchers can sharpen the identified set in proposition 3 and achieve a similar results as games with variation in action spaces. Assumption 6 states the conditions on exclusion restrictions.

Assumption 6. (a) *For each player i , there exists a variable $z_i \in \mathbb{R}$ that affects only her own payoff; moreover, z_i has exogenous variation over its support.*

(b) There exists a variable $s \in \mathbb{R}$ that affects each player's interactive effect $\delta_i(\cdot)$ but not the base return $\pi_i(\cdot)$; moreover, s has exogenous variation over its support.

As explained above, the existence of z_i is commonly assumed in literature. In the empirical application described in example 1, a plausible candidate for z_i would be the market's distance to chain i 's nearest distribution center. While this distance could substantially affect chain i 's delivery costs and profit, it has no direct impact on chain $-i$, though it may indirectly affect chain $-i$ through its impact on chain i .

In the same context, a plausible candidate for the other exclusion restriction s required by assumption 6 (b) could be the distance between the two chains' stores. If McD does not operate through the night, such distance would have no impact on KFC's night profit. In contrast, the interactive effect would be affected by this distance, since an opponent of closer proximity may have a larger impact than one that is further away. This type of horizontal differentiation created by distance has been studied in empirical games by Seim (2006), Zhu and Singh (2009), and Rennhoff and Owens (2012). They introduce distance between competitors as a plausible model specification instead of an instrument to facilitate identification and estimation. In contrast, this paper formally discusses the role of such a variable in identification without imposing the equilibrium assumption.¹⁵

Proposition 4. (a) Under assumption 1 to 5 and assumption 6 (a), $\frac{\delta_i(z_i, a_i=j, a_{-i})}{\delta_i(z_i, a_i=k, a_{-i})}$ is identified for any two actions j and k if there exist at least two realizations of z_{-i} , say z_{-i}^1 and z_{-i}^2 , such that $\mathbf{p}_i(z_i, z_{-i}^1) \neq \mathbf{p}_i(z_i, z_{-i}^2)$.

(b) Suppose further assumption 6 (b) holds and there exist at least two realizations of s , say s^1 and s^2 , such that $\frac{\delta_i(z_i, s^1, a_i=j, a_{-i})}{\delta_i(z_i, s^1, a_i=k, a_{-i})} \neq \frac{\delta_i(z_i, s^2, a_i=j, a_{-i})}{\delta_i(z_i, s^2, a_i=k, a_{-i})}$, then $\pi_i(z_i, a_i)$ and $\delta_i(z_i, s, a_i, a_{-i} = 1)g_i(z_i, z_{-i}, s)$ are point identified for every $a_i \in A_i$ and every z_i, z_{-i}, s .

Existing empirical applications usually restrict $\eta_i(\cdot)$ to be a positive function, and consequently $g_i(\cdot)$ is positive. Under such restriction, the sign of the interactive payoff is identified given the identification of

¹⁵There exist other plausible candidates for s in different empirical applications. For instance, consider an entry game such that two banks decide whether to open branches in a small town. If bank 2 does not open any branch, the profit for bank 1's branch in this small town will be unaffected by bank 2's existence/density in other cities, since depositors and lenders cannot do business with bank 2. In contrast, if bank 2 opens a branch, the profit for bank 1 will be affected by bank 2's network in surrounding regions, since bank 2 can steal more local consumers if it has a higher branch density in other regions. This is because consumers may care about convenience of withdrawal when they are traveling. This type of network effect is very prevalent in industries such as retail, fast food and airline, etc.

$\delta_i(\cdot)g_i(\cdot)$.¹⁶ Moreover, since the multiplicative separability mimics the feature in games with asymmetric actions, the exclusion restrictions studied in this subsection can be used to sharpen the identified set in proposition 1 (a) when actions spaces are constant across observations. Finally, this result can be generalized to a game with more than two players with the restriction that the number of player is smaller than the number of actions. Appendix D presents the identification result.

4 Empirical Application

4.1 Industry Background and Motivation

China's Western-style fast food industry is characterized as a duopoly of KFC and McD.¹⁷ KFC opened its first Chinese outlet in Beijing in 1987. Three years later, McD opened its first outlet in Shenzhen, Guangdong Province. After that, these two Western-style fast food chains expanded their business in the world's largest emerging economy. Despite its leading role in the world, McD expanded its business at a slower rate and its outlet stores are outnumbered by KFC by more than 2:1. Specifically, at the beginning of 2016, KFC operated 4,952 outlets across China, while McD only owned 2,231. In terms of geographic distribution, KFC operates in all 31 provinces of mainland China, while McD has only entered into 27 provinces.¹⁸ Unlike case in Western countries, KFC and McD are considered to be close substitutes in China. For instance, China's KFC serves hamburger, beef together with chicken products and McD also sells a variety of fried chicken. Moreover, both chains offer traditional Chinese food (e.g. congee) to attract Chinese consumers. Another important feature in the operation of these chains in China, in contrast to Western countries, is that the companies directly own and operate most of their outlets. At the end of 2014, about 15% of McD's stores were franchised. Similarly, KFC had less than 10% franchised

¹⁶Since $g_i(\cdot)$ represents subjective expectation instead of belief, it is not a valid probability distribution. Therefore, the lower bound of the interactive payoff is not identified.

¹⁷Other giant Western-style fast food companies have relatively small market shares in China. By 2014, Burger King had more than 300 stores while Subway owned about 600 stores. Most of these stores are located in the more developed areas of China. Some consider another brand called Dicos as a third player in the Chinese market. It serves very similar products as KFC and McD, and has a comparable size with McD. By the end of 2016, Dicos operated 2,092 stores and they are distributed across all 31 provinces in mainland China. However, only 2% of Dicos stores operate at night. In consequence, I do not model Dicos as a third player in the night market. Adding the number of Dicos stores as a control variable rarely changes my estimation results.

¹⁸The four provinces that McD has not yet entered are Xinjiang, Ningxia, Tibet, and Qinghai. These four provinces are excluded from my sample.

stores.¹⁹ On the logistics side, HAVI Logistics provides distribution services for McD and has seven distribution centers across China. The holding company of KFC, YumChina, provides logistic services itself and owns 16 distribution centers. For both chains, the distribution center supplies key raw materials (e.g. raw meat) to each store. Consequently, a market's distance to a chain's nearest distribution center substantially affects the logistic costs for that chain. In my analysis, this distance is used as an exclusion restriction on a single player's payoff. It facilitates the identification and estimation of players' payoffs and beliefs.

KFC and McD compete with each other through many dimensions such as entry/expansion, menu selection, pricing, location choice, etc. In this paper, I study their strategic interactions through store hours decision, conditional on the outlet structure in a market. As shown in subsection 3.2, variation in the number of outlets owned by the two chains across markets provides identification power for both payoff and belief functions. It allows me to distinguish whether decisions of operating hours are strategic substitutes or complements. At earlier stages of their business in China, neither chain operated at night. As China's economy grew, McD started to operate 24-hour stores in 2005 and KFC began in 2009.²⁰

In the theoretical literature, Inderst and Irmen (2005), Shy and Stenbacka (2006, 2008), and Wenzel (2010, 2011) view store hours as a strategic device that softens price competition. In addition, it is horizontally differentiated, since consumers may prefer to consume at different times of the day. In contrast, my discussions with industry experts suggest that extending operating hours enables the staff to better prepare for busy breakfast service, in addition to build brand value. Specifically, if a chain successfully attracts a consumer at night, it is more likely to also attract this consumer during the day time due to lower search costs or switching costs. Under this view, the choice of store hours has a component of vertical differentiation similar to quality choice. This view is taken by Ferris (1990) and Klemperer and Padilla (1997), and empirically studied by Kügler and Weiss (2016).²¹ Failing to consider

¹⁹For information of McD, see Sina News retrieved from <http://finance.sina.com.cn/chanjing/gsnews/2016-05-23/doc-ifxsktkr5912661.shtml>. For KFC's information, see People News retrieved from <http://finance.people.com.cn/n/2012/0927/c70846-19130602.html>.

²⁰A typical store that closes at night operates from 6 am to 11 pm. There is some variation in these operating hours. My model can accommodate to these variations if detailed data of each store's operation times are available.

²¹Kügler and Weiss (2016) employ a reduced form estimation for the strategic effect on store hours competition in the Austrian gasoline market. They find an insignificant interactive effect. In contrast, this paper employs a structural model and has several advantages. First, researchers can take potential biased beliefs into account, so the estimates are robust. Second, the estimates of interactive effects have structural interpretations and can be compared with the base return (e.g. monopoly profit). Third, the estimates can be used to construct counterfactual predictions.

this quality aspect would under-evaluate not only consumers welfare gains but also stores' business hours from policy deregulation of opening hours. Therefore, it is essential to quantify the extent of the vertical differentiation component in the choice of store hours.

As described above, both chains have been operating in a challenging and constantly changing industrial environment. For more details, several news agencies in China exposed a series of KFC food safety scandals. These scandals not only depreciated KFC's brand value in China, but also raised concerns about Western-style fast food restaurants in general. Euromonitor International reported that KFC's sales decreased after 2012 while McD's sales remained constant, amidst the rapid growth of China's catering industry in recent years.²² As reported by China Market Research Group, consumers have started to lose trust in both chains. This challenging and unprecedented situation poses a difficulty on each firm's strategic reasoning.

From a practical standpoint, if players have biased beliefs in empirical games, incorrectly imposing the equilibrium condition will bias the estimated magnitude or even the sign of the interactive effects. Additionally, investigating whether firms have rational expectations and how they form beliefs has its own interest, especially when the economic environment varies dramatically.

The identification results studied in this paper enable researchers to quantify the extent of store hours' quality aspect and it is robust to potential biased beliefs. Moreover, with variation in the other chain's characteristics, we can infer how a chain will adjust its beliefs and test the hypothesis of rational beliefs.

4.2 Data

I gather information on every KFC and McD outlet store in China through their respective official websites.²³ The information for each store includes its brand name, address, telephone number and other store characteristics such as 24-hour service, breakfast service, drive-through, etc. Due to consumers' travel costs, distances between stores within the same chain and between different chains can have an impact on the store hours decision. Such network structure is very challenging to control when the num-

²²As reported by the Ministry of Commerce of the P.R.C., the annual growth rate of sales in China's catering industry is 10.6% over the period 2010 to 2015.

²³The websites are: <http://www.kfc.com.cn/kfceda/storelist/> for KFC, and <http://www.mcdonalds.com.cn/top/map> for McD. The data were gathered on 28 April, 2016.

ber of stores is large, especially in Beijing or Shanghai where there are about 500 stores.²⁴ Therefore, instead of studying strategic interactions in large cities, I focus on small counties/districts in which the number of total stores of KFC and McD is considerably smaller.²⁵

Consumers are typically reluctant to travel long distances for fast food, especially at night. Therefore, I define market at a finer level. For every county where KFC or McD exists, I obtain a list of cinemas through Baidu map. I then derive the centroid of each cluster of cinemas, and define a market as an area that lies within a 5-km radius around the centroid.²⁶ This market definition does not necessarily assume that night consumers for fast food are customers in cinemas. Typically, cinemas are located in entertainment districts or densely populated residential areas in China; therefore, the centroid of cinemas cluster serves as an approximation for the location where night life is present. I believe that people who reside or go there are potential consumers of fast food. Based on this market definition, 95% of the stores in the sample are included in one of these local markets. Moreover, 39 out of 742 counties/districts encompass multiple markets while the rest of the counties have a unique market.²⁷ In this study, I focus on markets where the number of KFC stores is between 1 and 4 and McD has no more than 1 store. These markets comprise almost 90% of the total sample. Including markets with more stores expands McD's action spaces and raises KFC's payoff and belief dimensions. However, given few additional observations, the parameters for these extra dimensions and KFC's impact on McD would be imprecisely estimated. Therefore, I only study how KFC's action is affected by McD in this paper. Table 2 presents the joint distribution of the number of stores per market for these two chains. These numbers vary across markets; therefore, the identification results in subsection 3.2 can be applied to identify KFC's payoff and belief.

Demographic variables such as population and GDP are obtained through China Data Online, provided by the University of Michigan. I also collect night light data from the NOAA National Geophysical

²⁴Specifically, such network structure is completely characterized by a vector of distances between any two stores in the market. It then consists of $\frac{n(n-1)}{2}$ elements in a market with n stores. A complete characterization of large markets would yield very imprecise estimates due to high dimensionality.

²⁵The political hierarchy in China is: Nation→Province→City→County/District→Town. County is the smallest unit at which demographic data are richly available. Moreover, district is at the same level as county and is typically located in the central area of a city. Throughout this paper, I refer to districts as counties for the sake of brevity.

²⁶See Appendix E of how I construct the clusters. Since KFC and McD are typically located close to each other, a radius of 5 km is a very conservative definition; market rarely changes using a radius of 3 km as another definition.

²⁷Given these statistics, observations rarely change when markets are defined at the county level; therefore, the estimation results are robust to different market definitions.

Table 2: Distribution of the Number of Markets by the Number of KFC and McD Stores

KFC Stores	McD Stores		Total
	0	1	
1	390	34	424
2	107	42	149
3	36	32	68
4	23	17	40
Total	556	125	681

Data Center, provided by the U.S. Department of Commerce. Night light data measure the development level and population density of a local market. Furthermore, I collect information on the geographical location of every distribution center for both McD and KFC, and calculate their distances to each market. I also calculate the average minimum distance of stores of the same chain and of different chains within the same local market. Table 3 presents summary statistics.

Table 3: Summary Statistics on Local Markets

Variable	Definition	Mean	Std. Dev.	Min	Max
<i>Income</i>	GDP Per Capita, 10,000 RMB	4.83	3.37	0.59	31.57
<i>Pop</i>	Population, 100,000	7.15	3.94	0.28	28.5
<i>Center</i>	Dummy, =1 if market located at city center	0.20	0.40	0	1
<i>Light</i>	Night Light Density	56.51	6.76	28	63
<i>KFCDist</i>	Average Distance between KFC Stores, km	0.51	0.86	0	6.10
z_{KFC}	Distance to Nearest KFC's Distribution Center, 100 km	2.12	1.36	0.09	10.16
z_{MCD}	Distance to Nearest McD's Distribution Center, 100 km	2.69	1.76	0.31	9.91
<i>Cinemas</i>	Number of Cinemas	2.82	1.87	1	14
<i>s</i>	Distance between Two Chains' Centroids	0.80	0.75	0.01	3.73
<i>KFCStores</i>	Number of KFC Stores	1.59	0.89	1	4
<i>MCDStores</i>	Number of McD Stores	0.18	0.39	0	1
<i>KFC24h</i>	Number of KFC 24 Hour Stores	0.56	0.71	0	3
<i>MCD24h</i>	Number of McD 24 Hour Stores	0.38	0.49	0	1
<i>South</i>	Regional Dummy, =1 if in Guangdong or Hainan Prov.	0.07	0.26	0	1
<i>North East</i>	Regional Dummy, =1 if in Liaoning, Jilin or Heilongjiang Prov.	0.09	0.28	0	1
Observations		681			

Note: Statistics for z_{MCD} , *s* and *MCD24h* are calculated conditional on the existence of McD.

4.3 Reduced Form Estimation

As informed by fast food industry experts, it is the local/regional manager who decides the operating hours for every outlet in a market. Therefore, I model this situation as a static game, such that each chain simultaneously chooses how many of its current outlets to keep open at night. Specifically, given the market structure shown in table 2, KFC has three choices when it possesses more than one outlet, opening zero/one or two stores at night.²⁸ In contrast, McD only chooses between opening zero/one 24-hour store. Moreover, both chains' action spaces vary across markets. Therefore, the identification results in subsection 3.2 can be applied to identify KFC's profit function and beliefs. In addition to variation in action space, my data set also contains valid exclusion restrictions that facilitate identification and estimation. Specifically, these exclusion restrictions are z_{MCD} (i.e. distance to nearest McD's distribution center) and s (i.e. distance between the two chains' centroids).

Even though variation in players' action spaces provides identification power, this variation is likely to be endogenous, as each chain chooses which market to enter and how many outlets to operate. To address this sample selection problem, I first estimate a binary Logit model of each chain's entry decision (i.e. whether to open stores in a given market) and obtain generalized residuals advocated by Gourieroux (1987).²⁹ These generalized residuals are used as control variables in the estimation to account for selection bias.³⁰ See Guerre, Perrigne and Vuong (2009) for a justification of this method. Results of this reduced entry model are shown in table 8 in Appendix E.

Table 4 presents a reduced form Multinomial Logit model of KFC's decision concerning how many outlets to operate overnight. A comparison between the first and second column sheds light on the importance of sample selection controls. The estimated coefficients, especially for *Income* and *Center*, change significantly after controlling for generalized residuals. In addition, demographic variables such as *Income* and *Population* are statistically insignificant while *Light* explains a large fraction of variation in KFC's decisions. As market size (i.e. area) is substantially smaller than the unit for which *Income* and

²⁸I treat the operation of two and more than two 24-hour stores as the same action, because KFC operates more than two stores at night in only 1.5% of the markets. Furthermore, when KFC only possesses one store in a market, it cannot operate more than one 24-hour stores. In this case, opening more than one-24 hour store is treated as a strictly dominated action and will not be chosen.

²⁹The number of observations used to estimate the entry decision is 1,208.

³⁰Conditional on entry, I also add number of outlets each chain holds in a given market as a control variable for the profit function of operating 24-hour stores. The coefficients on these variables are identified with the help of exclusion restrictions z_i and s .

Population data are gathered, night light data consequently provides additional information at a finer level and substantially increases in-sample fitness. This is consistent with a growing literature that measures economic activity using outer-space data (e.g. Henderson, Storeygard and Weil (2012)). Moreover, the numbers of KFC stores and cinemas are added as proxies for local demand of fast food, their estimates are significantly positive as anticipated.

Given the market structure in table 2, $MCDDStores$ is a dummy that equals one if McD is present in the market. The estimated coefficient on $z_{MCD} \times MCDDStores$ suggests a significant negative impact of z_{MCD} on KFC's decision in markets where McD owns an outlet. Intuitively, z_{MCD} has a negative effect on McD's profit as it increases delivery cost. Moreover, it affects KFC's decision only through its impact on McD's decision, since the distance to McD's distribution center is irrelevant to KFC's own costs or revenues. Table 4 suggests that the negative impact of z_{MCD} on McD is transformed to a negative influence on KFC, which suggests that these chains' store hours decisions are strategic complements if KFC has rational expectations.

An alternative explanation for this empirical evidence is that z_{MCD} may be correlated with some unobserved market heterogeneity. For instance, a market that is far from McD's distribution center could be a bad market with low unobserved demand factors. If such correlation exists, z_{MCD} would no longer be a valid exclusion restriction. With the help of variation of McD's action sets, I can formally test the existence of this potential correlation, while such a test would be substantially difficult in other empirical games. Suppose that z_{MCD} is negatively correlated with unobserved market heterogeneity, it implies that z_{MCD} should negatively affect KFC's decision regardless of McD's presence. However, the coefficients on $z_{MCD} \times (1 - MCDDStores)$ shown in the third column indicate that z_{MCD} has a highly insignificant impact on KFC in markets where McD has zero outlet. This suggests that the potential correlation of z_{MCD} and unobserved heterogeneity is not a major driver of the statistical significance on $z_{MCD} \times MCDDStores$.

Unobserved market heterogeneity, if it exists, will invalidate my identification results and bias the estimates of payoff and belief functions. In this situation, McD's actual decision of store hours would reveal some information on unobserved market heterogeneity. Therefore, the number of McD's 24-hour stores should significantly affect KFC's decision after controlling for all other variables. However, the

Table 4: Multinomial Logit Regression: Number of KFC's 24-Hour Stores

	One 24h	Two 24 H	One 24 H	Two 24 H	One 24 H	Two 24 H
$\log(\text{Income})$	0.5026** (0.2121)	0.7490 (0.5118)	0.0137 (0.3741)	0.9439 (1.0697)	0.1670 (0.4372)	1.2521 (1.4226)
$\log(\text{Pop})$	0.1455 (0.2167)	0.3866 (0.5137)	-0.2041 (0.3166)	0.5381 (0.8866)	-0.1165 (0.3513)	0.8298 (1.1878)
$\log(z_{KFC})$	-0.1563 (0.1442)	-0.8924*** (0.3360)	-0.0931 (0.1647)	-0.7724* (0.4307)	-0.0302 (0.1799)	-0.7149 (0.4905)
$\log(1 + KFC\text{Dist})$	-0.9945* (0.5120)	-0.7416 (0.7688)	-1.0198* (0.5561)	-0.7313 (0.9236)	-1.0103* (0.5574)	-0.7442 (0.9387)
$KFC\text{Stores}$	0.6245* (0.3351)	2.0030*** (0.4060)	0.5908* (0.3594)	1.9856*** (0.4772)	0.6026* (0.3654)	2.0194*** (0.4963)
Center	0.5579* (0.3340)	-0.2809 (0.7133)	-0.1383 (0.5271)	-0.6059 (1.1126)	0.1894 (0.6739)	-0.0689 (1.9161)
Light	0.0517*** (0.0190)	0.1276** (0.0568)	0.0568*** (0.0206)	0.1211* (0.0661)	0.0618*** (0.0221)	0.1414* (0.0764)
Cinemas	0.1736** (0.0776)	0.1908 (0.1236)	0.1704** (0.0831)	0.1957 (0.1835)	0.1735** (0.0844)	0.2197 (0.1910)
MCDStores	0.9976 (0.6619)	-0.0477 (1.1278)	1.2816 (2.4012)	-1.5781 (3.3086)	0.6645 (2.6441)	-2.5796 (4.5422)
$z_{MCD} \times \text{MCDStores}$	-0.4757*** (0.1652)	-0.5249** (0.2474)	-0.4627* (0.2479)	-0.7398* (0.4121)	-0.4928* (0.2637)	-0.8148* (0.4593)
$\text{Center} \times \text{MCDStores}$	1.9506 (1.2067)	4.7926*** (1.7174)	2.2062* (1.3277)	5.8317** (2.4589)	1.9503 (1.4668)	5.7612** (2.6727)
$\log(1 + s)$	1.0972 (0.9566)	2.3547 (1.4466)	1.2389 (0.9732)	2.5966* (1.4803)	1.2177 (0.9963)	2.5588* (1.4982)
$\log(1 + s) \times \text{Center}$	-2.6855 (1.8136)	-4.3943** (2.2226)	-2.4801 (1.5563)	-4.1651* (2.3958)	-2.4442 (1.5525)	-4.2436* (2.4002)
$z_{MCD} \times (1 - \text{MCDStores})$					-0.0871 (0.1054)	-0.1593 (0.4292)
$\text{MCD}24h$					0.1557 (0.8659)	-0.5700 (1.1574)
Regional Dummies (3 Regions)	Yes		Yes		Yes	
Sample Selection Controls	No		Yes		Yes	
log-likelihood	-408.2954		-405.4208		-404.1815	
Observations	681					

Note: Sample selection controls include KFCResidual and MCDResidual. The standard error in second and third column is calculated by Jackknife. *, **, *** represent significant at significance level of 10%, 5% and 1% respectively.

coefficients are highly insignificant as shown in the third column. This indicates that unobserved market heterogeneity is not strong enough to generate a sizable bias on my estimates.

Variable s is another exclusion restriction that facilitates the identification of KFC's base return and interactive effects. It shows a significant impact on KFC's decision to open two 24-hour stores.³¹ Moreover, this impact differs by the locations of markets. For more details, recall that *Center* is a dummy variable that equals 1 if the market is located at the center of a city (i.e. downtown area).³² It influences KFC's decision in markets where McD is present, but has limited impact in markets where McD is absent. It is the only demographic variable that has this feature. One plausible explanation for this result is that McD's decision has a heterogeneous impact on KFC that differs across market locations. The econometric model estimated in the next subsection formally investigates this conjecture.

4.4 Structural Estimation of Empirical Games

Even though the reduced form estimates shed light on KFC's choice incentive, they quantify neither the competitive effect nor KFC's belief. In order to capture these latter two effects, I estimate an econometric model of games. KFC's base return is given by

$$\pi_{KFC}(\mathbf{x}, z_{KFC}, a_{KFC} = j) = (\mathbf{x}, \log(z_{KFC}))\alpha^j. \quad (7)$$

Recall that $a_{KFC} = j$ represents that KFC operates j stores at night. Vector \mathbf{x} contains control variables listed in table 3, a constant, and generalized residuals recovered from each chain's entry decision. The interactive effect is specified as

$$\delta_{KFC}(\mathbf{x}, s, a_{KFC} = j, a_{MCD} = 1) = \theta_1^j + \theta_2^j \log(1 + s) + \theta_3^j \text{Center} + \theta_4^j \log(1 + s) \times \text{Center}. \quad (8)$$

Intuitively, the interactive effect depends on s , the distance between the two chains' centroids. Furthermore, as shown in table 4, *Center* has a significant impact on KFC only when McD is present in a market. Therefore, this factor may influence the interactive effect and is controlled for in equation (8).

³¹Note that in table 4, s enters into the model through $\log(1 + s)$, which equals zero when McD is absent (i.e. $s = 0$). Therefore, it interacts with *MCDStores* by construction.

³²In this paper, city refers to administrative areas in China. Typically, a city contains a core urban area and satellite towns. The core area is usually much more developed than the satellite towns and there is generally rural area between them. Moreover, the distance between core area and satellite town is quite far such that it is implausible for a consumer to travel such a distance just for fast food. *Center* equals 1 if the market lies in a core area.

Other demographic variables are excluded in this interactive payoff to avoid imprecise estimates. As belief is an unknown, it multiplies by the interactive payoff. In this non-linear model, adding an additional parameter substantially affects estimation precision, compared to linear models. Moreover, other demographic variables are shown to have insignificant impact on the interactive payoff under the equilibrium condition.

As specified in equations (7) and (8), the econometric models put no restrictions on coefficients across KFC's different actions a_{KFC} . This therefore captures economies of scale and cannibalization effects in a flexible way. Moreover, these effects are allowed to vary when McD takes different actions. Finally, cautions should be exercised in the interpretation of payoff functions. When night-time market and day-time market are independent, equation (7) and (8) are interpreted as KFC's profit function in the night market. In contrast, when operating at night has positive spillover effects on the day-time profit, these payoff functions represent the net increase of KFC's *total* profit (for both night time and day time) by operating 24-hour stores.

In this paper, I specify a reduced Logit form for McD's choice probability. Specifically, McD's choice probability of opening one 24-hour store is

$$p_{MCD}^1(\mathbf{x}, z_{KFC}, z_{MCD}, s) = \frac{\exp [(\mathbf{x}, z_{KFC}, z_{MCD}, s, \mathbf{x} \cdot z_{MCD})\gamma]}{1 + \exp [(\mathbf{x}, z_{KFC}, z_{MCD}, s, \mathbf{x} \cdot z_{MCD})\gamma]}. \quad (9)$$

Equation (9) is consistently estimated using only McD's decision. In the literature that studies empirical games under BNE, $p_{MCD}^1(\cdot)$ would be used to approximate KFC's belief.³³

This paper allows KFC to have biased belief and treats KFC's belief as an unknown to be estimated. It takes a Logit form and does not need to have the same parameters as equation (9)

$$b_{KFC}^1(\mathbf{x}, z_{KFC}, z_{MCD}, s) = \frac{\exp [(\mathbf{x}, z_{KFC}, z_{MCD}, s, \mathbf{x} \cdot z_{MCD})\lambda]}{1 + \exp [(\mathbf{x}, z_{KFC}, z_{MCD}, s, \mathbf{x} \cdot z_{MCD})\lambda]}. \quad (10)$$

Equation (10) is estimated using KFC's decisions. If $\gamma = \lambda$, this implies that KFC has rational expectations. In contrast, if $\gamma \neq \lambda$, this suggests KFC has biased beliefs. Given the identification results in section 3 and Logit form of belief, λ is identified. However, it would be imprecisely estimated due

³³Due to small sample size, I exclude variables that are highly insignificant, for instance, z_{KFC} .

to small sample size. To reduce estimation burden, I restrict equation (9) and (10) to share the same coefficients on (\mathbf{x}, z_{KFC}, s) . Such restriction is equivalent to assuming that KFC has unbiased belief if $z_{MCD} = 0$. Therefore, imposing such restriction would not over-reject the null hypothesis of KFC's rational beliefs.

Since all proofs of the identification results in this paper are constructive, the two-step estimator commonly used in the estimation of incomplete information games can be applied. In this paper, I employ a slight variant of two-step pseudo maximum likelihood (two-step PML) to estimate the model. I assume that private information $\varepsilon_i(a_i)$ follows type 1 extreme value distribution and is independent across actions and players. Therefore, the expression of choice probability takes Logit form. The estimation method is outlined in the following steps.

1. Estimate KFC and McD's entry decision (i.e. whether each chain opens outlets in a given market or not) as a binary logistic regression of market observables and obtain generalized residuals. Such residuals are included in \mathbf{x} as controls for potential unobserved heterogeneity.
2. Estimate a binary logistic regression of McD's decision of operating 24-hour stores, with choice probability given by equation (9). Obtain the estimates of $\hat{\gamma}$.
3. Take the estimated coefficients of (\mathbf{x}, z_{KFC}, s) in equation (9) to be the coefficients of its counterpart in equation (10). Given equations of the base return (7), interactive payoff (8) and belief function (10), KFC's log-likelihood function of its 24-hour stores choices is well defined. I estimate α , θ , and λ by maximizing that log-likelihood function.

In step 3, there is an implementation issue related to the fact that the interactive payoff and belief functions are multiplied together. It implies that the log-likelihood function is not globally concave in θ and λ . I estimate these parameters by the Gauss-Seidel method (details are presented in appendix E). Moreover, the asymptotic variance of the estimator in the last step crucially depends on the estimates in the first two stages and is substantially difficult to derive. Therefore, I use Jackknife to calculate the standard error.³⁴

³⁴As a comparison, Bootstrap's performance is quite poor in this application. This is because some categories have very few observations and the entire category is very likely to be dropped in a single bootstrap sample. For instance, I control for regional dummies and only three markets in the North East area operate two 24-hour stores for KFC. It is very likely that those three markets are not chosen in a single bootstrap sample. In this sample, the estimate of the effect for the North East area would be negative infinity for the action of two 24-hour stores.

Table 5 presents the estimated coefficients and marginal effect of the belief function. The column titled “MCD’s Choice” represents equation (9), estimated using McD’s decision. Under the equilibrium assumption, this approximates KFC’s belief. The column titled “KFC’s belief” represents equation (10), estimated using KFC’s decision. Market characteristics in the interaction terms are taken as deviations from the sample means; therefore, the estimated effect of z_{MCD} represents its impact on an average market. Finally, as the equilibrium assumption is a nested special case of the unknown belief specification, a likelihood ratio test can be conducted to test the hypothesis of KFC’s unbiased beliefs. This hypothesis is rejected at the 1% significance level, as indicated in the last row.

Table 5: Estimates of KFC’s Belief (Logit Formula)

	MCD’s Choice		KFC’s Belief	
	Coefficient	Marginal Effect	Coefficient	Marginal Effect
z_{MCD}	-1.1183* (0.6455)	-0.0999 (0.0810)	-0.2038 (0.6219)	-0.0505 (0.1586)
$z_{MCD} \times (\text{Income} - \overline{\text{Income}})$	0.2253* (0.1160)	0.0201 (0.0209)	0.7188*** (0.2599)	0.1782*** (0.0597)
$z_{MCD} \times (\text{KFCDist} - \overline{\text{KFCDist}})$	0.4922* (0.2621)	0.0440 (0.0425)	-0.1547 (0.5131)	-0.0384 (0.1269)
Control Variables	Yes		Yes	
log-likelihood	-407.2865		-401.1085	
Median Market Choice Probability	0.0992		0.5452	
Unbiased Belief Test (p-value)			p=0.0063	
Observations			681	

Note: Standard error in parenthesis is calculated by Jackknife. Four Jackknife samples yield extremely high estimates and are deleted. The hypothesis of KFC’s unbiased beliefs is rejected in all these samples. The log-likelihood is the value for KFC’s decision when corresponding specification is used as an approximation of KFC’s belief.

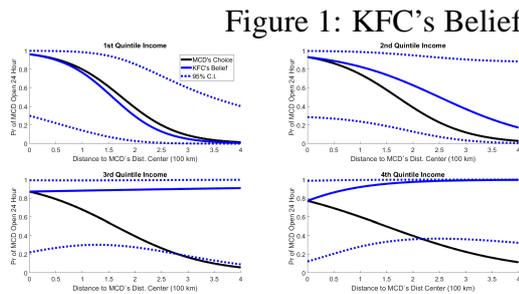


Figure 1: KFC’s Belief about McD’s Probability of Operation at Night

As shown in column “MCD’s Choice”, z_{MCD} significantly decreases McD’s probability of operating over night. Moreover, such negative impact is alleviated when a market has higher average income or

when KFC has a more sparse structure. A comparison with the “KFC’s Belief” column suggests that KFC over-evaluates the attenuation effect caused by higher *Income* as the interaction term exhibits a significantly larger magnitude. In contrast, no significant result suggests that KFC believes its network structure has a similar effect.

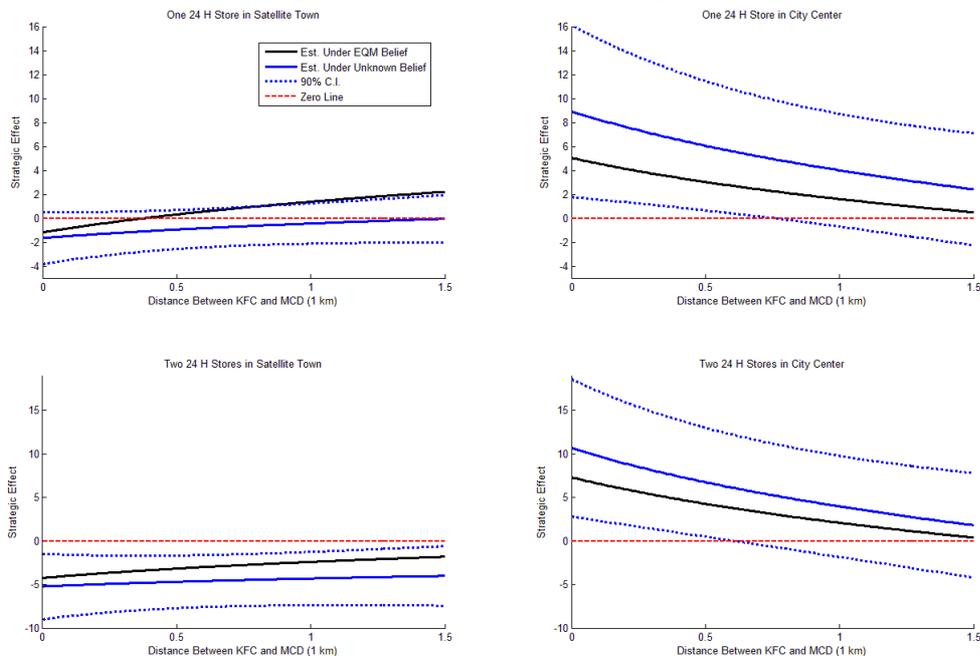
Figure 1 shows the impact of z_{MCD} on McD’s decision and KFC’s belief for different quintiles of *Income*. It is clear that in relatively poor markets, KFC tends to have rational expectations and its beliefs are almost identical to McD’s choice probabilities. In markets with higher average income, higher delivery costs have a lesser impact on McD’s decision as reflected by the flatter slope of the choice probability lines, moving from the 1st quintile (top left) to the 4th quintile (bottom right). However, KFC over-estimates McD’s discounting of delivery costs in wealthier markets. Specifically, KFC believes z_{MCD} has almost zero impact on McD’s choice in quite rich markets (e.g. markets in the 3rd and 4th quintile of *Income*). Table 7 in appendix E presents reduced form evidence of biased beliefs. When market has relatively high income, KFC’s decision is insignificantly influenced by z_{MCD} , indicating that KFC believes that z_{MCD} does not significantly affect McD’s decision when it in fact does.

Table 6 shows the estimates of KFC’s payoff function. Consistent with Aguirregabiria and Magesan (2017), these results suggest that incorrectly imposing the equilibrium assumption generates an attenuation bias on the interactive effect. In addition, figure 2 plots the estimate of the interactive payoff as a function of s . It is clear that the magnitudes of the estimated interactive payoffs under an unrestricted belief specification are larger than the ones under the equilibrium specification. Even such bias is insignificant, falsely imposing rational expectations can still lead to incorrect conclusions. As shown in figure 6 in appendix E, the estimates of the interactive payoffs are insignificant under the equilibrium condition. In contrast, figure 2 shows that store hours are strategic complements in markets located in city centers, and they are strategic substitutes in markets belonging to satellite towns. Furthermore, the magnitudes of these strategic effects attenuate with greater distance between the two competitors. For an average market located in a city center, the interactive effect is equivalent to a reduction in z_{KFC} from 1,000 km to 10 km.³⁵ Finally, the strategic complement nature of store hours suggests that extending operations to overnight service is a quality measure. Intuitively, the best response to a competitor’s quality

³⁵This reduction is equivalent to moving from the farthest market to the nearest market to KFC’s distribution center.

improvement is to increase one's own quality.³⁶ In contrast, decision of store hours will exhibit strategic substitute if it is horizontally differentiated.³⁷

Figure 2: KFC's Interactive Payoff



In a recent analysis, Shen and Xiao (2014) study KFC and McD entry and expansion decisions in China from 1987 to 2007. They find that a chain's market presence has a spillover effect on the other chain's entry/expansion decision. They offer and quantify two explanations, namely demand expansion and market learning.³⁸ Though these factors play important roles in entry/expansion, I believe that their impact on store hours decisions are limited. Suppose KFC and McD have equal market shares. For a pure demand expansion factor to generate positive interactive payoff, McD's decision to operate over night would need to more than double its market size, and it is very implausible for the night market to have such a large effect. Moreover, industry expert's opinion suggests that chains can gather sufficient knowledge about market characteristics through their outlets, which suggests that the informational spillover effect from the other chain's decision is negligible. Consequently, despite the positive interactive effect

³⁶In general, quality choice can exhibit either strategic substitute or complement depending on the model primitives. This can be seen in Brekke, Sicilliani and Straume (2010) who consider a general class of demand and cost functions to study firms' decisions on price and quality under spatial competition. Moreover, a simple model to demonstrate this point is available by the author upon request.

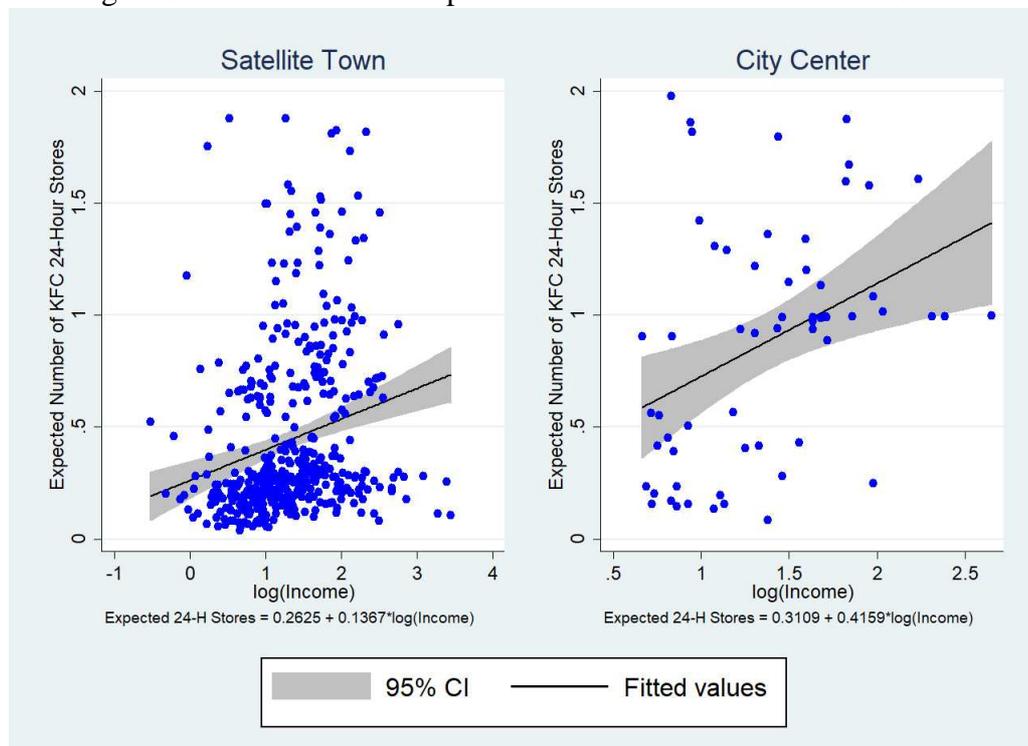
³⁷For an example, see table 1 and 2 in Shy and Stenbacka (2008) and figure 3 in Wenzel (2011).

³⁸Demand expansion refers to the effect of a firm's market presence on boosting the market size. Market learning refers to the situation where a firm can infer market conditions by observing the other firm's decision.

Table 6: Estimates of KFC's Payoff

	EQM Belief		Unknown Belief	
	One 24 H	Two 24 H	One 24 H	Two 24 H
<i>Interactive Payoff</i>				
Constant	-1.1974 (1.9817)	-4.2583 (2.8850)	-1.6616 (1.3249)	-5.2383** (2.2900)
$\log(1 + S)$	3.7034 (2.5254)	2.6606 (4.2432)	1.7571 (1.7484)	1.3283 (2.6425)
<i>Center</i>	6.2310* (3.5883)	11.5836** (5.1208)	10.5945** (4.8404)	15.9045*** (5.5316)
$\log(1 + S) \times \textit{Center}$	-8.6606** (3.9248)	-10.2263* (5.7437)	-8.8799* (4.5408)	-10.9990** (5.0870)
<i>Base Return</i>				
$\log(\textit{Income})$	0.1480 (0.3612)	0.7317 (0.9102)	0.1769 (0.3691)	1.4563 (1.0699)
$\log(\textit{Pop})$	-0.0942 (0.3182)	0.5608 (0.7963)	-0.0405 (0.3177)	1.1160 (0.8793)
$\log(z_{KFC})$	-0.2316 (0.1533)	-1.0863*** (0.3775)	-0.2647* (0.1480)	-1.0670*** (0.3932)
$\log(1 + \textit{KFCDist})$	-0.8477* (0.5112)	-0.4540 (0.9315)	-0.8349 (0.5352)	-0.8538 (0.9225)
<i>KFCStores</i>	0.5410* (0.3270)	1.9757*** (0.4515)	0.6947** (0.3512)	2.1630*** (0.4629)
<i>Center</i>	0.1074 (0.4957)	-0.4564 (1.0013)	0.1455 (0.4999)	-0.0660 (1.0658)
<i>Light</i>	0.0498** (0.0200)	0.1230* (0.0641)	0.0515** (0.0202)	0.1121 (0.0685)
<i>Cinema</i>	0.1599** (0.0780)	0.1810 (0.1751)	0.1564** (0.0798)	0.2048 (0.1905)
<i>Center</i> \times <i>MCDStores</i>	0.1852 (1.1243)	0.3010 (1.8682)	-1.4432 (1.3188)	-2.1125 (2.2870)
Regional Dummies (3 Regions)	Yes		Yes	
Sample Selection Controls	Yes		Yes	
log-likelihood	-407.2865		-401.1085	
Observations	681			

Figure 3: Counterfactual: Expected Number of KFC's 24-Hour Stores



found in this paper, McD's longer business hours are unlikely to generate a positive effect on KFC's profit. One plausible interpretation of such positive interactive payoff is the indirect business-stealing effect studied by Klemperer and Padilla (1997). Specifically, longer operation times by McD hurts KFC as it steals KFC's customers (i.e. both day-time and night-time consumers). However, this negative impact is alleviated if KFC also operates over night. As shown by Klemperer and Padilla (1997), with this indirect business-stealing effect, firms have strong incentives to expand their operation times upon the deregulation of store hours. This could be harmful, however, for some firms, especially for small ones that are not able to easily extend their business hours. It is also anticipated to increase consumer welfare. However, from a social planner's perspective, the deregulation of store hours could generate social loss overall.

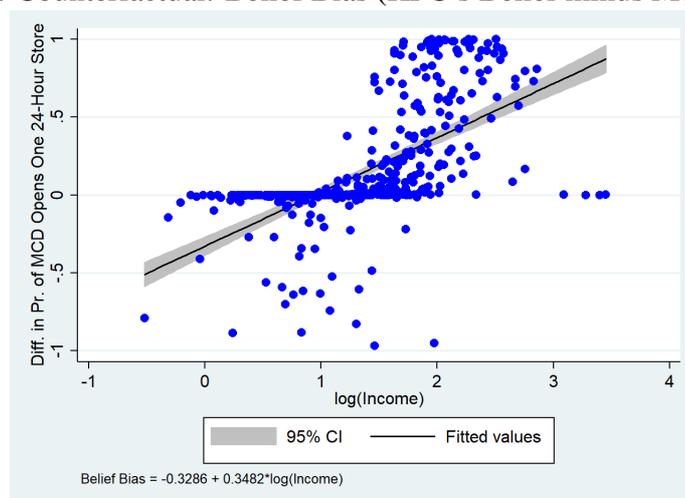
4.5 Counterfactual Analysis

In this subsection, I show that the strategic nature of store hours (e.g. strategic substitutes versus strategic complements) is the key in the evaluation of deregulation policy on store hours. Moreover, incorrectly imposing the equilibrium condition not only yields insignificant estimates of strategic effects as shown

in previous subsection, but also generates considerable bias on the counterfactual predictions. As shown in table 2, there are 556 markets where KFC does not have competition from McD. Given my focus on operating hours decision, these markets can be seen to have a regulation policy such that McD is forbidden to enter into the night market. In this subsection, I study KFC’s response if there were one McD store in those 556 markets. Equivalently, it can be seen as a deregulation policy that lifts the restriction on McD’s store hours.

In these counterfactual markets, I assume that KFC has a belief function as equation (10) with estimated coefficients in the “KFC’s Belief” column in table 5. Figure 3 shows the counterfactual prediction of the expected number of KFC’s 24-hour stores from above deregulation policy. The effect of deregulation policy differs by the locations of markets. Specifically, the average expected number of KFC’s 24-hour stores in city center (0.88) is twice as the one in satellite town (0.44). Even though KFC is willing to open more 24-hour stores in both markets when average income increases, the effect is much larger in the city center (the slope in the second plot is three times as the one in the first plot). These differential effects are due to the fact that store hours are strategic complements in city center; therefore, KFC has an extra incentive to operate over night when McD will do so. On the contrary, in satellite town where store hours are strategic substitutes, KFC is reluctant to open 24-hour stores together with McD.³⁹

Figure 4: Counterfactual: Belief Bias (KFC’s Belief minus McD’s CCP)

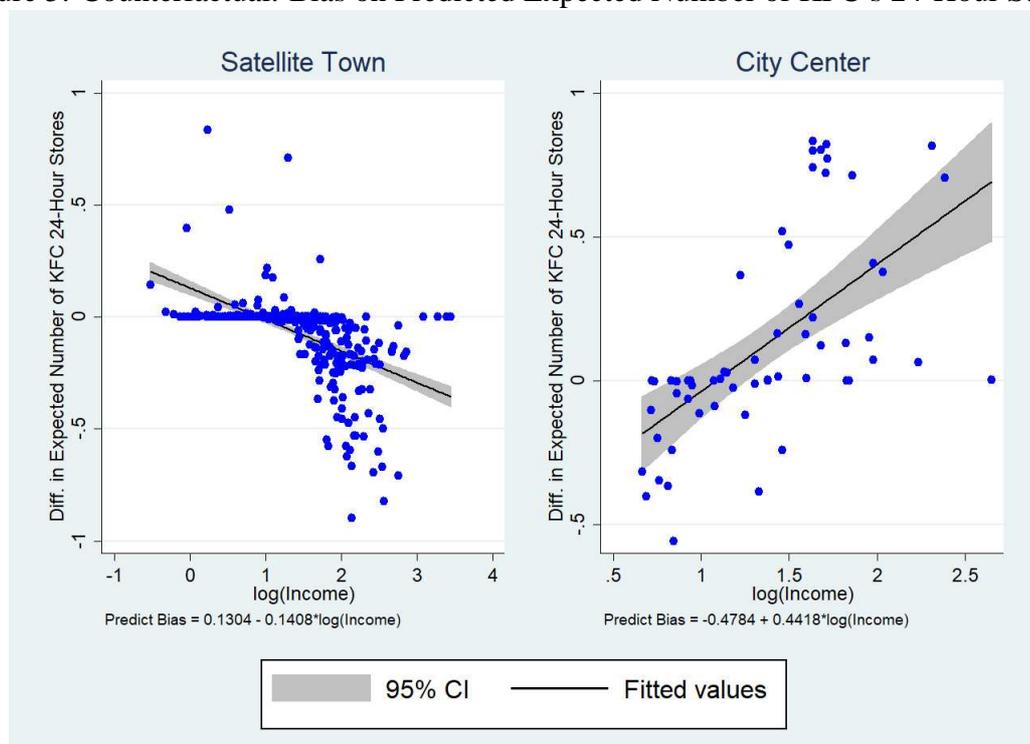


Incorrectly imposing the equilibrium condition would generate considerable bias for the prediction

³⁹Given estimation results in table 6, the effect of *Center* on the base return is highly insignificant both statistically and economically. Therefore, the reason of heterogeneous effects of deregulation policy is because that *Center* has a significant impact on the interactive payoff.

of above counterfactual deregulation policy. Figure 4 shows the bias of KFC’s belief. Consistent with estimation results in table 5, KFC over-predicts the McD’s operating hours in rich markets, while under-predicts it in poor markets. This belief bias transforms to a bias on the prediction of counterfactual. As shown in figure 5, in rich markets, the equilibrium condition over-estimates the number of KFC’s 24-hour stores in satellite town, while under-estimates it in city center. Again, this heterogeneity is caused by the fact that the interactive effect of store hours differs by the locations of markets.

Figure 5: Counterfactual: Bias on Predicted Expected Number of KFC’s 24-Hour Stores



Note: The difference is calculated as the expected number of KFC’s 24-hour stores under unrestricted belief specification minus the one under McD’s CCP estimated by equation (9).

5 Discussion

In this section, I discuss how to borrow recent techniques developed in the empirical games literature to extend the generality of my identification results. Specifically, I focus on three topics: unobserved heterogeneity, belief multiplicity, and dynamic interactions.

In subsection 3, I assume that researchers observe all the relevant state variables that affect a player’s payoff except each player’s private information. This assumption can be restrictive, as both players may

have common access to some information that is unobserved by econometricians. For instance, in the context of competition between KFC and McD, both firms may observe or learn at least part of the market demand shocks such as consumers' tastes toward Western-style fast food; however, such demand shocks are hardly observed by econometricians. The existence of unobserved market heterogeneity can potentially bias the estimation of players' payoff functions and the rational belief test.

Bajari et al. (2010), Ellickson and Misra (2012) and Marcoux (2017) discuss a control function approach in the estimation of games. Suppose researchers observe some variables that are informative about unobserved market heterogeneity, for instance, each firm's decision before the game starts. Controlling for such variables eliminates the bias, if unobserved heterogeneity is a deterministic function of these observables. This approach perfectly fits my framework wherein players are allowed to have biased beliefs. The identification results hold trivially as players' choice probabilities conditional on public information, including the variables on which unobserved heterogeneity depends, is non-parametrically identified.

As described in assumption 3, my framework assumes a player forms the same belief in two markets with same observables. It fails to include BNE when multiple equilibria are observed in the data. However, a modification of my framework renders the nesting of multiple BNEs. To see this, consider an extended specification of the belief function $\mathbf{b}_i(\mathbf{x}, k)$ where $k \in \{1, 2, \dots, K\}$ is known as a sunspot variable and can be seen as an index for the equilibrium being played. Its cardinality is allowed to depend on public information \mathbf{x} . This extension allows player i to form K different types of beliefs, which thereby, nests BNE with multiple equilibria, as long as K is no less than the number of equilibria in the game. In this extended framework, my identification results hold trivially if players' conditional choice probabilities for each type k is identified. The identification of choice probability is established by Aguirregabiria and Mira (2017).⁴⁰

This paper considers a static game while many empirical applications deal with dynamic interactions. In a dynamic game, a player's choice depends not only on her belief of the other player's current actions but also on her belief of the other player's and her own future actions. In general, this invalidates the asymmetry between the dimensionality of a player's actions and beliefs. Aguirregabiria and Magesan (2017) consider a class of dynamic games usually studied in applied research. Specifically, conditional

⁴⁰Aguirregabiria and Mira (2017) require a game with at least three players. Even though the main context of this paper focuses on a two-player game, appendix D generalizes the result to multi-player games. As a result, the conditions required in Aguirregabiria and Mira (2017) hold in my framework.

on current state variables and players' actions, the exclusion restriction has no impact on the transition probability of payoff relevant variables. In this situation, they show that a player's future belief is independent of the current realizations of the exclusion restriction. Therefore, this generates a dimension reduction on belief such that my identification results can be generalized to this type of dynamic game, with the help of usual exclusion restriction.

6 Conclusion

This paper investigates the identification of an incomplete information game without imposing the Bayesian Nash Equilibrium restriction. The econometric model imposes only weak assumptions on players' behaviors in the sense that each player's belief can be any probability distribution over the other player's action sets. I show that the asymmetry between the cardinality of players' action sets provides identification power for the payoff functions and characterize the identified set. Moreover, when players' action spaces have variation across observations, I show that players' base returns are identified. The interactive effect and belief are also identified up to scale normalizations. These identification results are obtained in absence of the commonly used exclusion restrictions, and provide researchers with empirical guidance on their choice of solution concept. In addition, it yields robust estimates of the payoff parameters of interest, even though BNE is rejected. Furthermore, researchers can infer how players adjust their beliefs, which sheds light on the process of player's belief formation. Finally, I also discuss the identification power of exclusion restrictions and the commonly used functional form of payoff.

I apply these identification results to study KFC and McD store hours decision in China. The estimation results reject the null hypothesis that KFC has rational beliefs on McD's decision. Specifically, KFC under-estimates the impact of McD's delivery costs on its operating hours in relatively rich markets. In addition, decision of store hours acts as strategic complement in markets located in city centers. It implies that store hours decision is vertically differentiated. This result is consistent with the indirect business-stealing effect studied by Klemperer and Padilla (1997). By operating through the night, a firm is likely to steal its opponent's customers during the day time. This suggests that the deregulation of store hours hurts the small retailers that are unable to easily extend their business hours, especially in city cores. Furthermore, failing to consider the vertical differentiation component will under-estimate not

only consumer welfare gains but also stores' operation hours of deregulation policy on store hours.

References

- [1] Aguirregabiria, V. and Magesan, A. (2017): "Identification and Estimation of Dynamic Games when Players' Beliefs Are not in Equilibrium." Working Paper, University of Toronto.
- [2] Aguirregabiria, V. and Mira, P. (2007): "Sequential Estimation of Dynamic Discrete Games." *Econometrica*, 75, 1-53.
- [3] Aguirregabiria, V. and Mira, P. (2017): "Identification of Games of Incomplete Information with Multiple Equilibria and Unobserved Heterogeneity." Working Paper, University of Toronto.
- [4] Aguirregabiria, V. and Xie, E. (2017): "Identification of Biased Beliefs in Games of Incomplete Information Using Experimental Data." Working Paper, University of Toronto.
- [5] Aradillas-Lopez, A. (2010): "Semiparametric Estimation of a Simultaneous Game with Incomplete Information." *Journal of Econometrics*, 157 (2): 409-431.
- [6] Aradillas-Lopez, A. (2012): "Pairwise Difference Estimation of Incomplete Information Games." *Journal of Econometrics*, 168 (1): 120-140.
- [7] Aradillas-Lopez, A. and Gandhi, A. (2016): "Estimation of Games with Ordered Actions: An Application to Chain-Store Entry." *Quantitative Economics*, 7 (3): 727-780.
- [8] Aradillas-Lopez, A. and Tamer, E. (2008): "The Identification Power of Equilibrium in Simple Games." *Journal of Business and Economic Statistics*, 26 (3): 261-283.
- [9] Asker, J., C. Fershtman, J. Jeon, and Pakes, A. (2017): "The Competitive Effects of Information Sharing." Working Paper. Harvard University.
- [10] Augereau, A., S. Greenstein and Rysman, R. (2006): "Coordination Versus Differentiation in A Standards War: 56K Modems." *Rand Journal of Economics*, 37 (4): 887-909.
- [11] Bajari, P., L. Benkard and Levin, J. (2007): "Estimating Dynamic Models of Imperfect Competition." *Econometrica*, 75: 1331-1370.
- [12] Bajari, P., Hong, H., Krainer, J. and D. Nekipelov (2010): "Estimating Static Models of Strategic Interactions." *Journal of Business and Economic Statistics*, 28 (4): 469-482.
- [13] Bajari, P., and A. Hortacsu (2003): "The Winner's Curse, Reserve Prices and Endogenous Entry: Empirical Insights From eBay Auctions." *Rand Journal of Economics*, 34, 329-355.
- [14] Bajari, P., and A. Hortacsu (2003): "Are Structural Estimates of Auction Models Reasonable? Evidence From Experimental Data." *Journal of Political Economy*, 113, 703-741.
- [15] Bernheim, B. (1984): "Rationalizable Strategic Behavior." *Econometrica*, 52, 1007-1028.
- [16] Besanko, D., U. Doraszelski, L. Lu, and Satterthwaite, M. (2010): "On the Role of Demand and Strategic Uncertainty in Capacity Investment and Disinvestment Dynamics." *International Journal of Industrial Organization*, 28 (4): 383-389.

- [17] Brock, W. and Durlauf, S. (2001): “Discrete Choice with Social Interactions.” *Review of Economic Studies*, 68: 235-260.
- [18] Brock, W. and Durlauf, S. (2007): “Identification of Binary Choice Models with Social Interactions.” *Journal of Econometrics*, 140 (1): 52-75.
- [19] Crawford, V. and Haller, H. (1990): “Learning How to Cooperate: Optimal Play in Repeated Coordination Games.” *Econometrica*, 58 (3): 571-595.
- [20] Delgado, M. and Hidalgo, J. (2000): “Nonparametric Inference on Structural Breaks.” *Journal of Econometrics*, 96: 113-144.
- [21] De Paula, A. and Tang, X. (2012): “Inference of Signs of Interaction Effects in Simultaneous Games With Incomplete Information.” *Econometrica*, 80 (1): 143-172.
- [22] Doraszelski, U., G. Lewis and Pakes, A. (2017): “Just Starting Out: Learning and Equilibrium in a New Market.” Working Paper, Harvard University.
- [23] Ellickson, P. and Misra, S. (2012): “Estimating Discrete Games.” *Marketing Science*, 30 (6): 997-1010.
- [24] Ferris, J. S. (1990): “Time, Space, and Shopping: The Regulation of Shopping Hours.” *Journal of Law, Economics, and Organization* 6 (1): 171-187.
- [25] Fershtman, C. and Pakes, A. (2012): “Dynamic Games with Asymmetric Information: A Framework for Empirical Work.” *Quarterly Journal of Economics*, 127 (4): 1611-1661.
- [26] Goeree, J. and Holt, C. (2001): “Ten Little Treasures of Game Theory and Ten Intuitive Contradictions.” *American Economic Review*, 91: 1402-1422.
- [27] Goldfarb, A. and Xiao, M. (2011): “Who Thinks about the Competition? Managerial Ability and Strategic Entry in US Local Telephone Markets.” *American Economic Review*, 101 (7): 3130-3161.
- [28] Gowrisankaran, G. and Krainer, J. (2011): “Entry and Pricing in a Differentiated Products Industry: Evidence from the ATM Market.” *Rand Journal of Economics*, 42 (1): 1-22.
- [29] Guerre, E., I. Perrigne and Vuong, Q. (2009): “Nonparametric Identification of Risk Aversion in First-Price Auctions Under Exclusion Restrictions.” *Econometrica*, 77 (4), 1193-1227.
- [30] Henderson, V., A. Storeygard and Weil, D. (2012): “Measuring Economic Growth from Outer Space.” *American Economic Review*, 102 (2): 994-1028.
- [31] Heinemann, F., R. Nagel and Ockenfels, P. (2009): “Measuring Strategic Uncertainty in Coordination Games.” *Review of Economic Studies*, 76: 181-221.
- [32] Hotz, J. and Miller, R. (1993): “Conditional Choice Probabilities and the Estimation of Dynamic Models.” *Review of Economic Studies*, 60: 497-529.
- [33] Inderst, R. and Irmen, A. (2005): “Shopping Hours and Price Competition.” *European Economic Review*, 49 (5): 1105-1124.
- [34] Jeon, J. (2017): “Learning and Investment under Demand Uncertainty in Container Shipping.” Manuscripts.

- [35] Kashaev, N. (2016): “Testing for Nash Behavior in Binary Games of Complete Information.” Working Paper, Western University.
- [36] Klemperer, P. and J. Padilla (1997): “Do Firms’ Product Lines Include Too Many Varieties?” *RAND Journal of Economics*, 28 (3): 472-488.
- [37] Kügler, A. and C. Weiss (2016): “Time as a Strategic Variable: Business Hours in the Gasoline Market.” *Applied Economics Letters*, 23 (15): 1051-1056.
- [38] Lewbel, A. (2000): “Semiparametric Qualitative Response Model Estimation with Unknown Heteroscedasticity or Instrumental Variables.” *Journal of Econometrics*, 97: 145-177.
- [39] Lewbel, A. and Tang, X. (2015): “Identification and Estimation of Games with Incomplete Information Using Excluded Regressors.” *Journal of Econometrics*, 189: 229-244.
- [40] Marcoux, M. (2017): “Games with Unobservable Heterogeneity and Multiple Equilibria: An Application to Mobile Telecommunications.” Manuscript.
- [41] Matzkin, R. (1992): “Nonparametric and Distribution-Free Estimation of the Binary Threshold Crossing and The Binary Choice Models.” *Econometrica*, 60 (2): 239-270.
- [42] Morris, S., and Shin, H. (2002): “Social Value of Public Information.” *American Economic Review*, 92: 1521-1534.
- [43] Morris, S., and Shin, H. (2004): “Coordination Risk and the Price of Debt.” *European Economic Review*, 48: 133-153.
- [44] Müller, H. (1992): “Change-Points in Nonparametric Regression Analysis.” *Annals of Statistics*, 20: 737-761.
- [45] Nishida, M. (2014): “Estimating a Model of Strategic Network Choice: The Convenience-Store Industry in Okinawa.” *Marketing Science*, 34 (1): 20-38.
- [46] Orhun, A. (2013): “Spatial Differentiation in the Supermarket Industry: The Role of Common Information.” *Quantitative Marketing and Economics*, 11: 3-37.
- [47] Pakes, A., M. Ostrovsky, and Berry, S. (2007): “Simple Estimators for the Parameters of Discrete Dynamic Games (with entry/exit examples).” *Rand Journal of Economics*, 38: 373-399.
- [48] Paarsch, H. (1992): “Deciding Between the Common and Private Value Paradigms in Empirical Models of Auctions.” *Journal of Econometrics*, 51, 191-215.
- [49] Pearce, D. (1984): “Rationalizable Strategic Behavior and the Problem of Perfection.” *Econometrica*, 52: 1029-1050.
- [50] Pesendorfer, M., and Schmidt-Dengler, P. (2008): “Asymptotic Least Squares Estimators for Dynamic Games.” *Review of Economic Studies*, 75, 901-928.
- [51] Rennhoff, A., and Owens, M. (2012): “Competition and the Strategic Choices of Churches.” *American Economic Journal: Microeconomics*, 4 (3): 152-170.
- [52] Seim, K. (2006): “An Empirical Model of Firm Entry with Endogenous Product-Type Choices.” *Rand Journal of Economics*, 37 (3): 619-640.

- [53] Shen, Q., and Xiao, P. (2014): “McDonald’s and KFC in China: Competitors or Companions?” *Marketing Science*, 33 (2): 287-307.
- [54] Shy, O., and Stenbacka, R. (2006): “Service Hours with Asymmetric Distributions of Ideal Service Time.” *International Journal of Industrial Organization*, 24 (4): 763-771.
- [55] Shy, O., and Stenbacka, R. (2008): “Price Competition, Business Hours and Shopping Time Flexibility.” *The Economic Journal*, 118: 1171-1195.
- [56] Skuterud, M. (2005): “The Impact of Sunday Shopping on Employment and Hours of Work in the Retail Industry: Evidence from Canada.” *European Economic Review*, 49 (8): 1953-1978.
- [57] Su, C. (2014): “Estimating Discrete-Choice Games of Incomplete Information: Simple Static Examples.” *Quantitative Marketing and Economics*, 12 (2): 167-207.
- [58] Sweeting, A. (2009): “The Strategic Timing Incentives of Commercial Radio Stations: An Empirical Analysis Using Multiple Equilibria.” *Rand Journal of Economics*, 40 (4): 710-742.
- [59] Tanguay, G., L. Vallge and Lanoie, P. (1995): “Shopping Hours and Price Levels in the Retailing Industry: A Theoretical and Empirical Analysis.” *Economic Inquiry*, 33 (3): 516-524.
- [60] Van Huyck, R. Battalio and Beil, R. (1990): “Tacit Coordination Games, Strategic Uncertainty, and Coordination Failure.” *American Economic Review*, 80 (1): 234-248.
- [61] Vitorino, M. (2012): “Empirical Entry Games with Complementarities: An Application to the Shopping Center Industry.” *Journal of Marketing Research*, 49 (2): 175-191.
- [62] Wan, Y. and Xu, H. (2014): “Semiparametric Identification and Estimation of Binary Decision Games of Incomplete Information with Correlated Private Signals.” *Journal of Econometrics*, 182 (2): 235-246.
- [63] Wenzel, T. (2010): “Liberalization of Opening Hours with Free Entry.” *German Economic Review*, 11 (4): 511-526.
- [64] Wenzel, T. (2011): “Deregulation of Shopping Hours: The Impact on Independent Retailers and Chain Stores.” *The Scandinavian Journal of Economics*, 113 (1): 145-166.
- [65] Xiao, R. (2015): “Identification and Estimation of Incomplete Information Games with Multiple Equilibrium.” Working Paper, Indiana University.
- [66] Xu, H. (2014): “Estimation of Discrete Games with Correlated Types.” *the Econometrics Journal*, 17: 241-270.
- [67] Zhu, T., and Singh, V. (2009): “Spatial Competition with Endogenous Location Choices: An Application to Discount Retailing.” *Quantitative Marketing and Economics*, 7 (1): 1-35.

A Proof of Proposition

Proof. Proposition 1:

According to equation (5), we then have following equations for any two actions j and k of player i

$$\pi_i(a_i = j) + \delta_i(a_i = j, a_{-i} = 1) \cdot b_i^1 = F_i^j(\mathbf{p}_i),$$

$$\pi_i(a_i = k) + \delta_i(a_i = k, a_{-i} = 1) \cdot b_i^1 = F_i^k(\mathbf{p}_i).$$

It is easy to see that we can cancel b_i^1 using previous two equations. It yields proposition 1 (a)

Now, suppose $J_{-i} = 0$ (i.e. player $-i$ has only one choice), then $b_{i, J_{-i}=0}^1 = 0$ and equation (5) turns

$$\pi_i(a_i = k) = F_i^k(\mathbf{p}_{i, J_{-i}=0}).$$

It yields identification of $\pi_i(a_i)$. Furthermore, for $J_{-i} = 1$, combining equation (5) and identification results of $\pi_i(a_i)$. It yields

$$\delta_i(a_i = k, a_{-i} = 1) b_i^1 = F_i^k(\mathbf{p}_{i, J_{-i}=1}) - F_i^k(\mathbf{p}_{i, J_{-i}=0}) \quad \forall 0 \leq k \leq J_i.$$

Therefore, the perceived interactive effect $\delta_i(a_i = k, a_{-i} = 1) b_i^1$ is identified. The above equation characterizes the identified set for the interactive effect and each player's belief. \square

Proof. Proposition 2:

Given proposition 1, we have following equations for any two alternatives j and k

$$\delta_i(a_i = j, a_{-i} = 1) b_i^1 = F_i^j(\mathbf{p}_{i, J_{-i}=1}) - F_i^j(\mathbf{p}_{i, J_{-i}=0}),$$

$$\delta_i(a_i = k, a_{-i} = 1) b_i^1 = F_i^k(\mathbf{p}_{i, J_{-i}=1}) - F_i^k(\mathbf{p}_{i, J_{-i}=0}).$$

Assuming $\delta_i(a_i, a_{-i} = 1) b_i^1 \neq 0$, we can take division and get

$$\frac{\delta_i(a_i = j, a_{-i} = 1)}{\delta_i(a_i = k, a_{-i} = 1)} = \frac{F_i^j(\mathbf{p}_{i, J_{-i}=1}) - F_i^j(\mathbf{p}_{i, J_{-i}=0})}{F_i^k(\mathbf{p}_{i, J_{-i}=1}) - F_i^k(\mathbf{p}_{i, J_{-i}=0})}.$$

Furthermore, as player i 's belief is a valid probability distribution, we have $0 \leq b_i^1 \leq 1$. It consequently yields

$$\text{sign}\{\delta_i(a_i = j, a_{-i} = 1)\} = \text{sign}\{F_i^j(\mathbf{p}_{i, J_{-i}=1}) - F_i^j(\mathbf{p}_{i, J_{-i}=0})\},$$

$$|\delta_i(a_i = j, a_{-i} = 1)| \geq |F_i^j(\mathbf{p}_{i, J_{-i}=1}) - F_i^j(\mathbf{p}_{i, J_{-i}=0})|.$$

Finally, for any $J'_i, J''_i \geq 1$, we have

$$\begin{aligned}\delta_i(a_i = j, a_{-i} = 1)b_{i,J'_i}^1 &= F_i^j(\mathbf{p}_{i,J'_i,J_{-i}=1}) - F_i^j(\mathbf{p}_{i,J'_i,J_{-i}=0}), \\ \delta_i(a_i = j, a_{-i} = 1)b_{i,J''_i}^1 &= F_i^j(\mathbf{p}_{i,J''_i,J_{-i}=1}) - F_i^j(\mathbf{p}_{i,J''_i,J_{-i}=0}).\end{aligned}$$

Taking division yields

$$\frac{b_{i,J'_i}^1}{b_{i,J''_i}^1} = \frac{F_i^j(\mathbf{p}_{i,J'_i,J_{-i}=1}) - F_i^j(\mathbf{p}_{i,J'_i,J_{-i}=0})}{F_i^j(\mathbf{p}_{i,J''_i,J_{-i}=1}) - F_i^j(\mathbf{p}_{i,J''_i,J_{-i}=0})}.$$

This completes the proof. □

Proof. Proposition 4:

Given player i 's expected payoff in equation (6), we then have following

$$\pi_i(z_i, a_i = j) + \delta_i(z_i, a_i = j, a_{-i} = 1)g_i(z_i, z_{-i}) = F_i^j(\mathbf{p}_i). \quad (11)$$

Given assumption 6 (a), we can find at least two realizations of z_{-i} , say z_{-i}^1 and z_{-i}^2 . Plug them separately into above equation and subtract to cancel $\pi_i(z_i, a_i = k)$, it becomes

$$\delta_i(z_i, a_i = j, a_{-i} = 1) \cdot [g_i(z_i, z_{-i}^1) - g_i(z_i, z_{-i}^2)] = F_i^j[\mathbf{p}_i(z_i, z_{-i}^1)] - F_i^j[\mathbf{p}_i(z_i, z_{-i}^2)] \quad \forall 0 \leq j \leq J_1.$$

Suppose $\mathbf{p}_i(z_i, z_{-i}^1) \neq \mathbf{p}_i(z_i, z_{-i}^2)$, it implies that $g_i(z_i, z_{-i}^1) - g_i(z_i, z_{-i}^2) \neq 0$.⁴¹ Note that the difference of subjective expectations does not depend on which action is taken by player i . Therefore, for any two actions j and k , we can cancel the term $g_i(z_i, z_{-i}^1) - g_i(z_i, z_{-i}^2)$ by taking division and get

$$\frac{\delta_i(z_i, a_i = j, a_{-i} = 1)}{\delta_i(z_i, a_i = k, a_{-i} = 1)} = \frac{F_i^j[\mathbf{p}_i(z_i, z_{-i}^1)] - F_i^j[\mathbf{p}_i(z_i, z_{-i}^2)]}{F_i^k[\mathbf{p}_i(z_i, z_{-i}^1)] - F_i^k[\mathbf{p}_i(z_i, z_{-i}^2)]}.$$

Since the terms on right hand side are known, $\frac{\delta_i(z_i, a_i = j, a_{-i} = 1)}{\delta_i(z_i, a_i = k, a_{-i} = 1)}$ is identified. Furthermore, given multiplicative separable assumption 5, $\frac{\delta_i(z_i, a_i = j, a_{-i} = 1)}{\delta_i(z_i, a_i = k, a_{-i} = 1)} = \frac{\delta_i(z_i, a_i = j, a_{-i} = 1)}{\delta_i(z_i, a_i = k, a_{-i} = 1)}$ for any a_{-i} . This proves proposition 4 (a).

⁴¹Conditional on z_i , player i 's payoff function $\pi_i(z_i, a_i)$ and $\delta_i(z_i, a_i = 1)$ is fixed, therefore, the only reason that player i 's choice probability varies as z_{-i} varies is because player i 's subjective expectation $g_i(z_i, z_{-i})$ varies as z_{-i} varies.

Given results in proposition 3, we have following equation for any two actions j and k

$$\frac{F_i^j[\mathbf{p}_i(z_i, z_{-i}, s)] - \pi_i(z_i, a_i = j)}{F_i^k[\mathbf{p}_i(z_i, z_{-i}, s)] - \pi_i(z_i, a_i = k)} = \frac{\delta_i(z_i, s, a_i = j, a_{-i} = 1)}{\delta_i(z_i, s, a_i = k, a_{-i} = 1)}.$$

Under assumption 6 (b), there must exist s^1 and s^2 , such that $\frac{\delta_i(z_i, s^1, a_i = j, a_{-i} = 1)}{\delta_i(z_i, s^1, a_i = k, a_{-i} = 1)} \neq \frac{\delta_i(z_i, s^2, a_i = j, a_{-i} = 1)}{\delta_i(z_i, s^2, a_i = k, a_{-i} = 1)}$,

therefore, previous equation turns to

$$\begin{aligned} \pi_i(z_i, a_i = j) - \frac{\delta_i(z_i, s^1, a_i = j, a_{-i} = 1)}{\delta_i(z_i, s^1, a_i = k, a_{-i} = 1)} \pi_i(z_i, a_i = k) &= F_i^j[\mathbf{p}_i(z_i, z_{-i}, s^1)] - \frac{\delta_i(z_i, s^1, a_i = j, a_{-i} = 1)}{\delta_i(z_i, s^1, a_i = k, a_{-i} = 1)} F_i^k[\mathbf{p}_i(z_i, z_{-i}, s^1)] \\ \pi_i(z_i, a_i = j) - \frac{\delta_i(z_i, s^2, a_i = j, a_{-i} = 1)}{\delta_i(z_i, s^2, a_i = k, a_{-i} = 1)} \pi_i(z_i, a_i = k) &= F_i^j[\mathbf{p}_i(z_i, z_{-i}, s^2)] - \frac{\delta_i(z_i, s^2, a_i = j, a_{-i} = 1)}{\delta_i(z_i, s^2, a_i = k, a_{-i} = 1)} F_i^k[\mathbf{p}_i(z_i, z_{-i}, s^2)] \end{aligned}$$

Since $\frac{\delta_i(z_i, s, a_i = j, a_{-i} = 1)}{\delta_i(z_i, s, a_i = k, a_{-i} = 1)}$ is identified, this is a linear equation system containing two unknowns (i.e. $\pi_i(z_i, a_i = j)$ and $\pi_i(z_i, a_i = k)$) and two equations. Given $\frac{\delta_i(z_i, s^1, a_i = j, a_{-i} = 1)}{\delta_i(z_i, s^1, a_i = k, a_{-i} = 1)} \neq \frac{\delta_i(z_i, s^2, a_i = j, a_{-i} = 1)}{\delta_i(z_i, s^2, a_i = k, a_{-i} = 1)}$, $\pi_i(z_i, a_i = j)$ and $\pi_i(z_i, a_i = k)$ are uniquely determined through this system. In addition, according to equation (11), $\delta_i(z_i, s, a_i = k, a_{-i} = 1) \cdot g_i(z_i, z_{-i}, s)$ is identified thereafter. Next, for an action $l \neq j, k$, it is clear that $\delta_i(z_i, s, a_i = l, a_{-i} = 1) \cdot g_i(z_i, z_{-i}, s)$ is identified as $\delta_i(z_i, s, a_i = k, a_{-i} = 1) \cdot g_i(z_i, z_{-i}, s) \cdot \frac{\delta_i(z_i, s, a_i = l, a_{-i} = 1)}{\delta_i(z_i, s, a_i = k, a_{-i} = 1)}$. Finally, researcher can uniquely determine the value of $\pi_i(z_i, s, a_i = l)$ according to equation (11) given the identification of $\delta_i(z_i, s, a_i = l, a_{-i} = 1) \cdot g_i(z_i, z_{-i}, s)$. This completes the proof \square

B Relaxation of Known Distribution of $G_i(\cdot)$

In the main text, player i 's private information is assumed to be independent across players and independent of public information \mathbf{x} . Moreover, the distribution of this private information is assumed to be known by researchers. The commonly used distributional assumptions in practice include i.i.d. type 1 extreme value distribution (i.e. Logit Model) and i.i.d. standard normal distribution (i.e. Multinomial Probit Model). These assumptions are restrictive in the sense that they restrict the private information among actions to be independent and homoscedastic. However, the distributional assumption of ε_i can be relaxed to capture heteroskedasticity and potential correlation among actions in a fairly flexible way. This subsection formally establishes this point with the help of exclusion restriction z_i and variation in players' action sets. First, consider an assumption 1' that is a weaker version of assumption 1.

Assumption 1'. (a) for each $i = 1, 2$, $\boldsymbol{\varepsilon}_i = (\varepsilon_i(0), \varepsilon_i(1), \dots, \varepsilon_i(J_i))'$ follows a CDF $G_i(\cdot; \boldsymbol{\beta}_{i,x})$ that is absolutely continuous with respect to Lebesgue measure in \mathbb{R}^{J_i+1} . $\boldsymbol{\beta}_{i,x} = (\beta_{1,i,x}, \dots, \beta_{L_i,i,x})'$ is a vector of parameters with $L_i < \infty$ dimensions. Moreover, researchers know the functional form of $G_i(\cdot)$ but not the parameters $\boldsymbol{\beta}_{i,x}$ for $i = 1, 2$.

(b) $\boldsymbol{\varepsilon}_i$ is independent of $\boldsymbol{\varepsilon}_{-i}$ conditional on (\mathbf{x}, z_1, z_2) . Moreover, conditional on \mathbf{x} , $\boldsymbol{\varepsilon}_i$ is independent of (z_1, z_2) for $i = 1, 2$.

Assumption 1' parametrizes the distribution of $\boldsymbol{\varepsilon}_i$ by a vector $\boldsymbol{\beta}_{i,x}$ which is unknown by researchers. Such parametrization relaxes assumption 1 in several directions. First, $\boldsymbol{\beta}_{i,x}$ can contain the standard deviation of $\varepsilon_i(a_i)$ for different a_i and potential correlation between $\varepsilon_i(j)$ and $\varepsilon_i(k)$ for $j \neq k$. As a consequence, it captures the possible heteroskedasticity and correlation of private information among different actions in a fairly flexible way. Second, assumption 1' allows the distribution of $\boldsymbol{\varepsilon}_i$ to be correlated with \mathbf{x} and such correlation is captured by the dependence of $\boldsymbol{\beta}_{i,x}$ on \mathbf{x} .⁴² Since I investigate identification conditional on \mathbf{x} , I write $\boldsymbol{\beta}_{i,x}$ as $\boldsymbol{\beta}_i$ for notation simplicity. Note that the identification results do not require exclusion restriction s .

An implication of assumption 1' is that the inverse of best response probability function $F_i(\cdot)$ will depend on $\boldsymbol{\beta}_i$. We then have following equations

$$\pi_i(z_i, a_i = j) + \sum_{k=1}^{J_i} \delta_i(z_i, a_i = j, a_2 = k) \cdot b_i^k(z_i, z_{-i}) = F_i^j[\mathbf{p}_i(z_i, z_{-i}); \boldsymbol{\beta}_i] \quad \forall 0 \leq j \leq J_1.$$

Given assumption 1', $F_i(\cdot, \boldsymbol{\beta}_i)$ is known by researchers up to the unknown parameters $\boldsymbol{\beta}_i$. Addition-

⁴²Lewbel and Tang (2015) generalize the *special regressor* approach considered in Matzkin (1992) and Lewbel (2000) to a two-player binary choice game with incomplete information. They show that if researchers can observe a variable that affects a player's payoff linearly, then the distribution of the error term is non-parametrically identified under the Bayesian Nash Equilibrium condition. Despite its power, this special regressor approach does not work in my framework since I non-parametrically specify the payoff function; as a result, I parametrize the function $G_i(\cdot)$.

ally, assumption 6 (a) implies that there exist h realizations of z_{-i} , say z_{-i}^1, z_{-i}^2 up to z_{-i}^h , such that

$$\begin{aligned}
F_i^1 [\mathbf{p}_{i,J_i=0}(z_i, z_{-i}); \boldsymbol{\beta}_i] &= \pi_i(z_i, a_i = 1), \\
&\vdots \\
F_i^{J_i} [\mathbf{p}_{i,J_i=0}(z_i, z_{-i}); \boldsymbol{\beta}_i] &= \pi_i(z_i, a_i = J_i), \\
F_i^1 [\mathbf{p}_{i,J_i=1}(z_i, z_{-i}^1); \boldsymbol{\beta}_i] &= \pi_i(z_i, a_i = 1) + \delta_i(z_i, a_i = 1, a_{-i} = 1) \cdot b_i^1(z_i, z_{-i}^1), \\
F_i^2 [\mathbf{p}_{i,J_i=1}(z_i, z_{-i}^1); \boldsymbol{\beta}_i] &= \pi_i(z_i, a_i = 2) + \delta_i(z_i, a_i = 2, a_{-i} = 1) \cdot b_i^1(z_i, z_{-i}^1), \\
&\vdots \\
F_i^{J_i} [\mathbf{p}_{i,J_i=1}(z_i, z_{-i}^h); \boldsymbol{\beta}_i] &= \pi_i(z_i, a_i = J_i) + \delta_i(z_i, a_i = J_i, a_{-i} = 1) \cdot b_i^1(z_i, z_{-i}^h).
\end{aligned} \tag{12}$$

Equation system (12) consists of $J_i(h+1)$ equations with $2J_i + h - 1 + L_i$ unknowns.⁴³ A necessary order condition for identification is $J_i(h+1) \geq 2J_i + h - 1 + L_i$ which yields $(J_i - 1)(h - 1) \geq L_i$. Moreover, denote $\mathbf{F}(z_i, \mathbf{z}_{-i}^{1:h}, J_i; \boldsymbol{\beta}_i) = (F_i^1[\mathbf{p}_{i,J_i=0}(z_i, z_{-i}); \boldsymbol{\beta}_i], \dots, F_i^{J_i}[\mathbf{p}_{i,J_i=1}(z_i, z_{-i}^h); \boldsymbol{\beta}_i])'$ as a $J_i(h+1) \times 1$ vector of inversion of choice probability, following assumption 7 establishes a sufficient condition for the identification of $\boldsymbol{\beta}_i$.

Assumption 7. *Conditional on $(\mathbf{x}, J_i > 1, z_i)$, there are $h \geq 2$ realizations of z_2 such that $(J_i - 1)(h - 1) \geq L_i$. Moreover, let $\frac{\partial \mathbf{F}(z_i, \mathbf{z}_{-i}^{1:h}, J_i; \boldsymbol{\beta}_i)}{\partial \boldsymbol{\beta}_i}$ be a Jacobian matrix with dimension $J_i(h+1) \times L_i$; such matrix has column rank L_i .*

Proposition 5. *Under assumption 6 (a), 7 and conditions met in proposition 1 with assumption 1 replaced by assumption 1', $\boldsymbol{\beta}_i$ is identified in a neighborhood of its true value.*

Proof. $\boldsymbol{\beta}_i$ does not enter into the right hand side of equation system (12); as shown in subsection 3.2, all unknowns on the right hand side are identified if researchers know the value of $\boldsymbol{\beta}_i$. Since there are $2J_i + h - 1$ unknowns on the right hand side, there still remain $(J_i - 1)(h - 1) \geq L_i$ restrictions that can be

⁴³There are J_i unknowns for $\pi_i(z_i, a_i = j) \forall 1 \leq j \leq J_i$, h unknowns for $\delta_i(z_i, a_i = 1, a_{-i} = 1) \cdot b_i^1(z_i, z_{-i})$, $(J_i - 1)$ unknowns for $\frac{\delta_i(z_i, a_i = j, a_{-i} = 1)}{\delta_i(z_i, a_i = 1, a_{-i} = 1)} \forall 1 < j \leq J_i$ and L_i unknowns for $\boldsymbol{\beta}_i$. Note that as shown in subsection 3.2, only $\delta_i(z_i, a_i = 1, a_{-i} = 1) \cdot b_i^1(z_i, z_{-i})$ is identified while $\delta_i(z_i, a_i = 1, a_{-i} = 1)$ and $b_i^1(z_i, z_{-i})$ are not distinguishable from each other, so I treat the perceived interactive effects as unknowns. In addition, $\delta_i(z_i, a_i = 1, a_{-i} = 1) \cdot b_i^1(z_i, z_{-i})$ and $\frac{\delta_i(z_i, a_i = j, a_{-i} = 1)}{\delta_i(z_i, a_i = 1, a_{-i} = 1)}$ perfectly determine the value of $\delta_i(z_i, a_i = j, a_{-i} = 1) \cdot b_i^1(z_i, z_{-i})$ and therefore $\delta_i(z_i, a_i = j, a_{-i} = 1) \cdot b_i^1(z_i, z_{-i}) \forall j \neq 1$ does not count as an unknown.

exploited to identify β_i . Under assumption 7, both the rank and order condition are satisfied and therefore β_i is locally identified. \square

Assumption 7 is a generic assumption. Suppose β_i is identified when researchers perfectly know player i 's payoff and belief, then the Jacobian matrix $\frac{\partial \mathbf{F}(z_i, \mathbf{z}_{-i}^{1:h}, J_i; \beta_i)}{\partial \beta_i}$ will have full column rank with probability one. This implies that researchers can capture heteroskedasticity and correlation as flexible as in the standard discrete choice model.

C Identification Result when $J_{-i} > 1$

In this section, I present the identification result of player i 's payoff function when player $-i$ has more than two actions. Same as the main text, the distribution of private information is assumed to be known by researcher in this section.

For some $k \leq J_i - J_{-i}$, define a $J_{-i} \times J_{-i}$ matrix of interaction effect $\Delta_i^{k:J_i+k-1}$ as

$$\begin{bmatrix} \delta_i(a_i = k, a_{-i} = 1), & \delta_i(a_{-i} = k, a_{-i} = 2), & \cdots, & \delta_i(a_i = k, a_{-i} = J_{-i}) \\ \delta_i(a_i = k + 1, a_{-i} = 1), & \delta_i(a_{-i} = k + 1, a_{-i} = 2), & \cdots, & \delta_i(a_i = k + 1, a_{-i} = J_{-i}) \\ \vdots & \vdots & \ddots & \vdots \\ \delta_i(a_i = k + J_{-i} - 1, a_{-i} = 1), & \delta_i(a_{-i} = k + J_{-i} - 1, a_{-i} = 2) & \cdots, & \delta_i(a_i = k + J_{-i} - 1, a_{-i} = J_{-i}) \end{bmatrix}.$$

Moreover, let $\boldsymbol{\pi}_i^{k:J_i+k-1} = (\pi_i(a_i = k), \dots, \pi_i(a_i = k + J_{-i} - 1))'$ and $\mathbf{F}_i^{k:J_i+k-1} = (F_i^k(\mathbf{p}_i), \dots, F_i^{k+J_{-i}-1}(\mathbf{p}_i))'$, we then have following proposition.

Proposition 6. (a) Under assumption 1 to 4 and suppose $\Delta_i^{k:J_i+k-1}$ is invertible for any k and data contain observations with $J_i > J_{-i}$; then for any $k', k \leq J_i - J_{-i}$, the identified set of player i 's payoff is given by the set of values that satisfy the following restrictions.

$$[\Delta_i^{k:J_i+k-1}]^{-1} [\mathbf{F}_i^{k:J_i+k-1} - \boldsymbol{\pi}_i^{k:J_i+k-1}] = [\Delta_i^{k':J_i+k'-1}]^{-1} [\mathbf{F}_i^{k':J_i+k'-1} - \boldsymbol{\pi}_i^{k':J_i+k'-1}].$$

(b) Suppose further that data also contain observations with $J_{-i} = 0$, then $\pi_i(a_i = k)$ is identified by $F_i^k(\mathbf{p}_{i,J_{-i}=0}) \forall k$. Furthermore, the identified set of player i 's interactive effect and belief is given by the

set of values that satisfy the following restriction

$$\sum_{j=1}^{J_i} \delta_i(a_i = k, a_{-i} = j) b_i^j = F_i^k(\mathbf{p}_{i, J_i}) - F_i^k(\mathbf{p}_{i, J_i=0}).$$

Proof. By construction, we have

$$\boldsymbol{\pi}_i^{k:J_i+k-1} + \Delta_i^{k:J_i+k-1} \mathbf{b}_i = \mathbf{F}_i^{k:J_i+k-1}.$$

Under assumption such that $\Delta_i^{k:J_i+k-1}$ is invertible, we then have the vector of belief equals to

$$\mathbf{b}_i = [\Delta_i^{k:J_i+k-1}]^{-1} [\mathbf{F}_i^{k:J_i+k-1} - \boldsymbol{\pi}_i^{k:J_i+k-1}].$$

Similarly, for another value of k' , we have $\mathbf{b}_i = [\Delta_i^{k':J_i+k'-1}]^{-1} [\mathbf{F}_i^{k':J_i+k'-1} - \boldsymbol{\pi}_i^{k':J_i+k'-1}]$. Consequently, it yields

$$[\Delta_i^{k:J_i+k-1}]^{-1} [\mathbf{F}_i^{k:J_i+k-1} - \boldsymbol{\pi}_i^{k:J_i+k-1}] = [\Delta_i^{k':J_i+k'-1}]^{-1} [\mathbf{F}_i^{k':J_i+k'-1} - \boldsymbol{\pi}_i^{k':J_i+k'-1}].$$

Furthermore, if there exist observations with $J_i = 0$. Then proposition 6 (b) follows a similar proof as proposition 1 (b) and is omitted. \square

D Identification in Games with Multiple Actions, Multiple Players

Consider a game with N players where $N > 2$. Player is indexed by $i, n \in \{1, 2, \dots, N\}$. Each player i has an action set $A_i = \{0, 1, \dots, J_i\}$. Consequently, Cartesian product $A = A_1 \times A_2 \cdots \times A_N$ represents the space of action profiles in this game. Each player i simultaneously chooses an action a_i from her action set A_i . Let $\mathbf{a} = (a_1, a_2, \dots, a_N) \in A$ be a realized outcome or action profile in this game. Player i 's payoff under \mathbf{a} is

$$\Pi_i(\mathbf{x}, \boldsymbol{\varepsilon}_i, \mathbf{a}) = \pi_i(\mathbf{x}, a_i) + \sum_{n=1, n \neq i}^N \delta_{i,n}(\mathbf{x}, a_i, a_n) \cdot \mathbb{1}(a_n \neq 0) + \varepsilon_i(a_i). \quad (13)$$

The term $\delta_{i,n}(\mathbf{x}, a_i, a_n)$ in equation (13) represents player n 's impact on player i . It allows different players to have heterogeneous interactive effects on player i but requires they are additive separable. When there is variation in players' choice sets, we can identify π_i and $\delta_{i,n}$ by studying the situation that $J_{i'} = 0 \forall i' \neq i, n$. This is essentially the game with two players considered in the main text and all identification results directly follow.

When players' action sets are constant across observations, I consider a restriction on the payoff function that allows for dimension reduction similar as assumption 5

$$\delta_{i,n}(\mathbf{x}, a_i, a_n) = \delta_{i,n}(\mathbf{x}, a_i, a_n = 1) \cdot \eta_{i,n}(\mathbf{x}, a_n) \text{ where } \eta_{i,n}(\mathbf{x}, a_n = 1) = 1.$$

Consequently, player i 's expected payoff for action a_i is

$$\begin{aligned} E[\Pi_i(\mathbf{x}, \boldsymbol{\varepsilon}_i, a_i)] &= \pi_i(\mathbf{x}, a_i) + \sum_{n=1, n \neq i}^N \delta_{i,n}(\mathbf{x}, a_i, a_n = 1) \left[\sum_{j=1}^{J_n} \eta_{i,n}(\mathbf{x}, a_n = j) b_{i,n}^j(\mathbf{x}) \right] + \varepsilon_i(a_i) \\ \Rightarrow E[\Pi_i(\mathbf{x}, \boldsymbol{\varepsilon}_i, a_i)] &= \pi_i(\mathbf{x}, a_i) + \sum_{n=1, n \neq i}^N \delta_{i,n}(\mathbf{x}, a_i, a_n = 1) \cdot g_{i,n}(\mathbf{x}) + \varepsilon_i(a_i). \end{aligned}$$

where $b_{i,n}^j(\mathbf{x})$ represents player i 's belief about the probability that player n will choose action $a_n = j$. Term $g_{i,n}(\mathbf{x}) = \sum_{j=1}^{J_n} \eta_{i,n}(\mathbf{x}, a_n = j) b_{i,n}^j(\mathbf{x})$ represents player i 's subjective expected value of $\eta_{i,n}$. Given the distribution of $\boldsymbol{\varepsilon}_i$, we can invert player i 's conditional choice probability

$$\pi_i(\mathbf{x}, a_i = j) + \sum_{n=1, n \neq i}^N \delta_{i,n}(\mathbf{x}, a_i, a_n = 1) \cdot g_{i,n}(\mathbf{x}) = F_i^j[\mathbf{p}_i(\mathbf{x})].$$

Suppose that $N \leq \min\{J_1, \dots, J_N\}$ (i.e. the number of players is smaller than the number of actions), the above equation shares same structure as the asymmetric game described in appendix C; for instance, a game with two players where player i has $J_i + 1$ actions while player $-i$ has N actions. Therefore, the result of proposition 6 (a) applies. Furthermore, when there exist exclusion restrictions z_i and s , the base

return $\pi_i(\cdot)$ and the perceived interactive payoff $\sum_{n=1, n \neq i}^N \delta_{i,n}(\cdot)g_{i,n}(\cdot)$ are point identified.⁴⁴

E Data Construction and Supplementary Tables

Construction of Cinema Clusters

I obtain a list of cinemas, including address, for each county. I then construct clusters of cinemas by following algorithm:

1. Start with an arbitrary cinema denoted by $cinema_i$ which is not assigned into any cluster, assign this cinema to a new cluster.
2. Draw a radius of 2 km around such $cinema_i$. If no other cinema exists in this area, the algorithm terminates and we start a new unassigned cinema by step 1. If there exist some cinemas in this area, assign those cinemas in the same cluster as $cinema_i$.
3. For each newly assigned cinema, repeat step 2.

Estimation Techniques

Given belief function (10), base return (7) and the interactive effect (8), KFC's expected payoff for action $a_{KFC} = j$ is

$$E[\tilde{\Pi}_{KFC}(a_{KFC} = j; \alpha, \theta, \lambda)] = \pi_{KFC}(a_{KFC} = j; \alpha) + \delta(a_{KFC} = j; \theta) \cdot b_{KFC}^1(\lambda).$$

Note that $\tilde{\Pi}_i(\cdot)$ represents the part of player i 's payoff function that is public information. For notation simplicity, I suppress the observed exogenous variable $(\mathbf{x}, z_{KFC}, z_{MCD}, s)$. Given i.i.d. type 1 extreme value distribution of $\varepsilon_{KFC}(a_{KFC} = j)$, KFC's choice probability of action $a_{KFC} = j$ is

$$p_{KFC}(j; \alpha, \theta, \lambda) = \frac{\exp\{E[\tilde{\Pi}_{KFC}(a_{KFC} = j; \alpha, \theta, \lambda)]\}}{1 + \sum_{l=1}^2 \exp\{E[\tilde{\Pi}_{KFC}(a_{KFC} = l; \alpha, \theta, \lambda)]\}}.$$

⁴⁴A formal proof is available from the author upon request.

Denote $a_{KFC,m}$ as KFC's actual choice in market m , then the log-likelihood function is defined as

$$\log(a_{KFC}; \alpha, \theta, \lambda) = \sum_{m=1}^M \sum_{j=0}^2 \mathbb{1}(a_{KFC,m} = j) \log [p_{KFC}(j; \alpha, \theta, \lambda)].$$

The Gauss-Seidel method is implemented through following iteration:

1. Start with an initial value of λ denoted by λ^1 , then solve $(\alpha^1, \theta^1) = \operatorname{argmax}_{\{\alpha, \theta\}} \log(a_{KFC}; \alpha, \theta, \lambda^1)$.
Given any λ^1 , such function is globally concave in α and θ .
2. Given value α^1 and θ^1 , update λ by solving $\lambda^2 = \operatorname{argmax}_{\lambda} \log(a_{KFC}; \alpha^1, \theta^1, \lambda)$. This log-likelihood function is not globally concave in λ and consequently a global search algorithm is required. However, it saves computational burden compared to a joint global search for α , θ and λ .
3. Continue update the values of λ through step 1 to 2 and terminate until two adjacent values, λ^k and λ^{k+1} , are arbitrarily close.

Upon convergence, this method attains local maximum. In my practice, this method always converge. However, it will not guarantee a global maximum. In estimation, I try lots of starting values.

When the dimension of $\lambda \in \Lambda$ is relatively small, researchers can exploit a brutal-force global search algorithm to find global maximum. It follows as:

1. Choose K equal distant points in the compact set Λ denoted by $\{\lambda^1, \lambda^2, \dots, \lambda^K\}$. In practice, K is a large number such that the distance between two adjacent points is extremely small.
2. For any k , find α^k, θ^k by solving $(\alpha^k, \theta^k) = \operatorname{argmax}_{\{\alpha, \theta\}} \log(a_{KFC}; \alpha, \theta, \lambda^k)$.
3. Let $k = \operatorname{argmax}_{1 \leq k \leq K} \log(a_{KFC}; \alpha^k, \theta^k, \lambda^k)$. Then α^k , θ^k and λ^k are the estimates.

In the estimation parts, I verify the estimates through Gauss-Seidel method by global search algorithm.

Supplementary Tables and Graphs

Table 7: Reduced Form Evidence: Biased Beliefs

	MCD's Choice	KFC's Choice	
		One 24 H	Two 24 H
$z_{MCD} \times IncomeQ_1$	-1.5752* (0.8869)	-1.2158* (0.6140)	-1.8364* (0.9897)
$z_{MCD} \times IncomeQ_2$	-1.3699* (0.8200)	-1.2800** (0.6326)	-1.4751 (1.0074)
$z_{MCD} \times IncomeQ_3$	-1.4179** (0.6803)	-0.6075 (0.7121)	-0.6013 (1.0046)
$z_{MCD} \times IncomeQ_4$	-1.1224* (0.6550)	-0.1599 (0.6555)	-0.2363 (0.8841)
$z_{MCD} \times KFCDist$	0.4592* (0.2682)	0.0773 (0.3164)	-0.0693 (0.4216)
Control Variables	Yes	Yes	
log-likelihood	-43.0099	-394.5789	
Observations	125	681	

Note: Standard error in parenthesis is calculated using Jackknife. One observation is kept in all Jackknife samples. $IncomeQ_i$ is a dummy that equals to one if the market lies in the i th quartile of income.

Figure 6: KFC's Interactive Payoff

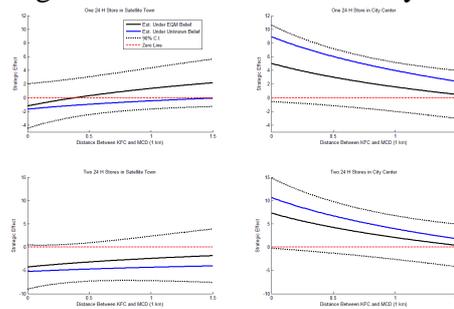


Table 8: Logit Estimates of Chains' Existence

	KFC's Existence	McD's Existence
<i>Income</i>	0.4914*** (0.1191)	0.4548** (0.2017)
<i>Income</i> ²	-0.0103*** (0.0029)	-0.0216** (0.0085)
<i>Pop</i>	0.0822 (0.1229)	0.1286 (0.1711)
<i>Pop</i> ²	-0.005 (0.0047)	-0.0002 (0.0065)
z_{KFC}	-0.0447 (0.3218)	0.1027 (0.6124)
z_{KFC}^2	0.0382 (0.0400)	0.0809 (0.0868)
z_{MCD}	-0.5105** (0.2332)	-0.5486 (0.4555)
z_{MCD}^2	0.0772*** (0.0280)	0.0772 (0.0524)
$z_{KFC} \times Income$	0.0102 (0.0227)	0.0615 (0.0396)
$z_{KFC} \times Pop$	0.0574** (0.0247)	0.0011 (0.0415)
$z_{MCD} \times Income$	-0.0209 (0.0174)	-0.0196 (0.0280)
$z_{MCD} \times Pop$	0.0000 (0.0172)	-0.0079 (0.0271)
$z_{KFC} \times z_{MCD}$	-0.0796 (0.0583)	-0.1525 (0.1082)
<i>Income</i> \times <i>Pop</i>	0.0384*** (0.0145)	0.0082 (0.0159)
<i>Center</i>	5.6754*** (1.2881)	1.5944*** (0.5457)
<i>Pop</i> \times <i>Center</i>	-0.3054*** (0.1032)	0.0176 (0.0613)
<i>Income Growth</i>	-0.0769*** (0.0217)	-0.0676* (0.0379)
log-likelihood	-605.7748	-257.2471
observations	1208	681

Note: Standard error in parenthesis. KFC acts as a potential entrant in markets such that at least one KFC, McD or Dicos exists. McD acts as a potential entrant in markets such that KFC operates. The variable *Income Growth* (i.e. average growth rate of GDP in past 15 years) is added as an exclusion restriction to exogenously vary each player's entry decision.