Misspecification in DSGE Models and Policy Implications: Empirical Evidence from the Euro Area

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Abstract

We investigate about possible misspecification on the estimated medium-scale New Keyenesian model for the EURO Area. In particular, we exploit the role of expectations and the implications for the monetary policy. The DSGE-VAR hybrid model is implemented to assess the sources of model misspecification. Our findings provide evidence about misspecification in using the rational expectations hypothesis which fails to capture the co-movements between macroeconomic variables and expectations. In a forecasting comparison, the DSGE-VAR outperforms the DSGE model.

Keywords: DSGE, Bayesian Estimation, Survey Professional Forecasts, Real Time Data, Misspecification

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1 Introduction

Dynamic Stochastic General Equilibrium (DSGE) models are the one of main tools in both theoretical and empirical macroeconomic literature. Introduced to satisfy the Lucas critique (Lucas, 1976), with respect to structural macroeconometrics approaches, DSGE models illustrate the business cycle relying on micro-economic foundations. Thanks to these features, DSGEs are particularly suited for policy evaluation and for forecasting analysis as discussed in Smets and Wouters (2003, 2004, and 2007), Del Negro and Schorfheide (2004), Christiano, Eichenbaum, and Evans (2005), and Adolfson, Laséen, Lindé, and Villani (2008) among others. Despite the recent computational contributions to improve the match these models to the observable data, the misspecification is still an important issue to investigate. As surveyed in Paccagnini (2017), there is a growing literature that aims at investigating the sources of misspecification. In particular, the recent Great Recession has brought new relevance to study the structural models (for example DSGEs) features to match the data to make policy evaluation and for forecasting purposes. The majority of the research studies about misspecification focus on the US DSGE models and as far as we know there is not significant contribution [1] discussing misspecified DSGE models estimated on the EURO Area.

This present paper seeks to contribute to the DSGE modelling misspecification literature proposing an empirical evidence using the EURO Area data. As discussed in Paccagnini (2017), there are several forms of the misspecification. In this study, we evaluate the misspecification in assumptions, in particular focusing on the role of the rational expectations. According to the Woodford (2003), Svensson (2004), Bernanke (2004), Blinder et al. (2008) among others, expectations play a crucial role in macroeconomic models which aim at explaining the monetary policy analysis and the expectation channel is recognized as the main channel for policy stabilization. As illustrated in Cole and Milani (2016), monetary DSGE models satisfy, in general, the Rational Expectations Hypothesis (REH). REH assumes that expectations are formed by agents according to all available information.

There are two important research questions we can investigate.

Are the DSGE models subject to the REH able to match the observed data? Is the REH a necessary component in the policy analysis?

[1]Despite there are several research papers proposing policy evaluation using DSGE models estimated on the EURO Area, see Smets and Wouters (2003, 2004, and 2005), Christoffel, Coenen, and Warne (2008), and Coenen, Straub, and Trabandt (2012 and 2013), there is not a deep investigation about the sources of misspecification testing DSGEs on the EURO Area data.

To reply to our research questions, we compare the macroeconomic performance of a medium scale DSGE model à la Smets and Wouters (2007) with the same medium scale DSGE model but augmented by the Survey Professional Forecasts (SPF) as observables. Moreover, we assess and investigate the sources of a possible DSGE misspecification relying on the DSGE-VAR introduced by Del Negro and Schorfheide (2004). This hybrid model allows the researcher to evaluate DSGE models between two extremes using a hyper-parameter, λ, which governs the weight on the theory restrictions. When $\lambda \to \infty$, the DSGE-VAR collapses to the rational expectations DSGE model which imposes the cross-equation restrictions existing under the REH. Meanwhile, when $\lambda = 0$, the DSGE-VAR collapses to the unrestricted VAR estimated only on the observable variables without imposing restriction from the theory or the REH. Our research investigation is closed to Cole and Milani (2016). But we study the role of the REH in a medium-scale DSGE model estimated for the EURO Area, focusing on the conventional and unconventional monetary policy. Starting from 2013, also the European Central Bank (ECB) has started to adopt the forward guidance as an extraordinary measure to enhancing the effectiveness of the monetary policy.[2] It grounds on the idea that monetary policy could have a larger effect on the longer-term interest rate if policymakers can commit themselves credibly to a path for future policy rate, as pointed out by Woodford (2012).[3]

We add a forecasting comparison where we show how the DSGE model with expectations outperforms the standard DSGE à la Smets and Wouters to forecast the inflation. Meanwhile, the DSGE-VAR à la Del Negro and Schorfheide (2004), augmented with expectations as illustrated in Cole and Milani (2016), reports the best predictions for the GDP growth rate and short term interest rate.

Our main findings suggest how a DSGE model augmented with expectations helps the researcher to evaluate whether or not we can explain the key macroeconomic variables, as GDP growth rate, Inflation, and short term interest rate, during the unconventional monetary policy period.

The exploiting of observed expectations in a DSGE model is rarely used in the literature,

[2]On 4^{th} July 2013 the Governing Council of the ECB states its intention to keep interest rate at prevailing or lower levels "for an extended period of time", in order to lower the future rates below the market expectations. On 6 March 2014 the Governing Council reinforced the qualitative guidance formulation by spelling out more precisely the conditions for a low interest rate policy. However, as stressed by Filardo and Hofmann (2014) forward guidance has been used in a small number of inflation targeting Central banks during the 1990s, among which Japan, Norway, Iceland and Sweden.

[3]Note that there is no agreement in the profession on the beneficial effect of forward guidance. For example Kool, Middeldorp, and Rosenkranz (2011) show that under near-risk-neutrality of market participants, a crowding out of private information occurs, reducing forecast precision. Brzoza-Brzezina and Kot (2008) show that the benefits of publishing interest rate forecasts are marginal once macroeconomic forecasts are provided.

with few exceptions. In this sense, our approach is akin to that adopted by Cole and Milani (2016), who exploit the SPF and real time data to estimate a DSGE model with friction to assess the empirical relationship between macroeconomic expectations and their realizations. They compare a DSGE-VAR à la Del Negro and Schorfheide (2004) to an unrestricted VAR and a DSGE model with cross-equation. They evidence that the DSGE model matches the data on expectation only by rejecting DSGE restrictions. On the other hand, Del Negro and Eusepi (2011) show that even if DSGE model fits well US macro data, it misspecifies in fitting the survey expectations.[4] Differently from them, we address the forward guidance policy evaluation in Euro Area as a failure of rational expectations hypothesis to capture the behaviour of expectations.

The remainder of the paper is organized as follows. Section 2 describes briefly the medium-scale DSGE model. Section 3 explains our empirical analysis. Section 4 evaluates the forecasting accuracy. Section 5 concludes. An appendix complements the paper by providing: Section A the sketch of the model while Section B the data description.

2 Model

Our model (SW, baseline) is based on Smets and Wouters (2007), which contains both nominal and real frictions affecting the choices of households and firms. Since the model is not the novelty of our paper, we briefly sketch its proprieties and we report the equations in the Appendix A.

The economy is composed of households, labour unions, labour packers, a productive sector and a monetary authority. Households maximize their utility that depends on their level of consumption relative to an external habit component and leisure. Labour supplied by households is differentiated by a union with monopoly power setting sticky nominal wages à la Calvo. Households rent capital to firms and decide how much capital accumulation depending in the capital adjustment costs.

Nominal frictions affect the supply side and include both sticky intermediate goods and wages introduced by Calvo-pricing, with partial indexation for those firms who do not re-optimize their prices. In particular, intermediate firms decide how much capital they use depending on the capital utilization adjustment costs. Intermediate firms also decide how much differentiated labour they hire to produce differentiated goods and set their prices à la Calvo. Moreover, both wages and prices are partially indexed to lagged inflation when they are not re-optimized,

[4]The issue of expectations formation is beyond the scope of this paper. Along this framework see Omeno and Molnar (2015), Milani (2007, 2011), Slobodyan and Wouters (2012) and Granziera (2014), who show the important role of expectations in explaining the inflation process by means of different learning assumptions.

introducing another source of nominal rigidity. Finally, monetary policy is set according to a Taylor rule and the government spending is exogenous.

The model contains 13 endogenous variables: output, consumption, investment, value of the capital stock, installed stock of capital, stock of capital, inflation, capital utilization rate, real rental rate on capital, real marginal cost, real wages, hours worked, and interest rate. Moreover, we consider 7 exogenous processes: total factor productivity, government spending, price and wage mark ups and monetary policy. All shocks are modelled as autoregressive processes with normal i.i.d. innovations, except for the price and wage mark-ups which are assumed to follow a first order autoregressive moving average process. Note that the model is detrended with respect to the deterministic growth rate of the labour- augmenting technological progress and linearised around the steady state of the detrended variables.

3 Empirical Analysis

To investigate empirically the effectiveness of the forward guidance we focus on quarterly time series from 1999 to 2015 for the Euro Area exploiting real time data.

Our empirical analysis is composed of two steps. As first step, we compare posteriors of the estimated parameters of SW model with the same model and same database but augmented with expectations (SW with expectations) as observables incorporating the Survey Professional Forecast (SPF). The estimation procedure is implemented using Bayesian techniques as described in Smets and Wouters (2007).

As second step, in a pseudo out-of-sample forecasting exercise, we assess the prediction ability of the two DSGE models (baseline and with expectations) from 2012:Q4 to 2015:Q4. In our horse-race, we include a Bayesian VAR model with priors à la Sims and Zha (1998) and the hybrid DSGE-VAR introduced by Del Negro and Schorfheide (2004).

3.1 Data

To estimate our models for the Euro Area we use as observables *real time* data on GDP (growth rate) and HICP inflation, collected from Eurostat starting from 1999:Q1.

Moreover, we consider the short term nominal interest rate (Euribor), employment, private consumption (growth rate), investments (growth rate) and compensation per employee from ECB Data Warehouse, as observable variables to be matched in the estimation. Output, con-

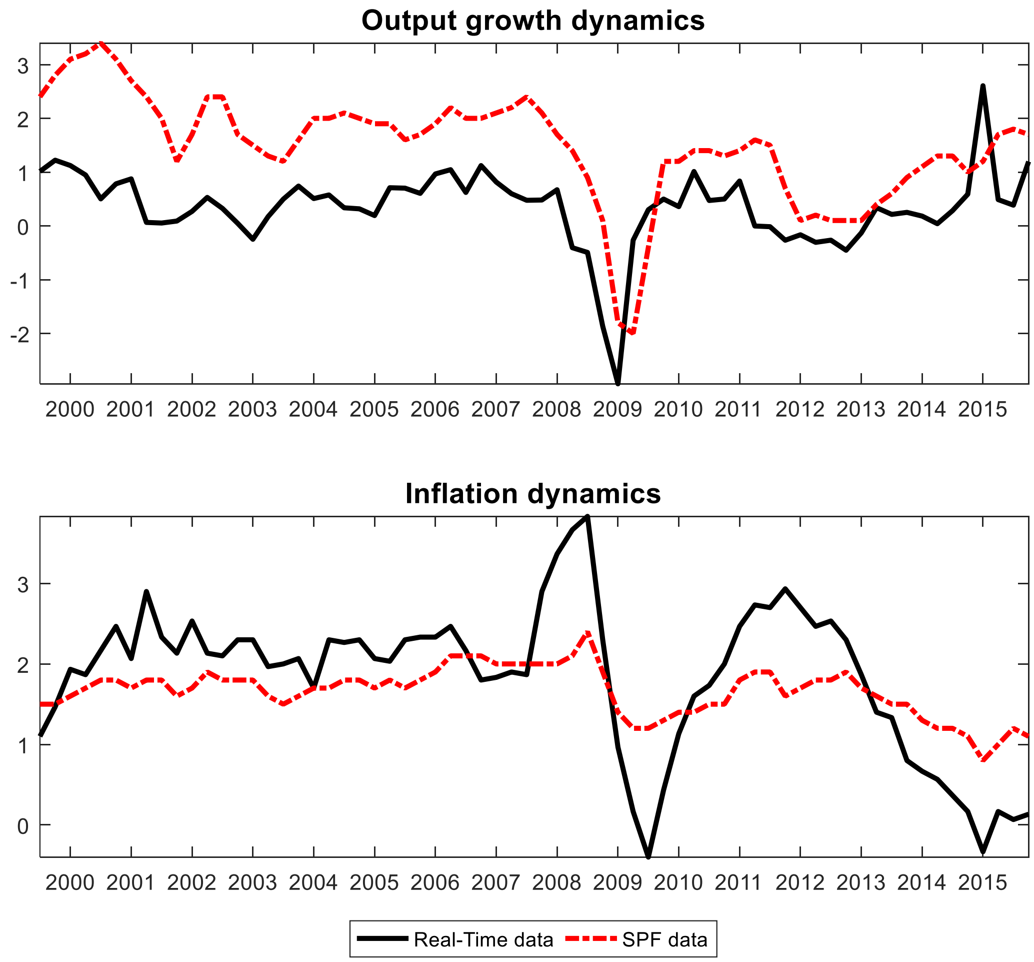

Figure 1: Figure 1: Realization and SPF series

sumption, investments, and wages are transformed in log differences; instead, total employment has been detrended with a HP trend.

Our survey measures are expectations for the one-year ahead inflation rate and real GDP growth from the Survey of Professional Forecasts (SPF). This survey is collected at quarterly series from 1999:Q1. So our data sample cover the period from 1999:Q2 to 2015:Q4.[5]

Figure 1 shows the different path between the median SPF inflation and real-time data. In particular, inflation realization appears to be quite different from SPF.

3.2 Estimation details

The DSGE model is estimated using Bayesian estimation technique, which combines a likelihood function of the data with a prior density to derive the posterior distribution of the structural parameters. Employing the Metropolis-Hasting algorithm, we run four chains of 500.000 draws

[5] We start the sample with the first realize of the Survey Professional Forecaster. A detailed description of the dataset used for estimation and transformations can be found in the Appendix B.

of all the possible realizations ξ for each parameter in order to obtain its posterior Kernel distribution.[6]

To avoid stochastic singularity the number of shocks equals the number of observables. The set of measurement equations, which relate the model to the observed variables, in case of the baseline is specified as follows:

$$Y_t = \begin{bmatrix} \Delta \ln y_t \\ \Delta \ln c_t \\ \Delta \ln i_t \\ \Delta \ln w_t \\ \ln e_t \\ \pi_t \\ R_t \end{bmatrix} = \begin{bmatrix} \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{e} \\ \overline{\pi}_* \\ \overline{r} \end{bmatrix} + \begin{bmatrix} y_t - y_{t-1} \\ c_t - c_{t-1} \\ i_t - i_{t-1} \\ w_t - w_{t-1} \\ e_t \\ \pi_t \\ r_t \end{bmatrix}, \tag{1}$$

where ln denotes 100 times log and $\Delta \ln$ refers to the log difference. $\overline{\gamma} = 100(\gamma - 1)$ is the common quarterly trend growth rate to real GDP, real consumption, real investment and real wages; $\overline{\pi}_* = 100(\overline{\pi} - 1)$ is the quarterly steady-state inflation rate, $\overline{r}$ is the steady-state quarterly nominal interest rate, and $\overline{e}$ is the steady-state employment, which is normalized at zero.[7]

In case of the SW estimated with SPF, the measurement equations set (1) includes two additional equations for expectations as follows:

$$\begin{bmatrix} E_t \Delta y_{t+1}^{obs} \\ E_t \pi_{t+1}^{obs} \end{bmatrix} = \begin{bmatrix} E_t y_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} + \begin{bmatrix} \hat{\varepsilon}_t^{y+1} \\ \hat{\varepsilon}_t^{\pi+1} \end{bmatrix},$$

where $E_t \Delta y_{t+1}^{obs}$ and $E_t \pi_{t+1}^{obs}$ denote respectively one-period-ahead real GDP growth expectations and inflation expectations. We interpret the survey data as a noisily measure of model consistent with rational expectations, following Cole and Milani (2016).

As argued by Ormeno and Molnar (2015), when survey data is used as an observable in the estimation, agent's expectations on inflation and output have to explain not only the model equations but also the SPF survey.

[6]The DSGE models are estimated using Dynare toolbox for Matlab. For more details on Bayesian estimation of DSGE models, see An and Schorfeide (2007).

[7]Following Christoffel et al. 2008, we relate the employment variable, e_t , to the unobserved worked-hours variable, h_t, by means of $\hat{e}_t = \frac{\beta}{1+\beta} E_t \hat{e}_{t+1} + \frac{1}{1+\beta} \hat{e}_{t-1} + \frac{(1-\xi_e)(1-\beta\xi_e)}{(1+\beta)\xi_e} \left(\hat{h}_t - \hat{e}_t\right)$, ξ_e determines the sensitivity of employment with respect to worked hours.

3.3 Calibration and priors

A number of parameters are calibrated to match stylized facts in the data, as indicated in Table 1. The time period is set, as in the sample, to one quarter. In particular, we set the discount factor β at the standard level 0.99, in line with a steady-state real interest rate of about 4%. The depreciation rate δ is 0.025 per quarter (approximately 10% per year). The Kimball aggregators in the goods and labor market are equal to 10, and the steady state gross wage and price mark-up is set respectively to 1.61 and 1.5.

Table 1: Calibrated parameters

	parameter	value
β	discount factor	0.99
δ	capital depreciation rate	0.025
η_p	Kimball aggregator in the goods markets	10
η_w	Kimball aggregator in the labour markets	10
λ_p	Gross steady state price markup	1.61
λ_w	Gross steady state wage markup	1.5

The remaining parameters governing dynamics of the model are estimated using Bayesian techniques. To make our economy as representative as possible, priors for parameters and shocks are set according to the literature, as in Smets and Wouters (2007, 2005) and Coenen et. al. (2012). Values are reported in table 2:

3.4 Alternative Estimation models

We compare the prediction ability of the two DSGE models considering a Bayesian VAR (BVAR) model and the DSGE-VAR à la Del Negro and Schorfheide (2004), which econometric properties are described below.

3.4.1 Bayesian VAR

As discussed by Smets and Wouters (2007), the BVAR à la Sims and Zha (1998) is a good alternative model to forecast macroeconomic series such as GDP growth rate, CPI, and interest rate. The main advantage of considering this kind of BVAR is that it combines a Minnesota-type prior as in Litterman (1981, 1986) with a unit-root prior which considers the degree of persistence and cointegration in the variables. In the comparison analysis, we estimate the BVAR à la Sims and Zha (1998) with two lags as suggested by Akaike and Schwartz Information Criteria.

Table 2: Priors

	Parameters	shape
σ_c	inverse of Intertemporal elastic. of subst.	N(1.5,0.37)
ϕ_l	inverse of Frish elasticity	N(2,0.75)
b	habits in consumption	B(0.7,0.1)
$\bar{\pi}_*$	SS inflation	G(0.62,0.1)
γ	SS output growth	N(0.5,0.05)
φ	investment adjustement cost	N(4,1.5)
α	capital share	N(0.3,0.05)
ξ_p	price rigidity	B(0.75,0.1)
ι_p	price indexation to past inflation	B(0.75,0.1)
ξ_w	wage rigidity	B(0.75,0.1)
ι_w	wage indexation to past inflation	B(0.75,0.1)
ξ_e	Calvo employment	B(0.5,0.15)
$\bar{e}$	SS employment	N(0.0,2.0)
Φ_r	interest rate smoothing	B(0.75,0.1)
Φ_π	Taylor rule parameter on inflation	N(1.5,0.25)
Φ_y	Taylor rule parameter on output	N(0.12,0.05)
$\Phi_{\Delta y}$	Taylor rule parameter on change in output	N(0.12,0.05)
	Shocks	
$\rho_a, \rho_b, \rho_i, \rho_w, \rho_r, \rho_p, \rho_g, \rho^e_\pi, \rho^e_y$	AR coefficient of shocks	B(0.5,0.15)
μ_w, μ_p	MA coefficient of shocks	B(0.5,0.15)
$\sigma_a, \sigma_b, \sigma_i, \sigma_w, \sigma_r, \sigma_p, \sigma_g, \sigma^e_\pi, \sigma^e_y$	standard deviation shocks	IG(0.1,∞)

3.4.2 DSGE-VAR à la Del Negro and Schorfheide (2004)

Based on the study of Ingram and Whiteman (1994), Del Negro and Schorfheide (2004) designed the DSGE-VAR approach to improve forecasting and monetary policy analysis with VARs. Their approach is to use the DSGE model to build prior distributions for the VAR. Basically, the estimation initializes with an unrestricted VAR of order p:

$$Y_t = \Phi_0 + \Phi_1 Y_{t-1} + ... + \Phi_p Y_{t-p} + u_t. \tag{2}$$

In compact format:

$$Y = X\Phi + U, \tag{3}$$

where Y is a $(T \times n)$ matrix with rows Y_t', X is a $(T \times k)$ matrix ($k = 1 + np, p$ =number of lags) with rows $X_t' = [1, Y_{t-1}', ..., Y_{t-p}']$, U is a $(T \times n)$ matrix with rows u_t' and Φ is a $(k \times n) = [\Phi_0, \Phi_1, ..., \Phi_p]'$.The one-step-ahead forecast errors u_t have a multivariate normal distribution $N(0, \Sigma_u)$ conditional on past observations of Y. The log-likelihood function of the data is a function of Φ and Σ_u :

$$L(Y|\Phi, \Sigma_u) \propto |\Sigma_u|^{-\frac{T}{2}} \exp\left\{-\frac{1}{2}tr\left[\Sigma_u^{-1}\left(\mathbf{Y}'\mathbf{Y} - \mathbf{\Phi}'X'Y - Y'X\Phi + \Phi'\mathbf{X}'\mathbf{X}\mathbf{\Phi}\right)\right]\right\}. \tag{4}$$

The prior distribution for the VAR parameters proposed by Del Negro and Schorfheide (2004) is based on the statistical representation of the DSGE model given by a VAR approximation. Let Γ^*_{xx}, Γ^*_{yy}, Γ^*_{xy} and Γ^*_{yx} be the theoretical second-order moments of the variables Y and X implied by the DSGE model, where:

$$\begin{aligned} \Phi^*(\theta) &= \Gamma^{*-1}_{xx}(\theta)\,\Gamma^*_{xy}(\theta)\,, \\ \Sigma^*(\theta) &= \Gamma^*_{yy}(\theta) - \Gamma^*_{yx}(\theta)\,\Gamma^{*-1}_{xx}(\theta)\,\Gamma^*_{xy}(\theta)\,. \end{aligned} \tag{5}$$

The moments are the dummy observation priors used in the mixture model. These vectors can be interpreted as the probability limits of the coefficients in a VAR estimated on the artificial observations generated by the DSGE model. Conditional on the vector of structural parameters in the DSGE model θ, the prior distributions for the VAR parameters $p(\Phi, \Sigma_u|\theta)$ are of the Inverted-Wishart (IW) and Normal forms:

$$\begin{aligned} \Sigma_u\,|\theta &\sim IW\left((\lambda T\Sigma^*_u(\theta)\,, \lambda T - k, n\right), \\ \Phi\,|\Sigma_u, \theta &\sim N\left(\mathbf{\Phi}^*(\theta)\,, \Sigma_u \otimes (\lambda T\Gamma_{XX}(\theta))^{-1}\right), \end{aligned} \tag{6}$$

where the parameter λ controls the degree of model misspecification with respect to the VAR: for small values of λ the discrepancy between the VAR and the DSGE-VAR is large and a sizeable distance is generated between the unrestricted VAR and DSGE estimators. Large values of λ correspond to small model misspecification and for $\lambda = \infty$ beliefs about DSGE misspecification degenerate to a point mass at zero. Bayesian estimation could be interpreted as estimation based on a sample in which data are augmented by a hypothetical sample in which observations are generated by the DSGE model, the so-called dummy prior observations (Theil and Goldberg, 1961; Ingram and Whiteman, 1994). Within this framework λ determines the length of the hypothetical sample. The posterior distributions of the VAR parameters are also of the Inverted-Wishart and Normal forms. Given the prior distribution, posterior distributions are derived by the Bayes theorem:

$$\Sigma_u\,|\theta, \mathbf{Y} \sim IW\left((\lambda+1)\,T\hat{\Sigma}_{u,b}(\theta)\,, (\lambda+1)\,T - k, n\right), \tag{7}$$

$$\Phi \left| \Sigma_u, \theta, \mathbf{Y} \right. \sim N\left(\hat{\mathbf{\Phi}}_b\left(\theta\right), \Sigma_u \otimes \left[\lambda T \Gamma_{XX}\left(\theta\right) + \mathbf{X}'\mathbf{X}\right]^{-1} \right), \tag{8}$$

$$\hat{\Phi}_b\left(\theta\right) = \left(\lambda T \Gamma_{XX}\left(\theta\right) + \mathbf{X}'\mathbf{X}\right)^{-1} \left(\lambda T \Gamma_{XY}\left(\theta\right) + \mathbf{X}'\mathbf{Y}\right), \tag{9}$$

$$\hat{\Sigma}_{u,b}\left(\theta\right) = \frac{1}{\left(\lambda+1\right)T}\left[\left(\lambda T \Gamma_{YY}\left(\theta\right) + \mathbf{Y}'\mathbf{Y}\right) - \left(\lambda T \Gamma_{XY}\left(\theta\right) + \mathbf{X}'\mathbf{Y}\right)\hat{\mathbf{\Phi}}_b\left(\theta\right)\right], \tag{10}$$

where the matrices $\hat{\Phi}_b\left(\theta\right)$ and $\hat{\Sigma}_{u,b}\left(\theta\right)$ have the interpretation of maximum likelihood estimates of the VAR parameters based on the combined sample of actual observations and artificial observations generated by the DSGE. Equations (7) and (8) show that the smaller λ is, the closer the estimates are to the OLS estimates of an unrestricted VAR. Instead, the higher λ is, the closer the VAR estimates will be tilted towards the parameters in the VAR approximation of the DSGE model ($\hat{\Phi}_b\left(\theta\right)$ and $\hat{\Sigma}_{u,b}\left(\theta\right)$). In order to obtain a non-degenerate prior density (6), which is a necessary condition for the existence of a well-defined Inverse-Wishart distribution and for computing meaningful marginal likelihoods, λ has to be greater than λ_{MIN} :

$$\begin{aligned} \lambda_{MIN} &\geq \frac{n+k}{T}; k = 1 + p \times n \\ p &= \text{lags} \\ n &= \text{endogenous variables.} \end{aligned}$$

Hence, the optimal lambda must be greater than or equal to the minimum lambda $\left(\hat{\lambda} \geq \lambda_{MIN}\right)$. Essentially, the DSGE-VAR tool allows the econometrician to draw posterior inferences about the DSGE model parameters θ. Del Negro and Schorfheide (2004) explain that the posterior estimate of θ has the interpretation of a minimum-distance estimator, where the discrepancy between the OLS estimates of the unrestricted VAR parameters and the VAR representation of the DSGE model is a sort of distance function. The estimated posterior of parameter vector θ depends on the hyperparameter λ. When $\lambda \to 0$, in the posterior of the parameters are not informative, so the DSGE model is of no use in explaining the data. Unfortunately, the posteriors (8) and (7) do not have a closed form and we need a numerical method to solve the problem. The posterior simulator used by Del Negro and Schorfheide (2004) is the Markov Chain Monte Carlo Method and the algorithm used is the Metropolis-Hastings acceptance method. This procedure generates a Markov Chain from the posterior distribution of θ and this Markov Chain is used for Monte Carlo simulations. The optimal λ is given by maximizing the log of the marginal data density

$$\hat{\lambda} = \arg\max_{\lambda \geqslant \lambda_{MIN}} \ln p(\mathbf{Y}|\lambda)$$

According to the optimal lambda, $\hat{\lambda}$, a corresponding optimal mixture model is chosen. This hybrid model is called DSGE-VAR and $\hat{\lambda}$ is the weight of the priors. It can also be interpreted as the restriction of the theoretical model on the actual data. We estimate the DSGE-VAR with two lags as suggested by Akaike and Schwartz Information Criteria.

4 Estimation results

Table 3 and 4 summarize estimation results for the baseline and the SW model augmented by expectations, reporting, the mean and the 5 and 95 percentiles of the posterior distribution of the parameters obtained by the Metropolis-Hastings algorithm.

Even if the posterior distributions of the SW baseline model for most of the parameters do not differ significantly from the literature, they demonstrates that there are some important changes in some of them. The steady state growth rate is estimated to be around 0.5, which is somewhat greater than the average growth rate of output over the sample. The posterior mean of the steady state inflation rate over the full sample is about 3.5% on an annual basis. The mean of the discount rate is estimated to be quite small (0.7% on an annual basis).

In the baseline model the productivity, the government spending, and the wage mark-up processes are estimated to be the most persistent with an AR(1) coefficient of 0.99, 0.85 and 0.71, respectively. The mean of the standard error of the shock to the productivity process is relatively high, meaning that at long horizons most of the forecast error variance of the real variables will be explained by this shock. In contrast, both the persistence and the standard deviation of inertia in monetary policy is relatively low (0.09). Policy reacts strongly to inflation expectations (1.52), but does not respond to output gap (0.049) and to change in the output-gap (0.18) in the short run.

Most source of endogenous persistence looses some of their importance when expectations are taken into account. The estimates of habit formation parameter, b, and the investment adjustment costs, φ, become smaller with compare to the Smets and Wouters's economy (respectively from 0.8 to 0.7 and 6.4 to 5.5). Similarly, the price and wage indexation as well as

Table 3: Posteriors distributions of parameters

Parameters		Priors	SW model+Expectations Mean	SW model+Expectations 90% interval	SW model Mean	SW model 90% interval
ϕ_l	inverse of Frish elasticity	N(2; 0.75)	1.290	(0.221; 2.224)	1.562	(0.721; 2.347)
b	habits in consumption	B(0.7; 0.1)	0.734	(0.671; 0.798)	0.827	(0.785; 0.871)
$\bar{\pi}_*$	steady state inflation	G(0.62; 0.1)	0.902	(0.755; 1.070)	0.508	(0.351; 0.662)
γ	SS output growth	N(0.5; 0.05)	0.412	(0.361; 0.455)	0.546	(0.476; 0.616)
φ	investment adjustment cost	N(4; 1.5)	5.567	(3.728; 7.942)	6.432	(4.881; 7.972)
σ_u	Capital utilization	B(0.5; 0.15)	0.780	(0.668; 0.901)	0.704	(0.562; 0.852)
α	capital share	N(0.3; 0.05)	0.271	(0.234; 0.308)	0.259	(0.203; 0.313)
ϕ_p	Fixed cost in production	N(1.25; 0.125)	1.830	(1.648; 2.000)	1.653	(1.512; 1.791)
ξ_p	price rigidity	B(0.75; 0.1)	0.927	(0.893; 0.953)	0.525	(0;437; 0.612)
ι_p	price indexation	B(0.75; 0.1)	0.336	(0.300; 0.396)	0.745	(0.599; 0.888)
ξ_w	wage rigidity	B(0.75; 0.1)	0.817	(0.753; 0.886)	0.741	(0.604; 0.890)
ι_w	wage indexation	B(0.75; 0.1)	0.394	(0.235; 0.569)	0.681	(0.504; 0.869)
ξ_e	Calvo employment	B(0.5; 0.15)	0.752	(0.705; 0.803)	0.815	(0.783; 0.848)
Φ_r	interest rate inertia	B(0.75; 0.1)	0.979	(0.968; 0.90)	0.965	(0.949; 0.981)
Φ_π	Taylor rule inflation	N(1.5; 0.25)	1.597	(1.322; 1.959)	1.525	(1.176; 1.852)
Φ_y	Taylor rule on GDP	N(0.12; 0.05)	0.161	(0.105; 0.229)	0.049	(-0.036; 0.160)
$\Phi_{\Delta y}$	Taylor rule change GDP	N(0.12; 0.05)	0.203	(0.157; 0.246)	0.184	(0.149; 0.220)
LDD			-415		-299	

Table 4: Posteriors distributions of shocks

Shocks		Priors	SW model+Expectations Mean	SW model+Expectations 90% interval	SW model Mean	SW model 90% interval
ρ_a	AR coeff. productity	B(0.5; 0.15)	0.980	(0.971; 0.990)	0.992	(0.988; 0.996)
ρ_b	AR coeff. preference	B(0.5; 0.15)	0.911	(0.870; 0.955)	0.584	(0.366; 0.930)
ρ_p	AR coeff. price markup	B(0.5; 0.15)	0.211	(0.026; 0.634)	0.603	(0.422; 0.790)
ρ_w	AR coeff. wage markup	B(0.5; 0.15)	0.556	(0.414; 0.669)	0.718	(0.502; 0.921)
ρ_g	AR coeff. governmet spending	B(0.5; 0.15)	0.888	(0.827; 0.949)	0.849	(0.781; 0.919)
ρ_i	AR coeff. investment-specific	B(0.5; 0.15)	0.392	(0.229; 0.531)	0.388	(0.222; 0.552)
ρ_r	AR coeff. monetary	B(0.5; 0.15)	0.310	(0.142; 0.475)	0.324	(0.167; 0.475)
ρ^e_{pic}	AR coeff. inflation expectations	B(0.5; 0.15)	0.722	(0.543; 0.979)	-	-
ρ^e_y	AR coeff. GDP expectations	B(0.5; 0.15)	0.987	(0.979; 0.995)	-	-
μ_p	MA coeff. price markup	B(0.5; 0.15)	0.384	(0.267; 0.654)	0.428	(0.236; 0.620)
μ_w	MA coeff. wage markup	B(0.5; 0.15)	0.852	(0.801; 0.915)	0.470	(0.280; 0.666)
σ_a	St. Dev coeff. productity	IG(0.4; ∞)	0.5555	(0.430; 0.685)	0.726	(0.553; 0.892)
σ_b	St. Dev preference	IG(0.4; ∞)	0.3498	(0.220;0.469)	0.345	(0.130; 0.559)
σ_p	St. Dev price markup	IG(0.4; ∞)	0.2729	(0.228;0.315)	0.224	(0.168; 0.278)
σ_w	St. Dev wage markup	IG(0.4; ∞)	0.3613	(0.278;0.440)	0.476	(0.364; 0.593)
σ_g	St. Dev governmet spending	IG(0.4; ∞)	0.0916	(0.076;0.107)	0.538	(0.458; 0.617)
σ_i	St. Dev investment-specific	IG(0.4; ∞)	0.635	(0.533;0,761)	0.332	(0.262; 0.400)
σ_r	St. Dev monetary inertia	IG(0.4; ∞)	0.7261	(0.606;0.841)	0.096	(0.076; 0.116)
σ^e_{pic}	St. Dev inflation expectations	IG(0.4; ∞)	0.0704	(0.052;0.135)	-	-
σ^e_y	St. Dev GDP expectations	IG(0.4; ∞)	0.7401	(0.787;0.877)	-	-

price rigidity are now much smaller.

Instead, the persistence of the interest rate inertia, government spending and preference are increasing. Overall, we observe that incorporating expectations implies a lower estimates of the parameters characterizing both endogenous and exogenous sources of persistence than the baseline model. Consequently, the expectation shocks play a crucial role in the estimation procedure.

These results are in line with the bounded rationality findings, such as Eusepi and Preston (2011) and Rychalovska and Slobodyan (2010).

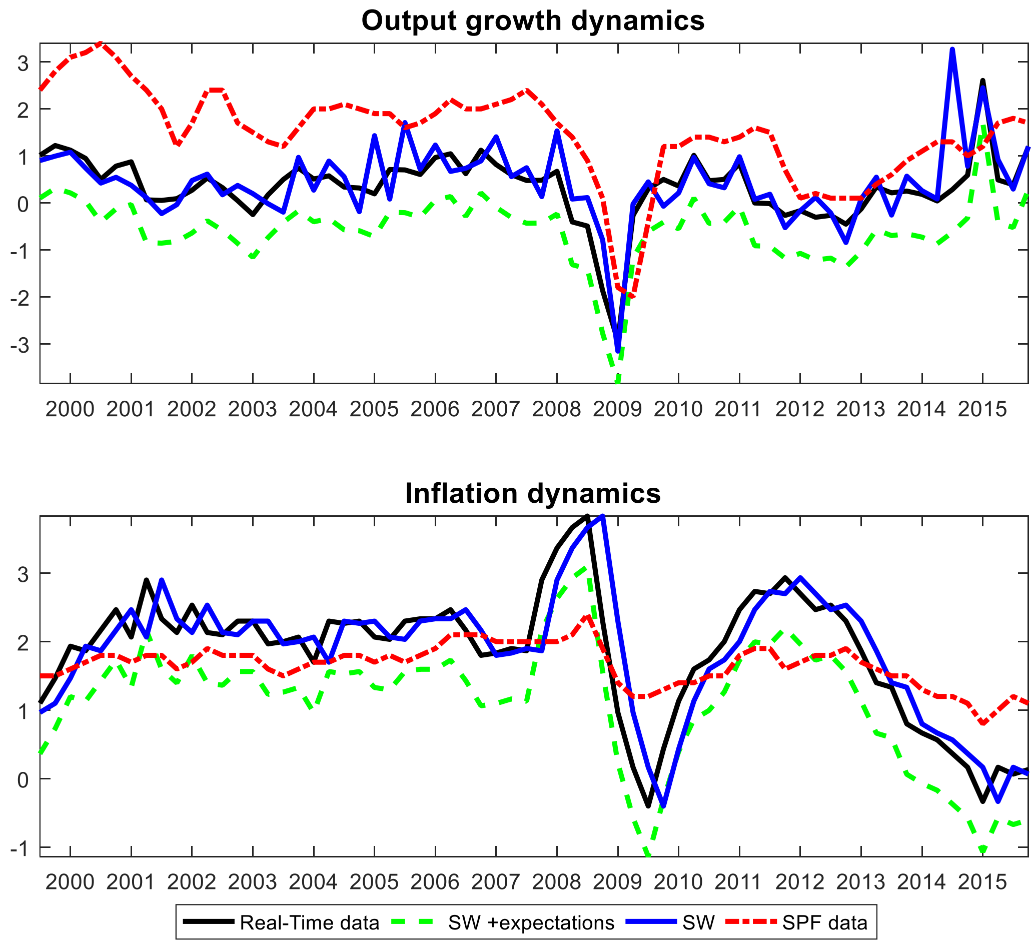

Figure 2: Figure 2: Survey data and model-implied series

Figure 2 outlines the relation between the median SPF inflation and output gap, the observations and the realization of our models. The paths generated by both models are quite different from the SPF data and more similar to real time data, especially in the baseline model for the output. However, the SW model augmented by expectations seems to fit better on average inflation dynamics.

5 Forecasting evaluation of the role of expectations

This section illustrates the main details about the forecasting exercises we conduct. Forecasting performance is an important criterion in the assessment of a model's credibility and usefulness for policy analysis. We perform a forecasting comparison among the two DSGE models, a BVAR, and DSGE-VAR models as described in Table 5. All these models are estimated from 1999:Q2 to 2012:Q3 and the pseudo out-of-sample is set from 2012:Q4 to 2015:4.

For the SW baseline and the SW with expectations, we generate unconditional forecasts taking each 20th draw from the final 250,000 parameter draws (with the first 50.000 draws used as burn-in period) produced by the Metropolis-Hastings algorithm, which gives us 10.000 draws from the posterior distribution. The point forecasts are calculated as means of these draws. For more technical details, see Kolasa et al. (2012) and Kolasa and Rubaszek (2014).

Table 5: Model implied forecasting

	2012q4	2013q4	2014q4	2015q4
GDP growth				
Observed	-0.45	0.25	0.59	1.2
SW model	-0.31	0.69	1.26	1.26
SW + expectations	-0.35	0.63	0.98	1.21
BVAR	-0.08	0.25	0.28	0.17
DSGEVAR	-0.31	0.79	1.08	0.97
DSGEVAR + expectations	-0.41	0.35	0.47	1.16
Inflation				
Observed	2.3	0.8	0.17	0.13
SW model	2.47	1.32	0.71	0.51
SW + expectations	2.31	0.8	0.3	0.47
BVAR	2.43	2.49	2.72	2.87
DSGEVAR	2.44	2.02	1.38	0.92
DSGEVAR + expectations	2.31	0.92	0.95	0.88
Interest rate				
Observed	0.15	0.13	0.08	0.02
SW model	0.32	0.22	0.42	0.81
SW + expectations	0.22	0.21	0.35	0.59
BVAR	0.17	0.10	0.09	0.06
DSGEVAR	0.19	0.09	0.13	0.10
DSGEVAR + expectations	0.22	0.08	0.10	0.08

Table 5 reports the observed values and the forecasted ones for three macroeconomic variables, GDP growth, Inflation, and interest rate, considering the periods: 2012:Q4, 2013:Q4, 2014:Q4, and 2015:Q4.

The forecasted values are reported for the SW baseline, the SW with expectations, the Bayesian VAR (BVAR), and the DSGE-VAR for both DGSE models.

At the first glance, focusing on the DSGE model forecasts, we note that the SW with expectations outperforms the SW baseline for all three key macroeconomic variables. In particular, for the inflation, the forecasted values are very close to the observed values especially in the short horizons, 2012:Q4 and 2013:Q4. Figure 3 shows the difference among the observed values and two forecasted ones. Graphically, we note that the DSGE models are weak to predict the interest rate at the long horizons. Several papers point out that the Bayesian VAR is the most suitable model to predict business cycle indicators, such as the GDP growth rate and the interest rate.

We estimate a BVAR à la Sims and Zha (1998) to provide a comparison with an alternative model.

As discussed for the Euro Area in Bekiros and Paccagnini (2016), the BVAR outperforms the DSGE models for the GDP growth rate and for the short term interest rates. For these two macroeconomic variables, it seems that the DSGE models fail to predict.

For this reason, we introduce in our forecasting comparison the DSGE-VAR à la Del Negro and Schorfheide (2004). The DSGE-VAR is an hybrid model combining information from the observed time series and from the theoretical DSGE model[8]. We estimate the DSGE-VAR for the SW baseline and the SW with expectations. In both cases, the hyper-parameter, λ, which indicates whether the posterior of the parameters are informative, is close to 1. Hence, we can conclude that the DSGE is far to be misspecified[9].

The DSGE-VAR with expectations outperforms the DSGE-VAR without expectations. In particular, the DSGE-VAR with expectations report similar forecasts as ones produced by the BVAR for the GDP growth and the interest rate.

As main findings, we can conclude that the inflation is well predicted by the SW with expectations, while the GDP growth and the interest rate are better predicted by the BVAR and the DSGE-VAR with expectations.

[8] As discussed in Sims (2007), the DSGE-VAR is a "Bayesian VAR" with the priors derived from a theoretical DSGE model. DSGE-VAR combines the advantage of the VAR model class to forecast with priors with model information. As pointed out by Sims (2007), DSGE-VAR does this "by modelling the data as a VAR — that is, without the tight parametric restrictions implied by a DSGE — but using a DSGE, and prior beliefs about the parameters of the DSGE, to generate a prior distribution for the parameters of the VAR".

[9] For a detailed discussion about the role of the λ hyperparameter, see Paccagnini (2010 and 2011), Bekiros and Paccagnini (2014) among other.

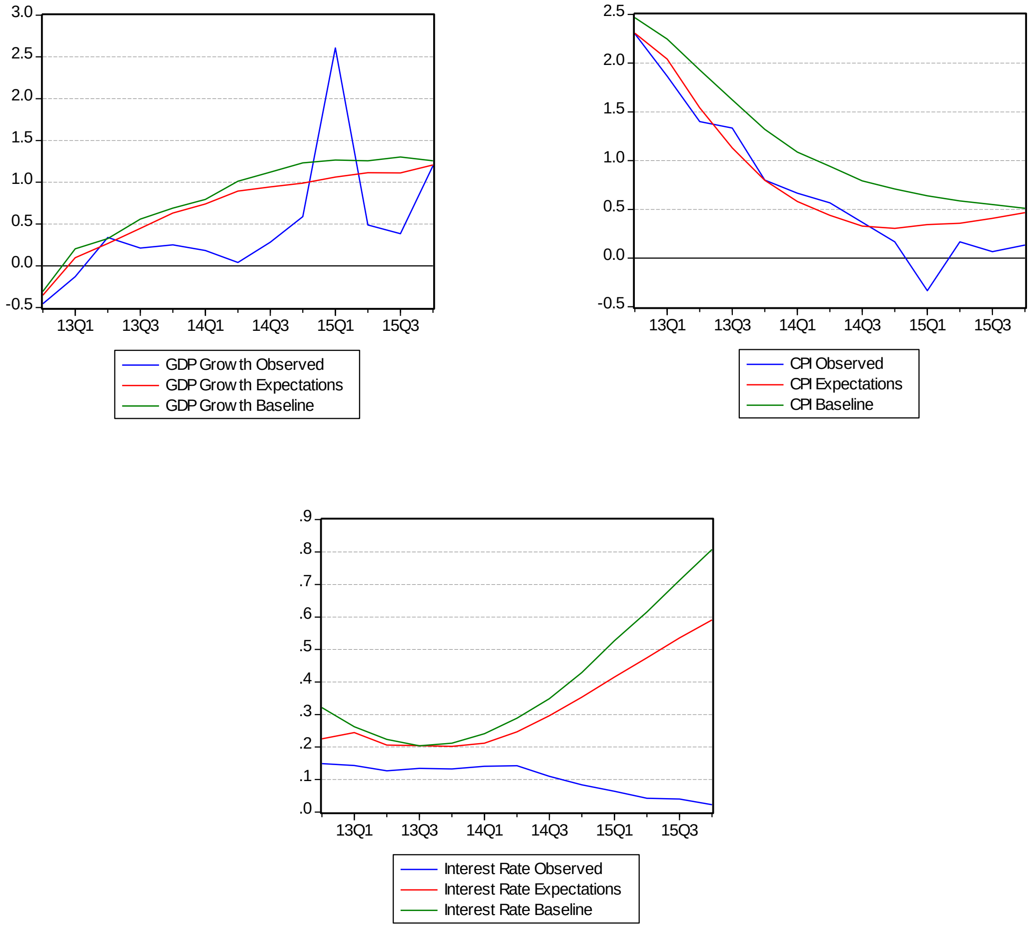

Figure 3: Figure 3: Forecasting comparison

6 Concluding Remarks

This paper investigates about possible misspecification on the estimated medium-scale New Keynesian model on the EURO Area data. Exploiting data on expectations from surveys, we show that incorporating expectations should be crucial in performance evaluation of models for the conventional and unconventional monetary policies. The DSGE-VAR hybrid model à la Del Negro and Schorfheide (2004) is implemented to assess the sources of model misspecification.

We conduct an exhaustive empirical exercise to compare the pseudo out-of-sample predictive performance of the estimated DSGE model with a Bayesian VAR and a DSGE-VAR models. DSGE model with expectations outperforms others for inflation; while for output and short term-interest rate the DSGE-VAR with expectations reports the best prediction.

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Appendix

A Sketch of the model

We consider a standard medium scale economy[10] *à la* Smets and Wouters (2007), which contains both nominal and real frictions affecting the choices of households and firms. All variables are log-linearized around their steady state balanced growth path.

The aggregates resource constraints is given by

$$\hat{y}_t = c_y \hat{c}_t + i_y \hat{\imath}_t + u_y \hat{u}_t + \hat{\varepsilon}_t^g \tag{11}$$

where $c_y = c_*/y_*$, $i_y = i_*/y_*$ and $u_y = z_*^k k_*/y_*$ represent respectively the steady-state consumption, investment and rental rate of capital. The output produces is absorbed by consumption, $\hat{c}_t$, investment, $\hat{\imath}_t$, capital utilization rate $\hat{u}_t$, and an exogenous government spending shock $\hat{\varepsilon}_t^g$ that follows:

$$\hat{\varepsilon}_t^g = \rho_g \hat{\varepsilon}_{t-1}^g + \eta_t^g \sim N\left(0, \sigma_g^2\right)$$

A continuum of households of mass unity populate the economy with identical preferences that depends on hours worked and consumptions. Their behavior is captured by the following Euler equation:

$$\widehat{c}_t = c_1 \widehat{c}_{t-1} + (1 - c_1) E_t c_{t+1} + c_2 (h_t - E_t h_{t+1}) - c_3 \left(r_t - E_t \pi_{t+1} + \varepsilon_t^b\right), \tag{12}$$

where $c_1 = \frac{b\gamma}{1+b\gamma}$, $c_2 = \frac{(\sigma_c - 1)(w_* h_*/c_*)}{\sigma_c(1+b\gamma)}$ and $c_3 = \frac{1-b\gamma}{\sigma_c(1+b\gamma)}$. Consumptions c_t is affected by the presence of external habits parameter $0 < b < 1$ and by the real interest rate. Parameter σ_c refers to the degree of intertemporal elasticity of substitution while the parameter γ captures the steady state growth rate.[11] The term ε_t^b is a preference shock affecting the subjective discount factor following an AR(1) process of the type:

$$\hat{\varepsilon}_t^b = \rho_b \hat{\varepsilon}_{t-1}^b + \eta_t^b \quad \eta_t^b \sim N\left(0, \sigma_b^2\right)$$

Households can move resources between periods by purchasing one period bonds and renting

[10]This DSGE model was proposed by Christiano, Eichenbaum and Evans (2005) and estimated by Smets and Wouters (2003 and 2007).

[11]Specifically, the model is detrended with a deterministic trend γ that represents a labor augmenting growth rate in the economy.

capital to firms. Households make a capital accumulation decision and decide how many units of capital services to rent firms. The accumulation of capital, $\hat{k}_t$, is not only a function of the flow of investment, $\hat{\imath}_t$, but also of the relative efficiency of these investment expenditures as captured by the investment-specific technology disturbance, $\widehat{\varepsilon}_t^i$:

$$\hat{k}_t = k_1 \hat{k}_{t-1} + (1 - k_1)\,\widehat{\imath}_t + k_2 \widehat{\varepsilon}_t^i, \tag{13}$$

where $k_1 = \frac{(1-\delta)}{\gamma}$ and $k_2 = [1 - \frac{(1-\delta)}{\gamma}]\left(1 + \beta\gamma^{1-\sigma_c}\right)\gamma^2\varphi$.

Capital adjustment is costly and it is a function of the change in investment. The optimal investment choice is described by the investment Euler equation:

$$\hat{\imath}_t = i_1 \hat{\imath}_{t-1} + (1 - i_1)\,E_t \hat{\imath}_{t+1} + i_2 \widehat{q}_t + \widehat{\varepsilon}_t^i \tag{14}$$

where $i_1 = \frac{1}{(1+\beta\gamma^{1-\sigma_c})}$ and $i_2 = \frac{i_1}{\gamma^2\varphi}$. The parameter φ is the elasticity of investment adjustment costs and e_t^i is an investment-specific technology shock following an AR(1) process with ρ_i the AR(1) coefficient

$$\widehat{\varepsilon}_t^i = \rho_i \widehat{\varepsilon}_{t-1}^i + \eta_t^i \quad \eta_t^i \sim N\left(0, \sigma_i^2\right)$$

The corresponding $\hat{q}_t$ equation measures the shadow price of a unit of investment good and takes the form of

$$\hat{q}_t = q_{1r} E_t \widehat{q}_{t+1} + (1 - q_{1r})\,E_t \widehat{z}_{t+1}^k - (\widehat{r}_t - \widehat{\pi}_{t+1}), \tag{15}$$

where $q_{1r} = \frac{(1-\delta)}{z_*^k + (1-\delta)}$ and δ is the depreciation rate of capital.

A labor union differentiates labor and sets wages in a monopolistically competitive market. Competitive labor packers buy labor services from the union, package and sell them to intermediate goods firms. Moreover, wages are staggered à la Calvo (1983). Union j receives permission to optimally reset the nominal wage with probability $(1 - \xi_w)$. Therefore, salary is set according to:

$$\widehat{w}_t = w_1 \widehat{w}_{t-1} + (1 - w_1)\left(E_t \widehat{w}_{t+1} + E_t \widehat{\pi}_{t+1}\right) - w_2 \widehat{\pi}_t + w_3 \widehat{\pi}_{t-1} - w_4 \widehat{\mu}_t^w + \widehat{\varepsilon}_t^w \tag{16}$$

where $w_1 = \frac{1}{1+\beta\gamma^{1-\sigma_c}}$, $w_2 = \frac{1+\beta\gamma^{1-\sigma_c}\iota_w}{1+\beta\gamma^{1-\sigma_c}}$, $w_3 = \frac{\iota_w}{1+\beta\gamma^{1-\sigma_c}}$ and $w_4 = \frac{\left(1+\xi_w\beta\gamma^{1-\sigma_c}\right)(1-\xi_w)}{(1+\beta\gamma^{1-\sigma_c})\xi_w[(\phi_w-1)e^w+1]}$. The parameter β represents the households discount factor, ξ_w indicates the Calvo probability of not adjusting nominal wages, ι_w denotes the degree of wage indexation of non-adjusting unions, $(\phi_w - 1)$ is the steady state labor market markup, and $\widehat{\varepsilon}_t^w$ is the curvature of the Kimball aggregator in the labor market which takes the form of an AR(1) process:

$$\widehat{\varepsilon}_t^w = \rho_w \hat{\lambda}_{t-1}^w + \eta_t^w \quad \eta_t^w \sim N\left(0, \sigma_w^2\right)$$

The wage mark-up is the difference between the real wages and the marginal rate of substitution between consumption and labor:

$$\widehat{\mu}_w = \widehat{w}_t - \left[\sigma_l \widehat{h}_t + \frac{1}{1-b}\left(\widehat{c}_t - b\widehat{c}_{t-1}\right)\right], \tag{17}$$

where σ_l is the elasticity of labor supply with respect to the real wage.

On the supply side, output is produced by a monopolistically a competitive sector with producers facing price rigidities. The aggregate production function takes the form of a standard Cobb Douglas.

$$\hat{y}_t = \phi_p \left[\alpha\left(\widehat{k}_{t-1} + \widehat{u}_t\right) + (1-\alpha)\widehat{h}_t\right] + \widehat{\varepsilon}_t^a \tag{18}$$

i.e. output is produced using capital, labor and capital utilization, $\hat{u}_t$. $\hat{\varepsilon}_t^a$ is the transitory technology shock following an AR(1) process, ρ_a is an autoregressive coefficient and $\mu_t^a \sim N(0, \sigma_a^2)$. The parameter ϕ_p represents one plus the share of fixed costs in production.[12], while α is the output elasticity to capital.

Cost minimization problem implies that:

$$\hat{u}_t + \hat{k}_t - \hat{h}_t - \hat{g}_{z,t} = \hat{w}_t - \hat{r}_t^k \tag{19}$$

where the degree of capital utilization is a positive function of the rental rate of capital:

$$\hat{r}_t^k = \frac{\sigma_u}{1-\sigma_u}\hat{u}_t \tag{20}$$

where σ_u represents the positive function of elasticity of the capital utilization adjustment cost.

Due to price stickiness as in Calvo (1983) and partial indexation to lagged inflation of those prices that can not be re-optimised profits maximization by price-setting firms lead to the following New-Keynesian Phillips curve

$$\hat{\pi}_t = \pi_1\hat{\pi}_{t-1} + \pi_2\hat{\pi}_{t+1} - \pi_3\widehat{\mu}_t^p + \widehat{e}_t^p, \tag{21}$$

[12]Note that from the zero profit condition in steady state, ϕ_p also represents the value of the gross steady state price markup.

where $\pi_1 = \frac{\iota_p}{1+\beta\gamma^{1-\sigma_c}\iota_p}$, $\pi_2 = \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}\iota_p}$, $\pi_3 = \frac{\left(1-\beta\gamma^{1-\sigma_c}\xi_p\right)(1-\xi_p)}{(1+\beta\gamma^{1-\sigma_c}\iota_p)\xi_p[(\phi_p-1)e^p+1]}$. ι_p represents the indexation parameter, ξ_p the degree of price stickiness in goods market and $\widehat{e}^p_t$ is the curvature of Kimball aggregator in the goods market. The price markup disturbance follows an ARMA(1,1) process, $\widehat{e}^p_t = \rho_p \widehat{e}^p_{t-1} + \varepsilon^p_t - \mu_p \varepsilon^p_{t-1}$, ρ_p is the AR(1) coefficient and $\varepsilon^p_t \sim N(0, \sigma^2_p)$. The term $(\phi_p - 1)$ is the steady-state markup in the goods market.

Marginal costs, $\widehat{mc}_t$, are affected by the factors costs and a productivity shocks

$$\widehat{mc}_t = -\hat{\varepsilon}^a_t + \alpha \hat{r}^k_t + (1-\alpha)\,\hat{w}_t \tag{22}$$

where the total factor productivity shocks follows an AR(1) process:

$$\hat{\varepsilon}^a_t = \rho_a \hat{\varepsilon}^a_{t-1} + \eta^a_t \sim N(0, \sigma^2_a)$$

The monetary authority sets the short-term interest rate according to a Taylor rule of the form:

$$\hat{r}_t = \phi_r \hat{r}_{t-1} + (1-\phi_r)\left[\phi_\pi \hat{\pi}_t + \phi_y\left(\hat{y}_t - \hat{y}^p_t\right)\right] + \phi_{\Delta y}\left[\left(\hat{y}_t - \hat{y}^p_t\right) - \left(\hat{y}_{t-1} - \hat{y}^p_{t-1}\right)\right] + \hat{\varepsilon}^r_t, \tag{23}$$

where $\hat{y}^p_t$ represents the level of output that would prevail under flexible prices and wages, ϕ_R, ϕ_π, ϕ_y, $\phi_{\Delta y}$ are policy parameters referring to interest-rate smoothing, and the responsiveness of the nominal interest rate to inflation, to the output gap and to changes in the output gap, respectively.

$$\hat{\varepsilon}^r_t = \rho_r \hat{\varepsilon}^r_{t-1} + \eta^r_t \sim N(0, \sigma^2_r)$$

Following Smets and Wouters (2003), we relate the employment variable, e_t, to the unobserved worked-hours variable, h_t, by means of the following relation:

$$\hat{e}_t = \beta E_t \hat{e}_{t+1} + \frac{(1-\xi_e)(1-\beta\xi_e)}{\xi_e}\left(\hat{h}_t - \hat{e}_t\right)$$

where ξ_e determines the sensitivity of employment with respect to worked hours.

We model is solved assuming that agents have perfect knowledge about the model, its parameters and the true stochastic processes of the economy. Using the approach laid out in Sims (2002), we can write the model mapping the expectational errors into the set of structural shocks:

$$X_t = \Gamma X_{t-1} + \Omega \Sigma_t \tag{24}$$

where X_t is a vector containing the endogenous variables of the model, Σ_t is the vector of the exogenous shocks, and matrices Γ and Ω contain the non-linear combinations of the model parameters. which yields the transition equation for our state space model. Note that 24 yields the transition equation for our state space model.

We estimate the preference shock, the technology shock, the wage and price markup shock, the monetary shock, government spending shock and the investment-specific technology shock. The innovations to these processes are structural shocks driving the model dynamics.

B Data description

Variable	Name	Source	Transformation
y	GDP at market prices	ECB Real-time DB	Log-difference; calendar and seasonally adjusted, converted in real term using GDP deflator
c_l	Final consumption expenditure	ECB Real-time DB	Log-difference; calendar and seasonally adjusted, converted in real term using GDP deflator
i	Gross fixed capital formation	Eurostat	Log-difference; calendar and seasonally adjusted, converted in real term using GDP deflator
π	HICP	Eurostat	-
w	Compensation for employees	Eurostat	Log-difference; calendar and seasonally adjusted, converted in real term using GDP deflator
r	Euribor	Eurostat	Divided by 4
e	Employment	Eurostat	HP detrendend
$E_t y_{t+1}^{obs}$	One-year ahed real GDP growth expectations	SPF, ECB	-
$E_t \pi_{t+1}^{obs}$	One-year-ahed inflation expectations	SPF, ECB	-