You can't always get what you want?
A Monte Carlo analysis of the bias and the efficiency of dynamic panel data estimators

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Abstract

We assess the bias and the efficiency of state-of-the-art dynamic panel data estimators by means of model-based Monte Carlo simulations. The underlying data-generating process consists of a theoretical growth model of income convergence based on capital accumulation. While we impose a true speed of convergence of around 5%, the results obtained with the different panel data estimators range from 0.03% to 17%. In terms of the squared percent error, the pooled OLS, fixed effects, random effects, and difference GMM estimators perform worst, while the system GMM estimator with the full matrix of instruments and the corrected least squares dummy variable (LSDVC) estimator perform best. The LSDVC estimator, initialized by the system GMM estimator with the full matrix of instruments, is the only one capturing the true speed of convergence within the 95% confidence interval for all scenarios. Other estimators yield values that are substantially different from the true ones.

Keywords: Monte Carlo Simulation, Dynamic Panel Data Estimators, Estimator Bias, Estimator Efficiency, International Income Convergence.

JEL codes: C10, C33, O41, O47.

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1. Introduction

Given that dynamic panel data estimators are widely used in economics, it might come as a surprise that even the most sophisticated methods are prone to large biases and inefficiencies. We use theory-based Monte Carlo simulations to uncover the direction of these biases, their magnitudes, and also the extent to which the simulated true speed of convergence lies outside the confidence intervals of the point estimates obtained with these estimators. In so doing, we rely on a standard theoretical model of income convergence as the true underlying data-generating process.

Since the publication of Islam (1995), dynamic panel data estimators have become a popular tool in the empirical analysis of the speed of income convergence. A very detailed overview of the literature and a thorough discussion of the problems that arise in these types of growth regressions is provided by Durlauf et al. (2005). While there seems to be a broad consensus in the profession that a reasonable estimate for the speed of convergence lies around 2%, the results of different empirical studies vary wildly: Abreu et al. (2005) analyze 48 articles with 619 estimated values for the speed of convergence and show that the estimates range from negative values to the maximum of 65.59%. This huge dispersion can be attributed partly to the use of different specifications, different control variables, and different sample sizes, the presence of measurement errors, and to endogeneity issues (see, for example, de la Fuente, 1997; Durlauf, 2001; Durlauf et al., 2005; Eberhardt and Teal, 2011, for discussions). However, purely methodological aspects also seem to play an important role: Abreu et al. (2005, p. 410) note that generalized method of moments (GMM) estimators and the corrected least squares dummy variable (LSDVC) technique yield substantially higher estimates of the speed of convergence than other approaches. This might be due to the fact that the biases of GMM-based estimators can be large as shown by Hsiao et al. (2002) in Monte Carlo studies, albeit in a different context.

From the perspective of growth economics, the large differences in the results delivered by the different estimation techniques urge for a thorough analysis of the biases and inefficiencies of the state-of-the-art estimators used in convergence regressions. Simulations based on a theoretical model as the "true" and known data-generating process offer an interesting opportunity to put the different methods to a test. Such an approach allows to abstract from complications that emerge in the real world such as measurement errors, omitted variables, different sample sizes, and endogeneity, i.e., it is essentially the same as running a controlled experiment. Hauk and Wacziarg (2009) were to our knowledge the first to provide a systematic analysis of the different biases involved with panel data estimators in growth regressions. Our study differs from theirs along the following lines: i) while Hauk and Wacziarg (2009) simulate the data based on estimated fixed effects, we simulate the trajectories of per capita GDP based on a Solow (1956) growth model with different deep parameters, such as the savings rate and the population growth rate, for the different countries. This yields simulated country-specific fixed effects without the need to rely on estimations obtained with the same statistical techniques that are later on tested for their overall performance;[3] ii) we include additional estimators: two versions of the system GMM (SYSGMM) estimator – one with the full matrix of instruments and one with the collapsed matrix of instruments – and the LSDVC estimator that has been proposed recently as an alternative to GMM-based estimators as a remedy for the Nickell (1981) bias. Including the LSDVC estimator is particularly important because we show that it outperforms all the other estomators; iii) we do not only analyze the extent of the bias of different estimators but also their efficiency by means of their confidence intervals. This yields the surprising insight that the true speed of convergence is outside of the 95% confidence intervals of almost all estimators and that the SYSGMM estimator – particularly the one with a collapsed matrix

[3]Note that we do not need to simulate "realistic" convergence processes. In fact, all we need is that the underlying true speed of convergence is known and that there are enough data points available for the estimation procedure.

of instruments – is highly inefficient. The SYSGMM estimator with the collapsed matrix of instruments is also the only estimator that yields insignificant results in some scenarios even though the simulated true rate of convergence is substantially larger than zero.

In our Monte-Carlo study we take into account the following state-of-the-art methods: the pooled least squares (POLS) estimator, the random effects (RE) estimator, the between estimator (BE), the fixed effects (FE) estimator, the difference GMM (DIFFGMM) estimator, the system GMM (SYSGMM) estimator with both the full matrix of instruments and the collapsed matrix of instruments, and the LSDVC estimator.[4] Knowing the true speed of convergence by design, we compare the point estimates delivered by the different methods as well as the confidence intervals of these point estimates to identify those estimators that are most promising for estimating the rate of convergence in practical applications. Clearly, our analysis indicates that the LSDVC estimator performs best and should therefore be among those estimators that are employed in empirical studies of the rate of convergence. Since even some of the allegedly unbiased estimators perform badly, we argue that researchers should not rely on only one estimator when assessing the speed of convergence, even if this estimator is deemed to be suitable for the different sources of biases involved in the given specifications and in the corresponding data set (for example the DIFFGMM or the SYSGMM estimators). A more cautious and appropriate strategy would be to compare the outcomes of different estimators in light of the results of Monte Carlo studies like this and the one of Hauk and Wacziarg (2009).

The remainder of the paper is organized as follows. In Section 2 we provide a short

[4]For the conceptual details of the different estimators and discussions regarding their advantages and disadvantages see Hurwicz (1950), Nickell (1981), Arellano and Bond (1991), Blundell and Bond (1998), Judson and Owen (1999), Wooldridge (2002), Bun and Kiviet (2003), Bruno (2005a), Durlauf et al. (2005), Hauk and Wacziarg (2009), Baltagi (2013), Hsiao (2014), Pesaran (2015), Ditzen and Gundlach (2016), and Hauk (2017).

discussion of important articles on convergence and we briefly describe some known biases of panel data estimators and the state-of-the-art solutions to cope with them. In Section 3 we provide a detailed explanation of the data-generating process and the different scenarios and trajectories that we simulate. In Section 4, we employ our generated data set to estimate the autoregressive coefficient of the dynamic panel data model with the different state-of-the-art methods. We report the point estimates and their confidence intervals for the different estimators and we compute the implied speed of convergence and the squared percent error for each estimator. This allows us to assess the biases of the estimators in terms of the deviations from the true speed of convergence and the efficiency of the estimators in terms of the range of their confidence intervals. Finally, in Section 5 we summarize our findings and conclude.

2. Panel data estimators and their known biases

While earlier studies of convergence relied on cross-sectional data (cf. Barro, 1991, 1997; Sala-i-Martin, 1997), progress has been made toward the use of panel data in the mid 1990s (cf. Islam, 1995; Caselli et al., 1996).[5] The main advantages of the use of panel data in this context are that i) the number of available observations increases substantially, ii) it becomes possible to control for unobserved heterogeneity that stays constant over time, and iii) dynamic relationships can be captured in a more accurate way by including the lagged dependent variable as regressor (see, for example, Baltagi, 2013; Hsiao, 2014; Pesaran, 2015, for detailed discussions).

While the inclusion of the lagged dependent variable in panel data growth regressions is crucial for the calculation of the speed of convergence, its introduction comes with a

[5]For recent applications see, for example, Cohen and Soto (2007), Esposti (2007), Hauk and Wacziarg (2009), Brückner (2013), Crespo-Cuaresma et al. (2014), Irmen and Litina (2016), and Gehringer and Prettner (2017).

substantial cost: the estimation of dynamic models is subject to the Hurwicz (1950) bias and endogeneity between fixed effects and the lagged dependent variable in FE estimation gives rise to the Nickell (1981) bias. While the Hurwicz (1950) bias can only be mitigated by increasing the time dimension of the panel data set, a number of estimators have been proposed to deal with the endogeneity between fixed effects and the lagged dependent variable: difference GMM (Arellano and Bond, 1991; Arellano and Bover, 1995), system GMM (Blundell and Bond, 1998) and the LSDVC estimator (Judson and Owen, 1999; Bun and Kiviet, 2003; Bruno, 2005a). In spite of the fact that the newer panel data estimators offer promising improvements over older ones (such as POLS, FE, and BE), there are still a number of known biases arising from these estimators. The sources of those biases that are relevant in our analysis are summarized in Table 1. Of course, the extent of the bias may be different from case to case.

[TABLE 1 here]

A very insightful overview of known biases of well-established panel data estimators is provided by Hauk and Wacziarg (2009): among other biases, they note that the omitted country-specific fixed effect may create a bias for BE and RE estimators and endogeneity of the lagged dependent variable would cause a bias for FE and RE estimators. Another issue is the problem of weak instruments as also noted by Hauk and Wacziarg (2009): this problem is particularly severe in DIFFGMM estimation because it only relies on lagged levels as instrument for current differences. The SYSGMM estimator was designed to alleviate this problem by relying on two types of instruments, lagged levels and lagged differences. Even if the instruments are not weak, however, there can simply be too many of them – this issue is described in detail by Roodman (2009) and referred to as the problem of instrument proliferation. In general, the validity of instruments is often not guaranteed in case of GMM-based estimators.

In our study we focus on the biases described in Table 1. However, there are other known sources for biases the analysis of which are beyond the scope of this paper and would require a different underlying data-generating process. For example, all of the estimators involved are exposed to the bias that arises because of measurement errors (Wooldridge, 2002, p. 311) and to the serial correlation of the error term (see Wooldridge, 2002, pp. 282–283 and 307).

3. The data-generating process

We design the data-generating process based on a Solow (1956) growth model. This is the simplest framework for simulating a theory-based convergence process in which we can adjust the true underlying speed of convergence by changing the model's parameters. The knowledge of the true speed of convergence allows us to assess the biases of the different state-of-the-art estimators by comparing it to the results of the parameter estimates obtained with the different models on our artificial data set. This strategy is essentially a controlled experiment for assessing the different estimators that does not require any parameter estimates for generating the data set. Since the data set is a purely theoretical construct kept distinct from the empirical part of assessing the performance of the different estimators, nothing – except for additional complexity – would be gained by using more sophisticated growth models with endogenous saving rates (as, for example, Ramsey, 1928; Cass, 1965; Koopmans, 1965; Diamond, 1965) or endogenous technological progress (as, for example, Romer, 1990; Jones, 1995; Segerström, 1998; Howitt, 1999) as baseline frameworks.

We introduce unobserved heterogeneity, μ_i, by assigning model-driven fixed effects though a random parametrization for each country based on the theoretical model equations. After initialization for the desired number of countries (the cross-country dimension, N), we generate the time series of the trajectories of per capita output (the time dimension, T). Finally, we introduce idiosyncratic distortions by means of stochastic shocks to account for the fact that there are deviations from the output series in the short run that are not explained by the underlying theoretical framework.

The mathematical implementation of the data-generating process is as follows. Suppose that time $t = 1, 2 \ldots, T$ evolves discretely and that we are observing $i = 1, 2, \ldots N$ different economies. Aggregate output of each economy is described by a Cobb-Douglas production function of the form

$$Y_{i,t} = K_{i,t}^{\alpha}(A_t L_{i,t})^{1-\alpha},$$

where $Y_{i,t}$ is aggregate output of country i at time t (which, by the national accounts identity, is equal to aggregate income), A_t refers to labor-augmenting technology that grows at the constant long-run rate g, $K_{i,t}$ is the physical capital stock (machines, production facilities, office buildings, etc.), $L_{i,t}$ is the amount of aggregate labor input, and α is the elasticity of aggregate output with respect to physical capital input. Households save a constant fraction s_i of their income $Y_{i,t}$ in each year, which implies that physical capital accumulation is given by the dynamic equation

$$K_{i,t+1} = s_i Y_{i,t} + (1 - \delta)K_{i,t},$$

where δ is the rate of depreciation. Note that this parameter is the same for all the countries, which is a standard assumption in the theoretical literature. Furthermore, country-specific differences in the rate of depreciation would anyway be perfectly captured by the fixed effects in the empirical analysis because δ does not vary over time. We denote per worker variables with lowercase letters such that per worker capital is given by $k_{i,t} = K_{i,t}/L_{i,t}$ and per worker output pins down to

$$y_{i,t} = Y_{i,t}/L_{i,t} = A_i^{1-\alpha} k_{i,t}^{\alpha}. \tag{1}$$

Altogether, we can derive the following approximation of the fundamental equation of the Solow (1956) model in terms of the evolution of capital per unit of effective labor $\hat{k}_{i,t} =$

$K_{i,t}/(A_{i,t}L_{i,t})$:

$$\hat{k}_{i,t+1} \approx s_i \hat{k}_{i,t}^{\alpha} + (1 - \delta - g - n_i)\hat{k}_{i,t}, \tag{2}$$

where n_i is the growth rate of the workforce. Since we abstract from unemployment, childhood, and retirement, per worker variables and per capita variables coincide, such that n_i is equivalent to the population growth rate. Note that, in continuous time, the differential equation counterpart to Equation (2) holds with equality. The approximation in case of discrete time becomes better the lower are the population growth rate and the rate of technological progress and the smaller is the time step between t and $t+1$. In our case, where t is measured in yearly terms, this is a reasonable approximation. It would be more difficult to defend this approximation in an overlapping generations framework in which a time step refers to one generation and therefore lasts for around 25 years.

The steady-state capital stock can be determined by setting $\hat{k}_{i,t+1} = \hat{k}_{i,t}$ in Equation (2) and is given by

$$\hat{k}_i^* = \left(\frac{s_i}{n_i + \delta + g}\right)^{\frac{1}{1-\alpha}}. \tag{3}$$

Steady-state output per unit of effective labor is then $\hat{y} = \hat{k}^{\alpha}$ such that output per capita is given by

$$y_i^* = A_i \cdot (\hat{k}_i^*)^{\alpha} = A_i \left(\frac{s_i}{n_i + \delta + g}\right)^{\frac{\alpha}{1-\alpha}}. \tag{4}$$

The true speed of convergence, $\lambda_{true,i}$, can then easily be derived for each country as (see Romer, 2006, pp. 25-26):

$$\lambda_{true,i} = (1 - \alpha)(n_i + \delta + g). \tag{5}$$

The average values of $\lambda_{true,i}$ over all countries are compared to the estimated speed of convergence from the different dynamic panel data estimators in Section 4. The variable that is crucial for generating convergence is the initial level of capital per unit of effective labor, $\hat{k}_{i,0}$. In case that we set $\hat{k}_{i,0}$ to a small value, we generate a poor country i that has a strong catch-up potential and will grow fast initially. By contrast, if we set $\hat{k}_{i,0}$ close to the steady-state value, we generate a rich country with a low catch-up potential that will grow sluggishly. To rule out the situation of convergence to the steady state from above (i.e., with negative growth rates)[6], we initialize the simulation by setting $\hat{k}_{i,0}$ to a level below the steady-state according to

$$\hat{k}_{i,0} = D_i \hat{k}_i^*, \tag{6}$$

where $D_i \in (0, 0.3]$ is the distance to the steady state as drawn from a truncated normal distribution[7] (see Tables 2 and 3 for an overview of the parameter values used in the different simulation scenarios). We set the upper bound of the relative position of the initial capital stock at 30% to ensure catch-up growth over a considerable time period. Equation (6) introduces model-driven heterogeneity in the growth rates between different countries and this is exactly the source of variation that we need to identify the parameter estimate for the convergence rate. Note, in this context, that we must not control for the differences in the capital stock between different countries because this would eliminate the source of the convergence process.

Instead of generating the data set for different countries by relying on estimated fixed effects from empirical specifications as in Hauk and Wacziarg (2009, p. 116), we create theory driven fixed-effects by generating different artificial countries, where the unobserved heterogeneity in the dataset, μ_i, follows from the different values of the deep parameters used for each single country. Although it is not required to use plausible parameter values

[6]It is often argued that the negative growth rates in the former countries of the Soviet Union in the 1990s can be attributed to a shrinking capital stock. While the Soviet Union had a very high forced investment rate that could not be sustained after the communist system collapsed, in our simulations the question would arise how a country could have built up a capital stock that is larger than its steady-state capital stock in the first place.

[7]Drawing from truncated normal distribution is based on the functions from Trautmann et al. (2014).

— because we could generate any data set we want and use it as our data-generating process as long as we can compute the true underlying speed of convergence — we think it is more digestible to use parameter values that are familiar from growth theory and/or that are empirically plausible. Most of the parameters of the Solow model are bounded in some way, for example, $s_i \in (0,1)$, $k_0 > 0$, $\alpha \in (0,1)$, and $\delta > 0$ cannot attain negative values and some cannot exceed 1. This provides theoretical restrictions that we impose on the parameter space by truncating the corresponding simulated distributions (see Robert, 1995; Robert and Casella, 2005). Second, we use mean values of the parameters that are reasonably close to the data observed in reality. We assume that α and δ are fixed and equal across countries, where we set $\alpha = 0.35$, which is broadly in line with the literature (cf. Acemoglu, 2009; Jones, 1995), and $\delta = 0.06$, which is in line with the findings of Fraumeni (1997).

We introduce country-specific heterogeneity via the savings rate s_i and the population growth rate n_i. It is very important to note that this unobserved heterogeneity is taken into account in the empirical part by those estimators that include country-specific fixed effects or that base the estimations on the first differences of the data. Consequently, even if we do not control for n_i and s_i directly, we are estimating a Solow (1956) model without the omitted variable bias. In determining the values of s_i and n_i we rely on World Bank (2016) data for 214 countries over the years 1966 to 2014, which suggests a mean gross savings rate of 27.97% and a mean population growth rate of 1.83%.[8] While we could easily introduce additional country-specific heterogeneity in the parameters g, α, and δ, this would merely complicate the analysis without leading to additional insights.[9]

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[8]Countries with negative average values for s and n over this time period were left out of the consideration.

[9]Altogether, the distributions from which we draw the underlying parameters for the simulation are independent from each other. It is possible to build in collinearity between the variables and to analyze the extent to which different estimators can cope with multicollinearity. While this is outside of the scope of

We simulate four scenarios, two deterministic and two stochastic ones, for 150 countries and 55 time steps, out of which we create our sample for the estimation. In contrast to the deterministic scenarios, which result in smooth and concave trajectories of output as it converges toward its steady-state level, the stochastic scenarios feature additional shocks over time on output, denoted by ε_y, on the savings rate, denoted by ε_s, and on the population growth rate, denoted by ε_n. Doing so introduces time-varying savings rates and population growth rates $s_{i,t}$ and $n_{i,t}$ (see Table 3, Scenario 4) without altering the underlying speed of convergence in a systematic way. The stochastic shocks ε_y, ε_s, and ε_n are simulated from a normal distribution such that these shocks can be considered as stochastic perturbations similar to unsystematic measurement errors or transient exogenous shocks. We leave out the first 5 time steps from the resulting series because the convergence effects are very strong for countries with a low value of D_i. Out of the resulting time series variables, we generate five-year averages to mimic the estimation strategy that is often employed to get rid of business-cycle effects in real-world data (cf. Islam, 1995; Crespo-Cuaresma et al., 2014). As a consequence, we have an artificial data set for 150 countries and 10 time periods (as five year averages) such that the dimensions of our panel data set are given by $N = 150$ and $T = 10$. These values are very common for panel data growth regressions.

The first scenario that we simulate involves a limited randomization relying on a truncated normal distribution only for D_i and s_i, whereas in the second scenario we also randomize the population growth rate n_i. In the third scenario we introduce stochastic shocks to Equation (1) for the dynamics of output, while the fourth scenario also features stochastic shocks on the savings rates and on the population growth rates such that $s_{i,t}$ and $n_{i,t}$ enter Equation (2) and the model dynamics in a time-varying manner.

[TABLE 2 here]

our paper, it is surely a promising avenue for further research.

[TABLE 3 here]

[FIGURE 1 here]

4. Estimation and comparative assessment of the estimators

In this section we estimate the speed of convergence that is implied by the different parameter estimates of the AR(1) term in the dynamic panel data growth regressions ($\lambda_{implied}$). We compare the resulting value to the true value (λ_{true}) that we know for each scenario from the simulations. Based on these values, we measure the error of each estimated value as captured by the relative distance of the implied estimated speed of convergence from the corresponding true speed of convergence. This allows us to compare the extent of the biases of the different estimators. Furthermore, we provide the confidence intervals for the different estimates of the AR(1) term and assess whether or not its true value is captured by them. Finally, we assess the efficiency of the different estimators by comparing the size of their confidence intervals. The equations that we estimate are standard and described in detail by, for example, Bond et al. (2001, p. 15) and Islam (1995, p. 1136):

$$y_{i,\bar{t}} = \gamma y_{i,\bar{t}-1} + \phi_{\bar{t}} + \mu_i + \upsilon_{i,\bar{t}}, \tag{7}$$

$$\gamma = e^{-\lambda_{implied} \cdot \tau}, \tag{8}$$

$$\lambda_{implied} = -\frac{\log(\gamma)}{\tau}, \tag{9}$$

where $y_{i,\bar{t}}$ is average per capita output of country i between time t and $t-4$ (because we take the average over five years), $y_{i,\bar{t}-1}$ refers to the corresponding lagged variable, $\phi_{\bar{t}}$ is a vector of time-specific fixed effects, μ_i is a vector of country-specific fixed effects, $\upsilon_{i,\bar{t}}$ is an idiosyncratic error term, γ refers to the auto-regressive coefficient, $\lambda_{implied}$ is the implied speed of convergence obtained via the estimate for γ, and τ is the number of periods captured by each time step, which is 5 in our case. Note that, in general, we control for country-specific fixed effects. Doing so implies that the corresponding estimators fully capture the differences between countries that are due to the time-independent construction of the unobserved heterogeneity via n, s, and D in Scenarios 1-3. Consequently, there is no omitted variable bias left: in case of estimators that control for unobserved heterogeneity, we are indeed estimating a model of conditional convergence and not of absolute convergence. Furthermore, note that we control for time-specific fixed effects in all specifications by including dummy variables for the 5-year periods. This captures the common influence of the long-run technological growth rate g on income convergence among all countries.

The POLS, FE, RE, and BE estimators are applied without the implementation of additional corrections/options. In case of LSDVC, DIFFGMM, and SYSGMM[10], we had to make further decisions. For both, DIFFGMM and SYSGMM, standard errors have been estimated with the small-sample correction proposed by Windmeijer (2005). In DIFFGMM and SYSGMM, the 5-year period dummies were used as variables and as instruments. In addition, for SYSGMM, we implemented two versions, one with the full matrix of instruments and one with the collapsed matrix of instruments, which reduces the number of instruments from 64 to 20. In this context, instrument proliferation (or "too many instruments") can lead to various problems as described in detail by Roodman (2009). Both versions of the estimates are presented here. The ones obtained with the collapsed matrix on instruments are marked by 'col'. In the initialization of the LSDVC estimator we use the SYSGMM estimator with the full matrix of instruments. Furthermore, we implement bias correction up to the third order as proposed by Bruno (2005a) and we report bootstrapped standard errors for this estimator based on 50 replications.

Before displaying the values of $\lambda_{implied}$ as obtained from our estimates, we first plot the

[10]For estimating LSDVC, we used the STATA functions of Bruno (2005b). For estimating DIFFGMM and SYSGMM, we used the STATA functions of Roodman (2003).

AR(1) coefficients with the corresponding confidence intervals in Figure 2. Since we know λ_{true}, we can derive the true AR(1) coefficient, which is indicated by the green dotted line for each scenario. Even if the estimated AR(1) coefficient is close to the true value, the confidence intervals can be very large such that even the cases of no convergence [with the AR(1) coefficient being equal to 1] and immediate convergence [with the AR(1) coefficient being equal to zero] might be inside the confidence interval. As can be seen in Figure 2, this is the case for SYSGMM col in the first two scenarios.

[FIGURE 2 here]

Our expectations regarding the different forms of biases and their direction (see Table 1) are met in case of the POLS, FE, RE, and BE estimators. The first three underestimate the true value of the AR(1) coefficient, whereas the latter overestimates it. Note that the BE estimator, while perfoming comparatively well to the first three estomtors, also performs poorly in the sense that its confidence interval does not include the true value. This stands in contrast to the findings of Hauk and Wacziarg (2009), where the BE estimator performs reasonably well because they use the estimator to estimate 4 parameters (3 of which are steady-state determinants). However, the poor performance of the BE estimator in our study is consistent with the explanations by Ditzen and Gundlach (2016) and Hauk (2017) who show that the BE estimator performs poorly when used to estimate only the convergence coefficient.

In general, the RE estimator performs slightly better than POLS and FE in Scenarios 1, 2, 3, and in Scenario 4 its performance is close to the one of the BE estimator (see Figure 3). Whereas in Scenario 1 only D_i and s_i are randomized, in Scenario 2, n_i is randomized as well and we have additional random shocks in Scenarios 3 and 4. By the design of our simulations, the variables that are responsible for the country-specific heterogeneity (D_i, s_i, and n_i) were sampled from truncated normal distributions with the mean being different from zero. At first glance it might seem that this construction provides an advantage for the RE estimator. Since the key assumption of the RE estimator is that $E(\mu_i|x_i) = E(\mu_i) = 0$ (Wooldridge, 2002, p. 257), i.e., that the country-specific effects are orthogonal to the explanatory variables, this is, however, not the case in our generated data set. This is shown in Table 4, which summarizes three results from diagnostic tests that are common for all of our scenarios: i) the country-specific effects correlate with the regressors; ii) the F test rejects the null of $\mu_i = 0$; and iii) the Hausman test indicates that the parameter estimates of the RE specification differ from the ones of the FE specification. For Scenarios 1-4 in Table 4, the Hausman test is conducted for the basic model with time dummies.

The Hausman test indicates that the parameter estimates of the RE specification differ from the ones of the FE specification. The result of the Hausman test is not problematic in our case because we know that the FE estimator itself is biased in the given setting. Altogether, while the results of the RE estimator are close to the target comparing to POLS and FE, all of them underestimate the true AR(1) coefficient. Therefore, POLS, FE and RE cannot be recommended.

[TABLE 4 here]

The GMM methods tend to yield estimates for the AR(1) coefficient that are quite far off the mark, except for SYSGMM with the full matrix of instruments. DIFFGMM and SYSGMM with the collapsed matrix of instruments underestimate the true value, whereas SYSGMM with the full matrix of instruments slightly overestimates it. As we observe in Figure 4, these discrepancies have the reverse effects on the implied speed of convergence, $\lambda_{implied}$: DIFFGMM yields a higher speed of convergence than the true value, whereas SYSGMM with the full matrix of instruments yields a lower one. The confidence intervals of the AR (1) coefficient estimated by SYSGMM with the full instrument matrix cover the true value of the given coefficient in Scenarios 1 and 3. However, the point estimate of the coefficient itself is considerably higher than the true value. The LSDVC estimator initialized by the SYSGMM with the full matrix of instruments is the only estimator that comes close to identifying the true AR(1) coefficient in case of all scenarios. Moreover, for all scenarios, the true AR (1) coefficient lies within the estimated confidence intervals of the LSDVC estimator. In general, therefore, the LSDVC estimator outperforms the other estimators. This can also be observed in Figure 3 that plots the squared persent error of all estimators for all scenarios.

For the deterministic Scenarios 1 and 2, the worst performers in terms of the squared percent error are the FE, POLS, DIFFGMM, and the RE estimators. The best performers are the LSDVC and the SYSGMM with the full matrix of instruments. It follows that the performance of the SYSGMM estimator is heavily exposed to the choice of instruments. In our case, collapsing the matrix of instruments makes SYSGMM perform worse.

[FIGURE 3 here]

The stochastic Scenarios 3 and 4 offer interesting information on the performance of the estimators after the introduction of stochastic shocks. In Scenario 3 only the time series for output is perturbed, while, in Scenario 4, s and n are also affected by shocks (see Table 3). For these scenarios, the POLS, FE, and DIFFGMM estimators perform as poorly as in the deterministic scenarios. The performance of the RE estimator improves slightly. DIFFGMM and the SYSGMM with the collapsed matrix of instruments still underestimate the true coefficient, whereas the SYSGMM estimator with the full matrix of instruments overestimates it. Yet, SYSGMM with the full matrix of instruments performs slightly better than with the collapsed matrix of instruments (see Figure 3). The worst performers remain the FE, the DIFFGMM, POLS, and RE estimators. For Scenario 3, LSDVC and SYSGMM with the full matrix of instruments yield the best results. Collapsing the matrix of instruments reduces the accuracy of the SYSGMM estimator similar to the cases of the deterministic scenarios. For Scenario 4 the situation is similar: LSDVC initialized by SYSGMM with the full matrix of instruments takes the lead (see Table 5 and Figure 3).

Finally, Table 6 provides the numerical values obtained by the different estimators for the implied speed of convergence and the true speed of convergence for comparison, while Figure 4 illustrates the discrepancies graphically. We observe that the implied speed of convergence ranges from barely above zero in case of the BE estimator to almost 17% in case of the POLS, FE, RE, and DIFFGMM estimators. Consequently, depending on the estimator that is used, the half life (the time it takes until half of the gap between current per capita GDP and steady-state per capita GDP is closed), ranges from around 4 years in case of the FE or DIFFGMM estimators to several hundred years in case of the BE estimator. This compares to the 14 years that are the true value according to our simulation design.

Altogether, our results suggest that a non-negligible part of the discrepancies found in the speed of convergence between different empirical studies might be due to the inherent biases of the different estimation methods that were employed. However, some estimators performed substantially better: SYSGMM with the full matrix of instruments and the LSDVC, initialized by the SYSGMM estimator with the full matrix of instruments. In fact, LSDVC is the only estimator which captures the true AR coefficient in all simulations, whereas the SYSGMM with the full matrix of instruments slightly overestimates the true value. It follows that using the LSDVC estimator based on SYSGMM estimator with a full matrix of instruments may be the most appropriate empirical strategy to estimate the implied speed of convergence. One has to note that using other estimators for the initialization of the LSDVC estimator did improve its performance.

The central conclusion of our paper follows immediate. Researchers should not rely on a single dynamic panel data estimator, even if the given estimator is deemed to be suitable for the different sources of biases involved in the empirical specification and in the corresponding data set. A better strategy is to compare the outcomes of different estimators and to keep their biases and inefficiencies from Monte Carlo studies in mind when drawing conclusions based on them.

[TABLE 5 here]

[TABLE 6 here]

[FIGURE 4 here]

5. Conclusions

We generated an artificial data set from the simulated growth trajectories of a Solow (1956) model for 150 countries over a time span of 55 years to construct a panel data set with the dimensions $N = 150$ and $T = 10$ (with the data being averaged over 5 years). This is a typical sample size of panel data growth regressions used to assess the speed of convergence. The resulting trajectories exhibit a rate of convergence that can be calculated and used as the true underlying rate of convergence in a controlled experiment to assess the biases and inefficiencies of different dynamic panel data methods. In the simulation exercise, we considered two deterministic scenarios, where the first assumes differences in initial capital stocks and savings rates between the different countries, the second allows for different population growth rates, the third introduces stochastic shocks on the per capita output series, and the forth allows for stochastic shocks on savings rates and population growth rates. We use a battery of standard dynamic panel data estimators to estimate the speed of converge and find that the estimated speed of convergence is typically far off the true speed of convergence. With the true rate being around 5% throughout the 4 scenarios, the estimated rate of convergence ranges from barely above 0% to almost 17%. This means that, while the true half life is around 14 years, the estimated half life ranges from 4 years to several hundred years.

Our analysis sheds some light on the performance of different estimators. This is crucial, given that the results of different econometric techniques regarding the analysis of panel data vary widely. For the sake of clarity, we did not include additional complications such as autocorrelated disturbances, multicollinearity, problems with small samples, and systematic measurement errors. These would have required a more elaborate simulation design with some additional arbitrary choices involved, which is outside the scope of the present paper. We think that analyzing these issues is a promising area for further research.

The immediate conclusion from our results is that it might not be a good strategy to rely on a single estimator in empirical studies. Using the simulated scenarios, we have shown that the estimated speed of convergence is highly dependent on the choice of the estimator: POLS, FE, RE, DIFFGMM, and SYSGMM with the collapsed matrix of instruments overestimate the true value substantially, whereas the BE estimator underestimates the true speed of convergence. SYSGMM with the full matrix of instruments exhibits a comparatively good performance, although it slightly underestimates the true speed of convergence and the estimator is inefficient in terms of the size of its confidence interval. Across all scenarios, the LSDVC estimator initialized by the SYSGMM estimator with the full matrix of instruments yields the most accurate estimates.

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Table 1: Biases of panel data estimators that we address in our study

Biases	POLS	FE	RE	BE	LSDVC	DIFFGMM	SYSGMM
Non-random heterogeneity	x		x				
Omitted group effects	x		x	x			
Endogeneity of y_{t-1}	x	x	x				
Validity of instruments					x	x	x

Sources: Hauk and Wacziarg (2009); Hayakawa (2007); Buddelmeyer et al. (2008) Fernández-Val and Vella (2011); Roodman (2009); Wooldridge (2002)

Table 2: Fixed parameter values and distributions from which the remaining parameters are drawn for the deterministic scenarios

Scenario	1	2
Distance to the steady state	$D \sim N(0.1, 0.15^2)$ $D \in [0.001, 0.3]$	$D \sim N(0.1, 0.15^2)$ $D \in [0.001, 0.3]$
s	$s \sim N(0.2797, 0.0919^2)$ $s \in [0.0266, 0.6109]$	$s \sim N(0.2797, 0.0919^2)$ $s \in [0.0266, 0.6109]$
n	0.0183	$n \sim N(0.0183, 0.0117^2)$ $n \in [0, 0.0837]$
g	0.01	0.01
α	0.35	0.35
δ	0.05	0.05
λ_{true}	0.0509	0.0521

Table 3: Fixed parameter values and distributions from which the remaining parameters are drawn for the stochastic scenarios

Scenario	3	4
Distance to the steady state	$D \sim N(0.1, 0.15^2)$ $D \in [0.001, 0.3]$	$D \sim N(0.1, 0.15^2)$ $D \in [0.001, 0.3]$
s	$s \sim N(0.2797, 0.0919^2)$ $s \in [0.0266, 0.6109]$	$s \sim N(0.2797, 0.0919^2)$ $s \in [0.0266, 0.6109]$
n	$n \sim N(0.0183, 0.0117^2)$ $n \in [0, 0.0837]$	$n \sim N(0.0183, 0.0117^2)$ $n \in [0, 0.0837]$
g	0.01	0.01
α	0.35	0.35
δ	0.05	0.05
ε_y	$\varepsilon_y \sim N(0, 0.03^2)$	$\varepsilon_y \sim N(0, 0.03^2)$
ε_s	-	$\varepsilon_s \sim N(0, 0.0008^2);\ s.t.\ s > 0$
ε_n	-	$\varepsilon_n \sim N(0, 0.00008^2);\ s.t.\ n > 0$
λ_{true}	0.0513	0.0513

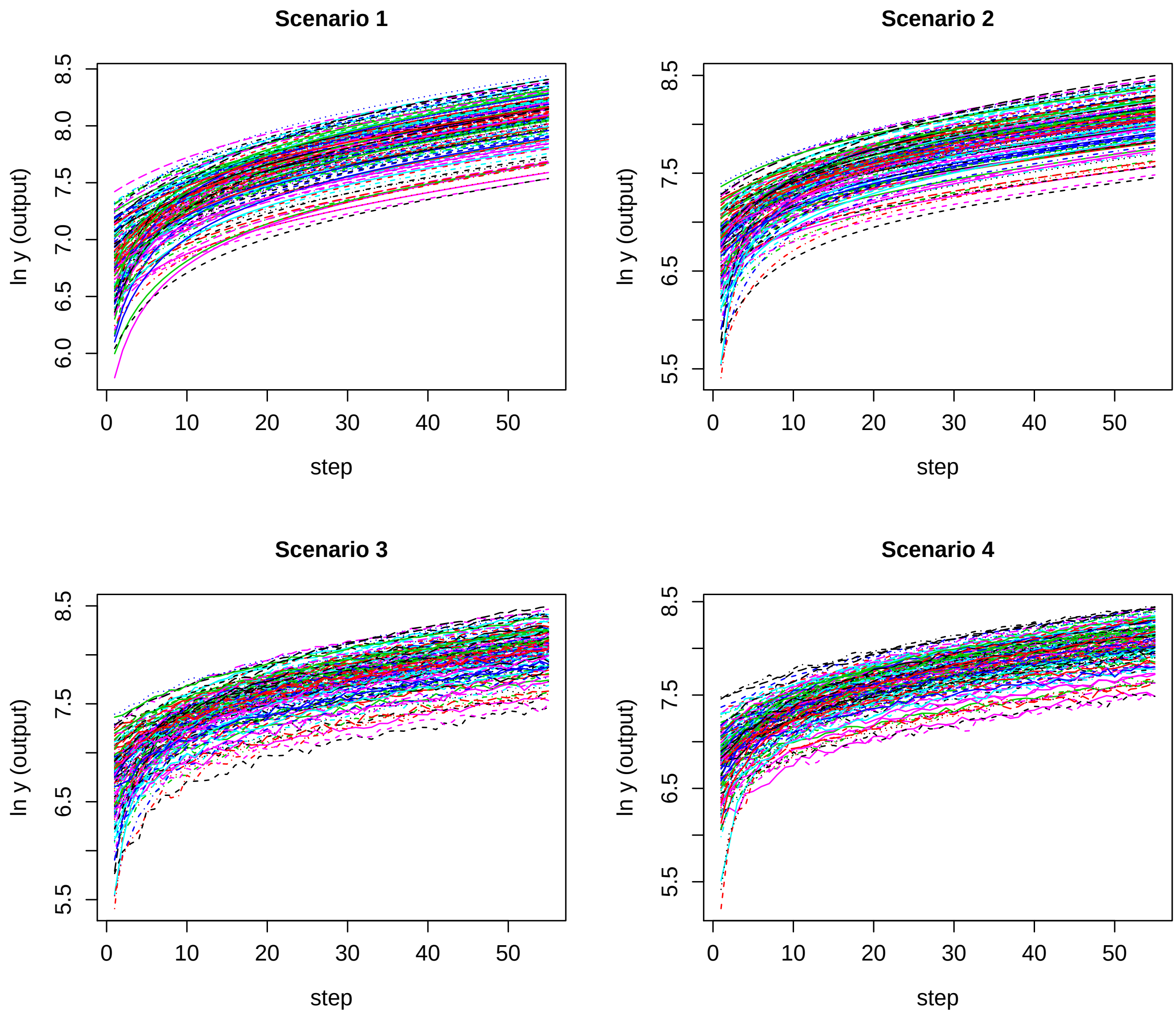

Figure 1: Convergence paths for 150 countries from the different simulated scenarios of the Solow (1956) model over 55 years (we excluded the first 5 years from the sample in the estimation part (see Section 3 for details). Scenario 1 considers deterministic paths, where D_i and s_i are allowed to differ between the different countries. In Scenario 2 also the population growth rate n_i is country-specific. Scenario 3 introduces a stochastic shock ε_y on the per capita output series. Scenario 4 allows for stochastic shocks also on the savings rate (ε_s) and on the population growth rate (ε_n).

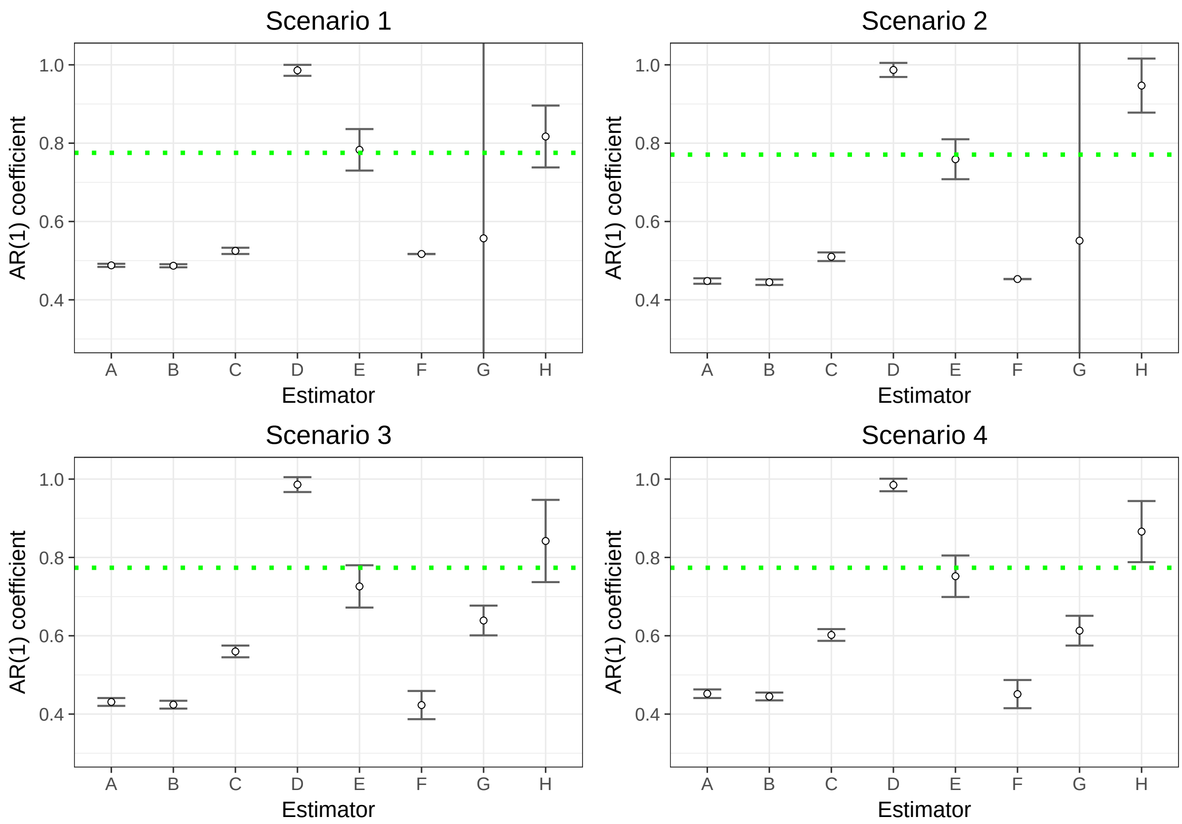

Figure 2: Estimated values of the AR(1) coefficient, γ. Note: The dotted green lines refer to the true value (γ_{true}) as calculated from the known speed of convergence (λ_{true}). The different estimators are denoted by the following list of letters A = POLS, B = FE, C = RE, D = BE, E = LSDVC, F = DIFFGMM, G = SYSGMM col, and H = SYSGMM. The circles indicate the point estimates for the corresponding parameters, while the whiskers refer to the 95% confidence intervals.

Table 4: A closer look at the fixed effects

Fixed effects inference	corr(μ_i, Xβ)	F test, H_0: $\mu_i = 0$ (p-values)	Hausman FE vs. RE (p-values)
Scenario 1	0.3426	0.0000	0.0000
Scenario 2	0.3230	0.0000	0.0000
Scenario 3	0.3094	0.0000	0.0000
Scenario 4	0.3257	0.0000	0.0000

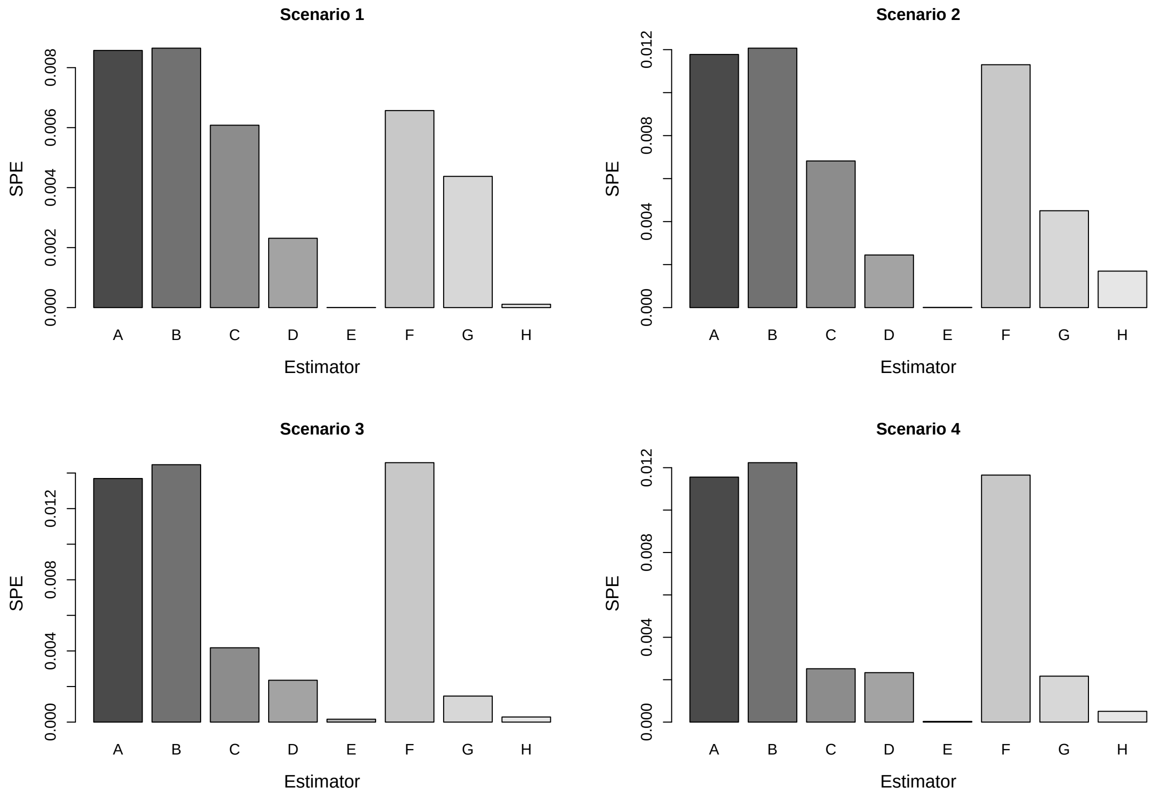

Figure 3: Squared Percent Error of the different estimators. Note: The estimators are referred to by the following letters; A = POLS; B = FE; C = RE; D = BE; E = LSDVC; F = DIFFGMM; G = SYSGMM, col; H = SYSGMM.

Table 5: Squared percent error (Fig. 3)

Estimator	Scenario 1	Scenario 2	Scenario 3	Scenario 4
PA	0.008573	0.01178	0.01369	0.01155
FE	0.008650	0.01207	0.01447	0.01224
RE	0.006080	0.00682	0.00418	0.00252
BE	0.002311	0.00245	0.00235	0.00233
LSDVC	0.000004	0.00001	0.00016	0.00003
DIFFGMM	0.006569	0.01130	0.01458	0.01165
SYSGMM, col	0.004375	0.00451	0.00146	0.00217
SYSGMM	0.000110	0.00170	0.00029	0.00051

Table 6: Estimates of the implied speed of convergence (Fig. 4)

Estimator	Scenario 1	Scenario 2	Scenario 3	Scenario 4
POLS	0.14349	0.16059	0.16833	0.15881
FE	0.14390	0.16194	0.17160	0.16194
RE	0.12887	0.13467	0.11596	0.10150
BE	0.00282	0.00262	0.00282	0.00302
LSDVC	0.04892	0.05515	0.06404	0.05700
DIFFGMM	0.13194	0.15837	0.17208	0.15926
SYSGMM, col	0.11704	0.11920	0.08957	0.09788
SYSGMM	0.04042	0.01089	0.03440	0.02877
True lambda	0.05090	0.05208	0.05132	0.05132
Simple average over all estimators	0.09468	0.10043	0.10235	0.09602

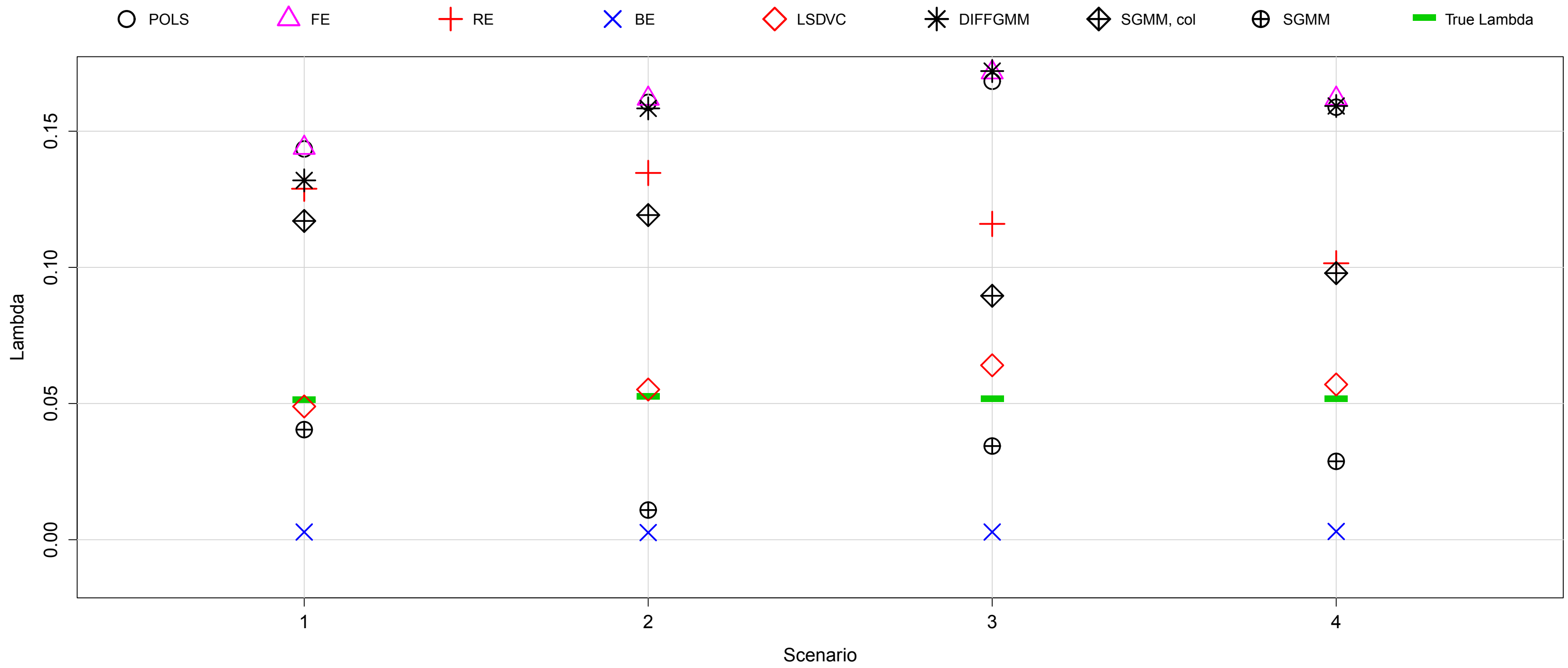

Figure 4: The estimated speed of convergence and the true value. Note: the green lines indicate the true speed of convergence, while the other signs refer to the different values obtained by the different estimators.