Measuring the Distributions of Public Inflation Perceptions and Expectations in the UK

Bayesian Analysis of a Normal Mixture Model for Interval Data with an Indifference Limen

Yasutomo Murasawa

Konan University

IAAE 2017

Plan

Plan

Why Inflation Expectatoins?

Important in monetary economics

- Level
- Disagreement

Why disagree?

- sticky information?
- bounded ratoinality?

Since Mankiw, Reis, and Wolfers (2004), there are many empirical works on the distributions of inflation expectations. They typically use numerical data (e.g., Michigan Survey), but many surveys collect only categorical data.

Quantificatoin of Qualitative Data

Ordinal data on inflation expectations (y_i^*)

$$y_i := \begin{cases} 1 & \text{if } y_i^* < 0 \\ 2 & \text{if } y_i^* \approx 0 \text{ (indifference limen)} \\ 3 & \text{if } y_i^* > 0 \end{cases}$$

Quantificatoin of Qualitative Data

Estimate the distribution of $\{y_i^*\}$ using $\{y_i\}$

Need strong assumptions; e.g., Carlson and Parkin (1975)
$\Longrightarrow$ Use interval data instead (if available).

Use of Interval Data

Murasawa (2013, OBES)

1. Estimate the distributions of inflation expectations among households in Japan
2. Fit various unimodal distributions (normal, skew normal, skew exponential power, and skew t)
3. Apply ML method

This paper

1. Estimate the distributions of inflation perceptions and expectations among individuals in the UK
2. Fit multimodal distributions (normal mixture)
3. Apply Bayesian method

Contributions

1. Bayesian analysis of a normal mixture model for interval data with an indifference limen (useful for measuring inflation expectations)

   Prior settings: hierarchical prior

   Posterior simulation: NUTS (No U-Turn Sampler)
2. Estimate the distributions of public inflation perceptions and expectations in the UK during 2001Q1–2015Q4
3. Illustrate a possible use of the estimated distributions by measuring information rigidity in the UK

Plan

BoE/GfK NOP Inflation Attitudes Survey

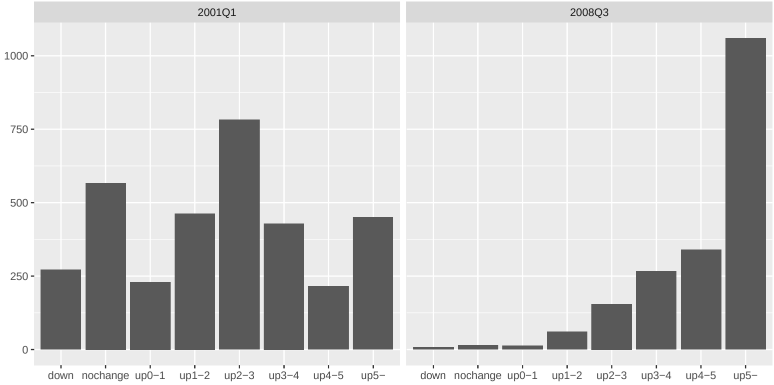
- Quarterly survey started in 2001Q1
- Quota sample of adults throughout the UK
- Quota is 4,000 (Q1) or 2,000 (others)
- Use 60 samples (2001Q1–2015Q4) separately

Intervals

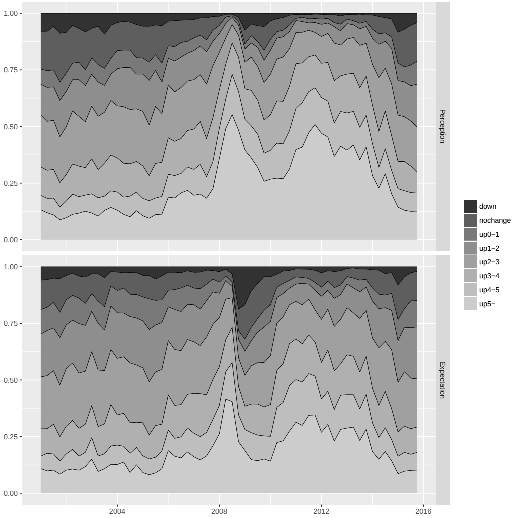
8 intervals (excluding 'No idea'):

1. Gone/Go down
2. Not changed/change (indifference limen)
3. Up by 1% or less
4. Up by 1% but less than 2%
5. Up by 2% but less than 3%
6. Up by 3% but less than 4%
7. Up by 4% but less than 5%
8. Up by 5% or more

Inflation Perceptions (2001Q1 and 2008Q3)



Relative Frequencies (2001Q1–2015Q4)



Plan

Interval/Ordinal Response Model

Interval/ordinal response model with an indifference limen

$$y_i := \begin{cases} 1 & \text{if } \gamma_0 < y_i^* \leq \gamma_1 \\ \vdots & \\ J & \text{if } \gamma_{J-1} < y_i^* \leq \gamma_J \end{cases}$$

where

- y_i^* is a latent variable
- $-\infty = \gamma_0 < \cdots < \gamma_l < 0 < \gamma_u < \cdots < \gamma_J = \infty$
- we know $\{\gamma_j\}$ except for an indifference limen $[\gamma_l, \gamma_u]$

Normal Mixture Model

Normal mixture model for y_i^*

$$y_i^* \sim \sum_{k=1}^{K} \pi_k \mathrm{N}\left(\mu_k, \sigma_k^2\right)$$

Parameters

- $\boldsymbol{\pi} := (\pi_1, \ldots, \pi_K)'$
- $\boldsymbol{\mu} := (\mu_1, \ldots, \mu_K)'$
- $\boldsymbol{\sigma} := (\sigma_1, \ldots, \sigma_K)'$
- $\boldsymbol{\gamma} := (\gamma_l, \gamma_u)'$

Plan

Why Bayesian?

ML may not work for normal mixture models

1. Given numerical data, the likelihood function is unbounded with a degenerate component; cf. Kiefer and Wolfowitz (1956)
2. Given interval data, the likelihood function is bounded, but the convergence problem remains; cf. Biernacki (2007)

Bayesian analysis seems safer.

Prior

Weakly informative priors (i.e., proper but vague) for the distribution parameters

$$\begin{aligned}
\boldsymbol{\pi} &\sim \mathrm{Dirichlet}(\boldsymbol{\imath}_K) \\
\mu_k &\sim \mathrm{N}(2.5, 100), \quad k = 1, \ldots, K \\
\sigma_k^2 &\sim \mathrm{Inv\text{-}Gam}(2, \beta_0), \quad k = 1, \ldots, K
\end{aligned}$$

A hierarchical prior for β_0 (Richardson and Green (1997))

$$\beta_0 \sim \mathrm{Gam}(.2, .1)$$

Flat priors for the indifference limen

$$\begin{aligned}
\gamma_l &\sim \mathrm{U}(-\infty, 0) \\
\gamma_u &\sim \mathrm{U}(0, 1)
\end{aligned}$$

Posterior Simulation

1) Problem with MCMC for cutoff points with large samples

- **Gibbs** Extremely slow
- **MH** Faster (with collapsing and reparametrization), but low acceptance rate (20–50%)
- **NUTS** Much faster with high acceptance rate (80–99%), easy to implement using Stan

2) Label switching problem

- Component labels may switch during MCMC $\Longrightarrow$ Mixture parameters may not converge
- Focus on permutation invariant parameters (moments and quantiles); cf. Geweke (2007)

Notes on Computation

- Apply NUTS using Stan 2.14.1 on R 3.3.2
- Generate four Markov chains in parallel
- For each chain, discard the initial 1,000 draws as warm-up, and use the next 1,000 draws for posterior inference (4,000 draws in total)
- Convergence diagnostics
  - potential scale reduction factor $\hat{R}$
  - effective sample size (ESS)
- Repeat MCMC for 60 quarters $\times$ 2 $=$ 120 times

Plan

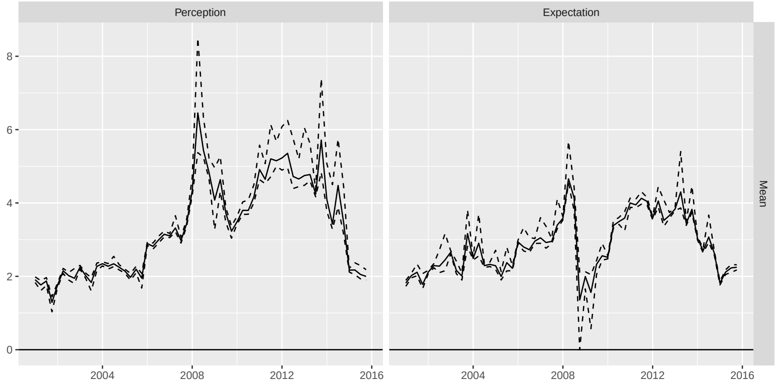
Results Using 6 or 8 Intervals

- Compare estimation using 6 intervals (no indifference limen) and 8 intervals (with an indifference limen)
- Estimation period: 2001Q1–2015Q4
- $K = 2$
- Focus on up to the 5th (central, standardized) moments
  1. mean
  2. s.d.
  3. skewness
  4. excess kurtosis
  5. asymmetry in the tails

Findings

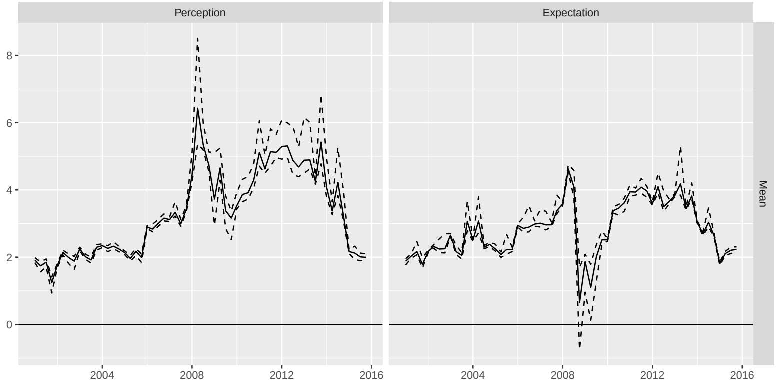
1. Estimates of the means are reasonably precise with narrow error bands.
2. Estimates of higher-order moments are imprecise.
3. Use of the prior information about the indifference limen ($\gamma_l < 0 < \gamma_u$) changes the results for higher-order moments.
4. Estimates of γ_l sometimes falls far below -1%.

Mean (6 Intervals)



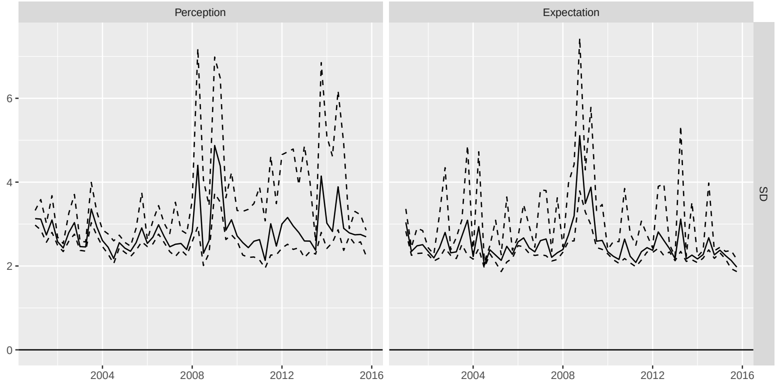
Posterior median and 68% error band

Mean (8 Intervals)



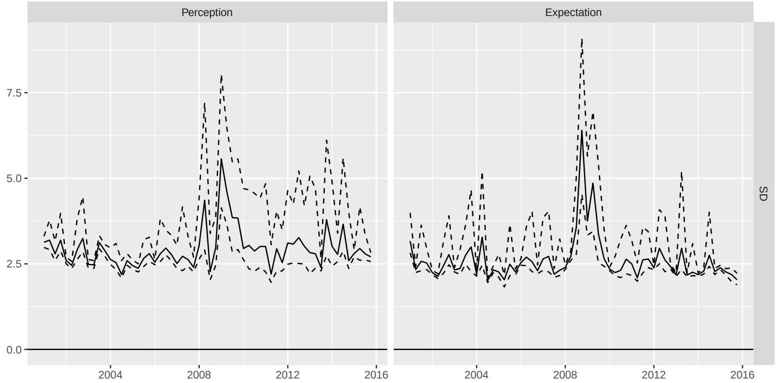
Posterior median and 68% error band

S.D. (6 Intervals)



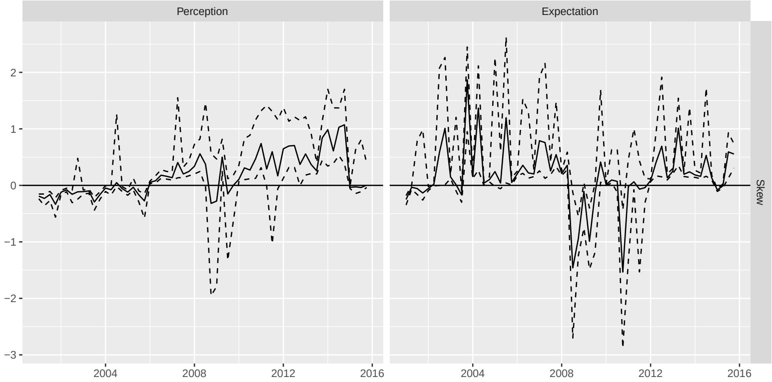
Posterior median and 68% error band

S.D. (8 Intervals)



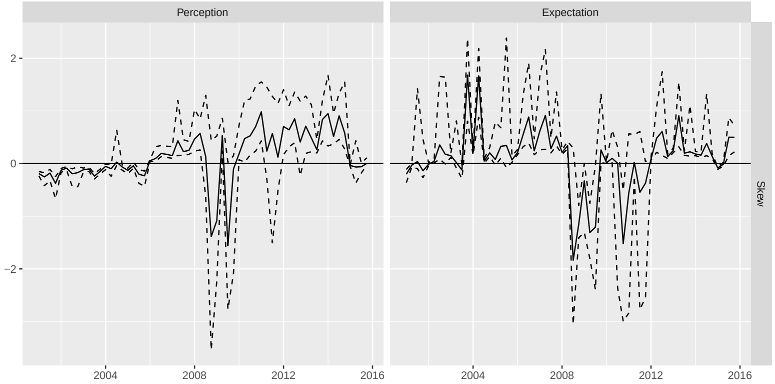
Posterior median and 68% error band

Skewness (6 Intervals)



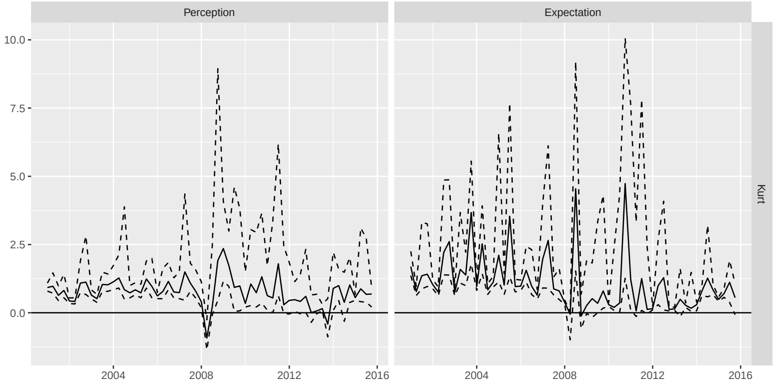
Posterior median and 68% error band

Skewness (8 Intervals)



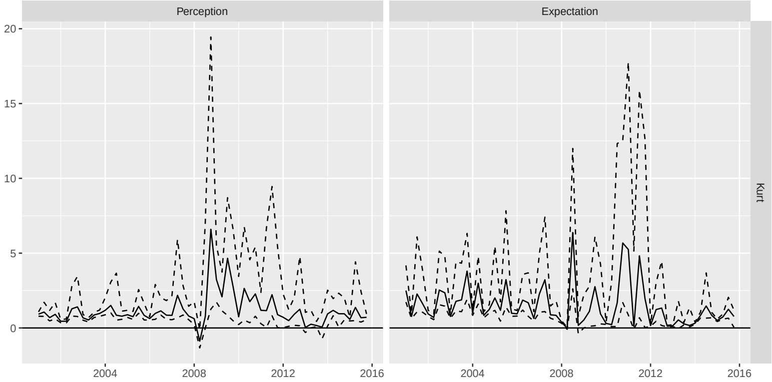
Posterior median and 68% error band

Excess Kurtosis (6 Intervals)



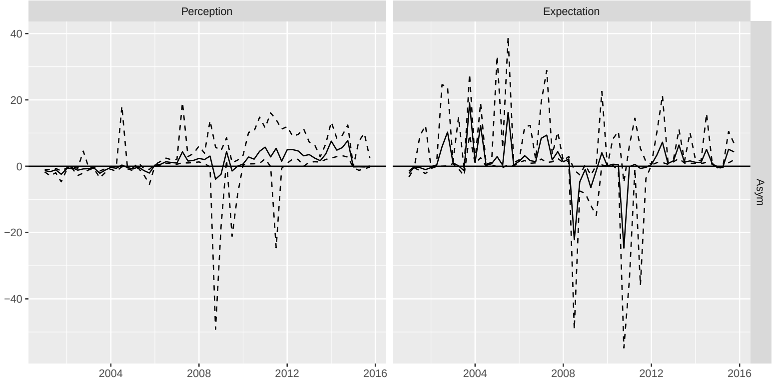
Posterior median and 68% error band

Excess Kurtosis (8 Intervals)



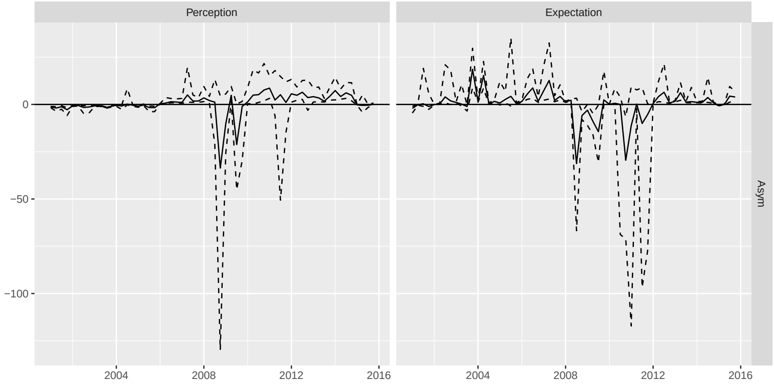
Posterior median and 68% error band

Asymmetry in the Tails (6 Intervals)



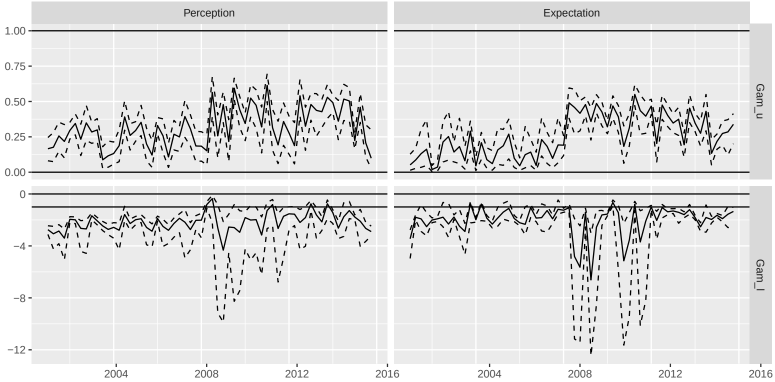
Posterior median and 68% error band

Asymmetry in the Tails (8 Intervals)



Posterior median and 68% error band

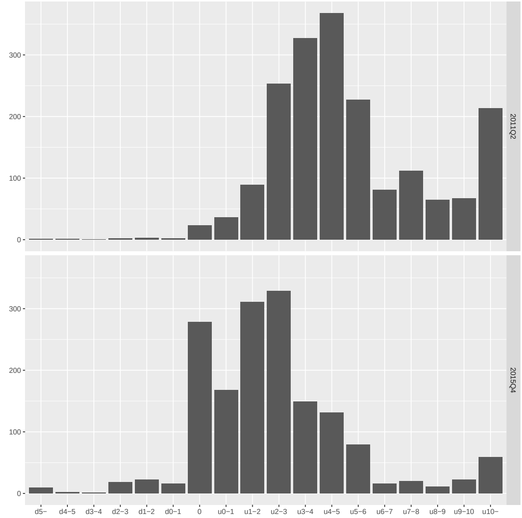
Indifference Limen



Posterior median and 68% error band

Plan

Inflation Perceptions (2011Q2 and 2015Q4)



Results Using 18 Intervals

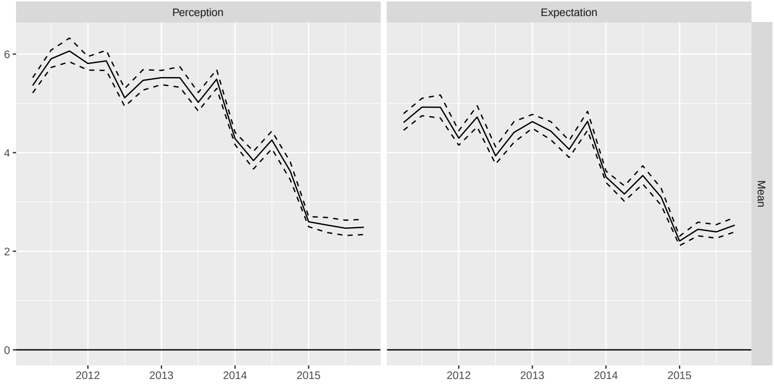
- Estimation period: 2011Q2–2015Q4
- $K = 2$
- New prior: $\gamma_I \sim \mathrm{U}(-1, 0)$

Findings

1. Posterior distributions are much narrower.
2. Estimates of the indifference limens are stable and reasonable.

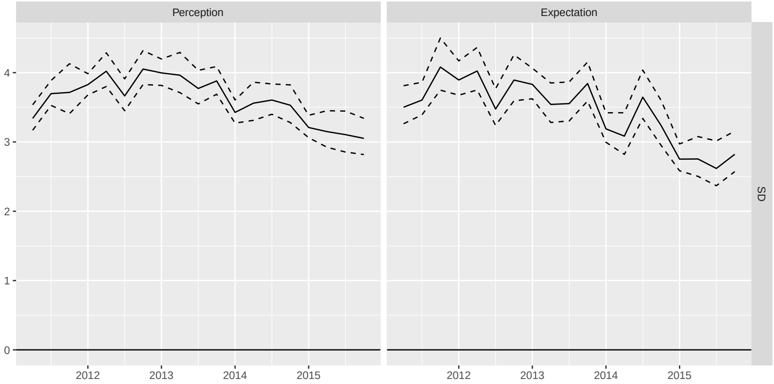
Lesson: Design of intervals is crucial, especially when collecting interval data repeatedly.

Mean



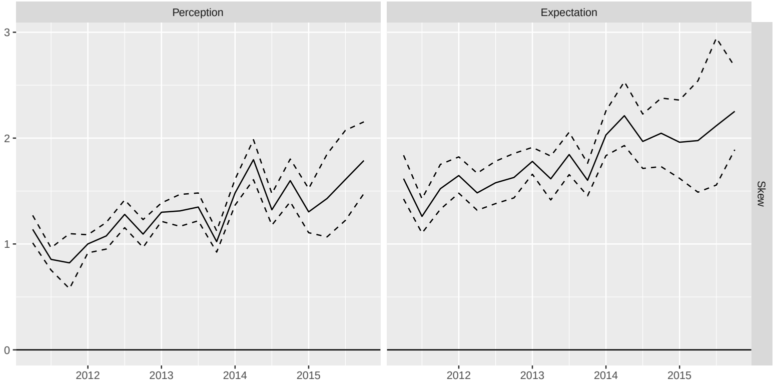
Posterior median and 95% error band

S.D.



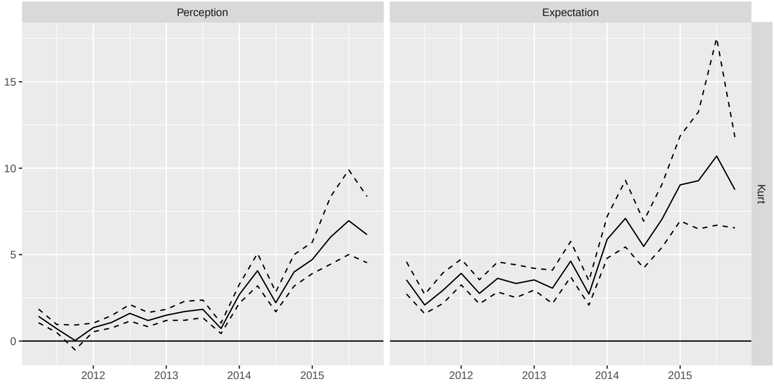
Posterior median and 95% error band

Skewness



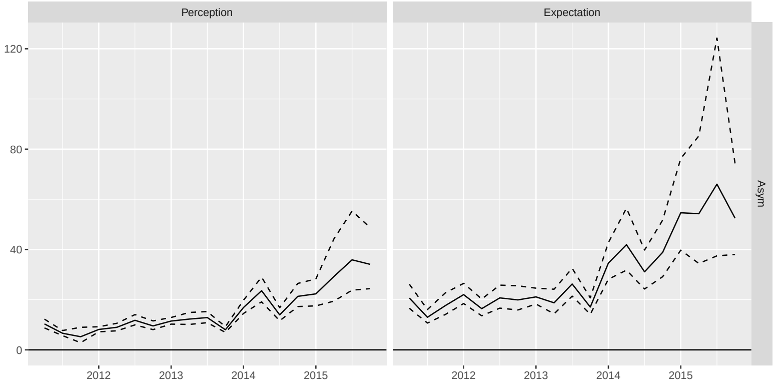
Posterior median and 95% error band

Excess Kurtosis



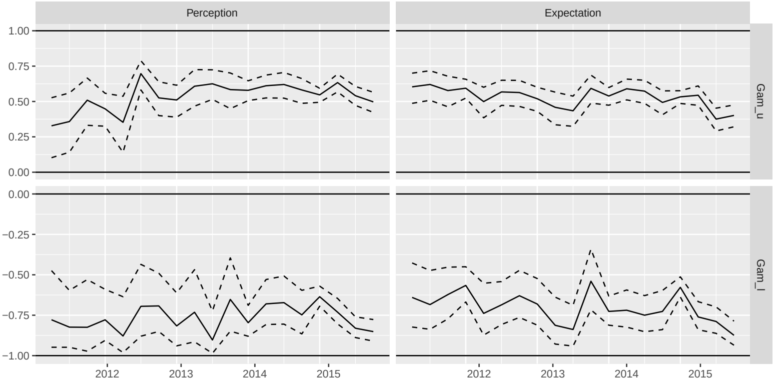
Posterior median and 95% error band

Asymmetry in the Tails



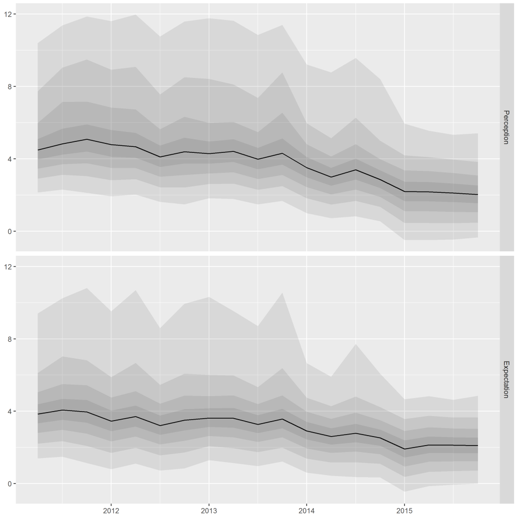
Posterior median and 95% error band

Indifference Limen



Posterior median and 95% error band

Deciles



Plan

Mean Inflation Expectations ($K = 1, 2$)

Use of the Estimated Means

Can use the mean inflation expectations for further analyses

- testing for rationality of expectations
- measuring information rigidity; cf. Coibion and Gorodnichenko (2015, AER)

The results depend on how we specify the distribution of inflation expectations $\Longrightarrow$ Use a flexible distribution
(Please see the paper for more details)

Plan

Summary

1. Bayesian analysis of a normal mixture model for interval data with an indifference limen

   Prior settings: hierarchical prior

   Posterior simulation: NUTS

2. Estimate the distributions of public inflation perceptions and expectations in the UK during 2001Q1–2015Q4

   8 intervals Estimated means are precise

   18 intervals Estimated distributions are precise

3. Illustrate a possible use of the estimated distributions by measuring information rigidity in the UK

Future Plans

Possible extensions

1. Heterogeneous indifference limens among individuals
2. Estimation of K (number of mixture components)
3. Time series analysis of repeated cross sections using a state space model

Use of individual data (now available!)

1. Determinants of inflation perceptions and expectations; cf. Blanchflower and MacCoille (2009)
2. Joint distribution of inflation perceptions and expectations

Biernacki, C. (2007). Degeneracy in the maximum likelihood estimation of univariate Gaussian mixtures for grouped data and behavior of the EM algorithm. *Scandinavian Journal of Statistics*, *34*, 569–586.

Blanchflower, D. G., & MacCoille, C. (2009). *The formation of inflation expectations: An empirical analysis for the UK* (Working Paper No. 15388). National Bureau of Economic Research.

Carlson, J. A., & Parkin, M. (1975). Inflation expectations. *Economica*, *42*, 123–138.

Coibion, O., & Gorodnichenko, Y. (2015). Information rigidity and the expectations formation process: A simple framework and new facts. *American Economic Review*, *105*, 2644–2678.

Geweke, J. (2007). Interpretation and inference in mixture models: Simple MCMC works. *Computational Statistics & Data Analysis*, *51*, 3529–3550.

Kiefer, J., & Wolfowitz, J. (1956). Consistency of the maximum likelihood estimator in the presence of infinitely many incidental parameters. *Annals of Mathematical Statistics*, *27*, 887–906.

Mankiw, N. G., Reis, R., & Wolfers, J. (2004). Disagreement about inflation expectations. In M. Gertler & K. Rogoff (Eds.), *NBER macroeconomics annual 2003* (Vol. 18, pp. 209–248). MIT Press.

Murasawa, Y. (2013). Measuring inflation expectations using interval-coded data. *Oxford Bulletin of Economics and Statistics*, *75*, 602–623.

Richardson, S., & Green, P. J. (1997). On Bayesian analysis of mixtures with an unknown number of components (with discussion). *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, *59*, 731–792.