KEEI and KU Green School Seminar

Testing for Continuous Structural Breaks in Cointegrated Error-Correction Model

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Motivation

This paper considers statistical inference of continuous structural breaks in the long-run equilibrium relationship of the vector error correction model.

The tests for continuous structural breaks are proposed using the continuous transition event window with unknown end points. We show the asymptotic distribution. The Monte Carlo simulation evidences improvement in finite sample performance of the proposed tests compared to those of one-time structural change tests with unknown break point.

An economic application to the environmental Kuznets curve is provided.

Continuous Structural Breaks

One-time immediate structural change

$y_t = \mu_1 + u_t$ for $t \le t^*$

$y_t = \mu_2 + u_t$ for $t > t^*$

Continuous structural breaks

$y_t = \mu_1 + (\mu_2 - \mu_1)F_t + u_t$

where $F_t : I \to [0, 1]$.

One-time immediate structural change is a special case of continuous structural breaks.

Continuous Breaks in Economic Relationship

The continuous transition seems reasonable considering the stylized facts of economy: the costs of adjustment, habit formation, and heterogeneous agents.

Continuous Breaks in Economic Relationship

$y_t = \beta x_t + w_t$ before the start of transition

$y_t = (\beta + \delta) x_t + w_t$ after the end of transition

$y_t = \beta x_t + \delta F_t x_t + w_t$ during transition

Alternatives: Nonlinear Relationship

Nonlinear relationship can be applied to explain economic relationship such as EKC.

$y_t = \beta x_t + w_t$ under the null of linearity

$y_t = \beta x_t + \delta' F(x_t) + w_t$ under the alternative of nonlinearity

Examples: EKC, Bubbles, Liquidity Trap, Target Zone

EKC: $y_t = \beta x_t + \delta x_t^2 + w_t$

Bubbles: $y_t = \beta x_t + \delta F(x_t) + w_t$

Nonlinear relationship is based on the function of variables while the continuous breaks assume the function of the deterministic trend.

Seo (2011) Nonparametric Testing for Linearity in Cointegrated Error-Correction Models, Studies in Nonlinear Dynamics and Econometrics.

Contents

Introduction

The economic relationships may involve structural breaks due to technological progress, supply and preference shocks, and policy and institutional changes.

The evidence of structural instability in economic relations has been well-documented in the literature, including Stock and Watson (1996), Ben-David and Papell (1998), and McConnell and Perez-Quiros (2000).

As structural instability affects the validity of estimation, inference, and forecasting seriously, statistical assessment of instability is crucial to avoid potential hazards in empirical research.

The continuous transition seems reasonable considering the stylized facts of economy: the costs of adjustment, habit formation, and heterogeneous agents.

Mortensen (1973 Ecta): Costs of Adjustment and Dynamic Factor Demand Theory

Sheshinski and Weiss (1977 RES): Inflation and Costs of Price Adjustment

Abel (1990 AER): Habit Formation and Equity Premium Puzzle

Krussel and Anthony (2006): Macro Models with Heterogeneous Agents

Previous studies have been relatively tightly focused on one-time immediate structural change, and the models with stationary variables have been assumed.

"While it may seem unlikely that a structural break could be immediate and might seem more reasonable to allow a structural change to take a period of time to take effect, we most often focus on the simple case of an immediate structural break for simplicity and parsimony." (Hansen, 2001)

The tests for structural change in the cointegrating relationship, which is composed of nonstationary variables, have been developed by Hansen (1992), Quintos (1993), and Seo (1998). These studies assumed immediate one-time shift in the relationship with unknown break date.

However, a structural change may take a period of time to take effect, and thus the transition can be gradual, rather than immediate, and takes time to complete.

In this paper, we explore testing for continuous structural breaks in the long-run relationship of the vector error correction model. It is natural and necessary to develop the tests for continuous structural breaks as the gradual transition seems relevant and it generalizes one-time immediate transition.

Related Issues

1. Nuisance Parameters

Unknown Break Point

- Quandt (1960)

- Davies (1977, 1987), Andrews and Ploberger (1994)

2. Nonstationary Variables

- Hansen (1992), Quintos (1997), Seo (1998)

3. Estimation

Break Dates

- Bai (1997), Bai, Lumsdaine, and Stock (1998)

4. Inference

Unit Root, Cointegration

- Perron (1989), Zivot and Andrews (1992), Saikkonen and Lutkepohl (2001)

The Model

Consider a p-dimensional nonstationary time series $x_t = (x_{1t}^{'}, x_{2t}^{'})^{'}$ generated by a VECM with continuous structural breaks in the cointegrating relationship as follows:

$$\Delta x_t = \alpha(x_{1t-1} + \beta^{'} x_{2t-1} + \delta^{'} x_{2t-1} F_{t-1}) + \sum_{i=1}^{k} \Gamma_i \Delta x_{t-i} + u_t. \quad (1)$$

The cointegration space is assumed known and r-dimensional. The cointegrating vector is normalized with respect to x_{1t}, which is r-dimensional. The cointegrating vector β and the shift parameter δ are $(p-r) \times r$ matrix, and the adjustment vector α is $p \times r$ full-column rank matrix. The error u_t is vector-valued sequence with $\Sigma = E(u_t u_t^{'}) < \infty$

Transition Function F_t

The transition function F_t determines structural change in the long-run relationship. The functional form can be specified in several ways, depending on the characteristics of transition dynamics.

A linear transition function, $F_t = t$, is employed in Farley and Hinich (1970) and Farley *et al.* (1975). A polynomial function is assumed in Ohtani *et al.* (1990) and the logistic smooth transition function is used in Luukkonen *et al.* (1988) and Lin and Terasvirta (1994).

1. $F_t = t$: Farley and Hinich (1970)

2. $F_t = \gamma_0 + \gamma_1 t + \cdots + \gamma_m t^m$ for $t \in [t_1, t_2]$: Ohtani, Kakimoto, and Abe (1990)

3. $F_t = [1 + \exp(-\lambda(t^m + \gamma_{m-1} t^{m-1} + \cdots + \gamma_0))]^{-1}$: Lin and Terasvirta (1994)

4. $F_t = [1 + \exp(-\lambda(t - \gamma))]^{-1}$: Luukkonen, Saikkonen and Terasvirta (1988)

Our tests consider a transition function with unknown dates of event window.

$$F_t = \begin{array}{ccc} 0 & \text{for} & t \in [1, t_1 - 1] \\ f(t) & \text{for} & t \in [t_1, t_2] \\ 1 & \text{for} & t \in [t_2 + 1, n] \end{array}$$

The transition function allows for a linear transition of Farley and Hinich (1970) and polynomial function of Ohtani, Kakimoto, and Abe (1990).

However, the period of structural change is specified as event window $[t_1, t_2]$, where the end date points t_1 and t_2 are unknown.

Event Window

The linear transition in our study is specified as $f(t) = k(t)$ for $0 \leq k(t) \leq 1$, where $k(t) = \frac{t-t_1}{t_2-t_1}$.

The polynomial transition adopts the Parzen window, $f(t) = 2k^3(t)$ for $0 < k(t) \leq 0.5$ and $f(t) = 1 - 6k(t) + 12k^2(t) - 6k^3(t)$ for $0.5 < k(t) \leq 1$.

Also, we consider the transition function based on the Blackman-Tukey window, $f(t) = 1 - 2a + 2a\cos((k(t) - 1)\pi)$ for $0 \leq k(t) \leq 1$ and $a = 0.25$.

As our specification involves an event window, a structural change may take a period of time to take effect.

We define the cointegrating relationship (or the long-run relationship) as

$$w_t = x_{1t} + (\beta + \delta F_t(\tau))' x_{2t}, \tag{2}$$

which is stationary (or $I(0)$).

The event window partitions the sample period into three sub-samples $t \in [1, t_1 - 1]$, $t \in [t_1, t_2]$, and $t \in [t_2 + 1, n]$.

Hence, the corresponding cointegrating vectors are β for the fist period, $\beta + \delta f(t)$ for the second period, and $\beta + \delta$ for the third period, respectively.

We treat t_1 and t_2 as fixed until we define optimal tests for unknown end dates of event window.

Denote $\tau = (\tau_1, \tau_2)$, $t_1 = [n\tau_1]$, and $t_2 = [n\tau_2]$.

Likelihood Function

The log-likelihood function, with the auxiliary condition that u_t is normally distributed, is given by

$$\mathcal{L}_n(\theta, \tau) = \sum_{t=1}^{n} l_t(\delta, \tau, \beta, \alpha, \Gamma, \Sigma), \tag{3}$$

where

$$l_t(\theta, \tau) = -\frac{1}{2} \log |\Sigma| - \frac{1}{2}\ tr\ u_t(\theta, \tau) u_t^{'}(\theta, \tau)\ \Sigma^{-1},$$

and $u_t = u_t(\theta, \tau)$ in equation (1).

Hypotheses

The null and alternative hypotheses for the stability of the cointegrating vector β are

$$\mathcal{H}_0^\beta \; : \; \delta = 0 \quad \text{and} \quad \mathcal{H}_1^\beta \; : \; \delta \neq 0.$$

Denote $\widehat{\theta}(= \widehat{\theta}_n(\tau))$ as the unrestricted MLE of θ for known $\tau = (\tau_1, \tau_2)$. That is,

$$\widehat{\theta}(\tau) = argmax \ _{\theta \in \Theta} \ \mathcal{L}_n(\theta, \tau). \tag{4}$$

Null Model

Under the null hypothesis, the model (1) reduces to the standard error correction model:

$$\Delta x_t = \alpha(x_{1t-1} + \beta^{'} x_{2t-1}) + \sum_{i=1}^{k} \Gamma_i \Delta x_{t-i} + u_t. \tag{5}$$

If we denote $\tilde{\theta}$ as the restricted MLE of θ, then

$$\tilde{\theta} = argmax_{\ \theta\in\Theta,\ \delta=0}\ \mathcal{L}_n(\theta). \tag{6}$$

Our tests use the restricted MLE. Methods to compute the restricted MLE $\tilde{\theta}$ have been suggested by Ahn and Reinsel (1988) and Box and Tiao (1977).

First Order Condition

The first order conditions of the unrestricted MLE are given by

$$\frac{\partial \mathcal{L}_n(\widehat{\theta},\tau)}{\partial \delta} = \sum_{t=1}^{n} F_{t-1}(\tau) x_{2t-1} \widehat{u}_t^{'} \widehat{\Sigma}^{-1} \widehat{\alpha} = 0, \tag{7}$$

$$\frac{\partial \mathcal{L}_n(\widehat{\theta},\tau)}{\partial \beta} = \sum_{t=1}^{n} x_{2t-1} \widehat{u}_t^{'} \widehat{\Sigma}^{-1} \widehat{\alpha} = 0, \tag{8}$$

$$\frac{\partial \mathcal{L}_n(\widehat{\theta},\tau)}{\partial \alpha^{'}} = \sum_{t=1}^{n} \widehat{w}_{t-1} \widehat{u}_t^{'} \widehat{\Sigma}^{-1} = 0, \text{ and} \tag{9}$$

$$\frac{\partial \mathcal{L}_n(\widehat{\theta},\tau)}{\partial \Gamma_i^{'}} = \sum_{t=1}^{n} \Delta x_{t-i} \widehat{u}_t^{'} \widehat{\Sigma}^{-1} = 0, \text{ for } i = 1, 2, \cdots, k, \tag{10}$$

where $\widehat{u}_t = u_t(\widehat{\theta})$ in equation (1), and $\widehat{w}_t = x_{1t} + (\widehat{\beta} + \widehat{\delta} F_t(\tau))^{'} x_{2t}$. Denote $\tilde{u}_t = u_t(\tilde{\theta})$ in equation (1), and $z_t = (\Delta x_t^{'}, \cdots, \Delta x_{t-k+1}^{'})^{'}$.

LM Statistic

Let

$$\lambda_n^\beta(\tau) = (\tilde{\alpha}'\tilde{\Sigma}^{-1} \otimes n^{-1} \sum_{t=1}^{n} F_{t-1}(\tau) x_{2t-1} \tilde{u}_t') \ vec(I).$$

We call $\lambda_n^\beta(\tau)$ the Lagrange multiplier (or the score). The score function is based on equation (7) evaluated at the null model estimator $\tilde{\theta}$.

We define the LM statistic for the null hypothesis $\mathcal{H}_0^\beta$ as follows:

$$LM_n^\beta(\tau) = \lambda_n^{\beta'}(\tau) \ [\ Est.\ Var(\lambda_n^\beta(\tau)) \ ]^{-1} \ \lambda_n^\beta(\tau).$$

The LM statistic is a simple function of the data and the restricted MLE $\tilde{\theta}$. Since we can use existing estimation methods, it is computationally easy and fast.

We have defined the LM statistics for fixed τ. This is appropriate when τ is known to the econometrician. More typically in applied work, it is natural that the break point τ is thought to be unknown. In this case, the testing procedure is nonstandard since a nuisance parameter τ appears only under the alternative hypothesis.

Optimal Tests

Tests with specific optimality properties have been proposed by Davies (1977, 1987), King and Shively (1993), Andrews (1993), and Andrews and Ploberger (1994). We follow Andrews (1993) and Andrews and Ploberger (1994) whose method is based on the weighted power criterion function with respect to the randomized nuisance parameter.

If we assume that $\tau = (\tau_1, \tau_2)$ lies in $\tau^* = [\underline{\tau}, \overline{\tau}]^2$, then the optimal tests are defined as follows:

$$\mathrm{Sup} - \mathrm{LM}_n^i = \mathrm{Max}\ _{(t_1,t_2)\in[\underline{t},\bar{t}]^2}\ LM_n([t/n]),$$

where $\underline{t} = [n\underline{\tau}]$, $\bar{t} = [n\overline{\tau}]$, and $i = L, P, B$ for the linear, Parzen, and Blackman-Tukey event window, respectively.

Distribution Theory

We define the following standard Brownian motions:

$$\begin{pmatrix} B_1(s) \\ B_2(s) \end{pmatrix} = \begin{pmatrix} (\alpha'\Sigma^{-1}\alpha)^{-1/2'}\alpha'\Sigma^{-1}W(s) \\ (C_2'(1)\Sigma C_2(1))^{-1/2'}C_2(1)W(s) \end{pmatrix}$$
$$= BM\begin{pmatrix} I_r & 0 \\ 0 & I_{p-r} \end{pmatrix}.$$

Theorem 1 *Under* $\mathcal{H}_0^\beta$,

$$LM_n^\beta(\tau) \Rightarrow tr\ \ G(\tau)^{b'}\ [V(\tau,\tau) - V(\tau)V^{-1}V(\tau)]^{-1}\ G(\tau)^b \ \equiv\ LM_1^\beta(\tau), \qquad (11)$$

where

$$G(\tau)^b = G(\tau) - V(\tau)V^{-1}G,$$
$$G(\tau) = \int_0^1 F(\tau)B_2(s)dB_1'(s),$$
$$V(\tau) = \int_0^1 F(\tau)B_2(s)B_2'(s)ds,\ \textit{and}\ V(\tau,\tau) = \int_0^1 F^2(\tau)B_2(s)B_2'(s)ds.$$

Hence,

$$\mathrm{Sup} - \mathrm{LM}_n \ \Rightarrow\ \mathrm{Max}_{\tau\in\tau^*}\ LM_1^\beta(\tau).$$

1. Asymptotic distributions are different from those found by Andrews (1993) and Andrews and Ploberger (1994).

- The functional vec $G(\tau)$ is distributed as mixed normal with covariance matrix $I \otimes V(\tau, \tau)$. However, vec $G(\tau)$ is not a Brownian bridge, as defined in the stationary case.

2. The transition function entails the event window with unknown end points, and thus our asymptotic distributions extend those of one-time immediate structural change tests of Hansen (1992) and Seo (1998).

3. The distribution of LM_1^β is chi-squared for a known transition function and τ, our tests for continuous structural breaks in the cointegrating vector are standard only if we know the transition function.

4. Given the transition function, the distribution depends only on the number of parameters and the admissible range of the break points. The empirical distribution and the associated asymptotic critical values can be generated by simulation.

Simulation Evidence

Suppose we have the local alternative hypothesis as follows:

$$\mathcal{H}_n^\beta : \delta_n = \bar{d}/n.$$

As the asymptotic power function is driven by the Lagrange multiplier under the local alternative hypothesis, the power of the tests mainly hinges on the decision error $\bar{d}$. If we know the true transition function and the true end points of event window, it removes uncertainty. This case generates the power envelope.

We consider the linear, Parzen, and Blackman-Tukey event windows with unknown end points in the testing. Obviously, uncertainty arises from unknown end points and unknown transition function, which generates the lack in power compared to the power envelop.

Power Performance of the Tests

Table 2 evaluates the asymptotic local power of the tests using the following bivariate model:

$$\begin{pmatrix} \Delta x_{1t} \\ \Delta x_{2t} \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ \beta + dF_{t-1}(\tau) \end{pmatrix}^{'} \begin{pmatrix} x_{1t-1} \\ x_{2t-1} \end{pmatrix} + \Gamma \begin{pmatrix} \Delta x_{1t-1} \\ \Delta x_{2t-1} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix},$$

where $\{e_{1t},\ e_{2t}\}^{'}$ are *i.i.d.* and normal with covariance ρ.

Monte Carlo experiments are based on a sample size of 250 and 1,000 replications for each $d = 0, 0.025, 0.050, \cdots, 0.200$. We set $\beta = -1$, $\Gamma = 0$, and $\rho = 0.0$. Each end point of $\tau = (\tau_1, \tau_2)$ is assumed to be uniformly distributed over $[.15, .85]$.

The chi-square test, which is based on the power envelop, rejects 61.2% of the null hypothesis at the 5% size when the deviation parameter d is 0.05 for the linear transition.

The Sup-LM test rejects 52.1%, 51.8%, and 51.7% using the linear, Parzen, and Blackman-Tukey windows, respectively. The power of the tests does not seem affected by the choice of event window. Although the end points of event window are unknown, the loss in power is the minimal compared to the nonparametric tests and the one-time structural break tests.

The one-time structural change tests with unknown break point reject 35.7% of the null of no structural change. The misspecified event window and the unknown parameter produce the gap in power compared to the power envelop.

The nonparametric tests, based on the polynomial approximation of the transition function, reject 48.0% of the null hypothesis. The nonparametric tests avoid the unknown parameters of break dates, and the asymptotic distribution is standard.

Table 2. Power Evaluation

$F_t(\tau)$	Tests	d 0.025	0.050	0.075	0.100	0.125	0.150	0.175	0.200
Linear	$SupLM_n^L$	0.230	0.521	0.715	0.822	0.903	0.939	0.968	0.985
	$SupLM_n^P$	0.229	0.518	0.714	0.820	0.902	0.938	0.967	0.985
	$SupLM_n^B$	0.229	0.517	0.715	0.820	0.903	0.938	0.968	0.985
	$SupLM_n^J$	0.155	0.357	0.523	0.621	0.676	0.715	0.734	0.743
	LM_n^{NP}	0.231	0.480	0.678	0.792	0.884	0.934	0.955	0.972
	LM_n^{PE}	0.310	0.612	0.794	0.890	0.940	0.964	0.984	0.991
Parzen	$SupLM_n^L$	0.256	0.537	0.725	0.833	0.907	0.943	0.974	0.986
	$SupLM_n^P$	0.255	0.534	0.723	0.832	0.907	0.941	0.974	0.985
	$SupLM_n^B$	0.256	0.535	0.724	0.831	0.907	0.941	0.974	0.985
	$SupLM_n^J$	0.168	0.379	0.550	0.632	0.683	0.716	0.739	0.741
	LM_n^{NP}	0.241	0.497	0.688	0.806	0.877	0.932	0.959	0.972
	LM_n^{PE}	0.332	0.635	0.804	0.894	0.944	0.969	0.989	0.994
Blackman-Tukey	$SupLM_n^L$	0.245	0.527	0.723	0.823	0.908	0.941	0.969	0.982
	$SupLM_n^P$	0.245	0.526	0.722	0.821	0.908	0.941	0.968	0.982
	$SupLM_n^B$	0.245	0.527	0.723	0.823	0.908	0.941	0.968	0.982
	$SupLM_n^J$	0.158	0.365	0.532	0.627	0.678	0.721	0.734	0.745
	LM_n^{NP}	0.233	0.487	0.680	0.793	0.880	0.934	0.958	0.972
	LM_n^{PE}	0.317	0.630	0.797	0.897	0.943	0.964	0.985	0.991

Size Behavior of the Tests

Table 3 reveals the size property of the tests for continuous structural breaks based on the following bivariate model:

$$\begin{pmatrix} \Delta x_{1t} \\ \Delta x_{2t} \end{pmatrix} = \begin{pmatrix} -\alpha \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ \beta \end{pmatrix}' \begin{pmatrix} x_{1t-1} \\ x_{2t-1} \end{pmatrix} + \Gamma \begin{pmatrix} \Delta x_{1t-1} \\ \Delta x_{2t-1} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix},$$

where $\{e_{1t}, e_{2t}\}'$ are *i.i.d.* and standard normal with covariance ρ.

Monte Carlo experiments are based on a sample size of 250 and 1,000 replications. We assess the sensitivity of the tests by varying parameters α, β, Γ, and ρ. As Table 3 shows, our proposed tests reveal the size behavior close to the nominal size across the various parameterizations. The choice of event window does not generate distinct size properties.

Table 3. Probability of Type I Errors

	α_2 / Γ	0 / Γ_0	-0.5 / Γ_0	0.5 / Γ_0	0 / Γ_1	0 / Γ_2
n	k	Size = 5%				
250	1	0.0549	0.0569	0.0512	0.0541	0.0523
250	2	0.0541	0.0568	0.0513	0.0526	0.0528
250	3	0.0545	0.0558	0.0475	0.0543	0.0548
250	4	0.0551	0.0566	0.0482	0.0552	0.0549
250	5	0.0553	0.0639	0.0478	0.0576	0.0571
2000	1	0.0517	0.0483	0.0491	0.0511	0.0518
2000	2	0.0477	0.0497	0.0501	0.0491	0.0475
2000	3	0.0497	0.0499	0.0519	0.0502	0.0504
2000	4	0.0481	0.0502	0.0495	0.0498	0.0490
2000	5	0.0512	0.0513	0.0523	0.0533	0.0532
n	k	Size = 10%				
250	1	0.1062	0.1106	0.1003	0.1065	0.1047
250	2	0.1067	0.1067	0.1013	0.1078	0.1068
250	3	0.1066	0.1128	0.0994	0.1063	0.1056
250	4	0.1073	0.1124	0.0990	0.1093	0.1079
250	5	0.1110	0.1177	0.0973	0.1135	0.1113
2000	1	0.1023	0.0977	0.1002	0.1034	0.1023
2000	2	0.1001	0.0985	0.1012	0.0990	0.0996
2000	3	0.0995	0.0998	0.1000	0.0992	0.1001
2000	4	0.0982	0.1060	0.0952	0.0985	0.0987
2000	5	0.1005	0.1035	0.1005	0.1031	0.1011

Economic Application

We apply the tests for continuous structural breaks to the environmental Kuznets curve. This experiment is motivated from the relationship between the per capita income and the CO2 emission explored by Grossman and Krueger (1992, 1995).

The quality of environment can be affected by technological progress and the change in industrial composition along with regulations, and thus the relationship between the per capita income and the environmental quality may reveal structural change.

Economic growth brings about structural change that shifts the center of gravity of the economy from low-polluting agriculture to high-polluting industry and eventually back to low polluting services. As the transition is gradual, rather than immediate, and takes time to complete, we set up the null hypothesis of no structural change against the alternative of continuous structural breaks.

The dataset used in the application is composed of per capita GDP and per capita CO2 emission series for the sample period 1950-2007. The dataset of per capital GDP is obtained from the Penn World Table, and per capita CO2 emission series is from CDIAC. We use the annual data: x_{1t} as per capital CO2 emission and x_{2t} as per capita GDP. All variables are in logarithms.

Table 4. Environmental Kuznets Curve

	$\text{Sup}-\text{LM}_n^L$	$\text{Sup}-\text{LM}_n^P$	$\text{Sup}-\text{LM}_n^B$	$\text{Sup}-\text{LM}_n^J$	LM_n^{NP}
U.S.	19.0460	19.0822	19.3476	14.2597	22.2329
U.K.	8.7300	8.7300	8.7300	5.4163	8.1081
Korea	4.9616	4.9616	4.9616	3.5520	16.6045
Japan	10.7927	10.7624	10.7624	6.8358	15.4972
Germany	17.5430	17.7698	17.7788	4.3720	18.4895
France	8.2795	7.7277	7.8223	6.6338	21.4323
China	5.8155	5.7745	5.7745	3.7891	6.7235
Canada	19.0902	18.7400	18.9532	13.0598	15.9568

Note: 5% critical values are 9.22 for $\text{Sup}-\text{LM}_n^L$; 9.09 for $\text{Sup}-\text{LM}_n^J$; 7.81 for LM_n^{NP}.

Table 4 indicates evidence of continuous structural breaks in the EKC relationship of the U.S. Japan, Germany, and Canada. The null hypothesis cannot be rejected for U.K. Korea, France, and China. The test statistics do not vary across event windows considered. One-time structural change tests evidence structural change in U.S. and Canada while nonparametric tests signify the evidence for all countries considered excepting China.

Concluding Remarks

This paper explores statistical inference of continuous structural beaks in the long-run relationship of the vector error-correction model. We have developed the test statistics based on the continuous transition event window with unknown end points. The asymptotic distribution of the test statistics are shown to follow nonstandard distribution, and simulation results evidence improvement in finite-sample performance.

Many previous studies have been relatively tightly focused on one-time immediate structural change, and the models with stationary variables have been assumed. Economic analysis frequently implies a continuous structural beaks in the long-run equilibrium relationship composed of integrated variables. As it proposes the tests for continuous structural breaks in the long-run relationship and the distribution theory, this study is necessary and fills the gap in the literature.