Optimal Monetary Policy in A New Keynesian Model with Bond and Credit Market Frictions*

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Abstract

We have built a medium-scale DSGE model that incorporates both bond and credit market frictions. This subsequently allows central bank asset purchases of long-term government bonds and private securities to have a real economic impact. We design joint optimal policy that combines both the short-term interest rate and the two types of asset purchases during both normal and crisis times. There are three key findings. First, while implementing both asset purchases can reduce welfare losses, it is the private asset purchases that more effectively offset the shocks hitting the economy than bond purchases. Second, when economy is hit by more severe shocks and the ZLB is met, the welfare gains from using asset purchases increases. Private purchase continues to outperform bond purchase in terms of welfare, but it increases frequencies of hitting ZLB than the latter. Third, we quantify the impact of the recent financial crisis and the optimality of policy responses using the realised shocks backed out from the data during 1987Q3- 2014Q2. We found significant welfare gains and less ZLB binding periods if joint optimal policy would have been in place prior to the crisis. When a Ramsey planner is allowed to use asset purchases more freely, the economy is able to escape from the liquidity trap.

Key Words: Bond Friction, Credit Friction, Ramsay Monetary Policy, Asset Purchase, Zero Lower Bound

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*All errors are ours.

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1 Introduction

The recent financial crisis has changed, in profound ways, the interpretation of macroeconomic fluctuations, and the operation of monetary policy. Central banks around the world have implemented multiple large-scale asset purchase (LSAP) programmes to stabilize national economies when short-term nominal interest rates were approaching the zero lower bound (ZLB). Accordingly, a growing body of literature has analyzed the macroeconomic effects of LSAP. To perform such analysis, various financial frictions are incorporated into standard DSGE models to break the result of Wallace's irrelevance theorem (Wallace, 1981). This allows LSAP programmes to have a real effect on economic outcomes.

Whilst studies of unconventional monetary policy have been fruitful, most have focused on frictions that only become significant during crisis time. Typical examples include those employing binding constraints and bank runs. However, financial markets do not work seamlessly in normal time. For example, Chen *et al.* (2012) identify a moderate degree of bond market friction, characterized as bond market segmentation, during 1987Q3-2009Q3. Moreover, for a similar sample period, Christiano *et al.* (2014) show that the severity of financial frictions, modelled as in Bernanke *et al.* (1999), accounts for substantial fluctuations in the US business cycle. The empirical relevance of these financial frictions suggests that conventional short-term interest rate policy, on its own, is unable to achieve full stabilization in normal times.[1] This raises a number of questions, such as whether asset purchases should be kept as monetary policy instruments even during normal times? Would short-term interest rates reach the ZLB when a financial crisis occurs if we had these additional policy tools in place beforehand? Finally, can these policies reduce welfare losses when crisis hits? This paper attempts to answer these questions by evaluating the efficacy of the Ramsey monetary policy, combining both conventional short-term nominal interest rates and asset purchases of long-term government and private securities.

To do so, we extend the Smets and Wouters (SW, 2007) workhorse model with frictions in bond and credit markets. Besides their empirical relevance in both normal and crisis times, the two types of frictions also correspond with some main features of the Fed's asset purchases in practice, e.g., LSAPs of long-term government bonds and private assets such as agency debt and mortgage backed securities (MBS). Specifically, the bond market friction is modelled by introducing market segmentation in the government bond market as in Andrés *et al.* (2004)

[1] several papers (See, e.g., Reavenna and Walsh 2006, Carlstrom et al 2010) have concluded a general result that, in the presence of financial frictions, policymakers are confronted with a trade-off between stabilizing output and stabilizing inflation even when the mark-up shock is absent.

and Chen *et al.* (2012). This segmentation takes the form of heterogeneous households who have distinct preferences, asymmetric adjustment costs and portfolio structure. As discussed in Andrés *et al.* (2004), this segmentation can be viewed as a stand-in for participants in financial markets, such as commercial banks, who have higher liquidity costs than others, for example pension funds and insurance companies. As a result, a term premium, defined as the spread between long- and short-term interest rates, emerges endogenously as a function of the relative supply of bonds of different maturities. This provides a role for central bank's asset purchase policy, as it effectively influences the relative supply of long-term to short-term bonds available to the public.

Our model is further augmented with the financial accelerator mechanism of Bernanke *et al.* (BGG, 1999), with a modification along the line of Del Negro *et al.* (2013) and Christiano *et al.* (2014). As discussed above, this friction fits well with the post-war US business cycle fluctuations and predicts the sharp contraction at the beginning of the financial crisis. At the centre of the BGG friction is the introduction of entrepreneurs and the assumption that they must borrow funds from external sources such as banks, to buy capital. This creates an agency problem between banks and entrepreneurs which induces an inefficiency in the capital market and gives rise to an external finance premium. If the central bank engages in asset purchases of private securities, defined as entrepreneurs' capital, it will increase capital demand, depress capital return and, in turn, lower external finance premium.

Importantly, the two frictions are interlinked. This allows us to understand the interplays of the two categories of LSAPs. For example, changes in relative holdings of government bonds can affect the long-term interest rate, which then affects how households value investment opportunities, by altering their stochastic discount factor. This implies that central bank purchases of long-term government bonds may also improve conditions in capital market. Conversely, central bank purchases of entrepreneurial capital boost capital demand, which again affects the asset pricing of households. This implies that capital returns and long-term bond returns may be correlated. They can be eased or worsen simultaneously by asset purchase policy. This then enables us to compare the efficacy of two asset purchases as two distinctive additional instruments of the central bank.

The degree of financial frictions that prevail in normal times ultimately determines the effectiveness of asset purchase policy. Therefore, we use Bayesian methods to estimate the importance of financial frictions from 1987Q1 till the recent financial crisis. We find a significant degree of credit friction in line with Del Negro et al. (2013) and Christiano et al. (2014), while our estimated bond friction larger than that of Chen et al. (2012) partly due to how the portfolio adjustment cost is modelled. This subsequently implies that bond purchase policy in our model can have bigger impact on the real economy compared to Chen et al's (2012) estimate. Based on our estimates, we derive the joint Ramsey policy of short-term interest rates and asset purchases. The latter are modelled as changing proportions of asset holdings by the central bank. This is similar to Gertler and Kiyotaki (2010), Gertler and Karadi (2011) and Harrison (2012). It allows us to examine not only the optimal path but also the optimal quantity of asset purchases as the instruments of monetary policy.

There are three main findings in our paper. First, the joint Ramsey policy that includes asset purchase of long-term government and private securities as additional instruments provides a sizable welfare gain during both normal and crisis times. While this result is not completely surprising, we find that the welfare gain associated with private asset purchase dominates government bond purchase. This is because private asset purchase is more effective in offsetting a number of shocks hitting the economy than bond purchase. The big differences of effectiveness emerge in responses to risk shocks and wage mark-up shocks. By contrast, bond purchase is only more efficient in stabilizing shocks that cause direct changes in relative bond holdings.

Second, when the Ramsey planner is allowed to use policy instruments freely, the Ramsey equilibria frequently hit the ZLB and two asset purchases can also go beyond their natural upper bound of one. Therefore, we focus our analysis on the so-called constrained Ramsey policy using smoothing terms of instruments to limit their extensive movements. We still find significant ZLB periods when the economy is hit by large shocks to mimic crisis time. Again, private purchase can achieve higher welfare level than bond purchase, even though the former induces higher frequency of hitting ZLB than the latter. This indicates that the gain of welfare from using private asset purchase outweighs the loss of welfare from hitting the ZLB.

Finally, we quantify the impact of the recent financial crisis using shocks backed out from a data sample of 1987Q1-2014Q2. The results again show a clear advantage of the joint Ramsey model with all three instruments over the Ramsey policy with interest rate only. In addition, under the constrained Ramsey policy that allows the Ramsey planner to move policy instruments consistent with the volatility observed in the data, the ZLB only binds for two/three quarters during the entire financial crisis period. Moreover, if we allow Ramsey planners to use asset purchases more aggressively (but still far from their upper bound of one), the economy can avoid hitting the ZLB altogether. This suggests the possibility of an escape from the liquidity trap if asset purchase policy would have had been in place before the crisis, illustrating the importance of asset purchase policy in normal time.

Our paper belongs to the strand of literature that incorporates the effects of financial frictions in DSGE models, and is particularly close to papers, such as Chen et al. (2012), Harrison (2012), Christiano et al. (2014), Gertler and Kiyotaki (2010), Gertler and Karadi (2011) and Kiyotaki and Moore (2012). What distinguishes our paper is that we model the interaction between different types of financial frictions and derive Ramsey monetary policy of multiple instruments that can improve welfare in both normal and crisis periods. In contrast, the above papers only consider one form of financial friction. This subsequently limits their models ability to distinguish between the different channels through which alternative asset purchases can impact on the economy.

Andrés *et al.* (2004), Chen *et al.* (2012) and Harrison (2012), for instance, only incorporate term structure into their models. Amongst these papers, we are closest to Chen *et al.* (2012), who include bond market segmentation in a medium-scale DSGE model of Smets and Wouters (2007) to fit post-war US data. Two key parameters that govern the degree of market segmentation and the elasticity of the term premium to the quantity of long-term bonds are identified from estimation. However, we deviate from Chen et al. (2012) in introducing portfolio adjustment costs which is modelled as a function of relative bond holdings of different maturities of unrestricted households only.[2] This requires keeping track of long-term bond holdings of each type of household which ensures the systemic segmentation of two types of households and asymmetric structure of adjustment costs across them. More importantly, this modification enables us to evaluate a consistent micro-founded welfare function encompassing these adjustment costs. This is important for analyzing optimal bond purchase.

Bernanke et al. (1999), Christiano et al. (2003, 2014), Kiyotaki and Moore (1997, 2012), Iacoviello (2005), Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) are the most well-known papers in the literature that introduce a micro-founded financial sector into standard models. Their models differ in the assumption about where the agency problem lies, whether it is between non-financial entrepreneurs and banks, or between credit-constrained households and banks, or within banks. Among various forms of financial frictions, the empirical relevance of the financial accelerator mechanism of Bernanke *et al.* (1999) is supported by numerous papers, such as Christensen and Dib (2008), De Graeve (2008), von Heideken (2009). This is the reason why we adopt this particular form of financial friction in our model.

Our paper is also related to the literature analyzing optimal monetary policy in the presence of financial frictions. A strand of the literature, see e.g., Bernanke and Gertler (2001), Reavenna and

[2] Chen et al. (2012) assumes an hoc reduced form of adjustment costs as a function of aggregate bond holdings of different maturities.

Walsh (2006), Faia and Monacelli (2006) and Gambacorta and Signoretti (2014) have conducted optimal policy with various forms of agency costs. However, these authors consider an augmented Taylor rule only. For conducting optimal policy under commitment and discretion, the most closely related papers are Curdia and Woodford (2008), Carlstrom *et al.* (2010), Brendon *et al.* (2011), Harrison (2012) and De Fiore and Tristani (2013). However, their models are very different from ours. For example, Brendon *et al.* (2011) focus on modelling a housing sector, whilst Carlstrom *et al.* (2010) and De Fiore and Tristani (2013) use an agency cost model without capital and financial frictions.

There are other papers that study the effectiveness of joint optimal monetary and macro-prudential policy in DSGE models with banks and credit market frictions, such as Angelini *et al.* (2012), Angeloni and Faia (2013), Lambertini *et al.* (2013), De Paoli and Paustian (2013) and Carrillo *et al.* (2016). These papers are related to ours in the sense that we consider multiple policy instruments. However, both nominal interest rates and asset purchase policy in our model are set by the central bank. Therefore, we do not consider strategic interaction between different policy makers and issues related to institutional design.

The remainder of this paper is organized as follows. Section 2 describes the model and the setup of Ramsey optimal policy. Then follows estimation of the model using US macroeconomic and finance data prior to crisis in Section 3. Section 4 presents the main results of Ramsey policy analysis. Section 5 concludes.

2 The model

The standard parts of the model are built on a medium-scale DSGE model similar with Christiano, Eichenbaum and Evans (CEE, 2005) and Smets and Wouters (SW, 2007). They encompass both nominal rigidities such as price stickiness and real rigidities such as habit formation and capital adjustment costs, to help fit data. The model is then extended with frictions in government bond and credit markets to allow for roles for government bond purchase and private asset purchase. To do so, we first set up a segmentation (following Andrés *et al.* 2004, Chen *et al.* 2012) which divides households into unrestricted and restricted groups with the former trading in both long- and short-term government bonds and the latter trading-in long-term bonds only. Second, we introduce a financial accelerator mechanism *(*BGG 1999) and modify it with nominal debt contract and an idiosyncratic risk shock along the work of Christiano *et al.* (2003, 2014) and Del Negro *et al.* (2013). This requires enriching the model structure by adding additional group of agents namely entrepreneurs who purchase capital from capital producers and sell to intermediate good producers. Finally, to specify the asset purchase policy suitable for conduct optimal policy analysis, we model the purchases of government bonds and private capital assets as time-varying proportions of these assets held by the central bank to total market value of them. The detailed structure of the model is discussed below.

2.1 Households

There is a large number of households, normalized by 1, who are divided into two groups. One group of households (measured by fraction $0 < \omega_u < 1$) are free arbitragers who can allocate savings in both short and long-term bonds, as well as bank deposits. These *unrestricted* households can make portfolio adjustments whenever they want, but they view assets differently in terms of their liquidity. In particular, unrestricted households are subject to liquidity costs, i.e., the costs due to holdings of any additional long-term bonds relative to holdings of more liquid assets, relative to their 'preferred habitat' holdings of the two assets. The intuition is that, more liquid asset is needed to compensate purchase of illiquid assets. This type of modelling strategy of liquidity cost is similar with Andrés *et al.* (2004), Harrison (2012) and Chen *et al.* (2012). In addition, unrestricted households also bear exogenous transaction costs, ε_t^{ζ}, which are fees they pay to financial intermediaries. The remaining fraction $\omega_r = 1 - \omega_u$ of households are *restricted* to hold long-term bonds only, but do not bear any liquidity costs or transaction costs.[3] Although this type of segmentation might not be strictly consistent with reality, it is a stand-in for heterogeneity among participants in financial markets. For example, unrestricted households in the model are owners of commercial banks who value liquidity more than other institutions. Restricted households stand in those latter institutions such as pension funds, insurances, who are less liquidity constrained and are more willing to hold securities to maturity. The are also more specialized in long-term securities and have less transaction costs. Besides those assumptions, we follow Chen *et al.* (2012) and also assume that the two groups of households have different preferences over patience, risk aversion and weights of labour supply in utility.

2.1.1 Unrestricted household

Unrestricted household supplies labour to labour unions and receives wage income $W_t^{h,u}$ per unit of labour. It then consumes a basket of goods C_t^u, pays lump-sum tax T_t^u, and then saves the remaining income in forms of commercial bank deposits D_t^u, and short- and long-term government

[3]It is noted that a looser condition for asymmetric asset substitutability among two types of households is that restricted households also face positive but lower liquidity costs than unrestricted, which is a closer situation to reality. However, this would result in mathematical complications without bringing more intuitions.

bonds, B_t^u and $B_t^{L,u}$. The budget constraint of household is given by:

$$\begin{aligned} & P_t C_t^u + B_t^u + \varepsilon_t^\zeta P_t^L B_t^{L,u} + D_t^u \\ = \; & R_{t-1} B_{t-1}^u + R_t^L P_t^L B_{t-1}^{L,u} + W_t^{h,u} L_t^u + \Pi_t^{l,u} + \Pi_t^{f,u} + \Pi_t^{cp,u} \\ & + \Pi_t^{fi,u} + (1-\Theta)(1-\widetilde{\gamma}) \Pi_t^{e,u} - T_t^u + R_{t-1}^d D_{t-1}^u \end{aligned} \tag{1}$$

where P_t is aggregate price level, R_{t-1} and R_{t-1}^d are the non-state contingent gross return of short-term bonds and deposits respectively, and P_t^L is the price of long-term bonds. The transaction cost shock ε_t^ζ alters the cost of holding long-term bonds and is used to capture variations in term premium (TP) that cannot be explained endogenously by relative bond quantities. Here it is assumed that only unrestricted household have bank deposits and they do not treat this kind of saving as risky. $\Pi_t^{f,u}$, $\Pi_t^{l,u}$, $\Pi_t^{e,u}$, $\Pi_t^{cp,u}$ and $\Pi_t^{fi,u}$ are the nominal profits of intermediate good producers, labour unions, entrepreneurs, capital producers and financial intermediaries, redistributed to unrestricted households.[4] It is assumed that all kinds of profits are equally distributed to all households despite their types. $W_t^{e,u}$ is a lump-sum tax to finance the entrepreneurs and T_t^u is the nominal lump-sum taxes levied on the same agent.

We model long-term bond as an asset with exponentially decaying coupon κ, following Woodford (2000). So R_t^L is the *ex post gross* yield to maturity

$$R_t^L = \frac{1}{P_t^L} + \kappa.$$

Thus, the value of $B_{t-1}^{L,u}$ can be written as $R_t^L P_t^L B_{t-1}^{L,u}$, and the average duration of these bonds is $R_t^L / \left(R_t^L - \kappa\right)$.

Unrestricted households derive utility from consumption, suffer disutility from supplying labour and a liquidity cost measured by a quadratic term of relative bond holdings. Specifically, the utility function of the unrestricted households is given by:

$$U^u = \varepsilon_t^{p,u} \left[\frac{\left(\frac{C_t^u}{Z_t} - h \frac{C_{t-1}^u}{Z_{t-1}} \right)^{1-\sigma_u}}{1-\sigma_u} - \frac{\chi_u \left(L_t^u\right)^{1+\eta}}{1+\eta} - \upsilon \left(B_t^u, P_t^L B_t^{L,u} \right) \right] \tag{2}$$

where h is the degree of internal habit, $\varepsilon_t^{p,u}$ is a preference shock whose steady-state level can differ from restricted households, χ_u is the weight of labour supply that can also differ from restricted households. The last term represents the cost of portfolio adjustment cost, measured in quadratic form $\upsilon \left(B_t^u, P_t^L B_t^{L,u} \right) = \frac{\upsilon}{2} \left(\delta_u \frac{B_t^u}{P_t^L B_t^{L,u}} - 1 \right)^2$ which captures the notion that unrestricted household's liquidity cost of holding long-term bonds can be compensated by holding some amount

[4] For simplicity, we assume that the financial intermediary provides intermediation services at zero cost.

of liquid short-term bonds.[5] Thus, there exists a 'preferred habitat' ratio $\frac{B_t^u}{P_t^L B_t^{L,u}} = \delta_u^{-1}$ that causes zero liquidity cost to unrestricted households. The parameter δ_u is set such that there is no liquidity cost in steady-state. The parameter υ governs the degree of liquidity cost, i.e., a larger value of it would cause unrestricted households to make less portfolio adjustments and thus larger term premium. So the larger is υ, the larger is the elasticity of term premium to relative bond holdings of different maturities. Finally, the steady-state value of $\varepsilon_t^{p,u}$ and values of χ_u and υ will be implied from estimation later to recover the preference of unrestricted households from data.

The consumption-saving decision of unrestricted households is solved by maximizing objective function (2) subject to budget constraint (1). For convenience, denote $\frac{\Xi_{t+s}^u}{\Xi_t^u}$ for $\{s = 0, 1, 2...\}$ as the nominal discount factor of the unrestricted household in the utility maximization problem. The corresponding real and detrended discount factor is $\frac{\xi_{t+s}^u}{\xi_t^u} = \frac{\Xi_{t+s}^u Z_{t+s} P_{t+s}}{\Xi_t^u P_t Z_t}$.

2.1.2 Restricted household

Restricted households supply identical labour as unrestricted household to labour unions, and consume a basket of goods C_t^r. However, they differ from unrestricted households by saving options and preferences. Due to market segmentation, restricted households only put their savings in long-term bond market and do not hold any short-term bonds or bank deposits. The budget constraint of the restricted households is given by:

$$\begin{aligned} & T_t^r + P_t C_t^r + B_t^{L,r} P_t^L \\ = \; & R_t^L P_t^L B_{t-1}^{L,r} + W_t^{h,r} L_t^r + \Pi_t^{f,r} + \Pi_t^{l,r} + \Pi_t^{cp,r} \\ & + (1-\Theta)(1-\widetilde{\gamma})\,\Pi_t^{e,r} + \Pi_t^{fi,r}, \end{aligned} \tag{3}$$

where the profits are redistributed to restricted households in the same way as these to unrestricted households.

The utility function of the restricted households is given by:

$$U^r = \varepsilon_t^{p,r} \left[\frac{\left(\frac{C_t^r}{Z_t} - h \frac{C_{t-1}}{Z_{t-1}} \right)^{1-\sigma_r}}{1-\sigma_r} - \frac{\chi_r \left(L_t^r\right)^{1+\eta}}{1+\eta} \right]. \tag{4}$$

Thus, restricted households differ from unrestricted households by not having liquidity costs, and

[5] We can also consider liquidity cost of this type $\frac{\upsilon}{2} \left(\delta_u \frac{(B_t^u)^s (D_t^u)^{1-s}}{P_t^L B_t^{L,u}} - 1 \right)^2$. As a result, we also need to estimate the parameter s, $0 < s < 1$.

also by different preference shock, $\varepsilon_t^{p,r}$ and weight of labour supply, χ_r. Again, the steady state value of $\varepsilon_t^{p,r}$ and the value of χ_r will be pinned down from estimation.

The consumption-saving decision of unrestricted households is solved by maximizing objective function (4) subject to budget constraint (3). For convenience, denote $\frac{\Xi^u_{t+s}}{\Xi^u_t}$ for $\{s = 0, 1, 2...\}$ as the nominal discount factor of restricted household in the utility maximization problem. The corresponding real and detrended discount factor is $\frac{\xi^r_{t+s}}{\xi^r_t} = \frac{\Xi^r_{t+s} Z_{t+s} P_{t+s}}{\Xi^r_t P_t Z_t}$. For the aggregation purpose, we define the *aggregate* multiplier as

$$\Xi_{t+s} = \omega_u \beta_u^s \Xi^u_{t+s} + \omega_r \beta_r^s \Xi^r_{t+s}.$$

2.1.3 Wage setting

Households, despite their types in bonds market, supply homogenous labour services to labour unions who then differentiate those labour services and allocate them to labour agencies. Finally, labour agencies package these labour services to composites and sell the composites to intermediate firms. The labour agencies are in perfect competition. We index the differentiated households labor by L_{lt}, $l \in (0, 1)$. The labour composite packaged by the labour agencies (labour packers) is in CES form:

$$L_t = \left[\int_0^1 L_{lt}^{\frac{\lambda_w - 1}{\lambda_w}} dl \right]^{\frac{\lambda_w}{\lambda_w - 1}}.$$

where λ_w is wage mark-up.

The profit maximization problem of the labour agency gives the following labour demand function:

$$L^j_{lt} = \left(\frac{W^j_{lt}}{W_t} \right)^{-\lambda_w} L_t, \quad \{j = u, r\} \tag{5}$$

Then the aggregate wage index is given by:

$$W_t = \left[\int_0^1 W_{lt}^{1-\lambda_w} dl \right]^{\frac{1}{1-\lambda_w}}. \tag{6}$$

The labour unions have monopolistic power and can set wages. However, they face nominal rigidities in doing so *á la* Calvo. That is, in each period, only a proportion of $(1 - \theta_p)$ of labour unions are randomly chosen to re-optimize their wages. Moreover, following SW(2007), The remaining proportion of θ_p unions who are not given the chance to re-optimize just adjust their wages by steady state inflation π (Yun 1996) and productivity growth γ (SW 2007).

The problem of the labour union is to maximize the labour income subject to the labour demand function (5) by labour agencies. The marginal cost of supplying an additional differentiated labour of unrestricted household L_{lt}^u, can be derived from solving the maximization problem:[6]

$$MaxE_t \sum_{s=0}^{\infty} \beta_u^s \left\{ \begin{array}{c} \varepsilon_{t+s}^{p,u} \left[-\frac{\chi_u \left(L_{lt+s}^u\right)^{1+\eta}}{1+\eta} \right] \\ +\Xi_{t+s}^u \left[\begin{array}{c} R_{t+s-1} B_{t+s-1}^u + R_{t+s-1}^d D_{t+s-1}^u + W_{t+s}^{h,u} L_{lt+s}^u + R_{t+s}^L P_{t+s-1}^L B_{t+s-1}^{L,u} + \\ \Pi_{t+s}^u + \Pi_{t+s}^{l,u} - T_{t+s}^u - P_{t+s} C_{t+s}^u - B_{t+s}^u - B_{t+s}^{L,u} P_{t+s}^L + D_{t+s}^u \end{array} \right] \end{array} \right\}.$$

The FOC w.r.t. L_t^u is given by

$$W_t^{h,u} = \frac{\varepsilon_t^{p,u} \chi_u \left(L_{lt}^u\right)^{\eta}}{\Xi_t^u}.$$

This marginal cost will be taken as given in the labour unions' profit maximization problem below. Similarly, the marginal cost of supplying an additional labour of restricted household is given by $W_t^{h,r} = \frac{\varepsilon_t^{p,r} \chi_r \left(L_{lt}^r\right)^{\eta}}{\Xi_t^r}$, which has the same form.

Thus, labour unions set an optimal wage which aims to maximizing expected net wage income:

$$\begin{aligned} & \underset{W_{lt}^j}{MaxE_t} \sum_{s=0}^{\infty} \theta_w^s \frac{\beta_j^s \Xi_{t+s}^j}{\Xi_t^j} \left[W_{lt}^j (\gamma\pi)^s - \varepsilon_{t+s}^{\lambda_w} W_{t+s}^{h,j} \right] L_{lt+s}^j \\ & \text{s.t. } L_{lt+s}^j = \left[\frac{W_{lt}^j X_{t,s}^w}{W_{t+s}} \right]^{-\lambda_w} L_{t+s}, \end{aligned}$$

where $\varepsilon_{t+s}^{\lambda_w}$ represents a wage mark-up shock, which is common to both types of households, and follows AR(1) process.[7] This maximization problem solves the optimal wage set by unions. A log-linearization of it together with evolution of aggregate wage (6) yields the evolution of wage inflation.

2.2 Final Good Producers

There is a large number of identical and perfectly competitive final good producers who combine intermediate goods to make final goods for households. Final good producer uses the following CES technology to produce a final good:

$$Y_t = \left\{ \int_0^1 Y_{it}^{\frac{\lambda_p - 1}{\lambda_p}} di \right\}^{\frac{\lambda_p}{\lambda_p - 1}}$$

where Y_t is the consumption basket, $Y_{i,t}$ is the goods as inputs produced by intermediate firm $i \in [0,1]$ and $\lambda_p > 1$ is the price markup of intermediate firms.

[6] Note that, L_{lt}^u denotes differentiated labour to labour agencies and firms, but it costs the same disutility as L_t^u. Thus L_{lt}^u enters the utility function in the same way as L_t^u.

[7] The way we add the wage mark-up shock differs from SW(2007), where wage mark-up λ_w is directly made stochastic. This is because we want to obtain recursive form of the FOC of the wage-setting problem, so that we can apply the second-order approximation.

The final good producers sell their products to consumers at the cost of buying intermediate goods. The profit-maximization problem of final good producer gives the demand function:

$$Y_{it} = \left[\frac{P_{it}}{P_t}\right]^{-\lambda_p} Y_t. \tag{7}$$

The evolution of aggregate price is given by:

$$P_t = \left[\int_0^1 P_{it}^{1-\lambda_p} di\right]^{\frac{1}{1-\lambda_p}}. \tag{8}$$

2.3 Intermediate Good Producers

Intermediate firm produces an individual good Y_{it} using the following Cobb-Douglas technology:

$$Y_{it} = \varepsilon_t^a K_{it}^\alpha (Z_t L_{it})^{1-\alpha}$$

where K_{it}^α is the effective capital (see below) rent from entrepreneurs, $Z_t L_{it}$ is the labour augmenting technology identical to all firms and α is the share of capital.

The productivity Z_t is a trend-stationary process,

$$\ln Z_t = \ln \gamma + \ln Z_{t-1} + \ln \varepsilon_t^z$$

and the stochastic part ε_t^z represents a temporary shock to the growth rate of productivity which follows an AR(1) process. In contrast, ε_t^a is a temporary shock to productivity, which also follows an AR(1) process.

2.3.1 Cost Minimization

Intermediate good producers first solve a cost minimization problem of the form:

$$\underset{L_{it},K_{it}}{Min} \left\{W_t L_{it} + R_t^k K_{it}\right\}$$
$$\text{s.t. } Y_{it} = \varepsilon_t^a K_{it}^\alpha (Z_t L_{it})^{1-\alpha}.$$

The FOCs with respect to L_{it} and K_{it} solves for capital demand by intermediate good producers and their marginal cost:

$$\frac{K_{it}}{L_{it}} = \frac{\alpha}{1-\alpha}\frac{W_t}{R_t^k}.$$

$$MC_t = (\varepsilon_t^a)^{-1} \alpha^{-\alpha} (1-\alpha)^{\alpha-1} \left(\frac{W_t}{Z_t}\right)^{1-\alpha} \left(R_t^k\right)^\alpha.$$

2.3.2 Profit Maximization

The intermediate goods market is assumed to be monopolistically competitive. Firms have market power and can set their prices. They also face Calvo type nominal rigidity, i.e., in each period, only a proportion of $(1-\theta_p)$ of firms are randomly chosen to re-optimize their prices. Moreover, following SW (2007), The remaining proportion of θ_p firms who are not given the chance to re-optimize just adjust their prices by steady state inflation, similar with the one used in wage-setting above. Therefore, θ_p is interpreted as the degree of rigidity in optimizing prices. When there comes the chance to change and re-optimize prices for firms, they know that the optimal price set might survive (with partial inflation indexation) for many periods in future with probability θ_p^s. Thus, they must set an optimal price which aims to maximizing expected future profits subject to the demand function of the product it produces (7):

$$\underset{P_{it}}{Max} E_t \sum_{s=0}^{\infty} \theta_p^s \beta^s \tfrac{\Xi_{t+s}}{\Xi_t} \left[P_{it}\pi^s - \varepsilon_{t+s}^{\lambda_p} MC_{t+s}\right] Y_{it+s}$$

$$\text{s.t. } Y_{it+s} = \left[\tfrac{P_{it}X_{t,s}}{P_{t+s}}\right]^{-\lambda_p} Y_{t+s}$$

This maximization problem solves the optimal price set by firms. A log-linearization of it together with evolution of aggregate price (8) yields the New Keynesian Phillips Curve.

2.4 Capital Producers

This is a large number of identical and perfectly competitive firms owned by households who produce capital and sell to entrepreneurs. It is also assumed that entrepreneurs can resell capital to capital producers. The production technology is that capital producers combine the last period capital purchased from the entrepreneurs x and the newly purchased investment I to produce the new capital x':

$$x' = x + \varepsilon_t^i \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{9}$$

where I_t is investment, $S(\cdot)$ is the cost of adjusting investment, with $S(1) = S'(1) = 0$, $S'(\cdot) > 0$ and $S''(\cdot) > 0.$, ε_t^i is the investment-specific technology shock which follows an exogenous AR(1).

Since capital producers are identical, we specify their problems in aggregate level, i.e., no need to denote a new index for each capital producer The capital producers' problem is to choose x and I_t to maximize the profits of selling capital to entrepreneurs subject to the producing technology

(9). The period nominal profit is given by:

$$\begin{aligned}\Pi_t^{CP} &= Q_t x' - Q_t x - P_t I_t \\ &= Q_t \varepsilon_t^i \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t - P_t I_t\end{aligned}$$

where Q_{t+s} is the price of the capital measured in terms of household's consumption, which the capital producers take as given. Note that the above expression implies that the choice of x does not matter for this maximization problem. We follow Christiano *et al.* (2010) and set $x = (1 - \delta_k)\overline{K}_t$ where δ_k is the depreciation rate of capital. The accumulation of capital thus follows:

$$\overline{K}_{t+1} = (1 - \delta_k)\overline{K}_t + \varepsilon_t^i \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t.$$

It is assumed that both types of households can assess the capital market via the capital producers such that he maximization problem is identical among all households and is given by:

Form the Lagrangian:

$$\underset{\{\overline{K}_{t+1}, I_t\}}{Max} E_t \sum_{s=0}^{\infty} \frac{\Xi_{t+s}}{\Xi_t} \left\{ Q_{t+s}\varepsilon_{t+s}^i \left[1 - S\left(\frac{I_{t+s}}{I_{t+s-1}}\right)\right] I_{t+s} - P_{t+s} I_{t+s} \right\}.$$

Solving this problem pins down the supply of capital in the economy.

2.5 Banks

Banks absorb deposits D_t from households at time t, promising them nominal interest rate R_t^d paying out at time t+1, and then lend funds to entrepreneurs. The nominal return for banks at time t+1 r_{t+1}^l is predetermined. Banks are in perfect competition. These two assumptions imply that their profit maximization is static and their profit is forced to be zero both ex ante and ex post, i.e., *a* time-to-time zero profit condition:

$$R_{t+1}^l = R_t^d.$$

2.6 Entrepreneurs

There is a large number of entrepreneurs who use their net worth plus bank loans to buy installed capital from capital producers. Following BGG (1999) and Christiano *et al.* (2014), the timing is as follows. Entrepreneurs obtain loans from the banks and use them together with their own net worth to purchase newly installed capital at the end of time t. Besides the systemic risk caused by the unknown aggregate shocks at time $t + 1$, there is a non-systemic risk to the value of entrepreneurial capital, namely an idiosyncratic shock ω_{t+1}^{i} which is specific to each entrepreneur indexed by i This idiosyncratic shock is asymmetric for entrepreneurs and banks in that the latter only observe it by paying monitoring costs. To overcome this agency problem, entrepreneurs and banks sign a one-period debt contract a la Townsend (1979) and BGG (1999).

Given the timing of events and the fact that net worth and purchased capital are for use at time $t+1$, we will use subscripts $t+1$ for these variables, following BGG (1999) and Christiano *et al.* (2014). The following constraint must hold for entrepreneurs:

$$Q_t\overline{K}_{t+1} = \left(B_t^e + B_t^{p,Fed}\right) + N_{t+1}$$

where N_{t+1} is their net worth, B_t^e and $B_t^{p,Fed}$ are loans obtained from private lenders and central bank respectively. Total loans are:

$$B_t^{LN} = B_t^e + B_t^{Fed}.$$

For convenience, define

$$\psi_t^p = \frac{B_t^{p,Fed}}{B_t^{LN}},\ 0 < \psi_t^p < 1 \tag{10}$$

as the proportion of central bank lending in total loans of firms. It is shown that

$$\begin{aligned} B_t^{p,Fed} &= \psi_t^p B_t^{LN} = \frac{\psi_t^p}{(1-\psi_t^p)} B_t^e. \\ B_t^e &= (1-\psi_t^p)\left(Q_t\overline{K}_{t+1} - N_{t+1}\right). \end{aligned}$$

It is useful to write the gross revenue of the entrepreneurs at time t as

$$\omega_t \widetilde{R}_t^k Q_{t-1}\overline{K}_t$$

where $\widetilde{R}_t^k$ is the nominal gross return of capital for entrepreneurs define as

$$\widetilde{R}_t^k = \frac{R_t^k u_t - P_t a\left(u_t\right) + \left(1-\delta_k\right) Q_t}{Q_{t-1}}.$$

We now derive the debt contract. It is assumed that ω follows a lognormal distribution,

$$\log\left(\omega_{t+1}\right) \backsim N\left(\mu_{\omega_t}, \sigma_{\omega,t}^2\right). \tag{11}$$

Following Christiano et al. (2014), we include the risk shock, $\sigma_{\omega,t}^2$ to capture the variations of financial conditions in credit market. It follows a AR(1) process. In designing this contract, the banks have to make two decisions. One decision is to set up a threshold, denoted by ϖ_{t+1}, of the value of the entrepreneurial capital, below which they will treat the loans as default and abstract its value net of monitoring cost. The meaning of the threshold is that those entrepreneurs shocked by $\omega_{t+1} \geq \overline{\omega}_{t+1}$ are able to pay back bank loans and earn positive profits, and the entrepreneur who is exactly at this threshold earns zero profit (but they are not forced out of market until they fall in the $(1-\widetilde{\gamma})$ portion). Another decision is to determine the size of the loan B_t^e. Denote R_{t+1}^c as the nominal interest rate for the loans and define the threshold as

$$\overline{\omega}_{t+1} \widetilde{R}_{t+1}^k q_t \overline{K}_{t+1} = B_t^e R_{t+1}^c$$

From this equation and the balance sheet equation, it is seen that once the size of the loan and the threshold are determined, the quantity of capital purchased and real contractual rate are also determined.

Banks earn revenues from entrepreneurs and pay promised interest payment to households. The participation constraint requires that the former should be no smaller than the latter. Meantime, assuming perfect competition of commercial banking, we can obtain a zero profit condition:

$$[1 - F_t(\overline{\omega}_{t+1})] \overline{\omega}_{t+1} \widetilde{R}_{t+1}^k Q_t \overline{K}_{t+1} + (1-\widetilde{\mu}) G_t(\overline{\omega}_{t+1}) \widetilde{R}_{t+1}^k Q_t \overline{K}_{t+1} \geq (1-\psi_t^p) \left(Q_t \overline{K}_{t+1} - N_{t+1}\right) R_t^d,$$

where

$$G_t(\overline{\omega}_{t+1}) = \int_0^{\overline{\omega}_{t+1}} \omega_{t+1} dF_t(\omega_{t+1}).$$

which is the participation constraint of commercial banks.

Now define

$$g(\overline{\omega}_{t+1}) = [1 - F_t(\overline{\omega}_{t+1})] \overline{\omega}_{t+1} + (1-\widetilde{\mu}) G_t(\overline{\omega}_{t+1}), \quad (12)$$

$$f(\overline{\omega}_{t+1}) = [1 - G_t(\overline{\omega}_{t+1})] - [1 - F_t(\overline{\omega}_{t+1})] \overline{\omega}_{t+1}, \quad (13)$$

as the return of entrepreneurs and banks respectively, and rewrite the participation constraint as:

$$g(\overline{\omega}_{t+1}) \widetilde{R}_{t+1}^k Q_t \overline{K}_{t+1} \geq (1-\psi_t^p) \left(Q_t \overline{K}_{t+1} - N_{t+1}\right) R_t^d. \quad (14)$$

2.6.1 The optimal debt contract

The expected return of the entrepreneurs at time t+1 is given by:

$$\Pi_t^e = E_t f(\overline{\omega}_{t+1}) \widetilde{R}_{t+1}^k Q_t \overline{K}_{t+1}. \quad (15)$$

The optimal debt contract is derived by maximizing entrepreneurs' expected return (15) subject to banks' participate constraint (14):

$$\max_{\{\overline{K}_{t+1}, \overline{\omega}_{t+1}\}} E_t f(\overline{\omega}_{t+1}) \widetilde{R}_{t+1}^k Q_t \overline{K}_{t+1} + E_t \lambda_{t+1} \left\{ g(\overline{\omega}_{t+1}) \widetilde{R}_{t+1}^k Q_t \overline{K}_{t+1} - (1-\psi_t^p) \left(Q_t \overline{K}_{t+1} - N_{t+1}\right) R_t^d \right\}.$$

where λ_{t+1} represents the Lagrangian multiplier associated with constraint. Note that the multiplier is written in form of $E_t\lambda_{t+1}$ rather than λ_t because the shadow value of the contract to entrepreneur at time t+1 can change when there is unexpected change in aggregate capital return in time t+1. This is due to the fact that the bank's return in the constraint is predetermined (non-state contingent) while the aggregate capital return is not.

An important relation obtained by combining the two FOCs of the above maximization problem is the determination of expected external finance premium (EFP):

$$E_t \frac{\widetilde{R}^k_{t+1}}{R^d_t} = E_t \left[\frac{\frac{f'(\overline{\omega}_{t+1})}{g'(\overline{\omega}_{t+1})}\left(1-\psi^p_t\right)}{\frac{f'(\overline{\omega}_{t+1})}{g'(\overline{\omega}_{t+1})} g\left(\overline{\omega}_{t+1}\right) - f\left(\overline{\omega}_{t+1}\right)} \right]. \tag{16}$$

Recall that $f\left(\overline{\omega}_{t+1}\right)$ and $g\left(\overline{\omega}_{t+1}\right)$ are defined in equations (13), (12) and (11). Therefore, the expected EFP is function of $\widetilde{\mu}$, $\overline{\omega}_{t+1}$, $\sigma_{\overline{\omega},t}$ and ψ^p_t:

$$E_t \frac{\widetilde{R}^k_{t+1}}{R^d_t} = E_t f\left(\widetilde{\mu},\, \overline{\omega}_{t+1},\, \sigma_{\overline{\omega},t},\, \psi^p_t\right). \tag{17}$$

2.6.2 Aggregate net worth evolution

At the end of each period, a portion of $(1-\widetilde{\gamma})$ entrepreneurs exist, the same portion of entrepreneurs enter such that the total number of entrepreneurs does not change over time. Thus, $0<\widetilde{\gamma}<1$ is the entrepreneurial survival rate. The same time, W^e_t is the transfer from households to those survived and newly entered entrepreneurs in each period to ensure that they always have net worth. We assume that the detrended real transfer is a constant: $W^e_t/\left(P_t Z_t\right) = w^e$, which is set small to avoid the possibility that the entrepreneurs become capable of self-funding. Each entrepreneur has the probability of $(1-\widetilde{\gamma})$ to exit the economy and consumes a portion $\Theta \in [0,1]$ of their value of capital,

$$C^{e,i}_t = \Theta f\left(\overline{\omega}^i_t\right) \widetilde{R}^k_t \varrho^i_t N^i_t.$$

and transfer the remaining $(1-\Theta) f\left(\overline{\omega}^i_t\right) \widetilde{R}^k_t \varrho^i_t N^i_t$ to households. At probability $\widetilde{\gamma}$, entrepreneur survives and accumulates net worth for next period:

$$N^i_{t+1} = f\left(\overline{\omega}^i_t\right) \widetilde{R}^k_t \varrho^i_t N^i_t + W^{e,i}_t.$$

In aggregation, it is:

$$N_{t+1} = \widetilde{\gamma}\left\{\left[1-\widetilde{\mu}G\left(\overline{\omega}_t\right) - g\left(\overline{\omega}_t\right)\right] \widetilde{R}^k_t Q_{t-1}\overline{K}_t\right\} + W^e_t.$$

2.6.3 Remaining capital renting problem

The remaining capital renting problem at time $t+1$ is straightforward. After observing the disturbance ω_{t+1} at the beginning of time $t+1$, the entrepreneurs choose the utilization rate and rent capital to firms. Given the notations, the effective capital rent to firms for production at time $t+1$ is

$$K_{t+1} = u_{t+1}\overline{K}_{t+1}.$$

At the end of period $t+1$, they sell the un-depreciated capital bank to capital producers. The capital utilization rate is chosen at the beginning of time $t+1$ to maximize the return of capital subject to an adjustment cost $a(u_{t+1})\overline{K}_{t+1}$:

$$\max_{u_{t+1}} \omega_{t+1}\left[R^k_{t+1}u_{t+1}\overline{K}_{t+1} - P_{t+1}a(u_{t+1})\overline{K}_{t+1} + (1-\delta_k)Q_{t+1}\overline{K}_{t+1}\right]$$

where $a(u_{t+1})$ is the cost function which is increasing and convex with $a(1)=0$, $a'(u_{t+1})>0$ and $a''(u_{t+1})>0$.

2.7 Competitive equilibrium

A stationary competitive equilibrium is obtained by detrending the equations described in above model setup, plus that we adopt stationary processes for three instruments $\{R_t, \psi^{bL}_t, \psi^p_t\}$, and plus an associated government budget constraint and aggregate resource constraint. The specifications of them are as follows.

2.7.1 Conventional policy

In competitive equilibrium, the monetary authority adopts a conventional interest rate feedback rule augmented with interest rate smoothing and money growth

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho}\left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi}\left(\frac{Y^m_t}{\gamma Y^m_{t-1}}\right)^{\phi_y}\right]^{1-\rho}\varepsilon^R_t$$

where variables without time subscripts denote their steady-state values, $0<\rho<1$ governs the smoothness of interest adjusting, $\phi_\pi>1$ is the weight put on inflation, $\phi_y\geq 0$ is the weight put on output growth (relative to its trend growth γ), ε^R_t is the innovations in monetary policy which follows an iid process. Following Christiano *et al.* (2013), we define Y^m_t as measured output that matches GDP data:

$$Y^m_t = C_t + I_t + G_t.$$

2.7.2 Asset purchase policy

We design asset purchases in the following way. First, we allow the central bank to hold a fraction of long-term bond denoted by ψ_t^{bL}. It affects quantities of long-term bond holdings of both types of households via the following market clearing condition:

$$\omega_u B_t^{L,u} + \omega_r B_t^{L,r} = \left(1 - \psi_t^{bL}\right) B_t^L.$$

The natural upper bound for ψ_t^{bL} is unity. However, we allow for negative values for ψ_t^{bL} since we use consolidated government budget constraint (shown below). The notion is that negative ψ_t^{bL} amounts to the contractionary asset purchase policy that central bank also takes the role of issuing (rather than buying) long-term bonds, for the quantity of $\psi_t^{bL} B_t^L$.

The purchase on private asset ψ_t^p has been specified earlier in (10) as a fraction of central bank holdings of private securities to total market value of private securities in the society. The upper bound for this private asset purchase is also unity. Similarly, we also allow for negative value for private asset purchase. The notion is that negative private purchase is analogous to a tax on the return of private assets. Finally, when computing competitive equilibrium, we assume that asset purchases $\left\{\psi_t^{bL}, \psi_t^p\right\}$ follow exogenous AR(1) processes.

2.8 Government budget and resource constraints

Given the setup of debt structure and central bank purchases, the budget constraint of the fiscal government is given by

$$B_t + \left(1 - \psi_t^{bL}\right) P_t^L B_t^L = R_{t-1} B_{t-1} + \left(1 - \psi_{t-1}^{bL}\right) R_t^L P_t^L B_{t-1}^L + B_t^{p,Fed} + P_t G_t - T_t.$$

To stabilize long-term debt, we follow Chen *et al.* (2012) and let relative bond supply follow AR(1) process:

$$\frac{P_t^L b_t^L}{b_t} = \chi_{B^L} \left(\frac{P_{t-1}^L b_{t-1}^L}{b_{t-1}}\right)^{\phi_{B^L}} \varepsilon_t^{B_L},$$

where $\varepsilon_t^{B_L}$ is a relative bond supply shock.

In addition, to make sure that total government debt (especially short-term bonds) is stable, we force the net transfer to respond to the outstanding government debt according to a feedback rule:

$$\frac{T_t}{P_t Z_t} - \frac{G_t}{Z_t} = \chi_T \left(\frac{\left(1 - \psi_{t-1}^{bL}\right) P_{t-1}^L b_{t-1}^L + b_{t-1}}{\left(1 - \psi^{bL}\right) P^L b^L + b}\right)^{\phi_T} \varepsilon_t^T,$$

where ε_t^T is an exogenous shock and $\phi_T > 1$ which ensures that the government raises taxes in response to rises in debt.

Moreover, the aggregate resource constraint is given by:

$$Y_t = \widetilde{\mu} G_{t-1}(\overline{\omega}_t) \widetilde{R}_t^k q_{t-1} \overline{K}_t + a(u_t) \overline{K}_t + \omega_u C_t^u + \omega_r C_t^r + I_t + G_t.$$

Finally, due to the way we model portfolio adjustment costs of unrestricted households, we also need to keep track of long-term bond holdings of both types of households. This implies that we need to retain one budget constraint of one type of households. We include the budget constraint of restricted households:

$$\omega_r \left(P_t C_t^r + P_t^L B_t^{L,r} \right) = \omega_r \left(\begin{array}{c} R_t^L P_t^L B_{t-1}^{L,r} + \omega_u \left(\varepsilon_t^{\zeta} - 1 \right) P_t^L B_t^{L,u} \\ + P_t C_t + P_t G_t - W_t L_t - T_t \end{array} \right) + wd_t^r W_t L_t \tag{18}$$

where wd_t^r is the wage dispersion of restricted households:

$$wd_t^r = \int \left[\frac{W_{lt}^r}{W_t} \right]^{1-\lambda_w}, \tag{19}$$

which emerges due to heterogeneity of households.

2.9 Ramsey equilibrium

In Ramsey equilibrium, the three instruments $\left\{ R_t, \psi_t^{bL}, \psi_t^p \right\}$ are optimally derived by a Ramsey planner. This will involve solving a recursive stationary system with Lagrangian multipliers that take care of lead and lag expectations of economic agents. This system will have unique solution as long as one picks up a specific steady-state which it can be approximated around. We will derive the Ramsey system and the optimal steady-state used to solve an unique Ramsey equilibrium.

2.9.1 Ramsey problem

The Ramsey optimal policy is obtained by maximizing the average of expected lifetime utility of households:

$$E_0 \sum_{t=0}^{\infty} \beta_j^t \varepsilon_t^{p,j} \left\{ \frac{\left(c_t^j - h c_{t-1} \right)^{1-\sigma_j}}{1-\sigma_j} - \frac{\chi_j \left(L_t^j \right)^{1+\eta}}{1+\eta} - v\left(B_t^j, P_t^L B_t^{L,j} \right)_{\{if\ j=u\}} \right\}.$$

Since we have two types of households with different utility functions, we follow Curdia and Woodford (2009) and maximize the weighted average of expected unconditional utility of households as Ramsey planner's value function:

$$\begin{aligned} V_t &= E_0 \sum_{t=0}^{\infty} \beta_u^t \varepsilon_t^{p,u} \left[\int_0^{\omega_u} \frac{\left(c_t^u - hc_{t-1}\right)^{1-\sigma_u}}{1-\sigma_u} - \int_0^{\omega_u} \frac{\chi_u \left(L_t^u\right)^{1+\eta}}{1+\eta} - \int_0^{\omega_u} v\left(B_t^u, P_t^L B_t^{L,u}\right) \right] \\ &+ E_0 \sum_{t=0}^{\infty} \beta_r^t \varepsilon_t^{p,r} \left[\int_0^{\omega_r} \frac{\left(c_t^r - hc_{t-1}\right)^{1-\sigma_r}}{1-\sigma_r} - \int_0^{\omega_r} \frac{\chi_r \left(L_t^r\right)^{1+\eta}}{1+\eta} \right]. \end{aligned}$$

Assuming perfect risk-sharing among households, we can rewrite it as:

$$\begin{aligned} V_t &= E_0 \sum_{t=0}^{\infty} \beta_u^t \varepsilon_t^{p,u} \left[\omega_u \frac{\left(c_t^u - hc_{t-1}\right)^{1-\sigma_u}}{1-\sigma_u} - \frac{\chi_u \left(wd_t^U L_t\right)^{1+\eta}}{1+\eta} - \omega_u v\left(B_t^u, P_t^L B_t^{L,u}\right) \right] \\ &+ E_0 \sum_{t=0}^{\infty} \beta_r^t \varepsilon_t^{p,r} \left[\omega_r \frac{\left(c_t^r - hc_{t-1}\right)^{1-\sigma_r}}{1-\sigma_r} -_t^R \frac{\chi_r \left(wdL_t\right)^{1+\eta}}{1+\eta} \right], \end{aligned}$$

where

$$\begin{aligned} wd_t^U &= \int_0^{\omega_u} \left(\frac{W_{lt}^u}{W_t}\right)^{-\lambda_w} \\ wd_t^R &= \int_0^{\omega_r} \left(\frac{W_{lt}^r}{W_t}\right)^{-\lambda_w} \end{aligned}$$

are wage dispersions associated with labor that supplied by unrestricted and restricted households respectively.

Now rewrite the objective function more succinctly as:

$$V_t = E_0 \sum_{t=0}^{\infty} \left[\omega_u \beta_u^t U_t^u + \omega_r \beta_r^t U_t^r\right], \tag{20}$$

with

$$\begin{aligned} U_t^u &= \varepsilon_t^{p,u} \left[\frac{\left(c_t^u - hc_{t-1}\right)^{1-\sigma_u}}{1-\sigma_u} - \frac{wd_t^U}{\omega_u} \frac{\chi_u \left(L_t\right)^{1+\eta}}{(1+\eta)} - v\left(B_t^u, P_t^L B_t^{L,u}\right) \right]. \\ U_t^r &= \varepsilon_t^{p,r} \left[\frac{\left(c_t^r - hc_{t-1}\right)^{1-\sigma_r}}{1-\sigma_r} - \frac{wd_t^R}{\omega_r} \frac{\chi_r \left(L_t\right)^{1+\eta}}{(1+\eta)} \right]. \end{aligned}$$

2.9.2 Recursive form of Ramsey policy

It is seen that the Ramsey problem in (20) cannot be written in recursive form due to differences in discount factors. The main difficulty is that neither discount factor is appropriate to discount model equations as constraints of Ramsey problem. In another view, even one can discount the constraint by one type of discount factor, it will cause long-run utility of the other type of households to disappear or explode.[8] To overcome this problem, we adopt an average discount factor for the Ramsey planner instead:

$$\beta = \omega_u \beta_u + \omega_r \beta_r,$$

and write the value function in the following recursive form:

$$V_t = U_t + \beta E_t V_{t+1}$$

where

$$U_t = \omega_u U_t^u + \omega_r U_t^r.$$

Then we can obtain a recursive form of the Ramsey problem:

$$\underset{\left\{R_t,\ \psi_t^{bL},\ \psi_t^p\right\}}{L} = E_0 \beta^t \left\{ \sum_{t=0}^{\infty} \left[\omega_u U_t^u + \omega_r U_t^r\right] + \lambda_t \left[constraints\right] \right\}. \tag{21}$$

This strategy is similar with Monacelli (2006), where they define above as recursive saddle-point stationary in the *amplified* state space. In Appendix, we also consider a robustness check where we simply set $\beta_u = \beta_r$. We find that the results obtained using this alternative setting is largely consistent with the benchmark case using average discount factor.

2.9.3 Welfare cost

We solve competitive and the Ramsey equilibria using a second-order approximation to calculate welfare up to the second-order. Following Schimitt-Grohe and Uribe (SGU, 2007), we use the Ramsey model with all three instruments $\left\{R_t,\ \psi_t^{bL},\ \psi_t^p\right\}$ as the benchmark case, and we then measure consumption equivalent welfare cost, λ^c, of an alternative policy relative to this benchmark. A consumption equivalent welfare cost can be solved using the following problem:

$$\begin{aligned}
& V_0^{ALT}\left(c_t^{u,ALT}, c_t^{r,ALT}, L_t^{u,ALT}, L_t^{r,ALT}, v\left(B_t^{u,ALT}, P_t^{L,ALT} B_t^{L,u,ALT}\right)\right) \\
= \; & V_0^{BM}\left(\left(1-\lambda^c\right) c_t^{u,BM}, \left(1-\lambda^c\right) c_t^{r,BM}, L_t^{u,BM}, L_t^{r,BM}, v\left(B_t^{u,BM}, P_t^{L,BM} B_t^{L,u,BM}\right)\right) \\
= \; & E_0 \beta^t \sum_{t=0}^{\infty} \left[\begin{array}{c} \omega_u U\left(\left(1-\lambda^c\right)\left(c_t^{u,BM} - h c_{t-1}^{BM}\right), L_t^{u,BM}, v\left(B_t^{u,BM}, P_t^{L,BM} B_t^{L,u,BM}\right)\right) \\ +\omega_r U\left(\left(1-\lambda^c\right)\left(c_t^{r,BM} - h c_{t-1}^{BM}\right), L_t^{r,BM}\right) \end{array} \right]
\end{aligned} \tag{22}$$

[8] For example, using discount factor of unrestircted households β_u to discount weighted average utility will cause Ramsey planner to gradually ignore long-run utility of restricted households since $(\beta_r/\beta_u) < 0$. For another, using discount factor of restricted households β_r to discount the above weighted average utility will cause long-run utility of unrestircted households to explode for the same reason.

where the terms have superscripts ALT and BM denote variables implied by an alternative policy and the benchmark Ramsey policy, respectively. We measure the welfare cost of an alternative policy as a deduction in consumption implied by the benchmark Ramsey policy, so that both policies deliver the same level of welfare.

3 Bayesian Estimation

3.1 Data

In order to derive the Ramsey policy based on structural parameters and the shocks hitting the economy has empirical relevance, we use eleven quarterly US data from the period 1987Q3 to 2008Q3 to estimate our model. These include seven variables used in Smets and Wouters (2007), i.e., GDP, consumption, investment, the real wage, hours worked, inflation and the federal funds rate.[9] The other four include the 10-year Treasury constant maturity yield, the interest rate on BAA-rated corporate bonds, the ratio between long-term and short-term US Treasury debt and the percentage of long-term US Treasury debt held by the Fed.[10] Details on the construction of the data set are provided in Appendix. The corresponding measurement equations that relate

[9] Chen et al. (2012) do not use consumption and investment observations to estimate their model. We think these data are important, particularly, in estimating the effect of credit market conditions on investment.

[10] We do not include the AR(1) process of private purchase for estimation as this policy was not implemented until 2008Q4. However, private purchase is included in the model and linked to the corresponding data in Section 4.4 when we conduct counterfactual simulation using realised shocks. In addition, as with Del Negro et al (2012), we decide not to include financial data on credit to non-financial firm and the Dow Jones Wilshire 5000 index as an indicator of entrepreneurial net worth. This is because the model cannot fit these variables well without adding sizable measurement errors.

the model variables to the observables are:

$$\begin{aligned}
dlGDP_t &= \overline{\gamma} + 100\left(\widehat{y}_{mt} - \widehat{y}_{mt-1} + \widehat{z}_t\right), \\
dlCONS_t &= \overline{\gamma} + 100\left(\widehat{c}_t - \widehat{c}_{t-1} + \widehat{z}_t\right), \\
dlINV_t &= \overline{\gamma} + 100\left(\widehat{i}_t - \widehat{i}_{t-1} + \widehat{z}_t\right), \\
dlWAG_t &= \overline{\gamma} + 100\left(\widehat{w}_t - \widehat{w}_{t-1} + z_t\right), \\
dlHOURS_t &= \bar{l} + 100\left(\widehat{l}_t\right), \\
dlP_t &= \overline{\pi} + 100\left(\widehat{\pi}_t\right), \\
FFR_t &= \overline{R} + 100\left(\widehat{R}_t\right), \\
(BaaCorporate_t - 10yearTreasury_t) &= \overline{EFP} + \left(\widetilde{R}^k_{t+1} - \widehat{R}_t\right), \\
(10yearTreasury_t - FFR_t) &= \overline{TP} + \left(\widehat{R}^L_{t+1} - \widehat{R}_t\right), \\
\left(\frac{P^L_t B^L_t}{B_t}\right) &= \overline{\frac{P^L B^L}{B}}\left(1 + \widehat{P}^L_t + \widehat{B}^L_t - \widehat{B}_t\right), \\
\left(\frac{P^L_t B^{L,Fed}_t}{P^L_t B^L_t}\right) &= \overline{\psi}^{bL} + 100\widehat{\psi}^{bL}_t,
\end{aligned}$$

where l and dl stand for 100 times log and log difference, respectively. $\overline{\gamma} = 100\left(\gamma_* - 1\right)$ is the common quarterly trend growth rate to real GDP, consumption, investment and wages. $\overline{\pi} = 100\left(\pi_* - 1\right)$ is quarterly steady-state inflation rate and $\overline{R} = 100\left(\beta_u^{-1}\gamma_*\pi_* - 1\right)$ is the steady-state of nominal interest rate. Finally, $\overline{EFP} = 100\left(EFP_* - 1\right)$ and $\overline{TP} = 100\left(\varepsilon^{\zeta} - 1\right)$ are quarterly steady-state of external finance and term premiums

3.2 Calibration and Prior Choice

We partition the model parameters into two groups. Table 1 gives the parameters that we fix a priori. Following Christiano *et al.* (2014), we fix the depreciation rate, capital share and inverse of the Frisch elasticity of labour supply at 0.025, 0.4 and 2, respectively. In addition, the steady-state value for quarterly inflation is set at 0.5% in line with the annual inflation target of 2% commonly assigned to the Federal Open Market Committee (FOMC), while the other steady-state parameters are fixed at data average, such that $\overline{\gamma} = 0.42$, $\overline{R} = 1.184$, $\overline{TP} = 0.33$, $\overline{\frac{P^L B^L}{B}} = 1.95$ and $\overline{\psi}^{bL} = 0.085$.[11] This, subsequently, underpins the discount factors of the unrestricted and restricted households which are set at $\beta_u = 0.997$ and $\beta_r = 0.994$. The entrepreneurs' steady-state default probability, $F\left(\overline{\omega}\right)$, and their survival rate, $\widetilde{\gamma}$, are fixed at 0.0075 and 0.99 as in Del

[11] Following Smets and Wouters (2007), $\bar{l}$ is steady-state hours which is normalized to zero.

Negro *et al.* (2012), while the parameters $\overline{EFP}$, $\varsigma_{ep,z}$, ρ_{σ_ω} and $\sigma_{\sigma_\omega}$are estimated.[12] This, in turn, implies the steady-state values of the risk shock, σ_ω and the monitoring cost, $\widetilde{\mu}$.

Table1: Calibrated parameters

η	Frisch elasticity	2
α	capital share	0.4
δ_k	depreciation rate	0.025
$F(\overline{\omega})$	default probability of entrepreneurs	0.0075
$\widetilde{\gamma}$	survival rate of entrepreneurs	0.99
λ_p	price markup	11
λ_w	wage markup	11
$\overline{\pi}$	steady-state inflation rate	0.5
$\overline{\gamma}$	trend growth rate	0.42
$\overline{R}$	steady-state of nominal interest rate	1.184
$\overline{TP}$	steady-state of term premium	0.33
$\overline{\frac{P^L B^L}{B}}$	steady-state of bond holdings	1.95
$\overline{\psi}^{bL}$	steady-state of percentage bond holdings of Fed	8.5

The second set of parameters, listed in Table 2a and 2b, is estimated using Bayesian methods, as surveyed in An and Schorfheide (2005). Prior choices for standard structural parameters, such as the investment adjustment cost convexity parameter S'', the utilization cost elasticity parameter a'', habit formation, h, and nominal rigidities θ_w and θ_p, are set at values fairly common in the literature (see Smets and Wouters 2007, Del Negro and Schorfheide, 2008).

In addition to the set of standard structural parameters, our model also contains parameters associated with bond and credit market frictions. As for the priors of financial friction parameters, $\overline{EFP}$, $\varsigma_{ep,z}$, ρ_{σ_ω} and σ_{σ_ω}, we set these following Del Negro *et al.* (2012), while the priors that relate to the bond market segmentation and heterogeneous households, such as ω_u, $\varsigma_{tp,b}$, σ_u, σ_r, $\frac{c^u}{c^r}$ and $\frac{\xi^u}{\xi^r}$, are set consistently with Chen *et al.* (2012). However, since our bond market friction differ slightly from Chen et al. (2012) in terms of measuring portfolio adjustment costs, we have an additional parameter δ_u to estimate. It is the steady state ratio of long-term bond holdings of unrestricted to restricted households. This parameter cannot be easily inferred from data and the prior of this parameter is also hard to set. We then decide to pin down δ_u indirectly through estimating the steady-state ratio of long-term bond holdings of restricted household to total long-term bonds available to society, δ_r, which lies in 0 and 1.[13]

[12] This is because the data is more informative about $\overline{EFP}$, $\varsigma_{ep,z}$, ρ_{σ_ω} and σ_{σ_ω}than $F(\overline{\omega})$ and $\widetilde{\gamma}$. Fixing $F(\overline{\omega})$ at 0.0075 is consistent with Bernanke et al. (1999) and also close to the value used in Fisher (1999).

[13] Note that the steady state ratio, $\varsigma_{w,u}$ estimated by Chen *et al.* (2012) needs not to be estimated in our model. This is because we keep track of the budget constraint of the restricted households (3) to pin down their wage dispersion wd^r defined in (**??**). The ratio $\varsigma_{w,u}$ is then derived as $1 - wd^r$.

Finally, except from the monetary policy shock, we set the other shocks to follow AR(1) processes and use loose priors for ρ_i and σ_i as shown in Table 2b.

3.3 Parameter Posterior Distribution

Table 2a and 2b also gives the mode, the mean, and the 10 and 90 percentiles of the posterior distribution of the parameters obtained by the Metropolis-Hastings algorithm. First, the estimated real and nominal rigidities are consistent with the DSGE literature incorporating financial friction, e.g., price rigidity is found to be higher than that of Smets and Wouters (2003 and 2007). As discussed by Del Negro et al. (2015), this result may reflect the intuition that, since financial variables are mainly driven by demand shocks in the model with financial frictions, inclusion of financial data into model makes demand (supply) shocks more (less) important. As a result, a flatter Phillips Curve is easier to fit model to data. Del Negro et al. (2016) also points out the role of higher nominal rigidities for magnifying the effects of financial shocks and private asset purchase policy.

Table 2a: structural parameters

	Priors			Posteriors			
	Dist	Mean	Std	Mode	Mean	10%	90%
σ_u	G	2.00	0.50	1.834	1.822	1.370	2.246
σ_r	G	2.00	0.50	2.068	2.340	1.472	3.222
ω_u	B	0.70	0.15	0.870	0.794	0.633	0.960
h	B	0.60	0.10	0.591	0.630	0.499	0.769
θ_p	B	0.70	0.10	0.791	0.798	0.747	0.846
θ_w	B	0.70	0.10	0.634	0.661	0.551	0.768
S''	G	4.00	1.00	3.005	3.247	2.202	4.330
a''	G	0.20	0.05	0.168	0.178	0.101	0.252
ρ_R	B	0.70	0.15	0.813	0.817	0.780	0.854
ϕ_π	G	1.75	0.25	2.006	2.087	1.791	2.381
ϕ_y	G	0.20	0.05	0.306	0.318	0.198	0.426
ϕ_T	G	1.44	0.20	1.413	1.444	1.115	1.756
ϕ_{B^L}	B	0.80	0.15	0.849	0.808	0.680	0.935
$100\varsigma_{tp,b}$	G	1.28	0.50	0.625	0.764	0.330	1.181
c^u/c^r	G	1.00	0.20	1.015	1.095	0.754	1.434
ξ^u/ξ^r	G	1.00	0.20	0.960	0.994	0.672	1.315
δ_r	B	0.50	0.10	0.493	0.496	0.415	0.582
$\varsigma_{ep,z}$	B	0.05	0.005	0.051	0.052	0.045	0.059

Table 2b: shock parameters

	Priors			Posteriors			
	Dist	Mean	Std	Mode	Mean	10%	90%
ρ_z	B	0.5	0.15	0.406	0.401	0.237	0.568
ρ_a	B	0.5	0.15	0.745	0.730	0.593	0.862
ρ_ζ	B	0.5	0.15	0.845	0.826	0.761	0.890
ρ_p	B	0.5	0.15	0.485	0.500	0.317	0.682
ρ_{λ_p}	B	0.5	0.15	0.771	0.754	0.661	0.848
ρ_{λ_w}	B	0.5	0.15	0.517	0.494	0.343	0.627
ρ_G	B	0.5	0.15	0.934	0.931	0.910	0.953
ρ_{σ_ω}	B	0.5	0.15	0.977	0.971	0.954	0.990
ρ_{B_L}	B	0.5	0.15	0.345	0.386	0.230	0.540
ρ_T	B	0.5	0.15	0.500	0.501	0.343	0.666
$\rho_{\psi^{bL}}$	B	0.5	0.15	0.952	0.950	0.933	0.967
ρ_i	B	0.5	0.15	0.378	0.405	0.243	0.572
σ_z	IGI	0.1	2.00	0.378	0.405	0.243	0.572
σ_a	IGI	0.1	2.00	0.419	0.437	0.351	0.516
σ_ζ	IGI	0.1	2.00	0.467	0.535	0.347	0.721
σ_p	IGI	0.1	2.00	1.004	0.920	0.045	1.455
σ_{λ_p}	IGI	0.1	2.00	0.476	0.509	0.380	0.636
σ_{λ_w}	IGI	0.1	2.00	0.709	0.885	0.429	1.365
σ_G	IGI	0.1	2.00	0.485	0.500	0.433	0.563
σ_{σ_ω}	IGI	0.1	2.00	0.058	0.059	0.052	0.067
σ_{B_L}	IGI	0.1	2.00	0.045	0.047	0.041	0.053
σ_T	IGI	0.1	2.00	0.082	0.155	0.040	0.298
$\sigma_{\psi^{bL}}$	IGI	0.1	2.00	0.329	0.337	0.292	0.381
σ_i	IGI	0.1	2.00	0.355	0.371	0.301	0.439
σ_R	IGI	0.1	2.00	0.154	0.156	0.132	0.180

Second, as for the model parameters related to the bond market friction and heterogeneous households, our estimate of the degree of market segmentation, ω_u, is larger than reported in Chen *et al.* (2012) but remains moderate with a mean of 0.794 and a mode of 0.87. However, the elasticity of term premium to relative bond supply, $\varsigma_{tp,b}$, is twice the size reported in Chen *et al.* (2012), with a mean estimated to be 0.625/100 and 90% interval of (0.330 1.181). This implies that government long-term bond purchase programmes would have a much larger impact on suppressing the term premium and the yields on long-term bonds than suggested by Chen *et al.* (2012). The discrepancy in this result, to some extent, reflects the difference in modelling the term premium. The term premium in our model is endogenously determined by the ratio of the market value of long-term bond holdings of the unrestricted household relative to their short-term bond holdings, while in Chen et al. (2012) it varies with the ratio of aggregate long-term bonds to short-term bonds. Since the unrestricted households suffer liquidity cost when making portfolio adjustments, the variation of their long- to short-term bond holdings might be smaller than that of aggregate long- to short-term bond holdings. This results in a larger elasticity $\varsigma_{tp,b}$ in our estimation. Our result also shows significant heterogeneity in the intertemporal elasticity of substitution of consumption for the two types of households, such that $\sigma_u = 1.822$ and $\sigma_r = 2.340$. These estimates suggest that consumption of the unrestricted household is more sensitive to changes in the short-term interest rates than that of the restricted household, as the short-term interest rates only have an indirect impact on the latter via affecting the long-term interest rates.

Finally, our posteriors of financial friction parameters ($\overline{EFP}$, $\varsigma_{ep,z}$, ρ_{σ_ω} and σ_{σ_ω}) are very close to Del Negro *et al.* (2012). These estimates imply that the steady-state values of the risk shock, σ_ω, the monitoring cost, $\widetilde{\mu}$, and the external finance premium, EFP_*, to be 0.45, 0.13, 1.0044, respectively.[14] Among these financial friction parameters, the elasticity of external finance premium to leverage ratio, $\varsigma_{ep,z}$, is particularly relevant to the private asset purchase by the central bank. Our estimate of 0.052 lies between the corresponding values of Del Negro *et al.* (2012) and Christiano *et al.* (2014).

As with Christiano *et al.* (2014), Table 3 reports the steady-state properties of our model when parameters are set to their posterior means. Our model matches the data well in a number of key ratios and it improves the equity-to-debt ratio compared to Christiano *et al.* (2014).

Table 3: Steady-State Properties of Model vs Data

variable	Model	Data
$\frac{i}{y}$	0.29	0.24
$\frac{c}{y}$	0.53	0.59
$\frac{g}{y}$	0.18	0.16
$\frac{k}{y}$	10.08	10.9
$\frac{n-k}{n}$	2.35	1.3-4.7
monitoring costs as percent of GDP	0.028	not known
Credit velocity	1.33	1.67

4 Results

We conduct Ramsey optimal policy which maximizes the weighted average of two types of households' lifetime utility making use of three types of instruments, i.e., nominal short-term interest

[14] Christiano et al (2014)chose to estimate a different set of financial friction parameters, such as $F(\overline{\omega})$ and $\widetilde{\mu}$. As a result, their monitoring cost and the external finance premium are slightly higher than ours. Our financial friction parameters are close to Bernanke et al. (1999).

rate, R_t, long-term government bond purchase, ψ_t^{bL}, and purchase on private asset, ψ_t^p. In this section, we present results of optimal policy under various scenarios, i.e., optimal policy in normal time with shocks size consistent with estimated value, optimal policy in crisis time where shock size is enlarged to generate a riskier economy to have ZLB binding frequently, and finally a scenario where we simulated Ramsey models using realized shocks backed out from data sample that includes both estimated and extended crisis periods from 1987Q1 to 2014Q2.

4.1 Ramsey steady-state

The Ramsey economy specified in Section 2 does not have an unique steady-state since Ramsey planner can chosen any long-run values for the three instruments, $\{R, \psi^{bL}, \psi^p\}$. This subsection focuses on deriving the Ramsey steady-state as it maximizes households' welfare. The Ramsey dynamics will be explored by taking a second-order approximation conditional on the Ramsey steady-state. We solve the Ramsey steady-state in (21) but switching off uncertainty of shocks. The solution encompasses the optimal choice of three instruments as well as multipliers associated with Ramsey problem.[15]

Note that private purchase, ψ^p, is the only relevant instrument in solving the Ramsey steady-state problem in our model. Given the use of full indexation in price-setting and wage-setting, any long-run value of nominal interest rate is optimal, so we fix $\overline{R}$ to the data average consistent with estimation. Moreover, since the steady-state of the term premium is fixed at data average which cannot be offset by any long-run value of government bond purchase, and thus $\overline{\psi}^{bL}$ is also set at the data mean.

Table 4 summarizes the main properties of the Ramsey steady-state. It can be seen that private asset purchase increases consumption, investment and capital. It is associated with a higher level of welfare, lower external financial premium marginal product of capital (MPK) and labour (MPL).

To better understand how the Ramsey steady-state is affected by different types of frictions, we re-calculate optimal private asset purchase by varying key parameter values governing the degree of frictions in the model. In steady-state, there are three blocks of frictions, imperfect competitions in goods and labour markets, segmentation of bond market, and the BGG friction in credit market. They induce inefficiencies that can be of different magnitudes, but importantly, they are correlated with each other. A way of analyzing these frictions is to view them as inefficiencies attached to factors of production, i.e., MPK and MPL. MPK can be derived using

[15] We follow SGU (2007) and find the solution using a least square method.

the definition of MPK, marginal cost and entrepreneurial capital return:

$$MPK = \frac{R^k}{\alpha MC} = \left[\frac{\widetilde{R}^k \left(\widetilde{\mu}, \overline{\omega}, \sigma_{\overline{\omega}}, \psi^p \right)}{\pi} - (1-\delta) \right] \left(\frac{\lambda_p}{\lambda_p - 1} \right) \alpha^{-1}. \tag{23}$$

MPK increases with price mark-up $\frac{\lambda_p}{\lambda_p-1}$ and decreases with capital return $\widetilde{R}^k$, where $\widetilde{R}^k$ is a function of parameters $(\widetilde{\mu}, \overline{\omega}, \sigma_{\overline{\omega}}, \psi^p)$ associated with the BGG friction and private purchase (see, e.g., (17)). In particular, $\widetilde{R}^k$ is increasing with the monitoring cost $\widetilde{\mu}$. Therefore, both imperfect competition and the BGG friction can make MPK higher than the value chosen by a Ramsey planner. As for MPL, it follows,

$$MPL = \frac{W}{(1-\alpha) MC} = (1-\alpha)^{-1} \left(\frac{\lambda_w}{\lambda_w - 1} \right) \left(\frac{\lambda_p}{\lambda_p - 1} \right). \tag{24}$$

Here we contemporarily assume that there is no bond market segmentation, so that the average wage, W, is equivalent to optimal wage in the steady-state, $\widetilde{W} = \left(\frac{\lambda_w}{\lambda_w - 1} \right)$. Therefore, both imperfect competitions in goods market and labour market can make MPL higher than the value chosen by a Ramsey planner.

We can then show the interaction of three frictions by plotting optimal private asset purchase conditional on grids of $\lambda_p, \lambda_w \in [2, 21]$, $\widetilde{\mu} \in [0, 0.8]$ and $\omega_u \in [0.1, 0.99]$.[16] How changes in these parameters affect the Ramsey steady-state of private asset purchase and external finance premium are shown in Figures 1 and 2, respectively. Several findings in Figure 1 are noteworthy. First, it is seen that more severe degree of imperfect competitions, as implied by lower values of λ_p and λ_w, call for higher level of private asset purchase. However, the impact of lowering λ_w is relatively moderate than that of λ_p on increasing the steady-state value of private purchase. This is because λ_p is related to private purchase directly via equation(23) and it also affects MPL in equation(24), thus it can generate bigger inefficiency overall. By contrast, there is no direct link between private purchase and λ_w in equations (23) and (24), but the numerical result suggests a weak indirect link. Second, monitoring costs have a significant impact on the Ramsey steady-state of private asset purchase, both in terms of magnitude and marginal effect. This is again evident from equation(23) where ψ^p responses to changes in $\widetilde{\mu}$. Third, the degree of bond market segmentation, ω_u, only has a mild impact on the steady-state value of private purchase. This may suggest that the segmentation itself does not lead to big inefficiencies.

An interesting question is whether there is trade-offs between these three inefficiencies. One way is to look at Figure 2 where the external finance premium is also plotted against the four

[16] We also conduct the numerical exercise by varying steady-state interest rate $\overline{R}$ and bond purchase $\overline{\psi}^{bL}$, but found they are insignificant in deriving optimal private asset purchase.

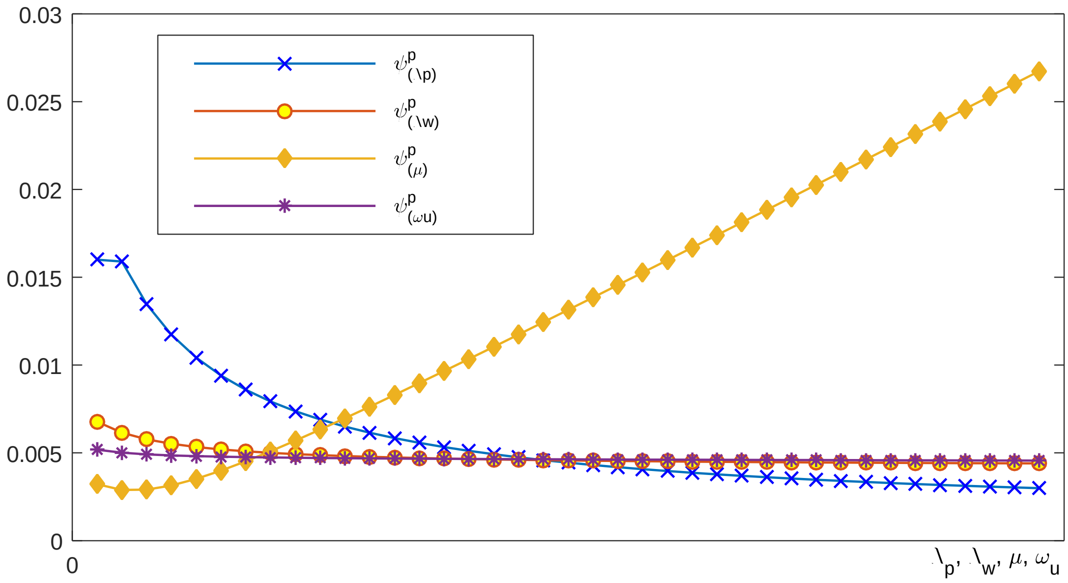
parameters on the same grids under optimal policy. We find that the external finance premium is not necessarily equal to zero, although it is close to zero when the four parameters are at their values used in estimation. The external finance premium will enlarge if price make-up increases. This implies the trade-offs between minimizing multiple steady-state inefficiencies using only one instrument ψ^p. This may also suggest the usefulness of a fiscal subsidy to offset inefficiency caused by imperfect competition in good market. However, we do not need such a subsidy to examine model dynamics since our second-order approximation can be taken around a distorted steady-state.[17]

In Subsection 4.2 below, we will analyze the Ramsey dynamics conditional on the Ramsey steady-state derived using parameter values consistent with estimation.

Table 4: Ramsey steady-state, selected variables.

ψ^p	$EFP(\%)$	V	c	i	k	n	b^e	MPK	MPL
0.0046	0.02	-595.07	2.45	1.71	58.65	41.33	17.49	5.38	2.02
0	0.45	-585.93	2.44	1.36	46.68	32.89	13.98	5.39	2.02

Note: The numbers in first row are obtained using benchmark parameterization, e.g., $\lambda_p = 11$, $\lambda_w = 11$, $\widetilde{\mu} = 0.1284$, $\omega_u = 0.794$. The last row presents the counterfactual absence of policy, i.e., $\psi^p = 0$, for reference.



[17] The welfare rankings among these grids of parameters are not presented since it is not straight forward to interpret them. In fact, because of the heterogeneity of households, the coefficients in objective function are also affected by these parameters considered in main text. For example, lower imperfect competition also lowers the coefficient χ_u in objective function and thus leads to lower welfare, seemingly counter-intuitive.

Figure 1: Optimal private asset purchase under alternative values for λ_p, λ_w, $\widetilde{\mu}$, ω_u. Horizontal axes are grids for these parameters, and vertical axes is optimal purchase.

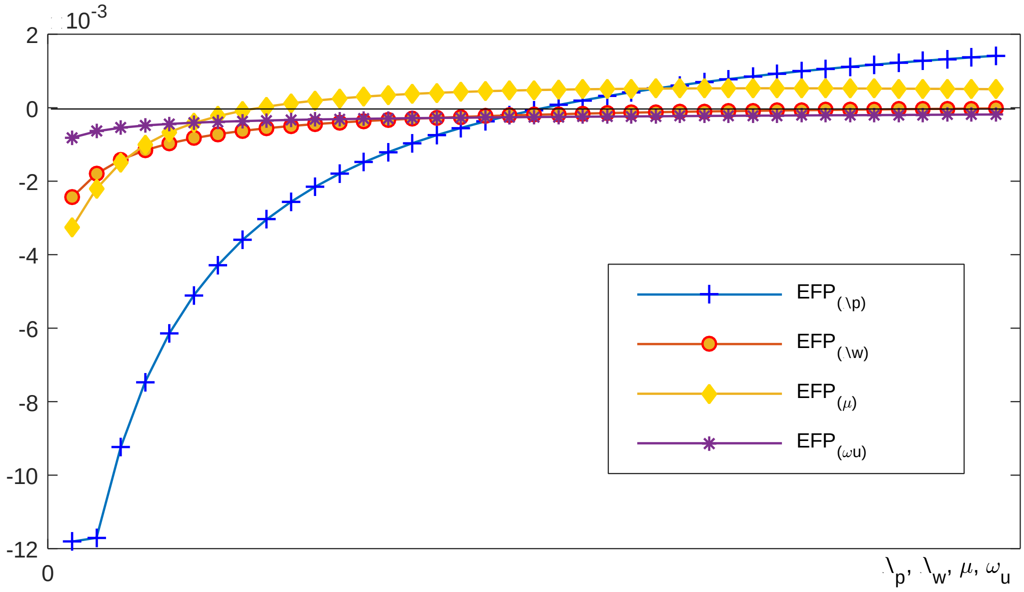


Figure 2: EFP in Ramsey steady state under alternative values for λ_p, λ_w, $\widetilde{\mu}$, ω_u. Horizontal axes are grids for these parameters, and vertical axes is EFP.

4.2 Ramsey dynamics

We now turn to dynamics of the Ramsey economy conditional on the Ramsey steady-state discussed in the previous subsection. We study Ramsey dynamics for different combinations of instruments. In all cases, the policymaker maximizes the expected average lifetime utility of both types of households. We calculate welfare of Ramsey equilibrium conditional on the initial state being the Ramsey steady-state and all shocks hitting the economy are fixed at the scale consistent with the estimation. In addition, we temporarily ignores the possible constraints on instruments, such as the ZLB of nominal interest rates.

We first derive the Ramsey model equilibrium conditions, and then solve for the Ramsey equilibrium numerically using the second-order approximation. Since the solution of the Ramsey problem involves an unit root in the dynamics of long-term government bond purchase, it causes explosive dynamics. We, therefore, remove it by putting a small weight on the smoothing term of government bond purchase in the objective function.[18]

[18] We set a smoothing term on government bond purchase in the objective function, with a mininal weight $1e-7$.

4.2.1 Welfare ranking

We summarize results of welfare rankings associated with different combinations of instruments in Table 5. The first column defines different monetary policy scenarios, while the first row labels what types of shocks hitting the economy. This analysis is helpful to identify sources of differences in welfare. We measure welfare costs of competitive equilibrium and Ramsey equilibria with less than three instruments by fractions of consumption deduced from the Ramsey planner with three instruments, so that they have the same level of welfare with the Ramsey planner. The details of this consumption equivalent welfare cost were discussed in section 2.9.3. For consistency, we set the same steady-state across different policy scenarios including Lagrangian multipliers for different Ramsey policies.

Table 5: Welfare comparisons across policies

	All shocks	$\sigma_{\overline{\omega},t}$	ε_t^{ζ}	$\varepsilon_t^{B_L}$	$\varepsilon_t^{\lambda_w}$
Taylor rule	0.6800	0.06	0.0048	0.0085	0.34
Ramsey R	0.1365	0.0042	0.0007	0.00422	0.125
Ramsey (R, ψ^{bL})	0.1298	0.0033	0.0004	0.0001	0.12
Ramsey (R, ψ^{p})	0.0355	0.00018	0.0003	0.0022	0.033
Ramsey (R, ψ^{bL}, ψ^{p})	0	0	0	0	0

Source: Authors' calculations. Numbers in this table are conditional welfare costs measured as a percentage (%) of consumption in Ramsey model with 3 instruments.

Several points are clear from Table 5. First and as expected, the Ramsey policy with all three instruments achieves the highest welfare regardless the types of shocks hitting the economy. However, when it compares with the Ramsey policies each using a single type of asset purchase the welfare gain can still be significant. Second, while employing any one type of asset purchase as an additional instrument to interest rate improves welfare, private purchase outperforms bond purchase in responses to most shocks hitting the economy. Third, the policy outcome differs substantially when the risk shock and the long-term bond supply shock hit the economy. In particular, the welfare cost is much lower when private purchase is used to offset the risk shock while the bond purchase is much more effective to deal with the inefficiency caused by the long-term bond supply shock. This suggests a division of tasks for the two types of asset purchases to offset inefficiencies in credit and bond markets, respectively. In addition, the welfare cost is much higher with bond purchase compared with private purchase when wage mark-up hits the economy. Importantly, the magnitudes of welfare costs induced by wage mark-up shock and risk

The welfare loss of it is negligible. This kind of unstable behavior is also found in optimal tax rate analysis as a result of costless adjustment of instruments relative to costly adjustment of the other endogenous variables, such as inflation.

shock are much larger than that other shocks. This implies that private purchase exceeds bond purchase mainly due to its efficacy of offsetting these two shocks.

The reason why private purchase is better offsetting risk shock and wage mark-up shock is understood in two ways. One is that the estimated elasticity of external finance premium to leverage is larger than that of term premium to relative bond supply. This implies that private purchase may also be more effective in correcting inefficiencies in financial market. Second, it is due to how these frictions are modelled. For example, the credit friction is closely related to production side of the economy, thus private purchase is potentially able to correct frictions in both labour and capital markets. Finally, Table 5 also shows that private purchase is able to correct part of inefficiencies in the bond market caused by the transaction cost shock. This is evidence that private asset purchase interacts with frictions other than credit market friction. In contrast, the link between bond purchase and credit market friction is much weaker.

4.2.2 Understanding policy responses (IRFs)

It is helpful to plot impulse response functions (IRFs) to better understand how different policies react to economics shocks and why they differ in welfare. To do so, we select three shocks, the transitory technology shock, the transaction cost shock and the risk shock. A second-order approximation is used in calculating these IRFs and pruning algorithm is always used so as to avoid undesired explosive dynamics.

The impulse response of various endogenous variables from different Ramsey models to a positive one standard deviation technology shock is plotted in Figure 3. In response to technology shock, all Ramsey policies choose to cut interest rates, in an attempt to avoid costly downward movement of inflation. However, the magnitude of cuts in interest rates is slightly different. Ramsey planner is able to cut interest rate less when asset purchases are available. Overall, the differences in model dynamics in response to the transitory technology shock are similar. Moreover, the impact of shock is short-lived. These imply that the transitory technology shock does not induce a large welfare trade-off for Ramsey planner.

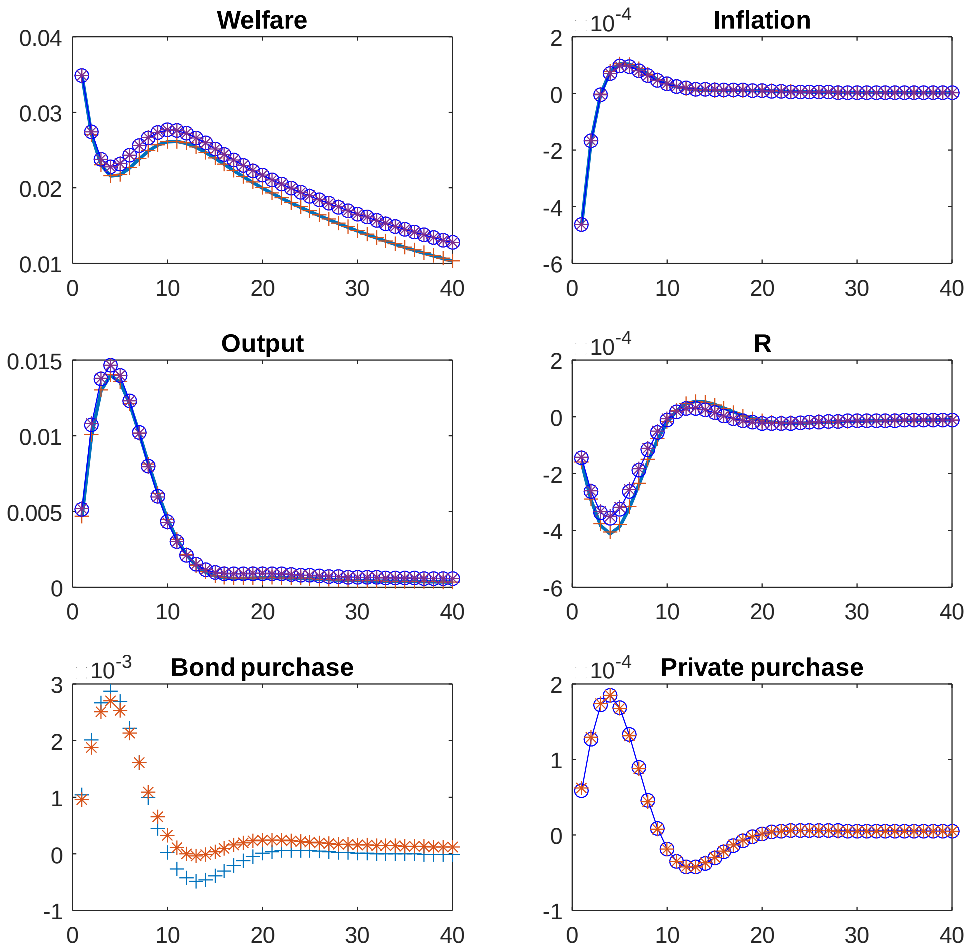


Figure 3: Impulse responses in Ramsey models to technology shock. In each graph, we use '-' for R_only policy, '+' for R+bond purchase, '-o' for R+private purchase and '*' for all 3 instruments.

Turning to Figure 4 where the IRFs of the same variables to a one standard deviation transaction cost shock are plotted. We observe much visible differences. A possible trade-off between stabilizing inflation and term premium is potentially in play. The transaction cost shock induces higher costs for the unrestricted household and downward pressure on inflation. However, the Ramsey planner initially chooses not to lower interest rates as the term premium is high. When asset purchases are available, the Ramsey planner immediately raises asset purchase to relax the trade-off between stabilizing the term premium and inflation. In medium term, the Ramsey planner with only interest rate available has to raise interest rate again since output is higher and there is upward pressure on prices, whilst with asset purchases as additional instruments, the task can be done by reducing asset purchases thus mitigating the need to raise interest rates. By using these combinations of multiple instruments, the economy is better stabilized. Note that while Figure 4 shows that welfare is lower when multiple instruments are used in the first 40 quarters, the longer-run welfare is indeed higher (which is not shown in Figure 4). In addition, Figure 4 also shows that the transaction cost shock has long-lived impact on welfare, a feature that is noteworthy.

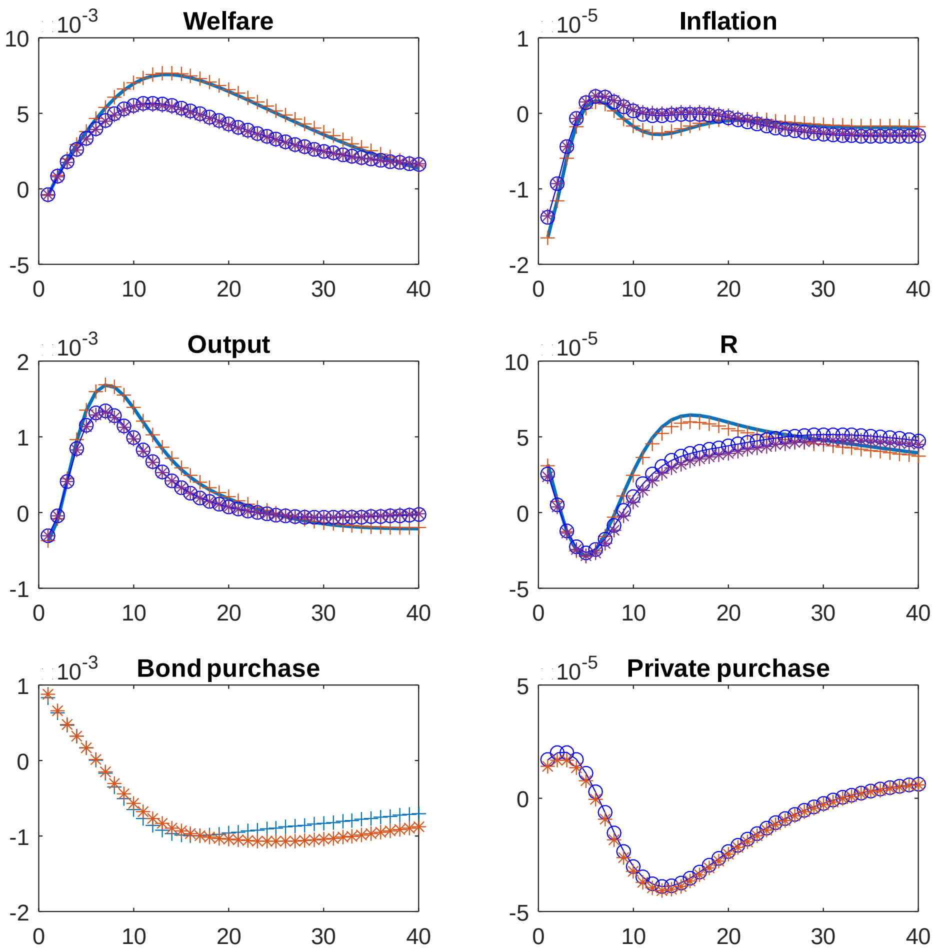


Figure 4: Impulse responses in Ramsey models to transaction cost shock. In each graph, we use '-' for R_only policy, '+' for R+bond purchase, '-o' for R+private purchase and '*' for all 3

instruments.

The differences in dynamics in response to a risk shock become remarkably large as shown in Figure 5. The importance of this shock in driving the US business cycle is highlighted by Christiano et al. (2014). Here we also show its prominent welfare impact. The Ramsey policies include private purchase as an instrument can achieve much higher welfare gain in response to this shock, while bond purchase is not particularly effective in offsetting most of inefficiencies in credit market caused by this shock. This is also reflected in the welfare ranking in Table 5. The IRFs in Figure 5 shows that the Ramsey planner has to cut the interest rate aggressively in response to the risk shock. However, when private purchase can be used as an additional instrument, it requires much less movements in interest rates and it reduces volatilities in inflation and output, and thus improves welfare.

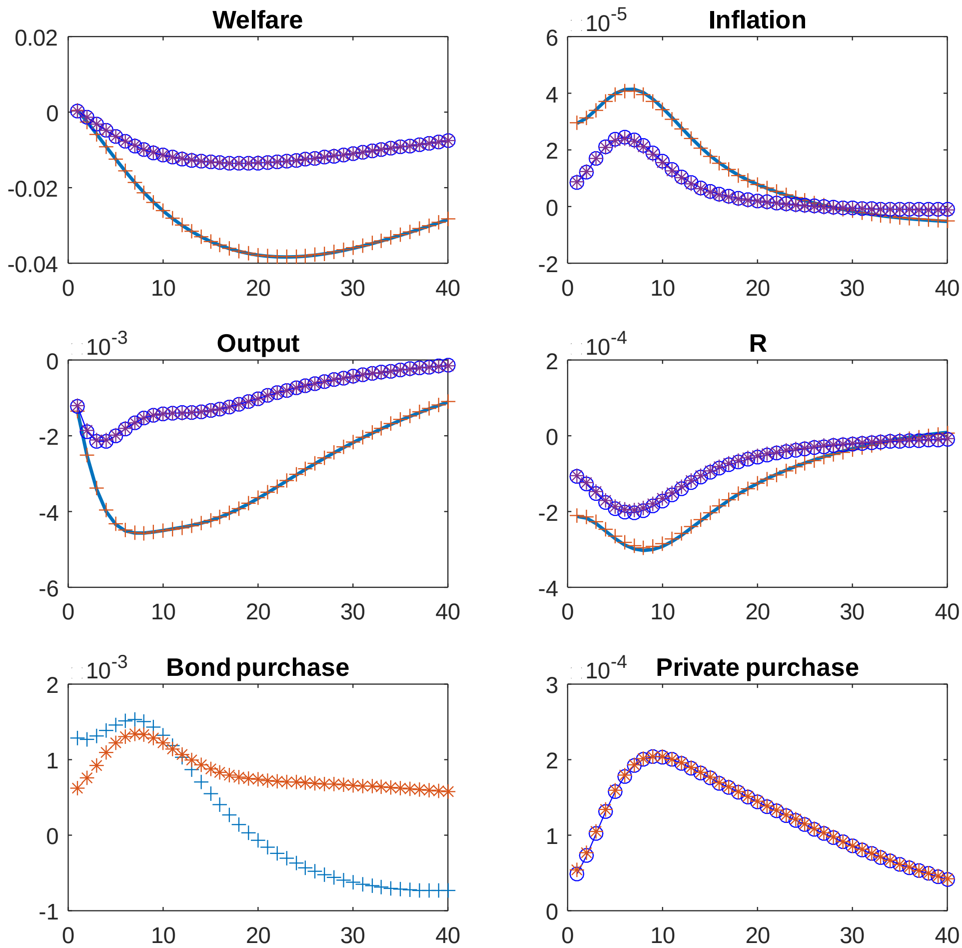


Figure 5: Impulse responses in Ramsey models to risk shock. In each graph, we use '-' for R_only policy, '+' for R+bond purchase, '-o' for R+private purchase and '*' for all 3 instruments.

4.2.3 Volatility of instruments and constrained Ramsey equilibria

An important aspect of the Ramsey policy is whether the optimal interest rate violates the ZLB constraint. To examine this, we conduct stochastic simulations using parameter values obtained from the estimation. We find that all our Ramsey equilibria discussed above frequently hit ZLB and two asset purchases can also go beyond their natural upper bound of one. This suggests that our DSGE model estimated using both macroeconomic and financial data with rich shocks and financial frictions imply substantial uncertainty to agents and the policy maker. One way to make optimal policy more implementable in reality is to put weights on instrument smoothing terms in the objective function as a way to limit the Ramsey policy maker's ability to move these instruments. In the following analysis we focus on a set of weights that they deliver volatilities in the interest rate and asset purchases to match those observed in the data. We consider this Ramsey policy with smoothing terms on instruments as the constrained Ramsey policy.

4.3 Ramsey policy with ZLB

4.3.1 Constrained Ramsey policy in normal time

This section focuses on the constrained Ramsey equilibria where the Ramsey policy maker's ability to use the three policy instruments are comparable with the policy implemented in reality. Although the frequency of interest rates hitting the ZLB is reduced under the constrained Ramsey policy, we still need to account for the ZLB. To do so, we apply the occasionally binding constraints (OBC) algorithm using "news"-type shocks proposed by Holden (2016).[19] The advantages of this algorithm are twofold. First, it produces higher-order approximations of the model which enable us to quantify welfare rankings across different policies. Second, the ZLB periods are endogenous to the shocks hitting the economy and policy responses.

Table 6 displays the welfare rankings for the constrained Ramsey equilibria. As with the estimated Taylor rule, all Ramsey equilibria avoid the ZLB and the two asset purchases are far away from their upper bound of unity when we fix the shock size consistent with the estimation using the sample period from 1987Q1-2008Q3. These results represent a normal time simulation.

[19] Global solution techniques that enable the Ramsey planner to incorperate ZLB in a recursive optimisation problem would be ideal. However, this approach is hindered by the size of model.

Table 6: Welfare comparisons across policies (Constrained Ramsey)

	All shocks	$\sigma_{\overline{\omega},t}$	ε_t^{ζ}	$\varepsilon_t^{B_L}$	$\varepsilon_t^{\lambda_w}$
Taylor rule	0.4970	0.05806	0.004	0.00565	0.4161
Ramsey R	0.0640	0.0072	0.000265	0.00190	0.0487
Ramsey (R, ψ^{bL})	0.0593	0.0069	0.00018	0.00007	0.0457
Ramsey (R, ψ^{p})	0.00412	0.0000335	0.000103	0.00185	0.0017
Ramsey (R, ψ^{bL}, ψ^{p})	0	0	0	0	0

Source: Authors' calculations. Numbers in this table represent average welfare costs measured as a percentage (%) of consumption in Ramsey model with 3 instruments. These are calculated from simulations using DynareOBC algorithm with ZLB. Smoothing terms used for (R, ψ^{bL}, ψ^{p}) are $25, 0.15, 32$ respectively. Simulation periods=20000.

The welfare rankings presented in Table 6 are largely consistent with these in Table 5 where the Ramsey planner is free to move their instruments without constraints. Again, the Ramsey policy with all three instruments outperforms the other policy alternatives. However, the welfare losses associated with using a smaller number of instruments are reduced due to the fact that the Ramsey planner uses asset purchases to a lesser extent. Private purchase, again, dominates government bond purchase in response to the risk shock and wage mark-up shock, while bond purchase is superior in offsetting inefficient changes in the relative bond holdings.

4.3.2 Larger shocks, ZLB and constrained Ramsey policy

In this subsection, we enlarge the shocks hitting the economy to mimic the crisis period. Specifically, we quadruple the standard deviations of four selected shocks, the temporary technology shock, investment-specific technology shock, the risk shock and the transaction cost shock, such that under the estimated Tayor rule the ZLB binds for 20% of periods in the stochastic simulation.[20] We then rank welfare costs of competitive and Ramsey equilibrium measured by fractions of consumption given up by Ramsey policy with three instruments, and also count for their

[20] It is argued (see, e.g., by Del Negro *et al.* 2013, Christiano *et al.* 2014) that the shocks $(\varepsilon_t^a, \varepsilon_t^i, \sigma_{\overline{\omega},t})$ are critical driving forces during this financial crisis.

corresponding periods of stay at ZLB. Table 7 summarizes results of this experiment.

Table 7: Welfare comparisons across policies (Larger Shocks)
The sizes of four shocks, $(\varepsilon_t^a, \varepsilon_t^i, \varepsilon_t^\zeta, \sigma_{\overline{\omega},t})$ are quadrupled to mimic a risky economy.

	Consumption equivalent welfare costs	ZLB hitting frequencies (% of simulation length)
Taylor rule	2.981	20.42%
Ramsey R	0.391	12.54%
Ramsey (R, ψ^{bL})	0.386	12.11%
Ramsey (R, ψ^p)	0.0044	14.26%
Ramsey (R, ψ^{bL}, ψ^p)	0	13.05%

Source: Authors' calculations. Numbers in this table represents average welfare costs measured as a percentage of consumption in Ramsey model with 3 instruments. These are calculated from simulations using DynareOBC algorithm with ZLB. Smoothing terms used for (R, ψ^{bL}, ψ^p) are $25, 0.15, 32$ respectively. Simulation periods=20000.

Two observations are noteworthy. First, when the economy is hit by four times larger of the four shocks mentioned above, the welfare costs are mostly six times larger in most cases compare with welfare costs in Table 6. This is not only caused by larger shocks but also bindings of ZLB in all policy scenarios. Second, the Ramsey policy faces further trade-offs due to the ZLB constraint. The results in Table 7 show again that the welfare ranking favors the use of private purchase. However, it also increases the probability of the interest rate hitting the ZLB. Bond purchase, on the other hand, despite mildly reduces the period of the ZLB, it only marginally improves welfare compared to the Ramsey policy with interest rate only. This finding implies that the gain from using private purchase outweighs the welfare loss due to hitting the ZLB. Overall, the use of multiple instruments especially private purchase avoids substantial welfare losses in face of larger shocks.

4.4 Realized shocks and policy responses

Our model is estimated over the sample period from 1987Q1 to 2008Q3, i.e before the short-term policy rate reaches the ZLB. Using the data from this sample period, and fixing model parameters at their posterior means, we can back out the exogenous shocks hitting the economy. We label these shocks as the realized shocks during the normal time. With these realized shocks, we can undertake various counterfactual analysis. For example, exploring what the outcomes would have been if the policy maker had adopted the Ramsey interest rate policy. We can also assess, how much further economic outcomes would have improved if the joint Ramsey monetary policy combining the short-term interest rate and asset purchases were implemented.

In addition to the normal time, it is also important to consider the performance of Ramsey policy during the recent financial crisis when the shocks hitting the economy become more severe. To back out the realized shocks during the crisis period, we extend our sample period till 2014Q2 and take into account monetary policy implemented at the time. These include the ZLB policy and the LSAP programmes on both long-term Treasury and private securities.

While the Fed's purchase of long-term Treasury debt can be considered as an extension to the standard open market operations, the purchase of private securities is a new development which first occurred during the crisis. This requires us to augment our model and the measurement equation to take into account the Fed's holdings of entrepreneurs' capital since 2008Q4 when the Fed started purchasing private securities during the first round of LSAP programmes. However, it is difficult to find an exact counterpart to entrepreneurs' capital held by the Fed. This is because the Fed's holdings of private securities covers a wide range, e.g., MBS and agency debt, and influence multiple financial markets. It is difficult to measure the exact percentage that the Fed's private security purchase effectively transformed into entrepreneurs' capital. We take a conservative estimate that 10 percent of the Fed's purchases of MBS and agency debt are transformed into entrepreneurs' capital. This number is subsequently divided by the total liabilities of nonfinancial firms. This gives rise to the percentage of the Fed's holdings of entrepreneurs' capital, $\frac{B_t^{p,Fed}}{B_t^{LN}}$. This variable is then linked to the exogenous AR(1) process of $\widehat{\psi}_t^p$ in the model:

$$\left(\frac{B_t^{p,Fed}}{B_t^{LN}}\right) = \overline{\psi}^p + 100\widehat{\psi}_t^p,$$

where the steady-state value of private securities purchase, $\overline{\psi}^p$, is set to zero given that this policy does not exist in normal time.

Furthermore, the short-term interest rate effectively reaches the ZLB since 2009Q1. We follow the solution method of Kulish and Pagan (2016) and Kulish *et al.* (2016) to solve for an equilibrium in the presence of a fixed interest rate regime. In particular, when the ZLB binds at t, the Fed sets its policy to zero, $\widehat{R}_t = -\overline{R}/100$, and the Fed communicates a plan to revert back to the Taylor rule at a later date, $t + d$. d is the expected duration of the ZLB at time t. We use the sequence of expected duration constructed by Kulish *et al.* (2016) based on two data sources, the Federal Reserve Bank of New York's Survey of Primary Dealers and the Blue Chip Financial Forecasts to impose the ZLB during period from 2009Q1 to 2014Q2. During the ZLB period the federal funds rate has no variance, we remove it as an observed variable following Kulish *et al.* (2016).

4.4.1 Realized shocks

Figure 7 and 8 plot four selected realized shocks that include two supply side shocks (i.e. the temporary productivity shock and the investment shock) and two financial shocks (i.e. the risk and transaction cost shocks). The movements of the two supply shocks are closely related to business cycle fluctuations. During downturns, both shocks become negative reflecting the reduction in production and private investment, respectively. During the post-crisis economic recovery, the temporary shock to productivity has become predominantly positive, while the improvement in the investment shock remains relatively uncertain till 2013.

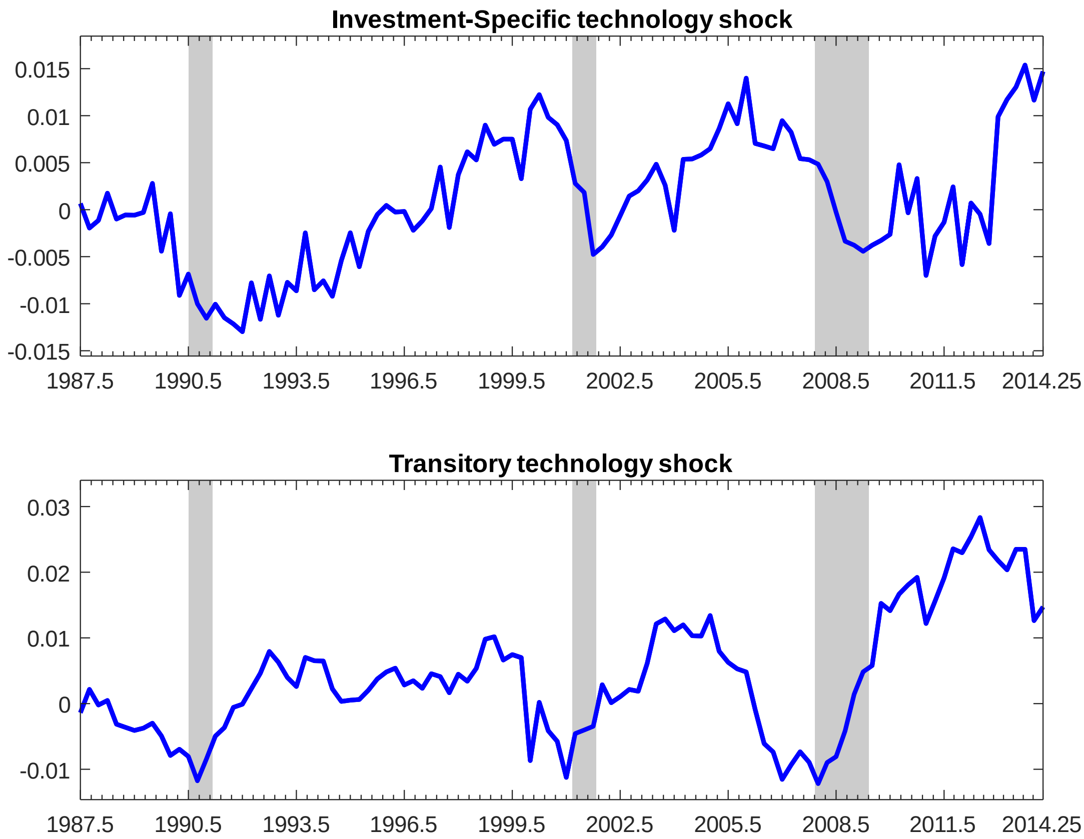
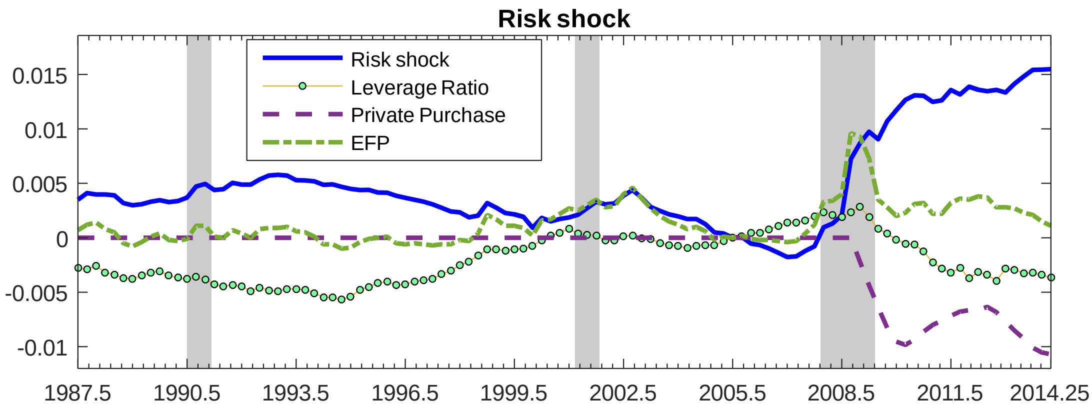


Figure 7: Realised investment shocks and technology shocks.

Turning to the two financial shocks (the risk and transaction cost shocks), they contribute to the fluctuations of the external financial premium and term premium, respectively. The external financial premium can be decomposed into three components: the risk shock, the Fed's private asset purchases and the component related to the leverage ratio. This can be seen from its log-linearised form:

$$\left(E_t \widehat{\widetilde{R}}^k_{t+1} - \widehat{R}^d_t\right) = \varsigma_{ep,z}\widehat{\varrho}_{t+1} + \varsigma_{ep,\sigma_\omega}\widehat{\sigma}_{\omega,t} - \varsigma_{ep,\psi^p}\widehat{\psi}^p_t$$

where $\varsigma_{ep,z}$ is the elasticity of the external financial premium to leverage. We find that the dynamics of realized risk shock (shown in Figure 7) are in line with Christiano *et al.* (2014) that for the risk shock series prior to 2008Q4 the shock has always positively contributed to the external financial premium and fully accounts for the initial jump in the premium when the crisis intensified during later 2008. In addition, we also show that during 2009Q1-2014Q2 the risk shock remains high, while the Fed's private purchase are effective in suppressing the external financial premium.



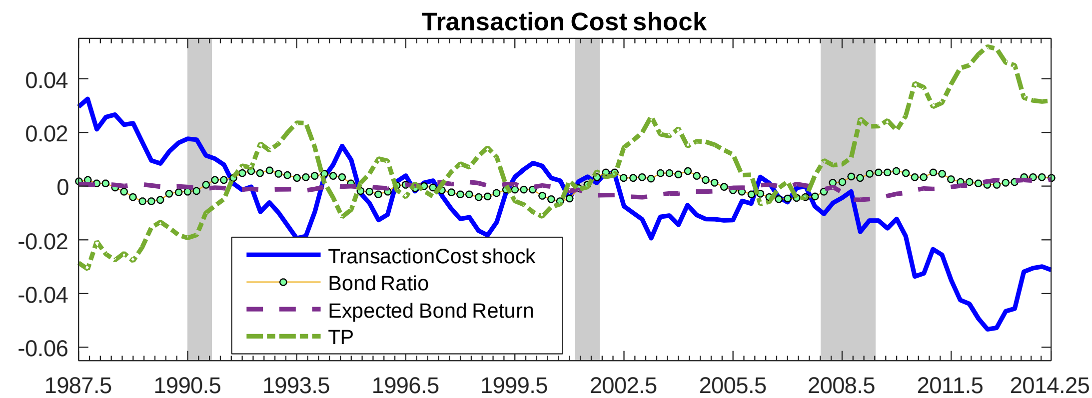


Figure 8: Realised risk shock and transaction cost shock.

As for the term premium, it can also be decomposed into three components, the transaction cost shock, the expected price changes of the long-term bonds and the component related to the relative bond holdings. Again this can be seen from log-linearization:

$$E_t \widehat{R}^L_{t+1} - \widehat{R}_t = \varsigma_{tp,b} \left(\widehat{b}^{L,u}_t + \widehat{P}^L_t - \widehat{b}_t \right) + \left(\widehat{P}^L_t - E_t \widehat{P}^L_{t+1} \right) + \widehat{\varepsilon}^{\zeta}_t.$$

where $\varsigma_{tp,b}$ is the elasticity of bond spread to relative bond holdings. The finding shown in lower panel of Figure 8 is consistent with this decomposition. Especially, it shows that the identified transaction cost shock follows a downward trend during the financial crises.

4.4.2 How optimal have monetary policies been during normal time and the Great Recession?

We insert these realized shocks back into our model under the actual policy with the LSAP programmes implemented since 2008Q4 and the alternative Ramsey policy scenarios. Figure 9 plots the Ramsey policy counterfactuals against the data. It shows that the Ramsey policy with interest rates, $RamseyRonly$, only hits the ZLB for 5 quarters during the crisis period, much less than the data. The constrained Ramsey policy with all three instruments, $Ramsey3$, can further reduce the ZLB periods to 2 quarters. Finally, if the Ramsey planner is allowed to use asset purchases more freely, $Ramsey3LS$, (not constrained to the level of volatility observed in the data), the weights placed on smoothing terms of asset purchases are relaxed. In this case, the result shows that the Ramsey planner can easily escape from the ZLB by using larger asset purchases without exceeding their upper bound of one.

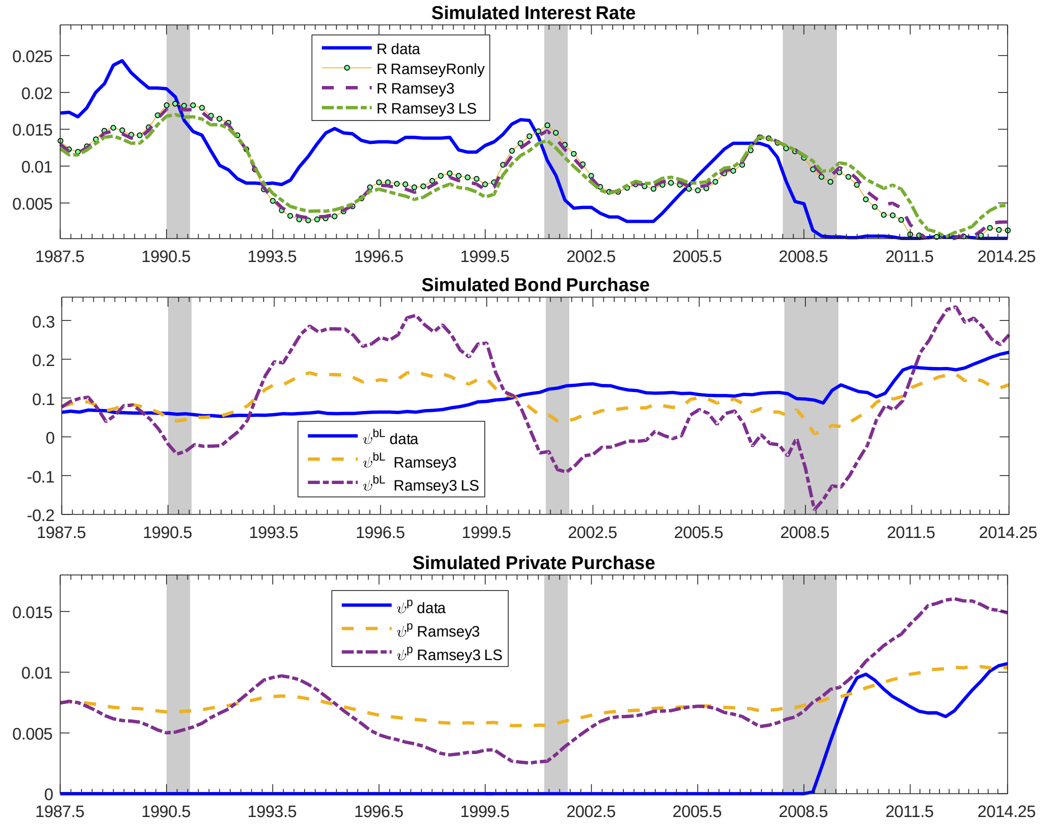


Figure 9: Simulated instruments for optimal polcies and data. Ramsey3 LS stands for Ramsey policy with 3 instruments that are used with larger scales.

Again, we use the Ramsey policy with three instruments as the benchmark to evaluate the welfare costs under alternative policy scenarios. The result is shown in Table 8. There are four main findings. First, it is shown that if the Fed had implemented the Ramsey policy of interest rates, while also keeping the other two asset purchases at their implemented levels in reality, it could have avoided over 85% (i.e., 0.0818 compared with 0.5472) the welfare costs of competitive equilibrium, over the whole sample period. Furthermore, if the Fed could apply the constrained Ramsey policy with asset purchases as additional instruments restrained at the volatility observed in the data, it could further improve welfare by 15%. These improvements are significant.

Second, the welfare costs of alternative policies are larger during crisis time than during normal time. This is caused by both larger size of shocks and frequent binding of ZLB. Third, in terms of comparison of two asset purchases, we find that private asset purchase still outperforms bond purchase. In particular, we find that private asset purchase performs extraordinary well when the economy is in crisis. In fact, the worse the economic condition, the closer it gets to Ramsey3LS. The effectiveness of bond purchase, on the other hand, is much weakened during crisis time. The superiority of private asset purchase over bond purchase during crisis may be explained by the fact that risk shock plays a key role during crisis and private purchase is particularly effective in offsetting it.

Finally, when Ramsey policy with three instruments is allowed to react more freely (but still far away from upper bound of 1), the welfare gains relative to benchmark is substantial, as can be seen from the last row of Table 8. This reflects the gain from escaping the liquidity trap since most portion of gain is attributable to crisis periods.

Table 8: Welfare comparisons across policies using realized shocks

	Consumption equivalent welfare costs		
	Whole Sample	Normal Time	Crisis Time
Taylor rule	0.5472	0.5342	0.5764
Ramsey R	0.0818	0.0715	0.1221
Ramsey (R, ψ^{bL})	0.0775	0.0672	0.1139
Ramsey (R, ψ^{p})	0.0028	0.00305	0.0019
Ramsey3 (R, ψ^{bL}, ψ^{p})	0	0	0
Ramsey3LS (R, ψ^{bL}, ψ^{p})	-0.086	-0.069	-0.138

Source: Authors' calculations. Welfare numbers in this table represents welfare costs measured as a percentage of consumption of Ramsey model with 3 instruments. DynareOBC algorithm is used to take account of ZLB.

5 Conclusion

The last decade witnessed the relevance of liquidity trap among most industrialized economies and the launch of widely used LSAP programmes. This paper investigates the possibility of using these asset purchases as regular instruments and the joint optimal design of monetary policy when they are combined with the short-term nominal interest rate. To do so, we build a medium-scale DSGE model with both bond market and financial market frictions to give asset purchases real effects. Combining the two frictions enables us to examine their joint degree of empirical relevance since they were estimated separately in the literature. In addition, it allows us to study optimal Ramsey policy with multiple instruments and examine the importance of using asset purchases during both normal and crisis times.

There are three main findings in our paper. First, the use of multiple instruments improves welfare, measured as the expected lifetime utility of households, in normal times due to their presence in the data. In addition, when comparing bond purchase with private asset purchase, the latter outperforms the former in offsetting the set of shocks hitting the economy. A detailed examination shows that private purchase has dominant advantage for stabilizing the risk shock and cost-push shock, while bond purchase performs much better for offsetting shocks that alter the quantity of bonds. This suggests a division of use of asset purchases.

Second, we focus on the constrained Ramsey policy when three instruments match their volatilities observed in data. Applying this constraint is realistic since the Ramsey policy can lead three instruments to go beyond their bounds such that they can be implemented in reality. We enlarge a number of selected shocks to simulate the crisis period as the economy is likely to be hit by more severe shocks than normal times. We find that welfare rankings across different combinations of instruments remain consistent during both normal and crisis periods. Private purchase is mostly useful in improving welfare when the economy becomes risky. However, the Ramsey policy with private purchase has a higher frequency of hitting ZLB than bond purchase, which may not be desirable in practice. Bond purchase, on the other hand, yields the lower frequency of hitting ZLB, but the associated welfare is lower than private purchase. This suggests a trade-off that policy makers face when using multiple instruments.

Furthermore, we perform a simulation exercise using realized shocks that are backed out from the data. The result shows that welfare gain is significant when monetary policy uses multiple instruments. In addition, if Ramsey planner is allowed to use asset purchases more freely, the economy shows an escape from liquidity trap during the recent financial crisis. This finding supports the possibility of using asset purchases as regular instruments in normal times as a way to guard the economy away from liquidity trap.

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