Implied volatility smile dynamics in the presence of jumps

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Aim: Analyze the dynamics of the S&P500 index option volatility smile around price jumps in comparison to no-jump periods.

Three main contributions:

- Study of the dynamics of implied volatility smile with option intraday data.
- Analyze the dynamics of volatility smiles in presence of jumps in the underlying.
- Provide evidence that jumps in the underlying affect the dynamics of the smile.

Motivation and related works:

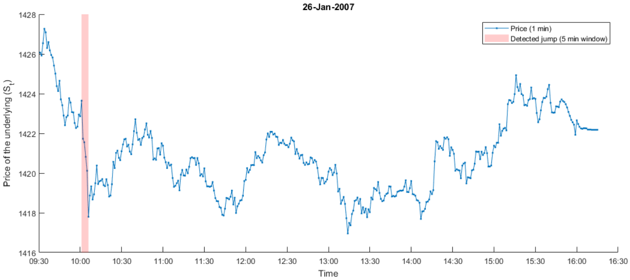
- Heynen [1994], Skiadopoulos et al. [2000], Cont and da Fonseca [2002], Fengler et al. [2003], Fengler [2006], Benko et al. [2009]
- Smile dynamics in the intra-day domain and in presence of jumps.
- Relevant for the literature on jump-diffusion option pricing models: empirical research exploring the independence between the price process and the and volatility diffusion.

Data

Data:

- Intraday SPX options for 1259 days (Jan-2006 to Dec-2010) provided by CBOE Livevol.
- Data contains the cross-sections of put and call prices and all the complementary option information (strike, maturity, rate, underlying price).
- Data is provided at *one minute* resolution, from 9:31:00 to 16:15:00 (405 records per trading day).
- By following the empirical option pricing and VIX literature, we focus on the out-of-money and at-the-money options (of most interest since traded the most and thus liquid).

Step I: Jump detection in the price



- Jump detection as in [Lee and Mykland, 2008].
- Detection windows of 5min (and 15min for robustness).
- About 85% of jumps occur in the first hour.
- We study the dynamics of smiles only in the *mornings* where jumps are detected (between 9:31:00 and 10:30:00).
- Two datasets: 290 days with jumps, 940 days without jumps (338 jumps in total).

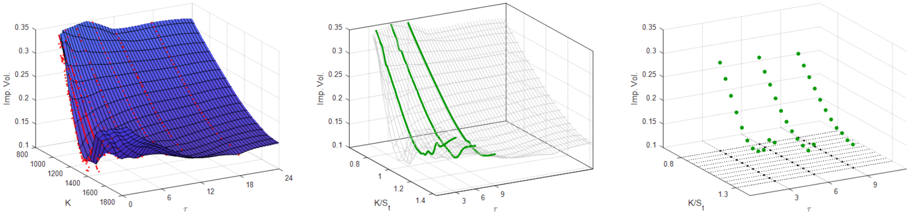
Step II: Implied volatility smile construction

- Get the implied volatility at time t:

$$\sigma_t(K,\tau) : P_t^{BS}\left(S_t, K, \tau, r_\tau, \sigma_t(K,\tau)\right) = P_t^{Mkt}(K,\tau)$$

$\sigma_t(\tau, K)$: spot volatility, S_t: underlying price, P_t^{BS}: Black-Sholes option price, K: strike, τ: time to maturity, r_τ: interest rate, $P_t^{Mkt}(K,\tau)$: market spot price.

- Fit of the implied volatility surface and smile sampling:



Step III: PCA

Complex multidimensional problem:

- 1230 (days) $\times$ 60 (minutes) $\times$ 3 (maturities) $=$ 221400 smiles.
- Each smile described by the IV sampled over 10 bins.

We apply PCA to conveniently characterized the surface with a limited number of variables [Skiadopoulos, 2001]:

- PCA on $(m_t, \Delta IV_t|\tau)$, where:

  $m_t = K/S_t$ is the moneyness

  ΔIV_t are the first differences in implied volatility ($\Delta IV_t = IV_t - IV_{t-1}$)

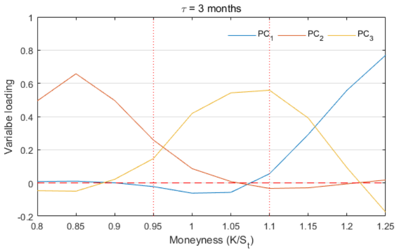
- $m_t \in [0.8, 0.85, 0.9, ..., 0.13]$, $\tau = 3, 6, 9$ months.

Step III: PCA (cont.)

- Three PCs capture most of the variability:

Maturity	PC_1 OTM Calls	PC_2 OTM Puts	PC_3 ATM	Total
3 months	65.3%	10.3%	9.4%	85.0%
6 months	59.8%	15.2%	11.8%	86.9%
9 months	54.7%	19.2%	12.7%	86.7%

- Components have a clear interpretation in terms of at-the-money and out-of-money calls and puts:



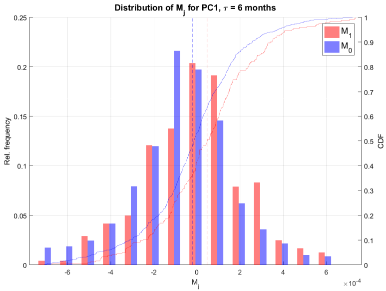
Dynamics of the volatility smile

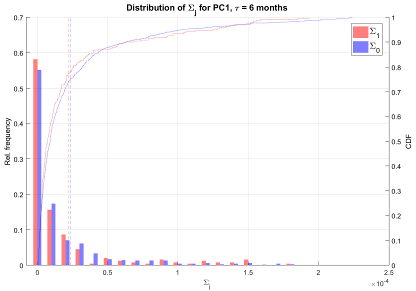
- For each component, we obtain two score matrices for the first 60 minutes, differentiating for jump and no-jumps mornings:

	minute$_1$	...	...	minute$_{60}$		M_j	Σ_j
Day 1	$s^j_{1,1}$	...	...	$s^j_{1,60}$	$\rightarrow$	μ_{1j}	ν_{1j}
...	...	...	...	...	$\rightarrow$	...	...
Day d	$s^j_{d,1}$	...	...	$s^j_{d,60}$	$\rightarrow$	μ_{dj}	ν_{dj}
...	...	...	...	...	$\rightarrow$	...	...
Day n_j	$s^j_{n_j,1}$	...	...	$s^j_{n_j,60}$	$\rightarrow$	$\mu_{n_j j}$	$\nu_{n_j j}$

- We extract the distribution of morning scores' means M_j and variances Σ_j for each component, when jumps occurred ($j = 1,\ n_1 = 290$) and not ($j = 0,\ n_0 = 940$).

Dynamics of the volatility smile (cont.)





i The Kolmogorov-Smirnov test is used to test for statistically significant differences in the distributions of M_0 and M_1, and in the distributions of Σ_0 and Σ_1.

ii The Welch-U test is used to compare the means of M_0 and M_1 and of Σ_0 and Σ_1.

Dynamics of the volatility smile (cont.)

We utilize six different hypotheses to address the dominance of the distributions and the direction of the inequalities and obtain their respective p-values.

- e.g. for the KS test:
  - $\mathbf{H_0^g}$: $\exists u : F_{M_1}(u) \leq F_{M_0}(u)$ (or simply $F_{M_1} \leq M_0$)
  - $\mathbf{H_1^g}$: $F_{M_1}(u) > F_{M_0}(u), \forall u$ (or simply $F_{M_1} > F_{M_0}$)
- e.g. for the Welch-U test:
  - $\mathbf{H_0^s}$: $\mu_{M_1} \geq \mu_{M_0}$
  - $\mathbf{H_1^s}$: $\mu_{M_1} < \mu_{M_0}$

Remind that the analyses are about the distributions of the *scores*. However, since the loadings of the PCs are largely positive we can easily re-interpret the results in terms of changes in implied volatility ΔIV.

Results

	Mean(ΔIV)		Var(ΔIV)	
	$\tau = 3$	$\tau = 6, 9$	$\tau = 3$	$\tau = 6, 9$
Out-of-money puts				
KS test	$+$	$+$	$+$	$+$
Welch-U test	$+$	$+$	$=$	$+$
At-the-money puts and calls				
KS test	$+$	$+$	$+$	$+$
Welch-U test	$+$	$+$	$=$	$+$
Out-of-money calls				
KS test	$+$	$+$	$=$	$=$
Welch-U test	$+$	$+$	$=$	$=$

Table: Effect of jumps on ΔIV. "+" indicates $F_{M_1} < F_{M_0}$ ($F_{\Sigma_1} < F_{\Sigma_0}$) or $\mu_{M_1} > \mu_{M_0}$ ($\mu_{\Sigma_1} > \mu_{\Sigma_0}$). "=" indicates no statistical evidence of differences in M_j (Σ_j) or μ_{M_j} (μ_{Σ_j}).

Conclusion

The maturity and moneyness -specific results depicted in the previous table suggest:

- A remarkable interconnection between jumps in the price of the underlying and implied volatility dynamics.
- That no matter what the price and maturity are, jumps impact the IV dynamics.
- A possible contradiction of the commonly used assumption about independence between jumps in underlying price and volatility diffusion.

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P-value tables

		PC1			PC2			PC3		
Maturity	Sign	H_0	H_0^s	H_0^g	H_0	H_0^s	H_0^g	H_0	H_0^s	H_0^g
Test for F_{M_j}										
3 months	pos	0.0816	0.0408	**0.9611**	0.0000	0.0000	**0.7926**	0.0001	0.0001	**0.7531**
6 months	pos	0.0004	0.0002	**1.0000**	0.0000	0.0000	**0.4358**	0.0000	0.0000	**0.8581**
9 months	pos	0.0000	0.0000	**0.8048**	0.0000	0.0000	**0.5213**	0.0000	0.0000	**0.6199**
Test for F_{Σ_j}										
3 months	pos	**0.7759**	0.7767	0.4181	0.0026	0.0013	**0.6939**	0.0740	0.0370	**0.8134**
6 months	pos	**0.3232**	0.8402	0.1623	0.0000	0.0000	**0.9967**	0.0003	0.0002	**0.8503**
9 months	pos	**0.9044**	0.7431	0.5254	0.0000	0.0000	**0.7555**	0.0000	0.0000	**0.9651**
Test for μ_{M_j}										
3 months	pos	0.0856	**0.9572**	0.0428	0.0009	**0.9995**	0.0005	0.0008	**0.9996**	0.0004
6 months	pos	0.0003	**0.9999**	0.0001	0.0008	**0.9996**	0.0004	0.0002	**0.9999**	0.0001
9 months	pos	0.0011	**0.9995**	0.0005	0.0007	**0.9997**	0.0003	0.0008	**0.9996**	0.0004
Test for μ_{Σ_j}										
3 months	pos	**0.7171**	0.3585	0.6415	**0.0937**	0.9532	0.0468	0.2687	**0.8657**	0.1314
6 months	pos	**0.2188**	0.1059	0.8941	0.0000	**1.0000**	0.0000	0.0000	**1.0000**	0.0000
9 months	pos	**0.6855**	0.3428	0.6572	0.0000	**1.0000**	0.0000	0.0000	**1.0000**	0.0000

Table: KS and Welch-U test p-values. Five minutes jump detection window.

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