What does the house price-to-income ratio tell us about the housing market affordability: A theory and international evidence *

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Abstract

While the house price-to-income ratio (PIR) is widely used as an indicator for affordability even housing market mispricing, formal analysis is relatively rare. This paper proposes a simple dynamic, stochastic general equilibrium (DSGE) model and shows that the PIR is correlated to the previous periods aggregate output. The model also predicts that the variance of PIR is positively correlated with the variance of output. We confirm these predictions and provide evidence for a long term, positive and significant relationship between PIR and output with cross-country data. Our tractable formulation of a stochastic money growth rule may carry independent research interest.

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1 Introduction

The importance of House Price-to-Income-Ratio (PIR), which is also called the House Price-to-Earning Ratio, can hardly be overstated. For instance, Demographia (2016) reports that "the Median Multiple (a house price-to-income ratio) is widely used for evaluating urban markets, and has been recommended by the World Bank and the United Nations and is used by the Joint Center for Housing Studies, Harvard University. Similar house price-to-income ratios... are used to compare affordability between markets by the Organization for Economic Cooperation and Development, the International Monetary Fund, international credit rating services, media outlets (such as *The Economist*) and others." Yet despite its importance, formal modeling of PIR is relatively rare.[1] This paper takes a preliminary step by constructing a simple dynamic, stochastic general equilibrium model (DSGE), where we can analytically link the PIR to the output growth. Since both variables are observable, we also bring this testable implication of the model to the data.

Clearly, this paper is also closely connected to the housing affordability literature. While "affordability" can carry different meanings in different contexts and hence can be measured differently,[2] PIR is one of the most commonly used measures of "affordability" (for instance, see National Housing Conference, 2012; National Housing Federation, 2012, among others). The reasons are clear. The data requirement for PIR is minimal and the ratio easy to calculate. Thus, PIR is often computed and compared across countries, or across regions or cities within the same country. In addition to the cross-sectional comparison, it is sometimes computed for a fixed region or country across different time periods, and used as an indicator of whether the housing affordability improves

[1] See Leung (2004), among others, for a review of the literature.
[2] See Hulchanski (1995), Quigley and Raphael (2004), among others.

(or deteriorates) over time. This paper attempts to complement the large literature in the following ways. First, we construct a simple DSGE model and derive the *equilibrium level of PIR* in the model. A merit of DSGE models is that both prices (e.g. house prices, wages) and quantities (e.g. physical capital stock, housing stock) are endogenous. Consequently, if agents are rational and forward-looking, they would adjust their consumption-saving decision, labor supply decision, as well as home purchase decision accordingly. Thus, the PIR is naturally tied to the movement of "economic fundamentals" at the equilibrium and to some extent, the dynamics of PIR become *predictable*. This may be important as there are recently concerns about "deterioration of housing affordability" (DHA) in different countries.[3] Thus, this study may be interpreted as an initial step towards building a theoretical framework which can enable us to assess issues such as DHA.[4]

Second, we show that the dynamics of PIR in the presence of a class of monetary policy is similar to the one without. While the flexibility of house price is generally accepted, the rigidity of wages is not. In fact, empirical studies confirm the existence of wage rigidity, which is consistent with the New Keynesian models.[5] A natural question to ask is, when the house price is flexible but the wage is not, would the dynamics of house price-to-wage ratio be different? We will therefore compare the case of sticky wages with the case of flexible wages. To keep the model tractable, we propose a formulation of the monetary policy, in which the money growth rate is no longer a constant, but instead a function of previous money growth rate and other macroeconomic variables.[6] It can be considered as an extension of the previous work on monetary policy, and hence

[3] Among others, see CBC News (2015), Moody (2015), RBC (2015).

[4] The word "deterioration" seems to suggest a comparison across different time periods. Hence, a dynamic model may be more appropriate for analyzing issues such as DHA.

[5] Clearly, it is beyond the scope of this paper to survey that literature. See Dickens et al. (2007), Bils et al. (2013), Barattieri et al. (2014), among others, for a review of the evidence.

[6] The previous literature tends to focus on the case of constant money growth rate, which is discussed by Friedman (1969). More discussion on this will be provided in a later section.

may carry an independent research interest.

Third, we confront our theoretical model with data. More specifically, we show a robust relationship between the PIR and real GDP in a dynamic panel data setting. We also perform a panel cointegration test across countries. Thus, it does not only provide an empirical validation, but also a suggestion for future research directions. In the past, housing affordability studies tend to focus on the cross-sectional difference. The dynamics of housing affordability, on the other hand, may be under-explored. This study complements the literature by considering the dynamics of PIR, which is a proxy of housing affordability, may therefore enrich the literature in that dimension. More specifically, we establish a coherent theoretical framework which is consistent with at least some stylized facts is crucial and find support from cross-country data. It may also suggest that a dynamic equilibrium perspective on housing policy could provide additional insights.

Clearly, this paper is related to a literature which focuses on the empirical determinants of housing prices. They include the efforts by Oikarinen (2009), Stadelmann (2010), Agnello and Schuknecht (2011), among others. While this paper is built on their insights, it has a very different focus. Rather than searching for the empirical determinants of housing price, which may include aggregate output, labor wage, monetary policy, etc., this paper attempts to relate the PIR and the output in a dynamic equilibrium model when all these variables are determined endogenously. We therefore consider this paper complements that literature as well.

The organization of this paper is simple. We first study a simple dynamic, stochastic general equilibrium (DSGE) model with flexible wage and examine how the house price-to-wage ratio will evolve in the model economy. We then compare the case with short-term sticky wages. We then confront the model predictions concerning the PIR and output dynamics with data. The last section concludes, with all proofs reserved in the appendix.

2 A Benchmark Model

This section will present a simple DSGE model. We will first provide an informal description, followed by the introduction of a mathematical model. Loosely speaking, the model developed here is a combination of Greenwood and Hercowitz (1991) and Benassy (1995) and hence a brief explanation will be sufficient.[7] Time is discrete and the horizon is infinite. The population is assumed to be constant to simplify the exposition and this assumption can be easily relaxed. There are several goods in the economy: the non-durable consumption goods C_t, the business (or physical) capital K_t, residential capital (or housing) H_t. The representative agent derives utility from the level of non-durable consumption, the amount of housing, and also the amount of labor hours L_t and the amount of real cash balance $\frac{M_t}{P_t}$.[8] The government print nominal money M_t according to certain money supply rule, which will be explained in more details later. Firms combine labor and business capital to produce output Y_t. All agents in the model economy maximize their utility or profit.

Our formal description of the model begins with the household side. The representative household in the model that maximizes the expected value of the discounted sum of utility:

$$\max . E_0 \sum_{t=0}^{\infty} \beta^t U \left(C_t, H_t + H_t^r, L_t, \frac{M_t}{P_t} \right), \tag{1}$$

where C_t is the amount of non-durable consumption, H_t is the amount of housing

[7] See also Leung (2007, 2014) for related studies.

[8] It can be easily justified as the transacton demand of money. There is a large literature on this and interested readers could consult Walsh (2010), among others, for more details.

stock, H_t^r is the amount of housing that are rented, L_t is the amount of labor hours (or efforts) devoted in goods production, $\frac{M_t}{P_t}$ is the real money balance, as M_t denotes the nominal money balance and P_t is the general price level. In this paper, a simple utility functional form is assumed,

$$U\left(C_t, H_t + H_t^r, L_t, \frac{M_t}{P_t}\right) = \ln C_t + \omega_1 \ln\left(H_t + H_t^r\right) + \omega_2 \ln\left(1 - L_t\right) + \omega_3 \ln \frac{M_t}{P_t}, \tag{2}$$

$\omega_i > 0$, $i = 1, 2, 3$, are parameters governing the relative importance of housing, leisure and money holding in the utility function. The maximization problem of the representative household is subject to several constraints. First, both the business capital K_t and housing stock H_t are durable and hence can only adjust gradually. The following equations captures this observation that the future amount of stock (whether the business capital or housing) depends positively, on the amount of current stock level as well as investment,

$$K_{t+1} = K_t^{1-\delta_k} I_{k,t}^{\delta_k}, \tag{3}$$

$$H_{t+1} = \left(H_t + H_t^m\right)^{1-\delta_h} I_{h,t}^{\delta_h}, \tag{4}$$

where δ_k, δ_h are the depreciation rates of business capital and housing stock, $0 \leq \delta_k, \delta_h \leq 1$, $I_{k,t}$, $I_{h,t}$ are the investment in business capital and housing. Second, this formulation also captures the idea that, holding fixed the amount of existing stock, the marginal rate of return of investment on future stock is diminishing, which can also be interpreted as a form of adjustment cost. Notice also that in (4), the amount of housing stock purchased form the secondary market, H_t^m, can also influence the amount of future housing stock. Thus, this formulation also captures the idea that housing stock can be accumulated through investment as well as direct purchase from the market. On top of the restrictions of (3) and (4), the household is also subject to the usual budget constraint,

$$R_tK_t + W_tL_t + \frac{\mu_t M_{t-1}}{P_t} \geq I_{k,t} + I_{h,t} + C_t + P_{ht}H_t^m + R_{ht}H_t^r + \frac{M_t}{P_t} \tag{5}$$

where R_t is real rental rate of capital, W_t is the real wage rate, P_{ht} is the real price of housing, R_{ht} is the real rental rate of housing, Following Benassy (1995), μ_t a stochastic multiplicative monetary shock.

Clearly, this dynamic optimization problem above can be solved using the Dynamic Programming method,

$$V\left(K_t, H_t, \frac{M_{t-1}}{P_t}\right) = \max U\left(C_t, H_t + H_t^r, L_t, \frac{M_t}{P_t}\right) + \beta E_t V\left(K_{t+1}, H_{t+1}, \frac{M_t}{P_{t+1}}\right),$$

subject to (3), (4), (5). The first order conditions are easy to derive and details are provided in the appendix.

The supply of goods in this economy is very simple. There is an aggregate production technology, which exhibits constant returns to scale in capital and labor,

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}, \tag{6}$$

where α is the capital share, $0 < \alpha < 1$, and A_t is the productivity shock. The logarithm of the productivity shock follows an AR (1) process,

$$a_t = \rho a_{t-1} + \varepsilon_{at}, \tag{7}$$

where $a_t = \ln A_t$, ρ measures the persistence of the productivity shock, $0 \leq \rho \leq 1$. To further simplify the exposition, we assume that $E(\varepsilon_{at}) = 0$, $Var(\varepsilon_{at}) =$

σ_a^2, which is a constant, and $E(\varepsilon_{at}\varepsilon_{as}) = 0$ whenever $s \neq t$. With competitive factor markets, the real rental rate and wage rate will be equalized to the corresponding marginal product,

$$R_t = \frac{\partial Y_t}{\partial K_t} = \alpha \frac{Y_t}{K_t}, \tag{8}$$

$$W_t = \frac{\partial Y_t}{\partial L_t} = (1-\alpha)\frac{Y_t}{L_t}. \tag{9}$$

And it is easy to see that the economic profit is zero in this model economy,

$$\Pi_t^d \equiv Y_t - R_t K_t - W_t L_t = 0. \tag{10}$$

Following Benassy (2002), we assume that the growth rate of the monetary stock is a random variable. We also assume that all the money printed by the government will be held by the household eventually. Mathematically, it means that

$$M_t = \mu_t M_{t-1}. \tag{11}$$

We will provide details on the monetary growth rate μ_t in a later section. At this point, we can take it as given.[9] Now, combining (6), (8), (9), (10) and (11), (5) can be simplified as:

$$Y_t = I_{k,t} + I_{h,t} + C_t. \tag{12}$$

To solve the model, we need to impose some market clearing conditions. Following Lucas (1978), there is no net trade among identical households, whether

[9] Since all prices are flexible in this section, monetary policy will be neutral. Among others, see Walsh (2010). In a following section with sticky wage, monetary policy will not be neutral and we will explicitly formulate the monetary policy.

in the sale market or rental market of housing. It follows that

$$H_t^m = H_t^r = 0. \tag{13}$$

For future reference, we use small letter to denote the natural log of capital letter variables. For instance, $c_t = \ln C_t$, $y_t = \ln Y_t$, $m_t = \ln M_t$, etc. The following proposition summarizes the equilibrium dynamics of the model (the proof can be found in the appendix).

Proposition 1 *With flexible prices and wages, the dynamic system depends on the joint dynamics of output and capital stock, which can be summarized by the following vector dynamics equation,*

$$\overrightarrow{y}_t = B_0 + B_1 \overrightarrow{y}_{t-1} + \overrightarrow{a}_t, \tag{14}$$

where B_0, B_1 are matrices of constant, the transpose of the vector $\overrightarrow{y}_t$ is (y_t, k_t), and $\overrightarrow{a}_t$ represents a vector of shock. In addition, we can show that $\overrightarrow{a}_t$ is serially correlated,

$$\overrightarrow{a}_t = \rho^j \overrightarrow{a}_{t-j} + \sum_{i=0}^{j-1} \rho^i \overrightarrow{\varepsilon}_{a,t-i}, \tag{15}$$

where $E\left(\overrightarrow{\varepsilon}'_{a,t-i}\right) = (0,\ 0)$, $E\left(\overrightarrow{\varepsilon}'_{a,t}\overrightarrow{\varepsilon}_{a,s}\right) = 0$ whenever $s \neq t$.[10]

In addition, we can also study the house price dynamics in this model.

Proposition 2 *In this model, the house price is positively correlated to the output and negatively to the housing stock. Formally, it is characterized by the following equation,*

$$p_{ht} = b^{ph} + y_t - h_t. \tag{16}$$

[10]Throughout this paper, we use $\overrightarrow{x}'$ to represent the transpose of the vector $\overrightarrow{x}$.

Notice that we have already derived the output level, the real wage and house price equation from the model. We can now derive a focus of this research, which is the real wage rate-to-house price ratio, which is often used as an "affordability index".[11]

Proposition 3 *The real wage-to-house price ratio (in log and in real terms) can be expressed as the follows,*

$$\ln\left(\frac{W_t}{P_{ht}}\right) = w_t - p_{ht} = b^{wp} + h_t, \tag{17}$$

where b^{wp} is a constant.

Notice that in log form, the usually used house price-to-income ratio (PIR) is simply $\ln\left(\frac{P_{ht}}{W_t}\right) = p_{ht} - w_t = -(w_t - p_{ht})$. Thus, our real wage-to-house price ratio will inform us directly about the widely discussed PIR. For mathematical convenience, we would proceed with the real wage-to-house price ratio.[12] Several observations are immediate from (17). First, even in a stationary environment, the real wage-house price is not a constant, but would vary according to the stock of housing. (17) also shows that if the stock of housing suddenly decreases (say, due to some unexpected natural disaster), the real wage-to-house price ratio will also decrease. The intuition is simple. If some housing stock is destroyed by some disasters, agents need to be re-allocated to existing shelters. However, housing stock cannot adjust soon enough to meet the demand, which results in

[11] The real wage rate-to-house price ratio measures how many hours (or any time units) of labor a household needs to give up to exchange for a unit of housing. Sometimes people would use the reciprocal of it, i.e. the house price-to-wage rate ratio.

[12] Some researchers argue that a more appropriate measure would be the wage income-to-house price ratio, i.e. $(W_t L_t / P_{ht})$, rather than he wage rate-to-house price ratio (W_t / P_{ht}). As we have shown in (31), the labor hours is constant at the flexible wage equilibrium. Hence, the two ratio would only differ by a fixed factor. We will revisit the difference between the two ratios when the wage is rigid in the short run.

an increase in the house price. Other things being equal, the real wage-to-house price ratio will drop.

In principle, one can estimate equations such as (14), (16), or (17) directly. Unfortunately, variables such as the business capital stock k_t and housing stock h_t are not available in some countries. Even when they are, they are typically measured infrequently and subject to larger measurement errors (relative to variables such as the GDP). Therefore, we need to derive other *testable implications* of the model. The following proposition takes a step towards this direction.

Proposition 4 *The real wage-to-house price ratio is related to lagged output level of the economy,*

$$w_t - p_{ht} = b^{wp\prime} + \delta_h \sum_{i=0}^{t-1} (1-\delta_h)^i \, y_{t-1-i} + (1-\delta_h)^{t-1} \, h_0, \tag{18}$$

where $b^{wp\prime}$ is a constant, h_0 is the amount of initial housing stock in the model economy.

Notice that as $t \longrightarrow \infty$, $(1-\delta_h)^{t-1} \, h_0 \longrightarrow 0$ as $0 < (1-\delta_h) < 1$. Thus, the importance of the initial stock of housing diminishes over time and the real wage-to-house price ratio will depend on the series of lagged output, $\{y_{t-1-i}\}$. In particular, an increase in the previous period real output will increase the real wage-to-house price in some subsequent period. The intuition is simple. Higher levels of previous periods real output will lead to both a higher demand of housing and a higher level of business capital, and the latter tends to lift up the wage. Given the assumptions made in this model economy, it happens that the wage effect dominates the housing demand effect, and he wage-to-house price ratio increases. We will further examine the robustness of this theoretical prediction in the following section, and then will confront the theory with data.

3 The case with short-term rigid wages

The previous section studies a model with perfectly flexible prices and wages, and hence the monetary policy is neutral. Recent evidence, however, suggest that nominal wages are sticky.[13] For instance, based on the micro-evidence from 1970s to early 2000s on twelve countries,[14] Dickens et al. (2007) conclude that the nominal wages are indeed sticky rather than flexible. Since our focus is on the house price-to-wage ratio, the differential flexibility between the house price and the wage may influence our conclusion. Therefore, we re-examine our results in a sticky wage environment in this section. To facilitate the comparison, the model economy we would consider is the same as the one in the previous section, except for the short-term nominal rigidity in wages. As shown in Walsh (2010), among others, monetary policy would affect the real economic activities if wages are sticky. Here our focus is how the equilibrium dynamics of PIR could interact with the monetary policy. To simplify the exposition, here we follow the formulation of Benassy (2002), which has been proved to be very tractable, in assuming that wages are *set one period in advance*. The nominal wage on the labor contract is pre-set at a level that *the labor market will clear ex ante*. More specifically, the contract wage is set equal to the expected value of the Walrasian wage ($E_{t-1}w_t^{n*}$), and thus the nominal wage becomes,

$$w_t^n = E_{t-1}w_t^{n*} = b^w + E_{t-1}m_t. \tag{19}$$

[13] The literature is too large to be reviewed here. Among others, see Bils et al. (2013).

[14] The twelve countries are Austria, Belgium, Denmark, Finland, France, Germany, Italy, Norway, Portugal, Sweden, Switzerland, and the United Kingdom.

We will provide more discussion on the details of expected value of money supply $E_{t-1}m_t$ later. We will proceed *as if* $E_{t-1}m_t$ is known to the agent. With short-term nominal wage rigidity, the real wage in the economy becomes,

$$w_t = w_t^n - p_t = E_{t-1}w_t^{n*} - p_t = b^w + E_{t-1}m_t - p_t. \tag{20}$$

After all the shocks realize, the firms will hire labor at the pre-committed wage. Profit maximization will imply that the factor returns (wage and capital rental rate) are still equalized to the corresponding marginal products, and hence (8), (9) are still valid. The following proposition dictates the equilibrium labor supply under this slight change in economic environment,

Proposition 5 *If (19) holds, the equilibrium labor supply under short-term wage rigidity depends on the "forecast error" in monetary supply,*

$$l_t = l + \varepsilon_{mt}, \tag{21}$$

where ε_{mt} *is the forecast error term in money supply at time* t*,*

$$\varepsilon_{mt} \equiv m_t - E_{t-1}m_t. \tag{22}$$

Furthermore, we can show that the joint dynamics of output and capital stock is summarized by the following vector dynamics equation,

$$\overrightarrow{y}_t = B_0 + B_1 \overrightarrow{y}_{t-1} + \overrightarrow{a_t^r}, \tag{23}$$

where B_0*,* B_1 *are matrices of constant, the transpose of the vector* $\overrightarrow{y}_t$ *is* (y_t, k_t)*, and* $\overrightarrow{a_t^r}$ *represents a vector of shock. And* $\overrightarrow{a_t^r}$ *is serially correlated.*

Notice that the form of (23) is identical to (14). The only difference is that the forecast error term in money supply at time t, ε_{mt}, is a part of the vector of shocks, $\overrightarrow{a_t^r}$. It is reasonable to expect that the forecast error term ε_{mt} depends on how the monetary policy is conducted. Thus, to fully understand the dynamics of the system, it is necessary for us to introduce the monetary policy formally.

4 Monetary policy

In the literature on monetary policy, a popular formulation of the monetary policy is to adopt a version of the Taylor rule (e.g., Walsh, 2010). While it has many merits, a drawback is that analytical solution is typically unavailable and the model would need to be solved numerically. To keep the model tractable, this paper proposes an alternative, which is to formulate the monetary policy as a money growth rule. In the literature, much have been discussed about the equivalence, of the lack of it, between the Taylor rule and a constant money growth rule, which was suggested by Friedman (1969).[15] On the other hand, a stochastic money growth policy rule (SMG) seems to be under-explored. Drawing lessons from the literature, our formulation of SMG is analogous to the Taylor rule. In particular, we assume that the growth rate of the nominal money supply μ_t depends on its own lag, the inflation rate P_t/P_{t-1}, and the output level Y_t, relative to the corresponding steady state values. Formally, it means that

$$\left(\frac{\mu_t}{\mu}\right) = \left(\frac{\mu_{t-1}}{\mu}\right)^{\rho^\mu} \left(\frac{P_t/P_{t-1}}{\pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y} e^{\varepsilon_{\mu t}}, \tag{24}$$

where μ, π, Y are the steady state level of $\mu_t, P_t/P_{t-1}, Y_t$ respectively, with ρ^μ, ϕ_π, ϕ_y being the policy parameters. For instance, if the inflation rate P_t/P_{t-1}

[15] For instance, see Carlstrom and Fuerst (1995), Schabert (2003), Auray and Feve (2008), among others. See also Nelson (2008) for a review.

deviates from the steady state inflation rate π, the money growth rate would respond and the parameter ϕ_π governs how much the money supply should react to the "excessive" inflation rate. Similarly, if the aggregate output level Y_t falls short of its steady state level Y, the money growth rate μ_t might increased relative to its steady state level μ, and the parameter ϕ_y captures how sensitive is the monetary growth rate to the deviation of the aggregate output from its steady state level. Clearly, if $\rho^\mu = \phi_\pi = \phi_y = 0$, the monetary growth rate is an i.i.d. process. Moreover, for simplicity, we assume that the "innovation term" of the monetary policy $\varepsilon_{\mu t}$ has a zero mean and a constant volatility, $E(\varepsilon_{\mu t}) = 0$, $Var(\varepsilon_{\mu t}) = \sigma_\mu^2$, which is a constant, and $E(\varepsilon_{\mu t}\varepsilon_{\mu s}) = 0$ whenever $s \neq t$. Furthermore, we assume that $\{\varepsilon_{at}\}$ and $\{\varepsilon_{\mu t}\}$ are independent. A merit of this formulation is that the model remains very tractable. Another merit of using SMG is that the gross monetary growth rate $\mu_t = M_t/M_{t-1}$ is by definition positive, and hence we do not worry about the zero lower bound (ZLB) of the nominal interest rate.[16] We prove the following lemma in the appendix.

Lemma 6 *Given (24) holds, the forecast error of money supply in (22), ε_{mt}, is shown to be an "weighted sum" of the forecast error of productivity, $(a_t - E_{t-1}a_t)$, and the monetary innovation term $\varepsilon_{\mu t}$,*

$$\begin{aligned} \varepsilon_{mt} &\equiv m_t - E_{t-1}m_t \\ &= \omega^a (a_t - E_{t-1}a_t) + \omega^\mu \varepsilon_{\mu t} \\ &= \omega^a \varepsilon_{at} + \omega^\mu \varepsilon_{\mu t}, \end{aligned} \tag{25}$$

where ω^a, ω^μ are constant.

[16] Recently, Hirose and Inoue (forthcoming) show that if we ignore the ZLB in an estimated DSGE model with the usual interest rate-rule-type monetary policy, the estimation of other parameters may be biased.

Equipped with these results, we can prove our main result, which related the real wage-to-house price ratio to the output dynamics and other random terms in this economy.

Proposition 7 *If (19) and (24) hold, the real wage-to-house price ratio (WPR) is related to lagged output level of the economy, as well as the forecast error of productivity shock* ε_{at}*, and that of money supply* $\varepsilon_{\mu t}$*,*

$$w_t - p_{ht} = b^{wp\prime} + \delta_h \sum_{i=0}^{t-1} (1-\delta_h)^i \, y_{t-1-i} + (1-\delta_h)^{t-1} h_0 - \widehat{\varepsilon}_t, \qquad (26)$$

where $b^{wp\prime}$ *is a constant,* h_0 *is the amount of initial housing stock in the model economy,* $\widehat{\varepsilon}_t$ *is a stochastic residual term.*

Similar to the flexible wage case, as $t \longrightarrow \infty$, $(1-\delta_h)^{t-1} h_0 \longrightarrow 0$ as $0 < (1-\delta_h) < 1$. In words, it means that the effect of the initial stock of housing on the real wage-to-house price ratio diminishes over time. And, under the monetary policy rule (24), the real wage-to-house price ratio will depend positively on lagged output, $\{y_{t-1-i}\}$, the current period "innovation term" in aggregate productivity and monetary policy, ε_{at} and $\varepsilon_{\mu t}$, as in the case of flexible wage.[17] An implication is that the PIR does not provide "extra information" about the housing market; it is merely a *"summary statistics"* of the aggregate output in previous periods. In principle, as both PIR and aggregate output are observable in the data, we will seek for empirical confirmation of (26). In practice, however, the right hand side of (26) contains the whole series of past output $\{y_t\}_{t=0}^{t-1}$, and it would pose a challenge in applied work. We therefore derive the following proposition from (26), which are much easier to implement in empirical works, and they provide a set of testable hypotheses that will be further discussed in the subsequent empirical section.

[17] In a sense, (18) and (26) are "observationally equivalent".

Proposition 8 *The real wage-to-house price ratio (WPR) can be characterized by the following equations:*

(a) The dynamics of the WPR is a "weighted average" of its lagged value and output level:

$$(w_{t+1} - p_{h,t+1}) = \widetilde{b^{wp\prime}} + \delta_h y_t + (1 - \delta_h)(w_t - p_{ht}) + [\widetilde{\varepsilon_{t+1}}], \tag{27}$$

where $\widetilde{b^{wp\prime}}$ is a constant term, $\widetilde{\varepsilon_{t+1}}$ is a stochastic term. It can be shown that $\widetilde{\varepsilon_{t+1}}$ is serially correlated, i.e. $E[\widetilde{\varepsilon_{t+1}}\widetilde{\varepsilon_t}] \neq 0$.

(b) The variance of the WPR is a "weighted sum" of the variance of output and some residual term,

$$var(w_t - p_{ht}) = \widetilde{\delta_t} var(y_t) + \widehat{\varepsilon_t}, \tag{28}$$

where $\widetilde{\delta_t} > 0$ depends on t and is non-stochastic.

Some researchers argue that a more appropriate measure would be the wage income-to-house price ratio, i.e. $(W_t L_t / P_{ht})$, rather than the wage rate-to-house price ratio (W_t/P_{ht}). The following corollary shows that the dynamics of the two ratios, in log form, are very similar.

Corollary 9 *In log form, the wage income-to-house price ratio is very similar to the wage rate-to-house price ratio,*

$$\begin{aligned} w_{t+1} + l_{t+1} - p_{h,t+1} &= b'' + \delta_h \sum_{i=0}^{t-1} (1-\delta_h)^i y_{t-1-i} + (1-\delta_h)^{t-1} h_0 \\ &= \widetilde{b''} + \delta_h y_t + (1-\delta_h)(w_t + l_t - p_{ht}). \end{aligned} \tag{29}$$

Thus, for practical purposes, the model proposed basically provide identical predictions on these ratios. Therefore, our empirical work, which will be presented in the next section, would depend on the data availability.

5 Empirical Evidence

The theoretical analysis have provided several testable implications and this section intends to verify them with international data. We begin this section with a description of our data set. Since the model presumes a well-functioned capital market, it may apply to more developed economies better. Moreover, time series data of house prices from developed countries are more accessible. Thus, our data on real GDP (in millions of US dollars) and wage index are collected from OECD.stat, while the housing price indices are obtained from Bank of International Settlements.[18] Altogether, there are 16 countries in our study, including Australia, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Spain, Sweden, UK, and US. Our data is in quarterly frequency and covers the period from 1997Q1 – 2013Q4. To be consistent, the seasonal component is removed from the time series.[19] This choice of such sampling period is a compromise of the maximization of the number of countries included in the study and the data requirement for some econometric tests.

For testable implications, we begin with (28). Notice that the stochastic residual term $\widehat{\varepsilon}_t$ is not directly observable. Thus, we can only say that, other things being equal, (28) predicts that the variance of wage-to-house price ratio and the variance of output are positively correlated. Since the original time series may not be stationary, we follow the business cycle literature and use the Hodrick-Prescott (HP) filter to extract the cyclical components for regression.[20]

[18] The data can be downloaded from https://www.bis.org/
[19] Among others, see Cooley (1995) for more discussion.
[20] See Hodrick and Prescott (1997) for more details.

Table 1 tests this prediction and confirms that the correlation is indeed positive and statistically significant at 5% level. Figure 1 provides a visualization. It means that if the output level of a country is volatile, its PIR also tends to be volatile. Thus, making policy decisions based on the PIR may not be easier than making policy decisions based on the output. It simply restates the message of (26) that the PIR is a "summary statistics" of previous periods output empirically.

(Figure 1, Table 1 about here)

Now, we turn to the relationship between PIR and output dynamics. First, we need to decide whether we will use (26) or (27) in our empirical investigation. While the two formulations are equivalent mathematically, it may be better to use (27). Notice that testing (26) directly would introduce many lags on the right hand, which makes the estimation difficult given the relatively short time series. On the other hand, equation (27), which express the wage-to-house price ratio as a weighted average of its own lag and lagged output level, require less lags and may provide more desirable results given the data availability.

Our second consideration is to choose an appropriate econometric methodology. We adopt the dynamic panel data approach (DPDA) and the reasons are clear. Panel data approach enable us to identify and measure effects that are simply not detectable in pure cross-section or pure time series data. It also allows us controlling for individual heterogeneity. As equation (27) contains a lagged dependent variable as explanatory variable, strict exogeneity of the regressors no longer holds (Hsiao, 2014). Hence, it will be proper to apply dynamic panel data model.[21] Notice also that equation (27) is directly derived

[21] Among others, see Arellano and Bond (1991), Baltagi (2013), Hsiao (2014) for more

from our simple DSGE model. Hence, our dynamic panel regression can be interpreted as a "structural estimation," although it may appear as a "reduced form estimation."[22] Table 2 shows that, the coefficients of lagged output and lagged dependent variable are found to be positive and significant. This confirms our theory proposed earlier that the increase in lagged real output will bring the wage having a faster growth than house price, resulting to an increase to wage-to-house price ratio (i.e. improvement in housing affordability). It is also worthy to note that DPDA requires instruments. Consistently, when we include different lag lengths of dependent variables as instruments, the probabilities of J-statistics are always between 0.3 and 0.4, and hence it is greater than 0.05, which suggest that they are all valid instruments.

(Table 2 about here)

Finally, we would like to examine a long-run relationship between log of wage-to-house price ratio and log of real GDP. The justification is clear. If there is indeed a long-run relationship between the wage-to-house price ratio and GDP, then it will be possible for us to detect "short-run deviations from the long-run relationship" in the data, and such deviations might be used one of proxies to measure whether the whole housing market "deviates" from its long-run situation.[23] To examine whether a long-run relationship exists between the wage-to-house price ratio and GDP (both in log), we proceed in two

discussion.

[22] In addition, to keep our model tractable, we intentionally abstract away from many aspects of the reality which may be important empirically. Institutional details vary significantly across countries, including whether public housing units and/or housing vouchers are provided, whether (and how much) mortgage payments are tax-deductable, what kind of mortgage contracts are available, etc. Clearly, it is beyond the scope of this paper to discuss these cross-country difference. Among others, see Green (2014), Malpezzi (1999b).

[23] Clearly, this is the idea behind the "error correction model." Among others, see Engle and Granger (1987), Malpezzi (1999).

steps. First, we check the stationarity of the series. As suggested by Cheng and Kwan (2000), Kwan (2007), performing panel unit root test is more powerful than individual time series version. In Table 3, all three panel unit root tests suggest that the two log series are indeed I(1). Second, we proceed to perform panel cointegration estimation. In our analysis, we adopt the group-mean fully-modified OLS method (MOLS) and group-mean dynamic OLS method (DOLS). DOLS involves augmenting the cointegrating regression with lags and leads of the change of variables so that the resulting cointegrating equation error term is *orthogonal* to the entire history of the stochastic regressor innovations.[24] FMOLS employs a *semi-parametric correction* to eliminate the problems caused by the long-run correlation between the cointegrating equation and stochastic regressors innovation.[25] Since both DOLS and FMOLS are widely used in the literature, we estimate both models and report the results in Table 4. The ADF statistic of Kao residual cointegration test is -3.26 (with a p-value of 0.0006), which rejects the null hypothesis of no cointegration. In order words, we have verified a positive and long-run relationship between PIR and the real GDP.

(Table 3, 4 about here)

6 Robustness Checks

In the previous section, we have provided empirical confirmations of our theoretical results. As our evidence comes from a panel dataset, a concern is the existence of "outlier(s)", which might have an impact of the results. Constrained

[24] DOLS is widely used in the literature. Among others, see Saikkonen (1992) and Stock and Watson (1993), Pedroni (2001) for more discussion and different variants of DOLS.

[25] MOLS can be interpreted as an extension of Phillips and Hensen (1990). Among others, see Pedroni (2000, 2001) for more discussion.

by the data availability,[26] we attempt to address this concern by perform the following tests. First, we re-run the dynamic panel regression, controlling for the size of government (which is captured by the ratio of government expenditure to GDP ratio). We also experiment with different sub-samples, including the G10,[27] European countries only, and a series of sub-samples, which excludes a country one-at-a-time. The results are presented in Table 5a. It is safe to conclude that the results from the dynamic panel regression are robust.

(Table 5a about here)

Another set of robustness checks is related to the panel cointegration. Again, we experiment with different sub-samples, in parallel to the previous robustness checks. The results are presented in Table 5b. The results from DOLS are fine. The results from MOLS are much stronger, highlighting the importance of semi-parametric correction. It might be that the true model involves nonlinearity that are not fully captured by our simple theoretical model, for instance, the existence of imperfect capital market and other government policies. These are important issues but we can only leave them for future research as they are beyond the scope of the current paper.

(Table 5a about here)

(To be added)

[26] As it is well known, many cross-country macroeconomic variables are in annual frequency, while our dataset is quarterly.

[27] The Group of Ten (G10) is made up of eleven industrial countries (Belgium, Canada, France, Germany, Italy, Japan, the Netherlands, Sweden, Switzerland, the United Kingdom and the United States) which consult and co-operate on economic, monetary and financial matters.

7 Concluding Remarks

The house price-to-income ratio (PIR) is widely used in the media as well as policy institutions an indicator of the property market. Yet formal modelling is disproportionately rare. This paper attempts to bridge the gap. First, it constructs a simple DSGE model and studies the endogenous dynamics of the house price-to-wage ratio. The model predicts that other things being equal, the volatility of the PIR should be positively correlated with the volatility of the real GDP. It also predicts that the PIR will be positively correlated to its own lag and previous period GDP. We confirm these predictions with the data of OECD countries. We further identify a long run relationship between PIR and GDP, suggesting that affordability can be "predicted" by the GDP. Our robustness checks also suggest the importance of semi-parametric correction, which means that some form of nonlinearity may exist in the data that our current model has yet to capture.

Clearly, the model can be extended in several ways. We can extend to an environment where some agents may be subject to the collateral constraints. We can also consider the possibility of accumulating inventory in a stick price environment. One can also consider models in which agents live in different cities or regions, or may have different income paths. In additions, agents may face informational frictions in both housing and labor markets. Obviously, one can also consider a richer set of government policies and compare their costs and benefits. Some of these possibilities are being pursued and would further enrich our understanding of housing affordability.[28]

[28] Among others, see Teo (2009), Leung and Teo (2011), Lubik and Teo (2012).

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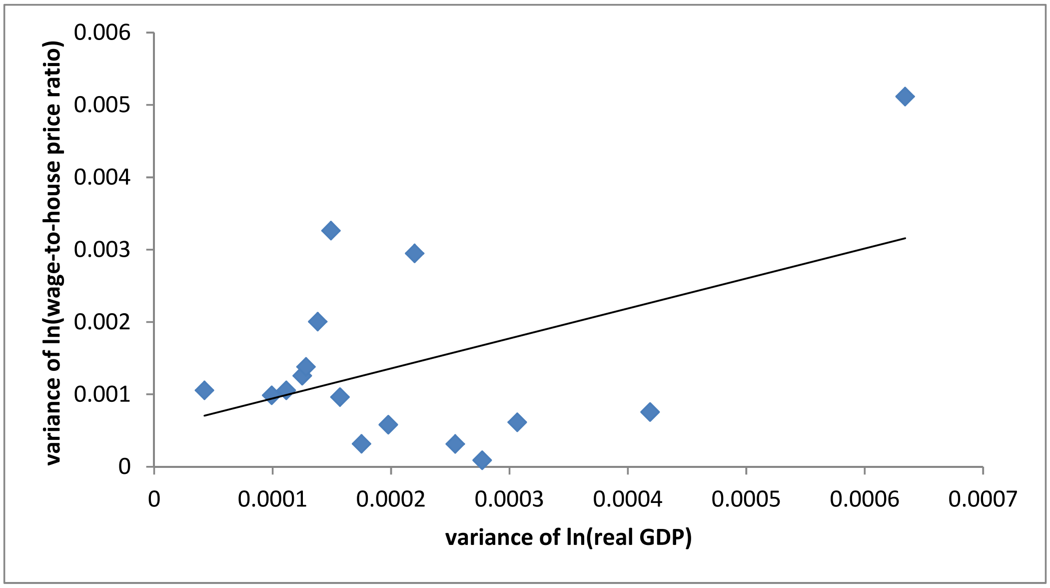
The appendix will be available online soon.

Figure 1 Scatter plot of Variance of PIR vs. Variance of real GDP



Note: Only the cyclical components are used. Sampling period is 1997Q1 – 2014Q4. The sample includes 16 countries: Australia, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Spain, Sweden, UK, and US.

Table 1 Estimation result of equation (35)

Dependent variable: $var(w_t - p_{ht})$

constant	0.0005
$var(y_t)$	4.1437 *
Adjusted R^2	0.15

* represent 10% significance level.

Note: Only the cyclical components are used. Sampling period is 1997Q1 – 2014Q4. The sample includes 16 countries: Australia, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Spain, Sweden, UK, and US.

Table 2 Dynamic Panel Data Regression result of equation (36)

Dependent variable: $w_{t+1} - p_{h,t+1}$

	Instruments – Lags of dependent variable included		
	Lags 2 to 3	Lags 2 to 4	Lags 2 to 5
$w_t - p_{ht}$	1.0049 ***	1.0113 ***	1.0077 ***
y_t	0.0692 ***	0.0825 ***	0.0751 ***
GMM weights	AB-n-step	AB-n-step	AB-n-step
J-statistics	14.32	16.69	14.98
Prob.(J-statistics)	0.43	0.34	0.45

***, ** represent 1% and 5% significance level respectively.

Table 3 Panel unit root tests

	$w_t - p_{ht}$	$\Delta(w_t - p_{ht})$	y_t	$\Delta(y_t)$
IPS W-statistic	1.530	-12.1842 ***	-0.042	-21.456 ***
ADF-Fisher Chi-Square	25.366	205.123 ***	27.531	331.011 ***
PP-Fisher Chi-square	11.746	249.487 ***	23.049	323.187 ***

Notes: H_0: Each country follows an individual unit root process. H_1: At least one country's process is trend stationary. Exogenous variables: individual effects, individual linear trends. ***denotes 1% significance level.

Table 4 Panel co-integration tests (between PIR and GDP)

Dependent variable: $w_t - p_{h,t}$

	Group-Mean Fully-modified OLS	Group-mean Dynamic OLS
y_t	0.0039 ***	0.0020 *

*** and * denote 1% and 10% significance level respectively.

Table 5a Dynamic panel data regression (Robustness check)

Dependent variable: $w_{t+1} - p_{h,t+1}$

		Instruments – Lags of dependent variable included			
		Lags 2 to 3			
		$w_t - p_{ht}$	y_t	$gov_{t/}$	Prob. (J-stat)
Full sample		0.9871 ***	0.0337 **	0.1153 ***	0.40
Members of G10 only		0.9862 ***	0.0349 **	0.0401 ***	0.47
European countries only		1.0044 ***	0.0975 ***	0.1260 ***	0.47
All countries except	Australia	0.9961 ***	0.0558 ***	0.0972 ***	0.40
	Canada	1.0448 ***	0.1292 ***	0.0070	0.04
	Denmark	0.9910 ***	0.0355 ***	0.0987 ***	0.43
	Finland	1.0009 ***	0.0564 **	0.0802 **	0.59
	France	1.0052 ***	0.0695 ***	0.1000 ***	0.48
	Germany	1.0050 ***	0.0601 ***	0.0810 **	0.49
	Ireland	0.9995 ***	0.0419 ***	0.1228 ***	0.03
	Italy	0.9928 ***	0.0432 ***	0.1407 ***	0.32
	Japan	0.9985 ***	0.0430	0.1225 ***	0.29
	Netherlands	0.9803 ***	-0.0150	0.1658 ***	0.35
	New Zealand	0.9978 ***	0.0472 ***	-0.0293	0.01
	Norway	0.9916 ***	0.0431 **	0.1325 ***	0.31
	Spain	1.0042 ***	0.0677 ***	0.1016 ***	0.43
	Sweden	0.9972 ***	0.0591 ***	0.0935 ***	0.53
	UK	1.0049 ***	0.0745 ***	0.0833 ***	0.60
	US	1.0099 ***	0.0691 ***	0.0453	0.63

Note: *** and ** represents 1% and 5% significance level respectively.

gov = ln (government expenditure / GDP)

Table 5b Panel co-integration test between PIR and GDP (Robustness check)

		Kao residual cointegration test	Fully modified OLS	Dynamic OLS
Full sample		-3.26 ***	0.0039 ***	0.0020 *
Members of G10 only		-2.02 **	0.0032 ***	0.0017
European countries only		-2.02 **	0.0039 ***	0.0029 ***
All countries except	Australia	-3.23 ***	0.0032 ***	0.0019 *
	Canada	-3.48 ***	0.0032 ***	0.0016
	Denmark	-3.06 ***	0.0040 ***	0.0020 *
	Finland	-3.17 ***	0.0038 ***	0.0018
	France	-3.10 ***	0.0036 ***	0.0018
	Germany	-3.21 ***	0.0043 ***	0.0023 **
	Ireland	-2.83 ***	0.0047 ***	0.0028 ***
	Italy	-3.18 ***	0.0038 ***	0.0019
	Japan	-3.14 ***	0.0047 ***	0.0027 **
	Netherlands	-3.17 ***	0.0039 ***	0.0020 *
	New Zealand	-2.99 ***	0.0037 ***	0.0018 *
	Norway	-3.33 ***	0.0037 ***	0.0017
	Spain	-3.16 ***	0.0035 ***	0.0016
	Sweden	-3.30 ***	0.0032 ***	0.0013
	UK	-3.26 ***	0.0037 ***	0.0019 *
	US	-2.91 ***	0.0045 ***	0.0026 **