Measurement of Common Risk Factors: A Panel Quantile Regression Model for Returns*

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Abstract

In this paper, we propose to measure the common risk factors using newly proposed Panel Quantile Regression Model for Returns. By exploring the fact that volatility crosses all quantiles of the return distribution, and employing penalized fixed effects estimator, we control for otherwise unobserved heterogeneity among financial assets and we focus on the commonalities in the quantiles of the returns in a selected portfolio. In the empirical application it is shown that our model perform significantly better than several benchmark models. Results of our research are important for correct identification of the sources of systemic risk, and will be particularly attractive for high dimensional applications.

JEL Classification: C14, C23, G17, G32
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1 Introduction

Proper risk identification is necessary for many financial applications including portfolio and risk management. The 2007-2009 global financial crisis serves as a prominent example that highlights this necessity. Nowadays with increased availability of ultra-high-frequency data we are able to obtain precise estimates of ex-post volatility, e.g. Realized Volatility (Andersen et al., 2003), that can be used for volatility modelling and forecasting. Although, the volatility forecasting is essential for many financial applications, it does not help us to specify conditional distribution of the future returns, which is crucial for proper risk evaluation. Conditional Autoregressive Value-at-Risk model of Engle and Manganelli (2004) is one of the few examples that focuses on the estimation of quantiles of various asset returns. A very different approach is explored by Žikeš and Baruník (2016) who combine the quantile regression framework with realized volatility resulting in simple to implement and flexible modeling strategy. In their work it has been shown that various realized measures are useful in forecasting quantiles of future returns without making assumptions about underlying conditional distributions.

While Žikeš and Baruník (2016) provided an important link between future quantiles of return distribution and its past variation, they worked with univariate time series. As the financial sector is highly connected and the co-movements in asset prices are common, there is a need for proper identification of dependencies in joint distributions. In the classical mean-regression framework, Bollerslev et al. (2016) showed that realized volatility of financial time series shares many commonalities. To the best of our knowledge there is no study dealing with estimates of conditional distribution of return series in a multivariate setting that explores ex-post information in volatility.

In this paper, we introduce Panel Quantile Regression Model for Returns that is able to control for unobserved heterogeneity among financial assets and allows us to exploit common factors in volatility series that directly affect future quantiles of return series. Moreover, using penalized fixed effect estimator we are able to disentangle overall market risk into systematic and idiosyncratic parts.

To evaluate performance of the model presented in the paper we conduct forecasting exercise in which absolute and relative performance are studied. For the absolute performance evaluation we use CAViaR test of Berkowitz et al. (2011) and our results suggest that the newly proposed model is dynamically correctly specified. To assess relative performance we employ standard Diebold-Mariano test for pair-wise comparison of expected loss functions from Value-at-Risk application as discussed in Clements et al. (2008). Relative performance is tested against 3 benchmark specifications - RiskMetrics (Longerstaey and Spencer, 1996) and 2 versions of Univariate Quantile Regression Model for Returns (Žikeš and Baruník, 2016). Overall we find that none of the benchmark specifications dominates our model. On the contrary, Panel Quantile Regression Model for Returns dominates RiskMetrics in all studied quantiles and Univariate Quantile Regression Model for Returns in economically important (5%,10% or 95%) quantiles.

The rest of the paper is structured as follows. Section 2 describes theoretical foundations of the methods applied in the paper. Section 3 presents our Panel Quantile Regression Model for Returns and Section 4 describes the dataset. In Section 5 evaluation framework together with benchmark models are presented. The empirical results are discussed in Section 6 and Section 7 concludes.

2 Theoretical background

In this section, we describe construction of realized measures that are used throughout the paper and theoretical foundations of panel quantile regression.

2.1 Realized Measures

We assume that the efficient logarithmic price process p_t evolves over time $0 \leq t \leq T$ according to the following dynamics

$$dp_t = \mu_t dt + \sigma_t dW_t + dJ_t, \tag{1}$$

where μ_t is a predictable component, σ_t is cadlag process, W_t is a standard Brownian motion, and J_t is a jump process.

The volatility of the logarithmic price process can be measured by quadratic return variation which can be decomposed into integrated variance(IV) of the price process and the jump variation(JV):

$$QV_t = \underbrace{\int_{t-1}^{t} \sigma_s^2 ds}_{IV_t} + \underbrace{\sum_{j=1}^{N_t} \kappa_{t,j}^2}_{JV_t}, \tag{2}$$

where N_t is total number of jumps during day t and $\sum_{j=1}^{N_t} \kappa_{t,j}^2$ represents magnitude of the jumps. As shown by Andersen et al. (2003) Realized Variance estimator can be simply constructed by squaring intraday returns:

$$\widehat{RV}_t = \sum_{k=1}^{N} \left(\Delta_k p_t\right)^2, \tag{3}$$

where $\Delta_k p_t = p_{t-1+\tau_k/N} - p_{t-1+\tau_{k-1}/N}$ is a discretely sampled vector of k-th intraday log-returns in $[t-1, t]$, with N intraday observations. Realized Variance estimator moreover converges uniformly in probability to QV_t as the sampling frequency goes to infinity

$$\widehat{RV}_t \xrightarrow[N\to\infty]{p} \int_{t-1}^{t} \sigma_s^2 ds + \sum_{j=1}^{N_t} \kappa_{t,j}^2$$

Building on the concept of Realized Variance Barndorff-Nielsen and Shephard (2004b) and Barndorff-Nielsen and Shephard (2006) introduced bipower variation estimator that is robust to jumps and thus able to consistently estimate IV_t. Furthermore, Andersen et al. (2011) adjust original estimator, which helps render it robust to certain type of microstructure noise:

$$\widehat{IV}_t^{BPV} = \mu_1^{-2} \left(\frac{N}{N-2}\right) \sum_{k=3}^{N} |\Delta_{k-2} p_t| |\Delta_k p_t|,$$

where $\mu_\alpha = E(|Z^\alpha|)$, and $Z \sim N(0,1)$. Having estimator of IV_t in hand jump variation can be consistently estimated[1] as a difference between Realized Variance and the bipower variation:

$$\widehat{RV}_t - \widehat{IV}_t^{BPV} \xrightarrow[N\to\infty]{p} \sum_{j=1}^{N_t} \kappa_{t,j}^2.$$

[1] Asymptotic behaviour and further details of the estimator can be found in Barndorff-Nielsen and Shephard (2006).

For many financial applications not only magnitude of the variation but also its sign is important. Therefore Barndorff-Nielsen et al. (2010) introduce innovative approach for measuring negative and positive variation in data called Realized Semivariance. They showed that Realized Variance can be decomposed to realized downside semivariance (RS^+) and realized upside semivariance (RS^-):

$$RV = RS^+ + RS^-,$$

where RS^+ and RS^- are defined as follows,

$$\widehat{RS}_t^+ = \sum_{k=1}^{N} (\Delta_k p_t)^2 I(\Delta_k p_t > 0) \xrightarrow{p} \frac{1}{2} IV_t + \sum_{j=1}^{N_t} \kappa_{t,j}^2 I(\kappa_{t,j} > 0) \tag{4}$$

$$\widehat{RS}_t^- = \sum_{k=1}^{N} (\Delta_k p_t)^2 I(\Delta_k p_t < 0) \xrightarrow{p} \frac{1}{2} IV_t + \sum_{j=1}^{N_t} \kappa_{t,j}^2 I(\kappa_{t,j} < 0). \tag{5}$$

Having briefly described realized measures used in our work we now turn to description of newly proposed panel quantile regression models for returns.

2.2 Panel Quantile Regression

Let us consider model

$$y_{it} = x_{it}^\top \beta + \alpha_i + u_{it}; \quad i = 1, ..., n; \quad t = 1, ..., T. \tag{6}$$

Quantile specific parameter estimates of 6 can be obtained by minimizing quantile function

$$Q_{y_{it}}(\tau | x_{it}) = \alpha_i + x_{it}^\top \beta(\tau), \quad \tau \in (0, 1). \tag{7}$$

Roger Koenker in his seminal work *Quantile Regression for Longitudinal Data* propose to use penalized fixed effects approach and solve

$$\min_{\alpha, \beta(\tau)} \sum_{d=1}^{q} \sum_{i=1}^{n} \sum_{t=1}^{T} \omega_d \rho_{\tau_d} \left(y_{it} - \alpha_i - x_{it}^\top \beta(\tau_d) \right) + \lambda \sum_{i=1}^{n} |\alpha_i|, \tag{8}$$

where $\rho_\tau(u) = u(\tau - I(u(< 0)))$ is the quantile loss function (Koenker and Bassett Jr, 1978), the ω_d are weights controlling relative effect of the q quantiles $\tau_1, ..., \tau_q$ on the estimation of α_i and $\sum_{i=1}^{n} |\alpha_i|$ is l_1 penalty that controls variability introduced by the large number of estimated parameters. Moreover, in this set-up individual fixed effects are consider to have pure location shift effect and are quantile independent.

Increased availability of panel data led to an intensive research of panel quantile methods. Nowadays, there are several works that extend the original model of Koenker (2004). Among other let us mention work of Lamarche (2010) who studied penalized quantile regression estimator, Galvao (2011) where fixed effects model with dynamic panels is introduced, Galvao and Montes-Rojas (2010) where it is shown that bias in dynamic panels can be reduced using penalty term for estimation, work of Canay (2011) who introduced simple two-step approach to estimation of panel quantile regression and showed consistency and asymptotic normality of the proposed estimator, or application of instrumental variables to quantile regression estimation (Harding and Lamarche, 2009). Other influential work developing theory of panel quantile methods are Galvao and Montes-Rojas (2015), Galvao and Wang (2015), Galvao and Kato (2015), Graham et al. (2015), Harding and Lamarche (2014) or Kato et al. (2012).

Panel quantile methods are attractive not only for researchers that further develop the theory but also for researchers concentrating on applied work. Areas of economics where the most active research is conducted are labour economics (Bargain and Kwenda (2011), Bartelsman et al. (2015), Billger and Lamarche (2015), Binder and Coad (2015),Cingano et al. (2015), Dahl et al. (2013), Kelchtermans and Veugelers (2011), Khandker et al. (2009), Matano and Naticchioni (2011), Nguyen et al. (2013), Toomet (2011)), banking and economic policy analysis (Covas et al. (2014), De Silva et al. (2009), Inanoglu et al. (2016), Klomp and De Haan (2012), Lin and Chu (2013)), economics of education (Arulampalam et al. (2012), Edwards (2012), Lamarche (2008), Lamarche (2011)), energy and environmental economics(Damette and Delacote (2012), Eaton (2013), You et al. (2015), Zhang et al. (2015)), international trade(Powell and Wagner (2014), Dufrenot et al. (2010), Foster-McGregor et al. (2014)), or real estate economics (Lee et al. (2012)). Panel quantile regression is also a useful tool for research conducted in the area of health economics and medicine (Abrevaya and Dahl (2008), Kim et al. (2007), Kniesner et al. (2010), Mayr et al. (2012), Ogasawara and Kobayashi (2015), Rithalia et al. (2009)).

Motivated by the usefulness of the panel quantile regression and the work of Žikeš and Baruník (2016) we propose to model quantiles of several return series using penalized fixed effects approach as originally proposed by Koenker (2004). To the best of our knowledge this is the first application of the panel quantile regression on the stock market data.

3 Panel Quantile Regression Model for Returns

We propose to model conditional quantiles of future returns as a linear function of various components of quadratic variation,

$$Q_{r_{i,t+1}}(\tau|v_{i,t}) = \alpha_i(\tau) + v_{i,t}^\top \beta(\tau), \tag{9}$$

where

$$r_{i,t+1} = p_{i,t+1} - p_{i,t}$$

and

$$v_{i,t} = \left(\widehat{QV}_{i,t}^{1/2}, \widehat{QV}_{i,t-1}^{1/2}, ..., \widehat{IV}_{i,t}^{1/2}, \widehat{IV}_{i,t-1}^{1/2}, ..., \widehat{JV}_{i,t}^{1/2}, \widehat{JV}_{i,t-1}^{1/2}, ... \right).$$

Following Koenker (2004), we solve following problem

$$\min_{\alpha(\tau),\beta(\tau)} \sum_{t=1}^{n} \sum_{i=1}^{t_i} \rho_\tau(r_{i,t+1} - \alpha_i(\tau) - v_{i,t}^\top \beta(\tau)) + \lambda \sum_{i=1}^{n} |\alpha_i(\tau)|, \tag{10}$$

however, we estiamete τ-dependent, individual distributional effects i.e. $\alpha_i(\tau)$. To the best of our knowledge all of the theoretical as well as applied work consider fixed effects α_i as a pure location shifts due to the problem of small time dimension T,[2] however majority of financial assets datasets consist of thousands of observations. In contrast to Koenker (2004) we are also not optimizing through several quantiles simultaneously rather each quantile is estimated separately.[3]

Penalty parameter, λ, is in our set-up chosen arbitrarily, $\lambda = 1$. We have tried $\lambda = 1$ as in Koenker (2004), Bache et al. (2008), Matano and Naticchioni (2011), Lee et al. (2012) and

[2]As detailed in Koenker (2004) it is not advisable to estimate τ-specific α_i in problems with small/medium T.

[3]For many financial applications we are interested only in specific quantiles e.g. 95% or 99% Value-at-Risk.

You et al. (2015), different values of λ from range $\langle 0.1; 1\rangle$ as in Damette and Delacote (2012) and Covas et al. (2014) and pure fixed effect model when $\lambda = 0$. We have found that choice of λ does not affect precision of β estimates. We address this finding to the structure and characteristics of the dataset (high time dimension T compared to low cross-section dimension N). Theoretical approach to λ selection can be found in Lamarche (2010) and Galvao and Montes-Rojas (2010).

4 Data

Empirical application of the proposed methodology is carried out using 29 U.S. stocks that are traded at New York Stock Exchange. These stocks have been chosen according to market capitalization and their liquidity. Final list of stocks is following: [4]

- Apple Inc. (AAPL), Amazon.com, Inc. (AMZN), Bank of America Corp (BAC), Comcast Corporation (CMCSA), Cisco Systems, Inc. (CSCO), Chevron Corporation (CVX), Citigroup Inc. (C), Walt Disney Co (DIS), General Electric Company (GE), Home Depot Inc. (HD), International Business Machines Corp. (IBM), Intel Corporation (INTC), Johnson & Johnson (JNJ), JPMorgan Chase & Co. (JPM), The Coca-Cola Co (KO), McDonald's Corporation (MCD), Merck & Co., Inc. (MRK),Microsoft Corporation (MSFT), Oracle Corporation (ORCL), PepsiCo, Inc. (PEP), Pfizer Inc. (PFE), Procter & Gamble Co (PG), QUALCOMM, Inc. (QCOM), Schlumberger Limited. (SLB), AT&T Inc. (T), Verizon Communications Inc. (VZ), Wells Fargo & Co (WFC), Wal-Mart Stores, Inc. (WMT), Exxon Mobil Corporation (XOM).

Sample we study spans from July 1, 2005 to December 31, 2015 and we consider trades executed within U.S. business hours (9:30 – 16:00 EST). In order to ensure sufficient liquidity and eliminate possible bias we explicitly exclude weekends and bank holidays (Christmas, New Year's Day, Thanksgiving Day, Independence Day). In total, our final dataset consists of 2613 trading days.

For estimation and forecasting purposes we use rolling window estimation with fixed length of 1000 observation [5], hence our model is always calibrated on a 4 years history. Our analysis is restricted to 5 minutes intraday log-returns that are used for computation of the daily returns and realized measures.

5 Benchmark Models & Forecast Evaluation

We evaluate absolute and relative performance of the newly proposed methodology and several benchmark models in the Portfolio Value–at–Risk forecasting exercise.

5.1 Forecast Evaluation

For absolute performance evaluation we use modified version of the Dynamic Quantile test (Engle and Manganelli, 2004), the CAViaR test of Berkowitz et al. (2011) In their work, Berkowitz et al. (2011) define "hit" variable in a way that

$$hit_{t+1} = \begin{cases} 1 & if \quad r_{t+1} \leq Q_{r_{t+1}}(\tau) \\ 0 & otherwise \end{cases}$$

[4] Stocks are sorted alphabetically according to their ticker.

[5] We have tried different length of rolling window with the qualitative results of our analysis remaining unchanged. These results are available from authors upon request.

i.e. hit_{t+1} is a binary variable taking values 1 if conditional quantile is violated and 0 otherwise. Hit series of a dynamically correctly specified series should be i.i.d Bernoulli distributed with parameter τ

$$hit_{t+1} \sim iid(\tau, \tau(1-\tau))$$

By construction, hit is a binary variable, therefore Berkowitz et al. (2011) propose to test the hypothesis of correct dynamic specification using following logistic regression

$$hit_t = c + \sum_{l=1}^{n} \beta_{1l} hit_{t-l} + \sum_{l=1}^{n} \beta_{2l} Q_{r_{t-l+1}}(\tau) + u_t$$

where u_t is assumed to have logistic distribution. We use likelihood ratio test to verify null hypothesis that β‘s are equal to zero and $\mathbb{P}(hit_t = 1) = \dfrac{e^c}{1+e^c} = \tau$. Exact finite sample critical values for the likelihood ratio test are obtained from Monte Carlo simulation as suggested by Berkowitz et al. (2011).

Relative performance of benchmark models is tested using expected tick loss for pairwise model comparison. Loss function is defined as

$$\mathcal{L}_{\tau,m} = E\left(\left(\tau - I\left(e^m_{t+1} < 0\right) e^m_{t+1}\right)\right),$$

where $I(\cdot)$ is indicator function, $e^m_{t+1} = r_{t+1} - Q^m_{r_{t+1}}(\tau)$ and $Q^m_{r_{t+1}}(\tau)$ is the m‘th model quantile forecast. Forecasting accuracy of two models is assessed using Diebold-Mariano test. Null hypothesis of the test that expected losses of two models are equal i.e. $H_0 : \mathcal{L}_{\tau,1} = \mathcal{L}_{\tau,2}$ is tested against general alternative.

5.2 Benchmark models

Benchmark models in our work includes popular and widely used RiskMetrics model that can be seen as the industry standard for the risk evaluation in high-dimensional problems and two applications of the Univariate Quantile Regression Model for Returns.

RiskMetrics

Based on Exponentially Weighted Moving Average, J.P. Morgan Chase in 1996 introduced new methodology for accessing the financial risk called RiskMetrics. For our benchmark purposes, we adopt the specification in its original form as defined in Longerstaey and Spencer (1996) with decay factor, λ set to 0.94. We assume a $q \times 1$ vector of daily returns $r_t = \sum_{k=1}^{n} (\Delta_k p_t)$ for $t = 1, ..., T$ such that $r_t \sim N\left(\mu_t, \sigma_t^2\right)$, where μ_t is conditional mean and σ_t^2 is conditional variance of daily returns. We also assume that $\mu_t = 0$ and therefore conditional covariance has the form

$$\sigma_{i,j,t} = \lambda \sigma_{i,j,t-1} + (1-\lambda) r_{i,t-1} r_{j,t-1},$$

where $\sigma_{i,j,t}$ denotes covariance between assets i and j at time t.

Univariate Quantile Regression Model for Returns

Žikeš and Baruník (2016) proposed to model quantiles of return series according to:

$$Q_{r_{t+1}}(\tau|v_t) = \alpha(\tau) + v_t^\top \beta(\tau), \tag{11}$$

where $r_{t+1} = p_{t+1} - p_t$ are return series of individual assets and

$$v_t = \left(\widehat{QV}_t^{1/2}, \widehat{QV}_{t-1}^{1/2}, ..., \widehat{IV}_t^{1/2}, \widehat{IV}_{t-1}^{1/2}, ..., \widehat{JV}_t^{1/2}, \widehat{JV}_{t-1}^{1/2}, ... \right)$$

are components of quadratic variation. Equation 11 is solved by minimizing following objective function:

$$\min_{\alpha(\tau),\beta(\tau)} \frac{1}{n} \sum_{t=1}^{n} \rho_\tau \left(r_{t+1} - \alpha(\tau) - v_t^\top \beta(\tau) \right), \tag{12}$$

where $\rho_\tau(u) = u(\tau - I(u < 0)))$ is the quantile loss function defined in Koenker and Bassett Jr (1978),

5.3 Portfolio Value–at–Risk

In general, percentage Value–at–Risk ($\%VaR$) of the given portfolio can be calculated as a

$$\%VaR_P = \sqrt{\sum_{i=1}^{N} (w_i \%VaR_i)^2 + 2 \sum_{i=1}^{N} \sum_{j=i+1}^{N} w_i w_j \%VaR_i \%VaR_j \sigma_{i,j}}$$

where $\%VaR_P$ denotes percentage portfolio VaR,w_i is the weight of asset i, $\%VaR_i$ is the percentage VaR of the i^{th} asset and $\sigma_{i,j}$ represents correlation between asset i and j. Previous equation can be rewritten into matrix form, as

$$\%VaR_P = \sqrt{(w^\top \odot \%VaR^\top) * \Omega * (w \odot \%VaR)} \tag{13}$$

where w is a vector of asset weights, $\%VaR$ is a vector of individual percentage VaR estimates, Ω stands for correlation matrix and $\odot$ is the Hadamar product.

In our setup we study Portfolio Value-at-Risk of the equally weighted portfolio. Concentrating on the equally weighted portfolio, we do not need to specify complicated weighting scheme. In total we compare 4 specifications:

- RiskMetrics,
- Panel Quantile Regression(PQR) Model for Returns,
- Univariate Quantile Regression(UQR) Model for Returns,
- portfolio version of Univariate Quantile Regression(Portfolio UQR) Model for Returns.

For calculation of Portfolio VaR using *RiskMetrics* approach equation 13 simplifies into

$$\%VaR_P = \sqrt{\gamma_\tau^2 * w^\top * \Sigma * w} \tag{14}$$

where Σ is covariance matrix obtained from RiskMetrics and γ_τ is a cut–off point of standard normal distribution at a given quantile τ.

In case of *PQR* and *UQR* forecasts of quantiles of return series are considered to be semi-parametric percentage VaR. Correlation matrix Ω is obtained from Realized Covariance matrix estimate (Barndorff-Nielsen and Shephard, 2004a), Σ, as

$$\Omega = (diag(\Sigma))^{-1/2} * \Sigma * (diag(\Sigma))^{-1/2}$$

and therefore equation 13 can be used for VaR calculation.

In contrast to previous approaches, *Portfolio UQR* is calculated in a different fashion. We firstly create portfolio returns and portfolio volatility series using individual returns and correlation structure obtained from Realized Covariance matrix, Σ, as

$$r_{t,P} = w^{\top} * r_t$$

and

$$\sigma_{t,P} = \sqrt{w^{\top} * \Sigma_t * w},$$

where $r_{t,P}$ and $\sigma_{t,P}$ is portfolio return and portfolio volatility at time t respectively and r_t is vector of individual returns at time t. Series $r_{t,P}$ and $\sigma_{t,P}$ are further modeled using Univariate Quantile Regression Model for Returns and the forecasts of the quantiles of the portfolio return series are considered to be semi-parametric percentage portfolio VaR.

6 Results

In this section, results of our empirical analysis are presented. We start with description of in-sample fit of PQR models and we continue with out-of-sample results of forecasting exercises where we concentrate on the portfolio Value-at-Risk application.

6.1 In-sample fit

We present estimation results of three specification of PQR models. In each specification quantiles of return series depends on different realized measures - Realized Volatility, Realized Semivariances and Realized Bi-Power Variation. Hence, we are estimating following Panel Quantile Regression Models for Returns specifications:

- *PQR-RV* for Realized Volaitlity, with quantile function defined as

$$Q_{r_{i,t+1}}(\tau) = \alpha_i(\tau) + \beta_{RV^{1/2}}(\tau) * RV_t^{1/2},$$

- *PQR-RSV* for Realized Semivariance, with quantile function defined as

$$Q_{r_{i,t+1}}(\tau) = \alpha_i(\tau) + \beta_{RS^{+1/2}}(\tau) * RS_t^{+1/2} + \beta_{RS^{-1/2}}(\tau) * RS_t^{-1/2},$$

- *PQR-BPV* for Realized Bi-Power Variation, with quantile function defined as

$$Q_{r_{i,t+1}}(\tau) = \alpha_i(\tau) + \beta_{BPV^{1/2}}(\tau) * BPV_t^{1/2} + \beta_{Jumps^{1/2}}(\tau) * Jumps_t^{1/2},$$

Detailed estimation results are presented in the Table 1 for 5%, 10%, 90% and 95% quantiles that are the most important one from the economic point of view. To get a better view of quantile dynamics we also report lower and upper quartile together with median.

Using lagged Realized Volatility as regressor we found out that parameters $\hat{\beta}_{RV^{1/2}}$ for all but median quantile are statistically highly significant. Moreover, signs of the estimated parameters correspond to our expectations – below/above median quantiles are negtive/positive. Furthermore, median parameter estimate that is not statistically significant confirm hypothesis about the randomness/unpredictability of the short-term returns. In the Table 1, we can also see that absolute values of parameter estimates are not symmetric around median which highlight the relative importance of the realized volatility on the estimation of the lower quantiles of return series. We arrive to a similar conclusion also when looking at the Figure 1 that compares and displays PQR estimates together with their corresponding 95% confidence intervals and individual UQR parameter estimates plotted in boxplots. Importantly, the Figure 1 shows that once we control for unobserved heterogeneity by PQR past volatility has larger influence on both the lower and the upper quantiles of returns than the majority of individual UQR. This is highlighted in far quantiles, e.g. coefficient of PQR in 5% quantile is -1.51 whereas median of individual UQR coefficient is -1.33 (mean -1.36) or 95% quantile PQR coefficient is 1.42 and median of individual UQR is only 1.30 (mean 1.31).

Next, as the regressor we use Realized Variance decomposed into realized downside (RS^-) and upside (RS^+) semivariance. For all the quantiles parameters RS^- and RS^+ are statistically significant. In the lower and upper tail parameters are highly significant and shows the importance of each component. However, influence of RS^- is far more important in upper quantiles where it dominates RS^+. On the contrary, in the lower quantiles values of parameters are close to each other and therefore we cannot draw the similar conclusion as in upper quantiles. Median performance is similar to Realized Volatility specification as coefficients sums to the same value of -0.01. Results of our analysis are also supported by the Figure 2. Similarly to PQR-RV specification we can see in the Figure 2 that controlling for unobserved heterogeneity among financial assets is important because influence of both downside and upside semivariance is greater in the lower quantiles than in individual UQR. For example in 5% quantile coefficients obtained by PQR-RSV are -0.97 and -1.18 for RS^+ and RS^- respectively, however median values of individual UQR are -0.82 (mean -0.84) for RS^+ and -0.95 (mean -1.1) for RS^-. Moreover, in the upper quantiles of negative semivariance (Figure 2b)PQR coefficients differs substantially from individual UQR (95% quantile PQR RS^- coefficient of 1.49 vs. individual UQR median/mean coefficient of 1.28/1.27), however, the opposite is true for RS^+ (95% quantile PQR RS^+ coefficient of 0.54 vs. individual UQR median/mean coefficient of 0.55/0.55). These findings support previous conclusion that RS^- influences future upper quantiles of returns more than RS^+.

In the last, PQR-BPV specification, we use the Bi-Power Variation and Jump components as regressors. In this set-up, the jump component play minor role in the upper quantiles. In the lower quantiles, however, jumps become significant and they attribute to overall performance although their influence is rather low. Bi-Power variation components is significant for all but median quantile. We can also see that the coefficient estimates are close to coefficients obtained by PQR-RV. We have to stress here the asymmetric influence of the jump component on the quantiles of return series because in the upper quantiles where jump component is not statistically different from zero Bi-Power Variation reduces to Quadratic Variation represented by Realized Volatility estimator. However, in the lower quantiles disentangling Quadratic Variation into Integrated Variance and Jump Variation helps to explained part of the data as sum of the BPV and Jump component is different from coefficients obtained by PQR-RV. The Figure 3 confirms our previous findings in a following way: the Figure 3a that displays coefficients from PQR-BPV model is close to the Figure 1 from PQR-RV specification. Therefore for some quantiles Quadratic Variation reduces to Integrated Variance. The Figure 3b supports this finding for quantiles above median where the confidence intervals of the jump component parameter estimates are wider and for many quantiles are statistically indistinguishable from zero. Opposite is true for quantiles bellow median where the confidence intervals are narrower and rarely contains zero.

Table 1: Coefficient estimates of Panel Quantile Regressions

τ	0.05	0.1	0.25	0.5	0.75	0.9	0.95
				PQR-RV			
const	0	0	0	0	0	0	0
	(-1.47)	(-1.27)	(-0.61)	(-0.58)	(0.9)	(1.64)	(2.08)
$\hat{\beta}_{RV^{1/2}}$	-1.51	-1.16	-0.6	-0.01	0.56	1.11	1.42
	(-24.24)	(-21.41)	(-16.36)	(-0.24)	(20.15)	(24.62)	(21.11)
				PQR-RSV			
const	0	0	0	0	0	0	0
	(-1.6)	(-1.21)	(-0.63)	(-0.44)	(0.97)	(2.07)	(2.41)
$\hat{\beta}_{RS^{+1/2}}$	-0.97	-0.74	-0.44	-0.15	0.18	0.41	0.54
	(-12.92)	(-13.02)	(-8.54)	(-2.9)	(2.82)	(4.39)	(4.3)
$\hat{\beta}_{RS^{-1/2}}$	-1.18	-0.91	-0.41	0.14	0.62	1.15	1.49
	(-11.14)	(-14.29)	(-10.12)	(2.78)	(9.23)	(13.91)	(10.06)
				PQR-BPV			
const	0	0	0	0	0	0	0
	(-1.4)	(-1.36)	(-0.6)	(-0.67)	(0.77)	(1.86)	(2.67)
$\hat{\beta}_{BPV^{1/2}}$	-1.55	-1.18	-0.62	0	0.59	1.15	1.44
	(-20.25)	(-17.58)	(-16.49)	(-0.15)	(24.16)	(22.79)	(25.91)
$\hat{\beta}_{Jumps^{1/2}}$	-0.24	-0.2	-0.14	-0.03	0.06	0.21	0.44
	(-3.2)	(-3.41)	(-3.41)	(-0.62)	(1.03)	(1.88)	(2.73)

Note: Table displays coefficient estimates with bootstraped t-statistics in parentheses. Individual fixed effects $\alpha_i(\tau)$ are not reported for brevity - they are available from authors upon request

6.2 Out-of-sample performance - Value at Risk application

We begin with a description of the absolute performance of the PQR models, followed by absolute performance of benchmark models and we finish with relative performance of PQR models with respect to the benchmark models. All the results are summarized in the Table 2.

In the *Panel A.1* we can see that unconditional coverage $\widehat{\tau}$ in case of all specifications of PQR models is close to nominal levels τ for all quantiles and the CAViaR test reveals that all model specifications for all quantiles are correctly dynamically specified at 5% significance level.

Panel A.2 summarize the performance of the benchmark models. Here we can see that unconditional coverage $\widehat{\tau}$ is close to nominal levels τ for all quantiles except median of RiskMetrics model. Median performance of RiskMetrics is also the only case of not dynamically correctly specified model according to the results of CAViaR test - null hypothesis of proper dynamic specification is strongly rejected given p-value<0.01. Poor median RiskMetrics performance can be attributed to the construction of equation 14 where cut-off point at 50% quantile, $\gamma_{50\%}$, is 0[6].

[6] Median of standard normal distribution is 0.

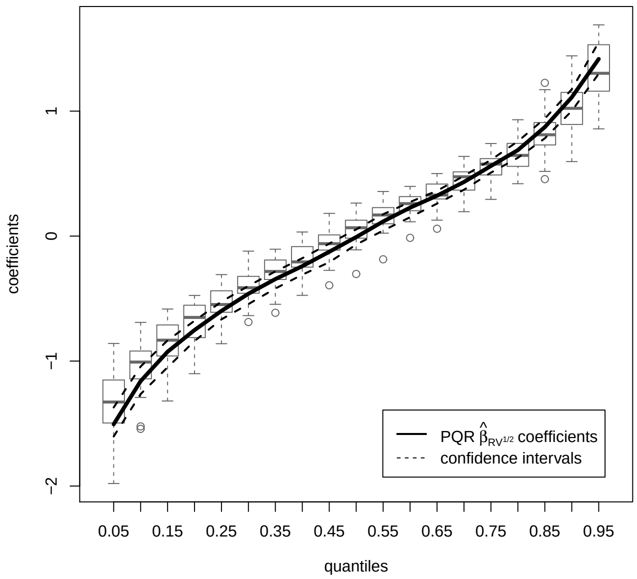
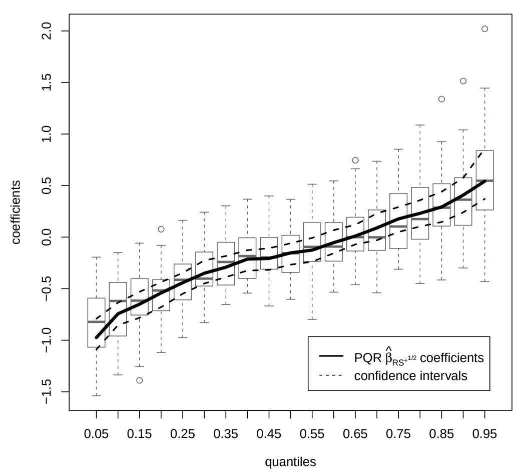


Figure 1: Parameter estimates with corresponding 95% confidence intervals from the PQR-RV specification are ploted by solid and dashed lines respectively. Individual UQR-RV estimates are ploted in boxplots

(a) $RS^{+1/2}$



(b) $RS^{-1/2}$

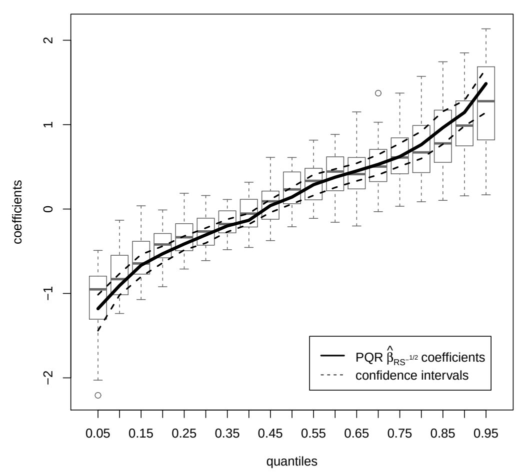


Figure 2: For both realized upside and downside semivariance parameters estimates with corresponding 95% confidence intervals are ploted by solid and dashed lines respectively. Individual UQR-RSV estimates are ploted in boxplots

Relative performance of the PQR models is summarized in the *Panel B*[7]. Results of our

[7]for brevity we report in Table 2 only pair-wise comparison against benchmark models, full matrix of pairwise

(a) $BPV^{1/2}$ (b) $Jumps^{1/2}$

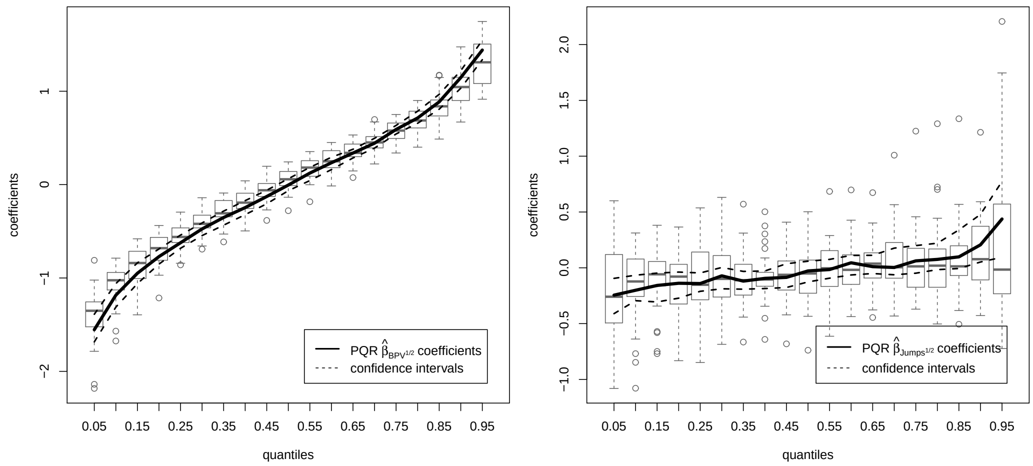


Figure 3: For both realized bi-power variation and jump component parameters estimates with corresponding 95% confidence intervals are ploted by solid and dashed lines respectively. Individual UQR-BPV estimates are ploted in boxplots

analysis indicate good relative performance of PQR models. All specifications, PQR-RV, PQR-RSV and PQR-BPV, significantly outperform RiskMetrics in all studied quantiles. Moreover, all PQR specifications consistently outperformed Portfolio UQR in upper quantiles and UQR in several quantiles i.e. PQR-RV outperform individual UQR estimates in 10% quantile, however performance of PQR-RSV is significantly better in 95% quantile and PQR-BPV delivers significantly more accurate forecasts than individual UQR in 5% and 10% quantiles.

If we concentrate on the full pair-wise comparison, the most interesting is the relative performance of the Portfolio UQR - Portfolio UQR outperformed RiskMetrics only at 5% and 10% quantiles. In contrast, UQR similarly to PQR outperform RiskMetrics in all studied quantiles. These results reveal the importance of the asset specific contribution to overall future portfolio risk as approach of firstly aggregating data and secondly modeling them is not able to capture dynamics creating variation in distribution of future portfolio returns.

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comparison is available from authors upon request

Table 2: Out-of-sample performance of various specifications of Panel Quantile Regression Model for Returns

		PQR-RV					PQR-RSV					PQR-BPV				
Panel A.1	τ	5%	10%	50%	90%	95%	5%	10%	50%	90%	95%	5%	10%	50%	90%	95%
	$\widehat{\tau}$	0.060	0.108	0.465	0.901	0.959	0.059	0.107	0.465	0.899	0.960	0.058	0.107	0.465	0.902	0.960
	$\widehat{DQ}$	8.917	3.373	10.157	6.939	5.686	8.180	3.339	10.129	1.476	9.152	7.956	4.625	10.157	5.298	6.210
	p-val	0.178	0.761	0.118	0.326	0.459	0.225	0.765	0.119	0.961	0.165	0.241	0.593	0.118	0.506	0.400
	$\widehat{L}$	0.078	0.126	0.254	0.107	0.065	0.078	0.126	0.254	0.107	0.065	0.077	0.126	0.254	0.107	0.065
		RiskMetrics					UQR					Portfolio UQR				
Panel A.2	τ	5%	10%	50%	90%	95%	5%	10%	50%	90%	95%	5%	10%	50%	90%	95%
	$\widehat{\tau}$	0.061	0.094	0.451	0.919	0.958	0.061	0.107	0.467	0.902	0.960	0.043	0.099	0.491	0.909	0.955
	$\widehat{DQ}$	9.652	3.096	<u>20.600</u>	9.452	10.899	8.323	3.041	9.067	7.174	6.796	9.426	5.988	3.273	4.507	3.238
	p-val	0.140	0.797	<u>0.002</u>	0.150	0.092	0.215	0.804	0.170	0.305	0.340	0.151	0.425	0.774	0.608	0.778
	$\widehat{L}$	0.083	0.132	0.255	0.110	0.067	0.078	0.126	0.254	0.107	0.065	0.078	0.127	0.254	0.108	0.066
		benchmark														
		RiskMetrics					UQR					Portfolio UQR				
Panel B	τ	5%	10%	50%	90%	95%	5%	10%	50%	90%	95%	5%	10%	50%	90%	95%
PQR-RV	DM	**-2.430**	**-2.259**	**-3.347**	**-2.127**	**-1.935**	0.125	**-1.734**	1.350	0.801	-0.362	-0.733	-1.590	-0.310	**-2.053**	**-2.260**
	p-val	0.008	0.012	0.000	0.017	0.027	0.550	0.041	0.911	0.788	0.359	0.232	0.056	0.378	0.020	0.012
PQR-RSV	DM	**-2.368**	**-2.249**	**-3.561**	**-2.367**	**-2.242**	1.244	-1.569	-0.438	-1.268	**-2.023**	-0.558	-1.580	-0.758	**-2.921**	**-3.099**
	p-val	0.009	0.012	0.000	0.009	0.012	0.893	0.058	0.331	0.102	0.022	0.289	0.057	0.224	0.002	0.001
PQR-BPV	DM	**-2.540**	**-2.422**	**-3.304**	**-2.055**	**-1.851**	**-1.796**	**-1.887**	1.424	0.839	0.703	-1.191	**-1.887**	-0.294	**-1.978**	**-1.705**
	p-val	0.006	0.008	0.000	0.020	0.032	0.036	0.030	0.923	0.799	0.759	0.117	0.030	0.384	0.024	0.044

Note: Table displays absolute and relative performance of PQR models for returns with RV, RSV and BPV as regressors and benchmark models.

Panel A.1 reports absolute performance of PQR models, *Panel A.2* reports absolute performance of benchmark models. For each model and quantile τ, unconditional coverage ($\widehat{\tau}$), the value of the CAViaR test for correct dynamic specification ($\widehat{DQ}$) with corresponding Monte Carlo based p-value and the value of loss function($\widehat{L}$) is displayed. Not correctly dynamically specified models are underlined.

Panel B reports relative performance of Panel Quantile Regression Models for Returns. For each specification and quantile τ we report Diebold-Mariano test statistics for pairwise comparison with benchmark models($\widehat{DM}$) with corresponding p-value. Significantly more accurate forecasts with respect to benchmark models at the 5% significance level are in bold. Full matrix of pairwise comparison is available from authors upon request

7 Conclusion

In this paper, we propose to employ panel quantile regression together with non-parametric measures of quadratic return variation to model conditional quantiles of financial assets return series. For estimation purposes we use penalized fixed effects estimator as introduced in Koenker (2004). Resulting Panel Quantile Regression Model for Returns inherit all favorable properties offered by panel data and quantile regression. A key attraction of the proposed methodology is the ability to control for otherwise unobserved heterogeneity among financial assets so it is possible to disentangle overall market risk into its systemic and idiosyncratic parts. Another attraction is the dimensionality reduction because the number of estimated parameters is always less than or equal to $k + n$, where k is the number of regressors and n number of assets. Last but not least, to the best of our knowledge this is the first application of the panel quantile regression on a data where $T >> N$, therefore we are able to obtain estimates of quantile specific individual fixed effects that accounts for unobserved heterogeneity and represents idiosyncratic part of the market risk.

We document accuracy of the proposed methodology using 29 highly liquid stocks from NYSE. In-sample model fit highlights relative importance of the different components of the quadratic variation for the various quantiles of return series. From the practitioners point of view model is highly attractive for portfolio and risk management because it is applicable to high-dimensional problems and can be easily used to obtain widely used Value-at-Risk measures of high-dimensional portfolios. This is supported by significantly better relative performance of proposed model in comparison to the benchmark models.

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