Cyclical Behavior of Government Spending Multipliers: A Markov-switching SVAR Approach

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Abstract

This paper investigates the state-dependent effects of government spending shocks. We explore a century-long data set with a Markov-switching structural VAR model that uses both quantitative and qualitative information for inference. We show that the business cycle accounts for most of the time-variation in spending multipliers with fiscal policy being considerably more effective in economic downturns than in booms. Specifically, GDP increases by \$1.9 (\$0.7) if government spending increases by \$1 in the first year of the shock that takes place during a recession (expansion). Our approach allows us to reconcile the main findings in the previous literature.

JEL codes: C15, C18, C32, C51, E32, E62

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1 Introduction

Measuring the size of fiscal multipliers is always a central issue in fiscal policy analysis. Many studies, including Barro and Redlick (2011) and Ramey (2011), have found government spending multipliers below unity on average, suggesting that increases in government purchases are unlikely to stimulate aggregate demand. However, to the extent that policy-makers are usually interested in the effects of fiscal policy at the time of implementation, the use of estimates of average fiscal multipliers is called into question. Some theories predict that the size of fiscal multipliers depends on the economic state such as the stance of monetary policy or the amount of slack in the economy. For example, classic Keynesian economics tells us that government spending shocks have larger effects in recessions than in expansions because private consumption and investment are less likely to be crowded out when resources are underutilized. Since many countries used discretionary fiscal policy to combat weak economic growth during the Great Recession, a key question is whether fiscal stimulus is considerably more effective in economic downturns than in booms.

Although some empirical works have been done to shed light on this question, the growing literature is far from reaching a consensus. Auerbach and Gorodnichenko (2012a,b), and Bachmann and Sims (2012) find much larger government spending multipliers in recessions than in expansions. Fazzari et al. (2015) also find that government spending multipliers become larger and more persistent during times of slack. On the contrary, Owyang et al. (2013) and Ramey and Zubairy (2014) do not observe higher multipliers when there is substantial economic slack in the United States. Bognanni (2013) and Alloza (2015) present evidence that the multipliers could even be smaller in recessions than in economic booms.

A key ingredient for gauging the size of state-dependent multipliers is the response of output to government spending shocks. In a nonlinear multivariate model that allows the economic state to switch, an individual set of model parameters is usually estimated for each state. As a result, how the parameter set transitions from one state to the other should be taken into account when constructing the impulse response functions. Auerbach and Gorodnichenko (2012a), Bachmann and Sims (2012), Bognanni(2013) and Alloza (2015) calculate impulse responses under the assumption that a recession remains throughout the duration of the response that usually lasts for more than 15 quarters. However, given the fact that the average duration of US recessions is about only one year, their constructed responses are conditional on an unreasonable economic path following the shock. In contrast, Auerbach and Gorodnichenko (2012b), Owyang et al. (2013), and Ramey and Zubairy (2014) calculate impulse responses using Jordà's (2005) local projections method, which amounts to a direct forecast of the effects of a government spending shock on future output conditional on the state when the shock occurs. In this case, any possible transitions between states after the shock are reflected in the impulse response estimates. Ramey and Zubairy (2014) argue, quite correctly in our view, that the crucial assumptions underlying the construction of impulse response functions can account for to a large extent why the leading study by Auerbach and Gorodnichenko (2012a) obtains multipliers as high as 2.5 for recessions.

An even more important issue is the inference about the states across which the multipliers may differ. A substantial part of the differences in conclusions of the literature is likely to lie in the definition of recession. In principle, different definitions of recession means different observations are used to inform the model parameters for each state. Auerbach and Gorodnichenko (2012a,b), and Bachmann and Sims (2012) assume that the transitions between recessions and expansions are characterized by a logistic function depending on the moving average of the output growth rate, and the function is calibrated to match the frequency of NBER dated recessions. Bognanni (2013) employs a Markov-switching VAR to jointly estimate when and how the output response to government spending shocks changes without imposing the timing a priori, and finds that the business cycle is the mostly likely source of variation in government spending multipliers. Instead of estimating the probability of recession for each period, Alloza (2015) uses NBER business cycle dates to measure recessions directly. Both Bognanni and Alloza provide evidence that the different ways to calculate the time-varying probability of occurrence for recession is the main reason why they obtain different results from Auerbach and Gorodnichenko (2012a). Some other studies define recessions as periods of slack. While Owyang et al. (2013) and Ramey and Zubairy (2014) use the unemployment rate directly as indicator of slack in the economy, Fazzari et al.

(2015) select capacity utilization out of a few variables, including the unemployment rate, as the preferred indicator of slack using Bayesian model comparison.

This paper applies a Markov-switching structural VAR model to explore the century data set from 1890-2013, developed by Ramey and Zubairy (2014), and finds a larger impact of government spending shocks on output in recessions than in expansions. To control for the timing of fiscal shocks in the spirit of Ramey (2011), we build a structural VAR to describe the dynamic relationship between military spending news, government spending, tax and output. Government spending multiplier is then defined as the integral of the output responses divided by the integral of the government spending responses to a military news shock. To allow for the state-dependence of spending multipliers, we assume there are two distinct unobserved regimes for the economy and different economic dynamics takes over when the regime switches from one to the other.

As in Bognanni (2013), our model allows the parameters for each regime to be jointly estimated with the timing of when regime changes occur, based on which an inference about the regimes can be made. However, the inference could actually be improved with some useful prior information outside the model. For example, the literature above shows that the multipliers probably vary over the business cycle or with the slack in the economy. Therefore we adopt the framework proposed by Jefferson (1998) to utilize two sources of qualitative information. First, we consider the recessions dated by the NBER. Second, we consider the measure by Ramey and Zubairy (2014) that the bad regime is the period when the unemployment rate is above 6.5 percent. From the maximum likelihood estimates, we find that the NBER dates contain much information while the measure by Ramey and Zubairy (2014) says little about the true regimes. This suggests why Ramey and Zubairy (2014) find no significant differences in multipliers between the high and low unemployment state.

Because the unemployment rate is much less informative, we keep only the NBER recession dates in our model. Our estimation results show that the two regimes are well characterized by the business cycle. To turn the estimates of parameters into impulse response functions, we propose a simulation approach that allows the regime to switch naturally. We find the responses to a military news shock smaller in expansions than in recessions. The multipliers for expansions are always below unity, while the multipliers for recessions do not fall below unity until two years after the shock. If government spending increases by \$1 in the first (four) year (years) of the shock that takes place during an expansion, the GDP will be \$0.7 (\$0.4) higher. However, if government spending increases by \$1 in the first (four) year (years) of the shock that takes place during a recession, the GDP will be \$1.9 (\$0.8) higher.

This paper contributes to the literature by providing new evidence that fiscal policy is indeed significantly more effective in economic downturns than in booms. By introducing the Markov-switching SVAR method into the field, we are also able to reconcile some evidence found in the previous studies. First, we argue that our model can include Ramey and Zubairy (2014) as a special case where the inference about the regimes is totally determined by the unemployment rate. For this case, we obtain very similar results as Ramey and Zubairy (2014) that the multipliers are always below unity, and the differences between the states are statistically insignificant. However, if we replace the unemployment rate with the business cycle that is favored by the model, the results are quite consistent with our main findings. Second, we illustrate how Alloza (2014) can be taken as a special case in our framework as well. Although they get the inference about the regimes correct, they impose some additional restrictions on the coefficients. Moreover, they convert the estimated coefficients into impulse response functions assuming that the regime stays in the one when the shock occurs. We estimate this special case using their sample period, and find similar results as the original paper that the multipliers for recessions are negative while the multipliers for expansions are positive. Then we show evidence that the restrictions they have on the coefficients are actually unreasonable. With the restrictions relaxed and using our method to compute impulse response functions, we obtain results that support our conclusion.

The remainder of this paper proceeds as follows. The methodology is presented in section 2. In section 3, we briefly describe the data used and show our empirical results. Section 4 shows how we reconcile some main findings in the previous studies. Section 5 concludes.

2 Methodology

Most empirical research on the effects of government spending shocks, such as Blanchard and Perotti (2002), relies on a VAR model that includes fiscal and macroeconomic variables. However, Ramey (2011) shows evidence that such VAR shocks to government spending can actually be predicted by war dates or professional forecasts. This is because there is usually a timing gap between the announcement of a certain fiscal policy and its actual implemetation, an issue known as fiscal foresight. From the standpoint of the neoclassical approach, an increase in government spending creates a negative wealth effect for the representative household. Thereby it is the change in the present discounted value of government purchases that really matters, and households react immediately once they learn the news about future purchases. Because the conventional VAR method captures shocks only when they occur, it misses the initial responses of the economy following the news.

To control for the timing of government spending shocks, we follow Ramey (2011) to incorporate an estimate of changes in the expected present value of military spending into a VAR. In particular, we are interested in the dynamics of

$$X_t \equiv \begin{bmatrix} \tilde{N}_t \\ \tilde{G}_t \\ \tilde{T}_t \\ \tilde{Y}_t \end{bmatrix} \equiv \begin{bmatrix} N_t/Y_t^* \\ G_t/Y_t^* \\ T_t/Y_t^* \\ Y_t/Y_t^* \end{bmatrix}, \tag{1}$$

where N_t is military spending news, G_t is government spending, T_t is tax revenue, Y_t is actual output, and Y_t^* is potential output. More details about the data will be presented in section 3. Following Gordon and Krenn (2010), we scale all the variables by potential output so that their responses are in the same dollar units. This allows us to calculate the spending multiplier directly as the integral of the output responses to a military news shock divided by the integral of the government spending responses. In contrast, most of the previous studies take logarithms of the variables, thus the ratio of the output response to the government spending response measures elasticity instead of the multiplier. The estimated elasticities are then converted to dollar equivalents, or the multipliers, using an *ex post* factor based on the sample average values of the ratio of output to government spending, Y/G. Ramey and Zubairy (2014) notice that in the post-WWII sample, the value of Y/G varies modestly from 4 to 7 with a mean of 5. However, in the full sample from 1890-2013, which is also used in this paper, the value of Y/G fluctuates heavily ranging from 2 to 24 with a mean close to 8, thereby the use of the average value as the conversion factor is no longer appropriate.

In order for the effects of government spending to vary across different economic states, we place a Markov-switching structure on the VAR. In the following subsections, we also introduce an algorithm for estimation as well as a framework by Jefferson (1998) which uses both quantitative and qualitative information for inference. Then we propose a simulation method to turn the estimated parameters into impulse response functions conditional on the state when the shock occurs, allowing the state to switch naturally after the shock.

2.1 A Markov-switching structural VAR

As the convention in the literature such as Auerbach and Gorodnichenko (2012a), we assume that government spending is the most exogenous and output is the most endogenous among the three fiscal and macroeconomic variables. This means that shocks in tax revenues and output can not affect government spending contemporaneously. Blanchard and Perotti (2002) justify this assumption by the observation that it usually takes policymakers more than a quarter to decide how government should change its spending in response to such shocks, pass the decisions through the legislature, and send them to implementation. Because there is no statistical evidence that military spending news depends on the past values of itself or the other variables, we assume it to be serially uncorrelated over time. We also allow all the other variables to react to military spending news immediately. Under these assumptions, the dynamics of X_t is modeled as a Markov-switching structural VAR (MS-SVAR hereafter):

$$A_0(s_t)X_t = C(s_t) + \sum_{j=1}^{4} A_j(s_t)X_{t-j} + u_t, \qquad u_t|d_t \sim \mathbf{N}\left(0_{4\times 1}, \Sigma(d_t)\right), \tag{2}$$

where the first rows of $A_j(s_t)$ $(j = 1, \cdots, 4)$ are filled with zeros. $A_0(s_t)$ is a lower-triangular matrix with ones on the diagonal. $\Sigma(d_t)$ is a diagonal matrix collecting the variances of structural shocks u_t. The unobserved state variable s_t is assumed to follow a two-state Markov chain with transition probabilities:

$$p_{ij} = Pr(s_t = j|s_{t-1} = i) \qquad i, j = 1, 2. \tag{3}$$

Hamilton (2008) shows that inference about the conditional mean can be severely affected if one does not consider heteroskedasticity in the data.[1] To take it into account, we assume there is a common unobserved state variable d_t^{GT} for variances of fiscal shocks, and another unobserved state variable d_t^{Y} for variance of output shock. d_t^{GT} and d_t^{Y} evolve according to independent two-state Markov chains, with transition probabilities:

$$q_{ij}^{GT} = Pr(d_t^{GT} = j|d_{t-1}^{GT} = i) \qquad i, j = 1, 2. \tag{4}$$

$$q_{ij}^{Y} = Pr(d_t^{Y} = j|d_{t-1}^{Y} = i) \qquad i, j = 1, 2. \tag{5}$$

Then we define the state variable d_t for the variance matrix Σ in (2) as

$$d_t = \begin{cases} 1 & \text{if } d_t^{GT} = 1, d_t^{Y} = 1 \\ 2 & \text{if } d_t^{GT} = 1, d_t^{Y} = 2 \\ 3 & \text{if } d_t^{GT} = 2, d_t^{Y} = 1 \\ 4 & \text{if } d_t^{GT} = 2, d_t^{Y} = 2 \end{cases}.$$

The Markov-switching properties of d_t can be easily derived from equation (4) and (5). In order

[1] Heteroskedasticity can be a big issue if there are outliers or high-variance episodes in the data.

for parsimonious estimation, we assume that s_t is independent of d_t^{GT} and d_t^{Y}.[2]

2.2 Filtering, smoothing, and estimation

Because our estimation of the model relies on calculating the smoothed probabilities which are based on filtering, we first introduce a basic filter, then a smoother, and finally the estimation method. To illustrate the statistical techniques, we consider a simple univariate regression which includes regime-switching to both coefficients and variance:

$$y_t = \mathbf{x}_t'\beta(s_t) + \sigma(d_t)\varepsilon_t, \qquad \varepsilon_t \sim i.i.d.N(0,1) \tag{6}$$

s_t and d_t are independent unobserved state variables, and each of them follows a two-state Markov chain with transition probabilities:

$$p_{ij} = Pr(s_t = j|s_{t-1} = i) \qquad i,j = 1,2, \tag{7}$$

$$q_{ij} = Pr(d_t = j|d_{t-1} = i) \qquad i,j = 1,2. \tag{8}$$

Let $\mathscr{Y}_T \equiv (y_T, y_{T-1}, \cdots, y_1)'$ denote the vector of observations, and the parameters to be estimated are collected by $\Theta = (\beta'(1), \beta'(2), \sigma^2(1), \sigma^2(2), p_{11}, p_{22}, q_{11}, q_{22})'$. The conditional likelihood function is

$$f(y_t|\mathbf{x}_t, s_t = j, d_t = j'; \Theta) = \frac{1}{\sqrt{2\pi\sigma(j')^2}} \exp\left\{-\frac{(y_t - \mathbf{x}_t'\beta(j))^2}{2\sigma(j')^2}\right\}, \qquad j, j' = 1,2. \tag{9}$$

2.2.1 Filtering

Hamilton (1989) provides a basic filter that could be used to draw probabilistic inference about the unobserved states s_t and d_t given observations of y through date t. Suppose $p(s_{t-1} = i, d_{t-1} = i'|\mathscr{Y}_{t-1}; \Theta)$ is known, it can be used as input to obtain $p(s_t = j, d_t = j'|\mathscr{Y}_t; \Theta)$ according to the

[2] In principle one can allow for correlation between the state variables. In that case, however, there will be 56 more parameters to estimate with probably not much gain in efficiency.

algorithm as follows:

- Step 1: Calculate

$$
\begin{aligned}
& p(s_t = j, d_t = j', s_{t-1} = i, d_{t-1} = i' | \mathscr{Y}_{t-1}; \Theta) \\
& = p(s_t = j | s_{t-1} = i) \cdot p(d_t = j' | d_{t-1} = i') \cdot p(s_{t-1} = i, d_{t-1} = i' | \mathscr{Y}_{t-1}; \Theta) \\
& = p_{ij} \cdot q_{i'j'} \cdot p(s_{t-1} = i, d_{t-1} = i' | \mathscr{Y}_{t-1}; \Theta)
\end{aligned}
$$

- Step 2: Calculate the joint conditional likelihood of y_t and the states:

$$
\begin{aligned}
& f(y_t, s_t = j, d_t = j', s_{t-1} = i, d_{t-1} = i' | \mathscr{Y}_{t-1}; \Theta) \\
& = f(y_t | s_t = j, d_t = j', \mathscr{Y}_{t-1}; \Theta) \times p(s_t = j, d_t = j', s_{t-1} = i, d_{t-1} = i' | \mathscr{Y}_{t-1}; \Theta)
\end{aligned}
$$

  where the first term of the product is just equation (9) and the second term comes from the previous step.

- Step 3: Then we have the conditional likelihood of y_t as

$$
f(y_t | \mathscr{Y}_{t-1}; \Theta) = \sum_{j=1}^{2} \sum_{j'=1}^{2} \sum_{i=1}^{2} \sum_{i'=1}^{2} f(y_t, s_t = j, d_t = j', s_{t-1} = i, d_{t-1} = i' | \mathscr{Y}_{t-1}; \Theta)
$$

- Step 4: By Bayes' rule

$$
\begin{aligned}
& p(s_t = j, d_t = j', s_{t-1} = i, d_{t-1} = i' | \mathscr{Y}_t; \Theta) \\
& = \frac{f(y_t, s_t = j, d_t = j', s_{t-1} = i, d_{t-1} = i' | \mathscr{Y}_{t-1}; \Theta)}{f(y_t | \mathscr{Y}_{t-1}; \Theta)}
\end{aligned}
$$

- Step 5: Finally we obtain

$$
p(s_t = j, d_t = j' | \mathscr{Y}_t; \Theta) = \sum_{i=1}^{2} \sum_{i'=1}^{2} p(s_t = j, d_t = j', s_{t-1} = i, d_{t-1} = i' | \mathscr{Y}_t; \Theta)
$$

2.2.2 Smoothing

The goal of smoothing is to obtain a more reliable inference about the states using full-sample information. Kim (1994) proposes a smoother that is easy to implement based on the filter above. Smoothed probabilities are calculated backward starting from the known probability $p(s_T = j, d_T = j'|\mathscr{Y}_T;\Theta)$. Suppose we know $p(s_{t+1} = j, d_{t+1} = j'|\mathscr{Y}_T;\Theta)$. Then we have

$$
\begin{aligned}
& p(s_t = i, d_t = i', s_{t+1} = j, d_{t+1} = j'|\mathscr{Y}_T;\Theta) \\
= \; & p(s_{t+1} = j, d_{t+1} = j'|\mathscr{Y}_T;\Theta) \cdot p(s_t = i, d_t = i'|s_{t+1} = j, d_{t+1} = j', \mathscr{Y}_t;\Theta) \\
= \; & \frac{p(s_{t+1} = j, d_{t+1} = j'|\mathscr{Y}_T;\Theta) \cdot p(s_{t+1} = j, d_{t+1} = j', s_t = i, d_t = i'|\mathscr{Y}_t;\Theta)}{p(s_{t+1} = j, d_{t+1} = j'|\mathscr{Y}_t;\Theta)} \\
= \; & \frac{p(s_{t+1} = j, d_{t+1} = j'|\mathscr{Y}_T;\Theta) \cdot p(s_t = i, d_t = i'|\mathscr{Y}_t;\Theta) \cdot p_{ij} \cdot q_{i'j'}}{p(s_{t+1} = j, d_{t+1} = j'|\mathscr{Y}_t;\Theta)}
\end{aligned}
$$

Therefore we obtain

$$
p(s_t = i, d_t = i'|\mathscr{Y}_T;\Theta) = \sum_{j=1}^{2} \sum_{j'=1}^{2} p(s_t = i, d_t = i', s_{t+1} = j, d_{t+1} = j'|\mathscr{Y}_T;\Theta)
$$

and also

$$
p(s_t = i|\mathscr{Y}_T;\Theta) = \sum_{i'=1}^{2} p(s_t = i, d_t = i'|\mathscr{Y}_T;\Theta)
$$

$$
p(d_t = i'|\mathscr{Y}_T;\Theta) = \sum_{i=1}^{2} p(s_t = i, d_t = i'|\mathscr{Y}_T;\Theta)
$$

2.2.3 Estimation

Hamilton (1990) proposes an expectation maximization (EM) algorithm for obtaining maximum likelihood estimates for a Markov-switching model where the parameter vector is usually of high dimension and the likelihood function is highly nonlinear. Because the EM algorithm usually requires a time-invariant conditional variance or that the conditional variance has the same state shifts as the conditional mean, it can not be applied to our model directly. In a separate paper by Lyu and Noh (2016), we develop an alternative expectation increasing (EI) algorithm, based on the

EM algorithm, to accomodate the conditional heteroskedasticity specified above.

Let $Q(\Theta_{l+1};\Theta_l,\mathscr{Y}_T)$ denote the expected log-likelihood which is parameterized by Θ_{l+1}. The expectation is taken with respect to a distribution parameterized by Θ_l, that is

$$\begin{aligned}&Q(\Theta_{l+1};\Theta_l,\mathscr{Y}_T)\\ &\equiv \sum_{s_T=1}^{2}\sum_{d_T=1}^{2}\cdots\sum_{s_1=1}^{2}\sum_{d_1=1}^{2}\log f(\mathscr{Y}_T,s_T,d_T,\cdots,s_1,d_1;\Theta_{l+1})\cdot f(\mathscr{Y}_T,s_T,d_T,\cdots,d_1,s_1;\Theta_l)\end{aligned}$$

For simplicity of notation, we partition the parameter vector into three subvectors, that is, $\Theta = (\beta,\sigma,\mathbf{P})$. The three subvectors collect coefficients, conditional variances and transition probabilities, respectively. The EI algorithm, which amounts to updating β,σ and $\mathbf{P}$ sequentially, is as follows:

- Step 1: Start with some arbitrary initial value $\hat{\Theta}_0$;
- Step 2: Choose $\hat{\beta}_{l+1}$ to maximize $Q\left(\Theta_{l+1};\hat{\Theta}_l,\mathscr{Y}_T\right)|_{\sigma_{l+1}=\hat{\sigma}_l,\mathbf{P}_{l+1}=\hat{\mathbf{P}}_l}$; then denote $\hat{\Theta}_{l+1,l} = (\hat{\beta}_{l+1},\hat{\sigma}_l,\hat{\mathbf{P}}_l)$ and calculate the smoothed probabilities $p\left(s_t=j,d_t=j'|\mathscr{Y}_T;\hat{\Theta}_{l+1,l}\right)$;
- Step 3: Choose $\hat{\sigma}_{l+1}$ and $\hat{\mathbf{P}}_{l+1}$ to maximize $Q\left(\Theta_{l+1};\hat{\Theta}_{l+1,l},\mathscr{Y}_T\right)|_{\beta_{l+1}=\hat{\beta}_{l+1}}$; denote $\hat{\Theta}_{l+1} = (\hat{\beta}_{l+1},\hat{\sigma}_{l+1},\hat{\mathbf{P}}_{l+1})$ and calculate the smoothed probabilities $p\left(s_t=j,d_t=j'|\mathscr{Y}_T;\hat{\Theta}_{l+1}\right)$;
- Step 4: Repeat step 2 and 3 until $\{\hat{\Theta}_{l+1}\}_{l=0}^{\infty}$ converges to a Θ^*.

Lyu and Noh (2016) prove that after each iteration, $\hat{\Theta}_{l+1}$ is associated with a higher likelihood value than is $\hat{\Theta}_l$, that is

$$f\left(\mathscr{Y}_T;\hat{\Theta}_{l+1}\right)\geq f\left(\mathscr{Y}_T;\hat{\Theta}_l\right)$$

thus Θ^* must be a (local) MLE. One good thing about the EI algorithm is that analytical solutions to the maximization problems during the iterations are available. For the example considered, the iteration equations are:

$$\hat{\beta}_{(l+1)}(j) = \left(\sum_{t=1}^{T}\mathbf{x}_t^{(l,j,*)}\mathbf{x}_t'^{(l,j,*)}\right)^{-1}\sum_{t=1}^{T}\mathbf{x}_t^{(l,j,*)}y_t^{(l,j,*)}, \qquad j=1,2, \tag{10}$$

$$\hat{\sigma}^2_{(l+1)}(j') = \frac{\sum_{t=1}^{T}\sum_{j=1}^{2}\left(y_t - \mathbf{x}_t'\hat{\beta}_{(l+1)}(j)\right)^2 p\left(s_t = j, d_t = j' | \mathscr{Y}_T; \hat{\Theta}_{l+1,l}\right)}{\sum_{t=1}^{T} p\left(d_t = j' | \mathscr{Y}_T; \hat{\Theta}_{l+1,l}\right)}, \quad j' = 1,2, \tag{11}$$

$$\hat{p}^{(l+1)}_{ij} = \frac{\sum_{t=2}^{T} p\left(s_t = j, s_{t-1} = i | \mathscr{Y}_T; \hat{\Theta}_{l+1,l}\right)}{\sum_{t=2}^{T} p\left(s_{t-1} = i | \mathscr{Y}_T; \hat{\Theta}_{l+1,l}\right)}, \qquad i,j = 1,2 \tag{12}$$

$$\hat{q}^{(l+1)}_{i'j'} = \frac{\sum_{t=2}^{T} p\left(d_t = j', d_{t-1} = i' | \mathscr{Y}_T; \hat{\Theta}_{l+1,l}\right)}{\sum_{t=2}^{T} p\left(d_{t-1} = i' | \mathscr{Y}_T; \hat{\Theta}_{l+1,l}\right)}, \qquad i',j' = 1,2 \tag{13}$$

where

$$\begin{aligned}
\mathbf{x}_t^{(l,j,*)} &= \mathbf{x}_t \cdot \sqrt{p\left(s_t = j | \mathscr{Y}_T; \hat{\Theta}_l\right)} \left(\sum_{j'=1}^{2} \hat{\sigma}_{(l)}(j') p\left(d_t = j' \middle| \mathscr{Y}_T; \hat{\Theta}_l\right)\right)^{-1}, \qquad j = 1,2, \\
y_t^{(l,j,*)} &= y_t \cdot \sqrt{p\left(s_t = j | \mathscr{Y}_T; \hat{\Theta}_l\right)} \left(\sum_{j'=1}^{2} \hat{\sigma}_{(l)}(j') p\left(d_t = j' \middle| \mathscr{Y}_T; \hat{\Theta}_l\right)\right)^{-1}, \qquad j = 1,2.
\end{aligned}$$

As a result, we just need to evaluate the smoothed probabilities at the end of each step and take them as inputs for the next one according to equations (10)-(13).

2.3 Inference about the states

With the model estimated, we can draw inference about the unobserved state s_t using all available information, that is, we can calculate $p(s_t = i|\mathscr{Y}_T;\Theta^*)$ using the techniques introduced above. While this inference uses only quantitative information contained in the observable time series of y, Jefferson (1998) notes that it can be improved by also taking advantage of qualitative information.

Jefferson (1998) assumes that there exists an observable indicator variable I_t which can be regarded as a proxy with measurement error for the true state s_t, that is:

$$g_{11} = p\left(I_t = 1 | s_t = 1\right), \quad 1 - g_{11} = p\left(I_t = 2 | s_t = 1\right) \tag{14}$$

$$1 - g_{22} = p\left(I_t = 1 | s_t = 2\right), \quad g_{22} = p\left(I_t = 2 | s_t = 2\right) \tag{15}$$

The parameters g_{11} and g_{22} indicate how reliable I_t is about s_t on average. In other words, g_{11} and

g_{22} close to one may suggest that the indicator variable contains much information about s_t. Let $\tilde{\mathbf{I}}_T = (I_T, I_{T-1}, \cdots, I_1)$ denote the values of the indicator variable. This qualitative information can be incorporated into the Markov-switching model easily by changing the conditional likelihood function (9) to:

$$f(y_t, I_t|\mathbf{x}_t, s_t = j, d_t = j'; \Theta) = p(I_t|s_t = j) \cdot f(y_t|\mathbf{x}_t, s_t = j, d_t = j'; \Theta) \qquad j, j' = 1, 2. \tag{16}$$

The filtering, smoothing and estimation algorithms can be revised accordingly to take into account the qualitative information I_t, and g_{11} and g_{22} are estimated jointly with the other parameters.

Within this general framework, two special cases are worth noting. First we consider $g_{11} = 1$ and $g_{22} = 1$, which amounts to assuming that I_t is exactly s_t with no measurement error. Then the inference about s_t becomes

$$p(s_t = i|\mathscr{Y}_T, \tilde{\mathbf{I}}_T; \Theta^*) = \begin{cases} 1 & \text{if } I_t = i \\ 0 & \text{if } I_t \neq i \end{cases} \tag{17}$$

This is the case in Owyang et al. (2013) and Ramey and Zubairy (2014), who define I_t by whether the unemployment rate is above 6.5 percent or not and take I_t as the true state.

The second case is $g_{11} + g_{22} = 1$, which implies $p(I_t = 1|s_t = 1) = p(I_t = 1|s_t = 2)$ and $p(I_t = 2|s_t = 1) = p(I_t = 2|s_t = 2)$. Because the value of the indicator variable is independent of s_t, the qualitative information is of no use for inference. It is not hard to prove that in this case, the smoothed inference collapses to the inference that uses only quantitative information, that is

$$p(s_t = i|\mathscr{Y}_T, \tilde{\mathbf{I}}_T; \Theta^*) = p(s_t = i|\mathscr{Y}_T; \Theta^*) \tag{18}$$

2.4 Generalized impulse response functions

In this subsection we present a simulation approach to constructing generalized impulse response functions. As discussed by Koop et al. (1996), a generalized impulse response function for a nonlinear system should depend on only the history of the economy and the size of the shock, averaging out all future scenarios. Because our model is linear in each state and the state transition is not affected by the shock, the impulse responses should be proportional to the size of the shock. However, since the parameter sets differ across the states, the impulse response functions are likely to vary according to the initial state when the shock occurs.

Note that the reduced-form of equation (2) can be expressed as:

$$X_t = B_0(s_t) + \sum_{j=1}^{4} B_j(s_t)X_{t-j} + B(s_t)u_t, \qquad u_t|d_t \sim \mathbf{N}\left(0_{4\times 1}, \Sigma(d_t)\right), \tag{19}$$

where $B_0(s_t) = (A_0(s_t))^{-1}C(s_t)$, $B_j(s_t) = (A_0(s_t))^{-1}A_j(s_t)$ $(j = 1,\cdots,4)$, and $B(s_t) = (A_0(s_t))^{-1}$. Then in companion form, the reduced-form can be rewritten as:

$$\mathbf{X}_t = \mathbf{C}(s_t) + \Phi(s_t)\mathbf{X}_{t-1} + \mathbf{B}(s_t)\mathbf{u}_t, \tag{20}$$

where $\mathbf{X}_t = (X_t^{'}, X_{t-1}^{'}, X_{t-2}^{'}, X_{t-3}^{'})'$, $\mathbf{C}(s_t) = (B_0(s_t)', 0_{12\times 1}^{'})'$, $\mathbf{u}_t = (u_t^{'}, 0_{12\times 1}^{'})'$, and

$$\Phi(s_t) = \begin{pmatrix} B_1(s_t) & B_2(s_t) & B_3(s_t) & B_4(s_t) \\ I_4 & 0_{4\times 4} & 0_{4\times 4} & 0_{4\times 4} \\ 0_{4\times 4} & I_4 & 0_{4\times 4} & 0_{4\times 4} \\ 0_{4\times 4} & 0_{4\times 4} & I_4 & 0_{4\times 4} \end{pmatrix}$$

$$\mathbf{B}(s_t) = \begin{pmatrix} B(s_t) & 0_{4\times 12} \\ 0_{12\times 4} & 0_{12\times 12} \end{pmatrix}$$

Let $x_{l,t}$ denote the lth element of X_t for $l = 1,...,4$. In order to measure the nonlinear effects of one unit structural shock in the kth variable $(k = 1,\cdots,4)$, we follow Koop et al. (1996) to define the generalized impulse response (*GIR* hereafter) of the lth variable at horizon h conditional on the state prevailing at the time of the shock, s_t , as:

$$GIR^{k,l}_{t+h|s_t} \quad = \quad E\left(x_{l,t+h}\middle|\,\mathbf{u}_t=\mathbf{e}_k,s_t,\Theta\right)-E\left(x_{l,t+h}\middle|\,\mathbf{u}_t=\mathbf{0},s_t,\Theta\right) \tag{21}$$

where $\mathbf{e}_k$ is the kth column of I_{16}, and Θ is the vector of parameters. From the reduced form (20), we know that the GIR can be expressed as a function of $\Phi(s_t)$ and $\mathbf{B}(s_t)$.

Observation 1.

$$GIR^{k,l}_{t+h|s_t} = \mathbf{e}_l' E\left(\prod_{i=1}^{h}\Phi(s_{t+i})\middle|\, s_t\right)\mathbf{B}(s_t)\mathbf{e}_k \tag{22}$$

Proof: See Appendix B.

Note that the *GIR* computed as equation (22) averages over future states that are allowed to switch naturally after the shock, and it coincides with the conventional impulse response function if there are no state shifts in coefficients throughout the duration of the response. Karamï¿œ (2010) notices that equation (22) can be calculated analytically by recursion, however, the magnitude of calculation becomes unwillingly for large h. We therefore propose a simulation procedure to estimate the *GIR* as follows:

- Step 1: Fix $s_t = j$.
- Step 2: According to the estimated Markov chain, simulate the ith set of future state path $\left\{s^{(i)}_{t+1}, s^{(i)}_{t+2}, \cdots, s^{(i)}_{t+h}\right\}$ conditional on $s_t = j$.
- Step 3: With the simulated state variables and the estimated coefficients, calculate

$$\prod_{i=1}^{h}\hat{\Phi}(s^{(i)}_{t+i})\middle|\, s_t = j.$$

- Step 4: Repeat step 2 and 3 for $i = 1, 2, \cdots, M$ and calculate

$$\hat{E}\left(\prod_{i=1}^{h}\hat{\Phi}(s_{t+i})\middle|\, s_t = j\right) = \frac{1}{M}\sum_{i=1}^{M}\left\{\prod_{i=1}^{h}\hat{\Phi}(s^{(i)}_{t+i})\middle|\, s_t = j\right\}$$

$$\hat{GIR}^{k,l}_{t+h|s_t} = \mathbf{e}_l' \hat{E}\left(\prod_{i=1}^{h} \hat{\mathbf{\Phi}}(s_{t+i}) \middle| s_t \right) \hat{\mathbf{B}}(s_t)\mathbf{e}_k$$

- Step 5: Repeat step 1 through 4 for $j = 1, 2$.

3 Main results

3.1 Data description

We use quarterly data from 1890Q1-2013Q4 developed by Ramey and Zubairy (2014) (RZ hereafter). Here we highlight some features of this long data set, and full details can be found in the data appendix of RZ.

The military spending news series (N) is initially constructed by Ramey (2011) and extended by RZ. In order to guarantee the news series is unanticipated and exogenous, the authors use narrative methods to estimate changes in the expected present discounted value of government spending that are related to military and political events, which are by nature very likely to be independent of the state of the economy.

Government spending (G) is defined as nominal government purchases including all federal, state, and local purchases, but net of transfer payments. Quarterly data since 1947 is from BEA NIPA. For 1889-1946, RZ first creat an annual series by splicing Kendrick's (1961) annual series starting in 1889 to the NIPA data from 1929. Then the annual series is interpolated by monthly federal outlay series from the NBER Macrohistory database and the 1954 quarterly NIPA data. Tax revenue (T) is the nominal value of federal government receipts, which is constructed in a similar way as government spending.

Output (Y) is nominal U.S. GDP. Quarterly GDP data since 1947 is from BEA NIPA. For 1889-1946, annual GDP data from BEA NIPA (1929-1946) and Historical Statistics of the United States (1889-1928) is interpolated by quarterly GNP data. Real potential GDP is constructed by splicing the February 2014 CBO estimates of real potential GDP since 1949 to an estimated cubic trend of real GDP from 1889-1950, excluding 1930 through 1946. We multiply real potential GDP by

GDP deflator to get nominal potential GDP (Y^*).

3.2 Evaluating two measures of recession

As mentioned in section 2, our methodology allows us to draw inference about the state of the economy using both quantitative and qualitative information. Based on data availability and the conventional wisdom, we consider two measures of recession: the dates identified by the NBER:

$$I_t^{NBER} = \begin{cases} 1 & \text{if the economy is identified in expansion by the NBER} \\ 2 & \text{if the economy is identified in recession by the NBER} \end{cases} \tag{23}$$

and the unemployment rate considered in RZ:

$$I_t^{UNEMP} = \begin{cases} 1 & \text{if unemployment rate } \leq 6.5\% \\ 2 & \text{if unemployment rate } > 6.5\% \end{cases} \tag{24}$$

Using the EI algorithm, we estimate the MS-SVAR model including these two indicator variables as qualitative information. Table 1 shows that the estimated values of $p\left(I_t^{NBER} = 1 \middle| s_t = 1\right)$ and $p\left(I_t^{NBER} = 2 \middle| s_t = 2\right)$ are close to 1, suggesting that the recession dates identified by the NBER serve as a reliable proxy for the true unobserved states across which the government spending multipliers may differ. On the other hand, the unemployment condition does not have much information about the true states, or at least not as good as the NBER dates.

In addition to the point estimates, we further explore some formal statistical evidence. As suggested in section 2.3, $p(I_t = i | s_t = 1) = p(I_t = i | s_t = 2)$ $(i = 1, 2)$ means that the value of the indicator variable I_t is independent of s_t, thus the indicator has no information about s_t. We run a likelihood-ratio test for this null hypothesis for I_t^{NBER} and I_t^{UNEMP}, respectively. The results in Table 2 show that we can definitely reject the null for I_t^{NBER} whereas we can not reject it for I_t^{UNEMP}. In other words, the NBER dates are very informative while the unemployment rate has little, if any, information about the true state s_t.

3.3 Cyclical behavior of fiscal multipliers

Because I_t^{UNEMP} is relatively uninformative, we keep only I_t^{NBER} in our model for parsimonious estimation. Figure XXX shows smoothed probability of s_t with $I_t = I_t^{NBER}$. The probability series closely follows NBER recession dates, which reflects high g_{11} and g_{22} of I_t^{NBER} as discussed in the previous subsection. The first half of this subsection is dedicated for presenting estimated generalized impulse-response function, and the remaining part is to discuss magnitude and time variation of fiscal multipliers.

3.3.1 Generalized impulse response

Figure 2 shows response of the output variable $\tilde{Y}$ to the news variable $\tilde{N}$. The left panel is the impulse-response function assuming no change in economic state and the right panel is generalized impulse-response function from the simulation method introduced in subsection 2.4. Both panels show significantly larger effect on the military news on output in recession. Considering low frequency of potential output, one can interpret the generalized impulse-response function that one dollar increase in defense spending increases approximately 0.17 (0.03) dollar in output on impact, conditional on current state being in recession (expansion). The effect becomes insignificant in 13 quarters in recessions, while it dies out only in 4 quarters in expansions.

Figure 3 depicts response of the government spending variable $\tilde{G}$ to the news variable $\tilde{N}$. As opposed to the response of $\tilde{Y}$ in Figure 1, the impulse-response method leads noticeable different pattern of the response of $\tilde{G}$ especially in recession periods. Assuming no change in economic state (left panel), the response of government spending is significantly larger in recession for the next 3 years of the military spending shock. However, when the possibility of regime changes is considered (right panel) there insignificant gap between the response of $\tilde{G}$ in 3 year after the shock.

We present the gaps in the generalized impulse-response functions of $\tilde{G}$ and $\tilde{Y}$ in Figure 4. The figure shows that the response of the output variable has higher gap in its response than the government spending variable, especially in two years after a military spending shock. Since a government spending multiplier is defined as ratio of the change in output to the change in government spending, the sharp difference in the magnitudes of the gaps of output and government spending responses across business cycle implies state-dependent fiscal multiplier. Moreover, larger output response in economic downturn than expansion presented in Figure 1 indicates higher fiscal multiplier in the bad time. In the next subsection, statistical evidence of regime switching fiscal multiplier is provided with estimated value of it in each state.

3.3.2 Government spending multiplier

We calculate $h-$quarter fiscal multiplier conditional on current state s_t as below:

$$h - \text{quarter multiplier}|s_t = \frac{\sum_{j=1}^{h} GIR_{t+j|s_t}^{N,Y}}{\sum_{j=1}^{h} GIR_{t+j|s_t}^{N,G}}. \tag{25}$$

As defined in (21), $GIR_{t+j|s_t}^{N,Y}$ and $GIR_{t+j|s_t}^{N,G}$ denote response of $\tilde{Y}$ and $\tilde{G}$ after j quarters to the military news shock given the state variable at period t. We use integral of responses of output and government spending instead of peak values, following Mountford and Uhlig (2009), Uhlig (2010) and Fisher and Peters (2010). This approach allows us to answer a policy question asking the magnitude of cumulative gain (or lose) of output in response to a change in cumulative government spending during a given period.

Table 3 shows estimation results of the fiscal multiplier. The table contrasts estimated fiscal multipliers under different assumptions in two dimensions. First, it compares the multipliers from the linear model, in which there is no regime switching in the model coefficients and the model with time-varying parameters. Fiscal multipliers in the linear model are presented in the second column of each panel. Second, the table shows differences in estimation results depending on impulse-response methods. The upper panel is for the estimates from generalized impulse response, and the lower panel shows fiscal multipliers assuming no change in economic states.

In the linear model, the multipliers are less than one for all impulse response horizons we consider. In contrast, fiscal multipliers from the nonlinear model are larger than one for two years after the shock that comes in the recession state ($s_t = 2$). According to our estimation, impulse-response method also affects to find sharp gaps of the multipliers depending on phases of the business cycle. Although the magnitudes of the estimated multipliers look similar in the upper and lower panels, only with GIR, the gap between the multiplier is significant. As a byproduct of our simulation-based generalized impulse response, we can obtain the simulated distribution of gaps of the multipliers between the two states, and calculate p-values of the null hypothesis that the multipliers in the different states are identical. [3] The results from generalized impulse response presented in the upper panel show significantly larger fiscal multiplier in recessions than expansions. On the contrary, if one ignores the possibility of changes in the economic state, the gap of the multiplier is not significant in 10% confidence level after one year of the shock. Qualitatively, our result is consistent to Bernbach and Gorodnichenko (2012, 2013), while it shows different implication from Ramey and Zubariy (2014), who adopt same shock identification strategy with our study but different regime identification.

3.4 Estimation without qualitative index

As a robust check, we estimated our MS-SVAR model without the qualitative index maintaining all other specifications unchanged from the main model. Figure 5 shows smoothed probability of s_t of the state variable inferred without I_t. The figure shows the state is still countercylcial - state 2 roughly corresponds to NBER recession date depicted with shaded areas. Table 4 supports the countercyclicality of the unobserved state variable. The average of real output growth rate weighted by the smoothed probabilities of the state variable indicates that the growth rate in state 1 is significantly higher than in state 2.

Table 5 shows estimated fiscal multipliers without employing the qualitative information. The value of the fiscal multiplier in each impulse-response horizon is similar to that from our main model with $I_t = I_t^{NBER}$, though the statistical evidence of larger recession multiplier is weaker than the results in Table 3. We interpret this result that adopting qualitative information allows more

[3] In order to calculate the p-value, we sorted the simulated gaps, marked the percentile of zero, and multiplied by two at that number.

efficient estimation and thus provides stronger evidence of state-dependent fiscal multipliers.

4 Reconciling the estimates of spending multipliers

Our results show that the effects of government spending stimulus are significantly larger in recessions than in expansions. However, this conclusion is not supported by all of the existing researches. Among the different voices, two leading studies are Ramey and Zubairy (2014) and Alloza (2015). The former provides extensive evidence that the government spending multipliers do not vary with the slack in the economy, while the latter claims larger policy effects in expansions than in recessions. We relate our empirical strategy to these two papers, and show how our model can reconcile the evidence they find.

4.1 Ramey and Zubairy (2014)

RZ adapt the local projection method proposed by Jordà (2005) to estimate a state-dependence model. To forecast the values of government spending and output directly allowing them to differ by the state of the economy when the military news shock hits, they estimate a set of regressions with a dummy variable I_t^{UNEMP} as defined in equation (24). In other words, they infer the economic state totally by the unemployment rate, which in our framework amounts to taking I_t^{UNEMP} as the only source of qualitative information for inference and imposing the restriction:

$$p\left(I_t^{UNEMP}=1\middle|\, s_t=1\right)=1, \quad p\left(I_t^{UNEMP}=2\middle|\, s_t=2\right)=1$$

To investigate the extent to which these assumptions may lead to the difference in conclusion between RZ and us, we estimate our MS-SVAR model under these assumptions first.[4] The results in Table 5 shows that the estimated spending multipliers are below unity and the differences between the two states are statistically insignificant in all cases, which are quite consistent with

[4]The Markov transition probabilities are unidentified in this case. We calculate it using the series of I_t^{UNEMP} for computation of generalized impulse responses.

RZ. Then we replace I_t^{UNEMP} with I_t^{NBER} that is used in our benchmark analysis, and maintain the restriction that

$$p\left(I_t^{NBER}=1\middle|\, s_t=1\right)=1, \quad p\left(I_t^{NBER}=2\middle|\, s_t=2\right)=1$$

We estimate this MS-SVAR and obtain similar results as in section 3. The multipliers for expansions are always below unity, and the multipliers for recessions are above unity for the first few years. The differences between the two states are significant at 10 percent level. As a consequence, we argue that it is the measure of the economic state that explains why we have different conclusions from RZ.

These results are not surprising if we recall the evaluation outcome of I_t^{NBER} and I_t^{UNEMP} in section 3.2, which reveals the reliability of the former indicator variable and how uninformative the latter is about the true state s_t. In fact, a typical recession dated by the NBER is usually featured by a climbing of the unemployment rate from its low point to its high point, and half of the official recession periods are of high unemployment. Bachmann and Sims (2012) show evidence that confidence is an important channel whereby government spending shocks may affect output in recessions. If confidence drops dramatically during the rise of the unemployment rate, a positive government spending shock is likely to stimulate confidence and hence stimulate economic activity. By focusing on only the unemployment status, RZ may miss a large part of this confidence channel beyond the unemployment dynamics.

4.2 Alloza (2015)

Alloza (2015) estimates a VAR that explicitly incorporates the structural shocks to military spending news ε_t^{ramey}:

$$\mathbf{X}_t = \mathbf{B}(L)\mathbf{X}_{t-1} + \mathbf{C}(L)H_t\varepsilon_t^{ramey} + \mathbf{D}(L)(1-H_t)\varepsilon_t^{ramey} + e_t \tag{26}$$

H_t is a dummy variable that takes a value of 1 during periods of NBER dated recessions and 0 otherwise. $\mathbf{X}_t$ includes government spending, output and tax revenue. $\mathbf{B}(L)$, $\mathbf{C}(L)$, $\mathbf{D}(L)$ are lag polynomials. Alloza estimates this system using quarterly data from 1948-2007, and computes impulse responses under the assumption that the economic state does not change throughout the responses. He finds that the government spending multipliers are negative for recessions and positive for expansions.

There are two main differences in empirical strategy between Alloza (2014) and us. First, he does not consider regime-switching to the coefficients on the variables other than the observable shocks, that is, $\mathbf{B}(L)$ is assumed to be time-invariant. Second, he does not allow the economic state to switch after the shock when constructing impulse response functions. To see how these differences may result in the discrepancy between our conclusions, we first try to obtain his results in our framework following his specification. To guarantee we have the same inference about the states as Alloza (2015), we include I_t^{NBER} in our model as the only qualitative information and assume

$$p\left(I_t^{NBER}=1 \middle| s_t=1\right)=1, \quad p\left(I_t^{NBER}=2 \middle| s_t=2\right)=1$$

Then we assume that in equation (2), all the structural parameters except for the (2,1), (3,1) and (4,1) elements of matrix A_j $(j=1,\cdots,4)$ are time-invariant. We estimate this model using the sample period 1948Q1-2007Q4, and compute impulse responses using both methods.

The upper-left panel of Table 8 shows that when we have the restrictions on the coefficients and compute impulse responses in Alloza's way, we obtain very similar results as him that the multipliers are negative for recessions and positive for expansions. The upper-right panel of Table 8 computes generalized impulse response functions as we propose while maintaining the restrictions on the coefficients. It shows that once the state of the economy is allowed to change, the estimated multipliers for recessions become positive, although smaller than those for expansions.

Then we consider whether the restrictions on the coefficients are reasonable or not. Table 9 shows that the restriction on the coefficients of output, government spending, and tax revenue is strongly rejected through likelihood ratio test. Relaxing the restrictions on the coefficients, if we assume fixed state in estimating the impulse-response function, the multiplier is smaller in recession with negative sign as Alloza (2015) asserts. This exercise shows that even without

Alloza's restriction which is rejected by the test, the the fiscal multiplier looks to be procyclical if the possibility of regime change is ignored in impulse-response analysis. However, the right column in the lower panel of Table 7 shows that once we compute generalized impulse response functions that allow the economic state to switch, we again reach the conclusion in this paper that the the effects of government spending shocks are larger in recessions than in expansions.

5 Conclusion

This paper explores whether government spending is more stimulative during recessions than expansions. To answer this question, we apply a Markov-switching structural VAR model that has two important features. First, following Jefferson (1998), we allow the model to use both quantitative and qualitative information to infer the unobserved state of the economy across which the spending multipliers may differ. We show that this method of inference is so general that it can include many previous studies as special cases. Second, we propose a simulation approach to constructing generalized impulse response functions that allow the economic state to change naturally.

Our results show that the business cycle is the most likely source of time variation in government spending multipliers. The multipliers are larger in recessions than in expansions, consistent with the prediction of the classic Keynesian theory. If the spending shock takes place during a recession, the multipliers will be larger than one in the first few years. However, if the shock takes place during an expansion, the multipliers will be always below unity. The difference in multiplier over the business cycle is also statistically significant.

Our approach allows us to reconcile some contradicting evidence found in the literature. First we consider Ramey and Zubairy (2014) who argue that the effects of government spending do not vary with the slack in the economy. We show that they reach this conclusion because they infer the economic state in a somewhat uninformative manner. Then we consider another leading study by Alloza (2015) who claims that the effects of government spending are negative in recessions and positive in expansions. We show that the reason why we obtain different results lies in some unrealistic assumptions they make.

This paper contributes to the literature by providing new evidence that fiscal policy could be significantly more effective during economic downturns than booms. Moreover, we provide a framework to answer many relevant questions, for example, if the effects of monetary policy depend on the business cycle. Some extensions to our model could also be made. A useful one might be to allow the regime-switching to be endogenous and allow the regime transition probabilities to be time-varying. This is left for future research.

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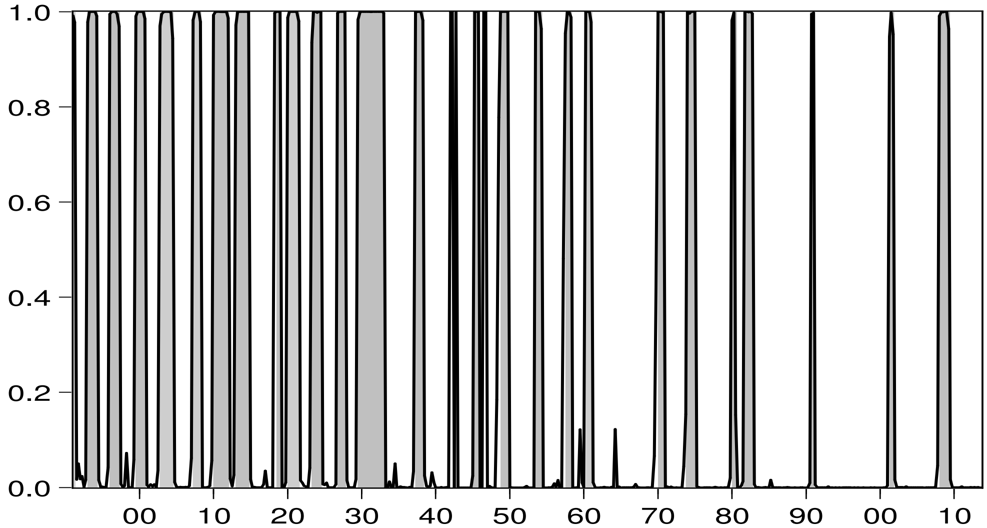
6 Figures and tables

Figure 1: Smoothed probability of s_t (with I_t=NBER)



Shaded area: NBER recession dates

Figure 2: Response of $\tilde{Y}$ to $\tilde{N}$ (with 90% confidence band)

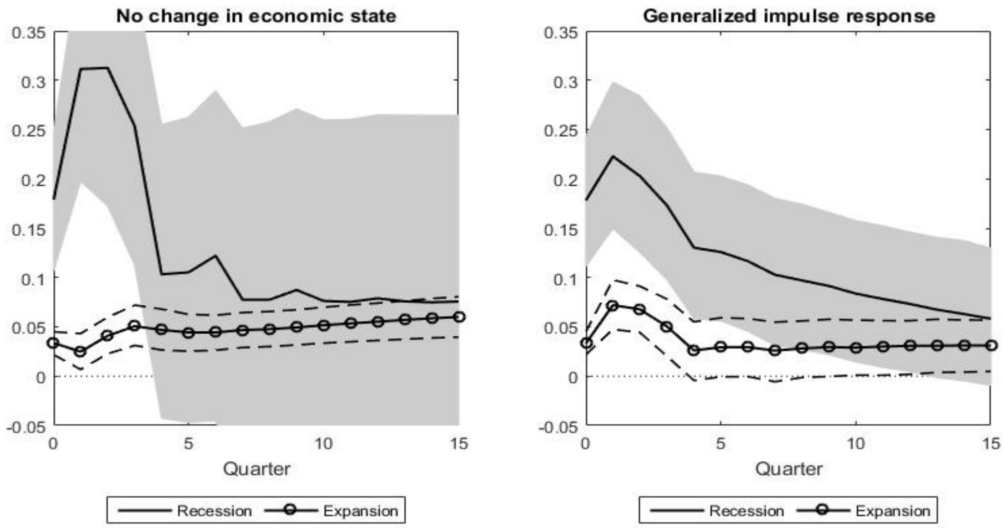


Figure 3: Response of $\tilde{G}$ to $\tilde{N}$ (with 90% confidence band)

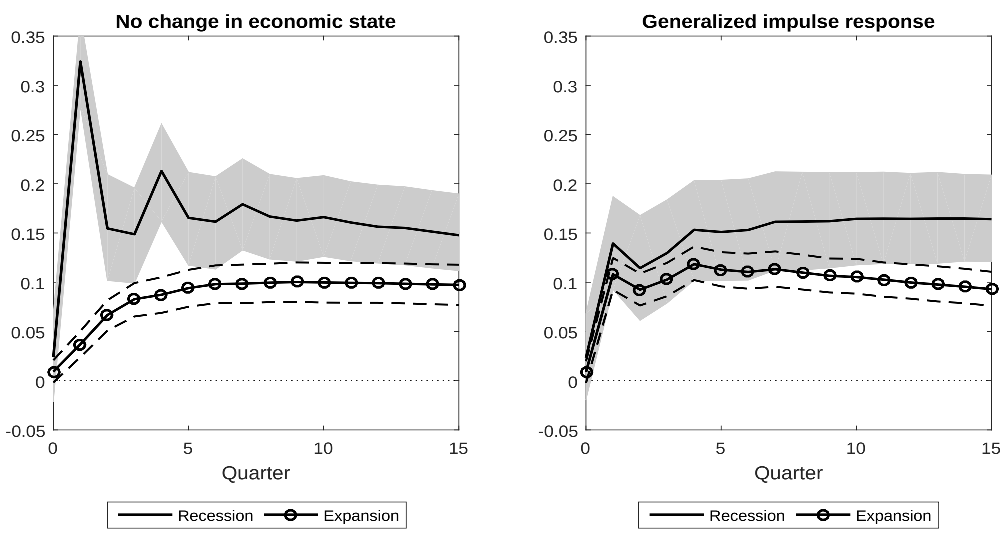
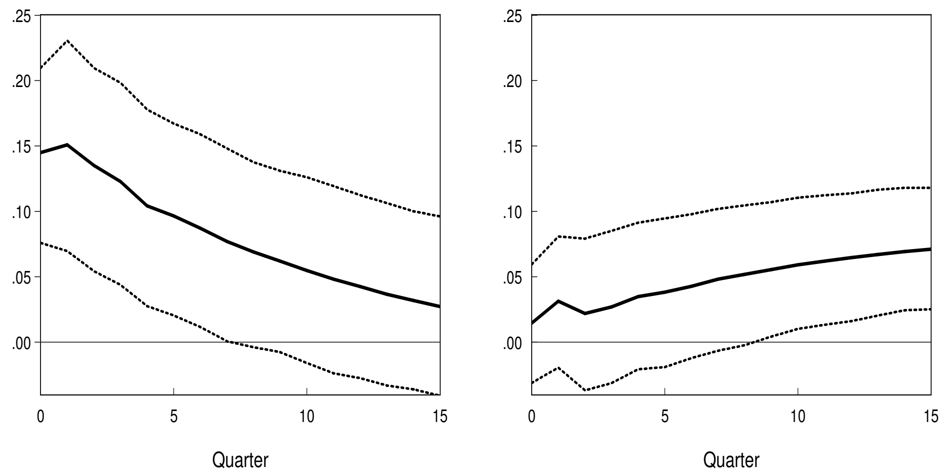


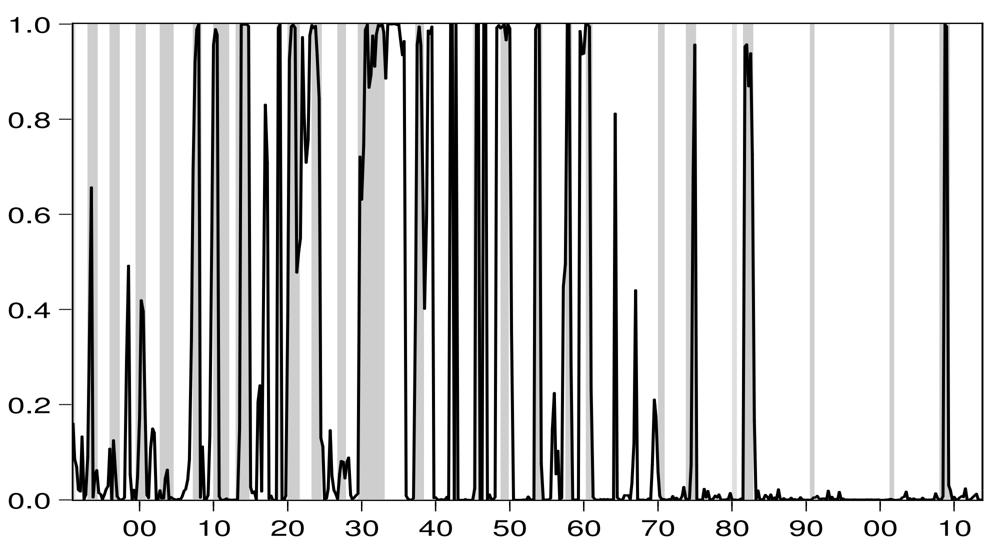
Figure 4: Gaps in GIR of $\tilde{Y}$ and $\tilde{G}$ across regimes

$GIR^{Y,G}_{t+h|s_t=2} - GIR^{Y,G}_{t+h|s_t=1}$ $\qquad$ $GIR^{N,G}_{t+h|s_t=2} - GIR^{N,G}_{t+h|s_t=1}$



Real line: generalized response, dotted line: 90% confidence line

Figure 5: Smoothed probability of s_t (without I_t)



Shaded area: NBER recession dates

Table 1: Reliability of NBER recession and unemployment dummy

	Estimation	Standard error
$p(s_t = 1 \mid s_t = 1)$	0.915	0.051
$p(s_t = 2 \mid s_t = 2)$	0.801	0.059
$p(I_t^{NBER} = 1 \mid s_t = 1)$	0.984	0.123
$p(I_t^{NBER} = 2 \mid s_t = 2)$	0.937	0.104
$p(I_t^{UNEMP} \mid s_{t-1} = 1)$	0.651	0.035
$p(I_t^{UNEMP} \mid s_{t-1} = 2)$	0.406	0.069

Table 2: Likelihood test for reliablity of I_t^{NBER} and I_t^{UNEMP}

H_0: $p(I_t^{NBER} = i \mid s_t = 1) = p(I_t^{NBER} = i \mid s_t = 2), \quad i = 1,2$		
lnL_0	lnL_1	χ^2-statistics (p-value)
-1758.863	-1644.666	228.394 (0.000)
H_0: $p(I_t^{UNEMP} = i' \mid s_t = 1) = p(I_t^{UNEMP} = i' \mid s_t = 2), \quad i' = 1,2$		
lnL_0	lnL_1	χ^2-statistics (p-value)
-1645.641	-1644.666	1.950 (0.163)

Table 3: Government spending multiplier ($I_t = I_t^{NBER}$, g_{11} and g_{22} free parameter)

$$h-\text{quarter multiplier}|\, s_t = \sum_{j=1}^{h} GIR_{t+j|s_t}^{N,Y} \Big/ \sum_{j=1}^{h} GIR_{t+j|s_t}^{N,G}$$

Generalized impulse response				
h	Linear	Expansion ($I_t^{NBER}=1$)	Downturn ($I_t^{NBER}=2$)	p-value for difference in multipliers across states
4	0.674	0.731	1.895	0.014
6	0.643	0.534	1.450	0.010
8	0.583	0.456	1.225	0.018
12	0.509	0.393	0.959	0.050
16	0.482	0.382	0.801	0.126
Impulse response assuming constant regime				
h	Linear	Expansion ($I_t^{NBER}=1$)	Downturn ($I_t^{NBER}=2$)	p-value for difference in mulipliers across states
4	0.674	0.772	1.619	0.050
6	0.643	0.643	1.218	0.172
8	0.583	0.581	1.059	0.284
12	0.509	0.549	0.867	0.508
16	0.482	0.559	0.768	0.676

Table 4: Conditional average of output growth and transition probability of s_t

	Esimation	Standard error
$\sum_{t=1}^{T} \Delta y_t \cdot p(s_t = 1 \vert \mathscr{Y}_T)$	1.528	0.108
$\sum_{t=1}^{T} \Delta y_t \cdot p(s_t = 2 \vert \mathscr{Y}_T)$	-0.890	0.166
p_{11}	0.890	0.020
p_{22}	0.714	0.051

Δy_t is growth rate of real ouput in percent. $p(s_t = i|\mathscr{Y}_T)$ is soothed probability of the $s_t = i$ and p_{ii} is transition probability from state i to i $(i = 1, 2)$.

Table 5: Government spending multiplier (without qualitative information)

$$h-\text{quarter multiplier}|\, s_t = \sum_{j=1}^{h} GIR_{t+j|s_t}^{N,Y} \Big/ \sum_{j=1}^{h} GIR_{t+j|s_t}^{N,G}$$

Generalized impulse response				
h	Linear	Unemp. rate $\leq 6.5\%$ ($I_t^{UNEMP} = 1$)	Unemp. rate $> 6.5\%$ ($I_t^{UNEMP} = 2$)	p-value for difference in multipliers across states
4	0.674	0.778	1.692	0.240
6	0.643	0.565	1.446	0.160
8	0.583	0.488	1.301	0.120
12	0.509	0.444	1.145	0.092
16	0.482	0.436	1.065	0.068

Table 6: Government spending multiplier ($I_t = I_t^{UNEMP}$, $g_{11} = g_{22} = 1$)

$$h-\text{quarter multiplier}|\, s_t = \sum_{j=1}^{h} GIR_{t+j|s_t}^{N,Y} \Big/ \sum_{j=1}^{h} GIR_{t+j|s_t}^{N,G}$$

Generalized impulse response				
h	Linear	Unemp. rate $\leq 6.5\%$ ($I_t^{UNEMP} = 1$)	Unemp. rate $> 6.5\%$ ($I_t^{UNEMP} = 2$)	p-value for difference in multipliers across states
4	0.674	0.680	0.623	0.868
6	0.643	0.717	0.697	0.904
8	0.583	0.691	0.693	0.972
12	0.509	0.680	0.713	0.868
16	0.482	0.696	0.746	0.696
Impulse response assuming constant regime				
h	Linear	Unemp. rate $\leq 6.5\%$ ($I_t^{UNEMP} = 1$)	Unemp. rate $> 6.5\%$ ($I_t^{UNEMP} = 2$)	p-value for difference in mulipliers across states
4	0.674	0.627	0.718	0.904
6	0.643	0.586	0.849	0.524
8	0.583	0.549	0.843	0.508
12	0.509	0.500	0.898	0.388
16	0.482	0.471	0.986	0.284

Table 7: Government spending multiplier ($I_t = I_t^{NBER}$, $g_{11} = g_{22} = 1$)

$$h - \text{quarter multiplier}|\, s_t = \sum_{j=1}^{h} GIR_{t+j|s_t}^{N,Y} \Big/ \sum_{j=1}^{h} GIR_{t+j|s_t}^{N,G}$$

Generalized impulse response				
h	Linear	Expansion ($I_t^{NBER} = 1$)	Downturn ($I_t^{NBER} = 2$)	p-value for difference in multipliers across states
4	0.674	0.610	1.758	0.092
6	0.643	0.373	1.426	0.088
8	0.583	0.307	1.254	0.072
12	0.509	0.218	1.056	0.088
16	0.482	0.181	0.938	0.100
Impulse response assuming constant regime				
h	Linear	Expansion ($I_t^{NBER} = 1$)	Downturn ($I_t^{NBER} = 2$)	p-value for difference in mulipliers across states
4	0.674	0.747	2.001	0.276
6	0.643	0.649	1.621	0.448
8	0.583	0.605	1.402	0.556
12	0.509	0.578	1.153	0.656
16	0.482	0.577	1.015	0.734

Table 8 : Government spending multiplier

($I_t = I_t^{NBER}$, $g_{11} = g_{22} = 1$, 1948Q1 - 2007Q4)

$$h-\text{quarter multiplier}|\,s_t = \sum_{j=1}^{h} GIR^{N,Y}_{t+j|s_t} \Big/ \sum_{j=1}^{h} GIR^{N,G}_{t+j|s_t}$$

Alloza's (2015) coefficient restriction

h	No chage in economic state Expansion ($s_t = 1$)	No chage in economic state Downturn ($s_t = 2$)	Generalized impulse response Expansion ($s_t = 1$)	Generalized impulse response Downturn ($s_t = 2$)
4	1.226	-1.541	1.140	0.658
6	0.822	-2.529	0.718	0.485
8	0.627	-3.357	0.525	0.385
12	0.487	-5.374	0.386	0.309
16	0.448	-9.563	0.350	0.291

No restrictions on coefficients

h	No chage in economic state Expansion ($s_t = 1$)	No chage in economic state Downturn ($s_t = 2$)	Generalized impulse-response Expansion ($s_t = 1$)	Generalized impulse-response Downturn ($s_t = 2$)
4	1.093	-1.120	1.149	2.330
6	0.698	-0.843	0.682	1.740
8	0.486	-1.074	0.484	1.481
12	0.336	-1.409	0.349	1.251
16	0.290	-1.710	0.314	1.140

Table 9: Coefficient restriction in Alloza (2015)

H_0: Coefficients of $\tilde{G}$, $\tilde{T}$, and $\tilde{Y}$ are not time-varying

H_1: All coefficients are time-varying

lnL_0	lnL_1	χ^2- statistics (p-value)
-168.829	-132.978	71.702 (0.003)

7 Appendix

A. Generalized impulse response

Starting from the companion form

$$\mathbf{X}_t = \mathbf{c}(s_t) + \Phi(s_t)\mathbf{X}_{t-1} + \mathbf{B}(s_t)\mathbf{u}_t, \qquad \mathbf{u}_t | d_t \sim \mathbf{N}\left(\mathbf{0}_{16}, \Sigma(d_t)\right), \tag{C.1}$$

we obtain following expression using recursive method:

$$\mathbf{X}_{t+h} = \mathbf{c}_{t+1,t+h} + \left(\prod_{i=1}^{h} \Phi(s_{t+i})\right)(\Phi(s_t)\mathbf{X}_{t-1} + \mathbf{B}(s_t)\mathbf{u}_t) + \mathbf{u}_{t+1,t+h} \tag{C.2}$$

where $\prod_{i=1}^{h} \Phi(s_{t+i}) = \Phi(s_{t+h}) \times \Phi(s_{t+h-1}) \times \cdots \times \Phi(s_{t+1})$ and

$$\mathbf{u}_{t+1,t+h} \equiv \left\{\sum_{m=1}^{h-1}\left(\prod_{j=1}^{m} \Phi(s_{t+h-j+1})\right)\mathbf{B}(s_{t+h-m})\mathbf{u}_{t+h-m}\right\} + \mathbf{B}(s_{t+h})\mathbf{u}_{t+h}$$

$$\mathbf{c}_{t+1,t+h} \equiv \left\{\sum_{m=1}^{h-1}\left(\prod_{j=1}^{m} \Phi(s_{t+h-j+1})\right)\mathbf{c}(s_{t+h-m})\right\} + \mathbf{c}(s_{t+h})$$

Note that given a future path of regimes $\{s_{t+1}, s_{t+2}, \cdots, s_{t+h}\}$, $\mathbf{u}_{t+1,t+h}$ has zero mean because of the linear structure:

$$E\left(\mathbf{u}_{t+1,t+h} \middle| s_{t+1}, s_{t+2}, \cdots, s_{t+h}\right) = \mathbf{0}_{16} \tag{C.3}$$

Our goal is to derive the response of $x_{l,t+h}$ (lth element of X_{t+h} and $\mathbf{X}_{t+h}$, $l = 1, ..., 4$) to the impulse u_{kt} (kth element of u_t and $\mathbf{u}_t$, one unit of structural shock, $k = 1, ..., 4$) given current regime s_t. In order to calculate $GIR_{t+h|s_t}$, we first need to have h-period ahead forecasting of $x_{l,t+h}$ conditional on the structural shock, current regime and the model parameters. Suppose Θ is known. Then we have

$$E\left(x_{l,t+h} \middle| \mathbf{u}_t, s_t, \Theta\right) = \mathbf{e}_l' E\left(E\left(\mathbf{X}_{t+h} | \mathbf{u}_t, s_t, s_{t+1}, \cdots, s_{t+h}\right) | \mathbf{u}_t, s_t, \Theta\right), \tag{C.4}$$

where $\mathbf{e}_l$ has one as its lth element and zero as the other. From (C.2) and (C.3),

$$\begin{aligned} & E\left(x_{l,t+h}\middle|\,\mathbf{u}_t=\mathbf{e}_k, s_t, s_{t+1},\cdots,s_{t+h},\Theta\right) \\ = \; & \mathbf{e}_l'\mathbf{c}_{t+1,t+h}+\mathbf{e}_l'\left(\prod_{i=1}^{h}\Phi(s_{t+i})\right)\{\Phi(s_t)\mathbf{X}_{t-1}+\mathbf{B}(s_t)\mathbf{e}_k\} \end{aligned}$$

$$\begin{aligned} & E\left(x_{l,t+h}\middle|\,\mathbf{u}_t=\mathbf{0}_{16}, s_t, s_{t+1},\cdots,s_{t+h},\Theta\right) \\ = \; & \mathbf{e}_l'\mathbf{c}_{t+1,t+h}+\mathbf{e}_l'\left(\prod_{i=1}^{h}\Phi(s_{t+i})\right)\Phi(s_t)\mathbf{X}_{t-1} \end{aligned}$$

Then (B.4) can be rewritten as below:

$$\begin{aligned} & E\left(x_{l,t+h}\middle|\,\mathbf{u}_t=\mathbf{e}_k, s_t,\Theta\right) \\ = \; & \mathbf{e}_l'\mathbf{c}_{t+1,t+h}+\mathbf{e}_l'E\left(\prod_{i=1}^{h}\Phi(s_{t+i})\middle|\,s_t\right)\{\Phi(s_t)\mathbf{X}_{t-1}+\mathbf{B}(s_t)\mathbf{e}_k\} \\ & E\left(x_{l,t+h}\middle|\,\mathbf{u}_t=\mathbf{0}_{16}, s_t,\Theta\right) \\ = \; & \mathbf{e}_l'\mathbf{c}_{t+1,t+h}+\mathbf{e}_l'E\left(\prod_{i=1}^{h}\Phi(s_{t+i})\middle|\,s_t\right)\Phi(s_t)\mathbf{X}_{t-1}, \end{aligned}$$

which give observation 2 in the main text:

$$\begin{aligned} GIR_{t+h|s_t}^{k,l} \; & \equiv \; E\left(x_{l,t+h}\middle|\,\mathbf{u}_t=\mathbf{e}_k, s_t,\Theta\right)-E\left(x_{l,t+h}\middle|\,\mathbf{u}_t=\mathbf{0}_{16}, s_t,\Theta\right) \\ & = \; \mathbf{e}_l'E\left(\prod_{i=1}^{h}\Phi(s_{t+i})\middle|\,s_t\right)\mathbf{B}(s_t)\mathbf{e}_k. \end{aligned}$$