Full Information Estimation of Household Income Risk and Consumption Insurance

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February 1, 2017 §

Keywords: Full information estimation, income risk, consumption insurance, panel unobserved components model.

JEL codes: E32; E22; C32

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§We thank James Hansen, and participants at the Workshop of the Australasian Macroeconomics Society (Brisbane), Annual Conference on Economic Growth and Development (Delhi), Sydney Macro Reading Group Workshop and Internal Brown Bag at University of Sydney for comments. We are grateful for the financial support from the Australian Research Council grant DE130100806 (Singh). The usual disclaimers apply.

Abstract

We develop a panel unobserved components model of household income and consumption that can be estimated using full information methods. Maximum likelihood estimates for a simple version of this model suggests similar income risk, but higher consumption insurance relative to the partial information moments-based estimates of Blundell, Pistaferri, and Preston (2008) using the same panel dataset. Bayesian model comparison supports this simple version of the model with a spillover from permanent income to permanent consumption only, no persistence in transitory components, and no cointegration. However, consumption insurance and income risk estimates are robust across different specifications.

1 Introduction

For panel data analysis, partial information generalized method of moments is often used to estimate parameters of interest. For time series analysis, by contrast, a full information likelihood approach is typically used. Our objective in this paper is to explore what happens when a full information likelihood approach is applied to a panel dataset.

Blundell, Pistaferri and Preston (2008) (BPP hereafter) construct a novel panel dataset of household income and consumption and consider partial information moments-based estimation of income risk and consumption insurance without imposing a particular structural model for household's behavior and decisions. They find a relatively low degree of consumption insurance in response to idiosyncratic shocks to permanent income. Their results have been challenged by Kaplan and Violante (2010) (KV hereafter), who argue that the BPP estimation strategy leads to a downward bias in consumption insurance, and the bias is more pronounced for households that are borrowing constrained.

In this paper, we propose an alternative estimation strategy for a more general panel model of household income and consumption. In particular, we consider full information methods of estimation for a panel unobserved components (UC) model. Despite data limitations such as missing observations and a short time dimension to the panel given annual data, we are able to estimate the panel UC model assuming a common distribution and independent shocks across households. Maximum likelihood estimates for a simple version of the model suggest similar income risk, but higher consumption insurance relative to BPP using the same panel dataset. Strikingly, our consumption insurance estimate is 17 percentage points higher than what was found by BPP using partial information moments-based estimation. Thus, a full information approach seems to help address KV's concern regarding downward bias in BPP'S estimation of the consumption insurance parameter.

In a full information environment, Bayesian methods allow us to easily compare different specifications of our model. In particular, we calculate marginal likelihoods to consider spillovers across different components, persistence in transitory income and consumption, and possible cointegration between income and consumption in driving our findings. This Bayesian model comparison supports the simple version of our model used in our MLE analysis as well that only has a spillover from permanent income to permanent consumption, no persistence in transitory components, and no cointegration. However, consumption insurance and income risk estimates are robust across the various specifications under consideration. At the same time, prior sensitivity analysis makes it clear that the degree of consumption insurance is not particularly well identified in the data, although it would take a highly informative and distorted prior to obtain the lower estimates previously found with partial information methods.

Subgroup estimation shows that our results regarding model specification are robust to different types of households and that the pattern of heterogeneity in consumption insurance estimates is intuitive. In particular, we find that consumption insurance is higher for older or more educated households. Furthermore, comparing our estimates with the corresponding BPP estimates, we find that the difference in estimates is larger for the subgroup that are potentially more borrowing constrained, for example the subgroup without college education, as argued by KV.

The rest of this paper is organized as follows: Section 2 presents the general panel UC model proposed in this paper. Section 3 describes the data. Section 4 reports empirical results. Section 5 provides discussion of how our results relate to the literature and concludes.

2 A Panel Unobserved Components Model

In this section we present the details of our panel UC model of household income and consumption. We also write the BPP model in a similar form to see how it compares with our model.

2.1 General Model Specification

Following Friedman and Kuznet (1954), the estimated household income process typically has a random walk permanent component, a transitory component that dies away, and zero correlation between movements in the two components. See, for example, Moffitt and Gottschalk (2002), Storesletten, Telmer, and Yaron (2004), Guvenen (2007), Blundell, Pistaferri, and Preston (2008), Primiceri and van Rens (2009), Low, Meghir, and Pistaferri (2010), and Heathcote, Perri, and Violante (2010), among many others. However, it is straightforward to show that, if the zero correlation is incorrectly assumed, the model mis-specification will bias the estimate of permanent risk, a key ingredient in heterogeneous agent quantitative macro models.[1] In the time series literature using aggregate U.S. quarterly real GDP data, Morley, Nelson, and Zivot (2003) clearly establish that the assumption of zero correlation between permanent and transitory movements can be rejected in the univariate case, while Morley (2007) finds evidence in favor of correlated movements using U.S. quarterly real GDP and consumption data in a multivariate unobserved components model.[2] Motivated by these results, the general model presented below allows for correlated movements in unobserved components of income and consumption, with random walk permanent components and persistent dynamics for the transitory components.

Our panel unobserved components model decomposes idiosyncratic income and consumption for household i (measured as residuals from regressions of household income and consumption on common observed factors) into permanent components and transitory deviations from the

[1] For example, see Ejrnaes and Browning (2014) for more details.

[2] Note, however, Morley (2007) considers total income, not just labor income, as is considered in this paper.

permanent components:[3]

$$y_{i,t} = \tau_{i,t} + (y_{i,t} - \tau_{i,t}), \tag{1}$$

$$c_{i,t} = \gamma_\eta \tau_{i,t} + \kappa_{i,t} + (c_{i,t} - \gamma_\eta \tau_{i,t} - \kappa_{i,t}). \tag{2}$$

The permanent components are specified as random walks with drift:

$$\tau_{i,t} = \mu_{\tau,i} + \tau_{i,t-1} + \eta_{i,t}, \qquad \eta_{i,t} \sim i.i.d.N(0, \sigma_\eta), \tag{3}$$

$$\kappa_{i,t} = \mu_{\kappa,i} + \kappa_{i,t-1} + u_{i,t}, \qquad u_{i,t} \sim i.i.d.N(0, \sigma_u), \tag{4}$$

while the transitory components are specified as ARMA(p,q) processes:

$$\phi_y(L)(y_{i,t} - \tau_{i,t}) = \lambda_{y\eta}\eta_{i,t} + \theta_y(L)\epsilon_{i,t}, \tag{5}$$

$$\phi_c(L)(c_{i,t} - \gamma_\eta \tau_{i,t} - \kappa_{i,t}) = \lambda_{c\eta}\eta_{i,t} + \lambda_{c\epsilon}\epsilon_{i,t} + \theta_c(L)v_{i,t}, \tag{6}$$

where $\phi_y(L) = (1 - \phi_{y,1}L - \phi_{y,2}L^2 - ... - \phi_{y,p}L^p)^{-1}$, $\phi_c(L) = (1 - \phi_{c,1}L - \phi_{c,2}L^2 - ... - \phi_{c,p}L^p)^{-1}$, $\theta_y(L) = (1 - \theta_{y,1}L - \theta_{y,2}L^2 - ... - \theta_{y,q}L^q)^{-1}$, and $\theta_c(L) = (1 - \theta_{c,1}L - \theta_{c,2}L^2 - ... - \theta_{c,q}L^q)^{-1}$ are lag polynomials that satisfy stationarity and invertibility constraints.

The permanent income shock in our empirical model is $\eta_{i,t}$ and it can be interpreted as reflecting shocks to health, promotion, or other idiosyncratic factors that result in a change in permanent income. Examples of permanent shocks to consumption, $u_{i,t}$, in addition to the permanent shocks to income, could be taste and preference shocks or other shocks to non-labor income, such as wealth shocks. The transitory consumption shock could capture measurement error which could be due to the imputation of non-durable consumption. We note that the model assumes time-invariant

[3] In particular, following BPP, we compute idiosyncratic income and consumption by removing the impact of observables such as education, race, family size, number of children, region, employment status, year and cohort effects, residence in large city, and income recipients other than husband and wife from total household disposable labor income.

volatilities of shocks, although we could, in principle, test for and allow structural breaks in these parameters.[4]

We note that, instead of directly specifying the shocks to be correlated as in Morley, Nelson, and Zivot (2003) and Morley (2007), we assume that permanent shocks can affect the transitory components according to impact coefficients $\lambda_{y\eta}$ and $\lambda_{c\eta}$. Thus, permanent and transitory movements will be correlated as in Morley, Nelson, and Zivot (2003) and Morley (2007). However, following Morley and Singh (2016), we explicitly assume the source of correlation as being due to the effects of permanent shocks on transitory components. Meanwhile, $\lambda_{c\epsilon}$ captures the response of consumption to transitory income shocks. For simplicity, we are assuming no corresponding effect of transitory consumption shocks on income.

Based on our panel UC model, we can solve for consumption growth for household i as follows:

$$\Delta c_{i,t} = \gamma_\eta \eta_{i,t} + u_t + (1-L)\phi_c(L)^{-1}(\lambda_{c\eta}\eta_{i,t} + \lambda_{c\epsilon}\epsilon_{i,t} + \theta_c(L)v_{i,t}), \tag{7}$$

which suggests that changes in consumption depend on the full history of permanent shocks to income.[5]

To calculate the implied consumption insurance based on our model, a change in consumption at date t due to the permanent income shock η_t is $\gamma_\eta + \lambda_{c\eta}$. Therefore, the consumption insurance coefficient is

$$\vartheta_c = 1 - (\gamma_\eta + \lambda_{c\eta}). \tag{8}$$

KV define the insurance coefficient with respect to permanent income shock as the share of the variance of the shock that does not translate into con-

[4]For example, BPP use the same panel data to look at changes in income and consumption inequality over time. We leave this for future research and focus on estimating average levels of income risk and degree of consumption insurance over the full sample period. Subsample analysis available from the authors upon request suggests that the full sample estimates are generally very close to averages of the subsample estimates.

[5]In KV's terminology, this means that there is no "short memory" in our model.

sumption growth such that

$$\vartheta_c = 1 - \frac{cov(\Delta c_t, \eta_t)}{var(\eta_t)} = 1 - \frac{cov(\gamma_\eta \eta_t + z_t^c, \eta_t)}{\sigma_\eta^2} = 1 - \frac{\gamma_\eta \sigma_\eta^2 + \lambda_{c\eta}\sigma_\eta^2}{\sigma_\eta^2}, \quad (9)$$

which the same as equation (8) for our empirical model. See Appendix B for more details and definition of z_t^c.

2.2 BPP Model

We re-write the BPP model in a similar form to our model for comparison (the state space representation is given in Appendix A). In particular, the BPP model has an implicit UC representation for income:

$$y_{i,t} = \tau_{i,t} + (y_{i,t} - \tau_{i,t}). \quad (10)$$

The permanent component of income is specified as follows:

$$\tau_{i,t} = \mu_i + \tau_{i,t-1} + \eta_{i,t}, \qquad \eta_{i,t} \sim i.i.d.N(0, \sigma_\eta). \quad (11)$$

The transitory component has a moving average, in particular, an $MA(1)$, specification as follows:

$$(y_{i,t} - \tau_{i,t}) = \epsilon_{i,t} + \theta\epsilon_{i,t-1}, \qquad \epsilon_{i,t} \sim i.i.d.N(0, \sigma_\epsilon). \quad (12)$$

Then, consumption growth, or change in logarithm of residual consumption has the following process:

$$\Delta c_{i,t} = \gamma_\eta \eta_{i,t} + \gamma_\epsilon \epsilon_{i,t} + u_{i,t} + \Delta u_{i,t}^\star, \qquad u_{i,t} \sim i.i.d.N(0, \sigma_u), \quad (13)$$

where η_t and ϵ_t are the permanent and transitory income shocks, u_t is the permanent shock to consumption, and $u_{i,t}^\star \sim i.i.d.N(0, \sigma_{u^\star})$ is measurement error for consumption. As discussed in BPP, the permanent shock to consumption captures taste and preference shocks, while the measurement error reflects errors of imputing non-durable consumption for the PSID.

Our panel UC model differs from the BPP specification in one key way. In particular, a transitory income shock can only impact transitory consumption in our model, while in the BPP model, transitory income shocks are assumed to have a completely permanent impact on consumption. To see this, we can rewrite the level of consumption, after suppressing the individual specific subscript for simplicity, as

$$c_t = \gamma_\eta \tau_t + \gamma_\epsilon Z_{\epsilon,t} + Z_{u,t} + u^{\star}_{i,t}, \tag{14}$$

$$Z_{\epsilon,t} = Z_{\epsilon,t-1} + \epsilon_t, \tag{15}$$

$$Z_{u,t} = Z_{u,t-1} + u_t, \tag{16}$$

We examine which specification has more support in the data in section 4.

3 Data

In this section we briefly describe the data created by BPP and look at autocorrelations to help motivate model specification in the next section. For full details of the data, we refer the reader to the BPP paper.

3.1 BPP Data

BPP use the Panel Study of Income Dynamics (PSID) sample from 1978-1992 of continuously married couples headed by a male (with or without children) age 30 to 65. The income variable is family disposable income which includes transfers. They adopt a similar sample selection in the Consumer Expenditure Survey (CEX). Since CEX has detailed non-durable consumption data, unlike PSID which primarily has food expenditure data, they impute non-durable consumption for each household

TABLE 1. SAMPLE ACF AND PACF

a_1	a_2	a_3	p_1	p_2	p_3
		Δy			
−0.29	−0.03	−0.01	−0.29	−0.13	−0.07
		Δc			
−0.34	−0.01	−0.02	−0.34	−0.14	−0.04

Notes: Autocorrelations and partial autocorrelations are calculated using 12041 observations.

per year by using the estimates of the food demand from CEX.The final dataset is a panel of income and imputed non-durable consumption. To get idiosyncratic (residual) income and consumption, they regress income and consumption for households on a vector of regressors including demographic and ethnic factors and other income characteristics observable/known by consumers. It is this residual idiosyncratic income and consumption that is modeled in section 2.

3.2 Sample Autocorrelations

To help motivate model specification in the next section, we compute the sample autocorrelation function (ACF) and partial autocorrelation function (PACF) for idiosyncratic income and consumption growth from the BPP dataset by pooling individuals of all ages and over all years. Table 1 reports the results.

Based on the sample ACFs and PACFs we can see that the ACFs cut off after 1 lag, but the PACFs tail off more gradually for both income growth and consumption growth. This pattern is consistent with an MA(1) process, not an MA(2) process, as would be implied for income growth by the BPP model. Moreover, this pattern is suggestive of a simple specification for the general panel UC model in section 2. In particular, it is consistent with a simple model in which both income and consumption follow a random walk plus noise. We start with this specification, but also conduct formal model comparison to determine the best specification.

4 Empirical Results

Instead of relying on a few moments to estimate key parameters of the permanent-transitory model of income and consumption, we use the entire likelihood for our estimation. A clear benefit of this full information approach is that it addresses possible extreme sensitivity of inferences to particular moments. For example, in the idiosyncratic income/wage risk literature Heathcote, Perri and Violante (2010) find that the estimates of the variance of the wage shocks are different if one uses moment conditions based on log residual wage growth or moment conditions based on log residual wage level.[6] Using the panel dataset of residual income and consumption, we report our empirical results using a full information approach in this section.

4.1 Maximum likelihood estimates for a simple version of the model

Motivated by the sample autocorrelations, we estimate a simple version of the model in section 2, which we refer to as the UC-WN model hereafter, using maximum likelihood. In this simple model, the transitory components have no persistence (i.e., the ϕ's in the general model are set to zero) and, for simplicity, we only consider a spillover from permanent income to permanent consumption, as captured by γ_η (i.e., the λ's in the general

[6] This inconsistency between the estimates has been reported by Brzozowski, Gervais, Klien and Suzuki (2010) for Canada; Fuchs-Schundeln, Krueger and Sommer (2010) for Germany, Floden and Domeij (2010) for Sweden and Chatterjee, Singh and Stone (2016) for Australia. However, due to differences in the dataset, sample selection and the estimated equation, our results may not be directly address the level versus growth puzzle. See Daly, Hryshko and Manovskiis (2016) who study this puzzle much more closely.

TABLE 2. MAXIMUM LIKELIHOOD ESTIMATES

	INCOME
σ_η	0.13 (0.002)
σ_ϵ	0.20 (0.002)
	CONSUMPTION
σ_u	0.09 (0.002)
σ_v	0.28 (0.002)
γ_η	0.42 (0.017)

Notes: The table reports maximum likelihood estimates with standard deviations reported in parentheses for UC-WN model.

model are set to zero). In spite of having a short time series for each individual in the sample, with a maximum time dimension of 14, maximum likelihood estimation procedure is feasible for this model because each observation effectively becomes an independent draw from the data generating process, making only the total sample size TN relevant for precision of inference rather than T mattering separately in addition to N for identifying parameters when there is dependence between observations across time.

Based on Table 2, the variance of the permanent income shock is 0.02 (0.13^2). This is similar to the estimate in BPP and also close to what one finds in the related idiosyncratic income risk literature. However, what is striking is that, using the same dataset as BPP but taking a full information approach, our estimate of consumption insurance, 1-γ_η, is 0.58 while the corresponding estimate in BPP is 0.36. This result provides some support for Kaplan and Violante (2010) who argue that BPP estimate of consumption insurance is likely to be biased downward.

It is not clear based on these results alone whether the differences in estimates are because of differences in model specification or because of the full information approach versus the partial information moments based approach. Moreover, despite the suggestion from the autocorrelations, is the UC-WN model really the right specification for the BPP dataset? We answer these questions in the subsequent subsections and use Bayesian methods to do so. We take a Bayesian approach because the general panel UC model in section 2 can suffer from weak identification or even be impossible to estimate via maximum likelihood given a small time dimension T when the transitory components of income and consumption are persistent, inducing dependence in observations for each individual household. By imposing reasonable priors for parameters based on past studies and *a priori* reasoning, we are able to estimate specifications of the panel UC model that imply time dependence, as well as compute marginal likelihoods to determine which model fits the data best. Another advantage of the Bayesian approach is that we can estimate the implied variances of idiosyncratic income and consumption growth, which are complicated functions of the model parameters, and compare them with the corresponding sample moments.

4.2 Bayesian model comparison

Much of the literature on earnings has moved away from a simple model in which the permanent component is a random walk and the transitory component is white noise, the UC-WN model, in recent years. It is now generally believed that the earnings dynamics are more complicated. MaCurdy (1982) and Abowd and Card (1982) find that the covariance matrix of earnings differences fits an MA(2), Gottschalk and Moffitt (1993) fit random walk plus ARMA(1,1) in levels which is an ARMA(1,2) in first differences, Heathcote, Storesletten and Violante (2010, 2014) employ a very persistent component and a white noise transitory component. We focus on two main specifications of the general model discussed in section 2.1, a UC-AR(2) model and the more traditional UC-WN model to encompass different views held in the literature.[7] Motivated by the findings for per-

[7] Following Morley, Nelson, Zivot (2003), the UC-AR(2) model is identified because $p = q + 2$ for the implied ARMA(p,q) process in first differences.

sistent autoregressive dynamics in the aggregate data found in the time series literature, we also investigate whether these dynamics play an important role in panel data and if income and consumption share a common trend like they do in the aggregate data.

We estimate our panel UC models using Bayesian posterior simulation based on Markov-chain Monte Carlo (MCMC) methods. We use multi-block random-walk chain version of the Metropolis-Hastings (MH) algorithm with 20,000 draws after a burn-in of 20,000 draws. To check the robustness of our posterior moments, we use different starting values. Our prior distributions are loosely motivated by the vast empirical literature on modeling income and consumption dynamics. First, the priors for the precisions (inverse variances) are $\Gamma(2.5, 2.5)$. Meanwhile, because there is no consensus in the literature regarding the estimate of the impact of permanent income shock on consumption, we choose an uninformative $U(0, 1)$ prior for γ_η. The priors for the impact coefficients, $\lambda_{y\eta}$, $\lambda_{c\eta}$, and $\lambda_{c\epsilon}$ are $TN_{[-1,1]}(0, 0.5^2)$–i.e., they are truncated to ensure that they lie between -1 and 1. The priors for autoregressive and moving-average coefficients are $TN_{|z|>1,\phi(z)=0}(0, 0.5^2)$ and $TN_{|z|>1,\theta(z)=0}(0, 0.5^2)$–i.e., they are truncated to ensure stationarity or invertibility.

Table 3 reports results for our different model specifications. First, estimates for the full UC-AR(2) model in column 2 suggest no persistent transitory dynamics, but permanent income shocks have an immediate positive impact on transitory income. This stands in contrast to some other studies, such as Hyrshko (2010) and Belzil and Bognanno (2008), which find a negative correlation between the permanent and transitory shocks. For both the income and consumption processes, transitory shocks are more volatile compared to permanent shocks. The implied variances of income and consumption closely match their corresponding counterparts in the data, 0.09 and 0.16. Second, when we shut down the additional permanent shocks to consumption beyond permanent income shocks in column 3, the implied variance of consumption is much lower than the variance of consumption in the data. Third, we set the impact coefficients to zero

TABLE 3. ESTIMATES OF PANEL UC MODELS

	UC-AR(2)			UC-WN
	full	$\sigma_u = 0$	$\lambda = 0$	
	INCOME			
ϕ_{y1}	-0.02 (0.01)	-0.10 (0.02)	-0.08 (0.01)	
ϕ_{y2}	-0.05 (0.01)	-0.10 (0.02)	-0.11 (0.01)	
σ_η	0.14 (0.01)	0.14 (0.01)	0.14 (0.01)	0.14 (0.01)
σ_ϵ	0.16 (0.02)	0.16 (0.02)	0.16 (0.02)	0.17 (0.01)
$\lambda_{y\eta}$	0.11 (0.01)	0.10 (0.01)		
	CONSUMPTION			
ϕ_{c1}	-0.12 (0.01)	-0.37 (0.01)	-0.30 (0.01)	
ϕ_{c2}	-0.07 (0.01)	-0.32 (0.01)	-0.20 (0.01)	
σ_u	0.13 (0.01)		0.14 (0.01)	0.13 (0.01)
σ_v	0.20 (0.02)	0.29 (0.02)	0.19 (0.02)	0.21 (0.02)
$\lambda_{c\eta}$	0.05 (0.01)	0.02 (0.01)		
$\lambda_{c\epsilon}$	-0.03 (0.01)	-0.02 (0.01)		
γ_η	0.47 (0.02)	0.43 (0.02)	0.47 (0.02)	0.47 (0.02)
	IMPLIED VARIANCE			
Δy	0.08 (0.00)	0.08 (0.00)	0.08 (0.00)	0.08 (0.00)
Δc	0.16 (0.00)	0.12 (0.00)	0.17 (0.00)	0.15 (0.00)
	MARGINAL LIKELIHOOD (IN LOGS)			
	-89595	-110416	-90295	-89041

Notes: The table reports posterior means of panel UC model parameters with posterior standard deviations reported in parentheses. The third panel reports the variance of residual income and residual consumption growth implied by the model and the marginal likelihood is in the bottom panel. The total number of households are 1765.

in column 4 and find relatively similar estimates for the other parameters as for the full UC-AR(2) model in column 2. Fourth, we shut down all dynamics and set all impact coefficients to zero in column 5, which corresponds to the UC-WN model, and find, again, that the estimates of the remaining parameters remain similar to the full UC-AR(2) model. In particular, the variance of income shocks and transitory shocks to consumption, as well as the pass-through of the permanent income shock to consumption, are quite similar across specifications.

Using Bayesian methods, we can compare different models to determine which model is supported by the data. We do so by computing the marginal likelihood following the method in Chib and Jeliazkov (2001). The last row in Table 3 clearly shows that the UC-WN model is preferred.[8] Note that the estimates of the key parameters of interest in the last column of Table 3, the permanent shock to income and the consumption insurance, are similar to the maximum likelihood estimates.

Based on our preferred model the variance of the permanent income shock is 0.02. Most importantly, we find that using uninformative prior, our estimate of consumption insurance is 17 percentage points higher than what was previously estimated. This fact that our estimate of consumption insuance is higher than BPP is consistent with the Kaplan and Violante (2010) who argue that BPP estimate of consumption insurance is likely to be biased downward. Moreover, our results of higher consumption insurance are also in line with Heathcote, Storesletten and Violante (2014) who take a structural approach. Finally, the implied volatilities of income and consumption growth for our preferred model are 0.08 and 0.15. The corresponding counterparts in the BPP dataset are 0.09 and 0.16.

Our analysis so far suggests that the UC-WN model is the correct spec-

[8]These results stand in contrast to those for the aggregate data (Morley (2007)), although this is perhaps not surprising given that common shocks have been removed from the data and idiosyncratic shocks are likely due to very different factors with different behaviors than the common shocks that drive the aggregate data. Also, we are using annual data, while Morley (2007) uses quarterly data, meaning that the transitory components for the aggregate data would be less persistent, as measured by, say, the sum of the AR(2) parameters, when considered at an annual frequency.

TABLE 4. BPP ESTIMATES USING BAYESIAN APPROACH

Prior γ_η	$TN_{[-1,1]}(0.65, 0.25^2)$	$TN_{[-1,1]}(0.65, 1^2)$	$U(0,1)$	BPP estimate/data
		INCOME		
θ	0.01(0.01)	0.00(0.01)	−0.04(0.02)	0.11
σ_η	0.14(0.01)	0.14(0.01)	0.15(0.01)	0.14
σ_ϵ	0.17(0.01)	0.17(0.01)	0.17(0.02)	0.17
		CONSUMPTION		
γ_η	0.64(0.01)	0.55(0.02)	0.46(0.02)	0.64
γ_ϵ	0.002(0.00)	−0.02(0.01)	−0.01(0.01)	0.05
σ_u	0.13(0.01)	0.13(0.01)	0.13(0.01)	0.11
$\sigma_{u^\star}$	0.21(0.02)	0.21(0.02)	0.21(0.02)	NA
		IMPLIED VERSUS ACTUAL VARIANCE		
Δy	0.08(0.00)	0.08(0.00)	0.08(0.00)	0.09
Δc	0.11(0.00)	0.11(0.00)	0.11(0.00)	0.16

Notes: The table reports posterior means of model parameters with posterior standard deviations reported in parentheses. The bottom panel reports the variance of residual income and residual consumption growth implied by the model versus the corresponding averages in the BPP data.

ification for the BPP data. In the next subsection we examine whether the estimates that we find are different due to differences in model specification or due to differences in approach, the full information approach versus the partial information approach.

4.3 Bayesian estimates for BPP model and prior sensitivity analysis

We employ the same methodology as in section 4.2 to estimate the BPP model. The priors for the additional parameters, θ and γ_ϵ are $TN_{[-1,1]}(0, 0.5^2)$, i.e., they are truncated to lie between -1 and 1.[9] In addition, we conduct the prior sensitivity analysis by varying the prior on γ_η in particular to investigate its role in determining the posterior estimate.

Table 4 reports the estimates of the BPP model using Bayesian methods. It is quite clear our estimation method can recover the volatility of

[9] All the other priors are the same as for our panel UC model.

income shocks and the consumption insurance parameter from BPP when the prior on γ_η is tight around the BPP estimate.[10] However, when the prior is less informative in the case of $TN_{[-1,1]}(0.65, 1^2)$ the estimate moves towards higher consumption insurance and similar to what we find with a uniform prior for our preferred UC-WN model. Note that in the last row of Table 4, the implied variance of residual consumption growth is 0.11, while the variance of residual consumption growth in the BPP sample is 0.16. This seems plausible as Figure 5 in BPP suggests that the process of consumption growth implied by their baseline model does not match the data all that well in the latter part of the sample. Finally, the marginal likelihood from the BPP model where γ_η is uniformly distributed, column 4 in Table 4, is -89946. This suggests that our UC-WN model with a higher marginal likelihood provides a better fit to the BPP dataset.[11]

We have demonstrated in this subsection that using the full information approach we can estimate the BPP model. Based on the prior sensitivity analysis we also find that consumption insurance is not particularly well identified in the data. As a result, it was necessary to impose a highly informative and distorted prior to obtain the lower estimates previously found with partial information methods.

5 Subgroup Estimates

In this section, we examine how estimates vary across different groups of households based on education and age. Since our results regarding model specification are robust to different types of households, we we report results for UC-WN specification in Tables 5 and 6.

From Table 5, it is clear that for the college educated the pass-through

[10] Note that the estimate of the standard deviation of the measurement error is not reported in Table 6 of BPP.

[11] Using simulated data we find that full information approach performs reasonably well when there is model mis-specification. In particular, if the DGP is UC-WN and we fit a BPP model, our full-information approach recovers the true parameters quite well. See Appendix C1 for more details.

TABLE 5. EDUCATION HETEROGENEITY

	No college	College
	INCOME	
σ_η	0.14 (0.02)	0.14 (0.02)
σ_ϵ	0.18 (0.02)	0.16 (0.02)
	CONSUMPTION	
σ_u	0.13 (0.02)	0.13 (0.02)
σ_v	0.24 (0.03)	0.19 (0.02)
γ_η	0.65 (0.03)	0.34 (0.02)

Notes: The table reports posterior means of model parameters with posterior standard deviations reported in parentheses. There are 883 households in the no college group and 882 in the college group.

TABLE 6. AGE HETEROGENEITY

	Young (30-47)	Old (48-65)
	INCOME	
σ_η	0.13 (0.01)	0.14 (0.02)
σ_ϵ	0.15 (0.01)	0.19 (0.02)
	CONSUMPTION	
σ_u	0.13 (0.01)	0.13 (0.02)
σ_v	0.21 (0.02)	0.20 (0.02)
γ_η	0.55 (0.03)	0.40 (0.03)

Notes: The table reports posterior means of model parameters with posterior standard deviations reported in parentheses. There are 1413 households for the young while the number of households for the old is 708.

of permanent income shocks to consumption is 34 percent which is approximately half of the pass through of income shocks to consumption for the households with no college education. Qualitatively, these results are similar to BPP, however the magnitudes are different. KV find that the downward bias in consumption insurance using BPP estimator is much more pronounced for households that are borrowing constrained and our results seem consistent with their analysis. To see this, consider households without college education that we think are likely to be more borrowing constrained than households with college education. We find that our estimate of consumption insurance for no college group is 6 times higher than BPP (.35 in ours versus .06 in BPP), while the apparent downward bias is not so large for households with college education (.66 in ours versus .58 in BPP).

Estimating our model on subgroups based on age, we find that the results are again intuitive. From Table 6 we can see that γ_η for the old is 0.40 while it is 0.55 for the young imply that older households can insure their consumption from fluctuations in income more relative to younger households. BPP mention that they find some evidence of an age profile in their estimate of the consumption insurance parameter but the estimate is imprecise.

Since the total sample size, TN, is smaller for the subgroups, we examine the sensitivity of our results to smaller sample size via simulations. See Table C2 in the Appendix. Our full information approach does fairly well, however based on the simulation results, the estimate of γ_η has an upward bias. Therefore our estimate of consumption insurance in section 4 could be seen to provide a lower bound for the true consumption insurance in the data. In our preferred UC-WN specification, consumption insurance is 53 percent. Heathcote, Storesletten and Violante (2014) find consumption insurance to be close to 60 percent in a structural model of income and consumption dynamics.

6 Discussion and Conclusion

There is a large literature that examines the amount of income risk households face and what fraction gets transmitted to household level consumption. Are there formal markets or informal arrangements that insure households against idiosyncratic and unexpected movements in their income or wealth? How does this vary across households and over their life-cycle even when borrowing and lending opportunities change? The literature has keenly focused on these issues, both to inform the incomplete-markets literature and prescribe policies after understanding the extent of market incompleteness.

From the simple test of consumption insurance of Cochrane (1991) and Townsend (1994)'s consumption insurance in village India to the measurement of consumption insurance using panel data on household income and (imputed) non-durable consumption and quasi structural approach suggested by Deaton (1997) and implemented by BPP(2008), to the structural approach of Heathcote, Storesletten and Violante (2014), the literature has made important advances that enhance our understanding of the extent of consumption insurance. It has clearly been established that for the U.S. economy at least, there is no evidence for the two extremes of full insurance or zero insurance. However, quantitative estimates differ significantly and so do the methods with respect to the identification of the permanent and transitory shocks to income. For example, Kruger and Perri (2011) simply compute the change in non-durable consumption because of a change in income. Others have proxied permanent and transitory income shocks, for example Souleles (1999).

In this paper we have developed a panel unobserved components model of household income and consumption that can be estimated using full information methods. Maximum likelihood estimates for a simple version of this model suggests similar income risk, but higher consumption insurance relative to the partial information moments-based estimates of BPP using the same panel dataset. Bayesian methods allows us to determine the most appropriate specification of our model in terms of spillovers between permanent and transitory components, persistence in transitory income and consumption, and possible cointegration between income and consumption. Marginal likelihood analysis supports the simple version of the model with only a spillover from permanent income to permanent consumption, no persistence in transitory components, and no cointegration, the UC-WN model. However, consumption insurance and income risk estimates are robust across different specifications, with consumption insurance always higher relative to the partial information estimates. At the same time, prior sensitivity analysis makes it clear that the degree of consumption insurance is not particularly well identified in the data, although it would take a highly informative and distorted prior to obtain the lower estimates previously found with partial information methods. Subgroup estimation shows that our results regarding model specification are robust to different types of households and that the pattern of heterogeneity in consumption insurance estimates is intuitive, with insurance being higher for older or more educated households.

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A STATE-SPACE REPRESENTATION

In this section we first present the state-space representation of our general model and the BPP model.

The state-space representation of our model is standard. The observation equation is

$$\tilde{\mathbf{y}}_t = \mathbf{H}\,\boldsymbol{\beta}_t$$

where

$$\tilde{\mathbf{y}}_t = \begin{bmatrix} y_t \\ c_t \end{bmatrix},\ \mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & \gamma_\eta & 1 \end{bmatrix} \text{ and } \boldsymbol{\beta}_t = \begin{bmatrix} y_t - \tau_t \\ y_{t-1} - \tau_{t-1} \\ \tau_t \\ c_t - \tau_t \\ c_{t-1} - \tau_{t-1} \\ \kappa_t \end{bmatrix}$$

The state equation is

$$\boldsymbol{\beta}_t = \boldsymbol{F}\boldsymbol{\beta}_{t-1} + \tilde{\boldsymbol{v}}_t$$

where

$$\mathbf{F} = \begin{bmatrix} \phi_{y,1} & \phi_{y,1} & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \phi_{c,1} & \phi_{c,2} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},\ \tilde{\boldsymbol{v}}_t = \begin{bmatrix} \lambda_{y\eta}\eta_t + \epsilon_t \\ 0 \\ \lambda_{c\eta}\eta_t + \lambda_{c\epsilon}\epsilon_t + v_t \\ 0 \\ \eta_t \\ u_t \end{bmatrix}$$

and the covariance matrix of $\tilde{\boldsymbol{v}}_t$, $\mathbf{Q}$, is given by

$$\mathbf{Q} = \begin{pmatrix} \lambda_{y\eta}^2\sigma_\eta^2 + \sigma_\epsilon^2 & 0 & \lambda_{y\eta}\lambda_{c\eta}\sigma_\eta^2 + \lambda_{c\epsilon}\sigma_\epsilon^2 & 0 & \lambda_{y\eta}\sigma_\eta^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_{y\eta}\lambda_{c\eta}\sigma_\eta^2 + \lambda_{c\epsilon}\sigma_\epsilon^2 & 0 & \lambda_{c\eta}^2\sigma_\eta^2 + \lambda_{c\epsilon}\sigma_\epsilon^2 + \sigma_v^2 & 0 & \lambda_{c\eta}\sigma_\eta^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_{y\eta}\sigma_\eta^2 & 0 & \lambda_{c\eta}\sigma_\eta^2 & 0 & \sigma_\eta^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_u^2 \end{pmatrix}$$

For the BPP model, the observation equation is

$$\tilde{\mathbf{y}}_t = \mathbf{H}\,\boldsymbol{\beta}_t$$

where

$$\tilde{\mathbf{y}}_t = \begin{bmatrix} y_t \\ c_t \end{bmatrix},\ \mathbf{H} = \begin{bmatrix} 1 & \theta & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & \gamma_\epsilon & \gamma_\eta & 1 \end{bmatrix} \text{ and } \boldsymbol{\beta}_t = \begin{bmatrix} \epsilon_t \\ \epsilon_{t-1} \\ u_t^\star \\ Z_{\epsilon,t} \\ \tau_t \\ Z_{u,t} \end{bmatrix}$$

The state equation is

$$\boldsymbol{\beta}_t = \boldsymbol{F}\boldsymbol{\beta}_{t-1} + \tilde{\boldsymbol{v}}_t$$

where

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},\ \tilde{\boldsymbol{v}}_t = \begin{bmatrix} \epsilon_t \\ 0 \\ u_t^\star \\ \epsilon_t \\ \eta_t \\ u_t \end{bmatrix}$$

and the covariance matrix of $\tilde{\boldsymbol{v}}_t$, $\mathbf{Q}$, is given by

$$\mathbf{Q} = \begin{pmatrix} \sigma_\epsilon^2 & 0 & 0 & \sigma_\epsilon^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{u^\star}^2 & 0 & 0 & 0 \\ \sigma_\epsilon^2 & 0 & 0 & \sigma_\epsilon^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_\eta 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_u^2 \end{pmatrix}$$

B IMPLIED VARIANCES

We first compute the variance of income and consumption growth in our general model and then the BPP model.

In our UC model income and consumption growth are given as follows:

$$\Delta y_t = \mu + \eta_t + z_t^y, \tag{B.1}$$

where $(1-\phi_{y,1}L-\phi_{y,2}L^2)z_t^y = (1-L)x_t^y$ and $x_t^y = \lambda_{y\eta}\eta_t + \epsilon_t$ and

$$\Delta c_t = \mu + \gamma_c\eta_t + z_t^c, \tag{B.2}$$

where $(1-\phi_{c,1}L-\phi_{c,2}L^2)z_t^c = (1-L)x_t^c$ and $x_t^c = \lambda_{c\eta}\eta_t + \lambda_{c\epsilon}\epsilon_t + v_t$.

We can then write a vector representation for z_t^y and z_t^c as

$$\mathbf{z}_t = \mathbf{K}\mathbf{z}_{t-1} + \mathbf{w}_t,$$

where

$$\mathbf{z}_t = \begin{bmatrix} z_t^y \\ z_{t-1}^y \\ z_t^c \\ z_{t-1}^c \\ x_t^y \\ x_t^c \end{bmatrix}, \mathbf{K} = \begin{bmatrix} \phi_{y,1} & \phi_{y,2} & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \phi_{c,1} & \phi_{c,2} & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \mathbf{w}_t = \begin{bmatrix} x_t^y \\ 0 \\ x_t^c \\ 0 \\ x_t^y \\ x_t^c \end{bmatrix}.$$

Let $\mathbf{W}$ be the covariance matrix of $\mathbf{w}_t$, with the following non-zero entries: $\mathbf{W}[1,1] = \mathbf{W}[1,5] = \mathbf{W}[5,1] = \mathbf{W}[5,5] = \lambda_{y\eta}^2\sigma_\eta^2 + \sigma_\epsilon^2$, $\mathbf{W}[1,3] = \mathbf{W}[3,1] = \mathbf{W}[1,6] = \mathbf{W}[6,1] = \mathbf{W}[3,5] = \mathbf{W}[5,3] = \mathbf{W}[5,6] = \mathbf{W}[6,5] = \lambda_{y\eta}\lambda_{c\eta}\sigma_\eta^2$, and $\mathbf{W}[3,3] = \mathbf{W}[3,6] = \mathbf{W}[6,3] = \mathbf{W}[6,6] = \lambda_{c\eta}^2\sigma_\eta^2 + \sigma_v^2$.

Since the $vec(var(\mathbf{z}_t)) = (I - \mathbf{K}\otimes\mathbf{K})^{-1}vec(\mathbf{W})$, the unconditional variance of output growth is given by

$$\begin{aligned} var(\Delta y_t) &= var(\eta_t + z_t^y) \\ &= \sigma_\eta^2 + var(z_t^y) + 2cov(\eta_t, z_t^y) \\ &= \sigma_\eta^2 + var(z_t^y) + 2\lambda_{y,\eta}\sigma_\eta^2 \end{aligned}$$

where $var(z_t^y)$ is the $[1,1]$ element of $var(\mathbf{z}_t)$. Similarly, unconditional variance of consumption growth is given by

$$var(\Delta c_t) = var(\gamma_c \eta_t + z_t^c)$$
$$= \gamma_\eta^2 \sigma_\eta^2 + var(z_t^c) + 2cov(\eta_t, z_t^c)$$
$$= \sigma_\eta^2 + var(z_t^c) + 2\lambda_{c,\eta}\sigma_\eta^2$$

where $var(z_t^c)$ is the $[3,3]$ element of $var(\mathbf{z}_t)$.

In BPP, computing these variances is rather simple. They are as follows:

$$var(\Delta y_t) = \sigma_\eta^2 + \sigma_\epsilon^2(1 + \theta^2 - \theta) \tag{B.3}$$

since $\Delta y_t = \epsilon_t - \epsilon_{t-1} + \theta\epsilon_{t-1} - \theta\epsilon_{t-2} + \eta_t$.

Similarly,

$$var(\Delta c_t) = \gamma_\eta \sigma_\eta^2 + \gamma_\epsilon \sigma_\epsilon^2 + \sigma_u^2 + +2\sigma_v^2 \tag{B.4}$$

since $\Delta c_t = \gamma_\eta \eta_t + \gamma_\epsilon \epsilon_t + u_t + \Delta v_t$.

C Simulation results

In this section we first examine if the full information approach can help us recover the key parameters of the income and consumption process that we are interested in such as the variance of the permanent shocks to income and consumption insurance if there is model mis-specification. To consider this we generate data from UC-WN model and fit the BPP model where all the priors are the same as in section 4 and γ_η uniformly distributed. We estimate the model using Bayesian methods. Our methodology does really well in recovering the key parameters even when the model is mis-specified. For example in the DGP, 45 percent of the permanent income shocks get transmitted to consumption and our estimate using full information is 43 percent. Our estimate of permanent income risk is also not impacted by model mis-specification,. In the partial information moments based approach, it has been documented in the level versus growth moment conditions literature that model mis-specification

TABLE C1. MODEL MIS-SPECIFICATION

	DGP	BPP model
		INCOME
θ		$-0.06(0.03)$
σ_η	0.14	0.15(0.02)
σ_ϵ	0.17	0.15(0.02)
		CONSUMPTION
γ_η	0.45	0.43(0.02)
γ_ϵ		0.005(0.00)
σ_u	0.13	0.15(0.02)
$\sigma_{u^\star}$	0.21	0.17(0.02)

Notes: The table reports posterior means of model parameters with posterior standard deviations reported in parentheses. For the DGP, N=700 and T=10.

can bias the estimate of permanent risk. See Domeij and Floden (2010) for more details.

We also examine the sensitivity of our results to small samples with N=700 and T=10. Since the preferred model is UC-WN and what matters is TN, we also report results for N=1400 and T=5. For both cases, we consider three values of γ_η. Table C2 reports our simulation results. Bayesian estimation is able to recover the true parameters quite well. In particular, the estimate of permanent income risk is close to the true parameter value and the estimate of γ_η appears to have an upward bias.

TABLE C2. SIMULATION RESULTS

	DGP	$\gamma_\eta = 0.25$	$\gamma_\eta = 0.45$	$T = 10, N = 700$ $\gamma_\eta = 0.65$	$\gamma_\eta = 0.25$	$T = 5, N = 1400$ $\gamma_\eta = 0.45$	$\gamma_\eta = 0.65$
				INCOME			
σ_η	0.14	0.14 (0.02)	0.14 (0.02)	0.14 (0.02)	0.14 (0.01)	0.14 (0.02)	0.14 (0.02)
σ_ϵ	0.17	0.15 (0.02)	0.15 (0.02)	0.16 (0.02)	0.15 (0.01)	0.15 (0.02)	0.15 (0.02)
				CONSUMPTION			
σ_u	0.13	0.15 (0.02)	0.15 (0.02)	0.15 (0.02)	0.15 (0.02)	0.15 (0.02)	0.15 (0.02)
σ_v	0.21	0.18 (0.02)	0.17 (0.02)	0.17 (0.02)	0.15 (0.02)	0.16 (0.02)	0.15 (0.02)
γ_η		0.28 (0.03)	0.47 (0.03)	0.72 (0.03)	0.30 (0.03)	0.50 (0.03)	0.75 (0.03)

Notes: The table reports posterior means of model parameters with posterior standard deviations reported in parentheses. In the simulated data, total number of observations (N*T) is equal to 7000.

D Additional material

We can establish the consequences of assuming zero correlation in a univariate setting, i.e. UC-ARMA model. We then show that such a model however cannot be identified unless we assume rich autoregressive dynamics which are hard to model given the short time series aspect of our dataset. This motivates the UC-VARMA model described in section 3.

D.1 Assuming zero correlation

Ignoring the individual specific subscript for simplicity, let the true UC-ARMA model be given by:

$$y_t = \tau_t + (y_t - \tau_t), \tag{D.1}$$

$$\tau_t = \mu + \tau_{t-1} + \eta_t, \tag{D.2}$$

and

$$\psi_p(L)^{-1}(y_t - \tau_t) = \psi_q(L)\epsilon_t, \tag{D.3}$$

where τ_t is the common trend for income y_t and the permanent and transitory shocks are η_t and ϵ_t respectively. Assume that $p = 1$ and $q = 0$,

TABLE A2 SUB-SAMPLE ESTIMATES

	1979-82	1983-1987	1988-1992
		INCOME	
σ_η	0.16 (0.01)	0.15 (0.00)	0.16 (0.00)
σ_ϵ	0.16 (0.00)	0.20 (0.00)	0.19 (0.00)
		CONSUMPTION	
σ_u	0.12 (0.00)	0.12 (0.00)	0.12 (0.00)
σ_v	0.22 (0.00)	0.28 (0.00)	0.28 (0.003)
γ_η	0.41 (0.03)	0.52 (0.02)	0.35 (0.02)

Notes: The table reports MLE estimates with standard deviations reported in parentheses.

therefore transitory sales have the following AR(1) dynamics $\psi_p(L)^{-1} = 1 - \psi_{y,1}(L)$. In this setting, if we solve for UC model parameters using moments, the variance of the permanent shock is biased when the we restrict the correlation between the two components to be zero. To see this, the first and second moments for the income process are given below

$$\gamma_0 = (1 + \psi_{y,1}^2)\sigma_\eta^2 + 2\sigma_\epsilon^2 + (1 + \psi_{y,1})\sigma_{\eta\epsilon} \tag{D.4}$$

$$\gamma_1 = -\psi_{y,1}\sigma_\eta^2 - \sigma_\epsilon^2 - (1 + \psi_{y,1})\sigma_{\eta\epsilon} \tag{D.5}$$

where $\sigma_{\eta\epsilon}$ is the correlation between the permanent and the transitory components. In the unrestricted case when $\sigma_{\eta\epsilon} \leq 0$, we get

$$\gamma_0 + 2\gamma_1 = (1 - \psi_{y,1})^2\sigma_\eta^2 - (1 + \psi_{y,1})\sigma_{\eta\epsilon}. \tag{D.6}$$

However in the restricted case, when $\sigma_{\eta\epsilon} = 0$

$$\gamma_0 + 2\gamma_1 = (1 - \psi_{y,1})^2\sigma_\eta^2. \tag{D.7}$$

This clearly shows that in the restricted case, the variance of the permanent shock will be biased given other parameters. If the correlation is negative, the estimate of σ_η^2 will be lower in the restricted case and and vice versa.