Identifying the Sources of Model Misspecification

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Abstract

In this paper we propose an empirical method for detecting and identifying misspecification in structural economic models. Our approach formalizes the common practice of adding "shocks" in the model, and identifies potential misspecification via forecast error variance decomposition and marginal likelihood analyses. The simulation results based on a small-scale DSGE model demonstrate that our method can correctly identify the source of misspecification. Our empirical results show that state-of-the-art medium-scale New Keynesian DSGE models remain misspecified, pointing to asset and labor markets as the sources of the misspecification.

Keywords: DSGE models, marginal likelihood, misspecification

1. Introduction

The recent financial crisis of 2007-2009 uncovered existing structural economic models' difficulties in explaining the data. The limitations of structural models in forecasting are also well known. For example, Edge and Gurkaynak (2010), among others, have shown that the forecast performance of the Smets and Wouters (2007) model is not better than a naïve constant growth-rate model during the Great Moderation period. Moreover, the

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severity and the prolonged duration of the Great Recession have challenged the adequacy of existing predictors and raised the possibility that these models might be misspecified. For example, Ng and Wright (2013) have argued that the features of "financial-crisis-induced" recessions (such as the Great Recession) are distinct from those of the "typical" recessions driven by supply or monetary policy shocks. This distinction may explain why the study of the Great Recession requires alternative models and predictors.

While the above limitations of the existing models are well known, it is unclear whether the recent financial crisis resulted from unexpected shocks or changes in the transmission mechanism. On the one hand, Stock and Watson (2012) argue that the transmission mechanism during the Great Recession was not different than that of any other post-war recession, showing that the larger shocks hitting the economy were the origin of the deep and prolonged recession. On the other hand, other researchers emphasize that existing macroeconomic models do not fully capture the mechanisms behind the Great Recession and argue that substantial modifications are necessary. For example, in an effort to improve the fit of structural economic models during the crisis, Del Negro and Schorfheide (2014) include information from inflation expectations, financial frictions and interest rate spreads: if this is the case, knowing the source of model misspecification will help economists develop alternative models. Therefore, in this paper, we examine the empirical importance of model misspecification in dynamic stochastic general equilibrium (DSGE) models and propose a methodology to shed light on the sources of misspecification responsible for their poor forecasting performance.

To examine misspecification in a structural model, we propose to broaden the treatment of the exogenous processes. Specifically, we consider two types of exogenous processes. The first type of exogenous processes represent structural shocks and are interpreted in the conventional way. Since these shocks are structural, they should be viewed as indispensable ingredients of the model at hand. In contrast, the second type of exogenous processes, which we call "margins", are not structural and their function is to check for model misspecification. In fact, by estimating the margins, we are able to assess: (i) where the misspecification might be located (that is, which parts of the model are most affected by the misspecification); and (ii) how qualitatively important it is.

The margins should not be confused with structural shocks. We introduce margins only after "traditional" structural shocks are included. Our goal is to evaluate the correct specification of the model that only includes the structural shocks. That is, we evaluate the misspecification *relative to a benchmark model*: if the benchmark model has shocks that are interpreted as structural shocks, we maintain those, and include margins on top of them. The specific way we incorporate the time-varying margins in the model is by including them into the agents' optimization problem (such as households' budget constraints): for example, the existence of margins in agents' optimization problems allows for deviations in the prices of relevant goods because the model misspecification will eventually lead to distorted relative prices. These margins can be used to measure both the nature and importance of misspecification. While it is common in practice to add shocks to a given model to get a better fit, our contribution is to develop this procedure into a formal methodology to evaluate model misspecification. We do not recommend adding shocks where our method identifies a margin: even if a margin is significant it does not necessarily mean it is a shock. Although some margins may have an economic interpretation, others do not, as the examples in our paper demonstrate. While our margins are uncorrelated with each other at all leads and lags, they should not be thought of as shocks, but as measures of misspecification: the uncorrelatedness of the shocks is important to make sure the researcher can identify the source of the misspecification, not because our margins should be thought of as shocks. In fact, it is difficult to interpret FEVD and marginal likelihood results if margins are correlated. This is because eliminating a margin may affect other margins, if the margins are correlated. In other words, we view our method as a diagnostic to detect which parts of a model are misspecified. The next step is then to improve the parts of the model that our methodology identifies as misspecified.

While the technique that we propose is very general, in this paper we focus on DSGE models given their widespread use in academia and central banks as standard tools for analyzing macroeconomic policies. To provide more intuition and better illustrate our framework, we consider a medium-scale DSGE model, embedded with most New Keynesian features. This model is mildly misspecified in the sense that: (i) the cross equation and equilibrium restrictions imposed by the model do not exactly hold in every time period; and (ii) the deviations from equilibrium are zero on average. We assume that when solving their optimization problems, the economic agents in the model (both firms and households) take into account the exogenous margin processes, which allow for deviations from equilibrium conditions. We interpret the variances of the margin processes as a measure of the degree of misspecification of the model. To examine where and how large the misspecification is, we conduct forecast error variance decompositions (FEVDs) and marginal likelihood comparison. Note that our goal is not to compare models but rather to evaluate the specification of a given model. While margins may have a structural interpretation, our goal is to start with a model of interest without margins and evaluate its specification by measuring the contribution of margins without necessarily giving the margins a structural interpretation. Finally, note that testing for over-identification would not be useful in our context: the J-test for overidentifying restrictions would allow researchers to determine whether a model's moment conditions are correctly specified, but would not shed light on the sources of misspecification if the model fails the test.

Our empirical application to a state-of-the-art DSGE model highlights two interesting findings. First, our technique points to misspecification in the model's labor demand component. Second, bond markets also show evidence of misspecification, which is persistent in nature. Our findings confirm the existing view that asset and labor markets in the New Keynesian DSGE model are misspecified and suggest that further work in these areas would be beneficial.[4]

Our method is related to several recent contributions. First, our paper is related to Sargent (1989), Ireland (2004), Del Negro and Schorfheide (2009) and Curdia and Reis

[4]Our findings are related to Del Negro, Schorfheide, Smets and Wouters (2007) who, among others, have recently showed that model misspecification cannot be ignored in policy analyses.

(2010), among others. Sargent (1989) and Ireland (2004) introduce errors in the measurement equations of the state space version of the model to assess whether the model is misspecified. Del Negro and Schorfheide (2009) develop a framework for Bayesian estimation of possibly misspecified DSGE models by using DSGE-implied parameters as priors for vector autoregressive (VAR) models. Their framework allows for model misspecification and produces the posterior distribution of structural parameters as well as the posterior structural impulse responses based on DSGE priors.[5] Our framework complements these works in that we can identify which parts of the model are misspecified. Curdia and Reis (2010) relax the restriction that exogenous disturbances in structural models are independent processes.[6] They argue that estimating models with correlated disturbances provides a useful check for model misspecification. We consider instead a different way to incorporate misspecification in the models by including independent disturbances in the equilibrium conditions of the model. We also suggest that the analysis of the estimated disturbances provides a way to identify the source of the misspecification and its importance over time. Note also that Watson (1993) proposed a measure of fit for calibrated models; we instead estimate the structural models and provide a different methodology to identify the sources of misspecification. This paper is also related to the literature on the challenges in the estimation of structural macroeconomic models. The challenge we focus on is misspecification, which has been considered by Corradi and Swanson (2007) and Canova and Ferroni (2011). Canova and Ferroni (2011) show that incorrect filtering results in model misspecification but, unlike us, do not search for the source of misspecification. Corradi and Swanson (2007) provide new tools for comparing the empirical joint distribution of historical time series with the empirical distribution of simulated time series based on structural macroeconomic models. Their focus is on detecting whether the whole distribution of a macroeconomic

[5]Note that they assume that DSGE models have a VAR representation in their implementation. Many DSGE models have VARMA representations and we do not need to assume a VAR representation although the error due to VAR approximations may be small if the lag is sufficiently large.

[6]See also Meyer-Gohde and Neuhoff (2015) for a VARMA generalization.

model is correctly specified, whereas our focus is on the first moments of the model and on providing guidance on the sources of model misspecification.

Other challenges that researchers face in the estimation of structural macroeconomic models are related to identification. Note that this paper does not deal with potential lack of identification or weak identification of the models' parameters. That is a different problem and has been analyzed by Dufour, Khalaf and Kichian (2013), Canova and Sala (2009), and Iskrev (2010), among others. If the models' parameters are not identified, then adding margins will not help. Indeed, it is possible that adding margins to an unidentified or weakly identified model may make the estimation more difficult.

Finally, our paper is related to Chari et al. (2007) and Brinca et al. (2016), who introduce time-varying "wedges" to account for deviations between the time series of the observables implied by the model and those actually observed in the data. There is a fundamental difference and two more minor differences between our work and theirs. The fundamental difference is that they focus on business cycle accounting while we focus on detecting model misspecification. In other words, Chari et al. view the model as correctly specified, and the (correlated) wedges are frictions introduced in the model to allow the model to account for business cycle fluctuations similar to those in the data. Our approach is substantially and philosophically different: we view the margins as a measure of model misspecification, and each margin independently measures the degree of misspecification in a specific part of the model (e.g. an equation). The other two more minor differences are as follows. One is that they consider a neoclassical stochastic growth benchmark model, while we consider a New Keynesian DSGE benchmark model which incorporates several frictions. Thus, our margins (i.e. distortions due to misspecification) reflect model misspecification that is not already accounted for by frictions built into the model. The other difference is that their analysis is based on calibrated parameter values, while ours is based on an estimated model, and thus is robust to incorrectly calibrating the parameter values.

2. An Illustrative Example

In this section, we focus on a simple consumption model to illustrate our method and compare it with those existing in the literature.

Suppose that we want to evaluate the following baseline model:

$$\max_{c_t} \; E_0[\sum_{t=0}^{\infty} \beta^t(\theta_0 c_t - \frac{\theta_1}{2} c_t^2)] \tag{1}$$

$$s.t. \; a_{t+1} = (1+r)(a_t + y_t - c_t), \tag{2}$$

$$y_t \;=\; y_{t-1} + \varepsilon_t, \tag{3}$$

where $a > 0$, $b > 0$, $\beta(1+r) = 1$, and ε_t is an independent and identically distributed (iid) mean zero random variable with variance σ_ε^2: $\varepsilon_t \overset{iid}{\sim} (0, \sigma_\varepsilon^2)$. We assume that the econometrician observes consumption (c_t), income (y_t) and assets (a_t).

The baseline model, eqs. (1)-(3), is potentially misspecified. We consider separately three different types of misspecification.

In the first example, transitory income is present in the true data generating process:

$$y_t^T \;=\; \rho_{y^T} y_{t-1}^T + \varepsilon_{y^T,t},$$

where $\varepsilon_{y^T,t} \overset{iid}{\sim} (0, \sigma_y^2)$. The solution can be described by:

$$c_t \;=\; \frac{r}{r+1} a_t + y_t^P + \frac{r}{1-\rho_{y^T}+r} y_t^T, \tag{4}$$

$$a_{t+1} \;=\; (1+r)(a_t + y_t - c_t), \tag{5}$$

$$y_t^P \;=\; y_{t-1}^P + \varepsilon_{y^P,t}, \tag{6}$$

$$y_t^T \;=\; \rho_{y^T} y_{t-1}^T + \varepsilon_{y^T,t}, \tag{7}$$

$$y_t \;=\; y_t^P + y_t^T. \tag{8}$$

In the second example, asset returns are uncertain and stochastic in the true data generating process, so that:

$$a_{t+1} \;=\; (1 + r_{t+1})(a_t + y_t - c_t), \tag{9}$$

where r_t is *iid* with $E(1+r_{t+1}) = 1/\beta$. Then the solution can be written as:

$$\begin{aligned} c_t &= \left(1 - \frac{1}{\kappa}\right) a_t + y_t, \qquad (10) \\ a_{t+1} &= (1+r_{t+1})(a_t + y_t - c_t), \qquad (11) \\ y_t &= y_{t-1} + \varepsilon_t, \qquad (12) \end{aligned}$$

where $\kappa = \beta E[(1+r_{t+1})^2]$.

In the third example, the asset data available to the econometrician contain measurement error: $\tilde{a}_t = a_t + \xi_t$, where $\xi_t = \rho_\xi \xi_{t-1} + \eta_{\xi,t}$. While the solution remains the same as in the baseline case, the econometrician erroneously fits

$$c_t = \frac{r}{r+1}\tilde{a}_t + y_t = \frac{r}{r+1}a_t + y_t + \frac{r}{1+r}\xi_t, \qquad (13)$$

and

$$\tilde{a}_{t+1} = (1+r)(\tilde{a}_t + y_t - c_t), \qquad (14)$$

which is equivalent to

$$a_{t+1} = (1+r)(a_t + y_t - c_t) + (1+r)\xi_t - \xi_{t+1}. \qquad (15)$$

2.1. Our Approach

Our approach to detect misspecification introduces "margins" in the optimization problem. We see the flexibility that characterizes our methodology in terms of choosing the location and the number of margins as a strength of our approach: in fact, the researcher can introduce as many margins as he/she wants. As we show in this example, this is not completely arbitrary, as we discipline the way we introduce margins by using the marginal likelihood; in fact, unnecessary margins will be eliminated by the marginal likelihood criterion that we suggest. The marginal likelihood has a built-in term that penalizes overparameterized models (Fernandez-Villaverde and Rubio-Ramirez, 2004). A model with an unnecessary margin will have a lower marginal likelihood value than a model without it.

Provided the researcher introduces enough margins, with our procedure he/she is likely to obtain the same model as another researcher who may introduce more margins.

To obtain a closed form solution that can be compared with eqs. (4)–(15), we introduce three margins, u_t, v_t and w_t, in the baseline model:

$$
\begin{array}{ll}
\max & \sum_{t=0}^{\infty} E_0\beta^t[(a+u_t)c_t - \frac{b}{2}c_t^2] \\
\text{subject to} & a_{t+1} = (1+r)(1+v_{t+1})(a_t + y_t - c_t + w_t), \\
& y_t \;=\; y_{t-1} + \varepsilon_t, \\
& u_t \;=\; \rho_u u_{t-1} + \eta_{u,t}, \\
& v_t \;=\; \eta_{v,t}, \\
& w_t \;=\; \rho_w w_{t-1} + \eta_{w,t},
\end{array}
$$

where $\beta(1+r) = 1$ and

$$
\begin{bmatrix} \varepsilon_t \\ \eta_{u,t} \\ \eta_{v,t} \\ \eta_{w,t} \end{bmatrix} \overset{iid}{\sim} \left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & 0 & 0 & 0 \\ 0 & \sigma_u^2 & 0 & 0 \\ 0 & 0 & \sigma_v^2 & 0 \\ 0 & 0 & 0 & \sigma_w^2 \end{bmatrix} \right).
$$

Note that u_t is placed in the linear part of the utility function and v_t is assumed to be iid for analytical tractability. Alternatively, one could add a margin to the utility function, i.e., $\theta_0 c_t - (\theta_1/2)c_t^2 + x_t$. However, this margin would not show up anywhere in the solution and, hence, would be of little help in identifying the sources of misspecification in this model. This observation highlights the importance that a researcher thinks carefully about how to add the margins. Again, we view this as a strength of our approach: margins should be included in a meaningful way.

The solution to this optimization problem is given by

$$
\begin{aligned}
c_t &= \left(1-\frac{1}{\phi}\right)a_t + y_t + \frac{1-\rho_u}{b(\phi-\rho_u)}u_t + \frac{\phi-1}{\phi-\rho_w}w_t, && (16)\\
a_{t+1} &= (1+r)(1+v_{t+1})(a_t+y_t-c_t+w_t), && (17)\\
y_t &= y_{t-1}+\varepsilon_t, && (18)\\
u_t &= \rho_u u_{t-1}+\eta_{u,t}, && (19)\\
v_t &= \eta_{v,t}, && (20)\\
w_t &= \rho_w w_{t-1}+\eta_{w,t}, && (21)
\end{aligned}
$$

where $\phi \equiv (1+r)E[(1+v_{t+1})^2]$.

2.2. How and Why Does Our Approach Work?

Suppose that one fits the above model (eqs. 16-21) when there is transitory income. Because the asset return is constant, v_{t+1} should be (close to) zero. So $\phi = 1+r$ and $(1-1/\phi) = r/(1+r)$. Furthermore, by comparing eqs. (5) and (17), the term w_t should be close to zero as well. Comparing (16) and (4), in the true model there is a term due to the presence of the transitory income component, $(r/(1-\rho_y^T+r))y_t^T$. In the fitted model, the total income is used as the permanent income and thus we have $y_t^P + y_t^T$. The difference $[(1-\rho_y^T)/(1-\rho_y^T+r)]y_t^T$ should be absorbed by either u_t or w_t on the right hand side of (16), but w_t is (close to) zero. Thus only u_t will be different from zero, significantly contributing to the FEVDs and successfully capturing the misspecification. Equivalently, comparing the marginal likelihood of models where each margin is removed one-at-a-time will also show that the likelihood decreases the most when u_t is removed while it is virtually unchanged when either w_t or v_t is removed.

In the second case, v_t will be significant because

$$1+r_{t+1} = (1+r)(1+v_{t+1}). \tag{22}$$

Once this equation is satisfied, the model is correctly specified and thus u_t and w_t will be insignificant. Hence, again, FEVDs and marginal likelihood analyses will correctly signal the source of the misspecification.

In the last case, the margin in the budget constraint, w_t, will capture the AR(1) measurement error that appears in both the consumption equation and budget constraint; however, given the way the serially correlated measurement error appears in eqs. (13)-(15), the margin in the preference parameter, u_t, might also be significant to pick up any remaining misspecification. v_t will be insignificant since v_t would yield a non-stationary error in the budget constraint that is different from the MA(1) error in equation (15). Either way, the misspecification will be successfully differentiated from the previous two cases.

Therefore, our method successfully distinguishes among the three types of misspecification: (i) misspecification due to erroneous serial correlation structures; (ii) misspecification due to the incorrect assumption of constant asset returns; and (iii) misspecification due to measurement error.

Note that the margins should not be confused with structural shocks. In fact, there is not necessarily a one-to-one mapping between margins and shocks, as this example shows, since in the first case the margin included the utility function u_t captures the absence of transitory income in the model but is not a preference/utility shock. However, if a researcher were interested in distinguishing between the two, he/she could estimate two models: a model with transitory income and without a preference shock, and a model with a preference shock but without transitory income; the marginal likelihood can distinguish between the two models since have different implications for the income processes.

2.3. Alternative Approaches

Sargent (1989) and Ireland's (2004) approach: Sargent (1989) and Ireland (2004) introduce (possibly serially correlated) errors in measurement equations of state space models. Although our baseline model is not a state space model, their idea can still be applied as follows:

$$\begin{aligned}
c_t &= \frac{r}{r+1}a_t + y_t + e_{1t}, &(23)\\
a_{t+1} &= (1+r)(a_t + y_t - c_t) + e_{2t}, &(24)\\
e_{1t} &= \rho_{e_1} e_{1,t-1} + \eta_{1t}, &(25)\\
e_{2t} &= \rho_{e_2} e_{2,t-1} + \eta_{2t}, &(26)
\end{aligned}$$

where

$$\begin{bmatrix} \eta_{1t} \\ \eta_{2t} \end{bmatrix} \overset{iid}{\sim} N(0, \Sigma).$$

In the first example, e_{1t} will be significant, but e_{2t} will not, since only the consumption equation is misspecified. In the last two examples, both e_{1t} and e_{2t} will have non-degenerate distributions because the two equations are both misspecified. Therefore their method cannot tell apart the source of the misspecification in the second and third examples.

Del Negro and Schorfheide's (2009) approach: In their (first) approach, they generalize the dependence structure. The only shock in our benchmark model is the shock to the permanent income process. To make the solution analytically tractable, we increase the dependence in the income process (which is equivalent to including a moving average in the error term) and consider the following generalization of the permanent income process:

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t, \qquad (27)$$

where $\phi_1 + \phi_2 = 1$. The solution is:

$$\begin{aligned}
c_t &= \frac{r}{r+1}a_t + \frac{r(1+r)}{(1-\phi_1+r)(1+r)-\phi_2}y_t + \frac{\phi_2 r}{(1-\phi_1+r)(1+r)-\phi_2}y_{t-1}, &(28)\\
a_{t+1} &= (1+r)(a_t + y_t - c_t), &(29)\\
y_t &= \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t. &(30)
\end{aligned}$$

Because the consumption equation is misspecified in all the examples, the error in the consumption equation will be captured by the more general structure of y_t, i.e., $\phi_2 \neq 0$. However, a researcher implementing Del Negro and Schorfheide's (2009) approach would be unable to identify the source of misspecification, even though their approach would correctly signal that the model is broadly misspecified.

Curdia and Reis' (2010) approach: Since we need two shocks to apply their method, we introduce transitory income and allow the permanent and the transitory income shocks to be correlated. Suppose that the econometrician observes the permanent and transitory income components, where the latter follows:

$$y_t^T = \rho y_{t-1}^T + \varepsilon_{y^T,t}, \tag{31}$$

where

$$\begin{bmatrix} \varepsilon_{y^P,t} \\ \varepsilon_{y^T,t} \end{bmatrix} \overset{iid}{\sim} \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{y^P}^2 & \rho\sigma_{y^P}\sigma_{y^T} \\ \rho\sigma_{y^P}\sigma_{y^T} & \sigma_{y^T}^2 \end{bmatrix} \right) \tag{32}$$

and $\varepsilon_{y^P,t}$ is the shock for the random walk permanent income process. A correlation different from zero, i.e., $\rho \neq 0$, indicates the presence of misspecification. Even if the shocks are correlated, the solution remains the same as eqs. (4)-(5):

$$\begin{aligned} c_t &= \frac{r}{r+1}a_t + y_t^P + \frac{r}{1+r-\rho}y_t^T, && (33) \\ a_{t+1} &= (1+r)(a_t + y_t - c_t), && (34) \\ y_t &= y_t^P + y_t^T, && (35) \\ y_t^P &= y_{t-1}^P + \varepsilon_{y^P,t}, && (36) \\ y_t^T &= \rho y_{t-1}^T + \varepsilon_{y^T,t}. && (37) \end{aligned}$$

Because the consumption equation will be misspecified in all the examples, the variance of the transitory income component will be positive. In the first example, because the permanent and transitory incomes are independent in the data generating process, the two shocks will be uncorrelated. When the interest rate is stochastic, that will yield persistent and transitory residuals because the asset equation includes both permanent and transitory income. That will likely make the two errors correlated. In the last example, the serially correlated measurement error in the Euler equation will be detected by the transitory income process. Because the measurement error is independent of the permanent income process, ρ will be zero.[7] One would notice that the model is misspecified, however, because the budget constraint is not satisfied. Thus this method can detect misspecification except for the first example. As with the Del Negro and Schorfheide method, however, it is not clear how to find out exactly which building blocks of the model are misspecified.

Chari, Kehoe and McGrattan's (2007) approach:

Our approach is fundamentally and philosophically different from the approach developed by Chari, Kehoe and McGrattan (2007) and further elaborated in Brinca, Chari, Kehoe and McGrattan (2016). Their method is not designed to detect misspecification; rather, to account for business cycle fluctuations. Their estimated wedges are calculated based on the difference between observables and their counterparts obtained from a model evaluated at given parameter values; thus, their wedges are typically correlated to one another and it is difficult to interpret the marginal contribution of a given wedge. In contrast, the exogenous process that we use in our methodology are crucially different objects from wedges: in fact, our margins are independent of each other and are introduced in the agents' optimization conditions. In our framework, FEVD and Marginal likelihood analyses provide statistically sound ways to detect which margins are significant and responsible for the misspecification. If one nevertheless treats their method as a method to detect misspecification, one may detect misspecification when deviations from equilibrium conditions are large. However, since they are evaluated at pre-specified parameter values rather than estimated, one could be mislead to conclude that the model is misspecified even when it is correctly specified if the parameters are not correctly calibrated.

2.4. Summary

The above discussion shows that our method can differentiate among the three types of misspecification, and hence detect the sources of misspecification, while existing methods fail to do so. In practice, however, the same margin may capture different types of misspecification. Thus applying our method only once may not uniquely identify a source of

[7]Since $c_t = \frac{r}{r+1}a_t + \frac{r}{r+1}\xi_t + y_t^P + \frac{r}{1+r-\rho}y_t^T$, then $y_t^T \approx -\frac{1+r-\rho}{r+1}\xi_t$, which is uncorrelated with y_t^P.

misspecification. We suggest repeating this process until no margin is found to be significant. For example, if the first margin is significant in the above example, one may replace the baseline model by a model with transitory income and re-estimate the model with additional margins. If the additional margins are insignificant, we suggest stopping there; otherwise, investigate another baseline model and proceed until no margin is significant.

3. The Proposed Methodology in Practice and Simulation Analysis

In this section, we provide Monte Carlo simulation evidence based on a simple New Keynesian model: we estimate several models (misspecified and correctly specified ones) and report FEVD analyses and marginal likelihoods to show how to detect the source of model misspecification using our method.

3.1. The Data-Generating Process (DGP)

Our DGP is the linearized simple New Keynesian model:[8]

$$\hat{Y}_t = E_t\left\{\hat{Y}_{t+1}\right\} - \frac{1}{\gamma}\left(\hat{R}_t - E_t\hat{\pi}_{t+1}\right) + \left(\hat{g}_t - E_t\hat{g}_{t+1}\right), \tag{38}$$

$$\hat{\pi}_t = \kappa\left\{(\gamma+\varphi)\,\hat{Y}_t - (\varphi+1)\,\hat{z}_t - \gamma\hat{g}_t + \hat{\chi}_t\right\} + \beta E_t\left\{\hat{\pi}_{t+1}\right\}, \tag{39}$$

$$\hat{R}_t = \rho_r\hat{R}_{t-1} + (1-\rho_r)\left\{\gamma_\pi\hat{\pi}_t + \gamma_y\hat{Y}_t\right\} + \hat{\nu}_t, \tag{40}$$

$$\hat{Y}_t = \hat{z}_t + \hat{L}_t, \tag{41}$$

where (38) is the dynamic IS curve, (39) is the New Keynesian Phillips curve (NKPC), (40) is the monetary policy rule, and (41) is the linearized aggregate production function. Note that in the NKPC, $\kappa = \frac{(1-\beta\xi)(1-\xi)}{\xi}$.

The structural shocks follow:

$$\hat{x}_{t+1} = \rho_x\hat{x}_t + \sigma_x\varepsilon_{x,t+1}, \quad \varepsilon_{x,t+1} \overset{iid}{\sim} N(0,1) \tag{42}$$

where $x \in \{z, \nu, g, \chi\}$ and $\hat{x}$ denotes the log deviation of x from its steady state value.

[8] A detailed description of the model can be found in Section A of the Not-for-Publication Appendix. See also An and Schorfheide (2007).

3.2. The Proposed Methodology

To demonstrate our methodology, we estimate a correctly specified model, labeled $\mathcal{M}_0$, and three misspecified ones, labeled $\mathcal{M}_l$, $\mathcal{M}_m$ and $\mathcal{M}_g$. It is worth noting that, in our simulation setting, we know exactly whether the models are misspecified and the sources of their misspecification; in reality, however, we do not have such valuable information. In order to mimic realistic circumstances, we assume that we do not know which models are misspecified and introduce several margins into the above four models. Note that there is more than one way to include the margins in the model. One might consider introducing margins in first-order conditions. We prefer introducing margins into agents' optimization problems rather than into first order conditions for two reasons. One reason is that there are many ways to write first-order conditions. Another reason is that introducing margins in agents' optimization problems allows us to interpret the margins more clearly. The models we consider are the following:

$\mathcal{M}_0$: The equilibrium conditions of the model $\mathcal{M}_0$ are highly similar to those of the DGP. The only difference is that the dynamic IS curve and the NKPC curve contain three time-varying margins: $\hat{\tau}_{b,t}$ is a bond market margin and $\hat{\tau}_{c,t}$ is a final good margin, both of which enter the household's budget constraint (the first multiplies the lagged bond and the second multiplies consumption in the budget constraint), whereas $\hat{\tau}_{l,t}$ is a time-varying labor margin (it multiplies labor in the cost minimization problem of the intermediate good firms). We assume that all margins follow independent AR(1) processes:

$$\hat{\tau}_{x,t+1} \;=\; \rho_x \hat{\tau}_{x,t} + \sigma_x \varepsilon_{x,t+1}, \quad \varepsilon_{x,t+1} \overset{iid}{\sim} N(0,1)$$

where $x \in \{l, c, b\}$. The corresponding equilibrium conditions are:

$$\hat{Y}_t \;=\; \mathrm{E}_t \hat{Y}_{t+1} - \frac{1}{\gamma}\left(\hat{R}_t - \mathrm{E}_t \hat{\pi}_{t+1}\right) + \left(\hat{g}_t - \mathrm{E}_t \hat{g}_{t+1}\right) - \frac{1}{\gamma}\left(\hat{\tau}_{c,t} - \mathrm{E}_t \hat{\tau}_{c,t+1}\right) - \frac{1}{\gamma}\mathrm{E}_t \tau_{b,t+1}, \tag{43}$$

$$\hat{\pi}_t \;=\; \kappa\left\{(\gamma+\varphi)\hat{Y}_t - (\varphi+1)\hat{z}_t - \gamma \hat{g}_t + \hat{\chi}_t\right\} + \beta \mathrm{E}_t\left\{\hat{\pi}_{t+1}\right\} + \kappa\left(\hat{\tau}_{c,t} + \hat{\tau}_{l,t}\right), \tag{44}$$

$$\hat{R}_t \;=\; \rho_r \hat{R}_{t-1}\left(1-\rho_r\right)\left\{\gamma_\pi \hat{\pi}_t + \gamma_y \hat{Y}_t\right\} + \hat{\nu}_t, \tag{45}$$

$$\hat{Y}_t \;=\; \hat{z}_t + \hat{L}_t. \tag{46}$$

In sum, model $\mathcal{M}_0$ is characterized by: (i) four equilibrium conditions: (43), (44), (45) and (46); (ii) four exogenous structural shock processes: $\hat{z}_t$, $\hat{\nu}_t$, $\hat{g}_t$ and $\hat{\chi}_t$; (iii) three exogenous margin processes: $\hat{\tau}_l$, $\hat{\tau}_c$ and $\hat{\tau}_b$.

$\mathcal{M}_l$: This model is misspecified since the labor supply shock, $\hat{\chi}_t$, is excluded from the set of structural shocks. In other words, we assume that there are only three structural shocks in the model. Thus, the NKPC becomes:

$$\hat{\pi}_t = \kappa \left\{ (\gamma + \varphi) \hat{Y}_t - (\varphi + 1) \hat{z}_t - \gamma \hat{g}_t \right\} + \beta \mathrm{E}_t \left\{ \hat{\pi}_{t+1} \right\} + \kappa \left(\hat{\tau}_{c,t} + \hat{\tau}_{l,t} \right). \tag{47}$$

In sum, model $\mathcal{M}_l$ is characterized by: (i) four equilibrium conditions: (43), (45), (46) and (47); (ii) three exogenous structural shock processes: $\hat{z}_t$, $\hat{\nu}_t$ and $\hat{g}_t$; and (iii) three exogenous margin processes: $\hat{\tau}_l$, $\hat{\tau}_c$ and $\hat{\tau}_b$.

$\mathcal{M}_m$: This model is misspecified because the non-systematic component of the nominal rate in the monetary policy decision, $\hat{\nu}_t$, is assumed to be *iid* rather than the AR(1) process described in the DGP. In other words, the model builder incorrectly assumes that the monetary policy shock follows:

$$\hat{\nu}_{t+1} = \sigma_\nu \varepsilon_{\nu,t+1}, \quad \varepsilon_{\nu,t+1} \overset{iid}{\sim} N(0,1). \tag{48}$$

In sum, model $\mathcal{M}_m$ is characterized by: (i) four equilibrium conditions: (43), (44), (45) and (46); (ii) four exogenous structural shock processes: $\hat{z}_t$, $\hat{g}_t$, $\hat{\chi}_t$ and $\hat{\nu}_t$, defined in eq. (48); and (iii) three exogenous margin processes: $\hat{\tau}_l$, $\hat{\tau}_c$ and $\hat{\tau}_b$.

$\mathcal{M}_g$: This model is misspecified because the government spending shock $\hat{g}_t$ is excluded from the set of the structural shocks. In this model, the exclusion of the government spending shock affects both the dynamic IS curve and the NKPC:

$$\hat{Y}_t = \mathrm{E}_t \left\{ \hat{Y}_{t+1} \right\} - \frac{1}{\gamma} \left(\hat{R}_t - \mathrm{E}_t \hat{\pi}_{t+1} \right) - \frac{1}{\gamma} \left(\hat{\tau}_{c,t} - \mathrm{E}_t \hat{\tau}_{c,t+1} \right) - \frac{1}{\gamma} \mathrm{E}_t \left\{ \tau_{b,t+1} \right\}, \tag{49}$$

$$\hat{\pi}_t = \kappa \left\{ (\gamma + \varphi) \hat{Y}_t - (\varphi + 1) \hat{z}_t + \hat{\chi}_t \right\} + \beta \mathrm{E}_t \left\{ \hat{\pi}_{t+1} \right\} + \kappa \left(\hat{\tau}_{c,t} + \hat{\tau}_{l,t} \right). \tag{50}$$

In sum, model $\mathcal{M}_g$ is characterized by: (i) four equilibrium conditions: (45), (46), (49) and (50); (ii) three exogenous structural shock processes: $\hat{z}_t$, $\hat{\nu}_t$, $\hat{g}_t$ and $\hat{\chi}_t$; and (iii) three exogenous margin processes: $\hat{\tau}_l$, $\hat{\tau}_c$ and $\hat{\tau}_b$.

Traditionally, FEVDs are used to evaluate the contribution of various structural shocks to the observables (e.g. Ireland, 2001). Usually, researchers assume that the model is correctly specified and all the variation in the observables are fully explained by the structural shocks. In contrast, we consider the possibility that all the models are potentially misspecified. Therefore, the FEVD analysis should reveal the effects of both structural shocks and margins since the model without margins is misspecified. We expect that the margin that plays an important role in the FEVD identifies the source of the misspecification. Regarding the use of the marginal likelihood, we expect that removing the margin that is most related to the type of misspecification has the largest impact on the marginal likelihood. Thus, we propose to remove one margin at a time and compare the marginal likelihoods of the model with all the margins with the model where one of the margins has been removed. Checking which margins reduce the marginal likelihood the most enables researchers to locate the source of model misspecification.

3.3. Simulation Results

We simulate data on hours worked $\hat{L}_t$, real output $\hat{Y}_t$, the inflation rate $\hat{\pi}_t$, and the nominal interest rate $\hat{R}_t$ using the DGP described in subsection 3.1 and estimate the four models described in subsection 3.2.[9] The sample size of the simulated data is 100.

The FEVDs are summarized in Table 1.[10] Panel (a) focuses on model $\mathcal{M}_0$. Note that the structural shocks, especially the labor supply shock, explain most of the FEVD. In contrast, the contribution of the margins is indeed negligible for any of the four variables, which indicates that the model is correctly specified, as expected: since model $\mathcal{M}_0$ is correctly specified, the three margins are redundant.

Panel (b) focuses on model $\mathcal{M}_l$. Clearly the contribution of the labor margin ($\hat{\tau}_l$) is substantial for all the observables. Comparing these results with those in panel (a), evidently the role of the labor supply shock is almost replaced by the labor margin. This

[9]The pseudo-true data are log deviations from steady-state values.

[10]We only report 12 period-ahead FEVDs. Unreported results for longer and shorter forecast periods are similar and are available upon request.

is because the omission of the labor supply shock distorts the wage in the labor market in model $\mathcal{M}_l$. Thus, the labor margin correctly identifies the source of the misspecification, which is the omission of the labor supply shock.

Panel (c) reports the FEVD of model $\mathcal{M}_m$. In this case, the bond market margin explains the majority of the forecast errors of all variables, while the labor supply and final good margins explain almost nothing. Again, the bond market margin correctly identifies the source of the misspecification which is related to the bond price, $\hat{R}_t$, and therefore points to misspecification in the monetary policy equation.

Finally, comparing the results of panels (d) and (a), it is clear that the final good margin somehow replaces the role of the government spending shock in panel (d), correctly pointing to the source of the misspecification even though the absolute magnitude of the contribution is not very large compared to the other two cases ($\mathcal{M}_l$ and $\mathcal{M}_m$). The latter may be due to the fact that, in our setting, the role of government spending is relatively small. The lack of the government spending in model $\mathcal{M}_g$ distorts the resource constraint which in turn causes distortion in the final goods market.

Table 2 reports the marginal likelihood of the models with the three margins as well as the likelihood of the models with one margin removed at a time. The first column shows that the marginal likelihood of several sub-models based on $\mathcal{M}_0$. Since the marginal likelihood does not change much regardless of which margin is dropped, the marginal likelihood correctly signals that margins are all negligible in the correctly specified model. However, when the labor supply shock is neglected (sub-models based on $\mathcal{M}_l$), the model without the labor margin has the lowest marginal likelihood. Similarly, when the process for the monetary policy shock (sub-model based on $\mathcal{M}_m$) is misspecified, the marginal likelihood is the lowest when the bond market margin is removed. When government spending is omitted from the model ($\mathcal{M}_g$), removing the final good margin leads to the lowest marginal likelihood, while the reduction is relatively minor due to the fact that government spending shocks play a smaller role in the DGP.

To provide intuition why our methodology works, let us examine the linearized equilibrium conditions in subsection 3.2. In model $\mathcal{M}_l$, the only equation affected by the misspecification is the NKPC (eq. 47), where the labor supply shock is missing relative to the correctly specified NKPC (eq. 39), and where two of the margins $(\hat{\tau}_{c,t}, \hat{\tau}_{l,t})$ show up. Note that the dynamic IS curve, eq. (43), where the margins $\hat{\tau}_{c,t}$ and $\hat{\tau}_{b,t}$ show up, is not affected by the misspecification. Thus, the misspecification in the Phillips curve must be captured by $\hat{\tau}_{l,t}$, which is the only margin that appears in the NKPC but not in the IS curve. This is exactly what we find. Regarding the model $\mathcal{M}_m$, the only equation affected by the misspecification is the Taylor rule. Since the nominal interest rate appears in the IS curve but not in the NKPC, the misspecification will be captured by the margin that appears in the IS but not in the NKPC, that is $\hat{\tau}_{b,t}$. In model $\mathcal{M}_g$, the misspecification affects both the NKPC and the IS equations; the only margin that enters in both is $(\hat{\tau}_{c,t})$, which will thus capture the misspecification.

The lesson we learn from the simulation exercise is that introducing margins in the model has the potential to capture the missing channels since the margins correctly reveal which structural shocks are missing from the model. To summarize, FEVD and marginal likelihood analyses provide useful tools for detecting and identifying model misspecification. When a model is misspecified, the contribution of a margin to the FEVD is substantial and is related to the misspecification, thus correctly signaling the possible cause of the misspecification. Moreover, when a margin captures an important aspect of model misspecification, removing it will have a large impact on the marginal likelihood.

3.4. Suggestions for Practitioners

In practice, one does not know which parts of a model are misspecified and may wonder where the margins should be included. In principle, while one can introduce a margin for each market in the model, such a strategy might yield an over-parameterized model even for Bayesian estimation methods. We suggest two approaches. One is to introduce margins in markets which the researcher suspects are misspecified. Another is to introduce margins everywhere, but impose tight priors on the AR(1) parameters and innovation variances of the margin where misspecification is unlikely. The first approach is suitable when the researcher has a strong view on which parts of the model are correctly specified and which parts are not. The second approach is more agnostic about the nature of the misspecification.

The parsimonious AR(1) specification of margin processes is chosen for estimation although one could consider more general specifications and our method would still be valid. By keeping the specification of the margin processes as simple as AR(1) processes, parameters are estimated more precisely and our procedure is expected to have a better chance in detecting misspecification.

One might worry that adding many margins into the model creates lack-of-identification problems. If that is the case, the researcher can use any of the existing methods for detecting identification problems. For example, if the researcher worries about weak identification, he/she could utilize the methods proposed by Canova and Sala (2009), Iskrev (2010) or Inoue and Rossi (2011). If the researcher worries about lack of local identification, he/she could utilize Komunjer and Ng (2011), while if the researcher worries about lack of global identification, he/she could utilize the method proposed by Qu and Tkachenko (2016). It should be noted that the marginal likelihood criterion can be used to detect misspecification even when some parameters are not identified, however. This is because the value of the marginal likelihood is identified even when parameters are not.

Finally, while in our examples FEVD and marginal likelihood analyses convey the same conclusions, that may not always be the case in practice. In fact, they allow researchers to evaluate different aspects of the model: FEVDs focus on how the margins affect the second moment properties of the observables while the marginal likelihood provides a more general assessment that includes other moments and general characteristics of the overall distribution. On the other hand, the marginal likelihood only assesses the joint performance of the margins on all the observables, while FEVDs provide more detailed information on which observables are affected by which margins. The marginal likelihood has a built-in penalty term for overparameterization. Hence, they both are useful in different ways and we recommend researchers to use both of them.

4. Empirical Application to a Medium-Scale New Keynesian Model

In this section, we consider potential misspecification in a medium-scale New Keynesian model. The model, based on Justiniano et al. (2010), is a stochastic neoclassical growth model with various real and nominal frictions, and is routinely used by researchers and policymakers. The frictions include imperfect competition in the intermediate goods and labor markets, sticky prices and wages, habit formation in consumption, investment adjustment cost and variable capital utilization. We explore whether this model may be misspecified by including several time-varying margins and evaluating their importance. See Sections B and C in the Not-for-Publication Appendix for the description of the model and for its linearized equilibrium conditions, respectively.

Following Justiniano et al. (2010), we estimate the model using seven quarterly aggregate U.S. time series from 1954:III to 2004:IV, namely: real output, real consumption, real investment, hours worked, inflation rate, and the Federal funds rate. Real output, real consumption, real investment and hours worked are per capita. Real output, investment and consumption per capita are obtained by dividing nominal GDP, investment and consumption by the population and the price index. Nominal consumption is defined as the sum of non-durable goods and services expenditures. Nominal investment is defined as the sum of private domestic investment and personal durable goods expenditure. Hours worked per capita are defined as total hours worked in the non-farm business sector divided by the population. Real wages are defined as non-farm business sector hourly compensation divided by the price index. We use the civilian non-institutional population as our population measure. The price index is the GDP deflator, and the quarterly inflation rate is its growth rate. Estimation results are based on 120,000 Markov Chain Monte Carlo (MCMC) draws, and the first 60,000 draws are discarded. Section D in the Not-for-Publication Appendix discuss the priors and posterior estimates of the parameters. For convenience, Table 3 summarizes the notation corresponding to all the margins and shocks in the model.

Forecast Error Variance Decomposition. Recall from Sections 2 and 3 that the margins that play the largest role in the FEVDs identify the sources of model misspecification. Table 4 reports the FEVD contribution of the shocks and margins (listed in the columns) to the overall variance of the observable variables (listed in the rows) at different forecast horizons ($H = 1, 4, 20$ quarters). Table 4 shows that the capital, homogenous labor market, and consumption good margins are extremely small for all the variables of interest. In contrast, the bond market, the intermediate goods demand, and the household labor margins contribute to explain the variability of several variables, sometimes substantially. For example, the household labor margin explains more than 60% of one quarter-ahead forecast error variance of wage growth. The effects of the household labor margin are persistent: it explains more than 40% of the wage growth variation after twenty quarters. This result indicates that the model without household labor margin could be misspecified in the wage growth dynamics. Our results differ from Justiniano et al. (2010), whose wage mark-up shocks explain about 56% of the wage growth variation at the business cycle frequency: according to our results, the wage mark-up shock does not explain more than 11% between 1 and 20 quarters. That is, the model without household labor margin might attribute wage fluctuations to the wage mark-up shock.

At the one-quarter ahead forecast horizon, the intermediate good margin (τ_q) explains around 20% of the variation in inflation. Actually, its contribution is even larger than that of technology shocks, which only explain about 14% of inflation fluctuations at the same forecast horizon. Note also that its importance decays as the forecast horizon increases. Since the intermediate good margin mainly affects short run fluctuations, our results warn against using the model for forecasting short run inflation if the model misspecification is not properly addressed. It is worth noting that while the intermediate good (τ_q) and household labor margins (τ_h) have crucial effects on nominal variables (that is, wage growth, inflation rates, and, to a lower extent, interest rates), they have hardly any effects on other variables.

The bond market margin τ_r mainly affects nominal interest rates fluctuations, and its contribution increases with the forecast horizon: at the twenty quarters horizon ($H = 20$), more than 40% of the interest rate variability can be attributed to fluctuations in the bond market margin. The dominant effect of the bond market margin comes at the expense of the wage mark-up and investment shocks, whose total contribution is merely around 30%. This result sharply differs from Justiniano et al. (2010), where about 67% of the interest rate variability is explained by wage mark-up and investment shocks. In addition, the bond market margin also contributes to output, investment, and consumption growth as well as hours worked fluctuations at various forecast horizons, although its effects are smaller in magnitude.

Based on the above FEVD analysis, we conclude that margins mainly affect wage growth, the inflation rate, and the interest rate; for other observables, our results are similar to Justiniano et al. (2010): that is, the investment shock is the main driving force behind output growth, investment growth, and hours worked, and the inter-temporal preference shock explains most of the variability in consumption growth.[11]

For comparison, we report the FEVD of the benchmark model without margins in Table 5. To simplify the discussion, we focus on the twenty quarter-ahead forecast horizon. Margins mainly affect the variability of wage growth, inflation, and the interest rate, and we discuss them in turn. Regarding the wage growth, the model without margins explains its fluctuations mainly by the technology shock (37.58%) and the wage mark-up shock (37.64%). However, once margins are taken into account, the contribution of the wage mark-up shock reduces to 6.8% only. In contrast, the household labor margin explains more than 41% of wage growth in the model with margins. Regarding the inflation rate, investment shocks are the key difference: in the model without margins, investment shocks explain around 11% of inflation variability, while their effects are negligible (1%) in the model with all margins. Finally, regarding the interest rate, the role played by investment shocks is very different depending on whether the model does or does not include margins: in the model without margins, investment shocks explain more than 60% of interest rate fluctuations; however, once margins are included, investment shocks only explain around 20%, and the bond market margin largely replaces the investment shock.

[11]See Table 3 of the technical appendix in Justiniano et al. (2010).

Overall, our FEVD results indicate that misspecification is mostly captured by the household labor, the bond market and the intermediate goods demand margins, which confirms the existing view that the asset and labor markets in the standard New Keynesian DSGE model are misspecified (see Krause, Lopez-Salido and Lubik, 2008, and Christiano, Motto and Rostagno, 2008, for example).

Marginal Likelihood. We compare the marginal likelihood (calculated by the modified harmonic mean estimator) of: (i) the model with all the margins; (ii) the model with no margins; (iii) the models that remove one margin at a time; and (iv) the model that removes the three margins that, according to the FEVD, are the source of the misspecification. Recall that the logic of this exercise is to investigate the relative importance of margins: if removing a particular margin considerably reduces the marginal likelihood value, it is a signal that the margin is crucial in explaining the dynamics of the observed data. Panel A in Table 6 summarizes the marginal likelihood values of the various versions of the benchmark model. First of all, the first two rows display the results for models with and without margins. Surprisingly, while the FEVDs of these two models are different, their marginal likelihood values are not very different. The marginal likelihood can be written as the product of one-step-ahead predictive densities that may be highly non-normal because parameters are integrated out in the marginal likelihood. Even if we focus on one-step-ahead predictions ($H = 1$), the densities contain more information than the second moments (i.e., FEVD). This may explain the difference between the results based on the marginal likelihood and those based on FEVDs. Interestingly, however, the model that removes the same three margins that FEVDs identify as the source of misspecification is the one with the lowest likelihood. This finding confirms that the latter are the sources of misspecification, while the small differences in the marginal likelihood suggest that the degree of misspecification may be mild.[12]

Figure 1 reports time series estimates of the margins. By examining the estimate of the

[12]We did not investigate all possible margin combinations, as that would result in too many models to consider. The scope of the marginal likelihood analysis was mostly to confirm the results of the FEVDs.

bond market margin over time ($\tau_{r,t}$), it is clear that the margin was especially large in the early 1980s, possibly due to the changes in monetary policy around that time, as well as during the latest financial crisis of 2007-2009.

Serial Correlation in the Shocks. DSGE models rely on a structural interpretation of the exogenous shocks. The exogenous shocks should be invariant to any policy changes (to be robust to the Lucas critique) as well as uncorrelated among each other. If a shock included in a DSGE model is in reality a combination of other shocks, that shock should be interpreted as a reduced-form, rather than structural, shock. Here, we assess the correct specification of the ARMA structure of the wage mark-up shock, as in Chari et al. (2009).

We consider a restricted model which is the same as the benchmark model except that the moving average coefficient of the wage mark-up shock is set to zero. We estimate several variants of the benchmark model, depending on which margins are included or removed, and compare their marginal likelihoods.

Panel B in Table 9 displays the marginal likelihood of the model with all margins except the household labor margin. Clearly, the marginal likelihood value is lower than that of the other models. One can interpret this finding in two ways. A first interpretation is that the wage mark-up shock indeed follows an ARMA process. Thus, by estimating the model with the wage mark-up shock following an AR process, we are actually estimating a misspecified model. By including the household labor margin into the model, the potential gap due to the labor market misspecification is filled by the labor market margin. This point can be seen clearly by comparing marginal likelihood values of the models with and without the household labor market margin. Given that the true model is that with an ARMA-type wage mark-up shock, our results indicate that the marginal likelihood values indeed help us detect the source of model misspecification. A second interpretation is that the wage mark-up shock instead follows an AR process. Since including the household labor margin improves the overall fitting of the model, then some factors related to the labor market must be missing. In other words, the models without the household labor margin are misspecified in the labor market. Either way, our exercise casts doubts on the structural interpretation of the wage mark-up shock.

5. Conclusion

This paper proposes empirical methods for detecting and identifying misspecification in structural economic models. Our approach is based on analyzing FEVDs and marginal likelihoods of DSGE models augmented with margins, where the margins are introduced in the agents' optimization problems to capture potential misspecification. Monte Carlo simulations demonstrate that our method can correctly identify the source of the misspecification. Our empirical results show that a medium-scale New Keynesian DSGE model that incorporates features in the recent empirical macro literature is still severely misspecified, and suggest that asset and labor markets are the sources of the misspecification.

We should note that there are three potential issues with implementing our method: exogeneity of the margins, over-parametrization and non-nesting misspecification. First, because the margin processes are assumed to be exogenous, our method might not correctly identify the location of misspecification if the misspecification was endogenous. One way to address this issue is to let the margins depend on state variables. In our simulation results, however, we are able to successfully identify the misspecification due to the serially correlated omitted frictions in the monetary policy reaction function, which suggests the usefulness of our approach in finding the omitted frictions in the model.

Second, when many markets are included in the model, there may be too many locations for introducing margins and ways for forming priors for these processes. We suggest to either use fewer margins when the prior on the location of the misspecification is strong (e.g. the researcher is confident that there is no misspecification in some parts of the model, but unsure whether there might be misspecification in others, and the researcher has strong opinions on where the misspecification is potentially located), or introduce many margins and impose prior information when the misspecification location is more uncertain (i.e., every part of the model can potentially be misspecified). When neither is possible, another approach is to use a Bayesian model averaging approach to take into account many margins, and let it provide information on the location of the margins.

Lastly, our method of comparing models via their marginal likelihood is clearly appropriate if the true model is one of the models with the margins. Otherwise, one could consider alternative ways to perform the comparison, such as cross-validation and out-of-sample predictions (see Bernardo and Smith, 2000, p.403 and Geweke, 2010). We leave these extensions to future research.

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Table 1: Forecast Error Variance Decompositions

Panel (a): Model $\mathcal{M}_0$

Variable	z	ν	g	χ	τ_l	τ_c	τ_b
Hours	9.15	57.78	10.06	21.41	0.44	0.89	0.26
Output	9.64	57.47	10.01	21.29	0.44	0.89	0.26
Inflation Rate	3.56	87.81	0.10	8.17	0.29	0.00	0.07
Interest Rate	21.21	27.99	1.20	47.53	1.16	0.21	0.70

Panel (b): Model $\mathcal{M}_l$

Variable	z	ν	g	χ	τ_l	τ_c	τ_b
Hours	9.40	60.67	8.93	–	19.73	1.13	0.15
Output	9.51	60.59	8.92	–	19.70	1.13	0.15
Inflation Rate	3.39	88.29	0.06	–	8.23	0.01	0.03
Interest Rate	21.25	30.04	1.51	–	46.62	0.27	0.31

Panel (c): Model $\mathcal{M}_m$

Variable	z	ν	g	χ	τ_l	τ_c	τ_b
Hours	11.96	14.87	8.55	22.18	0.50	0.95	40.99
Output	9.66	15.26	8.77	22.76	0.51	0.97	42.06
Inflation Rate	6.54	22.84	0.19	15.10	0.62	0.02	54.69
Interest Rate	2.94	3.81	0.06	6.97	0.14	0.02	86.06

Panel (d): Model $\mathcal{M}_g$

Variable	z	ν	g	χ	τ_l	τ_c	τ_b
Hours	9.46	59.89	–	22.49	0.51	6.59	1.05
Output	9.04	60.17	–	22.60	0.51	6.63	1.06
Inflation Rate	3.37	87.25	–	8.66	0.33	0.04	0.36
Interest Rate	18.64	27.56	–	47.04	1.23	1.19	4.33

Notes to the table. The table reports the median of the FEVD (in percentage) for the models we estimated. The forecast horizon is 12 periods.

Table 2. Marginal Likelihood Values of Models

	$\mathcal{M}_0$	$\mathcal{M}_l$	$\mathcal{M}_m$	$\mathcal{M}_g$
All margins	*-1412.3*	*-1415.4*	*-1497.1*	*-1412.2*
Remove τ_l	*-1413.0*	*-1429.1*	*-1497.6*	*-1412.6*
Remove τ_c	*-1411.8*	*-1414.4*	*-1496.4*	*-1413.3*
Remove τ_b	*-1411.7*	*-1414.6*	*-1506.7*	*-1412.6*

Notes to the table. The table shows values of the log marginal likelihood, calculated via the modified harmonic mean. Lower values of the likelihood denote models that are the most at odds with the data. Thus, the margins whose removal are associated with the lowest likelihood are the margins that are deemed the most necessary to explain the data.

Table 3. Margins and Shocks: Summary Table

Margin	*Description*	*Innovation*	*Reference Equation in the Not-for-Publication Appendix*
		Panel A. Margins	
τ_l	Homogeneous labor market margin	ε_l	Eq. (B.8)
τ_k	Capital market margin	ε_k	Eq. (B.7)
τ_c	Consumption margin	ε_c	Eq. (B.12)
τ_r	Bond market margin	ε_r	Eq. (B.10)
τ_q	Intermediate goods demand margin	ε_q	Eq. (B.3)
τ_h	Household labor margin	ε_h	Eq. (B.16)
		Panel B. Shocks	
η_{mp}	Monetary policy shock	ε_{mp}	Eq. (B.18)
z	Technology shock	ε_z	Eq. (B.6)
g	Government spending shock	ε_g	Eq. (B.17)
μ	Investment shock	ε_μ	Eq. (B.11)
λ_p	Price mark-up shock	ε_p	Eq. (B.2)
λ_w	Wage mark-up shock	ε_w	Eq. (B.15)
b	Intertemporal preference shock	ε_b	Eq. (B.13)

Notes to the table. The table summarizes the shocks and the margins in the model considered in Section 4. For simplicity, we removed time subscripts from the notation.

Table 4. The Forecast Error Variance Decomposition: Benchmark Model (All Margins included)

	Shocks							*Margins*					
Series	ε_{mp}	$\varepsilon_{z,t}$	$\varepsilon_{g,t}$	$\varepsilon_{\mu,t}$	$\varepsilon_{p,t}$	$\varepsilon_{w,t}$	$\varepsilon_{b,t}$	$\varepsilon_{l,t}$	$\varepsilon_{k,t}$	$\varepsilon_{c,t}$	$\varepsilon_{r,t}$	$\varepsilon_{q,t}$	$\varepsilon_{h,t}$
							Forecast Horizon: H = 1						
Output growth	5.79	12.41	10.04	51.79	0.83	0.06	13.71	0.00	0.00	0.00	3.32	0.03	0.03
Consumption growth	1.84	15.00	1.46	0.05	0.09	1.94	77.88	0.00	0.00	0.00	1.20	0.01	0.02
Investment growth	4.93	4.59	0.02	85.30	0.92	0.24	0.38	0.00	0.00	0.00	2.67	0.03	0.04
Hours	5.29	20.32	9.23	47.06	0.73	0.06	12.23	0.00	0.00	0.00	3.01	0.03	0.04
Wage growth	0.27	8.46	0.00	0.48	11.36	11.70	0.10	0.01	0.00	0.00	0.22	4.43	61.78
Inflation rates	2.83	13.53	0.10	1.19	36.27	15.11	0.42	0.06	0.00	0.00	4.78	19.60	1.15
Interest rates	53.29	6.49	0.62	12.39	1.05	0.96	13.65	0.03	0.00	0.00	6.22	1.75	0.36
							Forecast Horizon: H = 4						
Output growth	6.16	22.05	7.75	43.64	2.05	0.80	10.78	0.00	0.00	0.00	3.81	0.03	0.06
Consumption growth	2.10	26.60	2.57	0.34	0.26	4.48	60.75	0.00	0.00	0.00	1.47	0.01	0.02
Investment growth	5.56	8.72	0.02	78.04	2.31	0.27	0.41	0.00	0.00	0.00	3.20	0.03	0.06
Hours	10.90	7.11	4.44	54.39	3.62	1.16	7.77	0.00	0.00	0.00	7.09	0.01	0.13
Wage growth	0.32	26.54	0.00	0.77	13.90	7.38	0.07	0.01	0.00	0.00	0.26	3.72	45.40
Inflation rates	4.83	14.36	0.13	1.48	26.74	26.26	0.58	0.03	0.00	0.00	8.68	9.05	0.77
Interest rates	21.75	9.69	0.70	27.93	1.50	3.83	8.75	0.02	0.00	0.00	18.92	0.48	0.34
							Forecast Horizon: H = 20						
Output growth	6.10	22.57	6.90	42.63	2.47	2.65	10.02	0.00	0.00	0.00	3.63	0.03	0.08
Consumption growth	1.82	28.59	2.66	1.71	0.38	6.21	54.87	0.00	0.00	0.00	1.30	0.01	0.02
Investment growth	5.58	8.44	0.03	77.05	2.81	0.97	0.42	0.00	0.00	0.00	3.09	0.03	0.07
Hours	8.82	7.20	2.38	23.18	9.87	32.63	2.70	0.00	0.00	0.00	7.47	0.01	0.11
Wage growth	0.32	30.70	0.00	0.88	14.18	6.80	0.07	0.01	0.00	0.00	0.27	3.36	41.78
Inflation rates	5.29	9.52	0.13	1.02	16.63	41.26	0.54	0.02	0.00	0.00	12.85	5.25	0.49
Interest rates	10.07	5.70	0.46	19.56	1.15	10.53	4.54	0.01	0.00	0.00	40.14	0.22	0.17

Notes to the table. The table reports the median of the FEVD (in percentage) for the model with margins. The FEVD is calculated over the MCMC draws we do not discard (therefore, the sum of the percentages in a given row does not necessarily equal 100).

Table 5. The Forecast Error Variance Decomposition:
The Benchmark Without Margins

Series	ε_{mp}	$\varepsilon_{z,t}$	$\varepsilon_{g,t}$	$\varepsilon_{\mu,t}$	$\varepsilon_{p,t}$	$\varepsilon_{w,t}$	$\varepsilon_{b,t}$
			Forecast Horizon: H = 1				
Output growth	3.01	9.69	8.41	68.75	0.96	0.50	7.85
Consumption growth	1.81	17.20	2.29	1.75	0.23	2.79	72.90
Investment growth	1.60	2.44	0.01	94.23	0.72	0.01	0.70
Hours	2.89	12.86	8.14	66.08	0.90	0.52	7.57
Wage growth	0.30	10.82	0.00	2.25	21.72	63.68	0.33
Inflation rates	1.92	12.00	0.11	7.34	54.08	21.44	1.15
Interest rates	46.28	6.04	0.67	24.84	3.51	1.22	14.77
			Forecast Horizon: H = 4				
Output growth	3.00	16.24	6.45	63.18	1.66	1.90	6.31
Consumption growth	1.72	27.03	3.60	1.74	0.37	5.70	58.16
Investment growth	1.68	4.32	0.01	91.16	1.29	0.24	0.84
Hours	4.32	3.44	3.05	76.16	2.64	2.90	6.24
Wage growth	0.34	34.50	0.00	3.82	19.09	40.63	0.25
Inflation rates	3.56	12.96	0.15	11.96	29.93	35.86	1.90
Interest rates	15.49	6.89	0.53	55.86	2.38	3.60	12.59
			Forecast Horizon: H = 20				
Output growth	3.04	16.19	5.80	62.66	1.78	2.97	6.52
Consumption growth	1.44	25.19	3.33	7.66	0.33	6.17	53.53
Investment growth	1.69	4.04	0.01	90.89	1.39	0.59	0.86
Hours	3.41	4.09	1.98	48.17	5.01	32.53	2.94
Wage growth	0.33	37.58	0.01	4.30	18.44	37.64	0.31
Inflation rates	3.95	9.64	0.14	10.71	20.65	49.70	1.91
Interest rates	9.03	5.01	0.41	62.14	1.64	9.91	9.18

Notes to the table. The table reports the median of the FEVD (in percentage) for the model without margins. The FEVD is calculated over the MCMC draws we do not discard (therefore, the sum of the percentages in a given row does not necessarily equal 100).

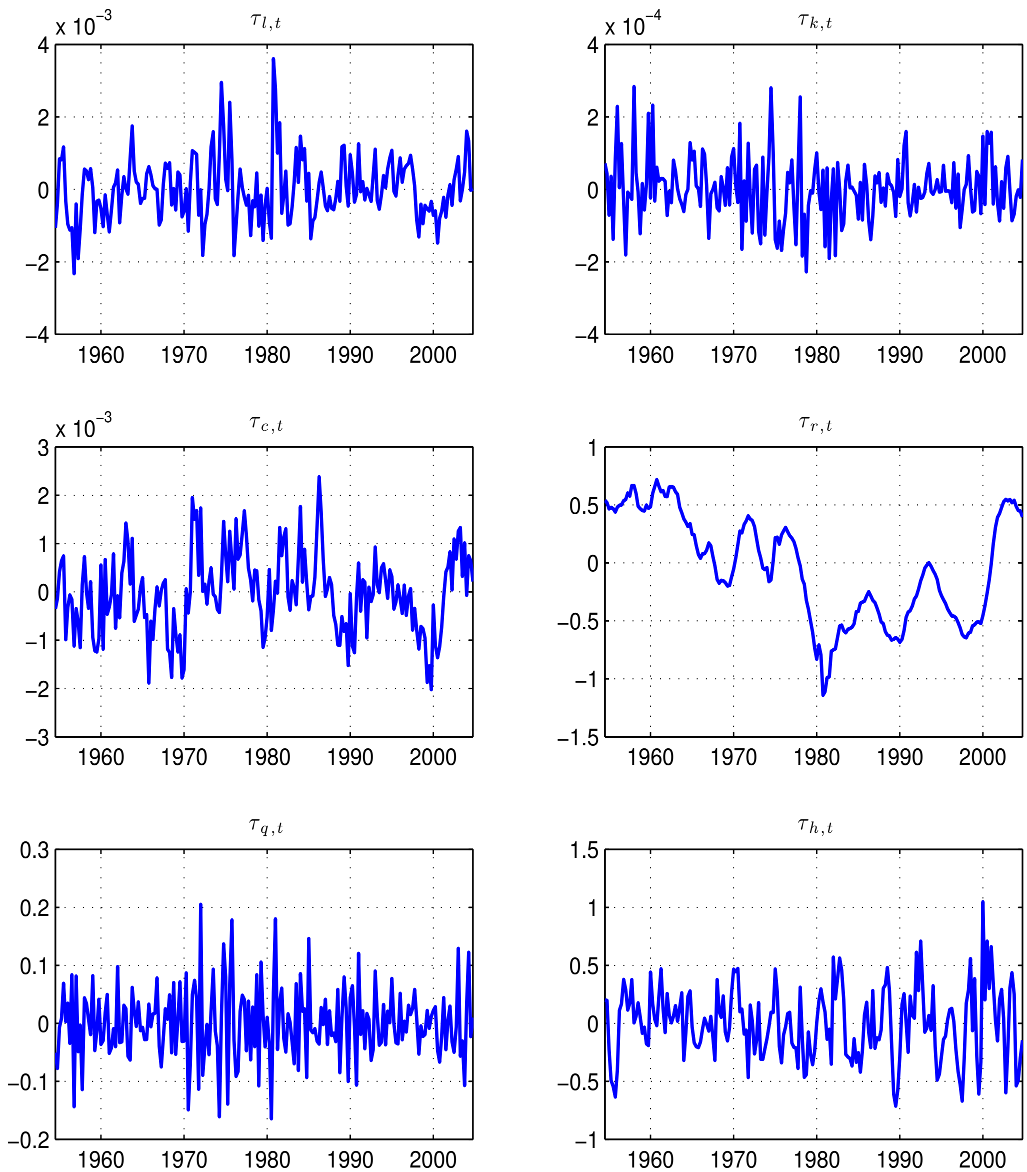
Table 6. Log Marginal Likelihood Values

Panel A. Variants of the Benchmark Model		
Models	*Log Marginal Likelihood*	*Ranking*
All margins	-1167.62	4
Remove all margins	-1166.99	3
Remove τ_l margin	-1169.13	7
Remove τ_k margin	-1166.19	1
Remove τ_c margin	-1170.86	8
Remove τ_r margin	-1166.66	2
Remove τ_q margin	-1168.87	6
Remove τ_h margin	-1168.25	5
Remove $\{\tau_r, \tau_q, \tau_h\}$ margins	-1171.01	9
Panel B. Model with a Simple AR Process for Wage Mark-up Shocks		
Models	*Log Marginal Likelihood*	*Ranking*
All margins	-1173.93	6
Remove all margins	-1192.49	7
Remove τ_l margin	-1170.18	1
Remove τ_k margin	-1171.52	3
Remove τ_c margin	-1170.19	2
Remove τ_r margin	-1172.53	4
Remove τ_q margin	-1173.09	5
Remove τ_h margin	-1207.99	8

Notes to the table. The table shows values of the log marginal likelihood. Lower values of the likelihood denote models that are the most at odds with the data. Thus, the margins whose removal are associated with the lowest likelihood are the margins that are deemed the most necessary to explain the data.

Figure 1. Margins Over Time (Smoothed)

700



Note to the figure. The figure plots the time series of the estimated margins over time.

Not-for-Publication Appendix to: Identifying the Sources of Model Misspecification by Atsushi Inoue, Chun-Hung Kuo and Barbara Rossi

Appendix A. ***The Small-Scale DSGE Model***

This section describes the small-scale DSGE model used in Section 3. In period t, the representative final good firm produces its output using the technology $Y_t = \left[\int_0^1 (Y_{j,t})^{\frac{1}{\lambda_f}} dj\right]^{\lambda_f}$, where $Y_{j,t}$ is the intermediate input purchased by the j-th intermediate good firm, $j \in [0,1]$. The competitive final good firm determines $Y_{j,t}$ by solving its profit maximization problem:

$$\max_{\{Y_{j,t}\}_{j\in[0,1]}} P_t Y_t - \int_0^1 P_{j,t} Y_{j,t} dj,$$

subject to its production technology, taking the prices of intermediate goods ($P_{j,t}$) and that of the final good (P_t) as given.

Following Calvo (1983), we assume that intermediate goods firms are subject to idiosyncratic shocks in setting their product prices, while they are monopolistic providers of differentiated goods. Specifically, in each period only a fraction $1-\xi$ of able to set the desired optimal price. As a consequence, period t pricing problem of the intermediate good firm is:[13]

$$\max_{\tilde{P}_t} \quad E_t \sum_{l=0}^{\infty} (\beta\xi)^l \, v_{t+l} \left\{\tilde{P}_t - P_{t+l} s_{t+l}\right\} \tilde{Y}_{t+l} \tag{A.1}$$

subject to $\tilde{Y}_{t+l} = \left(\frac{\tilde{P}_t}{P_t}\right)^{-\frac{\lambda_f}{\lambda_f - 1}} Y_{t+l} = Y_{l+t}$, where $\tilde{Y}_{t+l}$ is the demand of the intermediate good with price $\tilde{P}_t$, which is set in period t; v_{t+l} is the marginal value of per unit of money for households at time $(t+l)$, since households hold equities of intermediate good firms, and s_{t+l} is the real marginal cost in period $(t+l)$. Lastly, the presence of ξ^l in the objective function indicates the probability of not being able to re-optimize the price after l periods.

[13] Note that there is no need to specify the firm subscripts in this pricing problem since firms, which are able to re-optimize, face the same problem and make the same decision.

The intermediate goods firms' production function is: $Y_{j,t} = z_t L_{j,t}$, where $L_{j,t}$ is labor input and z_t is the technology shock z_t common to all intermediate firms, which follows: $\hat{z}_{t+1} = \rho_z \hat{z}_t + \sigma_z \varepsilon_{z,t+1}, \quad \varepsilon_{z,t+1} \sim iid\ N(0,1)$. Given the competitive nominal wage rate W_t, each intermediate good firm minimizes $W_t L_{jt}$ subject to its production function to determine its labor demand.

The representative household's lifetime utility maximization problem is:

$$\max_{\{C_t, L_t\}_{t=0}^{\infty}} E_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\gamma}}{1-\gamma} - \chi_t \frac{L_t^{1+\varphi}}{1+\varphi} \right\},$$

subject to the budget constraint:

$$P_t C_t + B_{t+1} = R_{t-1} B_t + W_t L_t + \mathsf{D}_t + \mathsf{T}_t,$$

where C_t is consumption, L_t is the labor supply (for which, in equilibrium, the aggregation condition $L_t = \int_0^1 L_{j,t} dj$ applies, since there exist a continuum of intermediate goods firms), B_{t+1} is the government bond, R_{t-1} is the gross nominal interest rate, D_t is the dividend paid by the intermediate goods firms, and T_t is the lump sum transfer from the government. The labor supply shock χ_t follows an AR(1) process: $\hat{\chi}_{t+1} = \rho_\chi \hat{\chi}_t + \sigma_\chi \varepsilon_{\chi,t+1}, \quad \varepsilon_{\chi,t+1} \sim iid\ N(0,1)$.

In addition to issuing bonds, the government consumes a fraction of final output spending $G_t = \zeta_t Y_t$. The stochastic fraction ζ_t is re-parameterized as $g_t \equiv \frac{1}{1-\zeta_t}$, where $\hat{g}_{t+1} = \rho_g \hat{g}_t + \sigma_g \varepsilon_{g,t+1}, \quad \varepsilon_{g,t+1} \sim iid\ N(0,1)$. The central bank determines the nominal interest rate by a Taylor rule: $\hat{R}_t = \rho_r \hat{R}_{t-1} + (1-\rho_r) \left\{ \gamma_\pi \hat{\pi}_t + \gamma_Y \hat{Y}_t \right\} + \hat{\nu}_t$, where ρ_r denotes the degree of interest rate smoothness, $\hat{\nu}_t$ is the interest rate shock, capturing the non-systematic part of the nominal rate decision, and it follows: $\hat{\nu}_{t+1} = \rho_\nu \hat{\nu}_t + \sigma_\nu \varepsilon_{\nu,t+1}, \quad \varepsilon_{\nu,t+1} \sim iid\ N(0,1)$.

The parameter values used to generate the data are summarized in the column labeled "DGP" in Table A and are set within widely accepted ranges. The model is estimated by Bayesian methods using the random walk Metropolis-Hastings algorithm (An and Schorfheide, 2007). The priors used for estimation are listed in the last three columns of Table A and are the same for each model. However, it is worth noting that depending on a particular model at hand, some parameters are not necessarily estimated. Specifically, in model $\mathcal{M}_l$, which excludes labor supply shocks, ρ_χ and σ_χ are set to 0; in model $\mathcal{M}_g$, which excludes government spending shocks, ρ_g and σ_g are set to 0; in model $\mathcal{M}_m$, where the non-systematic part of the Taylor rule is misspecified, ρ_ν is set to 0. Moreover, some parameters are not estimated in any of the models: the subjective discount factor β, the risk aversion coefficient γ, and the inverse elasticity of labor supply φ are all fixed at their true parameter values. In our estimation exercises, the number of MCMC draws are 120,000, and the first 60,000 draws are discarded.

Appendix B. *The Medium Scale DSGE Model*

This section describes the medium-scale New Keynesian DSGE model used in Section 4. Note that we cannot simply list the equilibrium conditions of the model by Justiniano et al. (2010). The reason is that we treat the margins as integral parts of agents' (including households and firms) decision problems. Thus, we have to specify the environment explicitly to show how these margins affect agents' decisions.

Appendix B.1. Final Good Firms

In the model, final goods are produced by a continuum of perfectly competitive final good firms. The production function of final good firms is

$$Y_t = \left[\int_0^1 Q_{i,t}^{\frac{1}{1+\lambda_{p,t}}} di \right]^{1+\lambda_{p,t}}, \tag{B.1}$$

where $Q_{i,t}$ is the intermediate good purchased from the i-th intermediate good firm, and $\lambda_{p,t}$ is an exogenous structural shock governing the elasticity of substitution among intermediate goods. In line with Smets and Wouters (2007), $\lambda_{p,t}$ follows the ARMA(1,1) process:

$$\log\left(1+\lambda_{p,t}\right) = \left(1-\rho_p\right)\log\left(1+\lambda_p\right) + \rho_p \log\left(1+\lambda_{p,t-1}\right) + \varepsilon_{p,t} - \theta_{p,t}\varepsilon_{p,t-1}, \quad \varepsilon_{p,t} \sim \mathcal{N}\left(0, \sigma_p^2\right). \tag{B.2}$$

In the literature, this shock is often called as the price mark-up shock, as it determines the time-varying desired mark-up of intermediate good firms.

Taking the final good price P_t and intermediate good prices $P_{j,t}$'s as given, the final good firms decide their intermediate good inputs ($Q_{i,t}$) by solving the profit maximization problem:

$$\max_{\{Q_{i,t}\}_{i\in[0,1]}} P_t Y_t - \int_0^1 \tau_{q,t} P_{i,t} Q_{i,t} di,$$

subject to the final good production function (B.1). Notice that we introduce a time-varying intermediate good margin $\tau_{q,t}$ into this problem, and it evolves according to:

$$\log \tau_{q,t} = (1-\rho_q)\log \tau_q + \rho_q \log \tau_{q,t-1} + \varepsilon_{q,t}, \quad \varepsilon_{q,t} \sim \mathcal{N}\left(0, \sigma_q^2\right), \tag{B.3}$$

with $\tau_q = 1$ in the steady state. The value of $\tau_{q,t}$ is assumed to be known by the final good firms when they make their decisions. The reason of introducing $\tau_{q,t}$ is to capture model misspecification in the demand of intermediate goods. To see this, consider the demand function of intermediate goods:

$$Q_{i,t} = \left(\frac{\tau_{q,t} P_{i,t}}{P_t}\right)^{\frac{-(1+\lambda_{p,t})}{\lambda_{p,t}}} Y_t.$$

The intermediate goods demand $Q_{i,t}$ is affected not only by the relative price $\frac{P_{i,t}}{P_t}$ but also by the intermediate good margin, $\tau_{q,t}$. Evidently, introducing $\tau_{q,t}$ helps capture the contribution of factors affecting final good firms' decisions other than $\frac{P_{i,t}}{P_t}$. The margin $\tau_{q,t}$ can be interpreted as a proxy that captures the fact that the model builder ignores that, in reality, firms are likely to consider more aspects than merely the relative prices of intermediate goods when they make their input decisions. Moreover, $\tau_{q,t}$ helps relaxing the tight restriction between intermediate good prices and the price of the final good. To see this, consider the following equilibrium condition:

$$P_t = \tau_{q,t} \left[\int_0^1 \left(P_{i,t}\right)^{-\frac{1}{\lambda_{p,t}}} di\right]^{-\lambda_{p,t}}, \tag{B.4}$$

which is obtained by substituting the intermediate good demand function into (B.1). Equation (B.4) implies that if the intermediate good prices ($P_{i,t}$) cannot characterize well the the dynamics of the overall price level P_t, the deviation is captured by $\tau_{q,t}$. Besides, our assumption of $\tau_q = 1$ in the steady-state indicates that we take the stand that the relationship between prices of intermediate goods and the final good is, on average, correctly specified.

Appendix B.2. Intermediate Goods Firms

We assume that there exists a continuum of intermediate goods firms, which are operating in imperfectly competitive markets. Thus, they have the ability to set their desired prices. Below, we adopt the typical two-stage procedure to describe the optimal behavior of intermediate goods firms. We first lay out the cost minimization problem and then the profit maximization one.

Cost Minimization. In period t, the i-th intermediate goods firm rents capital services $K_{i,t}$ and homogeneous labor services $L_{i,t}$ from perfectly competitive factor markets to produce differentiated intermediate goods, $Q_{i,t}$. Its production function function is defined as:

$$Q_{i,t} = \max\left\{A_t^{1-\alpha} K_{i,t}^{\alpha} L_{i,t}^{1-\alpha} - A_t F;\ 0\right\}, \tag{B.5}$$

where A_t denotes the time t technology level, which is common across all intermediate goods firms,[14] α is the capital elasticity of producing the intermediate good, and $A_t F$ is the fixed production cost in period t. Following Justiniano et al. (2010), we assume that the technology growth rate ($z_t = \Delta \log A_t$) follows a stationary AR(1) process:

$$z_t = (1-\rho_z)\,\gamma + \rho_z z_{t-1} + \varepsilon_{z,t}, \quad \varepsilon_{z,t} \sim \mathcal{N}\left(0, \sigma_z^2\right). \tag{B.6}$$

This assumption implies that the economy grows along the stochastic trend in A_t.

The cost minimization problem of the intermediate goods firms is

$$\min_{K_{i,t}, L_{i,t}} \tau_{k,t} R_{k,t} K_{i,t} + \tau_{l,t} W_t L_{i,t}$$

[14] Following Christiano et al. (2005), we assume that there is neither exit nor entry decision for intermediate goods firms, while theoretically some firms might not be able to produce anything due to high fixed costs.

subject to the production function (B.5). Here, $R_{k,t}$ and W_t are the nominal rental rate of capital and the nominal wage rate, respectively. Note that two time-varying margins are introduced in this problem to capture the model misspecification in the cost minimization problem. The first one is the capital market margin $\tau_{k,t}$, and the second one is the homogeneous labor market margin $\tau_{l,t}$. As in the case of the intermediate good margin ($\tau_{q,t}$), the two margins are assumed to be known when the intermediate goods firm makes the input decision in period t. To be parsimonious, the two exogenous margin processes both follow AR(1) processes:

$$\log \tau_{k,t} = (1-\rho_k)\log \tau_k + \rho_k \log \tau_{k,t-1} + \varepsilon_{k,t}, \quad \varepsilon_{k,t} \sim \mathcal{N}(0, \sigma_k^2), \tag{B.7}$$

$$\log \tau_{l,t} = (1-\rho_l)\log \tau_l + \rho_l \log \tau_{l,t-1} + \varepsilon_{l,t}, \quad \varepsilon_{l,t} \sim \mathcal{N}(0, \sigma_l^2). \tag{B.8}$$

The interpretation of the two margins is similar to that of the intermediate good margin $\tau_{q,t}$. That is, they capture the model misspecification in characterizing demand with respect to capital and labor inputs. For example, they can represent some missing factors affecting capital and labor demand. As in the case of the intermediate good margin $\tau_{q,t}$, the steady state values of these two margins are assumed to be one. Moreover, the cost minimization problem implies that the nominal marginal cost of the intermediate good firm is

$$S_t = \left(\frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}\right)(\tau_{k,t}R_{k,t})^{\alpha}\left(\frac{\tau_{l,t}W_t}{A_t}\right)^{1-\alpha},$$

which reveals how model misspecification in factor demands propagates to the marginal cost function.[15]

Profit Maximization. Since intermediate goods firms produce differentiated goods and have monopoly power on their own products, they seek to maximize the present value of the expected future profit flows by setting optimal product prices. Following Calvo (1983), for all intermediate goods firms, the probability of being able to re-optimize their prices is

[15]Because the cost minimization problem of intermediate good firms is essentially static, all firms make the same input renting decision, implying that the marginal cost function is the same across all firms. Thus, there is no need to attach the firm subscript i on the marginal cost function.

$(1-\xi_p)$, which is independent across firms and constant across time. Thus, we can view this uncertainty as idiosyncratic shocks facing intermediate goods firms. Any intermediate goods firm unable to re-optimize its desired price in period t resets its product price by a partial indexation scheme:

$$P_{i,t} = \pi_{t-1}^{\iota_p} \pi^{1-\iota_p} P_{i,t-1},$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$ denotes the period t gross inflation rate, and π is the steady-state inflation rate. Recently, this partial indexation scheme has become popular in New Keynesian models because it allows the dynamics of inflation to be both forward- and backward-looking. Thus, it is a convenient way of capturing the inflation rate persistence observed in the data. See Smets and Wouters (2007) and Justiniano and Primiceri (2008), for instance.

Any intermediate goods firm able to set its optimal price in period t, $\tilde{P}_t$,chooses it according to the profit maximization problem:[16]

$$\max_{\tilde{P}_t} E_t \left\{ \sum_{l=0}^{\infty} (\beta\xi_p)^l \frac{\Lambda_{t+s}}{\Lambda_t} \left[\tilde{P}_t \mathcal{F}_{t,l} - P_{t+l} s_{t+l} \right] \tilde{Q}_{t+l} \right\}$$

subject to:

$$\tilde{Q}_{t+l} = \left(\frac{\tau_{q,t+l} \mathcal{F}_{t,l} \tilde{P}_t}{P_{t+l}} \right)^{\frac{-\left(1+\lambda_{p,t+l}\right)}{\lambda_{p,t+l}}} Y_{t+l}, \quad \text{for } l = 0, 1, 2, \ldots .$$

Here, s_{t+l} is the real marginal cost of the firm in period $(t+l)$ and Λ_t is households' marginal value of nominal income in period t, since we assume that households hold equities of intermediate goods firms.[17] The constraints of the problem are the intermediate good demands in periods $t+l$, when the optimal price of the intermediate good is set in period t. $F_{t,l}$ is defined as

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[16]Since firms face exactly the same problem when they re-optimize their product prices, there is no need to distinguish them explicitly. We thus drop the firm index to simplify notation.

[17]One can obtain Λ_t by the Lagrange function associated with the household sector problem. See the next subsection.

$$\mathcal{F}_{t,l} = \begin{cases} \prod_{k=1}^{l} \pi_{t+k-1}^{\iota_p} \pi^{1-\iota_p}, & \text{if } l \geq 1; \\ 1 & \text{if } l = 0, \end{cases}$$

which captures the compound effect of partial indexation between periods t and $(t+l)$. Note that the intermediate good margin ($\tau_{q,t}$) in the final good firms affects the pricing decision of intermediate goods firms as well.

Appendix B.3. Households

There exists a continuum of infinitely lived households denoted by $j \in [0,1]$. Following Erceg, Henderson and Levin (2000) and Christiano, Eichenbaum and Evans (2005), we assume that households are monopolistic providers of differentiated labor. That is, they are able to set their own wage rates, $W_{j,t}$. The j-th household's budget constraint is

$$\begin{aligned} & \tau_{c,t} P_t C_t + P_t I_t + T_t + B_t + P_t a(u_t) \bar{K}_{t-1} \\ \leq \quad & \tau_{r,t} R_{t-1} B_{t-1} + W_{j,t} h_{j,t} + R_{k,t} u_t \bar{K}_{t-1} + D_t + F_t + A_{j,t}. \end{aligned} \tag{B.9}$$

The right hand side of (B.9) reflects various sources of the household's income in period t; $W_{j,t} h_{j,t}$ is the household's wage income. The household receives the dividend D_t from intermediate goods firms and the lump-sum transfer F_t from the government. We assume that there exists a complete set of Arrow securities, and $A_{j,t}$ is the return on holding such securities. The complete market assumption implies the all households make identical decisions on consumption C_t, investment I_t, capital $\bar{K}_t$, and bond holding B_t, while their labor income is heterogeneous.[18]

We assume that households can determine the utilization rate of capital, u_t. Hence, the effective capital service used in producing intermediate goods is $K_t = u_t \bar{K}_{t-1}$, and the nominal return of providing capital to intermediate goods firms is $R_{k,t} u_t \bar{K}_{t-1}$. By buying the government bond B_{t-1} in period $t-1$, the household receives (gross) interest income

[18]Thus, there is no need to add the firm index subscript i with respect to consumption, investment, capital, and the bond holding in the budget constraint.

$\tau_{b,t}R_{t-1}B_{t-1}$ in period t, where R_{t-1} is the nominal interest rate determined in period $t-1$. Note that we introduce a time-varying bond market margin $\tau_{r,t}$, which captures any potentially missing factors that might affect the demand for the bond. Similar to other margins in the model, it follows an AR(1) process (in logs):

$$\log \tau_{r,t} = (1-\rho_r)\log \tau_r + \rho_r \log \tau_{r,t-1} + \varepsilon_{r,t}, \quad \varepsilon_{r,t} \sim \mathcal{N}(0, \sigma_r^2). \tag{B.10}$$

It is worth noting that $\tau_{r,t}$ can also be interpreted as a structural shock: for instance, Smets and Wouters (2007) refer to $\tau_{r,t}$ as the risk premium shock. Because we are evaluating the specification of the medium-scale DSGE model without a risk premium shock, we interpret it as a margin.

The left hand side of (B.9) reflects the allocation of the household income for various purposes. First of all, B_t is the bonding holding to be carried over to period $t+1$. Second, the household spends $P_t I_t$ to accumulate physical capital. The evolution of physical capital can be expressed as:

$$\bar{K}_t = (1-\delta)\,\bar{K}_{t-1} + \mu_t \left[1 - \mathcal{S}\left(\frac{I_t}{I_{t-1}}\right)\right] I_t,$$

where $S(\cdot)$ reflects the investment adjustment cost and is restricted such that $S(1) = S'(1) = 0$ in the steady state.[19] Following Justiniano and Primiceri (2008), we introduce the investment shock μ_t, such that:

$$\log \mu_t = (1-\rho_\mu)\log \mu + \rho_\mu \log \mu_{t-1} + \varepsilon_{\mu,t}, \quad \varepsilon_{\mu,t} \sim \mathcal{N}\left(0, \sigma_\mu^2\right). \tag{B.11}$$

Moreover, the household faces the capital utilization cost $P_t a(u_t)\bar{K}_t$, which varies with the level of the capital utilization rate u_t. Finally, the household uses its income to consume the final good C_t with price P_t. When the household makes its consumption decision, it also considers the effects of a time-varying consumption margin $\tau_{c,t}$, which is designed to capture model misspecification in the consumption decision. While we treat $\tau_{c,t}$ as a margin, it can also be viewed as a structural shock. For example, Leeper at al. (2010)

[19]See Schmitt-Grohe and Uribe (2006) for a justification of the restriction.

interpret $\tau_{c,t}$ as the taxation shock on consumption. The consumption margin $\tau_{c,t}$ follows

$$\log \tau_{c,t} = (1-\rho_c)\log \tau_c + \rho_c \log \tau_{c,t-1} + \varepsilon_{c,t}, \quad \varepsilon_{c,t} \sim \mathcal{N}\left(0, \sigma_c^2\right). \tag{B.12}$$

As before, we assume $\tau_c = 1$.

The j-th household receives utility from consuming the final good C_t and disutility from providing differentiated labor $h_{j,t}$. Its behavior can be characterized by the following lifetime utility maximization problem:

$$\max E_t \sum_{s=0}^{\infty} \beta^{t+s} b_{t+s} \left\{ \log\left(C_{t+s} - \hbar C_{t+s-1}\right) - \varphi \frac{\left(h_{j,t+s}\right)^{1+\upsilon}}{1+\upsilon} \right\}$$

subject to its budget constraint (11) and the capital accumulation equation (13). Essentially, the household makes a sequence of decisions on consumption, capital accumulation, capital utilization, the bond holding and the desired wage. In this problem, β is the subjective discount factor, ν is the inverse of Frisch elasticity of labor supply, and φ controls the weight on labor disutility. The parameter $\hbar$ governs the extent of consumption habit formation.[20] In line with Justiniano et al. (2010), we assume that there exists an intertemporal preference shock b_t following

$$\log b_t = (1-\rho_b)\log b + \rho_b \log b_{t-1} + \varepsilon_{b,t}, \quad \varepsilon_{b,t} \sim \mathcal{N}\left(0, \sigma_b^2\right). \tag{B.13}$$

Labor-Packer Firms. A notable feature of New Keynesian models is that households provide differentiated labor service $h_{j,t}$, but intermediate good firms rent homogenous labor services. Following Smets and Wouters (2007), we introduce perfectly competitive labor-packer firms which are convenient to reconcile the above distinction. The representative labor-packer firm hires household labor services $h_{j,t}$'s and transforms them to the homogeneous labor service L_t by the following technology:

$$L_t = \left[\int_o^1 \left(h_{j,t}\right)^{\frac{1}{1+\lambda_{w,t}}} dj\right]^{1+\lambda_{w,t}}, \tag{B.14}$$

[20] Christiano et al. (2005) show that introducing habit formation helps to explain the hump-shaped response of consumption to a monetary policy shock.

where $\lambda_{w,t}$ governs the time-varying elasticity of substitution among heterogenous labor, $h_{j,t}$, and follows an ARMA(1,1) stochastic process:

$$\begin{aligned} \log\left(1+\lambda_{w,t}\right) &= \left(1-\rho_w\right)\log\left(1+\lambda_w\right)+\rho_w\log\left(1+\lambda_{w,t-1}\right)+\varepsilon_{w,t}-\theta_{w,t}\varepsilon_{w,t-1}, \quad \text{(B.15)} \\ \varepsilon_{w,t} &\sim \mathcal{N}\left(0,\sigma_w^2\right). \end{aligned}$$

Some economists interpret $\lambda_{w,t}$ as the wage mark-up shock, for it reflects households' desired wage mark-up over the marginal rate of substitution between consumption and leisure.

Taking the market wage rate W_t and idiosyncratic wage rates $W_{j,t}$ of households as given, the labor packer firm solves the following problem:

$$\max_{\{h_{j,t}\}_{j\in[0,1]}} W_t L_t - \int_0^1 \tau_{h,t} W_{j,t} h_{j,t} dj,$$

subject to (B.14). Here, we introduce a time-varying margin $\tau_{h,t}$ as follows

$$\log \tau_{h,t} = (1-\rho_h)\log\tau_h + \rho_l \log\tau_{h,t-1} + \varepsilon_{h,t}, \quad \varepsilon_{h,t} \sim \mathcal{N}(0,\sigma_h^2). \tag{B.16}$$

The first order condition of the above problem implies that the demand for the j-th household's labor service is:

$$h_{j,t} = \left(\frac{\tau_{h,t} W_{j,t}}{W_t}\right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} L_t.$$

Besides, it is easy to show that

$$W_t = \tau_{h,t}\left[\int_0^1 \left(W_{j,t}\right)^{-\frac{1}{\lambda_w}} dj\right]^{-\lambda_{w,t}},$$

which reveals how the model misspecification with respect to demand of $h_{j,t}$ affects the determination of the overall wage rate W_t.

Wage Setting. Following Erceg, Henderson and Levin (2000), we assume that not all households are able to set the desired optimal wage rate at every period of time. Specifically, the probability of being able to set the optimal wage rate in each period is $1-\xi_w$, which is independent across households and constant over time. On the other hand, a household unable to set its optimal wage rate in period t resets its wage rate according to the following partial indexation scheme:[21]

$$W_{j,t} = W_{j,t-1} \left(\pi_{t-1} e^{z_{t-1}}\right)^{\iota_w} \left(\pi e^{\gamma}\right)^{1-\iota_w}.$$

The households' wage setting decision can be represented by

$$\max_{\tilde{W}_t} E_t \left\{ \sum_{s=0}^{\infty} (\beta \xi_w)^s \left[-b_{t+s} \varphi \frac{\left(\tilde{h}_{t+s}\right)^{1+\nu}}{1+\nu} + \Lambda_{t+s} \left(\tilde{W}_t \mathcal{G}_{t,s}\right) \tilde{h}_{t+s} \right] \right\},$$

subject to the labor demand function:

$$\tilde{h}_{t+s} = \left(\frac{\tau_{h,t+s} \tilde{W}_t \mathcal{G}_{t,s}}{W_{t+s}} \right)^{-\frac{1+\lambda_{w,t+s}}{\lambda_{w,t+s}}} L_{t+s}.$$

Here, $\tilde{h}_{t+s}$ is the household labor demand in period $t+s$, and the optimal wage is determined in period t. In period $t+s$, the effective wage rate is $\tilde{W}_t G_{t,s}$, where

$$\mathcal{G}_{t,s} = \begin{cases} \prod_{k=1}^{s} \left[\pi_{t+k-1} e^{z_{t+k-1}}\right]^{\iota_w} \left[\pi e^{\gamma}\right]^{1-\iota_w}, & \text{if} \quad s \geq 1; \\ 1 & \text{if} \quad s = 0, \end{cases}$$

captures the compound effect of partial indexation from period t to period $t+s$. Note as well how $\tau_{h,t}$'s affect the labor demand functions. Moreover, Λ_{t+s} is current value of the marginal utility in period $t+s$, which is the Lagrange multiplier associated with the household budget constraint.

Appendix B.4. The Government

In this model economy, the government decides both fiscal and monetary policies. For the fiscal side, we assume that the government consumes a variable proportion of final output for non-productive purposes:

$$G_t = \left(1 - \frac{1}{g_t}\right) Y_t,$$

[21] As in the case of intermediate good prices, this partial indexation scheme makes the determination of the wage rate not only forward- but also backward-looking. Thus, it is convenient for capturing the persistence of wage rates.

where g_t follows

$$\log g_t = (1 - \rho_g) \log g + \rho_g \log g_{t-1} + \varepsilon_{g,t}, \quad \varepsilon_{g,t} \sim \mathcal{N}\left(0, \sigma_g^2\right). \tag{B.17}$$

We assume that the government fully finances its spending by lump-sum taxes and issuing government bonds. Thus, there is no need to explicitly specify the taxation reaction rule of the government.[22]

As for the monetary side, we assume that the government implements an interest rate rule a' la Taylor (1993). Specifically, we assume the government determines the short-run nominal (gross) interest rate by:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{X_t}{X_t^*}\right)^{\phi_x}\right]^{1-\rho_R} \left[\frac{X_t / X_{t-1}}{X_t^* / X_{t-1}^*}\right]^{\phi_{dX}} \eta_{mp,t}.$$

Here, ρ_R is a smoothing parameter capturing the nominal interest rate persistence. We define $X_t \equiv C_t + I_t + G_t$ as the measure of gross domestic product (GDP) of the economy. On the other hand, we denote $X_t^* \equiv C_t^* + I_t^* + G_t^*$ as the output level, in which prices (and wages) are fully flexible and with no price and wage mark-up shocks. Thus, X_t / X_t^* can be interpreted as the GDP gap. The last square bracket captures the change of the GDP gap. We assume that the monetary policy shock $\eta_{mp,t}$ follows

$$\log \eta_{mp,t} = \rho_{mp} \log \eta_{mp,t-1} + \varepsilon_{mp,t}, \quad \varepsilon_{mp,t} \sim \mathcal{N}\left(0, \sigma_{mp}^2\right). \tag{B.18}$$

In the literature, there exist various specifications of the interest rule, and our specification is in line with Del Negro et al. (2007) and Justiniano and Primiceri (2008).

Appendix B.5. Aggregation

Because of the complete markets assumption, the aggregation of the model economy is straightforward, even though the households and intermediate goods firms face idiosyncratic shocks due to the Calvo-type setting. By integrating the production function of

[22]See Christiano et al. (2005) for similar arguments.

the intermediate goods firms, we have the relationship between aggregate output and the aggregate factor inputs:

$$Y_t = A_t^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha} - A_t F.$$

Since our model is a closed economy, the total output of the economy can only be distributed to four outlets: household and government consumption, investment, and the capital utilization cost. Hence, the aggregate resource constraint is:

$$C_t + I_t + G_t + a(u_t)\bar{K}_t = Y_t.$$

We assume all innovations of structural and margin process are mutually independent over time.

Appendix C. Linearized Equilibrium Conditions

We estimate the structural parameters of the model as well as the time-varying margin processes by Bayesian methods. We first transform the original equilibrium conditions, which are non-stationary due to the growing technology level A_t, into stationary ones. Then, we linearize the transformed equilibrium conditions around the steady state values and estimate them.

To linearize the equilibrium conditions, we first define the following stationary variables

$$y_t = \tfrac{Y_t}{A_t}, \quad x_t = \tfrac{X_t}{A_t}, \quad k_t = \tfrac{K_t}{A_t}, \quad \bar{k}_t = \tfrac{\bar{K}_t}{A_t}, \quad c_t = \tfrac{C_t}{A_t}, \quad i_t = \tfrac{I_t}{A_t}, \quad \tilde{q}_t = \tfrac{\tilde{Q}_t}{A_t}$$

$$\tilde{p}_t = \tfrac{\tilde{P}_t}{P_t}, \quad w_t = \tfrac{W_t}{A_t P_t}, \quad s_t = \tfrac{S_t}{P_t}, \quad \pi_t = \tfrac{P_t}{P_{t-1}}, \quad \lambda_t = \Lambda_t A_t P_t, \quad \phi_t = \Phi_t A_t \quad \tilde{w}_t = \tfrac{\tilde{W}_t}{A_t P_t}$$

After that, we express the above stationary variables in terms of log deviation forms. Basically, for any variable ζ_t, we define its log deviation as $\hat{\zeta}_t = \log \zeta_t - \log \zeta$, where ζ denotes the steady-state value of ζ_t. However, there are three exceptions: We define $\hat{z}_t = z_t - \gamma$, $\hat{\lambda}_{p,t} = \log(1 + \lambda_{p,t}) - \log(1 + \lambda_p)$ and $\hat{\lambda}_{w,t} = \log(1 + \lambda_{w,t}) - \log(1 + \lambda_w)$. We classify the linearized equilibrium conditions into three categories: the endogenous part, the exogenous structural shocks, and the exogenous margin processes.

The Endogenous Part. This category consists of 16 equilibrium conditions, derived from the optimal decisions of agents in the economy. They are

1. The production function:

$$\hat{y}_t = \frac{y+F}{y}\left(\alpha\hat{k}_t + (1-\alpha)\hat{L}_t\right)$$

2. The capital-labor ratio:

$$\hat{k}_t - \hat{L}_t = (\hat{w}_t - \hat{r}_{k,t}) + (\hat{\tau}_{l,t} - \hat{\tau}_{k,t})$$

3. The real marginal cost:

$$\hat{s}_t = \alpha\left(\hat{\tau}_{k,t} + \hat{r}_{k,t}\right) + (1-\alpha)\left(\hat{\tau}_{l,t} + \hat{w}_t\right)$$

4. The New Keynesian Phillips curve (NKPC):[23]

$$\begin{aligned}\hat{\pi}_t \quad = \quad & \frac{\iota_p}{1+\iota_p\beta}\hat{\pi}_{t-1} + \frac{\beta}{1+\iota_p\beta}\mathrm{E}_t\left\{\hat{\pi}_{t+1}\right\} + \kappa_p\hat{s}_t + \hat{\lambda}^n_{p,t} \\ & - \left[\left(\frac{1}{1+\iota_p\beta}\right)\vartheta_q^{-1}\right]\hat{\tau}^n_{q,t-1} + \hat{\tau}^n_{q,t} - \left[\left(\frac{\beta}{1+\iota_p\beta}\right)\vartheta_q^{-1}\right]\mathrm{E}_t\left\{\hat{\tau}^n_{q,t+1}\right\}\end{aligned}$$

where

$$\begin{aligned}\hat{\lambda}^n_{p,t} &= \vartheta_p\hat{\lambda}_{p,t} \\ \hat{\tau}^n_{q,t} &= \vartheta_q\hat{\tau}_{q,t} \\ \vartheta_p &= \kappa_p = \frac{(1-\beta\xi_p)(1-\xi_p)}{(1+\iota_p\beta)\,\xi_p} \\ \vartheta_q &= \left(\frac{1+\beta\xi_p^2}{\xi_p\,(1+\iota_p\beta)}\right)\end{aligned}$$

5. Capital used in production:

$$\hat{k}_t = \hat{u}_t + \hat{\bar{k}}_{t-1} - \hat{z}_t$$

6. The capital evolution equation:

$$\bar{k}_t = (1-\delta)\,e^{-\gamma}\left(\bar{k}_{t-1} - \hat{z}_t\right) - \left(1-(1-\delta)\,e^{-\gamma}\right)\left(\hat{\mu}_t + \hat{\imath}_t\right)$$

[23] Following Justiniano et al. (2010), we normalize the price mark-up shock such that it enters the NKPC with a coefficient equal to one. We perform a similar transformation on $\hat{\tau}_{q,t}$.

7. The marginal utility of consumption:

$$\begin{aligned}\hat{\tau}_{c,t}+\hat{\lambda}_t &= \left(\frac{e^{\gamma}}{e^{\gamma}-\hbar\beta}\right)\left(\hat{z}_t+\hat{b}_t-\frac{e^{\gamma}}{e^{\gamma}-\hbar}\left(\hat{z}_t+\hat{c}_t\right)+\frac{\hbar}{e^{\gamma}-\hbar}\hat{c}_{t-1}\right)\\ &\quad -\left(\frac{\hbar\beta}{e^{\gamma}-\hbar\beta}\right)\left(\hat{b}_{t+1}-\frac{e^{\gamma}}{e^{\gamma}-\hbar}\left(\hat{z}_{t+1}+\hat{c}_{t+1}\right)+\frac{\hbar}{e^{\gamma}-\hbar}\hat{c}_t\right).\end{aligned}$$

8. The Euler equation:

$$\hat{\lambda}_t=\mathrm{E}_t\left\{\hat{\lambda}_{t+1}+\hat{\tau}_{r,t+1}+\hat{R}_t-\hat{z}_{t+1}-\hat{\pi}_{t+1}\right\}$$

9. The capital utilization rate:

$$\hat{r}_{k,t}=\chi\hat{u}_t$$

where $\chi=\frac{a''}{a'}=\frac{a''}{r_k}$

10. The capital supply:

$$\hat{\phi}_t=\left[1-\beta(1-\delta)e^{-\gamma}\right]\left(\hat{\lambda}_{t+1}-\hat{z}_{t+1}+\hat{r}_{k,t+1}\right)+\beta(1-\delta)e^{-\gamma}\left(\hat{\phi}_{t+1}-\hat{z}_{t+1}\right)$$

11. The investment:

$$\hat{\lambda}_t=\hat{\phi}_t+\hat{\mu}_t-\left(e^{2\gamma}S''\right)\left(\hat{\imath}_t-\hat{\imath}_{t-1}+\hat{z}_t\right)+\left(\beta e^{2\gamma}S''\right)\mathrm{E}_t\left(\hat{\imath}_{t+1}-\hat{\imath}_t+\hat{z}_{t+1}\right)$$

12. The wage equation:

$$\begin{aligned}\hat{w}_t &= \frac{1}{1+\beta}\left(\hat{w}_{t-1}\right)+\frac{\beta}{1+\beta}\mathrm{E}_t\left\{\hat{w}_{t+1}\right\}-\kappa_w\hat{g}_{w,t}\\ &\quad+\frac{\iota_w}{1+\beta}\left(\hat{\pi}_{t-1}+\hat{z}_{t-1}\right)-\left(\frac{1+\beta\iota_w}{1+\beta}\right)\left(\hat{\pi}_t+\hat{z}_t\right)+\frac{\beta}{1+\beta}E_t\left\{\hat{\pi}_{t+1}+\hat{z}_{t+1}\right\}\\ &\quad+\hat{\lambda}^n_{w,t}-\left[\frac{1}{1+\beta}\vartheta_h^{-1}\right]\hat{\tau}^n_{h,t-1}+\hat{\tau}^n_{h,t}-\left[\left(\frac{\beta}{1+\beta}\right)\vartheta_h^{-1}\right]\hat{\tau}^n_{h,t+1}\end{aligned}$$

where

$$\begin{aligned}\hat{\lambda}^n_{w,t} &= \vartheta_w\hat{\lambda}_{w,t}\\ \hat{\tau}^n_{h,t} &= \vartheta_h\hat{\lambda}_{h,t}\\ \vartheta_w &= \kappa_w=\left(\frac{1-\beta\xi_w}{\nu_w\left(1-\beta\right)}\right)\left(\frac{1-\xi_w}{\xi_w}\right)\\ \vartheta_h &= 1+\kappa_w\end{aligned}$$

13. The interest rate rule:

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\left[\phi_\pi \hat{\pi}_t + \phi_x \left(\hat{x}_t - \hat{x}_t^*\right)\right] + \phi_{dX}\left[\left(\hat{x}_t - \hat{x}_{t-1}\right) - \left(\hat{x}_t^* - \hat{x}_{t-1}^*\right)\right] + \hat{\eta}_{mp,t}$$

14. GDP:

$$\hat{x}_t = \hat{y}_t - \frac{r_k k}{y}\hat{u}_t$$

15. The resource constraint:

$$\frac{c}{y}\hat{c}_t + \frac{i}{y}\hat{\imath}_t + \frac{r_k k}{y}\hat{u}_t = \frac{1}{g}\hat{y}_t - \frac{1}{g}\hat{g}_t$$

16. The definition of $\hat{g}_{w,t}$:

$$\hat{g}_{w,t} = \hat{w}_t - \left(\nu \hat{L}_t + \hat{b}_t - \hat{\lambda}_t\right)$$

Structural Shocks. In terms of log deviations from steady state values, there are 7 structural shocks, containing (i) the technology shock $\hat{z}_t$, (ii) the monetary shock $\hat{\eta}_{mp,t}$, (iii) the government spending shock $\hat{g}_t$, (iv) the preference shock $\hat{b}_t$, (v) the investment shock $\hat{\mu}_t$, (vi) the price mark-up shock $\hat{\lambda}_{p,t}$, and (vii) the wage mark-up shock: $\hat{\lambda}_{w,t}$.

Margin Processes. In terms of log deviations, there are 6 margins in our model, containing (i) the labor demand margin $\hat{\tau}_{l,t}$, (ii) the capital demand margin $\hat{\tau}_{k,t}$, (iii) the consumption margin $\hat{\tau}_{c,t}$, (iv) the bond market margin: $\hat{\tau}_{r,t}$, (v) the labor market margin $\hat{\tau}_{h,t}$, and (vi) the intermediate good margin $\hat{\tau}_{y,t}$.

Appendix D. Priors for and Posterior Estimates of the Medium-Scale New Keynesian Model

It is well-known that some structural parameters in DSGE models are not well-identified, and they should be calibrated before estimating the model. Specifically, we calibrate the capital depreciation rate δ to 0.025, which is a value commonly used in the literature.[24] The steady-state ratio of government spending to output is set to 0.22, which is the average

[24] See Christiano et al. (2010) and Smets and Wouters (2007) for instance.

value in our sample. Moreover, we impose the zero profit condition on the intermediate goods firms' problem, and hence the fixed cost (F) is endogenously determined.

The remaining parameters can be classified into three groups. The first group contains the fundamental structural parameters of the model, such as the inverse Frisch elasticity of labor supply ν, the consumption habit parameter $\hbar$, etc. The second group contains the structural shocks parameters. Note that if we remove all margins from the model, it will be the same as Justiniano et al. (2010). Therefore, we use exactly their prior distributions with respect to parameters in the first and the second groups, which are broadly line with the literature. The third group contains the parameters of the exogenous margin processes. We do not have information on these parameters, and thus we set quite disperse prior distributions. For example, the prior distribution of ρ_l has mean 0.5 and standard deviation 0.2. As usual, the prior distributions are assumed mutually independent from each other. The prior distributions are summarized in Table B.[25]

The estimation results are summarized in the last four columns of Table B, which displays median, standard deviation, 5th and 95th percentiles of the posterior distribution for each of the estimated parameters. To simplify the discussion, we treat the posterior median as the point estimate. Overall, our results are similar to the existing literature with a few exceptions, which we discuss in turn. First of all, the point estimate of the consumption habit parameter $\hbar$ is 0.83, which is larger than in Christiano et al. (2005) and Smets and Wouters (2007). The point estimate of ν is 3.84, implying that the Frisch elasticity of labor supply is around 0.26. Comparing to Levin et al. (2006) and Smets and Wouters (2007), our Frisch labor supply elasticity value is a bit smaller. However, it is in line with empirical evidence from micro data.[26] For example, MaCurdy (1981) showed that the labor supply elasticity of white, married, prime-aged men is between 0.1 and 0.5.

Our point estimate of the Calvo price parameter is $\xi_p = 0.85$, implying that intermediate

[25]In Table B, "SS" denotes "steady state".

[26]The point estimates of inverse Frisch elasticity of labor supply are 1.49 and 1.92 in Levin et al. (2006) and Smets and Wouters (2007), respectively.

goods firms, on average, reset their optimal prices every 6.6 quarters. Our estimate of ξ_p is similar to that reported by Levin et al. (2006), according to whom it is 0.83.[27] Our estimate of the Calvo wage parameter is $\xi_w = 0.72$, which implies that average duration of a wage contract is around 4.5 quarters. Moreover, our results reveal that the extent of partial indexation on previous inflation and the wage rate is moderate: the point estimates are $\iota_p = 0.19$ and $\iota_w = 0.10$, respectively. The results of low partial indexation echo Del Negro and Schorfheide (2008), who claim that the model with indexation schemes on prices and wages are observationally similar to those with autocorrelated mark-up shocks: since our model includes the moving average terms for price and wage mark-up shocks, the contributions of ι_p and ι_w become smaller.

The point estimate of ρ_z, the persistence of technology shocks, is merely 0.24, which is much lower than the existing estimates. For example, the parameter estimates in Smets and Wouters (2007) and Levin et al. (2006) are 0.95 and 0.96. Moreover, the point estimate of σ_z is 0.87 percent, implying that the unconditional standard deviation of the technology shock is 0.92 percent. The implied unconditional standard deviation of technology shocks is about 60 and 40 percent of the values in Smets and Wouters (2007) and Levin et al. (2006), respectively. See also Ireland (2004) for further discussion about the role of technology shocks on several key macroeconomic variables in a New Keynesian DSGE model. It is worth to note that Del Negro and Schorfheide (2009) find that the persistence coefficient of the technology shock becomes smaller once model misspecification is taken into account.

The price and margin mark-up shocks are both quite persistent, and the point estimates of ρ_p and ρ_w are 0.94 and 0.99, respectively. Moreover, the moving average coefficients (θ_p and θ_w) both play non-trivial roles in shaping the dynamics of price and wage mark-up shocks. The government spending shock is highly persistent and behaves almost like a unit root process. Among the structural shocks and margin processes, the investment shock has the biggest innovation standard deviation: the point estimate of σ_μ is 5.24

[27] Altig et al. (2011) explain that the implied duration of price stickiness shortens once firm-specific capital is included in the model.

percent, which is about 6 and 52 times the innovation standard deviations of technology and preference shocks, respectively. Since the investment shock innovation has such a high standard deviation, we expect that investment shocks play a dominant role in determining the fluctuations of the endogenous variables of the model.

Our estimates also suggest that the margins are serially correlated. This suggests that the model misspecification cannot be captured by iid measurement errors in the measurement equations.

Finally, in order to understand whether including margins has substantial consequences on the parameter estimates, we report parameter estimates for the model without margins in Table C. By comparing Tables B and C, we conclude that estimating the model with the margins does not substantially modify the parameter estimates relative to those of a model without margins.[28]

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Table A. The DGP Parameter Values and the Prior Setting of Simulation Exercises

Parameter	*Description*	*DGP*	*Prior*		
			Type	*Mean*	*Std.*
β	Subjective discount factor	0.99	-	-	-
γ	Risk aversion coefficient	1.50	-	-	-
φ	Inverse Frisch elasticity	1.00	-	-	-
ξ	Calvo parameter	0.67	B	0.70	0.05
γ_π	Interest rate policy rule: inflation	1.80	G	2.00	0.05
γ_y	Interest rate policy rule: output	0.50	G	0.40	0.05
ρ_r	Persistence coef. of interest rate	0.70	B	0.75	0.10
ρ_z	Persistence of technology shocks	0.70	B	0.75	0.10
ρ_ν	Persistence of monetary shocks	0.70	B	0.75	0.10
ρ_g	Persistence of spending shocks	0.70	B	0.75	0.10
ρ_χ	Persistence of labor supply shocks	0.70	B	0.75	0.10
σ_z	Std. of innovation of technology shocks	0.10	I	0.30	2.00
σ_ν	Std. of innovation of monetary shocks	0.10	I	0.30	2.00
σ_g	Std. of innovation of spending shocks	0.10	I	0.30	2.00
σ_χ	Std. of innovation of labor supply shocks	0.10	I	0.30	2.00
ρ_l	Persistence of labor margin ($\tau_{l,t}$)	-	B	0.50	0.20
ρ_c	Persistence of consumption margin ($\tau_{c,t}$)	-	B	0.50	0.20
ρ_b	Persistence of bond market margin ($\tau_{b,t}$)	-	B	0.50	0.20
σ_l	Std. of innovation of labor margin	-	I	0.10	2.00
σ_c	Std of innovation of consumption margin	-	I	0.10	2.00
σ_b	Std of innovation of bond market margin	-	I	0.10	2.00

Notes to the table. The table describes the parameter values used in the simulation of the model (column labeled "DGP") as well as the distribution ("Type"), the mean ("Mean") and the standard deviation ("Std.") of the prior used in the estimation (columns labeled "Prior"). The distribution can be either Gaussian ("G"), Beta ("B"), Gamma ("G") or Inverse Gamma ("I").

Table B. Prior and Posterior Distributions of the Benchmark Model

Parameter	*Description*	*Prior*			*Posterior*			
		Type	Mean	Std	Median	Std	5%	95%
α	Capital share	Normal	0.30	0.05	0.17	0.01	0.16	0.18
ι_p	Price indexation	Beta	0.50	0.15	0.19	0.08	0.09	0.31
ι_w	Wage indexation	Beta	0.50	0.15	0.10	0.03	0.04	0.15
100γ	SS Technology growth rate	Normal	0.50	0.03	0.48	0.03	0.44	0.53
$\hbar$	Consumption habit	Beta	0.50	0.10	0.83	0.05	0.78	0.89
λ_p	SS price mark-up	Normal	0.15	0.05	0.23	0.04	0.17	0.29
λ_w	SS wage mark-up	Normal	0.15	0.05	0.14	0.05	0.06	0.22
$100\log(L_{ss})$	100 times log hours	Normal	0.00	0.50	0.12	0.51	-0.76	0.99
$100(\pi-1)$	SS inflation rate	Normal	0.50	0.10	0.63	0.12	0.48	0.79
$100(\beta^{-1}-1)$	Discount factor	Gamma	0.25	0.10	0.14	0.05	0.07	0.21
ν	Inverse Frisch elasticity	Gamma	2.00	0.75	3.84	0.80	2.58	5.03
ξ_p	Calvo prices	Beta	0.66	0.10	0.85	0.02	0.81	0.88
ξ_w	Calvo wages	Beta	0.66	0.10	0.72	0.05	0.63	0.81
χ	Capital utilization costs	Gamma	5.00	1.00	5.33	0.99	3.80	7.11
$\mathcal{S}'$	Investment adjustment costs	Gamma	4.00	1.00	2.23	0.50	1.37	3.12
ϕ_π	Taylor rule: inflation	Normal	1.70	0.30	2.03	0.17	1.69	2.39
ϕ_x	Taylor rule: output	Normal	0.13	0.05	0.06	0.02	0.03	0.09
ϕ_{dX}	Taylor rule: output growth	Normal	0.13	0.05	0.24	0.02	0.20	0.28
ρ_R	Interest rate smoothing	Beta	0.60	0.20	0.85	0.02	0.81	0.88
		Structural Shocks						
ρ_{mp}	Monetary policy	Beta	0.40	0.20	0.11	0.06	0.02	0.21
ρ_z	Technology growth	Beta	0.60	0.20	0.24	0.06	0.13	0.35
ρ_g	Government spending	Beta	0.60	0.20	1.00	0.00	1.00	1.00
ρ_μ	Investment	Beta	0.60	0.20	0.60	0.05	0.51	0.73
ρ_p	Price mark-up	Beta	0.60	0.20	0.94	0.02	0.90	0.97
ρ_w	Wage mark-up	Beta	0.60	0.20	0.99	0.01	0.98	1.00
ρ_b	Intertemporal preference	Beta	0.60	0.20	0.27	0.13	0.11	0.45
θ_p	Price mark-up MA	Beta	0.50	0.20	0.47	0.10	0.30	0.69
θ_w	Wage mark-up MA	Beta	0.50	0.20	0.90	0.03	0.85	0.94
$100\sigma_{mp}$	Monetary policy	InvGamma	0.10	1.00	0.23	0.01	0.21	0.25
$100\sigma_z$	Technology growth	InvGamma	0.50	1.00	0.87	0.05	0.80	0.95
$100\sigma_g$	Government spending	InvGamma	0.50	1.00	0.35	0.02	0.32	0.37
$100\sigma_\mu$	Investment	InvGamma	0.50	1.00	5.24	0.89	3.47	7.24
$100\sigma_p$	Price mark-up	InvGamma	0.10	1.00	0.06	0.02	0.03	0.08
$100\sigma_w$	Wage mark-up	InvGamma	0.10	1.00	0.09	0.02	0.05	0.12
$100\sigma_b$	Intertemporal preference	InvGamma	0.10	1.00	0.10	0.02	0.06	0.12
		Margin Processes						
ρ_l	Homogenous labor	Beta	0.50	0.20	0.52	0.28	0.20	0.84
ρ_k	Capital	Beta	0.50	0.20	0.53	0.28	0.21	0.85
ρ_c	Consumption	Beta	0.50	0.20	0.52	0.28	0.17	0.82
ρ_r	Bond	Beta	0.50	0.20	0.93	0.28	0.90	0.96
ρ_q	Intermediate good	Beta	0.50	0.20	0.39	0.30	0.10	0.71
ρ_h	Heterogenous labor	Beta	0.50	0.20	0.74	0.28	0.51	0.94
$100\sigma_l$	Homogenous labor	InvGamma	0.10	1.00	0.06	0.02	0.03	0.11
$100\sigma_k$	Capital	InvGamma	0.10	1.00	0.05	0.02	0.03	0.09
$100\sigma_c$	Consumption	InvGamma	0.10	1.00	0.07	0.02	0.03	0.15
$100\sigma_r$	Bond market	InvGamma	0.10	1.00	0.17	0.02	0.10	0.24
$100\sigma_q$	Intermediate good	InvGamma	0.10	1.00	0.11	0.03	0.07	0.14
$100\sigma_h$	Heterogenous labor	InvGamma	0.10	1.00	0.35	0.02	0.28	0.41

Table C. Prior and Posterior Distributions: The Benchmark Model with No Margins

		Prior			*Posterior*			
Parameter	Description	Type	Mean	Std	Median	Std	5 Pct	95 Pct
α	Capital share	Normal	0.30	0.05	0.17	0.01	0.16	0.18
ι_p	Price indexation	Beta	0.50	0.15	0.22	0.08	0.10	0.34
ι_w	Wage indexation	Beta	0.50	0.15	0.11	0.03	0.06	0.17
100γ	SS Technology growth rate	Normal	0.50	0.03	0.48	0.03	0.44	0.52
$\hbar$	Consumption habit	Beta	0.50	0.10	0.81	0.05	0.74	0.88
λ_p	SS price mark-up	Normal	0.15	0.05	0.23	0.04	0.17	0.29
λ_w	SS wage mark-up	Normal	0.15	0.05	0.15	0.05	0.07	0.22
$100\log(L_{ss})$	100 times log hours	Normal	0.00	0.50	0.19	0.51	-0.61	0.97
$100(\pi-1)$	SS inflation rate	Normal	0.50	0.10	0.64	0.12	0.50	0.79
$100(\beta^{-1}-1)$	Discount factor	Gamma	0.25	0.10	0.14	0.05	0.06	0.21
ν	Inverse Frisch elasticity	Gamma	2.00	0.75	3.93	0.80	2.68	5.24
ξ_p	Calvo prices	Beta	0.66	0.10	0.83	0.02	0.80	0.87
ξ_w	Calvo wages	Beta	0.66	0.10	0.68	0.05	0.58	0.76
χ	Capital utilization costs	Gamma	5.00	1.00	5.32	0.99	3.78	7.02
$\mathcal{S}'$	Investment adjustment costs	Gamma	4.00	1.00	2.72	0.50	1.91	3.60
ϕ_π	Taylor rule: inflation	Normal	1.70	0.30	2.08	0.17	1.80	2.39
ϕ_x	Taylor rule: output	Normal	0.13	0.05	0.07	0.02	0.04	0.10
ϕ_{dX}	Taylor rule: output growth	Normal	0.13	0.05	0.24	0.02	0.20	0.28
ρ_R	Interest rate smoothing	Beta	0.60	0.20	0.82	0.02	0.77	0.85
		Structural Shock						
ρ_{mp}	Monetary policy	Beta	0.40	0.20	0.16	0.06	0.06	0.27
ρ_z	Technology growth	Beta	0.60	0.20	0.25	0.06	0.15	0.34
ρ_g	Government spending	Beta	0.60	0.20	1.00	0.00	1.00	1.00
ρ_μ	Investment	Beta	0.60	0.20	0.71	0.05	0.64	0.79
ρ_p	Price mark-up	Beta	0.60	0.20	0.94	0.02	0.89	0.98
ρ_w	Wage mark-up	Beta	0.60	0.20	0.98	0.01	0.96	1.00
ρ_b	Intertemporal preference	Beta	0.60	0.20	0.58	0.12	0.43	0.74
θ_p	Price mark-up MA	Beta	0.50	0.20	0.72	0.08	0.57	0.83
θ_w	Wage mark-up MA	Beta	0.50	0.20	0.92	0.03	0.88	0.96
$100\sigma_{mp}$	Monetary policy	InvGamma	0.10	1.00	0.22	0.01	0.20	0.25
$100\sigma_z$	Technology growth	InvGamma	0.50	1.00	0.89	0.05	0.80	0.97
$100\sigma_g$	Government spending	InvGamma	0.50	1.00	0.35	0.02	0.32	0.38
$100\sigma_\mu$	Investment	InvGamma	0.50	1.00	5.84	0.89	4.24	7.34
$100\sigma_p$	Price mark-up	InvGamma	0.10	1.00	0.14	0.01	0.12	0.16
$100\sigma_w$	Wage mark-up	InvGamma	0.10	1.00	0.21	0.02	0.18	0.24
$100\sigma_b$	Intertemporal preference	InvGamma	0.10	1.00	0.05	0.01	0.03	0.07