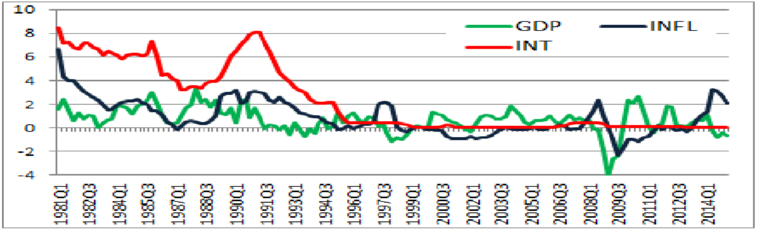
Zero interest rate policy and asymmetric price adjustment in Japan: an empirical analysis of a nonlinear DSGE model

Hirokuni IIBOSHI
Tokyo Metropolitan University

Mototsugu SHINTANI
University of Tokyo

22nd Computing in Economics and Finance
June 28, 2016, Bordeaux, France

Japanese Economy



	GDP: y_t		INFL: π_t		INT: r_t	
	mean	Vola	mean	Vola	mean	Vola
before 99:Q1	0.89	0.99	1.64	1.32	4.27	2.54
After 99:Q1	0.40	**1.10**	-0.01	**1.02**	0.09	0.14

- Under Zero Interest Rate Policy (After 99:Q1):
  Volatility of GDP: Up$\Uparrow$ & Volatility of INFL: Down$\Downarrow$

What we do

- We apply a non-linear (NL) New Keynesian (NK) DSGE extended
  - to two aspects;
    1. Zero Lower Bound (ZLB),
    2. Asymmetric Adjustment Cost (AAC),
  - to Japanese economy for the last thirty years (from 1981Q3 to 2015Q1) including, so called “Bubble boom” and “Lost decade”
  - using (1) *Projection method* and (2) *Particle Filter MH* oriented to a NL approach.

Summary

- From Marginal Likelihood, the model with both of ZLB and AAC is selected.
  - Downward rigidity is observed, even Japan faces deflation.
  - The adjustment cost near Zero Interest Rate Policy is 60.8% (=exp(0.475)) higher than *the inflation* period (non ZIRP),
- Estimated Policy Functions of the selected model shows
  1. Reactions of Output to TFP shock are steep, and Reactions of Inflation are moderate under ZIRP (low level of TFP shock). Under no ZIRP, *vice versa*.
     - These explain high volatility of GDP and low volatility of Inflation during deflation.
  2. Reactions of Inflation to Monetary Policy Shock are moderate under ZIRP (low TFP). Under no ZIRP, Reactions of Inflation are steep.
     - Even positive monetary policy has weak impact on inflation near ZIRP.

Related Literature

- Difference between NL and linear NK DSGE models under zero lower bound (ZLB) was studied by **Fernandez-Villaverde et al. (2015)**.
- In terms of balance between speed and precision, **Richter et al. (2014)** shows a *time iteration method with linear interpolation* (TL) performs best among projection methods to solve NL DSGE models under ZLB.
- **Gust et al. (2013)** estimated a NL NK DSGE under ZLB for US, using particle filter Metropolis-Hastings (PFMH) algorithm (see **Herbst and Schorfheide (2016)**).
- **Aruoba et al. (2014)** and **Aruoba and Schorfhide (2015)** estimated a log-linearized NK DSGE under ZLB for Japan.
- **Kim and Ruge-Murcia (2009)**, **Aruoba and Schorfhide (2015)** estimated downward rigidity for price and wage in USA using AAC.

Monetary Policy under ZLB

- Our model is extended from standard NK model by An & Schorfheide (2007). Aruoba et al (2014), Aruoba & Schorfhide (2015) are also based on them.
- Monetary policy rule is constrained by ZLB

$$R_t = \max(\, 1,\ R_t^{*1-\rho_R} R_{t-1}^{\rho_R} e^{\epsilon_{R,t}}\ ), \tag{1}$$

where R_t is the gross nominal interest rate. $\epsilon_{R,t}$ is monetary policy shock.

- R_t^* is nominal target rate

$$R_t^* = r\pi^* \left(\frac{\pi_t}{\pi^*}\right)^{\psi_1} \left(\frac{Y_t}{Y_t^*}\right)^{\psi_2}, \tag{2}$$

where π^* and Y_t^* is inflation target and output target, respectively.

Asymmetric Adjustment Cost (AAC)

- Cost shifted asymmetrically at threshold,π, is introduced to Rotemberg-type adjustment cost. π is a steady state of inflation.

$$AC_t(j) = \frac{\phi(\pi_t)}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - \pi\right)^2 Y_t(j), \tag{3}$$

$$\phi(\pi_t) = \begin{cases} \phi_1, & \text{if } \frac{P_t(j)}{P_{t-1}(j)} \; (= \pi_t) \; \geq \; \pi \\ \phi_2 = e^{\delta}\phi_1, \; \delta \in (-\infty, \infty), & \text{if } \frac{P_t(j)}{P_{t-1}(j)} \; (= \pi_t) \; < \; \pi \\ \doteqdot (1+\delta)\phi_1. & \end{cases},$$

where $\phi(\pi_t)$ is coefficient of the AC, and δ **is fraction of shift of the cost** when $\pi_t < \pi$. $\delta > 0$ **means downward rigidity.**

Asymmetric Adjustment Cost (AAC) (cont'd)

- Kim & Ruge-Marcia (2009), Arouba et al. (2013) used AAC as below (as introduced by Varian, 1974).

$$AC_t(j) = \phi \left(\frac{exp(-\psi(P(j)_t/P(j)_{t-1} - 1)) + \psi(P(j)_t/P(j)_{t-1} - 1) - 1}{\psi^2} \right),|$$

where $\phi > 0$. if $\psi > 0$, then downward rigidity, and if $\psi < 0$, then upward rigidity.

Agents

Firms

Monopolistically competitive intermediate goods producing firms

$$\prod = E_t \left[\sum_{s=0}^{\infty} \beta^s Q_{t+s|t} \left(\frac{P_{t+s}(j)}{P_t} Y_{t+s}(j) - W_{t+s} N_{t+s}(j) - AC_{t+s}(j) \right) \right],$$

s.t. (demand and product functions)

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-1/\nu} Y_t,$$

$$Y_t(j) = A_t N_t(j),$$

where β is discount factor. ν is inverse elasticity of demand for goods j. A_t and $N_t(j)$ are exogenous common productivity process and the labor input of firm j, respectively.

Agents (cont'd)

Households

$$U = E_t \left[\sum_{s=0}^{\infty} \beta^s \left(\frac{(C_{t+s}/A_{t+s})^{1-\tau} - 1}{1-\tau} + \chi_M \ln \left(\frac{M_{t+s}}{P_{t+s}} \right) - \chi_H H_{t+s} \right) \right],$$

s.t. (budget constraint)

$$P_t C_t + B_t + M_t + T_t = P_t W_t H_t + R_{t-1} B_{t-1} + M_{t-1} + P_t D_t + P_t SC_t,$$

where τ is inverse elasticity of intertemporal substitution of consumption.

Government

The government's budget constraint and expenditure

$$P_t G_t + R_{t-1} B_{t-1} + M_{t-1} = T_t + B_t + M_t,$$

$$G_t = \left(1 - \frac{1}{g_t} \right) Y_t. \tag{4}$$

Exogenous Shocks

- Aggregate productivity evolves as nonstationary process

$$A_t = \gamma\, A_{t-1}\, z_t, \;\; z_t = \; z_{t-1}^{\rho_z}\, e^{\epsilon_{z,t}} \; (> 0), \tag{5}$$

where γ stands for average technology growth rate, z_t is the exogenous fluctuation of the technology growth rate and $\epsilon_{z,t}$ is the productivity shock.

- The fraction of government to aggregate ouput

$$g_t/g \; = \; (\, g_{t-1}/g\,)^{\rho_g}\; e^{\epsilon_{g,t}} \; (> 0), \tag{6}$$

where $\epsilon_{g,t}$ is government shock.

- The steady state

$$\pi = \pi^*, \ r = \frac{\gamma}{\beta}, \ R = r\,\pi^*,$$
$$C/A = (1-\nu)^{1/\tau}, \ Y/A = g\, C/A = Y^*/A.$$

Because technology $\ln A_t$ evolves based on a random walk with drift $\ln\gamma$, consumption and output are detrended to obtain a steady state.

- The potential aggregate output

$$Y_t^* = (1-\nu)^{1/\tau} A_t g_t, \tag{7}$$

- The market clearing conditions

$$Y_t = C_t + G_t + AC_t, \ \ N_t = H_t, \tag{8}$$

Optimality Conditions

- Households:

$$1 = \beta E_t \left[\left(\frac{C_{t+1}/A_{t+1}}{C_t/A_t} \right)^{-\tau} \frac{A_t}{A_{t+1}} \frac{R_t}{2\nu} \right] \tag{9}$$

- Intermediate Firms:

$$\begin{aligned} 1 &= \phi(\pi_t)\,(\pi_t - \pi) \left[\left(1 - \frac{1}{2\nu}\right) \pi_t + \frac{\pi}{\pi_{t+1}} \right] \\ &- \beta E_t \left[\phi(\pi_{t+1}) \left(\frac{C_{t+1}/A_{t+1}}{C_t/A_t} \right)^{-\tau} \frac{Y_{t+1}/A_{t+1}}{Y_t/A_t} (\pi_{t+1} - \pi)\pi_{t+1} \right] \\ &+ \frac{1}{\nu} \left[1 - \left(\frac{C_t}{A_t} \right)^{\tau} \right]. \end{aligned} \tag{10}$$

The sizes of $\phi(\pi_t)$ and $\phi(E_t(\pi_{t+1}))$ depend on current inflation, π_t, and expected inflation, $E_t(\pi_{t+1})$, respectively.

Variation of Estimated Models

- We examine and estimate four models classified with respect to the two assumptions.
  - Model 1: in absence of the ZLB and the AAC
  - Model 2: in presence of the ZLB
  - Model 3: in presence of the AAC
  - Model 4: in presence of both the ZLB and the AAC

Estimation Method

Two Steps of MCMC of Bayesian Inference.

1. Calculation of Policy Functions: a *Time Iteration method with Linear Interpolation* (TL)
   - In terms of balance between speed and precision, **Richter et al. (2014)** shows TL performs best among projection methods to solve NL DSGE models under ZLB.
2. Estimation of Parameters: *Particle Filter MH algorithm* (Herbst & Schorfheide, 2015)
   - adopt 10,000 particles to calculate likelihood approximation using particle filter.
   - select the mode of posterior density out of parameters sampled from prior distribution.
   - obtain 25,000 draws of MCMC samplings after discarding the first 5,000 burn-in draws.

Time Iteration with Linear Interporation (TL)

- The model's decision rules (or policy functions)

$$Y_t/A_t = \mathcal{P}_{\dagger}(\underbrace{R_{t-1},\, g_t,\, z_t}_{3\, State\, Var} \;\; \underbrace{\varepsilon_{R,t}}_{1\, Shock}), \tag{11}$$

$$\pi_t = \mathcal{P}(\; R_{t-1},\, g_t,\, z_t,\, \varepsilon_{R,t} \;),$$

where Y_t/A_t, π_t, are control variables and R_{t-1}, g_t, z_t, and $\varepsilon_{R,t}$ are state variables or shocks.

- # of policy functions (11) = # of optimal conditions; (9) and (10)
- # of nodes.
    - $1715 (= 7 \times 7 \times 7 \times 5)$ nodes,
      becasue 7 grid points on 3 state variables and 5 on 1 shock.

State Space Form

- State Equations
  The policy functions, Eq.(11), with Eq. (1), (5) and (6),

$$s_t = \Phi(s_{t-1}, \varepsilon_t, \theta), \tag{12}$$

  where s_t is endogenous variables θ is the parametars.

- Measurement Equations

$$y_t = \psi(s_t, \theta) + \sigma_u \, u_t, \ \text{for } u_t \sim \text{i.i.d. } N(\,0,\ I\,)$$

$$\begin{bmatrix} YGR_t \\ INFL_t \\ INT_t \end{bmatrix} = \begin{bmatrix} 100 \times (\ln(Y_t/A_t) - \ln(Y_{t-1}/A_{t-1}) + \ln z_t + \ln \gamma) \\ 400 \times \ln \pi_t \\ 400 \times \ln R_t \end{bmatrix} + \begin{bmatrix} \sigma_{\Delta y} \;\; u_{y,t} \\ \sigma_\pi \;\; u_{\pi,t} \\ \sigma_r \;\; u_{r,t} \end{bmatrix}. \tag{13}$$

where y_t is observed variables, σ_u and u_t are standard deviation and disturbance term of the measurement error.

Data and Sample Period

Japan data on output growth (Δy_t), inflation (π_t), and nominal interest rate rates (R_t) **from 1985:Q3 through 2015:Q1.**

- real GDP figures from the Cabinet Office's National Accounts. release of benchmark year 2005 that cover the period 1994Q1-2015Q1. To extend, from the benchmark year 2000 release and spanned the period 1985Q3-2015Q1.
- For the price level, core CPI
- The nominal interest rate, the Bank of Japan's uncollateralized call rate and transformed monthly figures to quarterly averages over the sample period.

Prior Setting

parameters	definition	distribution	parameter 1	parameter 2
β	discount factor	beta	0.98	0.01
τ	inverse elasticity of substitution	gamma	2.0	0.1
ν	inverse elasticity of demand for goods	gamma	0.3	0.1
ϕ_2	coef. of adjustment costs	gamma	20.0	10.0
δ	Degree of asymmetric AC	beta	0.25	0.1
ψ_1	reaction of inflation	normal	1.5	0.5
ψ_2	reaction of output	normal	0.5	0.5
γ	average technology growth rate	normal	1.04	0.01
π	Steady state inflation rate	normal	1.0	0.01
ρ_r	persistence of MP shock	beta	0.75	0.1
ρ_g	persistence of Gov shock	beta	0.75	0.1
ρ_z	persistence of TFP shock	beta	0.75	0.1
σ_r	St.D. of MP iid shock	Inverse gamma	0.5	5
σ_g	St.D. of Gov iid shock	Inverse gamma	0.5	5
σ_z	St.D. of TFP iid shock	Inverse gamma	0.5	5

Model Selection

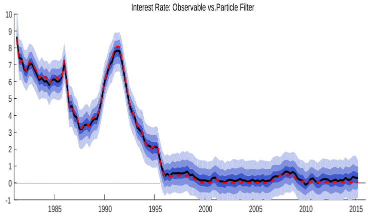
	ZLB	**AAC**	Log Marginal Likelihood	Posterior Model Prob
Model 1	No	No	-966.75	0.0000
Model 2	**Yes**	**No**	**-789.14**	**0.0004**
Model 3	No	Yes	-957.11	0.0000
Model 4	**Yes**	**Yes**	**-781.38**	**0.9996**

Notes: ZLB and AAC stand for zero lower bound and asymmetric adjustment cost, respectively.

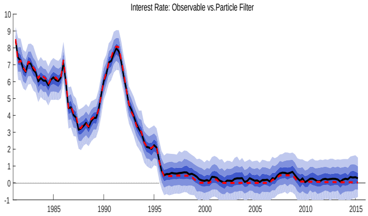
Posterior Estimates of Interest Rate

Actual: Red dashed line, Estimated: Black solid line, 68% bands: Blue shade area

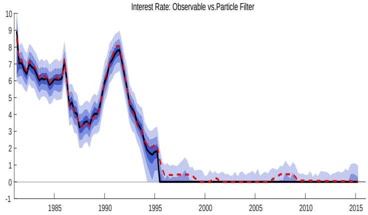
None (Model 1)



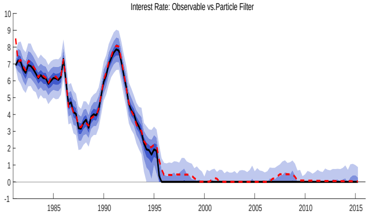
AAC (Model 3)



ZLB (Model 2)



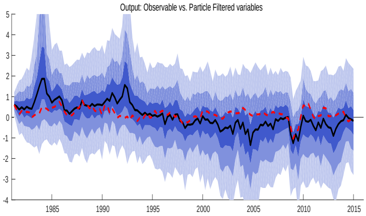
ZLB&AAC(Model 4)



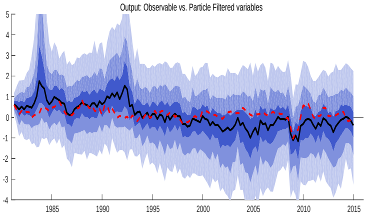
Posterior Estimates of Real GDP

Actual: Red dashed line, Estimated: Black solid line, 68% bands: Blue shade area

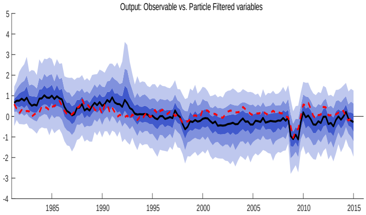
None (Model 1)



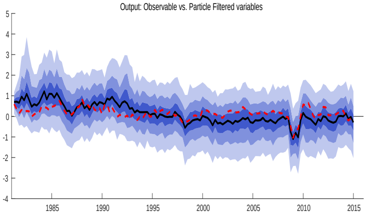
AAC (Model 3)



ZLB (Model 2)



ZLB&AAC(Model 4)



Estimated Parameters

$\delta = 0.475 > 0$ and $\exp(0.475) = 1.608$ indicates that

- Downward rigidity is observed, even Japan faces deflation.
- The adjustment cost near ZIRP is 60.8% higher than *the inflation* period (non ZIRP),

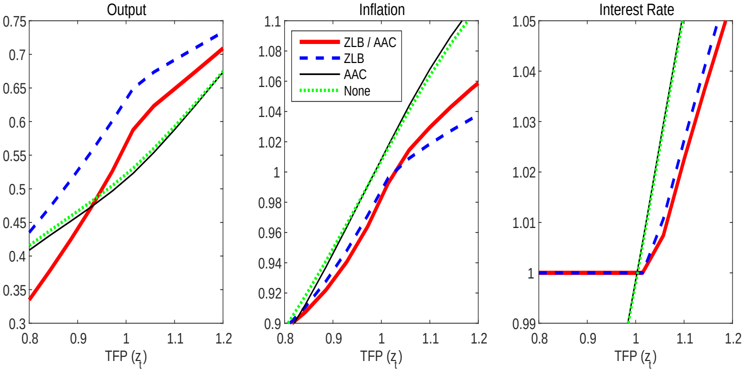
parameters		**Model 1**	**Model 2**	**Model 3**	**Model 4**
	ZLB	**No**	**Yes**	**No**	**Yes**
	AAC	**No**	**No**	**Yes**	**Yes**
β	discount factor	0.994	0.995	0.995	0.995
τ	inverse elasticity of substitution	1.755	1.566	1.819	1.617
ν	inverse elasticity of demand for goods	0.423	0.562	0.418	0.494
ϕ_1	coef. of adjustment costs	16.85	16.74	16.91	16.97
δ	**Degree of asymmetric AC**	**N.A.**	**N.A**	**-0.058**	**0.475**
ψ_1	reaction of inflation	1.60	1.67	1.61	1.66
ψ_2	reaction of output	0.61	0.63	0.60	0.58
γ	average technology growth rate	1.0048	1.0005	1.0056	1.0003
π	Steady state inflation rate	1.0017	1.0034	1.0024	1.0017

Estimated Parameters (cont'd)

parameters		Model 1	Model 2	Model 3	Model 4
	ZLB	**No**	**Yes**	**No**	**Yes**
	AAC	**No**	**No**	**Yes**	**Yes**
ρ_r	persistence of MP shock	0.657	0.635	0.667	0.689
ρ_g	persistence of Gov shock	0.639	0.837	0.644	0.838
ρ_z	persistence of TFP shock	0.851	0.768	0.857	0.791
σ_r	St.D. of MP iid shock	1.538	0.921	1.478	0.872
σ_g	St.D. of Gov iid shock	0.731	1.765	0.681	1.744
σ_z	St.D. of TFP iid shock	1.565	0.768	1.506	0.781
$\sigma_{\Delta y}$	St.D. of measument error of output	0.552	0.576	0.573	0.571
σ_π	St.D. of measument error of inflation	0.882	0.861	0.874	0.851
σ_R	St.D. of measument error of R	1.241	1.242	1.259	1.241

Posterior Estimation of PFs for TFP Shock

- Reactions of Output to TFP shock are steep, and Reactions of Inflation are moderate under ZIRP (low level of TFP shock). Under no ZIRP, *vice versa*.
  - These explain high volatility of GDP and low volatility of Inflation during deflation.

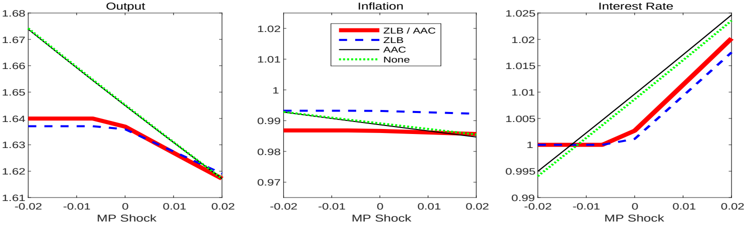


Notes: The PFs are calculated from the posterior means of parameters. We fix as $Gov_t = 1.7, \;\; R_{t-1} = 1.01, \;\; MP_t = 0.$

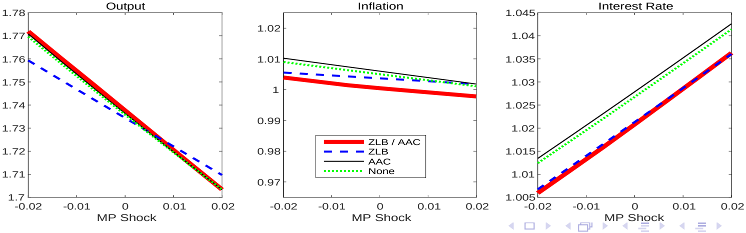
Posterior Estimation of PFs for MP Shock

- Reactions of Inflation to Monetary Policy Shock are moderate under ZIRP (low TFP). Under no ZIRP, the Reactions of Inflation are steep.

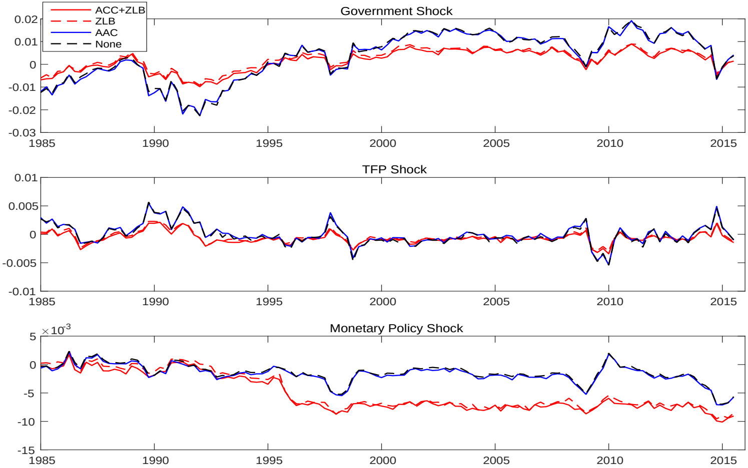
(a) Around ZIRP (small TFP) $TFP_t = 0.97, \quad Gov_t = 2.6, \quad R_{t-1} = 1.03$



(b) Far from ZIRP (large TFP) $TFP_t = 1.01, \quad Gov_t = 2.6, \quad R_{t-1} = 1.03$

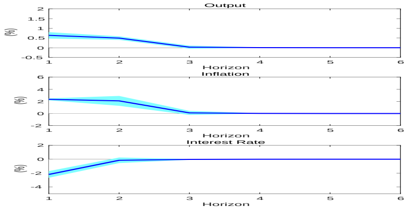


Posterior Estimates of Exogenous Shocks

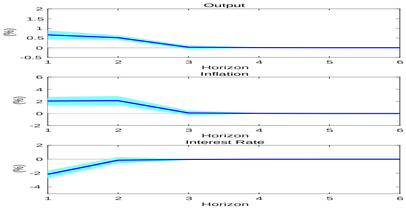


IRF of Negative Monetary Policy Shock

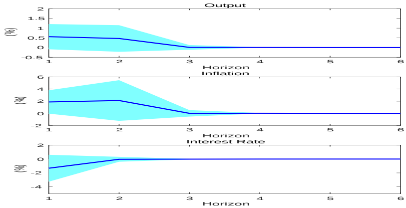
(a) None



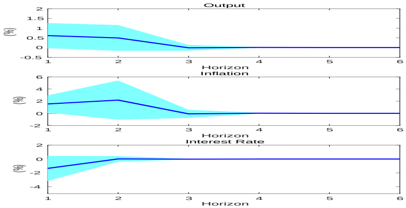
(c) AAC



(b) ZLB



(d) ZLB & AAC



Conclusion

- From Marginal Likelihood, the model with both of ZLB and AAC is selected.
  - Downward rigidity is observed, even Japan faces deflation.
  - The adjustment cost near Zero Interest Rate Policy is 60.8% (=exp(0.475)) higher than *the inflation* period (non ZIRP),
- Estimated Policy Functions of the selected model shows
  1. Reactions of Output to TFP shock are steep, and Reactions of Inflation are moderate under ZIRP (low level of TFP shock). Under no ZIRP, *vice versa*.
     - These explain high volatility of GDP and low volatility of Inflation during deflation.
  2. Reactions of Inflation to Monetary Policy Shock are moderate under ZIRP (low TFP). Under no ZIRP, Reactions of Inflation are steep.
     - Even positive monetary policy has weak impact on inflation near ZIRP.