Heterogenous Production Functions, Panel Data, and Productivity Dispersion

Jeremy T. Fox[1] Vitor Hadad[2] Stefan Hoderlein[3]
Amil Petrin[4] Robert P. Sherman[5]

[1]Rice University and NBER

[2]Boston College

[3]Boston College

[4]University of Minnesota and NBER

[5]California Institute of Technology

Heterogeneity in Supply **and** Demand
December 2016

Panel Data

- Today's talk is motivated by somewhat large panel datasets
- Large enough to use nonparametric methods
- Not require multiple servers, big data software
- **Online marketing**: One popular website, single advertising campaign
  - Panel ID: browser cookies or required to log in
- **Productivity**: firm or plant level data for a medium to large country
  - Panel ID: government statistical agency collects data
  - Application *eventually* in today's paper
- Also present **nonparametric identification** results of theoretical interest
- Identification could motivate parametric estimation approaches

Uses of Panel Data Sets

- *Some* **non-structural** machine learning
  - Predictive nonparametric regression
  - Polynomials, splines, trees, nearest neighbor, etc.
- *Some* **structural** demand estimation with consumer panel data
  - Recover unobservable preference heterogeneity (random coefficients)
  - Explore dynamics of consumer decision making (learning, stockpiling)
- *Some* theoretical **econometricians**
  - Allow **correlation of unobservables and explanatory variables**
  - Minimizing **conduct** assumptions in model for explanatory variables
  - Conduct: no "reduced form" model of firm advertising choice, say
  - Most common paradigm: **fixed effects**

Correlated Random Coefficients

- **Correlated random coefficients** combine
  - one aspect from structural consumer panels research
  - fixed effects line of thought from theoretical econometrics
- Unobserved heterogeneity from, say, consumer panels or production data
  - Include **random coefficients** in a regression setting
  - Alternatively, call it *treatment effect heterogeneity*
  - Further, random coefficients are **not fixed over time** for individual or firm
- Allow random coefficients to be **correlated with explanatory variables**
  - Firms with more return to capital might use more capital
  - Consumers that are more responsive to ads might be targeted by more ads
  - No explicit **conduct** model of say capital choice, advertising FOCs
  - Although certainly behavioral assumptions

Potential Applications

- Consumer panels for demand estimation
  - Many endogeneity issues with non randomized marketing exposures
  - Possible application: detergent in Germany
- Current paper: **productivity in India**
  - In process of applying these methods to plant level productivity data on India
  - Debate about low, dispersed productivity in India
  - With random coefficients, more general notation of productivity
  - No empirical results today, no more about India specifically

Correlated Random Coefficients

- Linear equation with random coefficients

$$Y_{i,t} = A_{i,t} + B_{i,t} X_{i,t}$$

- $Y_{i,t}$ outcome variable of interest
- $X_{i,t}$ explanatory variable of interest
- $A_{i,t}$ additive unobservable
- $B_{i,t}$ random slope
- Call *both* $A_{i,t}, B_{i,t}$ **random coefficients**
- Want to identify **moments of distribution**

$$F_t\left(A_{i,t}, B_{i,t} \mid X_{i,t}\right)$$

- Random coefficients $A_{i,t}, B_{i,t}$ can be **correlated** with $X_{i,t}$

Productivity Dispersion Motivation

- Traditional Cobb Douglas production function

$$Y_{i,t} = A_{i,t} + B^K K_{i,t} + B^L L_{i,t}$$

- **Productivity dispersion**: standard deviation of $A_{i,t}$ across plants or firms in same industry
- Syverson (2011) surveys empirical findings on productivity dispersion
- Typical finding: some plants produce more than twice as much output for same inputs
- *Assuming* $\left(B^K, B^L\right)$ *is the same across plants in a 4- or 2-digit industry classification may be too restrictive*
- If $\left(B^K_{i,t}, B^L_{i,t}\right)$ vary by plant i and time t and above specification is estimated, much of this heterogeneity enters into $A_{i,t}$

One Input Under Constant Returns to Scale

- Impose **constant returns to scale** (CRS)
- Skip simple algebra to get

$$Y_{i,t} = A_{i,t} + B_{i,t} X_{i,t}$$

- $Y_{i,t}$ outcome variable of interest, log output minus log labor under CRS
- $X_{i,t}$ log capital minus log labor
- $A_{i,t}$ total factor productivity
- $B_{i,t}$ input elasticity for capital, **heterogeneous** in this paper
- **Random coefficients** $A_{i,t}, B_{i,t}$ can be **correlated** with $X_{i,t}$
- Want to identify **moments of distribution**

$$F_t \left(A_{i,t}, B_{i,t} \mid X_{i,t} \right)$$

Moments to Identify

- **Cobb Douglas production** under constant returns to scale

$$Y_{i,t} = A_{i,t} + B_{i,t} X_{i,t}$$

- $E[A_{i,t}]$ **mean of log TFP**, could see how log TFP varies over time, across countries
- $E[A_{i,t} \mid X_{i,t}]$ do firms with **more inputs** / capital intensity have higher TFP? **Allocation of inputs**
- $\mathrm{SD}[A_{i,t}]$ how **dispersed is log TFP** within, say, a country? See huge literature
- $\mathrm{SD}[A_{i,t} \mid X_{i,t}]$ how dispersed is log TFP for firms using **same capital intensity**
- $E[B_{i,t}]$ **mean capital elasticity**: percentage increase in output for percentage increase in capital
- $E[B_{i,t} \mid X_{i,t}]$ do firms with **more capital intensity** have higher parameters on capital?
- $\mathrm{SD}[B_{i,t}]$ how **dispersed is technology for capital** across firms?

Endogeneity Concerns

- **Cobb Douglas production** under constant returns to scale

$$Y_{i,t} = A_{i,t} + B_{i,t} X_{i,t}$$

- Firms with higher capital coefficient $B_{i,t}$ might use more capital, be more capital intensive
- Firms with more log TFP $A_{i,t}$ might use more inputs

Two Time Periods

- Leave productivity example
- Simplify: periods $t = 1$ and $t = 2$ (fixed T at 2)
- Regression with random coefficients

$$Y_{i,1} = A_{i,1} + B_{i,1} X_{i,1}$$
$$Y_{i,2} = A_{i,2} + B_{i,2} X_{i,2}$$

- Identify **moments of distribution**

$$F\left(A_{i,1}, A_{i,2}, B_{i,1}, B_{i,2} \mid X_{i,1}, X_{i,2}\right)$$

- Random coefficients $(A_{i,t}, B_{i,t})$ **correlated** with $X_{i,1}$, $X_{i,2}$
- $F_t\left(A_{i,t}, B_{i,t} \mid X_{i,t}\right)$ can shift over time!

Innovations to Random Coefficients

- Add in time series process for **innovations to random coefficients** (shocks)

$$
\begin{aligned}
Y_{i,1} &= A_{i,1} + B_{i,1}X_{i,1} \\
Y_{i,2} &= A_{i,2} + B_{i,2}X_{i,2} \\
A_{i,2} &= A_{i,1} + U_{i,2} \\
B_{i,2} &= B_{i,1} + V_{i,2}
\end{aligned}
$$

- Drop i subscripts in what follows, almost everything depends on i in this paper!
- A_t and B_t are **random walks with drift**
  - Some AR(1) results in paper
- U_2 and V_2 are innovations to production functions
- Innovations not necessarily mean 0, allow drift in random coefficients distribution!
- Seasonality, learning, productivity growth, etc.

Identification Assumptions

- Recall dropping i subscript

$$
\begin{aligned}
Y_1 &= A_1 + B_1 X_1 \\
Y_2 &= A_2 + B_2 X_2 \\
A_2 &= A_1 + U_2 \\
B_2 &= B_1 + V_2
\end{aligned}
$$

- **Most important identification assumptions**

$$
\begin{aligned}
&(U_2, V_2) \perp (A_1, B_1) \mid X_1, X_2 \\
&(U_2, V_2) \perp (X_1, X_2)
\end{aligned}
$$

Productivity: Identification Assumptions

- **Most important identification assumptions**

$$(U_2, V_2) \perp (A_1, B_1) \mid X_1, X_2$$
$$(U_2, V_2) \perp (X_1, X_2)$$

- Productivity: **timing assumption**
- Firms said to choose inputs X_2 in previous period 1 before (U_2, V_2) seen
- Story has economic content: could verify by talking to firms about when they choose inputs

Productivity: Timing Assumption vs Proxy Variable Literature

- **Most important identification assumptions**

$$(U_2, V_2) \perp (A_1, B_1) \mid X_1, X_2$$
$$(U_2, V_2) \perp (X_1, X_2)$$

- Related timing assumption to literature **without random coefficient** on capital
- Olley & Pakes (1996), Levinsohn & Petrin (2003), Wooldridge (2009), Ackerberg et al (2015), Gandhi et al (2015)
- Some inputs chosen in period $t-1$ with knowledge of only $t-1$ production function
- A_t often nonparametric Markov, so distribution is $\Pr(A_t \mid A_{t-1})$
- We have random walk, AR(1) extension
- We do not allow i.i.d. output error not in A_t (proxy variable methods require scalar unobservables)

No Conduct Assumptions Like Static FOCs

- Estimate B as expenditure share on capital input
- Based on static profit maximizing FOC for Cobb-Douglas
- Treats input as having no adjustment costs, opposite of our approach
- Based on strong **conduct assumption**: static profit maximization
- We do not impose profit maximization, important for studying unproductive firms
- We allow unobservables outside production function to affect input choice
  - Firm-specific input prices
  - Adjustment costs
  - Product demand

Consumer Panels: Analogous Issues

- **Most important identification assumptions**

$$(U_2, V_2) \perp (A_1, B_1) \mid X_1, X_2$$
$$(U_2, V_2) \perp (X_1, X_2)$$

- **Targeting** based on previous behavior, not current period
- Targeting can be a complex function of past X's
- No explicit targeting FOC / supply side model
- Higher frequency data?
- Activity bias?
- Consumer preferences evolve over time

Correlated Random Coefficients Literature

- Literature on **correlated random coefficients** in panel data
  - Chamberlain (1982), Arellano and Bonhomme (2011), Graham and Powell (2012), Evdokimov (2011), etc.
- Models have **time invariant** random coefficients (A_i, B_i)

$$Y_{i,t} = A_i + \bar{A}_{i,t} + B_i X_{i,t}$$

- Correlation with $X_{i,t}$ usually only arises for time invariant components (A_i, B_i)
- Sometimes, $\bar{A}_{i,t}$ assumed to be independent of $\bar{A}_{i,t-1}$
- Time invariance assumptions impede investigating firm or industry growth
- Our results only for scalar $X_{i,t}$

Recent Papers on Heterogeneous Production Functions

- Kasahara, Shrimpf and Suzuki (2016)
  - Kasahara and Shimotsu (2009) finite mixture approach
- Balat, Brambilla and Suzuki (2016)
  - Extend proxy variable approaches but require variables to be chosen in period $t-1$ with only static considerations, as in FOCs

All Identification Assumptions

- Model, dropping *i* subscript

$$
\begin{aligned}
Y_1 &= A_1 + B_1 X_1 \\
Y_2 &= A_2 + B_2 X_2 \\
A_2 &= A_1 + U_2 \\
B_2 &= B_1 + V_2
\end{aligned}
$$

- **Most important identification assumptions**

$$
\begin{aligned}
&(U_2, V_2) \perp (A_1, B_1) \mid X_1, X_2 \\
&(U_2, V_2) \perp (X_1, X_2)
\end{aligned}
$$

- **Other identification assumptions**
  - X_1 and X_2 have continuous, common support
  - X_1 and X_2 not linearly dependent

Will Identify Conditional Moments

- **Conditional distribution of random coefficients**

$$F(A_1, A_2, B_1, B_2 \mid X_1, X_2)$$

- **Means** $E[A_1 \mid X_1, X_2]$, $E[B_1 \mid X_1, X_2]$, $E[A_2 \mid X_1, X_2]$, $E[B_2 \mid X_1, X_2]$
- **Standard deviations** $\mathrm{SD}[A_1 \mid X_1, X_2]$, $\mathrm{SD}[B_1 \mid X_1, X_2]$, $\mathrm{SD}[A_2 \mid X_1, X_2]$, $\mathrm{SD}[B_2 \mid X_1, X_2]$
- Can then identify **unconditional moments** as in

$$E[A_1] = E_{X_1, X_2}[E[A_1 \mid X_1, X_2]]$$

- Paper: identification entire joint distribution of random coefficients (A_1, A_2, B_1, B_2)
  - Characteristic functions

Identification & Estimation Steps

1. Identify moments of innovations

$$E[U_2], E[V_2]$$

2. Identify conditional moments of outcomes like

$$E[Y_1 \mid X_1, X_2]$$

   - Directly from data
   - Nonparametric regression

3. Identify first period conditional moments of random coefficients like

$$E[A_1 \mid X_1, X_2]$$

4. Identify second period conditional moments like

$$E[A_2 \mid X_1, X_2]$$

5. Form unconditional moments like

$$E[A_1] = E_{X_1, X_2}[E[A_1 \mid X_1, X_2]]$$

Step 1: Means of Innovations

- Identify means of innovations U_2, V_2
- Focus on (X_1, X_2) such that $X_1 = X_2 = x$: input choices remain constant
- Difference time periods using panel data, giving

$$Y_2 - Y_1 = (A_2 - A_1) + (B_2 - B_1)x = U_2 + V_2 x$$

- Take expectations

$$E[Y_2 - Y_1 \mid X_1 = X_2 = x] = E[U_2] + E[V_2]x$$

- Uses mean independence relaxation of

$$(U_2, V_2) \perp (X_1, X_2)$$

- Local kernel regression, local to $X_1 = X_2 = x$, identifies $E[U_2]$, $E[V_2]$
- Estimation: with optimal kernel bandwidth, rate 2/5 (less than 1/2), normality

Step 1: Intuition for Identifying Innovation Moments

- Look at plants that do not change input X
- Mean of change in output only due to innovations

Step 2: Outcome Moments

- Data directly identify conditional moments of outcomes like

$$E[Y_1 \mid X_1, X_2], E\left[Y_1^2 \mid X_1, X_2\right], E[Y_1 \cdot Y_2 \mid X_1, X_2]$$

- Estimation: nonparametric regression
- Kernel with optimal bandwidth gives rate 1/6, asymptotic normality
- Have also simulated splines, polynomials with Lasso
- Interests of time, show Monte Carlos only for polynomials with Lasso although nothing sparse about DGP in Monte Carlos

Step 3: Two Linear Equations for Means

- Production functions

$$Y_1 = A_1 + B_1 X_1$$
$$Y_2 = A_2 + B_2 X_2$$

- Then

$$E[Y_1 \mid X_1, X_2] = E[A_1 \mid X_1, X_2] + E[B_1 \mid X_1, X_2] x_1$$
$$E[Y_2 \mid X_1, X_2] = E[A_2 \mid X_1, X_2] + E[B_2 \mid X_1, X_2] x_2$$

- Rewrite period 2 production function at $X_1 = x_1$ and $X_2 = x_2$

$$E[Y_2 \mid X_1, X_2] =$$
$$E[A_1 + U_2 \mid X_1, X_2] + E[B_1 + V_2 \mid X_1, X_2] x_2 =$$
$$E[A_1 \mid X_1, X_2] + E[B_1 \mid X_1, X_2] x_2 + E[U_2] + E[V_2] x_2$$

Step 3: Means of Random Coefficients

- Fix $X_1 = x_1$ and $X_2 = x_2$
- Two linear equations

$$E[Y_1 \mid X_1, X_2] = E[A_1 \mid X_1, X_2] + E[B_1 \mid X_1, X_2] x_1$$
$$E[Y_2 \mid X_1, X_2] = E[A_1 \mid X_1, X_2] + E[B_1 \mid X_1, X_2] x_2 + E[U_2] + E[V_2] x_2$$

- $E[U_2]$, $E[V_2]$, $E[Y_1 \mid X_1, X_2]$, $E[Y_2 \mid X_1, X_2]$ previously identified
- Two unknowns: $E[A_1 \mid X_1, X_2]$ and $E[B_1 \mid X_1, X_2]$
- Linear equations have unique solution if $x_1 \neq x_2$
- So identify **conditional means of random coefficients** $E[A_1 \mid X_1, X_2]$ and $E[B_1 \mid X_1, X_2]$ for $x_1 \neq x_2$
- **Step does not use variation in X_1, X_2 because of panel data solution to endogeneity**

Step 4: Second Period Means of Random Coefficients

- Identified for $X_1 = x_1$ and $X_2 = x_2$

$$E[A_1 \mid X_1, X_2], E[B_1 \mid X_1, X_2], E[U_2], E[V_2]$$

- Now identify conditional means of second period random coefficients for at $X_1 = x_1$ and $X_2 = x_2$

$$E[A_2 \mid X_1, X_2] = E[A_1 + U_2 \mid X_1, X_2] = E[A_1 \mid X_1, X_2] + E[U_2]$$
$$E[B_2 \mid X_1, X_2] = E[B_1 + V_2 \mid X_1, X_2] = E[B_1 \mid X_1, X_2] + E[V_2]$$

Step 5: Unconditional Moments

- Now identify unconditional moments

$$E[A_1] = E_{X_1,X_2}[E[A_1 \mid X_1, X_2]]$$

- Identified if points $X_1 = X_2$ have measure zero
- For many sample sizes, estimation error jumps up dramatically close to $x_1 = x_2$
- One insanely mis-estimated $E[A_1 \mid X_1, X_2]$ for x_1 close to x_2 can wreck $E[A_1]$
- Monte Carlos also report censored means, where 10% closest $|x_1 - x_2|$ dropped from $E[A_1]$ calculation
- Finally, Monte Carlos report no-censoring-needed median of conditional means

$$\text{median}_{X_1,X_2}[E[A_1 \mid X_1, X_2]]$$

Use of Means of Random Coefficients

- Identified

$$E[A_1 \mid X_1, X_2], E[B_1 \mid X_1, X_2], E[U_2], E[V_2]$$

- Can explore growth in mean TFP and mean input elasticity (say capital elasticity) over time
- Compare means of random coefficients across industries, countries
- Conditional means useful to see if inputs **allocated to most productive firms**

Asymptotic Distribution

- Estimator for
$$E\left[A_1 \mid X_1 = x_1, X_2 = x_2\right]$$
- Use local constant kernels in first stage, rectangular kernel
- If second-stage bandwidth h_n satisfies certain properties,
$$\sqrt{nh_n^2}\left[\begin{array}{c} \hat{E}\left[A_1 \mid x_1, x_2\right] - E\left[A_1 \mid x_1, x_2\right] \\ \hat{E}\left[B_1 \mid x_1, x_2\right] - E\left[B_1 \mid x_1, x_2\right] \end{array}\right]$$

 converges to a multivariate normal with variance matrix

$$V\left(x_1, x_2\right) = \frac{1}{\left(x_2 - x_1\right)^2 \cdot 4f\left(x_1, x_2\right)}\left[\begin{array}{cc} x_2 & -x_1 \\ -1 & 1 \end{array}\right] \cdot \left[\begin{array}{cc} \mathrm{Var}\left(Y_1 \mid x_1, x_2\right) & \mathrm{Cov}\left(Y_1, Y_2 \mid x_1, x_2\right) \\ \mathrm{Cov}\left(Y_1, Y_2 \mid x_1, x_2\right) & \mathrm{Var}\left(Y_2 \mid x_1, x_2\right) \end{array}\right]\left[\begin{array}{cc} x_2 & -1 \\ -x_1 & 1 \end{array}\right]$$

Monte Carlo: Confidence Region Coverage

	Feasible		Infeasible	
(X_1, X_2) value	$E[A_1 \mid X_1, X_2]$	$E[B_1 \mid X_1, X_2]$	$E[A_1 \mid X_1, X_2]$	$E[B_1 \mid X_1, X_2]$
(-.1, .1)	0.946	0.953	0.949	0.952
(-.5, .5)	0.937	0.915	0.937	0.937
(-1, 1)	0.936	0.921	0.933	0.934
(-1.5, 1.5)	0.908	0.777	0.926	0.909

- Infeasible: true asymptotic variance used, feasible: variance matrix estimated
- 100,000 observations, 100 Monte Carlo replications, local constant, rectangular kernel, bandwidth $\propto n^{(-1/4)}$

Second Moments: Productivity Dispersion

- Our empirical focus is on **productivity dispersion**
- Productivity dispersion refers to **second moment** of productivity
- In our model, productivity refers to both TFP and capital input elasticity (random coefficients)
- How different in TFP / capital input elasticities are firms?
- Identify **standard deviation** and **correlation** of TFP and input elasticity…

$$\mathrm{SD}(A_1), \mathrm{SD}(B_1), \mathrm{Corr}(A_1, B_1), \mathrm{Corr}(A_1, A_2), \ldots$$

- Identification also conditional on inputs X_1, X_2, as in $\mathrm{SD}(A_1 \mid X_1, X_2)$

Step 1: Variances, Covariances of Shocks (U_2, V_2)

- Like with first moments, focus on $X_1 = X_2 = x$: input choices remain constant
- Earlier algebra showed

$$Y_2 - Y_1 = (A_2 - A_1) + (B_2 - B_1)x = U_2 + V_2 x$$

- Take conditional variance and exploit independence from X of second moments of innovations

$$\mathrm{Var}[Y_2 - Y_1 \mid X_1 = X_2 = x] = \mathrm{Var}[U_2] + \mathrm{Var}[V_2]x^2 + 2\mathrm{Cov}(U_2, V_2)x$$

- Local variation in x identifies $\mathrm{Var}[U_2]$, $\mathrm{Var}[V_2]$ and $\mathrm{Cov}(U_2, V_2)$

Step 3: Variances, Covariances of Random Coefficients

- Skip steps 2, 4, 5 for conciseness
- Fix $X_1 = x_1$ and $X_2 = x_2$
- Algebra and our main independence assumption show

$$\mathrm{Var}(Y_1 \mid X_1, X_2) = \mathrm{Var}(A_1 \mid X_1, X_2) + \mathrm{Var}(B_1 \mid X_1, X_2)\, x_1^2 + 2\mathrm{Cov}(A_1, B_1 \mid X_1, X_2)\, x_1$$

$$\mathrm{Var}(Y_2 \mid X_1, X_2) = \mathrm{Var}(A_1 \mid X_1, X_2) + \mathrm{Var}(B_1 \mid X_1, X_2)\, x_2^2 + 2\mathrm{Cov}(A_1, B_1 \mid X_1, X_2)\, x_2 + \mathrm{Var}(U_2 + V_2 x_2)$$

$$\mathrm{Cov}(Y_1, Y_2 \mid X_1, X_2) = \mathrm{Var}(A_1 \mid X_1, X_2) + \mathrm{Var}(B_1 \mid X_1, X_2)\, x_1 x_2 + \mathrm{Cov}(A_1, B_1 \mid X_1, X_2)(x_1 + x_2)$$

- Three linear equations, three unknowns
- Identify

$$\mathrm{Var}(A_1 \mid X_1 = x_1, X_2 = x_2),\ \mathrm{Var}(B_1 \mid X_1 = x_1, X_2 = x_2),\ \mathrm{Cov}(A_1, B_1 \mid X_1 = x_1, X_2 = x_2)$$

- Again, no identification when $x_1 = x_2$

Use of Second Moments

- Literature on **productivity dispersion within an industry** all about **second moments**
- Here, identify **dispersion in production functions** instead of just dispersion in total factor productivity, A_t
- Generalize the output level and output growth decompositions popular in empirical literature for higher dimensional notion of productivity
- Decompositions involve productivity and allocation of inputs to firms
- We identify the second moments conditional on $X_1 = x_1, X_2 = x_2$
- Important for questions on the allocation of inputs

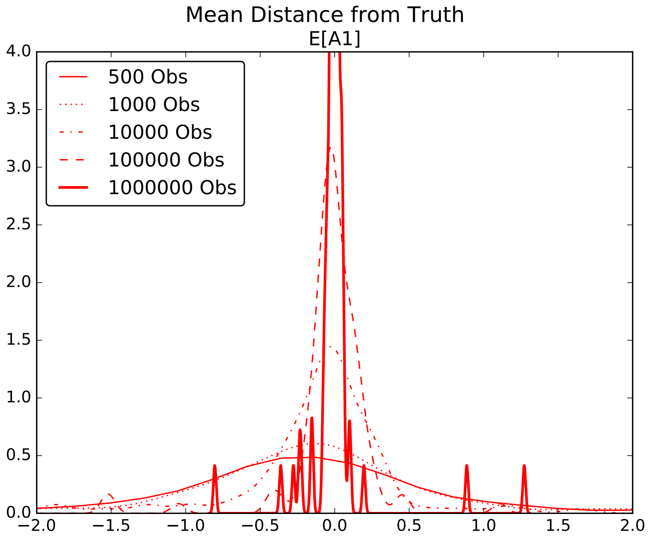
Estimation Steps Recap

1. Estimate moments of innovations U_2, V_2
   - Kernel local to $x_1 = x_2$
2. Identify conditional moments of outcomes like $E[Y_1 \mid X_1, X_2]$ directly from data
   - Nonparametric regression: Monte Carlos show polynomials with Lasso
3. Identify first period conditional moments of random coefficients like $E[A_1 \mid X_1, X_2]$
4. Identify second period conditional moments like $E[A_2 \mid X_1, X_2]$
5. Form unconditional moments like $E[A_1] = E_{X_1,X_2}[E[A_1 \mid X_1, X_2]]$
   - Uncensored
   - Censoring 10% observations of closest $|x_1 - x_2|$
   - Alternative: $\text{median}_{X_1,X_2}[E[A_1 \mid X_1, X_2]]$

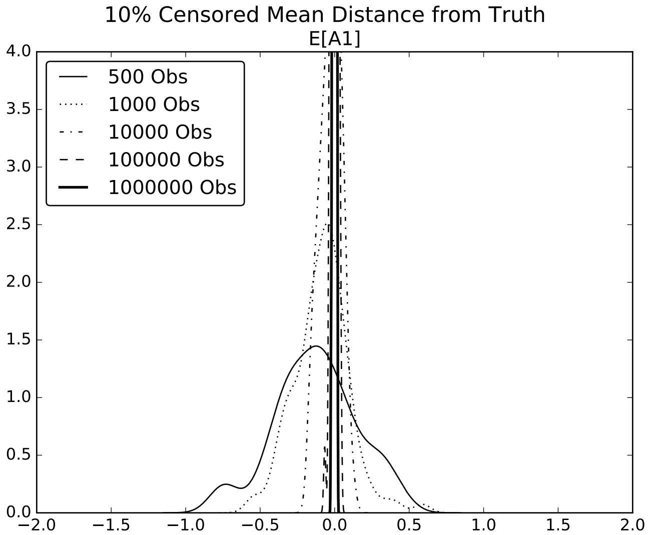
Monte Carlo: Productivity Inspired DGP

- Value added production function
- Constant returns to scale (subtract log labor from log output, log capital)
- Everything multivariate normal
- Autocorrelation in capital intensity X_1 and X_2, 0.8
- Capital coefficient B_1 correlated with capital intensity X_1
- Correlation 0.43 between log TFP A_1 and capital elasticity B_1
- Capital elasticity B_1 has mean of 0.4
- High productivity dispersion, $\mathrm{SD}(A_1)$ of 3.0

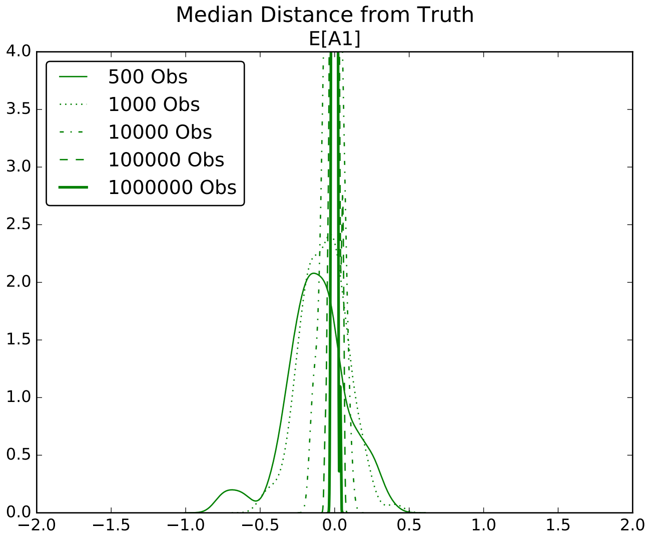
MC: $E[A_1]$ Uncensored Mean of log TFP



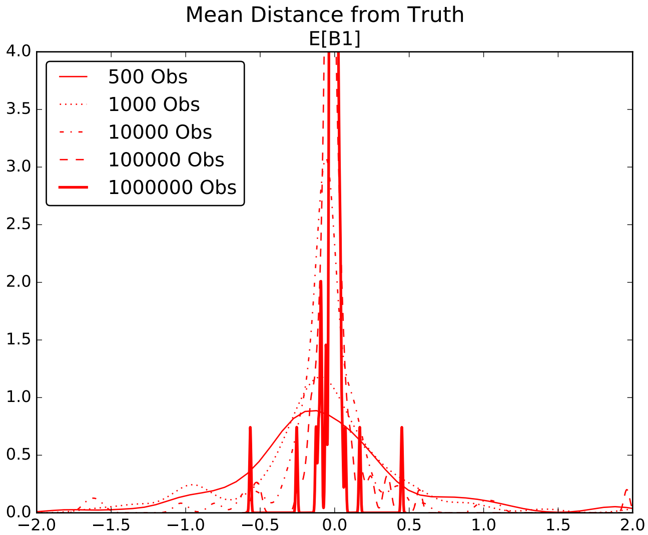
MC: Censored Mean of log TFP



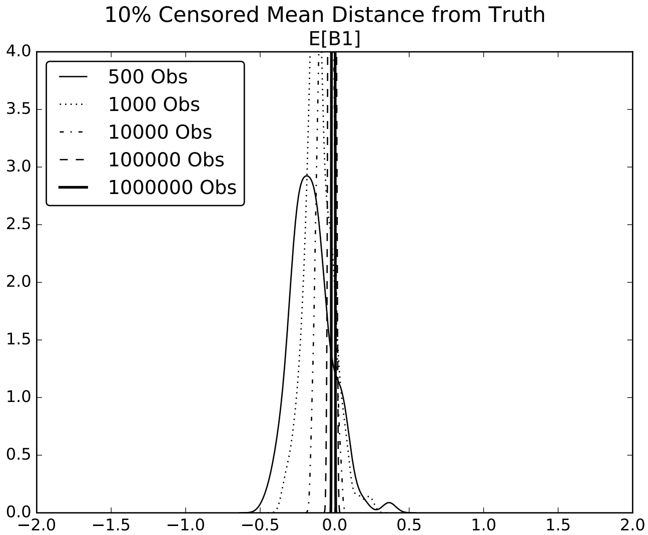
MC: $\text{median}_{X_1,X_2}\left[E\left[A_1 \mid X_1, X_2\right]\right]$



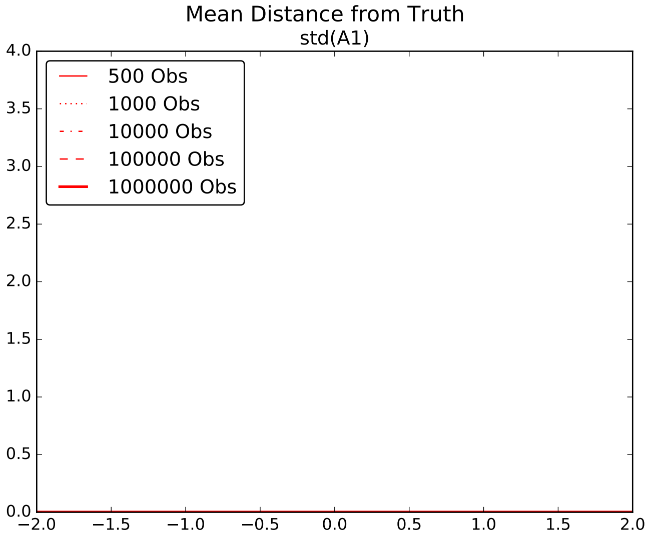
MC: $E[B_1]$ Uncensored Mean of Capital Elasticity



MC: Censored Mean of Capital Elasticity

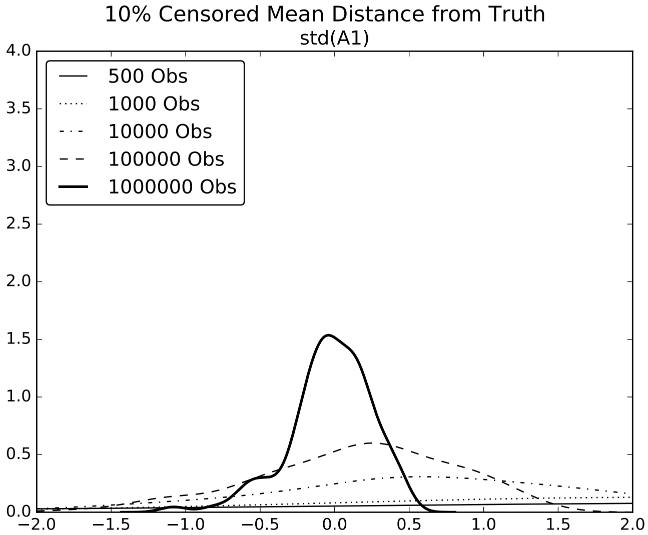


MC: $\mathbb{SD}[A_1]$ Uncensored SD of log TFP



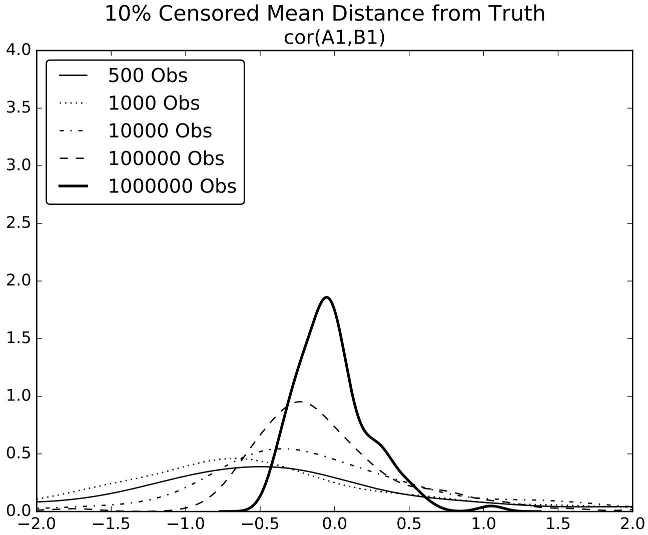
- Productivity dispersion

MC: Censored SD of log TFP



- Productivity dispersion

MC: Censored Correlation of TFP, Capital Elasticity $\mathrm{Corr}(A_1, B_1)$



Conclusions

- Random coefficients
  - Consumer panels with heterogeneous consumers
  - Production function parameters (A, B) vary across firms in same industry
- Levels of random coefficients (A, B) can be **correlated with inputs** (X_1, X_2)
- Estimators allow **arbitrary correlations** of production function parameters (A_1, B_1) with inputs (X_1, X_2)
- **Timing assumptions** with **random coefficients** (A, B)
  - Input decisions chosen in period $t-1$ with knowledge of $t-1$ production function