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Decomposing the Volatility Structure of Inflation

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Motivations

A generic forward-looking Taylor rule with feedback effect says

$$r_t = (1-\gamma)r^* + \gamma r_{t-1} + \theta[E_t\pi_{t+j} - \pi^*].$$

- $r_t = i_t - E_t\pi_{t+1}$ the short-term ex-ante interest rate.
- π^* the inflation target, such as 2% annually.
- r^* interest rate equilibrium.

Apparently, inflation forecast $E_t\pi_{t+j}$ is important in choosing the policy instrument parameter γ and θ.

Literature in macro

DSGE model

- Bodenstein et al. (2008): Objective function penalizing core inflation volatility $\Rightarrow$ welfare maximization. Related to Mishkin (2007).
- Del Negro and Schorfheide (2013), Diebold et al. (2015): Inflation with SV, and technology shock with SV.

Volatility response to macro shocks

- Goodfriend and King (1997), King and Wolman (1999), Aoki (2001): Welfare loss $\Leftarrow$ price change volatility in the CPI sticky components.
- Erceg et al. (2000), FOMC (2009), BOE (2013): Discrepancy of core and headline inflation is due to different volatility response to macro shocks, such monetary policy shock and energy crisis.

Should we and how can we detrend and deseasonalize?

- Aadland (2005): A review.

Literature in econometrics

Time-varying volatility is a key nonlinearity in macroeconomic time series

- Sims and Zha (2006), Justiniano and Primiceri (2008), Bloom (2009), Clark (2011), Fernandez and Rubio (2013), Curdia et al. (2014): fit of VAR, factor model, and ARIMA model are largely improved with SV.

Inflation forecasting models have time-inconsistent performance without SV

- Stock and Watson (2007, 2008) showed that Gordon(1990)'s "triangle" model, Harvey (1990)'s local level model, Atkeson and Ohanian (2001)'s random walk model, and many other univariate as well as multivariate models show episodes of good performance in forecasting.

Stock and Watson's LoL-SV

Stock and Watson (2007, 2008) showed good forecasting performance of the following local level model with SV (LoL-SV)

$$y_t = \mu_t + \exp(\frac{1}{2} h_t^y)\epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0,1),$$
$$\mu_{t+1} = \mu_t + \exp(\frac{1}{2} h_t^\mu)\eta_t, \quad \eta_t \sim \mathcal{N}(0,1),$$

and SV h_t^y, h_t^μ are modelled as random walks with correlated innovations, i.e.

$$h_{t+1}^y = h_t^y + \sigma_y \zeta_t^y, \quad h_{t+1}^\mu = h_t^\mu + \sigma_\mu \zeta_t^\mu,$$
$$\begin{bmatrix} \zeta_t^y \\ \zeta_t^\mu \end{bmatrix} \sim \mathcal{N}(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}).$$

Novelty: SV in both observation and the state equation.
Difficulty: They did not estimate but "calibrated" $\sigma_y = \sigma_\mu = 0.2$ **and** $\rho = 0$**.**

ML estimation

LoL-SV is non-linear state space model without analytical likelihood function.

We propose a simulated likelihood method for estimating this model, using importance sampling to integrate out the

- SV h_t^y (transitory volatility) in the observation equation, and
- SV h_t^μ (permanent volatility) in the state transition.

Conditional on h_t^y and h_t^μ, this model is a linear Gaussian state space model. Kalman filter can efficiently evaluate its likelihood.

ML estimation

The conditional log-likelihood can be written as

$$\log p(Y_T | H_T^y, H_T^\mu) = \sum_{t=1}^{T} \log p(y_t | Y_{t-1}, H_{t-1}^y, H_{t-1}^y),$$

and the conditional log-likelihood contribution is

$$\log p(y_t | Y_{t-1}, H_{t-1}^y, H_{t-1}^y) = \log \Phi(v_t; 0, F_t),$$

where $\Phi(.; /mu, /sigma^2)$ denotes the Gaussian density function, and v_t and F_t are produced by Kalman filter given h_{t-1}^y and h_{t-1}^μ.

ML estimation

We can show that the likelihood function can be written as

$$\begin{aligned} L(Y_T;\sigma_\mu,\sigma_y,\rho) &= \int\int p(Y_T|H_T^y,H_T^\mu)p(H_T^y,H_T^\mu)dH_T^y dH_T^\mu \\ &= g(Y_t)\int\int \frac{p(Y_T|H_T^y,H_T^\mu)}{g(Y_T|H_T^y,H_T^\mu)} g(H_T^y,H_T^\mu|Y_T)dH_T^y dH_T^\mu. \end{aligned}$$

For estimation, we maximise

$$L^*(Y_T;\sigma_\mu,\sigma_y,\rho) = g(Y_T)\frac{1}{M}\sum_{m=1}^{M}\frac{p(Y_T|H_T^{y,(m)},H_T^{\mu,(m)})}{g(Y_T|H_T^{y,(m)},H_T^{\mu,(m)})},$$

We propose the importance density or importance model to be a linear Gaussian state space model, so that

- $g(Y_t)$ is easily computed using prediction error decomposition
- $H_T^{y,(m)}, H_T^{\mu,(m)}$ can be easily drawn from $g(H_T^y,H_T^\mu|Y_T)$ using simulation smoother.

ML estimation

The parameters of the importance model are determined via a modified Numerically Accelerated Importance Sampling algorithm of Koopman et al. (2015).

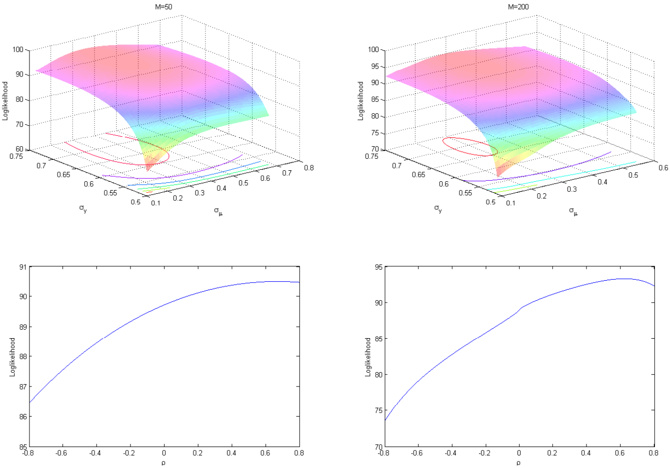

Figure: Log-likelihood as a function of (hyper)parameters.

Estimation and signal extraction of LoL-SV

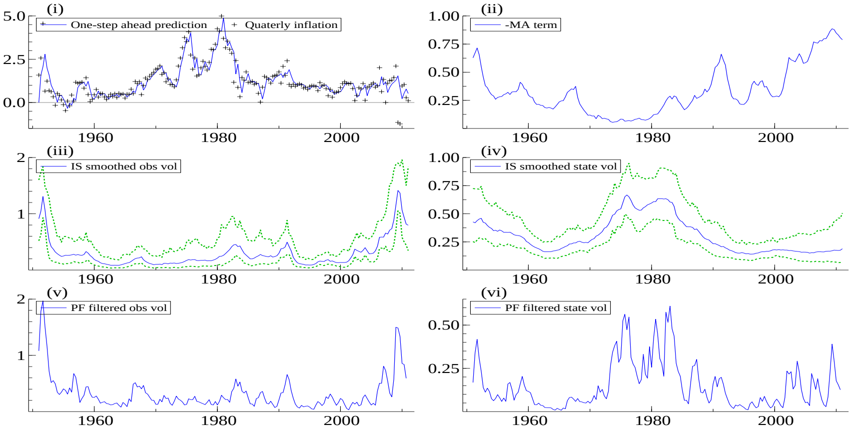

Figure: Main fit from the U.S. quarterly inflation series. (i) Inflation series and one-step ahead forecast; (ii) -MA term indicating memory and persistence of y_t; (iii) $\mathbb{E}(\exp(\frac{1}{2}h^y_{1:n})|y_{1:n};\psi)$; (iv) $\mathbb{E}(\exp(\frac{1}{2}h^\mu_{1:n})|y_{1:n};\psi)$; (v) $\{\mathbb{E}(\exp(\frac{1}{2}h^y_t)|y_{1:t};\psi)\}_{t=1}^n$; (vi) $\{\mathbb{E}(\exp(\frac{1}{2}h^\mu_t)|y_{1:t};\psi)\}_{t=1}^n$.

Example: LLS-OTSSV

Monthly core inflation from 1957:1-2015:1, not seasonally adjusted

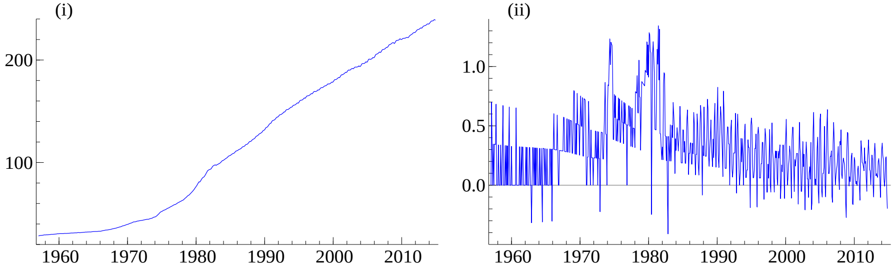

Figure: (i) The U.S. core monthly CPI and (ii) the first difference of log CPI (inflation).

Example: LLS-OTSSV

The model is local level plus seasonal model with SV (LLS-OTSSV),

$$y_t = \mu_t + \gamma_t + \exp(\frac{1}{2}h_t^y)\epsilon_t, \qquad h_{t+1}^y = \alpha_y + \phi_y h_t^y + \sigma_y \zeta_t^y$$

$$\mu_{t+1} = \mu_t + \exp(\frac{1}{2}h_t^\mu)\eta_t^\mu, \qquad h_{t+1}^\mu = h_t^\mu + \sigma_\mu \zeta_t^\mu,$$

$$\gamma_{t+1} = -(\gamma_t + \gamma_{t-1} + ... + \gamma_{t-10}) + \exp(\frac{1}{2}h_t^\gamma)\eta_t^\gamma, \quad h_{t+1}^\gamma = h_t^\gamma + \sigma_\gamma \zeta_t^\gamma,$$

with $(\epsilon_t^y, \eta_t^\mu, \eta_t^\gamma)$ being uncorrelated standard Gaussian random variables and independent on $(\zeta_t^y, \zeta_t^\mu, \zeta_t^\gamma)$, for $t = 1, ..., n$ and $n = 695$. Furthermore, we also model correlations among SV series by

$$\mathbb{E}(\zeta_t^y \zeta_t^\mu) = \rho_{y\mu}, \quad \mathbb{E}(\zeta_t^\mu \zeta_t^\gamma) = \rho_{\mu\gamma}, \quad \mathbb{E}(\zeta_t^y \zeta_t^\gamma) = \rho_{y\gamma}.$$

There are eight parameters in the model, namely

$$\psi = (\alpha_y, \phi_y, \sigma_y, \sigma_\mu, \sigma_\gamma, \rho_{y\mu}, \rho_{\mu\gamma}, \rho_{y\gamma})'.$$

Example: LLS-OTSSV

Parameter	LLS	LLS-D	LLS-OTSSV	LLS-OTSSV-D
α			-4.377 [-5.150, -3.604]	-4.346 [-5.057, -3.635]
ϕ_y			0.984 [0.975, 0.993]	0.975 [0.967, 0.983]
σ_y	0.144 [0.133, 0.156]	0.131 [0.120, 0.142]	0.172 [0.126, 0.218]	0.182 [0.135, 0.228]
σ_μ	0.043 [0.031, 0.054]	0.043 [0.032, 0.053]	0.150 [0.082, 0.217]	0.149 [0.082, 0.216]
σ_γ	0.027 [0.020, 0.033]	0.029 [0.022, 0.037]	0.122 [0.092, 0.152]	0.109 [0.079, 0.139]
$\rho_{y\mu}$			0.638 [0.496, 0.780]	0.594 [0.426, 0.761]
$\rho_{\mu\gamma}$			-0.081 [-0.382, 0.220]	-0.103 [-0.428, 0.222]
$\rho_{y\gamma}$			-0.119 [-0.494, 0.257]	-0.126 [-0.485, 0.233]
D_1 (1974:2)		0.419 [0.107, 0.731]		0.436 [0.123, 0.7450]
D_2 (1974:11)		-0.267 [-0.576, 0.042]		-0.282 [-0.587, -0.024]
D_3 (1980:7)		-1.213 [-1.521, -0.906]		-1.160 [-1.469, -0.852]
D_4 (1981:9)		-0.425 [-0.732, -0.117]		-0.413 [-0.723, -0.103]
D_5 (1982:8)		-0.421 [-0.730, -0.113]		-0.437 [-0.745, -0.128]
Normality	0.000	0.000	0.850	0.296
Box-Ljung	0.000	0.000	0.091	0.024
H(n/3)	0.000	0.000	0.339	0.137
LL	179.934	220.300	373.762	394.074

Example: LLS-OTSSV

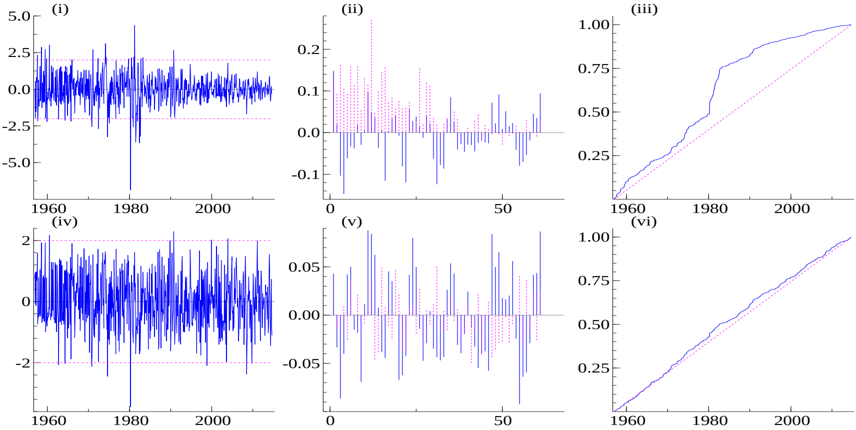

Figure: Graphic diagnostics for LLS and LLS-OTSSV based on standardized residuals: (i): LLS standardized residuals; (ii) LLS autocorrelogram of residuals (solid lines) and squared residuals; (iii) LLS scaled cumulative sum of squared residuals; (iv)-(vi) LLS-OTSSV counterparts.

Example: LLS-OTSSV

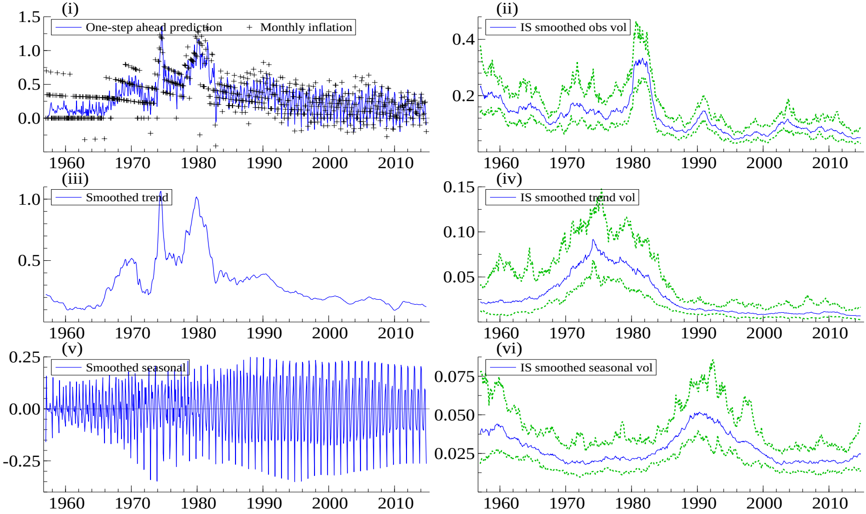

Figure: Main fit from the U.S. monthly inflation series.

LLS-OTSSV Model Variants

Parameter	LLS-TSSV	LLS-OSV	LLS-OSSV	LLS-OTSV
α		-4.362 [-5.374, -3.351]	-4.403 [-5.437, -3.368]	-4.347 [-5.157, -3.537]
ϕ_y		0.987 [0.978, 0.996]	0.988 [0.983, 0.993]	0.985 [0.968, 0.999]
σ_y	0.104 [0.095, 0.112]	0.186 [0.127, 0.246]	0.180 [0.135, 0.224]	0.169 [0.103, 0.234]
σ_μ	0.314 [0.216, 0.411]	0.022 [0.016, 0.028]	0.024 [0.018, 0.031]	0.192 [0.089, 0.295]
σ_γ	0.144 [0.074, 0.213]	0.028 [0.021, 0.035]	0.089 [0.047, 0.130]	0.027 [0.022, 0.033]
$\rho_{y\mu}$				0.594 [0.426, 0.761]
$\rho_{\mu\gamma}$	0.351 [0.176, 0.490]			
$\rho_{y\gamma}$			-0.154 [-0.365, 0.057]	
Normality	0.867	0.059	0.001	0.148
Box-Ljung	0.000	0.002	0.002	0.019
H(n/3)	0.116	0.106	0.766	0.203
LL	331.895	335.42	349.251	365.701

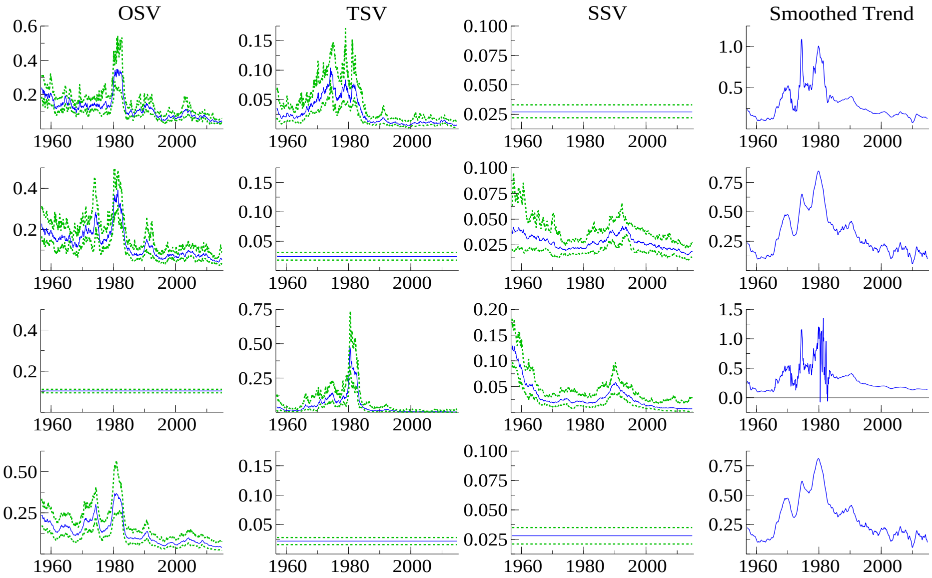

Figure: Main fit from the U.S. monthly inflation series under LLS-OTSSV variants.

Forecasting evaluation

Four model specifications

- No SV at all, LLS;
- Only transitory volatility: LLS-OSV;
- Only permanent volatility: LLS-TSSV;
- Both: LLS-OTSSV;

Four forecasting horizons h

- Monthly, $h = 1$;
- Quarterly: $h = 3$;
- Semiannually: $h = 6$;
- Annually: $h = 12$.

Three types of forecast

- Point forecast;
- Density forecast.

Point forecast

	MFE				MAFE				RMSE			
	$h=1$	$h=3$	$h=6$	$h=12$	$h=1$	$h=3$	$h=6$	$h=12$	$h=1$	$h=3$	$h=6$	$h=12$
LLS	-0.006	-0.005	-0.009	-0.014	0.109	0.117	0.108	0.098	0.144	0.153	0.146	0.141
LLS-OSV	-0.004	-0.004	-0.010	-0.011	0.091	0.109	0.098	0.092	0.137	0.142	0.140	0.137
LLS-TSSV	-0.005	-0.006	-0.008	-0.009	0.092	0.113	0.096	0.094	0.134	0.150	0.141	0.137
LLS-OTSSV	-0.004	-0.004	-0.008	-0.010	0.087	0.097	0.094	0.092	0.128	0.131	0.124	0.122

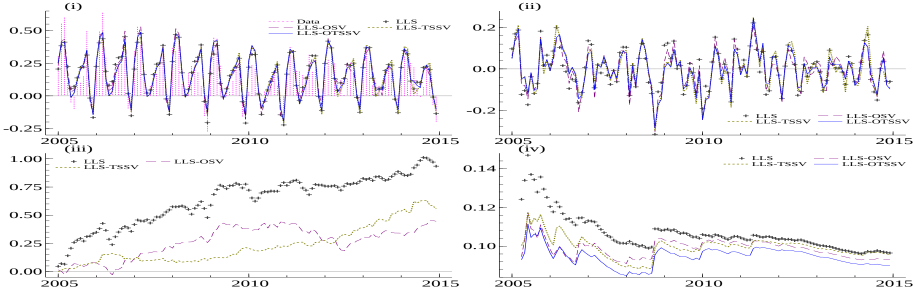

Figure: (i) One-step ahead forecast of four models. (ii) Forecast errors. (iii) Cumulative difference of absolute forecast errors. (iv) Recursive standard deviation plot of forecast errors.

Density forecast

- We firstly consider overall calibration based on PIT, similar to Diebold et al. (1998).
- Let $d_t^h(.)$ denote an h-step ahead forecasting density with distribution function $D_t^h(.)$.
- The PIT of h-step ahead forecast $y_{t+h|t}$ is

$$
\begin{aligned}
D_t^h(y_{t+h}) &= \int_{-\infty}^{y_{t+h}} d_t^h(s)ds \\
&\approx \sum_{m=1}^{M} \frac{p(y_t|Y_{t-1}, H_{t-1}^{y,(m)}, H_{t-1}^{\mu,(m)})}{g(y_t|Y_{t-1}, H_{t-1}^{y,(m)}, H_{t-1}^{\mu,(m)})} \Phi(v_{t+h}^{(m)}; 0, F_{t+h}^{(m)}).
\end{aligned}
$$

If the forecasting density $d_t^h(.)$ is correctly calibrated, then $D_t^h(.)$'s are uniformly distributed random variables in the unit interval.

Density forecast

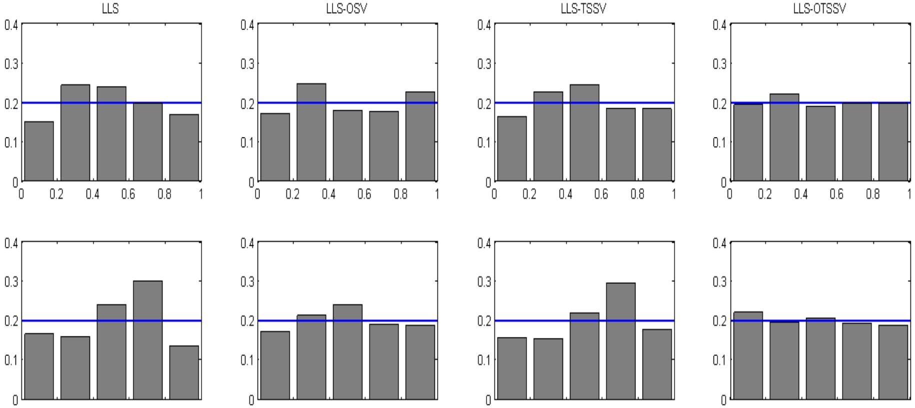

Figure: Histograms of one-step and three-step ahead forecasting density PIT $D_t^1(y_{t+1})$ and $D_t^3(y_{t+3})$. Top to bottom row are one- and three-step ahead PIT. We group PIT's into five equal-sized bins each of which should contain exactly 20% of PIT's under uniformity.

Conclusion

- One should take into account of SV when forecasting inflation. Static models usually show episodes of satisfactory performance.
- We propose a structural state space model which explicitly decomposes a time series into unobserved components with SV.
- An efficient simulated likelihood estimation procedure is developed.
- Besides good forecasting performance, this model provides a natural way for de-trending and de-seasonalisation.