The Role of Time Preferences in Educational Decision Making

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February 2015

– Preliminary –

Abstract

We analyze the relevance of time-inconsistent preferences in educational decision making and corresponding policies within a structural dynamic choice model. Using a novel identification approach based on exclusion restrictions, we exploit the variation in average years invested in degree attainment through various educational reforms to identify the discount factor of a decision maker with hyperbolic time preferences. We make two important research contributions. First, we estimate our model using data from the German Socioeconomic Panel (SOEP) and provide quantitative evidence for time-inconsistent behavior in educational decision making. Second, we evaluate the relevance of time-inconsistent behavior for the effectiveness of education policies. For this purpose, we simulate policies where time preferences may play an important role: (1) an increase in the state grant for students (BaföG) as a way to affect short-run costs while at school and (2) an increase in the state grant as a loan which will have to be payed back five years after the end of the education.

JEL:D91, I21, I28, D03

Keywords: time preferences, hyperbolic discounting, structural estimation, dynamic discrete choice, education

1 Introduction

The empirical literature typically reports annual returns to schooling between 6-16%, depending on the instrument (Card, 1999).[1] Despite these high positive returns we observe wide heterogeneity in educational decisions that is hard to explain by conventional models of economic decision making. However, a good understanding of the underlying mechanisms of educational choices is crucial to the development of successful educational policies. In this study, we contribute to this gap in our understanding by investigating the role of time discounting within a structural dynamic model of educational decision making. While previous studies on this topic investigated the role of uncertainty and constant information updates during the time spent in education citepEckstein1999,Heckman2005, Belzil2007, we extend this literature by deviating from the assumption of exponential discounting and allow for time-inconsistent preferences through hyperbolic discounting. In the spirit of Magnac and Thesmar (2002), Fang and Wang (2015) and Chan (2013), we use a novel identification method to not only identify an exponential discount factor, but also an additional parameter that captures hyperbolic discounting.

The analysis of potential deviations from standard assumptions in economic models that describe the *homo oeconomicus* have received increasing interest in the empirical literature. One formalization of time-inconsistent preferences is hyperbolic discounting (Laibson, 1997; O'Donoghue and Rabin, 1999a; O'Donoghue and Rabin, 1999). While the hyperbolic discounter discounts exponentially between any two subsequent payoffs made in a distant future, she might change her preferences and put a higher weight on the more immediate payoff once she approximates the point in time when the first payoff is made. Behavioral responses to policy measures hinge on intertemporal preferences because individuals trade off short-term costs against potential future returns on the educational investments. Hence, the way people discount is likely to have an important impact on the effectiveness of policies that aim at easing the short-term cost of education or policies that aim at increasing the long-term benefits of education.

In this paper, we make two important research contributions. First, we use the German Socioeconomic Panel (SOEP), to estimate a dynamic structural life-cycle model of educational choices that allows for hyperbolic discounting. Following Magnac and Thesmar (2002), Fang and Wang (2015) and Chan (2013), we achieve identification by imposing exclusion restrictions that affect educational choices indirectly through their impact on the transitions probabilities of relevant state variables but have no impact on current utility flows. These restrictions reveal information on the discount-

[1] See Heckman et al. (2005) for a review on IV-approaches to estimating the returns to schooling.

ing behavior of the observed individuals. We use birth cohorts that were affected by different educational policy reforms as instruments and show their relevance on the time invested for the attainment of educational degrees. Furthermore, agents are assumed to face two kinds of uncertainty: (1) there is uncertainty over whether an additional invested schooling year will, in fact, be successful and lead to a degree; and (2) there is uncertainty over the returns to the degree earned when exiting education. The estimation of the structural parameters of our choice model indicates time-inconsistent behavior and provides quantitative evidence to its relevance.
Our second contribution to the literature refers to an evaluation of the relevance of time-inconsistent behavior for the effectiveness of education policies. For this purpose, we simulate policies where time preferences may play an important role: (1.) an increase in the state grant for students (BaföG) as a way to affect short-run costs while at school and (2.) an increase in the state grant as a loan which will have to be payed back five years after the end of the education. The interesting policy question is, whether, when allowing for present-biased preferences, educational decisions differ depending on whether financial support during the educational time is a grant or a loan. Given that loans will only have to be payed back long after the decision has been made, this loss in consumption should play a smaller roll for the hyperbolic discounter as opposed to the exponential discounter.

To the best of our knowledge, there is no previous empirical study that investigates the role of time-inconsistent preferences in the context of educational policies. Yet, a number of experimental results in behavioral economics indicate hyperbolic behavior in intertemporal decision making (see Giné et al. (2010) for a review). There is a number of papers that implement quasi-hyperbolic discounting – an approximation to hyperbolic discounting in discrete time (Laibson, 1997)– in dynamic discrete choice models. Magnac and Thesmar (2002) show that basic dynamic structural models are underidentified if the discount factor is estimated together with the other structural parameters. A large amount of the literature deals with this identification problem and tries to estimate the discount factor in exponential as well as a hyperbolic settings. For instance, Fang and Silverman (2009) estimate the discount factor with a specific focus on identification in a dynamic choice model of labor supply and welfare take-up of single mothers. Their results suggest a significant present bias factor and a better fit to the data when allowing for hyperbolic discounting. Similarly, Paserman (2008) rejects the hypothesis of exponential discounting for low-wage workers when implementing and estimating hyperbolic discounting in a job search model. Further literature on the implementation and estimation of hyperbolic discounting include Laibson et al. (2007) who estimate short and long term discount rates in a struc-

tural buffer-stock consumption model, DellaVigna and Paserman (2005) who study the role of time preferences in job search and Gustman and Steinmeier (2012) in the context of retirement.
Using a structural estimation approach, this study follows the line started by Willis and Rose (1979) that control for the endogeneity problem in schooling choices by taking a structural approach to the data. There are two advantages of dynamic structural decisions models that we exploit in this study: (1) the possibility to investigate potential channels that drive the schooling decision and (2) the simulation and analysis of counterfactual policy changes.

We implement a life-cycle model in a dynamic discrete choice framework. In education economics, this approach has been pioneered by Keane and Wolpin (1997). The authors used their parameter estimates to simulate the effect of a college fee subsidy on educational decisions while risk attitudes or time preferences remain unidentified. Our study relies on the basics of the model formulated by Belzil and Hansen (2002). It is an optimal stopping model in which the agents make annual decisions of remaining in education or exiting to the labor market. Furthermore, Belzil and Hansen (1999) and Oosterbeek and van Ophem (2000) study the role of the discount factor in education in its relation to the socioeconomic background but without deviating from an exponential specification.
The rest of this paper is structured as follows. The model is developed in the following chapter while identification, solution and estimation are discussed in the subsequent chapters. We present the estimation results and model fit in Chapter 6. Chapter 7 presents the policy simulations. Chapter 8 concludes.

2 Model

In this chapter we introduce a dynamic discrete choice model with annual schooling decisions. The basic model setup is based on Belzil and Hansen (2002). Individuals have rational expectations and maximize their present discounted value of expected lifetime utility, making annual decisions between continuing to go to school and exiting. Exit from school is defined as an absorbing state, classifying this as an optimal stopping problem. We distinguish between actual years of schooling and successful years of schooling. That is, individuals face uncertainty over whether an additional year of schooling translates into successful schooling years. Within the model, every individual has left school after having spent at most 26 actual years in education or having achieved a university degree as the highest observed degree in the sample which corresponds to 18 successful schooling years.
We also control for individual specific unobserved schooling preferences and returns

to schooling.

In order to focus only on the decision individuals face during the schooling period, we abstain from the explicit implementation of any labour market processes into the model. Individuals' beliefs on income after school depict a distribution over potential lifetime income profiles with the level depending on the degree obtained. After exiting school, the individual will receive a draw from the distribution of potential profiles which will be their lifetime income profile. Thus, income evolves deterministically after exiting school.

At the beginning of each period t, the individual's information set consists of her age, cohort, the number of years past the last degree earned, the number of degrees earned as well as the collected successful schooling years up to period t. The uncertainty the decision maker faces is twofold. On the one hand, she faces uncertainty over if and how the next period spent in school will be successful. On the other hand, she faces uncertainty over her returns to schooling when she decides to exit education due to the heterogeneity at this point.

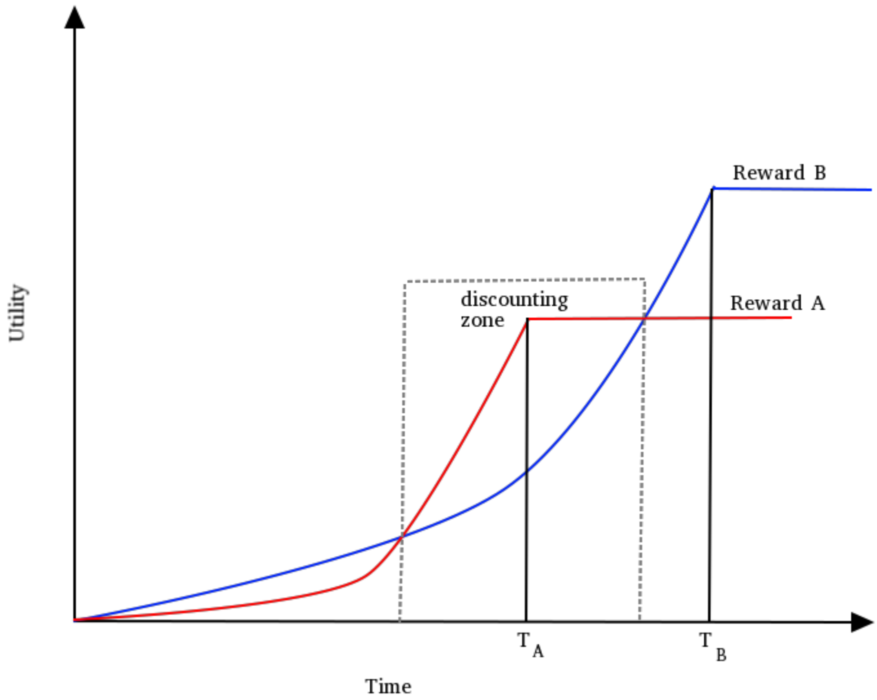
2.1 Objective Function

We specify a dynamic discrete model of individual schooling decisions, where individuals maximize their present discounted utility streams over the life-cycle. Individuals are indexed by n and discrete time,the agent's age, is indexed by t. The utility flow at age t, $(U(S^U_{n,t}, d_{n,t})$ depends on a vector of state variables that affect the flow utilities $S^U_{n,t}$ and the agent's choice $d_{n,t}$. Decisions can only be made until the individual exits schooling or reaches the maximum schooling age (age 33). The following objective function differs from the standard objective function in dynamic programming models to the extend that it allows for hyperbolic discounting.

$$EU_t = U(S^U_{n,t}, d_{n,t})\beta E_t[\sum_{j=t+1}^{T} \delta^{j-t} U(S^U_{n,j}, d_{n,j})] \quad (1)$$

We distinguish between a short-term discount factor β, also known as the *present-bias factor* and a long-term discount factor δ (O'Donoghue and Rabin, 1999b). Between the current and the next period the individual discounts with $\beta\delta$ while she discounts with δ between any two future adjacent periods. This is known in the literature as *β-δ-preferences* (Laibson, 1997). The standard model with exponential or time-consistent discounting is a special case of *β-δ-preferences* with $\beta = 1$ while the individual exhibits hyperbolic discounting if $\beta \in (0, 1)$. That is, the individual weights this period's payment more heavily than next period's payment but does not distinguish as heavily between any two payments in adjacent periods in the future.

Figure 1: Illustration of Hyperbolic Discounting



red: utility from reward A, blue: utility from reward B

Figure 1 illustrates the way a hyperbolic discounter evaluates rewards over time. We consider two rewards in the future, a sooner low reward A and a later high reward B. At an early point in time, the time delay between both rewards does not affect the utility derived from both rewards such that $B \succ A$. However, at a critical proximity to the reward A the discounter reverses the preference relationship. Since preferences for the same rewards change over time, hyperbolic discounting is a form of time-inconsistent preferences.

2.2 Utility Function

Utility depends on consumption and individual-specific preferences for schooling. In terms of model fit, the different peak years at which large shares of individuals leave school impose a problem. An easy way around this would be to include different additional intercepts for different school years. The drawback of this method is that the coefficients of these variables yield no economic theory in explaining the process behind the schooling decisions. Instead of using one dummy variable for each peak year, we only use one for the years nine to twelve with twelve years being the obligatory minimum in some German states under certain age restrictions. The utility function exhibits constant relative risk aversion (CRRA) and fulfills additional separability across time. It takes up the following form:

$$
\begin{aligned}
U_{n,t} =& (\phi_1 + \theta_{1g}\mathbb{1}[d_{n,t} = 1])\frac{C(S^U_{n,t}, d_{n,t})^{1-\rho} - 1}{1-\rho} \\
&+ \phi_2 \mathbb{1}[sc^y_{n,t} \leq 12] + \epsilon_{n,t}(d_{n,t})
\end{aligned} \tag{2}
$$

where $\epsilon(d_{n,t})$ follows a type 1 extreme value distribution. $C(S^U_{n,t}, d_{n,t})$ is the level of consumption which depends on the vector of state variables S_{nt} and whether the individual is at school ($d_{n,t} = 1$) or not ($d_{n,t} = 0$). θ_{1g} represents an unobserved individual-specific preference for schooling. ϕ_1 is a scaling parameter for consumption preferences. The distribution of this term is explained further down. ϕ_2 is a dummy variable that shifts the level of schooling utility for the schooling years ≤ 12. the coefficient of relative risk aversion is represented by ρ, where higher levels of ρ correspond to higher levels of risk aversion. The decision maker is risk-loving if $\rho < 0$

2.2.1 Consumption at School

Consumption at school is a function of household income, adjusted according to the OECD modified equivalence scale and basic rules of the German state grant system for students (BaföG). The idea of BaföG is that each student should have enough net income to cover her basic costs of living, where the level of these costs is set by the national government to a specific monthly amount that has been increased infrequently since its implementation. Thus, the share of the net household income including transfers ($HHinc_{n,t}$) according to the OECD scale is adjusted by the number of siblings and whether or not the individual lives with her parents ($X_{n,t}$). If this amount falls below the minimum cost of living, we set the income at school to the maximum BaföG amount ($B_{n,t}$) in the respective year.

$$
C(d_{n,t} = 1) = \begin{cases} f(HHinc_{n,t}, X_{n,t}) & \text{if } f() \geq B_{n,t} \\ B_{n,t} & \text{if } f() < B_{n,t} \end{cases} \tag{3}
$$

2.2.2 Consumption after School

The definition of *income* in this model deviates from the standard interpretation of labour market income as in the human capital investment model. Instead of only considering earnings as returns to human capital, individual income is defined as a share of actual household income adjusted by the OECD modified equivalence scale [2]. This definition of income has the advantage that individuals' expectations on future levels of income are not exclusively formulated as the labour market returns to human capital investment but also comprise the sociological effect of their educational decision. For instance, studies have shown that individuals are more likely to match with a partner with the same educational level (Mare, 1991). That is, even in the case of a highly educated person being unemployed, the higher education could still generate a higher income through the increased probability of being in a relationship with an equally educated person that earns a higher income for both.

We never observe the full lifetime income of each individual in the sample on which this analysis is based. Therefore, prior to estimating the parameters of an income equation in the structural model, we impute missing income observations based on a linear random effects regression, using a less restricted sample with also older cohorts. The details of this imputation will be discussed further below but note that we control for the path dependency of lifetime income through a lagged dependent variable in the imputation regression.

Individuals' beliefs on income after school form potential lifetime income profiles with heterogeneous returns to schooling across individuals. After exit from school, individuals receive a draw from the distribution of the returns to schooling which will determine the level of their income profile. From this point onwards, income evolves deterministically. We specifically deviate from the standard human capital investment model by using a share of household income rather than labour market earnings as an outcome that is affected by education. However, just like labour earnings, the age returns are decreasing over time. The income equation is then defined as

$$\log(inc_{n,t}) = \theta_{2g} sc_n^d + \alpha_1 + \alpha_2 age_{n,t} + \alpha_3 age_{n,t}^2 + \eta_{n,t} \tag{4}$$

where η_n is a normally distributed uncorrelated random error. α is a constant and θ_{2g} is a random coefficient that represents the individual-specific returns to successful years of schooling(sc_n^d).

[2]OECD Equivalence Scales

2.3 Unobserved Heterogeneity

In order to control for the fact that schooling and labour market ability are correlated (**citation**), we allow for correlation between the unobserved schooling preferences (θ_{1g}) and the returns to schooling in the income equation (θ_{2g}). These parameters follow a Heckman and Singer (1984) type mass-point distribution with $g = 3$ mass points κ_g with $\sum_1^3 \kappa_g = 1$. Correlation is assured by allowing for only one mass point for each pair of θ.

2.4 State Transitions

The number of actual years of schooling increases by one for every decision $d_{n,t} = 1$. However, whether or not an additional year spent in education translates into a degree is subject to uncertainty. The probability to attain a degree that translates into additional successful years of schooling is described in an ordered logit model as follows:

$$\begin{aligned} deg_{n,t+1} =& \gamma_0 yslt_{n,t} + \gamma_1 numdeg_{n,t} + \gamma_2 cohort2_n + \gamma_3 cohort3_n \\ &+ \gamma_4 cohort4_n + \zeta_{n,t} \end{aligned} \tag{5}$$

Where $deg = \{0, 1, \cdots, 6\}$ indicates the number of successful years of schooling for the degree achieved. We summarize the variables that enter the transition equation (5) by S^Γ as the set of state variables for the transition probabilities. Every period, $sc^y_{n,t}$ increases by 1 and $sc^d_{n,t}$ increases by $\Gamma(S^\Gamma_{n,t}) \cdot deg$, where Γ is an array that describes the discrete probability distribution for $deg_{n,t}$ that results from equation (5). Given that $deg_{n,t}$ has $M = 7$ distinct values, we estimate $M - 1 = 6$ corresponding cut points: $\gamma_5 - \gamma_{10}$. $yslt_{n,t}$ is the number of years in education since the last degree was earned. This variable increases by one for every $d_{n,t} = 1$ but falls back to a value of one if $sc^d_{n,t}$ has increased in the previous period. Each degree earned translates into a certain number of years that will be added to the number of successful years of schooling. With every additional degree earned, the variable $numdeg_{n,t}$ increases by one. Furthermore, this probability model depends on cohort groups that are sorted as presented in table 1.

Every cohort group is subject to different educational policy regimes which are summarized in table 7 in the appendix. Therefore, we assume that different cohorts have different beliefs about the probabilities of attaining a degree after having spent a certain amount of time in education. To account for these differences, we included

Table 1: Distribution of cohort groups

cohort group	Perc %
1955-1964	6.64
1965-1974	38.02
1975-1984	44.51
1985-1992	10.82
Total	100.00

cohort group indicators in this function. These dummies are also central to the identification of the discount factor.

From this model we construct an array Γ that indicates the probability of increasing the number of successful years of schooling by $deg_{n,t+1}$ conditioned on the covariates in the model. The parameters of equation (5) are summarized in the vector γ and we compute this probability in an ordered logit model as follows:

$$Pr(deg_{n,t+1} > deg_j) = \frac{exp(X\gamma - m_j)}{1 + exp(X\gamma - m_j)}, j = 1, 2, \cdots, M-1, \tag{6}$$

such that

$$\begin{aligned} Pr(deg_{n,t+1} > deg_1) =& 1 - \frac{exp(X\gamma - m_1)}{1 + exp(X\gamma - m_1)} \\ Pr(deg_{n,t+1} = deg_j) =& \frac{exp(X\gamma - m_{j-1})}{1 + exp(X\gamma - m_{j-1})} \\ & - \frac{exp(X\gamma - m_j)}{1 + exp(X\gamma - m_j)}, j = 2, \cdots, M-1 \\ Pr(deg_{n,t+1} = deg_M) =& \frac{exp(X\gamma - m_{M-1})}{1 + exp(X\gamma - m_{M-1})} \end{aligned} \tag{7}$$

When constructing Γ, it should also be considered that once the individual has reached 13 successful years of schooling, the probability to increase this variable by six years is zero since the highest achievable level is 18. We adjust the array accordingly, such that the remaining probabilities over the potential increases in the degree still sum up to one.

2.5 Value Functions: Hyperbolic Discounting

This chapter deals with the construction of the expected value functions of this model under hyperbolic discounting. For readability we suppress the transition array in this chapter and stick to a uniform matrix of state variables S. We will return to the implementation of the transition array when discussing the solution of the model. We assume that state transitions follow a Markov process which, by Bellman's principle of optimality, allows us to break down the value function to a two-period decision problem between current and the present discounted value of future utility. In the basic exponential case the value function has the form:

$$V_t(S_{n,t}, d_{n,t}) = U(S_{n,t}, d_{n,t}) + \delta E_t[max\{V^1_{t+1}, V^0_{t+1}\}] \tag{8}$$

where

$$\begin{aligned} V^1_{t+1} &= V(S_{n,t+1}, d_{n,t+1} | S_{n,t} = s_{n,t}, d_{n,t} = 1) \\ V^0_{t+1} &= V(S_{n,t+1}, d_{n,t+1} | S_{n,t} = s^-_{n,t}, d_{n,t} = 0) \end{aligned} \tag{9}$$

Following the terminology in Fang and Wang (2015), we call equation (8) the *perceived long-run value function*. Since the discount factor is constant over time in the exponential discounting case, there is no difference between how the decision maker perceives her discounting behavior in the future and how she discounts in the present, such that expected payoffs over the short and long run are discounted equally, leading to one value function covering all aspects of forward looking behavior.

In the case of hyperbolic discounting, we distinguish between two aspects of forward looking behavior: *1*) The individual's *actual* discounting behavior for the short and long run. *2*) The individual's *perceived* discounting behavior for the short and long run.

Therefore, we introduce a second value function that describes the discounting behavior of the hyperbolic discounter for the short run:

$$W_t = U(S_{n,t}, d_{n,t}) + \beta\delta E_t[max\{V^S_{t+1}, V^W_{t+1}\}] \tag{10}$$

We call this function the *current value function*. If $0 < \beta < 1$, all non-immediate periods are discounted heavier than in the exponential discounting case which is why this type of discounting is also referred to as *present bias*. Note that if $\beta = 1$ the decision maker exhibits the basic case of exponential discounting. Thus, exponential discounting is embedded in the model as a corner solution.

For a better understanding of hyperbolic discounting, it helps to look at the decision maker in each period as a different period's self, i.e. the decision maker in this period is called current period self, while the decision maker in the next period is called next period self.

Note that, with hyperbolic discounting the decision problem expands from a two-period to a three-period problem since we now also consider how the current period self perceives the next period self to discount subsequent utility flows. In addition to this, except for the corner case $\beta = 1$ the current period self perceives its next period self to discount differently.

The literature on hyperbolic time preferences distinguishes between *naïve* and *sophisticated* decision makers that differ in the perception of discounting behavior for future periods (Strotz, 1955; Pollak, 1968; **?**; O'Donoghue and Rabin, 1999b). The sophisticated discounter is aware of the fact that she will have present-biased preferences also in future periods while the naïve discounter remains ignorant of this fact. That is, while both discount in the current period according to equation (10), the sophisticated discounter perceives her next period selves to discount according to (10), while the naïve hyperbolic discounter perceives her next period selves to discount according to equation (8).[3]

The focus of this paper lies on the extensive margin effect of generally allowing for another form of discounting. An evaluation of the intensive margin is subject to future research. Without any commitment mechanism, the sophisticated type would discount heavier, than the naïve type, making the former a stronger deviation from the standard model. We focus on the naïve hyperbolic discounter as a lower bound of the effect of time-inconsistent preferences.

3 Identification of Time Preferences

Rust (1994) shows that the discount factor in dynamic discrete choice models with infinite horizon is generally not identified. While implementing further restrictions such as an absorbing state and a finite horizon helps with the identification of parameters in the utility flows, Magnac and Thesmar (2002) show that these assumptions still not lead to the identification of the discount factor. However, they further show that the non-identification issue can be resolved with certain exclusion restrictions. In addition to this, Fang and Wang (2015) show that additional restrictions with the same requirements can be used to identify the additional time-preference parameters that come with a model of hyperbolic discounting. Formally, the exclusion restriction has to fulfill the following conditions:

There exist state variables $x_1 \in S^\Gamma$ and $x_2 \in S^\Gamma$ with $x_1 \neq x_2$ such that:

(i) for all $j \in J, U_j(x_1) = U_j(x_2)$

[3] There is also an intermediate case where a *partially naïve discounter* perceives future period selves to have a short-run discount factor of $\tilde{\beta}$, where $\beta < \tilde{\beta} < 1$.

(ii) for some $j \in J, \Gamma(S^{\Gamma'}|x_1, j) \neq \Gamma(S^{\Gamma'}|x_2, j)$

where $j \in J$ represents the choice made. That is, in order to fulfill this exclusion restriction, there exists one variable that does not affect the current utility flows for any $j \in J$ but may still affect the choices made through its effect on the transition probabilities of state variables.

The differences in the individuals' choices at different values of the exclusion restrictions will give us information about the discount factor. For each additional parameter that describes discounting in the model, we will need at least one additional exclusion restriction (Fang and Wang, 2015). Furthermore, we rely with our identification on a setting with at least three consecutive decision periods. The idea behind this is, as described in chapter 2.5, that under hyperbolic discounting the value function in a dynamic choice model expands from a two- to a three-period problem. An exponential discounter discounts payoffs one period from now by δ and two periods from now by δ^2 while the hyperbolic discounters discounts payoffs one period from now by $\beta\delta$ and two periods from now by $\beta\delta^2$. This means, while we are able to identify a value for the discount factor in a two-period setting, we cannot distinguish hyperbolic discounting from exponential discounting with a low discount factor. The behavioral difference between these two types of discounting is only exhibited in the discounting of two consecutive future periods which turn out to be time-inconsistent in the case of hyperbolic discounting but time-consistent in the exponential case.

In choosing the right exclusion restriction for our model, the fact that we account for schooling preferences in our utility function imposes a problem similar to that of the infamous ability bias: We need a variable that is uncorrelated with schooling preferences but causes variation in the accumulated successful schooling years. This precludes variables that describe the socioeconomic background of the decision maker since these affect both unobserved schooling preferences and the probability of acquiring schooling degrees. We argue that the cohort group indicators fulfill this requirement because each cohort group was subject to different educational policy regimes that had an effect on the time spent in education. Due to the staggered implementation of these reforms, we can also ensure that two cohort groups affected by one reform differ in the intensity of reform treatments. Furthermore, since the unobserved schooling preferences are individual specific but not cohort specific, the cohort does not affect the current utility function through the schooling preference parameter. Thus, there is no reason to assume a relation between the distribution of the unobserved heterogenous preference for school and the cohort.

4 Solution and Estimation

So far, we presented the three major parts of the model: The utility function, the income equation and the transition equation. Prior to estimating the parameters of the utility function, it is necessary to solve the dynamic decision problem. The solution of this problem with hyperbolic discounting does not differ substantially. First, given the finite horizon of the decision problem, the solution can be computed by backwards induction starting from the utility flow in the last decision period. Secondly, exit from schooling is an absorbing state after which each period's utility flow does not change. This yields the expected choice specific exit value of:

$$E_t[W^0_{n,t}] = \bar{W}^0_{n,t} = \beta \sum_{j=t+1}^{T} \delta^{j-(t+1)} U^0_{n,j}(S^U_{n,t} = s^{\bar{U}}_{n,t}, d_{n,t} = 0) \tag{11}$$

Furthermore, given that $\epsilon(d_{n,t})$ follows a type one extreme value distribution, we can derive a closed form solution for the expected maximum of future choice specific value functions (Rust, 1987):

$$\begin{aligned} E_t[W^1_{n,t}] = \bar{W}^1_{nt} =& U(S^U_{n,t}, d_{n,t}) + \Gamma(S^\Gamma_{n,t})\beta\delta \\ &\cdot \log\{\exp(E_t[V^1_{t+1}]) + \exp(E_t[V^0_{t+1}])\} \end{aligned} \tag{12}$$

where,

$$\begin{aligned} E_t[V^1_{n,t+1}] = \bar{V}^1_{n,t+1} =& U(s_{n,t+1}, d_{n,t+1}) + \Gamma(S^\Gamma_{n,t})\delta \\ &\cdot log\{\exp(E_{t+1}[V^1_{t+2}]) + \exp(E_{t+1}[V^0_{t+2}])\} \end{aligned} \tag{13}$$

Note that for the backward induction with naïve hyperbolic discounting, we compute the value functions recursively until $t+1$ using the perceived long-run value function and compute the last step from $t+1$ to t using the current value function.
Rust (1987) shows that with the assumption of additive separability in utility over time and conditional independence, the probability of continuing to go to school in period t has the following logit-type form:

$$Pr(d_{n,t} = 1) = \frac{exp(\bar{W}^1_{n,t})}{\exp(\bar{W}^0_{n,t}) + exp(\bar{W}^1_{n,t})} \tag{14}$$

Hence, the probability to remain in school for $\bar{t}$ years (after grade 9) for individual n is:

$$L_{1n} = \sum_{\theta} \kappa_g \{\prod_{t=1}^{\bar{t}} Pr(d_{n,t} = 1)) \cdot Pr(d_{n,\bar{t}+1} = 0)\} \tag{15}$$

where we take the sum weighted by κ_g over the discrete distribution of the individual heterogeneity in the returns to schooling and schooling preferences.
Finally we formulate the full log-likelihood of the decision problem by multiplying the likelihood in equation (15) with the income density computed from the income equation (4) (L_{2n}) and the likelihood for the transition probabilities presented in equation (5) (L_{3n}) :

$$\sum_{n=1}^{N} \{log(\sum_{\theta_n} \kappa_g \{L_{1n} \cdot L_{2n}\}) + log(L_{3n})\} \tag{16}$$

The parameters of this problem are estimated with the maximum likelihood method.

Generally, the solution of the model results in the estimation of the unknown parameter vector Ψ which contains the structural parameters of the utility function $(\rho, \phi_1, \phi_2, \beta, \delta)$, the income equation $(\alpha_1, \alpha_2, \alpha_3)$, the transition equation $(\gamma_1 - \gamma_{10})$, the standard deviations of the random errors $(\sigma_\eta, \sigma_\zeta)$, and the weights and values of the mass point distribution for the schooling preferences and returns to schooling $(\theta_{1g}, \theta_{2g}, \kappa_g)$. To avoid excessive computation time, we follow Rust (1987) and Rust and Phelan (1997) by estimating Ψ in a two-step procedure. First, we estimate $\gamma_1 - \gamma_{10}, \sigma_\zeta$ with the partial likelihood L_{3n} and then use these parameters to estimate the remaining parameters with the partial likelihood $\sum_{n=1}^{N} \{log(\sum_{\theta_n} \kappa_g \{L_{1n} \cdot L_{2n}\})\}$. The trade-off with this estimation procedure is a loss of efficiency for a reduction in computational time whereas the efficiency loss is suggested to be rather small according to Rust and Phelan (1997).

5 Data

This study is based on data from the German Socio-Economic Panel (SOEP), a survey which, since 1984, collects annual individual- and household-level information from

12,000 households on socio-economic and demographic aspects (Wagner et al., 2007). Due to the structural break in the German labour market and education system following the reunification, the income information is restricted to the years 1992-2014 and educational degrees obtained prior to the reunification only to west-german degrees. The sample is restricted to individuals whose educational transitions including vocational education is observed from the point of finishing primary school to their final educational degree. Furthermore, we exclude disabled individuals due to their special life-cycle income paths originating in particular social security programmes only for the disabled and people without any educational degree. In addition, we exclude individuals that remain in education for more than 26 years as outliers which constitute less than 2% of the sample. Since missing income information will be imputed, we further reduce the sample to individuals with a minimum of two observed periods of household income. The final sample is a balanced panel with 2680 individuals. Table 2 shows summary statistics of the variables used. Note that we distinguish between actual years of schooling and successful years of schooling.
We defined the following variables for our model:

years of schooling Years of schooling are defined as the last age the individual is observed in any educational institution minus seven, the age most individuals turn in the year of enrollment.

successful years of schooling Most studies based on the SOEP and equivalent household survey data sets use as a schooling variable the predefined variable "Number of Years of Schooling". However, this variable does not inform about the actual number of schooling years but is better described as an index assigned to the educational degrees achieved. We define this as successful years of schooling. We do not observe the assigned schooling variable but the degree earned at each respective educational transition. The method used to assign years of schooling to a degree follows closely (Couch, 1994).

Number of transitions in education This counts the number of times the variable *successful years of schooling* changes during the time spent in school.
lifetime income Individual income is computed by adjusting total net income (also covering government transfers) of all household members according to the OECD modified equivalence scale[4]. The estimation of our model parameters requires a measurement of total lifetime income which naturally is not observed for our sample cohorts. Thus, we create counterfactual lifetime incomes conditioned on individuals'

[4]OECD Equivalence Scales

Table 2: Summary Statistics

	Mean	Std.	Min	Max
age	25.00	4.90	17.00	33.00
female	0.52	0.50	0.00	1.00
birth cohort	1975.33	7.33	1955.00	1992.00
successful years of school	13.62	2.47	9.00	18.00
actual years of school	15.51	3.17	9.00	26.00
number of educ. transitions	2.05	0.37	1.00	4.00
net HH income(10,000 e)	2.98	2.13	0.79	56.71
number of siblings	1.65	1.45	0.00	11.00

Note: Authors' calculations based on GSOEP.

educational decisions and other socio-demographic characteristics using predictions from a linear regression model. This imputation process is based on a SOEP sample that includes observations of people up to age 70. Here we rely on the assumption that individuals form their expectations about their future lifetime incomes based on what individuals from earlier cohorts earned given the choices they made and the characteristics they had, so called *reference group expectations* (Manski, 1991). In order to capture path dependency in life-cycle incomes and age effects, we assume that annual earnings over the life-cycle follow an AR(1) process. Furthermore, we include age as well as successful years of schooling as a second order polynomials. Besides, we set the minimum annual income of an individual to the average social security minimum of the respective year in order to avoid any bias due to unobserved transfers in the income variable. Figure 3 in the appendix shows the imputed life-cycle earnings paths divided by education levels. The hump-shaped form as well as the level differences by years of education as known in the literature suggest useful imputation results. Moreover, figure 4 shows box-plots of average lifetime incomes, i.e. the average annual income over an individual's life-cycle. The figure indicates the well known positive relationship between lifetime income and education, a crucial aspect to the discrete choice model developed in this paper.

Cohort groups The cohorts range from 1965 to 1992. We define four cohort groups as follows: Before 1965, 1965 to 1974, 1975 to 1984, after 1984. Each cohort group was subject to a slightly different educational regime, which is important for the identification of the discount factors.

6 Results

6.1 Transition Probabilities

We start by discussing the results of the transition equation. All variables that enter this model are highly significant. Particularly, the coefficients of the cohort group

Table 3: Ordered Logit Model of Transition Equation

	coefficient	**se**
yslt	0.2294***	0.0060
degree	1.4665***	0.0141
$cohort_2$	0.3550***	0.0047
$cohort_3$	0.3605***	0.0096
$cohort_4$	0.5831***	0.0379
cut_1	5.0655***	0.0201
cut_2	5.3163***	0.0093
cut_3	6.4954***	0.0188
cut_4	7.0185***	0.0281
cut_5	8.1182***	0.0407
cut_6	8.6229***	0.0467
$N1$	2680	
ll	-16988.8749	

Note: Standard errors in parentheses. *,** and *** denote significance level of 10%, 5% and 1%, respectively.

indicators are significant which suggests a high relevance for the exclusion restriction.

6.2 Flow Utility and Income

Table 4: Schooling and Utility Parameters

	Exponential		Hyperbolic	
	parameter	**se**	**parameter**	**se**
β	1	-	0.3758	-
δ	0.96	-	0.96	-
ρ	1.125***2	0.0419	0.9063	0.0135
α	0.9995***	0.0442	1.1923	0.0580
ϕ	0.7319***	0.0649	1.3852	0.0981
$\mu_{\theta_{1n}}$	-2.1885***	0.1731	-3.7723	0.2030

Please note that the model with identified time preferences is currently being estimated. In order to illustrate what can be expected, we show results from an earlier version of this model in which we estimated two versions of the model, one with an exponential, one with a hyperbolic discounting and compared the results. In this early version, $\delta = 0.96$ and β is identified with an alternative identification strategy. We used SOEP survey responses to a question in which respondents were asked to rate their patience in order to identify β. This earlier version also exhibits continuous instead of discrete heterogeneity in schooling preferences and returns to schooling.

All estimated parameters are significant.

Table 5: Income process

	Exponential		Hyperbolic	
	parameter	**se**	**parameter**	**se**
γ_{0n}	5.9060	0.0435	5.0167	0.0625
age	0.0783	0.0012	0.0808	0.0015
age^2	-0.0005	0.0000	-0.0006	0.0000
$\mu_{\theta_{2n}}$	0.1380	0.0028	0.1993	0.0040
σ_η	0.2426	0.0010	0.2379	0.0012
N	2,000		2,000	
ll	-9,891.6812		-6,860.8811	
$p - lratio$		0.0000		

The results for the wage equation do not differ significantly under the different specifications. This is not surprising given that the parameters in the wage equation are only identified in the flow utility after schooling exit and should not be affected by an intertemporal parameter such as the discount factor.

We also find heterogeneity in the returns to schooling.

6.3 Model Fit

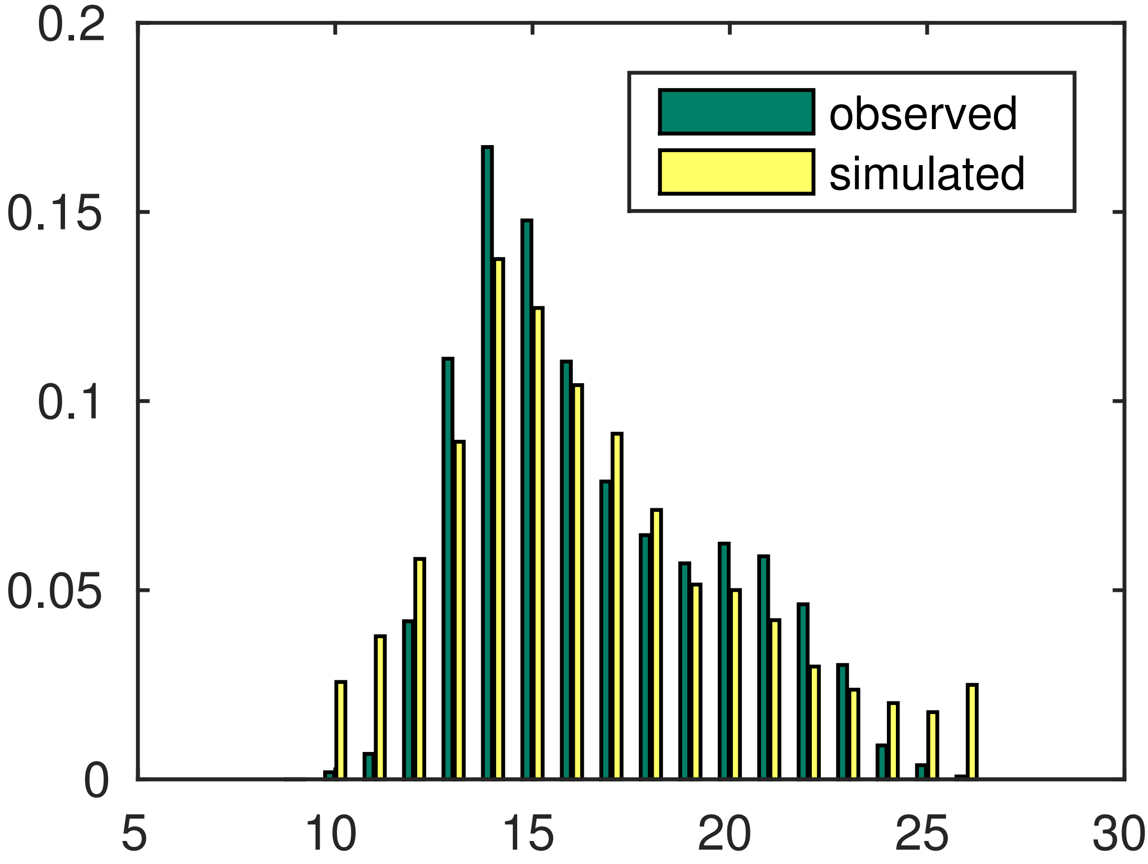
We test the fit of the models by comparing the estimated educational choice probabilities with choice probabilities from the sample, which we obtain by non-parametric

Table 6: Variance-covariance matrix of schooling preferences and returns to schooling

	Exponential		Hyperbolic	
	parameter	**se**	**parameter**	**se**
$chol_{11}$	0.0402	0.0005	0.0492	0.0009
$chol_{21}$	-0.8642	0.0774	-1.6537	0.0834
$chol_{22}$	0.0018	0.0252	0.0082	0.0309

estimation. Note that figure 2 depicts a comparison of sample probabilities with probabilities based on simulated samples using the estimated model parameters. The plot does not differ from the plot with predicted probabilities and the simulated samples will be used as departure points for the policy analysis in the following chapter.

Figure 2: Educational Decisions: Observed and Simulated



Choice probabilities are non-parametrically estimated based on simulated samples.

The model fits the the overall shape of the distribution in the sample reasonably well. We overpredict the exit probabilities at the tails of the distribution and slightly underpredict them in the middle. Note that, we refrain from trying to improve the in-sample fit artificially by including several indicator variable for almost every value of the decision variable since these variables do not provide any additional information for economic inference. The only dummy variable that we included is $\phi_2 \mathbb{1}[sc^y_{n,t} \leq 12]$ which is motivated by the institutional background. The focus of this study is on comparing the effects from policy analysis. Given that these are based on simulations using the corresponding parameter estimates for each model respectively, this imperfect sample fit should not affect the following results.

7 Policy Analysis

The discount factor affects how individuals evaluate utility streams intertemporally. That is, changing the discount factor not only affects utility in one period but triggers dynamic effects over time. Therefore, the policies we look at specifically affect individual utility streams at different points in time over their decision horizon. Particularly, we look at the effects of an increase in the state-supplied student loans (BaföG) which affects individual utility in the short-run. This is simulated through an increase of the lower bound $B_{n,t}$ in equation (3).

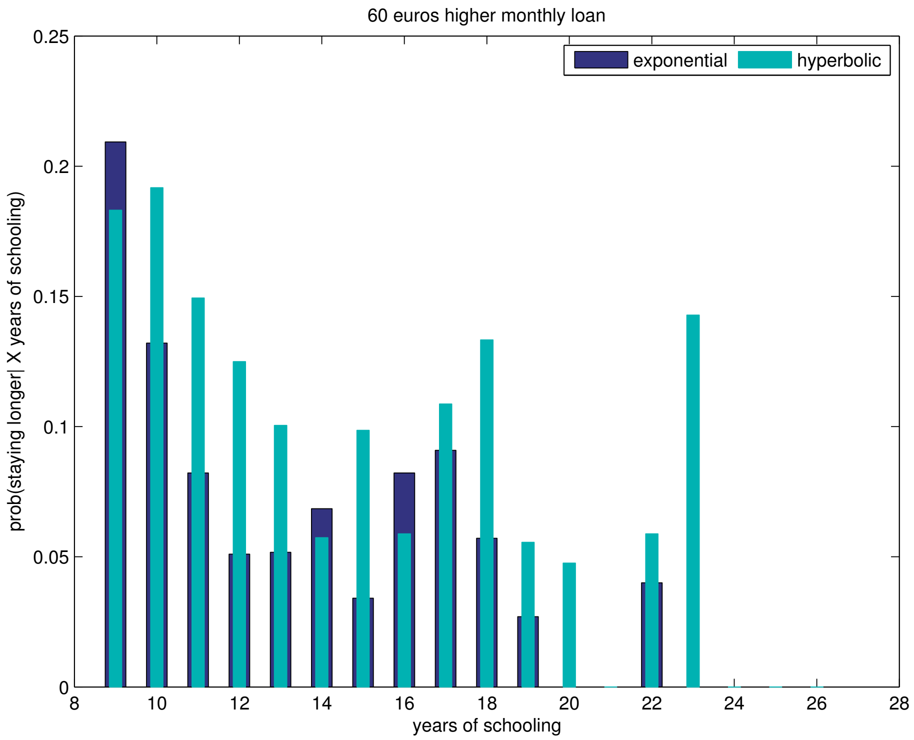
Before we introduce any policy changes we simulate one sample with $N = 2680$ observations using the parameters estimated in our model. For the simulation of income at school we fit a normal distribution over the sample distribution truncated at the minimum income in 2010 (585 €monthly)[5] that students from low income households would receive as a state grant. Lifetime earnings are predicted using the wage equations and educational decisions are simulated in the optimal stopping, utility maximization framework: individuals choose in every period the option that maximizes their respective objective function as presented in equation (1). For every additional year invested in education, we adjust the variables in the transition equation accordingly and draw a value for the new successful schooling variable from the discrete distribution with respective transition probabilities. The algorithm stops once the individual decides to exit education. The standard exponential benchmark of our results are simulations generated from the same parameters with β set to one since the corner solution $\beta = 1$

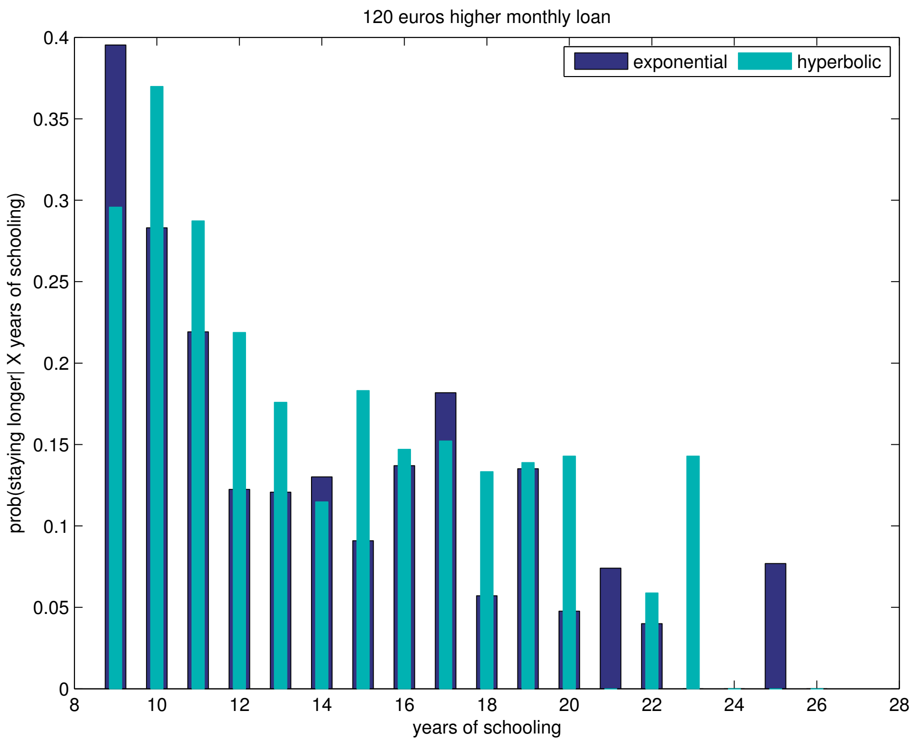
7.1 Student Grant

In the first scenario, the increase will be a grant, i.e. the short-run gain does not correspond to a long-run gain. Naturally, we can expect significant positive behavioural responses to this policy independent of the specification of the discount factor. We simulate an increase in student loans by gradually increasing the minimum threshold of income at school ($B_{n,t}$) by 40€ per month and reiterating the respective model with the new parameter specification. Note that an increase in the minimum state grant should not lead to parents (or other care providers) adjusting their payments to the new state grant rendering the policy change ineffective. State grant adjustments in Germany occur based on recalculations of living expenses rather than changes in the parental share of the provision. Therefore, an increase in the provided state grant

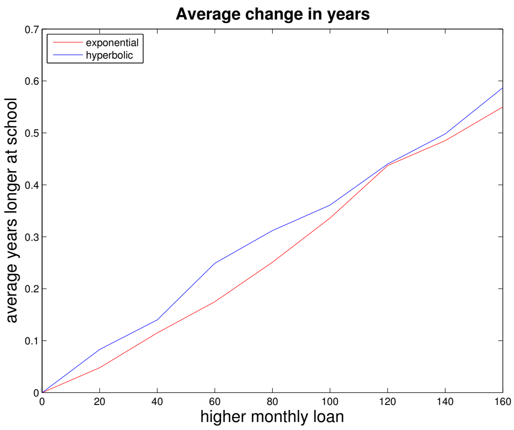
[5]All income variables are adjusted by the consumer price index in 2010. Since the simulated samples could not be adjusted by observed years we use the value from 2010 with the implicit assumption that the purchasing power parity of the state grant did not change in the observed time interval.

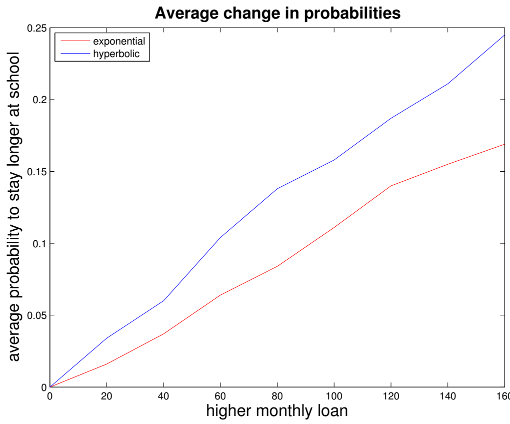
should feed directly into the minimum income level for students.





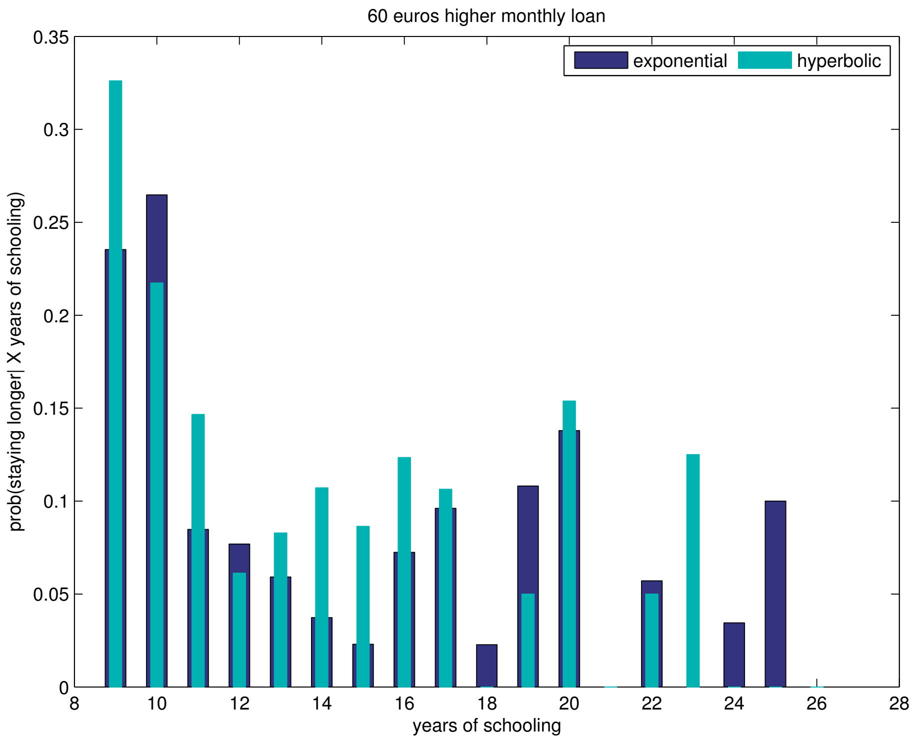
The hyperbolic discounter reacts stronger, than the exponential discounter. While the exponential discounter reacts more in lower levels of schooling and also in higher levels of schooling, when the grant is high, the hyperbolic discounter shows a stronger overall effect.



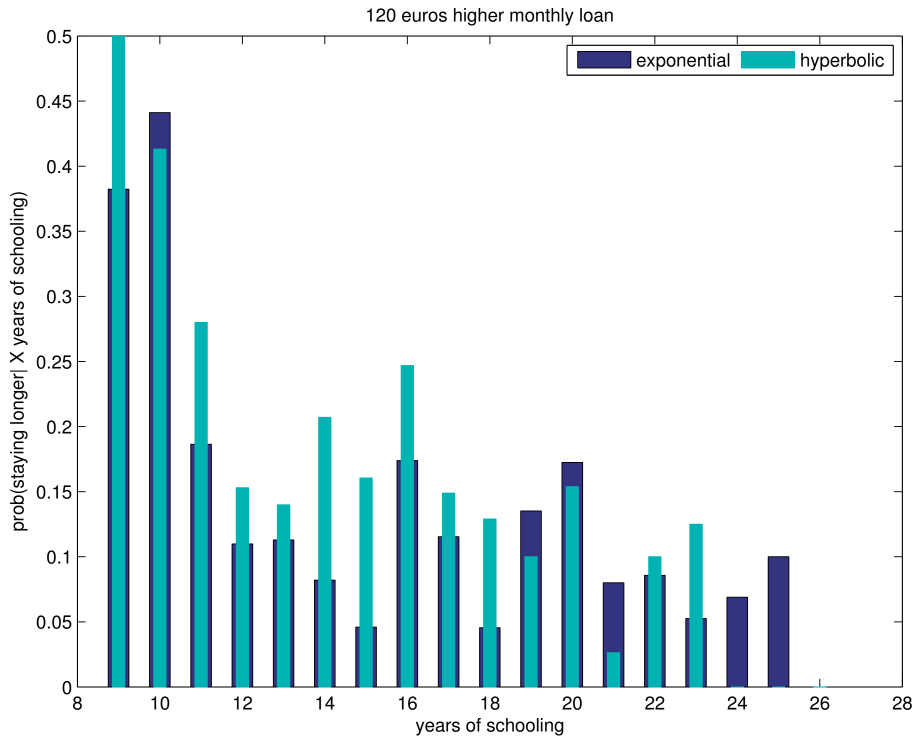


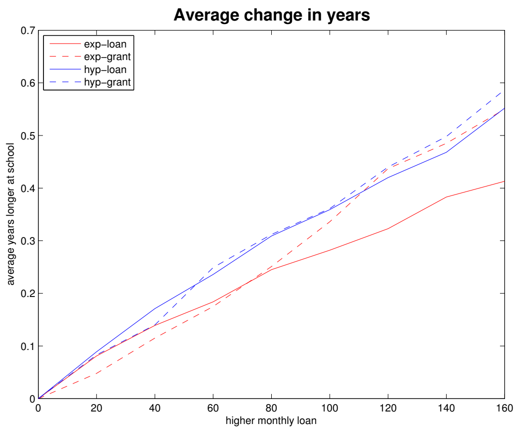
Compared to exponential discounters, there is a larger share of hyperbolic discounters that react to this policy. However, it seems like the response in *"more years of schooling"* is higher for an exponential discounter that reacts as opposed to a hyperbolic discounter. The extensive margin effect here is stronger for the hyperbolic discounter but the intensive margin effect is stronger for the exponential discounter.

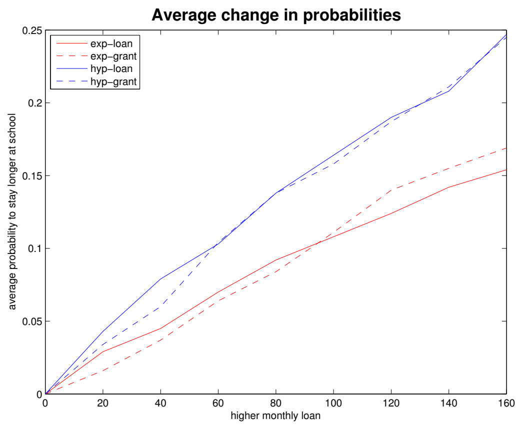
7.2 Student Loan



Again, the exponential discounters reacts more in the higher levels of education, while hyperbolic discounters exhibit a larger overall response.
When changing the policy simulation from a grant to a loan, we can see that the exponential discounter seems more likely to adjust her behavior downwards at the







extensive, but more so at the intensive margin. The hyperbolic discounter does not significantly adjust her positive behaviour downwards when switching from a grant subsidiy to a loan subsidy. This is due to the fact, that the hyperbolic discounter puts more weight on the presence and discounts the future more heavily.

Although these differences are arguably very small, they only show a lower bound of the actual effect since only allowing for a variation in the discount factor in the hyperbolic case without adjustment of schooling preferences and the case of sophisticated hyperbolic discounting should yield stronger differences.

8 Conclusion

In this paper we formulate a dynamic structural life-cycle model of educational decisions in order to investigate the role of time-preferences in policy analysis. Using a novel identification approach based on exclusion restrictions, we exploit the variation in average years invested in degree attainment through various educational reforms to identify the discount factor of a decision maker with hyperbolic time preferences. We compare the results from subsequent policy analyses to exponential discounting, the benchmark from neoclassical theory. Furthermore, we allow for unobserved heterogeneity in the returns to and the preferences for schooling. Furthermore, agents are assumed to face two kinds of uncertainty: (1) there is uncertainty over whether an additional invested schooling year will, in fact, be successful and lead to a degree; and (2) there is uncertainty over the returns to the degree earned when exiting education. The estimation of the structural parameters of our choice model indicates time-inconsistent behavior and provides quantitative evidence to its relevance.
Due to the effect of the discount factor on intertemporal evaluations of utility streams we compare the results from policies that affect the utility streams at separably different points over the decision horizon: (1) an increase in the state grant for students (BaföG) as a way to affect short-run costs while at school and (2) an increase in the state grant as a loan which will have to be payed back five years after the end of the education.
The differences suggest, that for policy-simulations that affect short-term payoffs of education, exponential may be too restrictive and should be tested against a model that allows for time-inconsistencies.

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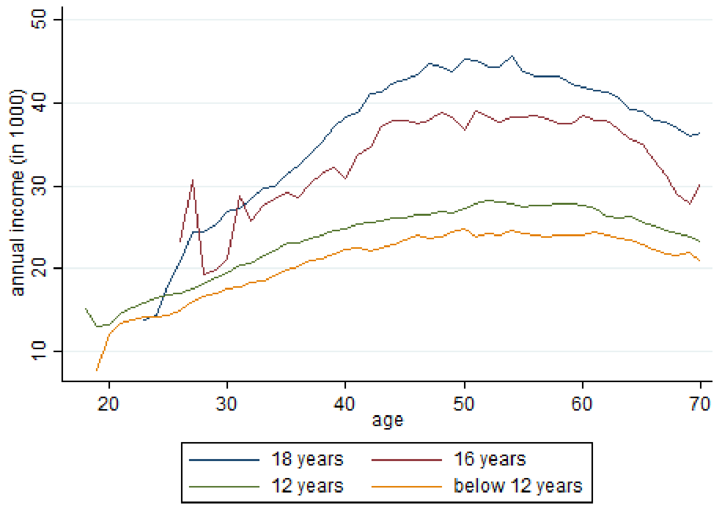
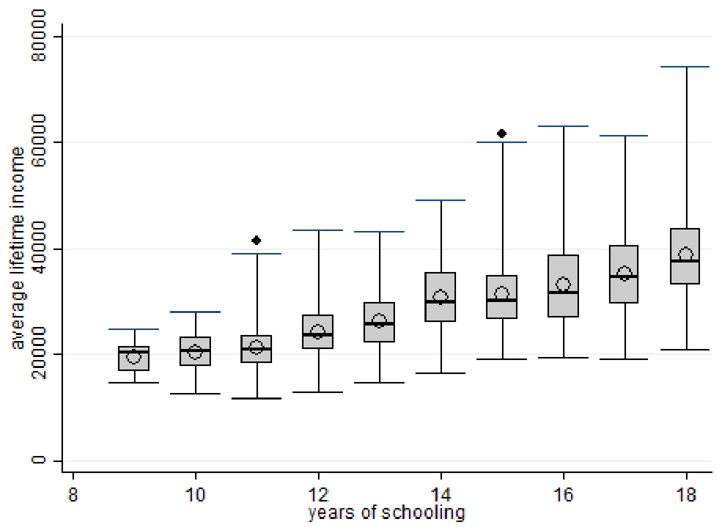
A Imputation Results for Lifetime Income

Figure 3: Life-cycle income profiles by education



Profiles are divided by counted years of schooling

Figure 4: Average lifetime income by education



Boxplots are divided by counted years of schooling

B Reforms

Table 7: Summary of Educational Reforms

reform introduction	reform content	affected cohort groups
1968	Implementation of advanced technical colleges	1,2
1972	Reorganization of the upper secondary level	1,2
1973	Implementation of integrated comprehensive schools parallel to the three-tier school system	1,2
1997	Implementation of nation-wide educational standards	3,4
1999	Signing of Bologna Declaration marks transition from former 5-year university programmes to 3+2 years	3,4
2005	Implementation of Bologna Declaration structures	3,4
2007	Shortening of upper secondary level education by one year	4