Hours and Employment Over the Business Cycle *

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Abstract

We estimate with likelihood methods and U.S. data a general equilibrium model that includes labor-market search frictions and a standard neoclassical hours-supply decision to study the cyclical behavior of hours and employment. We first show that this benchmark model cannot account for the unconditional cyclicality of the margins of labor adjustment, and their empirical covariance with macroeconomic time series. Two additional features reconcile the model with the data: non-separable preferences that exhibit a weak short-run wealth effect on hours supply and hours adjustment costs. In addition, we show that the comovement of employment and hours per worker varies with aggregate shocks, reconciling the empirical observation that the covariance between the margins of labor adjustment can be either positive or negative during specific historical episodes. Model counterfactuals demonstrate that if hours per-worker could not adjust in response to aggregate shocks, the dynamics of employment and output would have been significantly affected during U.S. cyclical recoveries.

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1 Introduction

The study of labor market fluctuations plays a central role in business cycle research since at least Kydland and Prescott (1982).[1] A vast literature addresses the cyclical behavior of the labor market in the context of the Mortensen-Pissarides search and matching model (Mortensen and Pissarides, 1994, and Pissarides, 2000), arguably the standard theory of equilibrium unemployment today.[2] Despite this interest, the majority of works in this literature abstract from labor supply considerations. Omitting the compositional adjustment of total hours worked—fluctuations in average hours per worker (the intensive margin) versus movements in and out of employment (the extensive margin)—is not without loss of generality. Changes in hours per worker are about as large as changes in employment in many OECD countries (Ohanian and Raffo, 2012). Even in the U.S., the volatility of the intensive margin accounts for approximately one-third of the variability of aggregate hours, showing that fluctuations in the intensive margin affect total hours.[3] Moreover, the relationship between hours per worker and employment displays significant variation over time; in specific business cycle episodes, the two margins covary either positively or negatively, and the relative importance of the intensive and extensive margins is time-varying. Determining the quantitative significance of these empirical observations requires a model that can account for both labor margins.

In this paper, we take up the challenge of accounting for and explaining the cyclical behavior of the margins of labor adjustment and their comovement with the rest of the economy. Motivated by the aforementioned evidence, we address two main issues. First, we determine under which conditions a benchmark search and matching model augmented with a standard neoclassical hours supply decision can account for the cyclicality of labor adjustment observed in U.S. data.[4] Second, we provide a structural assessment of the contribution of the intensive margin of labor adjustment to aggregate dynamics, shedding new light on the sources of labor market fluctuations in post-war

[1] Seminal contributions include Hansen (1985), Benhabib, Rogerson, and Wright (1991), Christiano and Eichenbaum (1992), and Gali (1999).

[2] See, among others, Andolfatto (1996), den Haan, Ramey, and Watson (2000), Gertler and Trigari (2009), and Shimer (2005).

[3] This percentage is larger than suggested by Cho and Cooley (1994) but consistent with the estimates of Hansen (1985). Section 2 discusses the data and robustness of these computations. In addition, we document the positive covariance between hours per worker and employment is a significant contributor to total hours variation, highlighting an additional indirect effect of the intensive margin through employment.

[4] As is common practice in the literature, we assume that hours per worker adjust to equate the marginal rate of substitution between hours and consumption to the value marginal product of labor. See, among others, Andolfatto (1996), Arseneau and Chugh (2008), Christiano, Trabandt, and Walentin (2011), Merz (1995), Ravenna and Walsh (2012), and Trigari (2009).

U.S. business cycles.

To this end, we estimate with Bayesian methods a general-equilibrium model from the class that Smets and Wouters (2007) and Christiano, Eichenbaum, and Evans (2005) develop, augmented with a rich labor market structure: search and matching frictions, endogenous fluctuations in hours per worker, and shocks that affect both margins of labor adjustment.[5] Our full information procedure provides an ideal laboratory to study the compositional adjustment of aggregate hours for at least three reasons. First, it allows us to encompass most of the views on the sources of business cycles found in the literature, thus giving disturbances other than the neutral technology shock a fair chance to account for labor market adjustments. Second, it allows us to study the empirical performance of the model relative to a large set of moments, without restricting the focus to pure labor market outcomes. Third, the abundant assortment of shocks and transmission mechanisms allows us to uncover the economic forces that drive the relationship between hours per worker and employment during specific business cycle episodes.

Our analysis yields three main results. First, the benchmark model cannot jointly account for the cyclicality of the margins of labor adjustment. In particular, the model cannot reproduce the positive unconditional covariance between employment and hours per worker, and it generates counterfactual volatilities for both margins of labor adjustment. Moreover, the model cannot account for the empirical covariance between hours per worker and macroeconomic time series.[6] These results hold regardless of the number of labor-market observables included in the estimation—either total hours alone or hours and employment together—and the shocks that affect labor adjustment.[7] The specific source of amplification for employment fluctuations is also irrelevant. While the estimated benchmark model features a prominent role for wage stickiness, our results are virtually unaffected when we consider an alternative version of the model in which wage adjustment is flexible, and employment volatility stems from a higher value of the flow value of unemployment (similarly to Hagedorn and Manovskii, 2008). Finally, the counterfactual behavior of the benchmark model is not intrinsically linked to a specific value of the Frisch elasticity of labor supply. While our esti-

[5] The model has the key features that many have found useful for explaining the data, including habit formation, costs of adjusting the flow of investment, variable capital utilization, and nominal price and wage rigidities. Importantly, while wage-setting frictions affect job creation, wage rigidity does not have a direct impact on on-going worker-employer relations (and thus on the adjustment of hours per worker). As a result, the setup is not vulnerable to the Barro (1977) critique.

[6] Gertler, Sala, and Trigari (2008) find that the benchmark model with only the extensive margin is able to reproduce the joint dynamics of one labor margin and macroeconomic variables. Estimates of a version of our benchmark model with only the extensive margin are consistent with this result.

[7] When we use aggregate hours as the only labor market observable, we either consider a standard bargaining power shock or a shock that affects the hours margin. When we include hours and employment as observables, we consider simultaneously the bargaining power shock and a shock to the intensive margin.

mates for this elasticity are in general closer to microeconometric estimates, the inability of the model to reproduce the margins of labor adjustment persists even when we calibrate the Frisch elasticity to values used in the macroeconomic literature.[8]

Our second contribution is to show two key features that reconcile the model with the data: non-separable preferences that exhibit a weak short-run wealth effect on hours supply and hours adjustment costs, a reduced-form cost capturing various technological frictions that constrain the ability of firms to adjust hours per worker (for instance, set-up costs and coordination issues). We introduce parameterized wealth effects in households' preferences following Jaimovich and Rebelo (2009), since their specification allows us to study the limiting case of no wealth effects considered by Greenwood, Hercowitz, and Huffman (1988), while preserving the existence of balanced growth in the model. The weakening of wealth effects eliminates the negative comovement between hours per worker and employment in response to TFP and demand shocks, while non-separability increases the comovement between hours per worker and consumption, which in turn helps the model to reproduce the empirical covariance of the intensive margin with output and investment. In addition, the presence of hours adjustment costs prevents excessive variability in hours per worker, a second key dimension to reproduce the cyclical behavior of both margins of labor adjustment.

Finally, we examine the behavior of hours and employment in post-war U.S. recoveries using the estimated model. We show that the contribution of hours per worker to the employment recovery depends on the historical innovations that were responsible for labor market fluctuations. When employment recoveries feature shocks that induce a negative covariance between employment and hours per worker—such as shocks that result in higher bargained wages as in 1975 and 1982—lack of adjustment along the intensive margin would unambiguously boost employment, since firms would no longer have an incentive to substitute from the more costly labor input. By contrast, when employment recoveries feature shocks that induce a positive covariance between intensive and extensive margin (such as in 1970, 1991 and 2001), the contribution of hours to employment can be either positive or negative, depending on whether hours adjustment works toward increasing or reducing producers' total labor input.[9] Thus, our results indicate that hours per worker

[8] A large literature examines the gap between the microeconometric estimates of the Frisch elasticity and the values used in macroeconomic models. Rogerson and Wallenius (2009) demonstrate in a simulated model that due to different treatment of the extensive margin, the macro and micro Frisch elasticities are conceptually different and can lead to large differences in their values. However, empirical studies have generally been unable to reconcile the gap by relaxing these restrictions (see Peterman, 2012, and references therein).

[9] This is because, conditional on shocks that induce positive comovement, nominal rigidities imply that firms are forced to adjust their labor force to meet a given demand. Notice that, to avoid the zero lower bound on monetary policy, we exclude the Great Recession for estimation.

and employment are not systematically substitutes or complements, of central importance are the particular disturbances responsible for specific episodes.

While we estimate the model on U.S. data, the results of our paper are broader in scope. First, as documented by Ohanian and Raffo (2012), hours and employment positively comove in several economies (for instance, in the U.K., Canada, and Japan), suggesting that the inability of the benchmark model to account for the margins of labor adjustment is not limited to the U.S. economy. Second, parametrized wealth effects and hours adjustment costs introduce enough flexibility to allow the model to match a broad array of empirical covariances between hours per worker and employment, including potentially negative ones as observed in some European economies.

Methodologically, this paper relates to several strands of the literature. First, since Shimer (2005), a large literature addresses the ability of the search and matching model to replicate the cyclical behavior of vacancies and employment. While the debate has for the most part focused on calibrated versions of the search model, a few recent contributions examine the issue in the context of quantitative, estimated models (Gertler, Sala, and Trigari, 2008, and Justiniano and Michelacci, 2012).[10] We contribute to this literature by documenting the inability of the model to jointly reproduce the cyclical behavior of hours per worker, employment, and their empirical covariances with macroeconomic time series. Moreover, we show how to amend the benchmark model to address these shortcomings.

This paper also relates to the literature addressing the behavior of employment in U.S. cyclical recoveries. In particular, an active strand of research addresses the so-called "jobless recoveries" following the past three U.S. recessions (of 1991, 2001, and 2009), where aggregate employment continued to decline for years following the turning point in aggregate income and output.[11] Our results provide additional insights to the debate. First, our counterfactual experiments show that employment growth in jobless recoveries would have not been unambiguously stronger in the absence

[10] Christiano, Trabandt, and Walentin (2011) estimate a medium-scale model featuring search and matching frictions and endogenous hours per worker using Swedish data. Their focus is on understanding the quantitative effects of financial shocks on output and inflation, and the spillover effects of financial market disturbances to unemployment in a small open economy. For this reason they do not address the merit of the model in capturing fluctuations in the intensive and extensive margin of hours worked. Altug, Kabaca, and Poyraz (2011) show that financial frictions in the form of working capital requirements are important to explain observed movements in employment and hours per worker in a small open economy model calibrated to match features of emerging economies.

[11] No consensus has yet emerged regarding the source of jobless recoveries. Some attribute the occurrence of this phenomenon to fundamental changes in the underlying economic structure, such as higher uncertainty in the presence of a more flexible labor force due to temporary and part-time contracts and overtime (Schreft, Singh, and Hodgson, 2005), or a change in the speed of sectoral reallocation (Groshen and Potter, 2003). Others focus on cyclical explanations, such as the intensive margin of labor adjustment in the wake of a short and shallow recession (Bachmann, 2012). Jaimovich and Siu (2012) show that jobless recoveries in the aggregate are accounted for by jobless recoveries in the middle-skill occupations that are disappearing because of job polarization.

of hours adjustment. This result suggests that policies aimed at increasing flexibility in hours per work, such as those advocated by the so-called "Hartz reforms" adopted in Germany, may or may not further delay the recovery of employment, depending on the shocks that affect the margins of labor adjustment. In addition, we find negative labor supply shocks and sluggish aggregate demand were important drivers of jobless recoveries, similar to (Aaronson, Rissman, and Sullivan, 2004),(Bernanke, 2003), and Gali, Smets, and Wouters (2012).

Additionally, some contributions in the early 1990s develop calibrated models in which the supply of total hours adjust along both the intensive and extensive margins, but abstract from search and matching frictions.[12]

The rest of the paper is organized as follows. Section 2 reviews the empirical relation of U.S. hours and employment. Section 3 outlines the benchmark model. Section 4 describes the approach to inference, and discusses the cyclical behavior of the margins of labor adjustment in the estimated model. Section 5 presents the alternative model featuring parameterized wealth effects and hours adjustment costs. Section 6 studies the performance of the alternative model and discusses the cyclical behavior of hours per worker and employment in post-war U.S. recoveries. Section 7 evaluates the robustness of the results to alternative model specifications. Section 8 concludes.

2 Hours and Employment in the Data

We begin with a review of stylized facts about U.S. hours per worker, employment, and total hours worked. In contrast to previous work, we use measures of total hours worked and employment for the entire economy constructed by the BLS mainly from the Current Employment Statistics (CES) survey.[13] Francis and Ramey (2009) show this economy-wide total hours series is less sensitive to sectoral shifts than nonfarm business sector measures. First, we find that fluctuations in hours per worker account for up to 30 percent of the variation in total hours. Second, hours per worker and employment positively co-move, and their positive covariance is a substantial contributor to the variability of total hours. Third, both the comovement and the relative contribution of the

[12] For instance, Cho and Cooley (1994) assume that agents face a fixed cost associated with each day of work, such that the number of employed agents is endogenous. Conditional on employment, agents face a standard hours supply decision. Kydland and Prescott (1991) assume that there is a cost associated with moving people between the household sector and the production sector, letting employed workers choose among workweeks of different lengths. Hansen and Sargent (1988) consider a model in which agents choose among not to work, to work regular time, or to work regular plus overtime. Bachmann (2012) extends the framework to a dynamic stochastic general equilibrium model with heterogeneous establishments that use both margins of labor services.

[13] Although this data is not officially published in the BLS databases, it is now publicly available from the BLS website at www.bls.gov/lpc/special_requests/us_total_hrs_emp.xlsx.

intensive margin varies in specific business cycle episodes such as cyclical recoveries. We highlight the robustness of these facts across alternative labor data sets and discuss their importance for explaining fluctuations in aggregate hours.

We use quarterly data over the period 1965:1-2007:4, which corresponds to the estimation sample period in section 4. Hours per worker is constructed from the total hours and employment series. Total hours and employment are divided by the civilian non-institutional population to express in per capita terms. All variables are expressed in logs and multiplied by 100. We consider several alternative detrending methods. Our preferred method removes a linear trend from each series, which corresponds to the series used for estimation in section 4.[14] In addition, we apply a HP filter with smoothing parameters of 1600 and 10^5 and a band pass filter as in Christiano and Fitzgerald (2003).

To assess the contribution of the intensive margin to labor adjustment, we consider two standard decompositions of the variance of total hours. The first decomposition exploits the fact that

$$\text{var}(TH_t) = \text{cov}(TH_t, h_t) + \text{cov}(TH_t, L_t),$$

where TH_t is total hours worked, h_t is hours per worker, and L_t is employment. Using this decomposition, we compute the shares of the variance attributed to hours per worker and employment as

$$\beta_{cov,h} \equiv \frac{\text{cov}(TH_t, h_t)}{\text{var}(TH_t)}, \quad \beta_{cov,L} \equiv \frac{\text{cov}(TH_t, L_t)}{\text{var}(TH_t)}.$$

In addition, we consider the following alternative decomposition:

$$\text{var}(TH_t) = \text{var}(h_t) + \text{var}(L_t) + 2\text{cov}(h_t, L_t),$$

and define the shares of the variance attributed to hours per worker, employment, and the covariance term respectively as

$$\beta_h \equiv \frac{\text{var}(h_t)}{\text{var}(TH_t)}, \quad \beta_L \equiv \frac{\text{var}(L_t)}{\text{var}(TH_t)}, \quad \beta_{cov} \equiv \frac{2\text{cov}(h_t, L_t)}{\text{var}(TH_t)}.$$

Table 1 displays these variance shares for the alternative detrending methods. In addition, it

[14] Over the sample period, hours per worker exhibits a downward trend while employment exhibits an upward trend. When these (logged) variables are linearly detrended, their sum almost perfectly matches the original, demeaned total hours series (their correlation is 0.9999). Thus, the linear filtering appears to account for the low-frequency structural features of employment and hours per worker while preserving the original properties of the total hours series.

documents the shares using a longer data sample from 1965:1-2014:4. While employment accounts for the largest share of variation in total hours, the intensive margin plays a nontrivial role. The first decomposition shows that the covariance between hours per worker and total hours ($\tilde{\beta}_{cov,h}$) accounts for up to one-third of the total variation in TH_t. The second decomposition shows that the positive covariance between hours and employment (β_{cov}) explains approximately one-third of the variability in total hours. Thus, fluctuations in the intensive margin affect total hours both directly and indirectly through employment.

Table 1: Components of the Variance of Total Hours

Filtering	$\tilde{\beta}_{cov,h}$ $\left(\frac{\text{cov}(TH_t,h_t)}{\text{var}(TH_t)}\right)$	$\tilde{\beta}_{cov,L}$ $\left(\frac{\text{cov}(TH_t,L_t)}{\text{var}(TH_t)}\right)$	β_h $\left(\frac{\text{var}(h_t)}{\text{var}(TH_t)}\right)$	β_L $\left(\frac{\text{var}(L_t)}{\text{var}(TH_t)}\right)$	β_{cov} $\left(\frac{2\text{cov}(h_t,L_t)}{\text{var}(TH_t)}\right)$
			1965:1-2007:4		
Linear	0.33	0.67	0.18	0.51	0.31
HP 1600	0.21	0.79	0.09	0.67	0.23
HP 10^5	0.25	0.75	0.10	0.60	0.30
BP	0.23	0.77	0.10	0.63	0.27
			1965:1-2014:4		
Linear	0.15	0.85	0.11	0.81	0.08
HP 1600	0.21	0.79	0.09	0.67	0.24
HP 10^5	0.21	0.79	0.08	0.65	0.27
BP	0.24	0.76	0.09	0.61	0.30

Table A.1 in the Appendix documents the robustness of these results to two alternative data sources. The first uses labor variables from the Current Population Survey (CPS) which are augmented with armed forces data to provide an alternative economy-wide measure. CPS total hours data exhibit less pronounced low-frequency behavior than CES measures, as shown by Frazis and Stewart (2010) and Ramey (2012). Our results are robust to unfiltered measures of these variables, providing further robustness towards the choice of filtering. In addition, we document the robustness of the results to the labor variables of Smets and Wouters (2007), which are widely employed in the DSGE estimation literature. In this case, hours per worker contributes up to 48% of the variation in total hours. See the Appendix for details of the data sources.

While Table 1 documents an unconditional positive correlation between hours per worker and employment, the comovement varies in specific episodes. To illustrate this, figure 1 plots total hours, hours per worker, and employment during five recoveries: 1970:1, 1975:1, 1982:4, 1991:1, and 2001:4.[15] For reference, the figure displays the first difference of the natural logarithm of

[15] The literature comparing employment measures in jobless recoveries suggests preference for CES data measures

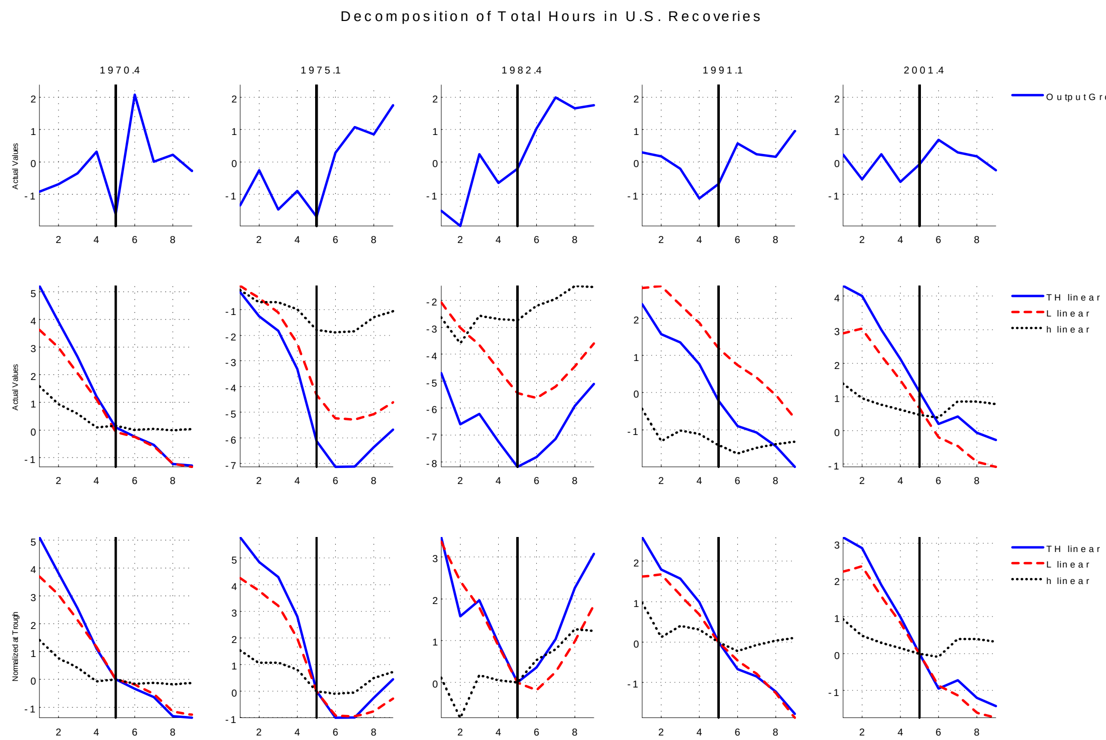

Figure 1. U.S. cyclical recoveries. Row 1 and 2: actual values; Row 3: normalized relative to GDP trough. Solid vertical lines indicate the troughs. Labor data are measures for the entire economy (see the Appendix for details).

GDP as well (top row). We display labor market variables relative to trend (middle panel) and relative to GDP's trough (bottom panel). Hours per worker and employment positively co-move in some recoveries, such as 1982:4, but negatively co-move in other episodes, as in 1991:1.[16] In addition, hours per worker was quantitatively important for aggregate hours in several recoveries. For instance, at the 1982:4 trough, the difference in employment and total hours relative to trend was over two percentage points, whereas four quarters later the gap shrunk to a difference of about one percentage point (see the middle row, column three). The closing of the gap was due to hours per worker, which was rising on average over the period. Note that these magnitudes are partially masked when the labor market variables are plotted relative to GDP's trough (bottom row). Likewise, in the recovery of 2001:4, total hours and hours per worker exhibited a short increase two periods after GDP's trough, while employment steadily declined over the whole episode.

similar to those used here. See Bachmann (2012) for a review of the literature.

[16]These results hold independently of the detrending procedure, as labor variables exhibit the same trends with alternative filtering methods.

In the subsequent sections, we focus on developing a model consistent with these patterns in the data.

3 The Model

This section outlines a benchmark medium-scale, dynamic stochastic general equilibrium model that features labor-market search and matching frictions and a standard neoclassical hours-supply decision. The model shares salient details that many have found useful for capturing features of the data. These include habit formation, costs of adjusting the flow of investment, variable capital utilization, and nominal price and wage rigidities. We abstract from monetary frictions that would motivate a demand for currency and model a cashless economy following Woodford (2003). Below, variables without a time subscript denotes steady-state values.

Household Preferences

The economy is populated by a representative household with a continuum of members along the unit interval. In equilibrium, some family members are unemployed, while others are employed. As is common in the literature, we assume that family members perfectly insure each other against variation in labor income due to changes in employment status, so that there is no *ex post* heterogeneity across individuals in the household (see Andolfatto, 1996, and Merz, 1995).

The representative household maximizes the expected intertemporal utility function

$$W_t \equiv E_t \sum_{s=t}^{\infty} \beta^{s-t} \bar{\beta}_s \left[\log(C_s - h_C C_{s-1}) - \int_0^{L_s} \frac{h_{j,s}^{1+\omega}}{1+\omega} dj \right], \tag{1}$$

where $\beta \in (0,1)$ is the discount factor, C_t is aggregate consumption, h_C is the degree of habit formation, L_t is the number of employed workers, and h_{jt} denotes hours worked by the employed member j. $\bar{\beta}_t$ denotes an exogenous shock to the discount factor, which evolves according to $\log \bar{\beta}_t = \rho_{\bar{\beta}} \log \bar{\beta}_{t-1} + \varepsilon_{\bar{\beta}t}$ with $\varepsilon_{\bar{\beta}t} \overset{iid}{\sim} N\left(0, \sigma^2_{\bar{\beta}}\right)$. Utility is logarithmic to ensure the existence of a balanced growth path in the presence of non-stationary technological progress.

The consumption basket C_t aggregates differentiated consumption varieties, $C_{\omega t}$ in Dixit-Stiglitz form: $C_t = \left[\int_0^1 C_{\omega t}^{(\bar{\theta}_t - 1)/\bar{\theta}_t} d\omega\right]^{\bar{\theta}_t/(\bar{\theta}_t-1)}$, where $\bar{\theta}_t > 1$ is the exogenous elasticity of substitution across goods. We assume that $\bar{\theta}_t$ follows the stochastic process $\log \bar{\theta}_t = \rho_{\bar{\theta}} \log \bar{\theta}_{t-1} + (1-\rho_{\bar{\theta}}) \log \bar{\theta} + \varepsilon_{\bar{\theta}t}$, where $\varepsilon_{\bar{\theta}t} \overset{iid}{\sim} N\left(0, \sigma^2_{\bar{\theta}}\right)$, which, following the literature, we refer to as a price markup shock. The corresponding price index is given by: $P_t = \left[\int_0^1 P_{\omega t}^{1-\bar{\theta}} d\omega\right]^{1/\left(1-\bar{\theta}\right)}$, where $P_{\omega t}$ is the price of variety ω.

Production

There are two vertically integrated production sectors. In the upstream sector, perfectly competitive firms use capital and labor to produce a homogenous intermediate input. In the downstream sector, monopolistically competitive firms purchase intermediate inputs and produce the differentiated varieties that are sold to consumers. This production structure is common in the search and matching literature featuring nominal rigidities and monopolistic competition, as it greatly simplifies the introduction of labor market frictions in the model; see, for instance, Gertler, Sala, and Trigari (2008), Ravenna and Walsh (2011), and Trigari (2009).

Intermediate Input Producers

There is a unit mass of perfectly competitive intermediate producers. Production requires capital and labor. Within each firm there is a continuum of jobs; each job is executed by one worker. Capital is perfectly mobile across firms and jobs and there is a competitive rental market in capital. All jobs produce with identical exogenous productivity $\bar{A}_t$. We assume that the growth rate of technology, $\bar{g}_{At} \equiv \bar{A}_t/\bar{A}_{t-1}$, follows the stochastic process: $\log \bar{g}_{At} = \rho_{\bar{g}_A} \log \bar{g}_{At-1} + (1-\rho_{\bar{g}_A}) \log \bar{g}_A + \varepsilon_{\bar{g}_A t}$, where $\varepsilon_{\bar{g}_A t} \overset{iid}{\sim} N\left(0, \sigma_{\bar{g}_A}^2\right)$.

A filled job i in the representative firm j produces $\left(k_{jt}^i\right)^a \left(\bar{A}_t h_{jt}^i\right)^{1-\alpha}$ units of output, where k_{jt}^i is the stock of capital allocated to the job i and h_{jt}^i is the corresponding number of hours worked. Since all jobs produce with identical aggregate productivity $\bar{A}_t$, all existing matches produce the same amount of output using the same capital and hours inputs. Thus, we omit the job-specific index i henceforth. Total producer's output exhibits constant returns to scale in total hours and capital:

$$Y_{jt}^I = K_{jt}^{\alpha} \left(\bar{A}_t L_{jt} h_{jt}\right)^{1-\alpha}, \tag{2}$$

where L_{jt} is the measure of jobs within the firm and $K_{jt} \equiv L_{jt} k_{jt}$.[17]

The relationship between a firm and a worker can be severed for exogenous reasons. We denote by λ the fraction of jobs that are exogenously destroyed in each period.[18] Job creation is subject

[17] This stems from the fact that $Y_{jt}^I = L_{jt} \left(k_{jt}\right)^a \left(\bar{A}_t h_{jt}\right)^{1-\alpha} = K_{jt}^{\alpha} \left(\bar{A}_t L_{jt} h_{jt}\right)^{1-\alpha}$.

[18] Hall (2005) and Shimer (2005) argue that, in the U.S. data, the separation rate varies little over the business cycle, although part of the literature disputes this position; see Davis, Haltiwanger, and Schuh (1998) and Fujita and Ramey (2009).

to matching frictions. To hire a new worker, firms have to post a vacancy, incurring a real cost $\bar{A}_t \kappa_{jt}$, where $\kappa_{jt} \equiv \kappa V_{jt}^{\tau} / (1+\tau)$. This specification implies that total vacancy costs are convex in the number of posted vacancies, V_{jt}, an assumption that is consistent with the evidence in Merz and Yashiv (2007).[19] We let the vacancy cost drift with the level of technology to ensure balanced growth; otherwise, κ_{jt} would become a smaller fraction of labor income as the economy grows. The probability of finding a worker depends on a constant returns to scale matching technology, which converts aggregate unemployed workers U_t and aggregate vacancies V_t into aggregate matches $M_t = \chi U_t^{\varepsilon} V_t^{1-\varepsilon}$, where $0 < \varepsilon < 1$. Each firm meets unemployed workers at a rate $q_t \equiv M_t / V_t$.

The timing of events in the labor market proceeds as follows. The firm j begins a period with a stock of L_{jt-1} workers, which is immediately reduced by exogenous separations. Then, the firm posts vacancies V_{jt} and selects the total capital stock, K_{jt}.[20] Once the hiring round has been completed, wages and hours per worker are determined, and production occurs.[21] The law of motion of employment is given by:

$$L_{jt} = (1-\lambda) L_{jt-1} + q_t V_{jt}. \tag{3}$$

As in Arseneau and Chugh (2008), we use Rotemberg's (1982) model of a nominal rigidity and assume that firms face a quadratic cost of adjusting the hourly nominal wage rate, w_{jt}^n.[22] The real, per-worker cost of changing the nominal wage between period $t-1$ and t is

$$\Gamma_{w_j t} \equiv \frac{\phi^w \bar{A}_t}{2} \left(\frac{w_{jt}^n}{w_{jt-1}^n} \pi_C^{\iota_w - 1} \pi_{Ct-1}^{-\iota_w} - \bar{g}_A \right)^2,$$

[19] Our results are robust to considering convex hiring costs as in Gertler, Sala, and Trigari (2008).

[20] With full capital mobility and price-taker firms in the capital market, it is irrelevant whether producers choose the total stock of capital K_{jt}, or, instead, determine the optimal capital stock for each existing job, k_{jt}. Moreover, as noted by Cahuc, Marque, and Wasmer (2008), the specific timing of the capital decision is immaterial for the equilibrium allocation, since capital can be costlessly adjusted within each firm—firms can always re-optimize K_{jt} within a given a period. See Cahuc, Marque, and Wasmer (2008) for a model in which capital is predetermined at the producer level.

[21] Thus, labor-market matching occurs within a period, which, as noted by Arsenau and Chugh (2012), is empirically descriptive of U.S. labor-market flows at quarterly frequencies.

[22] Alternatively, we could follow Gertler, Sala, and Trigari (2008) and assume staggered (Calvo) nominal wage bargaining. The advantage of assuming a quadratic wage adjustment cost is a more convenient model aggregation. Notice that these alternative sources of wage rigidity are not observationally equivalent, even in a first-order approximation to the model policy functions around a deterministic steady state with zero net inflation. The reason is that, as discussed by Gertler and Trigari (2009), the wage dispersion implied by staggered Nash bargaining generates a spillover effect on the average wage that is absent with convex wage adjustment costs. However, as already shown by Gertler and Trigari (2009), the quantitative importance of such an externality is very modest. Accordingly, the implied model dynamics are remarkably similar across the two alternative specifications (results are available upon request).

where $\phi^w \geq 0$ is in units of consumption, $\pi_{Ct} \equiv P_t/P_{t-1}$ is the gross CPI inflation rate, and $\iota_w \in [0,1]$ measures the degree to which nominal wage adjustment is indexed to previous price inflation. If $\phi^w = 0$, there is no cost of wage adjustment. As for the vacancy cost, the wage adjustment cost is tied to the level of technology $\bar{A}_t$ to ensure balanced growth. We center the wage adjustment cost at $\bar{g}_A$ to ensure zero wage-adjustment costs along a balanced growth path with zero net price inflation.

Intermediate input producers sell their output to final producers at a real price φ_t in units of consumption. The present discounted value of the stream of profits is given by:

$$\Pi_{jt}^I \equiv E_t \left\{ \sum_{t=s}^{\infty} \beta_{s,s+1} \left[\varphi_s Y_{js}^I - \frac{w_{js}^n h_{js}}{P_s} L_{js} - \Gamma_{w_j,s} L_{js} - r_{K,s} K_{js} - \kappa \bar{A}_s \frac{V_{js}^{1+\tau}}{1+\tau} \right] \right\}, \tag{4}$$

where $\beta_{t,t+1} \equiv \beta u_{Ct+1}/u_{Ct}$ is the household stochastic discount factor. Equation (1) implies that the marginal utility of consumption u_{Ct} is defined by

$$u_{Ct} \equiv \frac{\bar{\beta}_t}{C_t - h_C C_{t-1}} - h_C \beta E_t \left(\frac{\bar{\beta}_{t+1}}{C_{t+1} - h_C C_t} \right).$$

The representative producer chooses V_{jt}, L_{jt}, and K_{jt} to maximize (4) subject to (2) and (3). When making these decisions, the firm anticipates that both the hourly wage w_{jt} and hours per worker h_{jt} do not depend on the scale of the firm, so that $\partial w_{jt}^n/\partial L_{jt} = \partial h_{jt}/\partial L_{jt} = 0$. As shown below, these results obtain under the standard assumptions of individual Nash wage bargaining and neoclassical determination of hours per worker.

The first-order condition for K_{jt} equates the marginal revenue product of capital to its rental cost:

$$\varphi_t \alpha \left(\frac{K_{jt}}{\bar{A}_t L_{jt} h_{jt}} \right)^{\alpha-1} = r_{Kt}, \tag{5}$$

implying that the capital-total hours ratio is symmetric across producers, since it only depends on aggregate variables. Let S_{jt}^f denote the Lagrange multiplier on the constraints (3), representing the value to the firm of hiring an extra worker. The first-order condition for L_{jt} implies:

$$S_{jt}^f = (1-\alpha)\,\varphi_t \left(\frac{K_{jt}}{\bar{A} h_{jt} L_{jt}} \right)^{\alpha} \bar{A}_t h_{jt} - \frac{w_{jt}^n h_{jt}}{P_t} - \Gamma_{w_j t} + E_t \beta_{t,t+1} (1-\lambda)\, S_{jt+1}^f. \tag{6}$$

Intuitively, the value of a job to the firm corresponds to the expected, present discounted value of the streams of profits from the match—the difference between the value of the marginal product and the wage payment to the worker minus the cost of adjusting the nominal wage. Finally, the first-order condition for vacancies equates the cost of filling a vacancy to the value of a filled position:

$$\kappa \bar{A}_t \frac{V_{jt}^{\tau}}{q_t} = S_{jt}^{f}. \tag{7}$$

Equation (6) and (7) imply a standard job creation condition:

$$\frac{\kappa \bar{A}_t V_{jt}^{\tau}}{q_t} = (1-\alpha)\,\varphi_t \left(\frac{K_{jt}}{\bar{A} h_{jt} L_{jt}}\right)^{\alpha} \bar{A}_t h_{jt} - \frac{w_{jt}^{n} h_{jt}}{P_t} - \Gamma_{w_j t} + \kappa\,(1-\lambda)\, E_t \beta_{t,t+1} \frac{\bar{A}_{t+1} V_{jt+1}^{\tau}}{q_{t+1}}.$$

Forward looking iteration of the job creation equation implies that, at the optimum, the expected discounted value of the stream of profits generated by a match over its expected lifetime is equal to the cost of filling a vacancy, $\kappa \bar{A}_t V_{jt}^{\tau}/q_t$.

Hours Determination

We assume that hours per worker adjust to the point where the worker's marginal cost of working an extra hour is equal to the firm's marginal benefit, as is common practice in the literature.[23] This requires that the worker's marginal rate of substitution between consumption and leisure is equal to the value of the marginal value product of an extra hour worked, leading to the condition:

$$-\frac{W_{h_j t}}{u_{Ct}} = (1-\alpha)\,\varphi_t \left(\frac{K_{jt}}{\bar{A} h_{jt} L_{jt}}\right)^{\alpha} \bar{A}_t, \tag{8}$$

where $W_{h_j t} \equiv \partial W_{h_j t}/\partial h_{jt} = \bar{\beta}_t h_{jt}^{\omega}$. Using the first-order condition for capital, the optimality condition in (8) can be written as

$$\frac{\bar{\beta}_t h_{jt}^{\omega}}{u_{Ct}} = (1-\alpha)\,\varphi_t \left(\frac{r_{Kt}}{\varphi_t \alpha}\right)^{\frac{\alpha}{\alpha-1}} \bar{A}_t, \tag{9}$$

which shows that h_{jt} only depends on aggregate conditions, i.e., $h_{jt} = h_t$ is invariant to the scale of the firm. Moreover, hours per worker do not directly depend on the hourly wage w_{jt}.

Wage Bargaining

The nominal wage is the solution to an individual Nash bargaining problem, and the wage payment divides the match surplus between workers and firms. Due to the presence of nominal rigidities, we

[23] Alternatively, we could assume that firms have the right to manage hours. The disadvantage of such a theoretical framework is twofold. First, the choice of hours is not privately efficient from the perspective of each firm-worker match. Second, under right-to-manage, wage stickiness affects fluctuations in hours worked. As a consequence, this framework is subject to the Barro (1977) critique, given that firms and workers have an ongoing relationship.

assume that bargaining occurs over the nominal wage rather than the real wage, as in Arseneau and Chugh (2008), Gertler, Sala, and Trigari (2008), and Thomas (2008). With zero costs of nominal wage adjustment ($\phi^w = 0$), the real wage is identical to the one obtained from bargaining directly over the real wage. This is no longer the case in the presence of wage adjustment costs. As is standard practice in the literature, the wage bargaining is atomistic, implying that the firm and the worker take K_{jt} and L_{jt} as given at the bargaining stage. Moreover, both parties account for the fact that $\partial h_t / \partial w_{jt} = 0$, as shown above.

Let $\bar{\eta}_t \in (0, 1)$ be the weight given to the worker's individual surplus in Nash bargaining. We assume that $\bar{\eta}_t$ follows the process: $\log \bar{\eta}_t = \rho_{\bar{\eta}} \log \bar{\eta}_{t-1} + (1 - \rho_{\bar{\eta}}) \log \bar{\eta} + \varepsilon_{\bar{\eta}t}$, where $\varepsilon_{\bar{\eta}t} \overset{iid}{\sim} N\left(0, \sigma^2_{\bar{\eta}}\right)$. Exogenous fluctuations in the worker's bargaining power are the counterpart of wage-markup shocks typically assumed in the estimation of benchmark New Keynesian models that abstract from search and matching frictions.[24] The firm and the worker maximize the Nash product

$$\left(S^f_{jt}\right)^{1-\bar{\eta}_t} \left(S^w_{jt}\right)^{\bar{\eta}_t},$$

where S^f_{jt} is defined as in (12) and S^w_{jt} denotes the worker surplus:

$$S^w_{jt} = \frac{w^n_{jt}}{P_t} h_t - b\bar{A}_t - \frac{\bar{\beta}_t h_t^{1+\omega}}{(1+\omega)\, u_{Ct}} + E_t \left[\beta_{t,t+1} (1-\lambda)\, S^w_{jt+1} \left(1 - \frac{M_{t+1}}{U_{t+1}}\right)\right]. \tag{10}$$

The worker's surplus corresponds to the expected present discounted value of wage payments over the lifetime of the match minus the expected present discounted value of the flow value of unemployment, including unemployment benefits from the government $b\bar{A}_t$ (financed with lump sum taxes), and the utility gain from leisure in terms of consumption.

The first-order condition with respect to w^n_{jt} implies the following sharing rule:

$$\eta_{w_j t} S^f_{jt} = (1 - \eta_{w_j t}) S^w_{jt}, \tag{11}$$

[24] Up to a first-order approximation, wage markup shocks are isomorphic to hours supply shocks in the benchmark New Keynesian model. Such equivalence breaks down in the presence of labor-market search and matching frictions. For this reason, in section 7 we consider an alternative version of the model that features exogenous hours-supply shocks. We show there that the inability of the standard search and matching model to jointly explain fluctuations in hours per worker and employment does not depend on the specific labor-market shock we consider.

where η_{w_jt} is the *effective* bargaining share of workers:

$$\eta_{w_jt} \equiv \frac{\bar{\eta}_t \left(\partial S^w_{jt}/\partial w^n_{jt}\right)}{\bar{\eta}_t \left(\partial S^w_{jt}/\partial w^n_{jt}\right) + (1-\bar{\eta}_t)\left(\partial S^f_{jt}/\partial w^n_{jt}\right)} = \frac{\bar{\eta}_t h_t}{\bar{\eta}_t h_t + (1-\bar{\eta}_t)\left(\partial S^f_{jt}/\partial w^n_{jt}\right)}.$$

(See the Appendix for the expression of $\partial S^f_{jt}/\partial w^n_{jt}$.) As in Gertler and Trigari (2009), the effective bargaining share is time-varying due to the presence of wage adjustment costs. Absent these costs, we would have an exogenous bargaining share $\eta_{w_jt} = \bar{\eta}_t$. Importantly, wage rigidity implies that η_{w_jt} is countercyclical, amplifying employment fluctuations in response to aggregate shocks as first noted by Gertler and Trigari (2009).

It is straightforward to verify that w^n_{jt} does not depend on the scale of the firm. To see this, substitute equation (9) into the definition of the worker's and firm's surplus, S^f_{jt} and S^w_{jt}, and use the first-order condition for capital to eliminate the capital-labor ratio in S^f_t:

$$S^f_{jt} = (1-\alpha)\,\varphi_t \left(\frac{r_{Kt}}{\varphi_t \alpha}\right)^{\frac{\alpha}{\alpha-1}} \bar{A}_t h_t - \frac{w^n_{jt} h_t}{P_t} - \Gamma_{w_jt} + E_t \beta_{t,t+1} (1-\lambda)\, S^f_{jt+1}. \tag{12}$$

Since all the intermediate firms produce with identical technology $\bar{A}_t$, there is a symmetric equilibrium in which $K_{jt} = K_t$, $L_{jt} = L_t$, $h_{jt} = h_t$, $V_{jt} = V_t$, and $w^n_{jt} = w^n_t$. In turn, nominal hourly wage inflation, defined by $\pi_{wt} \equiv w^n_t/w^n_{t-1}$ is linked to CPI inflation by $\pi_{wt} = (w_t/w_{t-1})\,\pi_{Ct}$, where $w_t \equiv w^n_t/P_t$ denotes the real hourly wage. Finally, searching workers in period t are equal to the mass of unemployed workers: $U_t = 1 - (1-\lambda)\, L_{t-1}$.

Final Goods Production

A continuum of monopolistically competitive final-sector firms produce differentiated varieties using the intermediate input. The producer ω faces the following demand: $Y^C_{\omega t} = (P_{\omega t}/P_t)^{-\bar{\theta}_t}\, Y^C_t$, where Y^C_t denotes aggregate demand of the final consumption basket, inclusive of sources besides household consumption.[25]

We introduce price-setting frictions by following Rotemberg (1982) and assume that final producers must pay a quadratic price adjustment cost.[26] We also allow for price indexation by assuming

[25] Aggregate demand takes the same CES form as the consumption basket, with the same elasticity of substitution $\bar{\theta}_t$ across consumption varieties. This ensures that the consumption price index is also the price index for aggregate demand of the final basket.

[26] Since we solve the model with a first-order approximation to the model policy function and set $\pi_C = 0$ along the balanced growth path, our model yields identical dynamics to assuming staggered (Calvo) price setting.

that final producers index price changes to past CPI inflation, so that price adjustment costs take the form:

$$\frac{\phi^p}{2}\left(\frac{P_{\omega t}}{P_{\omega t-1}}\pi_C^{\iota_p-1}\pi_{Ct-1}^{-\iota_p}-1\right)^2 P_{\omega t}Y_{\omega t}^C,$$

where $\phi^p \geq 0$ determines the size of the adjustment cost (prices are flexible if $\phi^p = 0$) and $\iota_p \in [0,1]$ is the indexation parameter. Since we assume zero net trend inflation along the balanced growth path, the price adjustment cost is zero absent aggregate shocks.

The producer ω maximizes the present discounted value of the expected stream of (real) profits:

$$\Pi_{\omega s}^F = E_t\sum_{s=t}^{\infty}\beta_{s,s+1}\left\{\frac{P_{\omega s}}{P_s}\left[1-\frac{\phi^p}{2}\left(\frac{P_{\omega s}}{P_{\omega s-1}}\pi_C^{\iota_p-1}\pi_{C,s-1}^{-\iota_p}-1\right)^2\right]Y_{\omega s}^C-\varphi_s Y_{\omega s}^C\right\},$$

subject to the demand schedule $Y_{\omega t}^C = (P_{\omega t}/P_t)^{-\bar{\theta}_t} Y_t^C$. Let $\pi_{\omega t} \equiv P_{\omega t}/P_{\omega t-1}$. Optimal price setting implies that the (real) output price $P_{\omega t}/P_t$ is equal to a markup over the real marginal cost φ_t:

$$\frac{P_{\omega t}}{P_t} = \frac{\bar{\theta}_t}{\left(\bar{\theta}_t-1\right)\Xi_{\omega t}}\varphi_t,$$

where

$$\Xi_{\omega t} \equiv 1-\frac{\phi^p}{2}\left(\pi_{\omega t}\pi_{Ct-1}^{-\iota_p}\pi_C^{\iota_p-1}-1\right)^2 + \frac{\phi^p}{\bar{\theta}_t-1}\left\{\begin{array}{c}\pi_C^{\iota_p-1}\left(\pi_{\omega t}\pi_{Ct-1}^{-\iota_p}\pi_C^{\iota_p-1}-1\right)\pi_{\omega t}\pi_{Ct-1}^{-\iota_p}\\ -E_t\left[\beta_{t,t+1}\left(\pi_{\omega t+1}\pi_{Ct}^{-\iota_p}\pi_C^{\iota_p-1}-1\right)\pi_{Ct+1}^{-1}\pi_{\omega t+1}^2\pi_{Ct}^{-\iota_p}\frac{Y_{\omega t+1}^C}{Y_{\omega t}^C}\right]\end{array}\right\}.$$

There are two sources of endogenous markup variation in our model. First, the cost of adjusting prices gives firms an incentive to change their markups over time in order to smooth price changes across periods. Second, exogenous shocks to the firm market power result in time-varying markups even in the absence of price stickiness. In the symmetric equilibrium, $P_{\omega t} = P_t$ and $\Xi_{\omega t} = \Xi$. As a consequence, $\pi_{\omega t} = \pi_t = \pi_{Ct}$.

Household Budget Constraint and Optimal Intertemporal Decisions

The household enters period t with nominal private bond holdings B_t, earning a gross interest rate i_t. The household also accumulates the physical capital and rents it to intermediate input producers in a competitive capital market. Investment in the physical capital stock, I_{Kt}, requires the use of the same composite of all available varieties as the basket C_t. We introduce convex adjustment costs in physical investment and variable capital utilization. The utilization rate of capital is set by the household. Thus, effective capital rented to firms, K_t, is the product of physical capital, $\tilde{K}_t$, and the utilization rate, u_{Kt}: $K_t = u_{Kt}\tilde{K}_t$. Increases in the utilization rate are costly because higher utilization rates imply faster capital depreciation. We assume a standard convex depreciation function: $\delta_{Kt} = \delta_0 + \delta_1 (u_{Kt} - 1) + \delta_2 (u_{Kt} - 1)^2$. Physical capital, $\tilde{K}_t$, obeys a standard law of motion:

$$\tilde{K}_{t+1} = (1 - \delta_{Kt}) \tilde{K}_t + \bar{P}_{Kt} \left[1 - \frac{\nu_K}{2} \left(\frac{I_{Kt}}{I_{Kt-1}} - \bar{g}_A \right)^2 \right] I_{Kt}, \tag{13}$$

where $\nu_K > 0$ is a scale parameter, and $\bar{P}_{Kt}$ is an investment specific shock. The latter is a source of exogenous variation in the efficiency with which the final good can be transformed into physical capital, and thus into tomorrow's capital input.[27] The investment shock evolves via the process $\log \bar{P}_{Kt} = \rho_{\bar{P}_K} \log \bar{P}_{Kt-1} + \varepsilon_{\bar{P}_K t}$, where $\varepsilon_{\bar{P}_K t} \overset{iid}{\sim} N\left(0, \sigma^2_{\bar{P}_K}\right)$.

The per-period household's budget constraint is:

$$P_t C_t + P_t I_{Kt} + B_{t+1} = i_t B_t + w_t^n h_t L_t + r_{Kt} P_t K_t + b \bar{A}_t (1 - L_t) P_t + P_t \Pi_t^I + P_t \int_0^1 \Pi_t^F (i)\, di + T_t^g, \tag{14}$$

where T_t^g is a nominal lump-sum tax from the government. In equilibrium, $T_t^g = b\bar{A}_t U_t P_t + G_t P_t$, where G_t denotes real government spending.

The household maximizes its expected intertemporal utility subject to (13) and (14). The Euler equation for capital accumulation requires: $\zeta_{Kt} = E_t \{\beta_{t,t+1} [r_{t+1} u_{Kt+1} + (1 - \delta_{Kt+1}) \zeta_{Kt+1}]\}$, where ζ_{Kt} denotes the shadow value of capital (in units of consumption), defined by the first-order condition for investment I_{Kt}:

$$\begin{aligned} \zeta_{Kt}^{-1} = & \left[1 - \frac{\nu_K}{2} \left(\frac{I_{Kt}}{I_{Kt-1}} - 1 \right)^2 - \nu_K \left(\frac{I_{Kt}}{I_{Kt-1}} - 1 \right) \left(\frac{I_{Kt}}{I_{Kt-1}} \right) \right] \\ & + \nu_K \beta_{t,t+1} E_t \left[\frac{\zeta_{Kt+1}}{\zeta_{Kt}} \left(\frac{I_{Kt+1}}{I_{Kt}} - 1 \right) \left(\frac{I_{Kt+1}}{I_{Kt}} \right)^2 \right]. \end{aligned}$$

The optimal condition for capital utilization implies: $r_{Kt} = \zeta_{Kt}[\delta_{K1} + \delta_{K2}(u_{Kt} - 1)]$. Finally, the Euler equation for bond holdings implies: $1 = i_t E_t [\beta_{t,t+1} / (1 + \pi_{Ct+1})]$.

[27] Justiniano, Primiceri, and Tambalotti (2009) show that this variation might stem from technological factors specific to the production of investment goods, but also from disturbances to the process by which these investment goods are turned into productive capital.

Market Clearing

In the symmetric equilibrium, bonds are zero in net supply: $B_t = B_{t+1} = 0$. Thus, the household's budget constraint (14) yields the following aggregate resource constraint:

$$Y_t^C \left[1 - \frac{\nu}{2}\left(\pi_{Ct}\pi_C^{\iota_p - 1}\pi_{Ct-1}^{-\iota_p} - 1\right)^2\right] = C_t + I_{Kt} + \kappa_t \bar{A}_t V_t + G_t + \frac{\phi^w \bar{A}_t}{2}\left(\pi_{wt}\pi_C^{\iota_w - 1}\pi_{Ct-1}^{-\iota_w} - \bar{g}_A\right)^2 L_t. \tag{15}$$

Intuitively, total output produced by firms must be equal to the sum of market consumption, investment in physical capital, the costs associated to job creation, the purchase of goods from the government, and the real cost of changing prices and wages. Finally, labor market clearing implies $Y_t^C = Y_t^I$. We define model GDP as $Y_t \equiv C_t + I_{Kt} + G_t$.

The Government

The monetary authority sets the nominal interest rate following a feedback rule of the form

$$\frac{i_{t+1}}{i} = \left(\frac{i_t}{i}\right)^{\varrho_i}\left[\left(\frac{\pi_{Ct}}{\pi_C}\right)^{\varrho_\pi}\left(\frac{Y_{gt}}{Y_g}\right)^{\varrho_Y}\right]^{1-\varrho_i}\left(\frac{Y_{gt}}{Y_{gt-1}}\right)^{\varrho_{dY}} \bar{\imath}_{it}, \tag{16}$$

where i is the steady state of the gross nominal interest rate. The interest rate responds to deviations of inflation and the GDP gap from their long-run targets, as well as to deviations of the growth rate of the GDP gap, Y_{gt}/Y_{gt-1}. Consistent with Woodford (2003), we define Y_{gt} as the deviation of model GDP from its level prevailing under flexible prices and wages and absent inefficient shocks (i.e., absent markup and bargaining power shocks). The monetary policy rule is subject to a shock, $\bar{\imath}_{it}$, which evolves according to $\log \bar{\imath}_{it} = \rho_{\bar{\imath}} \log \bar{\imath}_{it-1} + \varepsilon_{\bar{\imath}t}$, with $\varepsilon_{it} \overset{iid}{\sim} N\left(0, \sigma_{\bar{\imath}}^2\right)$.

Fiscal policy is fully Ricardian. The government finances its budget deficit with lump-sum taxes each period. Public spending is determined exogenously, $G_t = \bar{g}_t$, where the exogenous government spending shock $\bar{g}_t$ follows the process $\log \bar{g}_t = \rho_{\bar{g}} \log \bar{g}_t + (1 - \rho_{\bar{g}}) \log \bar{g} + \varepsilon_{\bar{g}t}$, with $\varepsilon_{\bar{g}t} \overset{iid}{\sim} N\left(0, \sigma_{\bar{g}}^2\right)$.

Table A.2 in the Appendix summarizes the equilibrium conditions of the model. The table contains 15 equations that determine 15 endogenous variables: i_t, π_{Ct}, π_{wt}, C_t, L_t, V_t, M_t, h_t, w_t, φ_t, $\tilde{K}_{t+1}$, I_{Kt}, ζ_{Kt}, u_{Kt}, r_{Kt}, and 10 definitions (U_t, S_t^f, S_t^w, q_t, u_{Ct}, δ_{Kt}, κ_t, η_{wt}, Ξ_t, and Y_{gt}). Additionally, the model features 8 exogenous disturbances: $\bar{g}_{At}$, $\bar{\beta}_t$, $\bar{h}_t$, $\bar{\theta}_t$, $\bar{\eta}_t$, $\bar{P}_{Kt}$, $\bar{\imath}_t$, and $\bar{g}_t$.

Model Solution

Consumption, investment, capital, the real wage, and GDP, (together with Y_t^C, S_t^f, S_t^w, and u_{Ct}) fluctuate around a stochastic balanced growth path, since the level of technology has a unit root. Therefore, the solution involves the following steps. First, rewrite the model in terms of detrended variables. Second, compute the non-stochastic steady state of the transformed model, and log-linearly approximate it around this steady state. The details of these steps can be found in the Appendix. Third, solve the resulting linear system of rational expectation equations to obtain the transition equations, which are combined with an observation equation to form the state-space model used in the estimation procedure.

4 Estimation

We estimate the model with U.S. quarterly data from 1965:1 to 2007:4. The sample starting period reflects the initial availability of some wage measures we consider. For the benchmark estimation, we end the estimation prior to the recent zero lower bound episode.[28] Our initial estimation includes seven observables commonly employed in the literature. To avoid stochastic singularity, we include seven structural shocks. To facilitate comparison with the literature (i.e., Christiano, Trabandt, and Walentin, 2011, and Gertler, Sala, and Trigari, 2008), our benchmark specification assumes that shocks to the exogenous component of the worker's bargaining power, $\bar{\eta}_t$, are the only disturbance directly affecting the labor market.[29] The seven observables include the log difference of aggregate consumption, investment, output, and real wages, the log difference of the GDP deflator, the Federal Funds rate, and the log of economy-wide total hours worked. The literature widely employs measures of these seven series for estimated DSGE models.[30]

In addition, we estimate the model including the hours supply shock, $\bar{h}_t$, and one ancillary observable, the log of economy-wide employment.[31] Using information on both margins of labor adjustment helps identify key labor parameters such as the Frisch elasticity. Moreover, the inclusion of the hours supply shock gives the model a better chance to match the dynamics of the labor

[28] See Hirose and Inoue (2015) for a discussion of how the ZLB can bias estimates of log-linearized model parameters. Our results are robust to including this recent period and are available upon request.

[29] In section 7, we discuss the alternative possibility of focusing on stochastic fluctuations in the disutility of hours worked.

[30] Examples include Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2007), Del Negro, Schorfheide, Smets, and Wouters (2007), Gertler, Sala, and Trigari (2008), and Justiniano, Primiceri, and Tambalotti (2010).

[31] This is observationally equivalent to estimating the model using hours per worker and employment as observables, since we abstract from measurement error.

margins.

As already noted in Section 2, economy-wide total hours and employment are constructed by the BLS mainly from the Current Employment Statistics (CES) survey. Both series are linearly detrended. Details of the data construction and linkages to observables are presented in the Appendix.

We use Bayesian inference methods to construct the parameters' posterior distribution, which is a combination of a prior density for the parameters and the likelihood function, evaluated using the Kalman filter. We take 1.5 million draws from the posterior distribution using the random walk Metropolis-Hastings algorithm. For inference, we discard the first 500,000 draws and keep one every 50 draws to remove some correlation of the draws.[32]

Prior Distributions

We impose dogmatic priors for some parameters. The household discount factor β is set to 0.99, α is 0.3, and depreciation δ is 0.025. The steady state price markup is set at 1.1. Steady state government spending is fixed at 20% of GDP, which equals the post-war average for all levels of government spending. Following standard practice in the literature, we use independent evidence for the average quarterly separation rate λ and the elasticity of matches to unemployment, ε. In particular, we choose $\lambda = 0.105$ based on the observation that jobs last about two and half years. We set ε to be equal to 0.5, the midpoint of the evidence typically cited in the literature and within the range of plausible values (0.5 to 0.7) reported by Petrongolo and Pissarides (2006). Finally, we set the cost of posting a vacancy, κ, and the matching efficiency parameter, χ, to match the quarterly average job finding probability, M/U, and the average probability of filling a vacancy, q. The former is equal to 0.95, while the latter is 0.9 (Shimer, 2005).

Table 2 lists the prior distributions for the remaining parameters. Our priors for common New Keynesian parameters are similar to those in Smets and Wouters (2007). The estimated labor market parameters include the steady-state value of the workers' bargaining power $\bar{\eta}$, the replacement rate b/wh, and the degree of convexity in the cost of posting vacancies τ. The first two have priors similar to those in Gertler, Sala, and Trigari (2008). The prior for τ covers the range of hiring costs typically reported in the literature. Finally, the bargaining power, price markup, investment, and hours production shocks are normalized to enter with a unitary coefficient in the log-linearized

[32] We set the step size to ensure the acceptance rate is in the range of 20 to 40 percent for all variations of the estimated model. Convergence diagnostics include cumulative sum of draws (cumsum) statistics and Geweke's Separated Partial Means (GSPM) test. Results are available from the authors.

equations that determine wages, inflation, investment, and hours per worker respectively. The priors for the standard deviations of shocks are chosen to generate similar volatilities between the variables they directly impact and their data counterparts, as is common practice in the literature.

Table 2: Priors

Parameter	**Prior**			
	dist.*	mean	std.	90% int.
Preferences				
h_C, habit formation	B	0.5	0.1	[0.34, 0.66]
ω, inverse Frisch	G	2	0.5	[1.25, 2.89]
Frictions and Production				
$100 \log \bar{g_A}$, steady state growth rate	N	0.4	0.03	[0.35, 0.45]
ν_K, investment adj. cost	N	4	1.5	[1.53, 6.47]
ϕ_h, hours adj. cost	N	4	1.5	[1.53, 6.47]
ς, capital utilization	B	0.5	0.1	[0.34, 0.66]
$\bar{\eta}$, workers' bargaining power	B	0.5	0.1	[0.34, 0.66]
$b/(w * h)$, replacement rate	B	0.5	0.1	[0.34, 0.66]
τ, covexity vacancy cost	G	2	0.5	[1.25, 2.89]
$\omega^w/1000$, wage stickiness	N	2	0.5	[1.18, 2.82]
ι_w, wage partial indexation	B	0.5	0.15	[0.25, 0.75]
$\omega^p/100$, price stickiness	N	5	1.5	[2.53, 7.47]
ι_p, price partial indexation	B	0.5	0.15	[0.25, 0.75]
Monetary policy				
ϱ_π, interest rate resp. to inflation	N	1.7	0.3	[1.21, 2.19]
ϱ_Y, interest rate resp. to output gap	G	0.125	0.1	[0.02, 0.32]
ϱ_{dY}, interest resp. to output gap growth	N	0.13	0.05	[0.05, 0.21]
ϱ_i, resp. to lagged interest rate	B	0.75	0.1	[0.57, 0.90]
Shocks				
ρ_{g_A}, technology	B	0.5	0.2	[0.17, 0.83]
ρ_β, preference	B	0.5	0.2	[0.17, 0.83]
ρ_{P_K}, investment	B	0.5	0.2	[0.17, 0.83]
ρ_θ, price markup	B	0.5	0.2	[0.17, 0.83]
ρ_η, bargaining	B	0.5	0.2	[0.17, 0.83]
ρ_g, govt cons	B	0.5	0.2	[0.17, 0.83]
$\rho_{\bar{\iota}}$, monetary shock	B	0.5	0.2	[0.17, 0.83]
ρ_h, hours shock	B	0.5	0.2	[0.17, 0.83]
$100\sigma_{g_A}$, technology	IG	0.5	1	[0.01, 0.19]
$100\sigma_\beta$, preference	IG	1	1	[0.01, 0.19]
$100\sigma_{P_K}$, investment	IG	0.1	1	[0.01, 0.19]
$100\sigma_\theta$, price markup	IG	0.1	1	[0.01, 0.19]
$100\sigma_\eta$, bargaining	IG	1	1	[0.01, 0.19]
$100\sigma_g$, govt cons	IG	0.5	1	[0.01, 0.19]
$100\sigma_{\bar{\iota}}$, monetary shock	IG	0.1	1	[0.01, 0.19]
$100\sigma_h$, hours supply shock	IG	0.5	1	[0.01, 0.19]
$100\sigma_{\alpha_h}$, hours production shock	IG	0.1	1	[0.01, 0.19]

*Distributions: N: Normal; G: Gamma; B: Beta; IG: Inverse Gamma.

Posterior Estimates

Table 3 reports the posterior estimates of the benchmark model presented in section 3. As previously discussed, we estimate two versions of this model. The first includes seven observables and seven shocks: TFP, investment, preference, government spending, interest rate, price markup, and bargaining shocks. Parameter estimates from this version are listed under the column "7 obs." The second version includes an additional observable, employment, and an additional labor market shock, the hours-supply shock $\bar{h}_t$. Parameter estimates from this version are listed under the headings "8 obs" and "Benchmark" in Table 3.

Table 3: Posterior Distributions for Estimated Parameters.

Parameter	**Posterior**									
	7 obs		**8 obs**							
			Benchmark Model		**No jr pref**		**No adj cost**		**Preferred Model**	
	mean	90% int	mean	90% int	mean	90% int	mean	90% int	mean	90% int
Preferences										
h_C, habit formation	0.79	[0.74, 0.84]	0.68	[0.64, 0.72]	0.72	[0.68, 0.76]	0.79	[0.73, 0.84]	0.79	[0.73, 0.84]
ω, inverse Frisch	3.33	[2.50, 4.31]	7.03	[5.88, 8.29]	2.57	[1.70, 3.66]	5.53	[4.60, 6.56]	2.73	[1.93, 3.67]
Frictions and Production										
$100 \log \bar{g_A}$, steady state growth rate	0.41	[0.36, 0.45]	0.40	[0.36, 0.44]	0.40	[0.36, 0.45]	0.40	[0.36, 0.44]	0.41	[0.36, 0.45]
ν_K, investment adj. cost	4.87	[3.15, 6.89]	7.04	[5.56, 8.63]	7.27	[5.76, 8.83]	7.27	[5.64, 8.98]	7.86	[6.22, 9.58]
ϕ_h, hours adj. cost	n.e.		n.e.		8.84	[7.06, 10.65]	n.e.		6.14	[4.52, 7.84]
ς, capital utilization	0.53	[0.44, 0.61]	0.51	[0.43, 0.59]	0.51	[0.43, 0.59]	0.42	[0.34, 0.51]	0.45	[0.36, 0.52]
$\bar{\eta}$, workers bargaining power	0.76	[0.64, 0.86]	0.57	[0.44, 0.68]	0.47	[0.33, 0.60]	0.58	[0.45, 0.69]	0.50	[0.38, 0.62]
$b/(w*h)$, replacement rate	0.59	[0.47, 0.69]	0.56	[0.41, 0.70]	0.50	[0.37, 0.61]	0.55	[0.41, 0.68]	0.47	[0.35, 0.59]
τ, convexity vacancy cost	1.26	[0.80, 1.82]	2.71	[2.07, 3.44]	2.93	[2.24, 3.73]	2.60	[1.99, 3.30]	2.78	[2.12, 3.54]
$\omega^w/1000$, wage stickiness	3.19	[2.54, 3.86]	2.68	[2.05, 3.34]	2.65	[2.01, 3.31]	2.79	[2.17, 3.44]	2.77	[2.16, 3.42]
ι_w, wage partial indexation	0.77	[0.60, 0.90]	0.68	[0.52, 0.83]	0.68	[0.51, 0.83]	0.72	[0.57, 0.85]	0.71	[0.56, 0.85]
$\omega^p/100$, price stickiness	5.23	[3.59, 7.09]	6.76	[4.87, 8.79]	6.72	[4.91, 8.68]	6.47	[4.70, 8.42]	6.60	[4.84, 8.57]
ι_p, price partial indexation	0.12	[0.05, 0.20]	0.12	[0.05, 0.21]	0.13	[0.06, 0.22]	0.13	[0.06, 0.22]	0.13	[0.06, 0.22]
Monetary policy										
ϱ_π, interest rate resp. to inflation	1.77	[1.54, 2.03]	1.27	[1.11, 1.46]	1.30	[1.13, 1.49]	1.32	[1.17, 1.49]	1.37	[1.21, 1.56]
ϱ_Y, interest rate resp. to output gap	0.06	[0.02, 0.10]	0.09	[0.06, 0.14]	0.10	[0.06, 0.14]	0.05	[0.02, 0.09]	0.06	[0.02, 0.10]
ϱ_{dY}, interest resp. to output gap growth	0.35	[0.29, 0.41]	0.31	[0.25, 0.37]	0.30	[0.24, 0.36]	0.29	[0.23, 0.34]	0.28	[0.22, 0.34]
ϱ_i, resp. to lagged interest rate	0.77	[0.72, 0.81]	0.73	[0.68, 0.77]	0.73	[0.69, 0.78]	0.73	[0.68, 0.77]	0.75	[0.70, 0.79]
Shocks										
ρ_{g_A}, technology	0.14	[0.06, 0.24]	0.06	[0.02, 0.13]	0.08	[0.02, 0.15]	0.09	[0.03, 0.17]	0.10	[0.03, 0.18]
ρ_β, preference	0.69	[0.59, 0.79]	0.84	[0.77, 0.89]	0.82	[0.75, 0.88]	0.68	[0.54, 0.80]	0.65	[0.50, 0.78]
ρ_{P_K}, investment	0.84	[0.77, 0.89]	0.20	[0.10, 0.30]	0.17	[0.08, 0.26]	0.23	[0.14, 0.33]	0.20	[0.11, 0.30]
ρ_θ, price markup	0.86	[0.80, 0.92]	0.84	[0.79, 0.89]	0.85	[0.80, 0.90]	0.86	[0.81, 0.91]	0.87	[0.82, 0.91]
ρ_η, bargaining	0.36	[0.23, 0.50]	0.15	[0.06, 0.26]	0.18	[0.08, 0.28]	0.14	[0.05, 0.23]	0.15	[0.06, 0.25]
ρ_g, govt cons	0.99	[0.98, 0.99]	0.99	[0.98, 0.99]	0.99	[0.98, 0.99]	0.99	[0.98, 0.99]	0.98	[0.98, 0.99]
$\rho_{\bar{i}}$, monetary shock	0.12	[0.05, 0.22]	0.14	[0.05, 0.24]	0.14	[0.05, 0.24]	0.16	[0.07, 0.26]	0.16	[0.06, 0.26]
ρ_h, hours shock	n.e.		0.97	[0.94, 0.98]	0.96	[0.94, 0.98]	0.96	[0.94, 0.98]	0.97	[0.96, 0.99]
$100\sigma_{g_A}$, technology	0.83	[0.75, 0.91]	1.02	[0.92, 1.12]	1.03	[0.93, 1.14]	1.05	[0.96, 1.16]	1.08	[0.98, 1.19]
$100\sigma_\beta$, preference	2.48	[2.04, 3.04]	2.04	[1.76, 2.36]	2.20	[1.89, 2.55]	2.64	[2.15, 3.29]	2.87	[2.29, 3.63]
$100\sigma_{P_K}$, investment	0.71	[0.60, 0.84]	1.37	[1.18, 1.57]	1.41	[1.23, 1.61]	1.31	[1.13, 1.50]	1.37	[1.19, 1.58]
$100\sigma_\theta$, price markup	0.06	[0.05, 0.07]	0.06	[0.05, 0.07]	0.06	[0.05, 0.07]	0.06	[0.05, 0.07]	0.06	[0.05, 0.07]
$100\sigma_\eta$, bargaining	4.12	[3.40, 4.88]	4.94	[4.33, 5.58]	4.78	[4.17, 5.44]	5.02	[4.43, 5.65]	4.90	[4.30, 5.54]
$100\sigma_g$, govt cons	1.47	[1.34, 1.61]	1.53	[1.39, 1.67]	1.54	[1.40, 1.69]	1.55	[1.41, 1.70]	1.57	[1.43, 1.72]
$100\sigma_{\bar{i}}$, monetary shock	0.24	[0.21, 0.26]	0.24	[0.22, 0.27]	0.24	[0.22, 0.27]	0.24	[0.22, 0.26]	0.24	[0.21, 0.26]
$100\sigma_{\bar{h}}$, hours supply shock	n.e.		3.51	[3.00, 4.08]	5.00	[4.26, 5.81]	2.54	[2.16, 2.97]	3.50	[2.92, 4.15]

Since, as shown below, the benchmark model cannot account for the margins of labor adjustment, we do not extensively comment on the estimated parameter values here. We postpone the discussion to section 5, after having presented our alternative preferred model. Here, we limit ourselves to noting that posterior estimates with seven observables for the inverse Frisch elasticity ω and value of the workers' bargaining power $\bar{\eta}$ are significantly different from those estimated with eight observables. In the seven observable case, the Frisch is estimated to be in the mid-end of microeconomic estimates, which range between 0.1 and 0.6 (see Card, 1991, for a survey). By contrast, in the eight observable scenario, the value is closer to macroeconomic estimates.[33] Concerning the worker's bargaining power, in the seven observable model, the posterior mean for $\bar{\eta}$ is 0.76, slightly above the range commonly used in calibrated models, 0.4 and 0.6. Interestingly, in the eight observable specification, η's posterior mean drops to 0.44, in the ballpark of the estimates by Flinn (2006). All together, these results suggest that the inability of the model to account for the margins of labor adjustment is not intrinsically linked to specific parameterizations of these two labor market parameters.

Model Fit

To understand how well the model fits the data, we compare a set of statistics implied by the model to their data counterparts. Figure 2 plots the correlogram for several aggregate macroeconomic and labor market variables in the data (solid lines), as well as the 90 percent posterior intervals implied by both parameter and small sample uncertainty from the seven observable case (dotted lines) and the eight observable case (dashed lines).[34] We discuss the results of each case in turn.

The literature shows that the benchmark model with only the extensive margin and seven observables–including either total hours or employment–is able to reproduce the joint dynamics of one labor margin and macroeconomic variables (see for instance Gertler, Sala and Trigari (2008)). Estimates of a version of our benchmark model with only the extensive margin are in line with these

[33] Ceteris paribus, ω also affects the disutility of labor which, together with the replacement rate, determines the outside option of the worker. The latter evaluated at the posterior mean is 0.83 with seven observables, while it is 0.78 with eight observables. The larger outside option increases the sensitivity of the surplus and employment to aggregate shocks, other things equal. A high workers' bargaining power also increases employment's response to innovations. Ceteris paribus, a high workers' bargaining power reduces the firm's surplus, making the latter more sensitive to shocks—a mechanism in spirit of Hagedorn and Manovskii (2008).

[34] We sample 10,000 draws from the posterior. For each parameter draw, we generate 100 samples of the observable variables from the model with the same length as our dataset, after first discarding 100 initial observations. We compute statistics for each of these samples. The correlogram for the remaining observables is provided in the Appendix.

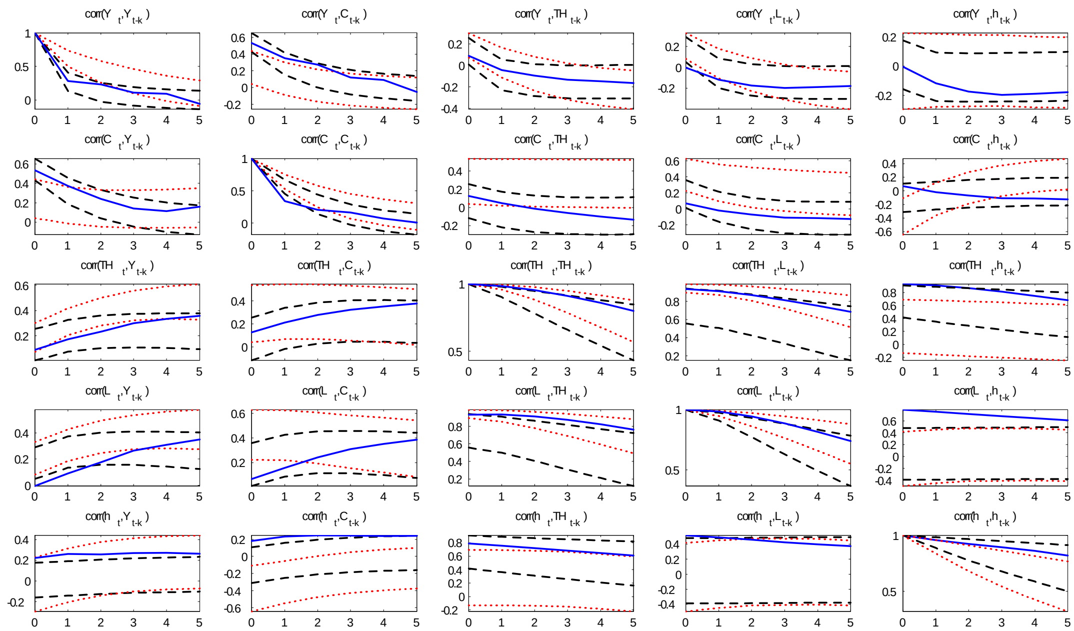

Figure 2. Correlograms from the data (blue solid lines) and 90 percent posterior intervals from 1) the benchmark model with seven observables (red dotted lines) and 2) the benchmark model with eight observables (black dashed lines).

results.[35] However, when the intensive margin is introduced in the model, its ability to account for the correlations between labor market variables and aggregate macroeconomic series is significantly impaired, as evidenced by comparing the data (solid lines) and model (dotted lines) statistics in figure 2. The model overstates the relationship between the growth rate of output with total hours or employment. Even though it correctly reproduces the correlogram between total hours and consumption growth, it does so with a counterfactual comovement of the individual margins with respect to consumption. In addition, the model does not capture the positive correlation between hours and employment nor the relative contributions of the labor margins to the variance of total hours. The model assigns an almost exclusive role to employment, as the 90 percent posterior bands for the share of the labor margin to the variance of total hours (β_L) are between 0.65 and 1.22, while the data counterpart is only 0.51. The β_{cov} ranges from -0.36 to 0.25, well short of the positive comovement (0.31) between hours and employment observed in the data.

Prima facie, the poor performance of the model with seven observables is not too surprising, given the model is estimated with only one labor market observable. However, simply adding

[35] Results available from the authors upon request.

information about the labor market by increasing the set of observables to include simultaneously employment (or hours per worker) and total hours does not improve the performance of the model. The dashed lines of figure 2 report the 90 percent posterior correlogram bands for the benchmark model when employment data and an hours supply shock are incorporated in the estimation.[36] The correlation of hours per worker and consumption growth is still too low relative to the data, while the correlation between employment and output growth is instead too high. Despite providing more information about labor market dynamics, the model still fails to deliver the positive correlation between hours and employment, and the β_{cov} ranges from -0.44 to 0.37. In addition, this version of the model tends to overstate the importance of hours per worker relative to the data, as the posterior for β_h ranges from 0.15 to 0.74, whereas its value is 0.18 in the data. All in all, the benchmark model—independently of the shocks considered or the observables included in the estimation—is unable to replicate satisfactorily the correlation structure between the aggregate macroeconomic series and the labor market variables.

The main issue is that hours per worker tends to be too countercyclical in the model. This result stems from the wealth effect on hours supply. Other things equal, the wealth effect reduces the response of hours as an increase in consumption induces households to value less additional hours worked. To address the shortcomings of the benchmark model, in the next section we propose two modifications that reconcile the model with the data. First, we introduce preferences with a flexible parametrization of the strength of the short-run wealth effect on hours supply. In addition, we also assume adjustment costs to the intensive margin to help dampen the movement in hours. These two ingredients provide a parsimonious strategy to reproduce the correlation of the labor market variables and the macroeconomic series.

5 Alternative Model

Parametrized Wealth Effects in Labor Supply

We modify the period utility function in equation (1) to encompass an alternative preference specification that features a flexible parameterization of the strength of the short-run wealth effects on the labor supply. We consider the class of preferences first introduced by Jaimovich and Rebelo (2009) (JR henceforth). Following Schmitt-Grohe and Uribe (2007), we modify the original JR

[36] The type of shock is not paramount to this result. Alternatively, we could introduce a shock to the efficiency of hours in production and obtain the same results. The key issue, as discussed in section 6, is that the benchmark model's transmission mechanisms work against the covariance of hours per worker and employment observed in the data.

specification to allow for internal consumption habit formation. The period utility function of the representative household is now given by:

$$\frac{1}{1-\sigma}\left(C_t - h_C C_{t-1} - \bar{h}_t X_t \int_0^{L_t} \frac{h_{jt}^{1+\omega}}{1+\omega} dj\right)^{1-\sigma} - \frac{1}{1-\sigma}, \tag{17}$$

where $\gamma \in (0,1]$ and $X_t = (C_t - h_C C_{t-1})^{\gamma} X_{t-1}^{1-\gamma}$. The parameter γ governs the magnitude of the wealth elasticity of labor supply. As $\gamma \to 0$, the argument of the period utility function becomes linear in habit-adjusted consumption and a function of hours worked by employed family members. In the absence of habit formation, and abstracting from time variation in the number of employed family members, given by L_t, this is the preference specification considered by Greenwood, Hercowitz, and Huffman (1988). This special case induces a supply of labor that is independent of the marginal utility of consumption. As a result, when γ is small, anticipated changes in income will not affect the current labor supply. As γ increases, the wealth elasticity of labor supply rises. In the polar case in which γ is unity, per-period utility becomes a product of habit-adjusted consumption and a function of hours worked.

Notice that the term X_t makes preferences non–time-separable in consumption and hours worked. In representative-agents models, the lack of separability does not pose a challenge for the aggregation of preferences across agents, since in the symmetric equilibrium aggregate welfare coincides with the welfare of the representative agent. However, in our model, there are ex-ante two types of agents: employed and unemployed workers. Thus, even in the presence of full risk-sharing within the household, the specification in equation (17) cannot be obtained by aggregating primitive utility functions for employed and unemployed workers. In the Appendix, we present an alternative version of the model that features JR preferences for employed workers and a distinct utility function for unemployed family members. We then aggregate across agents, maintaining the assumption of full risk sharing within the household. In contrast to the specification in equation (17), we must rely on external (rather than internal) habits in consumption in order for the model to preserve its ex-post representative-agent representation.[37]

A key advantage of JR preferences is that they are compatible with long-run balanced growth provided that $\sigma = 1$, which we assume from now on. Thus, the representative household maximizes

[37] As shown in the Appendix, the only difference relative to the benchmark specification in equation (17) is that both the marginal utility of consumption and the value of leisure for an employed worker no longer depend on aggregate employment, L_t. Nevertheless, none of our results are significantly affected.

the expected intertemporal utility function

$$W_t \equiv E_t \sum_{s=t}^{\infty} \beta^{s-t} \bar{\beta}_s \left[\log \left(C_s - h_C C_{s-1} - \bar{h}_s X_s \int_0^{L_s} \frac{h_{js}^{1+\omega}}{1+\omega} dj \right) \right] \tag{18}$$

subject to the sequence of budget constraints given by equations (13) and (14). This alternative preference specification affects the household's stochastic discount factor, since now the marginal utility of consumption, u_{Ct}, is given by:

$$u_{Ct} = \bar{\beta}_t \Psi_t^{-1} + \gamma \mu_t \left(C_t - h_C C_{t-1} \right)^{\gamma-1} X_{t-1}^{1-\gamma} - \beta h_C E_t \left(\bar{\beta}_{t+1} \Psi_{t+1}^{-1} \right) - \gamma \beta h_C E_t \left[\mu_{t+1} \left(C_{t+1} - h_C C_t \right)^{\gamma-1} X_t^{1-\gamma} \right], \tag{19}$$

where

$$\Psi_t \equiv C_t - h_C C_{t-1} - \bar{h}_t X_t \int_0^{L_t} \frac{h_{jt}^{1+\omega}}{1+\omega} dj \tag{20}$$

and

$$\mu_t \equiv -\bar{\beta}_t \Psi_t^{-1} L_t \frac{\bar{h}_t h_{jt}^{1+\omega}}{1+\omega} + (1-\gamma) \beta E_t \left[\mu_{t+1} \left(C_{t+1} - h_C C_t \right)^{\gamma} X_t^{-\gamma} \right].$$

In turn, the alternative preference specification affects the marginal rate of substitution between consumption and leisure, since the marginal disutility of hours for a generic worker j is now given by:

$$W_{h_j t} \equiv \frac{\partial W_t}{\partial h_{jt}} = -\Psi_t^{-1} \bar{\beta}_t \bar{h}_t h_{jt}^{\omega} X_t, \tag{21}$$

while the marginal utility of consumption $\partial W_t / \partial C_t = u_{Ct}$ is defined by equation (19). Notice that the marginal rate of substitution between hours and consumption for worker j, $-W_{h_j t}/u_{Ct}$, only depends on aggregate variables, with the exception of hours worked, h_{jt}.

Hours Adjustment Costs

We modify the production function in equation (2) by introducing hours adjustment costs, capturing various frictions that may constrain the ability of firms to adjust hours per worker, for instance, technological constraints due to set-up costs and coordination issues. We maintain the assumption that each producer is of measure zero relative to the size of the economy.

A filled job in firm j produces

$$(k_{jt})^a \left\{ \bar{A}_t h_{jt} \left[1 - \frac{\phi_h}{2} \left(h_{jt} - h_j \right)^2 \right] \right\}^{1-\alpha}$$

units of the intermediate input, where $\phi_h \geqslant 0$ denotes the cost of adjusting hours per worker, and h_j is the value of hours-per worker along the balanced growth path. Since, as in the benchmark model, all workers produce with identical productivity, we continue to omit the worker-specific index in our notation.

Let $\tilde{h}_{jt}$ denote effective hours used as an input of production:

$$\tilde{h}_{jt} = h_{jt} \left[1 - \frac{\phi_h}{2} (h_{jt} - h_j)^2 \right],$$

such that the job production function can be written more compactly as $(k_{jt})^a \left(\bar{A}_t \tilde{h}_{jt} \right)^{1-\alpha}$. The value of the marginal product of an hour per worker is now given by

$$(1 - \alpha) \varphi_t \left(\frac{k_{jt}}{\bar{A} \tilde{h}_{jt}} \right)^{\alpha} \bar{A}_t \Delta_{\tilde{h}_j t},$$

where

$$\Delta_{\tilde{h}_j t} \equiv \frac{\partial \tilde{h}_{jt}}{\partial h_{jt}} = \frac{\tilde{h}_{jt}}{h_{jt}} - \phi_h h_{jt} (h_{jt} - h_j).$$

Hours per Worker

Optimality in hours per worker, h_{jt}, continues to equate the worker's marginal rate of substitution between consumption and leisure to the value of the marginal value product of an extra hour worked:

$$-\frac{W_{h_j t}}{u_{Ct}} = (1 - \alpha) \varphi_t \left(\frac{k_{jt}}{\bar{A} \tilde{h}_{jt}} \right)^{\alpha} \bar{A}_t \Delta_{\tilde{h}_j t},$$

where $W_{h_j t} \equiv \partial W_t / \partial h_{jt}$ is defined by equation (21). Owing to perfectly mobile capital across jobs, the optimal capital allocation for each job continues to equate the value of the marginal product of capital to its marginal cost:

$$\alpha \varphi_t \left(\frac{k_{jt}}{\bar{A} \tilde{h}_{jt}} \right)^{\alpha - 1} = r_{Kt}. \quad (22)$$

Therefore, hours per worker satisfy the following optimality condition:

$$\Psi_t^{-1} \bar{\beta}_t \bar{h}_t h_{jt}^{\omega} X_t = (1 - \alpha) \varphi_t \left(\frac{r_{Kt}}{\alpha \varphi_t} \right)^{\frac{\alpha}{\alpha - 1}} \bar{A}_t \Delta_{\tilde{h}_j t}. \quad (23)$$

Equation (23) implies that hours per worker, h_{jt}, continue to depend only on aggregate conditions, so that $h_{jt} = h_t$ (and thus $\tilde{h}_{jt} = \tilde{h}_t$).[38] Thus, hours per worker do not depend on firm-level employment, i.e., $\partial h_{jt}/\partial L_{jt} = 0$. Notice also that equation (22) implies that $k_{jt} = k_t$. Thus, total output exhibits constant returns to scale in total effective hours, $L_{jt}\tilde{h}_t$, and capital:

$$Y_{jt}^I \equiv \int_0^{L_{jt}} (k_{jt})^a \left(\bar{A}_t \tilde{h}_t\right)^{1-\alpha} dj = K_{jt}^{\alpha} \left(\bar{A}_t L_{jt} \tilde{h}_t\right)^{1-\alpha}, \tag{24}$$

where $K_{jt} = L_{jt}k_t$ is the total amount of capital used by the intermediate input producer j.

Job Creation and Wage Bargaining

The representative producer j chooses V_{jt}, L_{jt}, and K_{jt} to maximize equation (4) subject to equations (3) and (24). The first-order condition for L_{jt}, which as before defines the marginal value of hiring an extra worker, implies:

$$S_{jt}^f = (1-\alpha)\,\varphi_t \left(\frac{K_{jt}}{\bar{A}L_{jt}\tilde{h}_t}\right)^{\alpha} \bar{A}_t \tilde{h}_{jt} - \frac{w_{jt}^n h_t}{P_t} - \Gamma_{w_j t} + E_t \beta_{t,t+1} (1-\lambda)\, S_{jt+1}^f.$$

Notice that, as in the benchmark model, the optimality condition above exploits the fact that $\partial h_{jt}/\partial L_{jt} = \partial w_{jt}^n/\partial L_{jt} = 0$. The reason why the hourly wage remains independent of the scale of the firm is that the firm and worker surplus, S_{jt}^f and S_{jt}^w, do not depend on firm-level employment, L_{jt}. To see this, notice first that using again the optimality condition for capital, the firm's surplus can be written as

$$S_{jt}^f = (1-\alpha)\,\varphi_t \left(\frac{r_{Kt}}{\varphi_t \alpha}\right)^{\frac{\alpha}{\alpha-1}} \bar{A}_t \tilde{h}_t - \frac{w_{jt}^n h_t}{P_t} - \Gamma_{wt} + E_t \beta_{t,t+1} (1-\lambda)\, S_{jt+1}^f.$$

Moreover, the worker's surplus is given by

$$S_{jt}^w = \frac{w_{jt}^n}{P_t} h_t - b\bar{A}_t - \Psi_t^{-1} \bar{\beta}_t \bar{h}_t h_t^{\omega} X_t + E_t \left[\beta_{t,t+1} (1-\lambda)\, S_{jt+1}^w \left(1 - \frac{M_{t+1}}{U_{t+1}}\right)\right].$$

Thus, the equilibrium wage differs from what is implied by the sharing rule in equation (11) only because of the different definition of the value of the marginal product of labor and the different flow value of unemployment implied by the parametrized wealth effect on the labor supply. In the symmetric equilibrium, $K_{jt} = K_t$ and $L_{jt} = L_t$, as in the benchmark model.

[38] The term Ψ_t depends on aggregate employment, L_t. Since the firm is of measure zero relative to the economy, $\partial L_t/\partial L_{jt} = 0$.

As shown in Table A.3 in the Appendix, hours adjustment costs and the alternative preference specification affect three equilibrium conditions—equations (4), (14) and (15) in Table A.2, and three definitions—equations D.4-D.6 in Table A.2. Moreover Table A.3 contains the definitions for the variables μ_t, Ψ_t, X_t, $\tilde{h}_t$, and $\Delta_{\tilde{h}t}$ presented above.

6 Hours and Employment in Post-War U.S. Business Cycles

This section contains the econometric analysis of the model with JR preferences and hours adjustment costs, which we reference as our preferred model. We first discuses the prior and posterior distributions of parameters as well as the ability of the model to fit the data. Next, we study the propagation of structural disturbances and present a Bayesian counterfactual experiment to assess the importance of the intensive margin in U.S. recoveries.

Estimation and Model Performance

We estimate the model with the eight observables, including total hours and employment data. To our knowledge, we are the first to incorporate a form of adjustment costs on hours per worker in an estimated model. For symmetry, we employ the same prior for hours adjustment costs as for investment adjustment costs, a normal distribution centered at 4 with a standard deviation of 1.5. This prior is diffuse enough to allow positive mass over a wide range of low and high adjustment cost values. We use a dogmatic prior for the parameter governing the strength of the wealth effect in labor supply, setting $\gamma = 0.01$. This value is sufficiently small to approach the limiting case of no wealth effects.[39]. The priors for the remaining parameters are the same as those discussed in Section 4.

Table 3 lists the posterior mean and 90 percentile estimates of the model under the column headings "8 obs" and "Preferred Model." For comparison, we also list estimates when only one additional feature is included—either JR preferences or the hours adjustment cost. First, notice that the estimate of the Frisch elasticity in the preferred model, in contrast to the benchmark model with eight observables, is close to micro estimates. Second, the posterior mean of the hours adhjustment cost, ϕ_h, implies that to increase $\tilde{h}_t$—effective hours in the production function—by one percent, hours per worker need to increase by an additional 0.025 percent because of the

[39] We have estimated a version of the model with a Beta prior for γ centered at 0.5 with a standard deviation of 0.1. The posterior mean for γ in this case is 0.14, outside the 90 percent prior bands. Lowering the prior mean of γ results in lower posterior estimates and similar transmission mechanisms as our calibrated version.

adjustment cost. Notice that, although both ϕ_h and the (higher) values of the Frisch elasticity tend to reduce the sensitivity of hours worked to changes in the value of the marginal product of hours, the two parameters are not observationally equivalent. In particular, ϕ_h only enters the equation that determines hours, while the Frisch elasticity also affects the outside option of the worker.

The posterior mean of the replacement rate $b/(wh)$ is 0.50 in the preferred model, while its 90 percentile interval includes the 0.4 value commonly assumed by some (see for instance Shimer, 2005). In addition, the posterior interval is tighter relative to the prior. The posterior mean of the flow value of unemployment—the sum of the unemployment benefit and the real value of leisure—relative to the steady-state wage is larger and equal to 0.75, closer to the value assumed by Hall (2008). By contrast, in the benchmark model, the replacement rate tends to be larger, with a 90 percentile interval between 0.51 and 0.80. The estimate for the bargaining power remains close to the value proposed by Flinn (2006).

The other estimated parameters are affected little across the different specifications reported in table 3 with eight observables. The shock processes in the preferred model in general have lower persistence and larger standard deviations, for instance the process for the preference and the investment-specific shock. Overall the variability of the exogenous variables is similar across specifications.

Figure 3 plots the correlogram for several aggregate macroeconomic and labor market variables in the data (solid lines), as well as the 90 percent posterior intervals implied by both parameter and small sample uncertainty from this preferred model (dashed lines) and the benchmark model with eight observables (dotted lines). In almost all cases, the correlogram bands for the preferred model encapsulate the data counterparts, whereas the benchmark model often fails to account for the cross-correlation structure of labor variables and macroaggregates.

The preferred model's preferences significantly improve the performance of the model through two channels. First, a smaller wealth effect implies that an increase in consumption induces household to value more additional hours worked, raising the correlation between hours and consumption. Second, the preferred model features non-separable preferences. In this case, the marginal utility of consumption depends upon the disutility of labor. When the two margins of labor increase, the marginal utility of consumption also rises, prompting households to consume more. This reinforces the comovement between hours and consumption. A reduction in the wealth effect also contributes to more volatile hours, realigning its relative contribution to total hours. This, in turn, explains the data's preference for large adjustment cost to hours, as they readjust the variability of hours to be

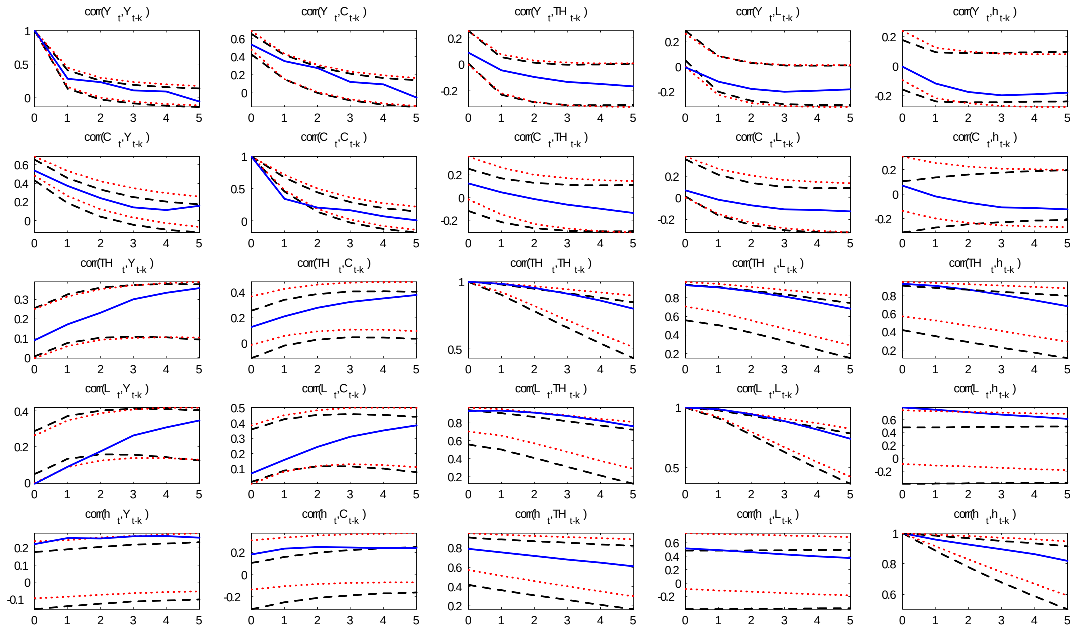

Figure 3. Correlograms from the data (blue solid lines) and 90 percent posterior intervals from 1) the preferred model with JR preferences and hours adjustment costs (red dotted lines) and 2) the benchmark model with eight observables (black dashed lines).

comparable to the data. Additionally, this preferred specification implies variance decompositions of total hours more united with the data counterparts: β_h ranges from 0.12 to 0.54, β_L from 0.20 to 0.72, and β_{cov} from -0.05 to 0.44.

Table 4 reports the log marginal data densities and Bayes factors for various specifications of our model. Bayes factors quantify the relative support of two competing specifications given the observed data and are calculated from marginal data densities, see Kass and Raftery (1995). Log marginal data densities are computed using Geweke's (1999) modified harmonic mean estimator with a truncation parameter of 0.5.[40] Higher log marginal data density values imply greater fit. Kass and Raftery (1995) suggest that if twice the natural logarithm of the Bayes factors is greater than 2, then there is positive evidence in favor of the first model. Values greater than 10 suggest very strong evidence. All cases from Table 4 have values substantially larger than 10, suggesting the data have strong preference for the model with JR preferences and hours adjustment costs.

[40] Model rankings are invariant to alternative truncation parameter choices. We restrict analysis to the parameter subspace that delivers a unique rational expectations equilibrium and denote this subspace as Θ_D. Let $\mathcal{I}\{\theta \in \Theta_D\}$ be an indicator function that is one if the parameter vector θ is in the determinacy region and zero otherwise. Then, the joint prior distribution is defined as $p(\theta) = \frac{1}{c}\tilde{p}(\theta)\mathcal{I}\{\theta \in \Theta_D\}$, where $c = \int_{\theta \in \Theta_D} \tilde{p}(\theta) d\theta$ and $\tilde{p}(\theta)$ denotes the joint prior density.

The Bayes factors also suggest that respectively these two model features substantially improve the model's fit relative to the benchmark specification.

Table 4: Log Marginal Data Densities and Bayes Factors

Specification	Log Marginal Data Density	$2\ln$(Bayes Factor) vs. $\mathcal{M}_1$
Benchmark Model, $\mathcal{M}_1$	-1020	0
Model without hours adjustment costs, $\mathcal{M}_2$	-1039	38
Model without JR preferences, $\mathcal{M}_3$	-1037	34
Preferred Model, $\mathcal{M}_4$	-1069	98

As a final check on the performance of our preferred model, we perform the following counterfactual. First, we use the posterior mean estimates from the benchmark model estimated with seven observables to obtain the model's predicted series for the seven structural shocks (TFP, investment, preference, government spending, interest rate, price markup, and bargaining power) using the two-sided Kalman filter. Next, these filtered seven structural shocks are used to simulate variables from two models: (1) the benchmark seven shock model and (2) the preferred model at its posterior mean estimates. Figure 4 displays the labor market variables and output growth generated from the benchmark model (dashed lines), and the preferred model (solid lines), as well as the data (dotted-dashed lines). Since the benchmark model includes total hours and output growth as observables, by construction the two-sided Kalman filter ensures the benchmark model perfectly matches these data series. However, the benchmark model matches total hours only with fictitious employment and hours per worker series. In contrast, the preferred model's implied employment and total hours series track the data well. It is important to note that the preferred model series are not generated using the two-sided Kalman filter, but rather from the benchmark model's seven structural series. Thus, it is not surprising that the preferred model does not perfectly match the total hours or output growth series. Nonetheless, it matches these series quite well in the counterfactual while additionally improving the fit of the individual labor margins. This result confirms the preferred model's fit stems from internal propagation, as opposed to being induced from the addition of an hours supply shock.

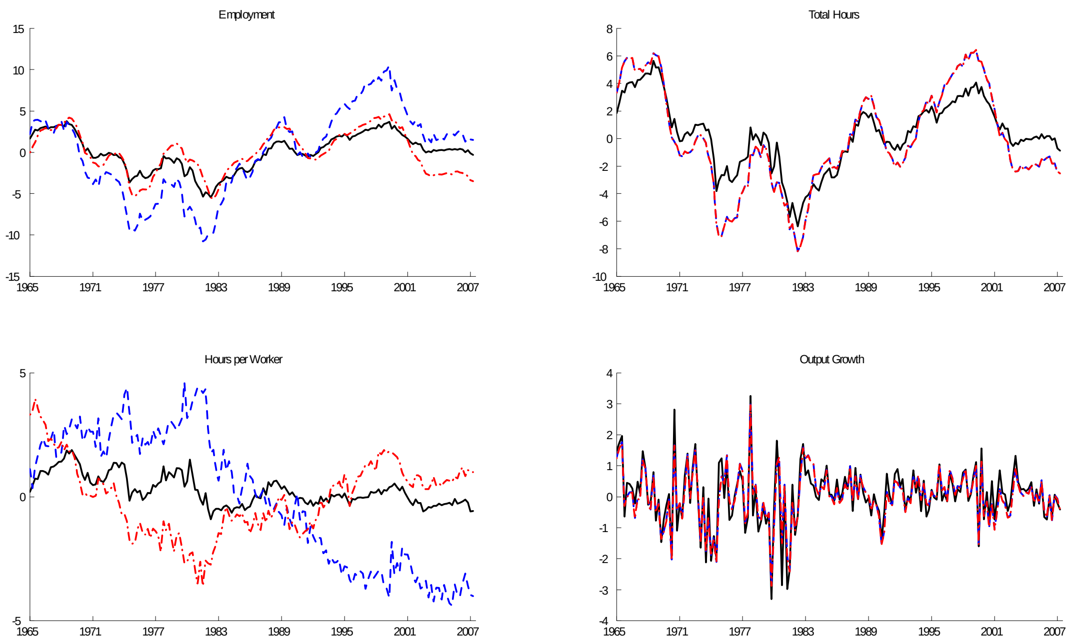

Figure 4. Fitted and counterfactual variables. Blue dashed lines are simulated from the posterior mean estimates of the benchmark model with seven structural shocks. Black solid lines are simulated from the posterior mean estimates of the preferred model using the benchmark model's seven shock series. Red dotted-dashed lines denote the data.

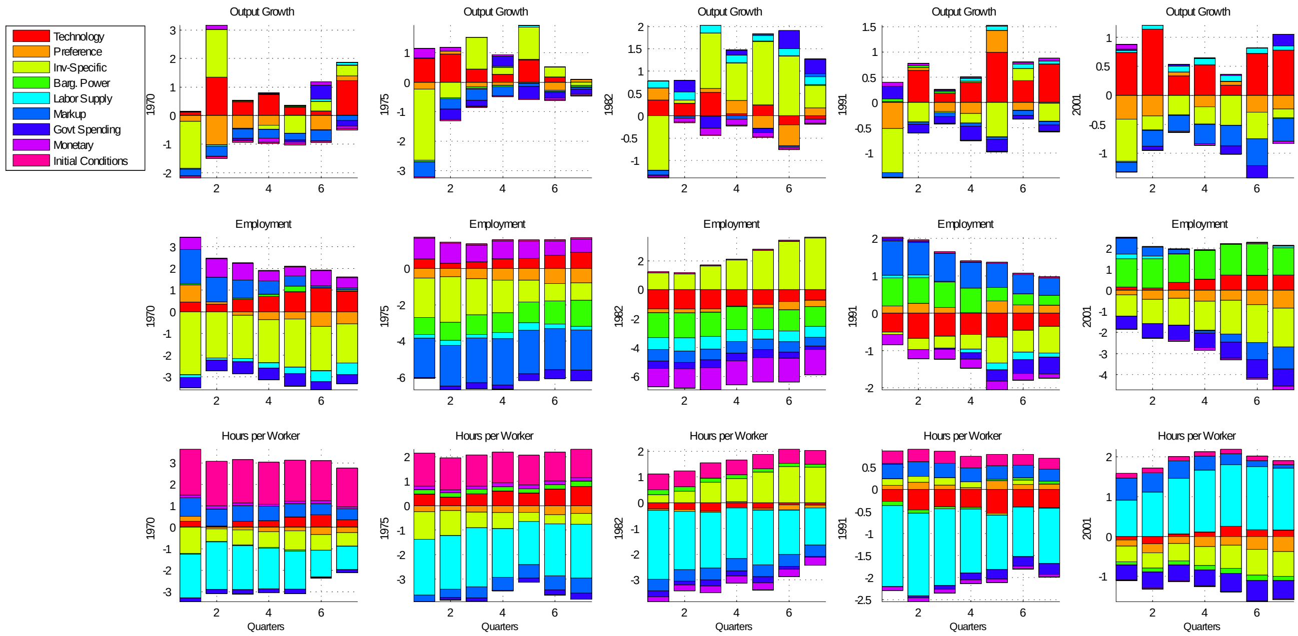

Figure 5. Historical decomposition for US business cycle recoveries.

Aggregate Shocks and the Margins of Labor Adjustment

To further examine the differences in the preferred and benchmark models' transmission channels, we examine the propagation mechanism of individual shocks, focusing on the adjustment of the two labor margins. For the two model specifications, we focus on the dynamics following innovations to aggregate TFP, investment-specific productivity, preference, worker's bargaining power and to the nominal interest rate. These shocks are important drivers of economic fluctuations since they account for 90 percent of the variance of the growth rate of output, consumption and investment on impact and 10 periods after the shocks. For total hours, the contribution is 80 percent on impact and 60 percent after 10 periods.[41] (The Appendix presents the full details about variance decompositions.) In addition, these shocks are important driving forces in U.S. recoveries. To see this, figure 5 plots the historical decomposition of labor market variables and output growth using the posterior mean estimates of the preferred model. The decompositions identify productivity shocks, aggregate or investment-specific, and bargaining power shocks as the most significant forces to labor market recoveries.

[41] Markup shocks account for 25 percent of the variance in total hours at period 10. We do not report the impulse responses following an innovation to the elasticity of substitution across goods because they are qualitatively and quantitatively similar across the preferred and the benchmark model.

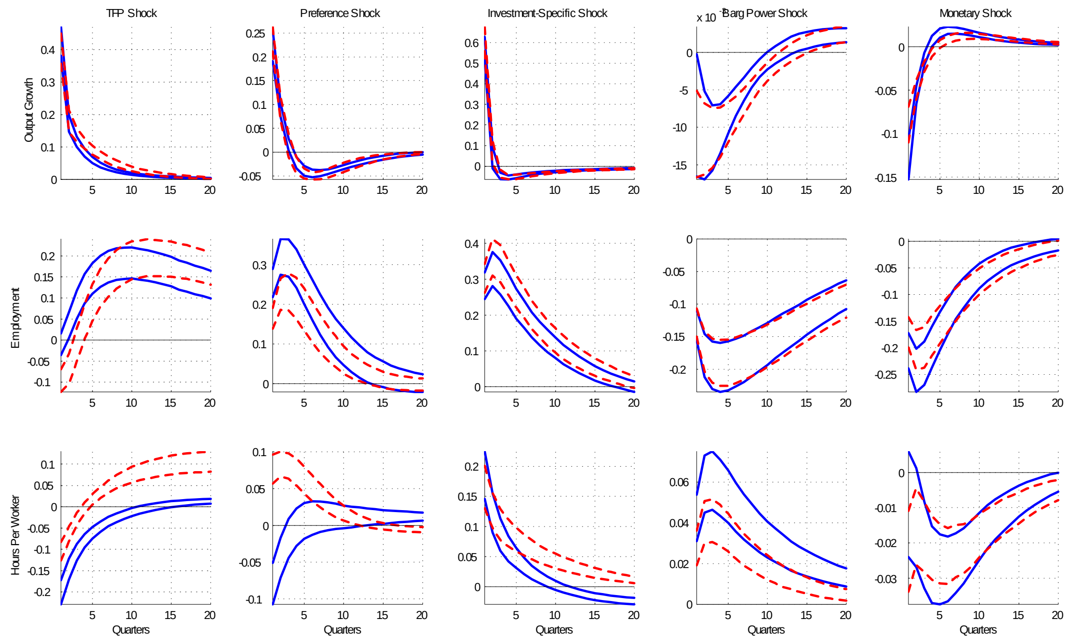

Figure 6. Impulse response following a standard deviation innovation. Bands represent 90 percent confidence intervals.

Figure 6 reports the 90 percent posterior intervals for the impulse responses of output growth, employment, and hours per worker to aggregate TFP, investment-specific productivity, preference, worker's bargaining power and nominal interest rate shocks. Solid lines denote the responses of the benchmark model estimated with eight observables, while dashed lines correspond to the preferred framework. In all cases, responses are computed following a one standard deviation shock. As reported in Table 3, the estimated persistence and standard deviations of innovations are very similar across the benchmark and preferred specifications, suggesting that the improved fit can be traced to an improvement in the propagation mechanism rather than to different estimates of the shock processes.

The first column displays the responses following a positive shock to the growth rate of aggregate productivity. Other things equal, price stickiness induces lower labor demand, rather than lower goods prices. However, in the benchmark model, the brunt of the impact adjustment of total hours is on the intensive margin, as higher productivity induces a positive wealth effect that reduces labor supply. By contrast, employment is virtually unaffected initially. The decrease in hours per worker reduces the flow value of unemployment, leading to wage moderation. As a consequence, the surplus of hiring a worker increases, leading to higher employment after the first period. The relative contribution of the two margins is altered in our preferred model. JR preferences reduce the wealth effect on the labor supply, causing hours per worker to drop less on impact. This, in turn, reduces its effect on the firm's surplus, leading employment to decline on impact as well. Thus, reducing the wealth effect on the labor supply induces positive comovement between the intensive and the extensive margin. A similar mechanism is at work following an increase in the degree of impatience of households—i.e., the preference shock (column two of figure 6). In this case, households substitute from investment to consumption. Higher aggregate demand boosts employment in both models. However, in the preferred model, due again to the limited wealth effect, the expansionary demand shock results in an increase in hours per worker (rather than in a fall, as in the benchmark model), and thus implies a positive comovement with employment. The same logic applies to the monetary shock as well (column five), with the exception that the increase in the policy rate translates into reductions in demand, as the real interest rate increases.

An increase in productivity specific to the production of the investment good displays positive comovement between the labor margins in both specifications (column three of figure 6). In this case, the wealth effect is small, independently of the particular form of preferences assumed because of the low estimated persistence of the shock. The limited persistence implies a short-lived increase in output growth with little effect on permanent income and consumption. As a result, the wealth effect is not large enough to induce a negative comovement between hours per worker and employment on impact.

The four shocks analyzed thus far directly affect both labor margins. In contrast, the exogenous disturbance to the workers' bargaining power directly affects only employment (column four of figure 6). Following an increase in the workers' bargaining power, workers appropriate a larger share of the surplus through higher wages. Firms have less incentives to create jobs and shift resources toward a relatively cheaper intensive margin. The shock is recessionary as it increases the cost of production, leading output, investment and consumption to decline. The impulse responses are qualitatively similar in the benchmark and preferred models, although hours per worker in the preferred model, insulated by the wealth effect, tends to respond less.

Employment and Hours in U.S. Cyclical Recoveries

In this section, we use the preferred model to empirically assess the role of hours per worker in determining employment outcomes. We focus on U.S. business cycle recoveries — i.e. the progression of the economy after having hit the trough of a recession — since the topic recently has received attention in policy circles due to the so-called jobless recoveries (see Bernanke, 2003). Our model provides an ideal laboratory to quantify the contribution of the intensive margin for employment outcomes. Toward this goal, we perform the following counterfactual. First, we use the posterior mean estimates of the preferred model and the two-sided Kalman filter to construct smoothed estimates of the structural shocks and model variables. We then construct a counterfactual time series in each recovery where hours are held constant at their steady state value starting at the trough. In each episode, we initialize the economy using the smoothed estimates and then compare the actual path to the hypothetical one where hours per worker are constant. Our results indicate that the contribution of hours per worker to the employment recovery — i.e. whether hours per worker and employment displayed substitutability or complementarity—depends upon the structural disturbances that were responsible for labor market fluctuations.

The Recoveries of 1975 and 1982

Figure 7 contrasts the actual values of the growth rate of GDP, employment and hours per worker (dashed lines) with the model counterfactual values (dotted-dashed lines).[42] Figure 7 shows that the contribution of hours adjustment during U.S. cyclical recoveries is significant. Importantly, the direction of this effect can be either positive or negative. In the recoveries of 1970, 1975, 1982, and 2001, employment would have been, on average, half of a percentage point higher in the absence of any adjustment along the intensive margin. In the recovery of 1991, employment would have been 0.4 percentage points lower without hours adjustment.

To understand these results, notice that the channel through which the intensive margin affects employment outcomes ultimately depends on the nature of the shocks driving employment fluctuations. During the recoveries of 1975 and 1982, employment was driven in part by labor market disturbances, namely negative shocks to the bargaining power of workers that depressed employment. This shock generates a negative comovement between hours per worker and employment. Since an increase in the bargaining power of workers reduces the surplus of the firm, producers shift to the (relatively cheaper) hours margin, which reduces the surplus of the firm and employment even further. In the counterfactual economy with constant hours, this secondary effect is shut down, leading employment to be higher.

In all the other recoveries, employment was driven by standard supply and demand shocks,

[42] Since the growth rate of GDP, employment, and total hours are observables, the smoothed estimates of these variables from the two-sided Kalman filter, as well as hours per worker, perfectly match the data by construction.

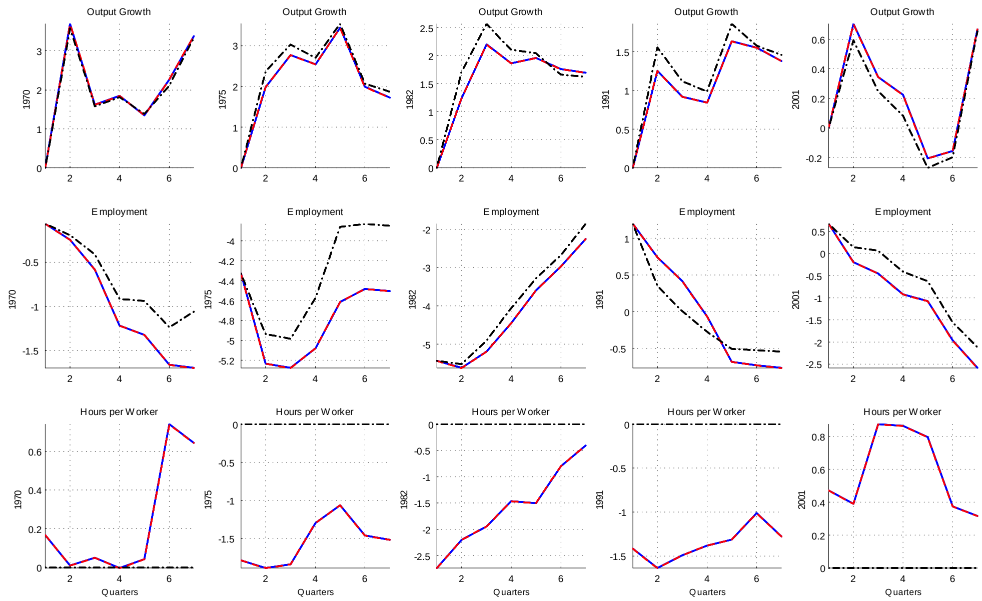

Figure 7. Recoveries relative to GDP trough. Blue solid lines: actual data. Black dotted-dashed lines: Counterfactual with hours per worker constant at his trend level from the trough-on. Red dashed lines: preferred model. Output growth is normalized to zero at the trough.

which result in positive comovement of the margins of labor. In this case, whether the presence of the intensive margin boosts or reduces employment depends on whether the absence of hours adjustment increases or reduces the total labor input. This is because, conditional on shocks that induce positive comovement, nominal rigidities imply that firms are forced to adjust their labor force to meet a given demand. In 1970 and 2001, the absence of hours adjustment would have reduced aggregate hours. As a result, firms would have reacted by further increasing employment. In 1991, the scenario is reversed.

Our results demonstrate that in order to evaluate the contribution of hours per worker to employment, one needs to account for the particular disturbances driving the economy in specific episodes.

7 Sensitivity Analysis

To be updated

We investigate the robustness of our results under several alternative specifications. The results of these robustness checks are summarized in table 5. To get a sense of how well the model accounts for the labor market variables, we report for each specification the shares of the variance of total hours attributed to hours per worker, employment, and their covariance. We discuss each case in turn.

		$\tilde{\beta}_{cov,h}$ $\left(\frac{cov(TH_t,h_t)}{var(TH_t)}\right)$	$\tilde{\beta}_{cov,L}$ $\left(\frac{cov(TH_t,L_t)}{var(TH_t)}\right)$	β_h $\left(\frac{var(h_t)}{var(TH_t)}\right)$	β_L $\left(\frac{var(L_t)}{var(TH_t)}\right)$	β_{cov} $\left(\frac{2cov(h_t,L_t)}{var(TH_t)}\right)$
Data		**0.33**	**0.67**	**0.18**	**0.51**	**0.31**
	Preferred Model, no wage obs					
	Preferred Model, mix wage obs					
	7 obs, $\bar{h}$ shock	[0.33, 0.73]	[0.27, 0.67]	[0.17, 0.58]	[0.11, 0.53]	[0.15, 0.43]
	Preferred Model, hours production shock	[0.19, 0.61]	[0.39, 0.81]	[0.11, 0.53]	[0.25, 0.79]	[-0.12, 0.42]
CPS Data		**0.15**	**0.85**	**0.07**	**0.78**	**0.15**
	Preferred Model					

Robustness checks for the Shares of the Variance of Total Hours from Alternative Estimated Specifications. 90 percent posterior intervals reported in parenthesis.

Wage Data

Disparate views exist in the literature as to the appropriate empirical counterpart for the model wage. Thus, we document the robustness of the results to the wage observable. Towards this end, we first consider a specification where we drop wages and the bargaining power shock from the set of observables. In this case, we further assume that wage adjustment is flexible, and employment volatility stems from a higher value of the flow value of unemployment. Row "Preferred Model, no wage obs" of table 5 displays the total hours variance shares in this case. The 90 percent bands are quite similar to the estimated model with wage observables and wage stickiness, demonstrating the irrelevance of the results to the specific wage adjustment process.

In addition, we estimate a version of the preferred model in which three measures of the wage are simultaneously included in the observables. This strategy has been recently used by several papers in the literature (see for instance Boivin and Giannoni (2006), Gali, Semts and Wouters (2011), and Justiniano, Primiceri and Tambalotti (2013)). The first is the measure described in section 4, which is the BLS' hourly compensation for the nonfarm business sector. The second measure is the BLS' average hourly earnings of production and nonsupervisory employees. The third measure is the quality adjusted wage series of Haefke, Sonntag and van Rens (2013), which adjusts for individual-level characteristics. Additional details of the data and estimation are relegated to the Appendix. We assume that each series represents an imperfect measure of the model wage according to:

$$\begin{bmatrix} \text{Comp Wage}_t \\ \text{Earn Wage}_t \\ \text{Quality Wage}_t \end{bmatrix} = \begin{bmatrix} 1 \\ \Gamma_1 \\ \Gamma_2 \end{bmatrix} (\hat{w}_t - \hat{w}_{t-1} + \hat{g}_{At}) + \begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \end{bmatrix}$$

where e_{it} for $i = 1, 2, 3$ denote i.i.d. observation errors. Row "Preferred Model, mix wage obs" of Table 5 displays the total hours variance shares in this case. Again, the model bands well encompass the data.

Alternative Shocks

We explore the sensitivity of the results to the inclusion of alternative structural shocks. First, we estimate the benchmark model with seven observables when the hours supply shock is included as opposed to the bargaining power shock. Hours supply shocks can potentially improve the model's fit with respect to the labor market variables, as they directly affect the intensive labor margin. The total hours variance shares in this case are listed in row "7 obs, $\bar{h}$ shock" of table 3. While the hours supply shock does ensure the model matches the covariance of employment and hours per worker, it does so with a counterfactually high volatility of hours per worker. Indeed the relative ranking of the contribution of hours per worker and employment are reversed, as $\tilde{\beta}_{cov,h}$'s bands range from 0.33 to 0.73, while the bands of $\tilde{\beta}_{cov,L}$ only range from 0.27 to 0.67.

Next, we consider the performance of our preferred model with an alternative labor market shock. Instead of a shock to hours supply, we consider a shock to the efficiency of hours per worker in production (see the Appendix for model details). The results are presented in row "Preferred Model, hours production shock" of table 3. In this case, the posterior bands for the model's βs well-encompass their data counterparts.

Alternative Labor Market Variables

We check whether our results are sensitive to the labor market measures used for the estimation. We estimate the model using CPS labor market variables, as in Ramey (2012).[43] In this case, neither total hours nor employment are linearly detrended; the two variables are demeaned. Parameter estimates in this case are comparable to those in table 3. Log marginal data densities suggest strong preference for the model with JR preferences and hours adjustment costs as well. As shown in table 5, the posterior bands for the model's βs well-encompass their data counterparts. In addition, these results are robust to using the Smets and Wouters (2007) labor market observables for estimation.[44]

[43] See section A for a description of this labor market data.

[44] Results available from the authors upon request.

8 Conclusions

Aggregate hours vary with adjustments to two margins—fluctuations in average hours per worker (the intensive margin) and movements in and out of employment (the extensive margin). Changes in hours per worker account for up to 30 percent of the variation in total hours in the U.S., and the positive covariance of hours per worker and employment is a substantial contributor to the variability of total hours as well. We document the robustness of these findings and show that both the co-movement and the relative contribution of the intensive margin varies in specific, cyclical U.S. recoveries.

To determine the quantitative significance of these empirical observations, we estimate a Mortensen-Pissarides search and matching model augmented with endogenous fluctuations in hours per worker and shocks that affect both margins of labor adjustment. We show that this benchmark model is unable to replicate the correlation structure between aggregate macroeconomic series and the labor market variables. Two proposed modifications reconcile the model with the data: adjustment costs to the intensive margin and a flexible parametrization of the strength of the short-run wealth effect on hours supply, as first introduced by Jaimovich and Rebelo (2009). We use the modified model to structurally assess the contribution of the intensive margin of labor adjustment to aggregate dynamics. We find the contribution of hours adjustment during U.S. cyclical recoveries is significant and can be either positive or negative depending on the innovations in the economy. Our results demonstrate that in order to evaluate the contribution of hours per worker to employment, one needs to account for the particular disturbances driving the economy in specific episodes.

While we estimate the model on U.S. data, our model introduces enough flexibility to allow the model to match a broad array of empirical covariances between hours per worker and employment, including potentially negative ones as observed in some European economies. Discerning the role of the intensive margin for other countries, as well the introduction and study of country-specific labor market policies, are important avenues for future research.

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Appendix

A Data Description

We consider three alternative sets of labor market variables. The estimation in the main text uses the first set, while the alternative sets are considered for robustness. We apply the following transformation to Total Hours and Employment: $\ln\left(\frac{x}{Pop}\right) * 100$ where Pop is the civilian noninstitutional population (series LNU00000000 of the BLS). We apply the following transformation to hours per worker: $\ln(h) * 100$.

1. Current Employment Statistics (CES) data.

Total Hours, TH is economy-wide total hours measure of the BLS (`www.bls.gov/lpc/special_requests/us_total_hrs_emp.xlsx`).

Employment, L is the economy-wide employment series of the BLS (from same source as total hours).

Hours, h is average weekly hours calculated as $(TH/L)/52$.

2. Current Population Survey (CPS) data.

Total Hours, TH is an economy-wide measure constructed by combining the total hours series of Cociuba, Prescott, and Ueberfeldt (2012), which is constructed from the BLS' CPS data for all industries, with total hours of the armed forces, taken from `www.bls.gov/lpc/special_requests/us_total_hrs_emp.xlsx`. Ramey (2012) constructs a total hours series in a similar manner.

Employment, L is an economy-wide employment series constructed from combining CPS employment for all industries with armed forces employment (from same sources as total hours).

Hours, h is average weekly hours calculated as $(TH/L)/52$.

3. Smets-Wouters (SW) data.

Hours, h is defined as the index for nonfarm business, all persons, average weekly hours duration, 2009 = 100, seasonally adjusted (from the Major Sector Productivity and Cost series PRS85006023 of the BLS).

Employment, L is civilian employment for all industries for ages sixteen years and over, seasonally adjusted (from the CPS series LNS12000000Q of the BLS).

Total Hours, TH is calculated as $h * L$.

In addition to the labor market variables, the following data are used for estimation. Unless otherwise noted, data are from the National Income and Product Accounts tables of the Bureau of Economic Analysis.

GDP. Gross domestic product (Table 1.1.5 line 1). Output (y) growth is $100\left[\ln\left(\frac{y_t}{gdpp_t pop_t}\right) - \ln\left(\frac{y_{t-1}}{gdpp_{t-1} pop_{t-1}}\right)\right]$ where $gdpp$ is the GDP deflator (Table 1.1.4, line 1) and Pop is the civilian noninstitutional population (series LNU00000000 of the BLS).

Consumption. Total personal consumption expenditures on nondurables and services (Table 1.1.5, lines 5 and 6). Consumption (c) growth is $100\left[\ln\left(\frac{c_t}{gdpp_t pop_t}\right) - \ln\left(\frac{c_{t-1}}{gdpp_{t-1} pop_{t-1}}\right)\right]$ where $gdpp$ is the GDP deflator (Table 1.1.4, line 1) and Pop is the civilian noninstitutional population (series LNU00000000 of the BLS).

Investment. Gross private domestic investment (Table 1.1.5, line 7) and personal consumption expenditures on durables (Table 1.1.5, line 4). Investment (i) growth is $100\left[\ln\left(\frac{i_t}{gdpp_t pop_t}\right) - \ln\left(\frac{i_{t-1}}{gdpp_{t-1} pop_{t-1}}\right)\right]$ where $gdpp$ is the GDP deflator (Table 1.1.4, line 1) and Pop is the civilian noninstitutional population (series LNU00000000 of the BLS).

Wage Rate. The wage rate w is the index for hourly compensation for nonfarm business, all persons, 2009 = 100 (from the Major Sector Productivity and Cost series PRS85006103 of the BLS). Wage growth is $100\left[\ln\left(\frac{w_t}{gdpp_t}\right) - \ln\left(\frac{w_{t-1}}{gdpp_{t-1}}\right)\right]$ where $gdpp$ is the GDP deflator (Table 1.1.4, line 1).

Inflation. The gross inflation rate is the log first difference of the GDP deflator (Table 1.1.4, line 1).

Interest Rate. The nominal interest rate is the average of daily figures of the Federal Funds Rate (from the Board of Governors of the Federal Reserve System) divided by 4.

Inflation and the interest rate are demeaned, while total hours and employment are linearly detrended. Figure 1 illustrates our preference for linearly detrended data. Over the sample period, hours per worker exhibits a downward trend while employment exhibits an upward trend. When these (logged) variables are linearly detrended, their sum almost perfectly matches the original, demeaned total hours series (their correlation is 0.9999). Thus, the linear filtering appears to account for the low-frequency structural features of employment and hours per worker while preserving the original properties of the total hours series. In contrast, HP filtered hours per worker and employment change the properties of a total hours measure. GDP, consumption, investment, and wages are neither demeaned nor detrended. Observables are linked to model variables in the following manner:

$$\begin{bmatrix} \text{GDP}_t \\ \text{Cons}_t \\ \text{Inv}_t \\ \text{Wage}_t \\ \text{TotalHour}_t \\ \text{Emp}_t \\ \text{Infl}_t \\ \text{FedFunds}_t \end{bmatrix} = \begin{bmatrix} 100 \log \bar{g_A} \\ 100 \log \bar{g_A} \\ 100 \log \bar{g_A} \\ 100 \log \bar{g_A} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} + \hat{g}_{At} \\ \hat{c}_t - \hat{c}_{t-1} + \hat{g}_{At} \\ \hat{\imath}_t - \hat{\imath}_{t-1} + \hat{g}_{At} \\ \hat{w}_t - \hat{w}_{t-1} + \hat{g}_{At} \\ \hat{TH}_t \\ \hat{L}_t \\ \hat{\pi}_t \\ \hat{R}_t \end{bmatrix}$$

Hours and Employment with Alternative Data

We document the robustness of the results of section 2 of the main paper to alternative measures of the labor market variables. Table 1 displays the shares of hours per worker and employment for the variance of total hours for the two alternative labor market data sets described above, based on CPS data and Smets and Wouters (2007) observables. The shares are calculated after applying various transformations on the data and for two alternative sample periods. In all but one case, the covariance of hours per worker and employment is positive. Hours per worker accounts for 15-48% of the variance of total hours.

B Wage Bargaining

The firm and worker maximize the Nash product

$$\left(S_t^f\right)^{1-\bar{\eta}_t} \left(S_t^w\right)^{\bar{\eta}_t},$$

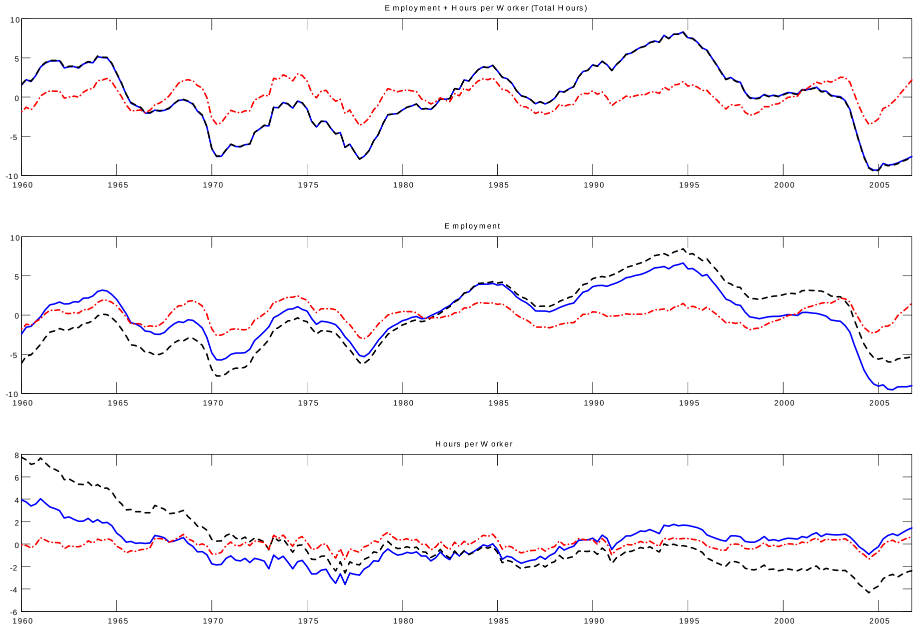

Figure 1. CES labor market variables. Black dashed lines: demeaned data; blue solid lines: linearly detrended data; Red dotted-dashed lines: HP filtered with smoothing parameter of 1600.

where, as detailed in the main text:

$$S_t^f = (1-\alpha)\,\varphi_t \left(\frac{K_t}{\bar{A}h_t L_t}\right)^{\alpha} \bar{A}_t h_t - \frac{w_t^n h_t}{P_t} - \frac{\phi^w \bar{A}_t}{2}\left(\frac{w_t^n}{w_{t-1}^n}\pi_C^{\iota_w - 1}\pi_{Ct-1}^{-\iota_w} - \bar{g}_A\right)^2 + E_t \beta_{t,t+1}(1-\lambda)\, S_{t+1}^f$$

and

$$S_t^w = \frac{w_t^n}{P_t} h_t - b\bar{A}_t - \frac{\bar{\beta}_t \bar{h}_t h_t^{1+\omega}}{(1+\omega)\, u_{Ct}} + (1-\lambda)\, E_t\left[\beta_{t,t+1} S_{t+1}^w \left(1 - \frac{M_{t+1}}{U_{t+1}}\right)\right].$$

The first-order condition with respect to w_t^n implies

$$(1-\bar{\eta}_t)\, S_t^w \frac{\partial S_t^f}{\partial w_t^n} + \bar{\eta}_t S_t^f \frac{\partial S_t^w}{\partial w_t^n} = 0, \tag{A-25}$$

where

$$\frac{\partial S_t^f}{\partial w_t^n} = -\frac{h_t}{P_t} - \phi^w \bar{A}_t \left(\frac{w_t^n}{w_{t-1}^n}\pi_C^{\iota_w-1}\pi_{Ct-1}^{-\iota_w} - \bar{g}_A\right)\frac{\pi_C^{\iota_w-1}\pi_{Ct-1}^{-\iota_w}}{w_{t-1}^n} + (1-\lambda)\, E_t\left(\beta_{t,t+1}\frac{\partial S_{t+1}^f}{\partial w_t^n}\right) \tag{A-26}$$

and

$$\frac{\partial S_t^w}{\partial w_t^n} P_t = h_t.$$

Components of the Variance of Total Hours for Alternative Data Sources

	Filtering	$\tilde{\beta}_{cov,h}$ $\left(\frac{\text{cov}(TH_t,h_t)}{\text{var}(TH_t)}\right)$	$\tilde{\beta}_{cov,L}$ $\left(\frac{\text{cov}(TH_t,L_t)}{\text{var}(TH_t)}\right)$	β_h $\left(\frac{\text{var}(h_t)}{\text{var}(TH_t)}\right)$	β_L $\left(\frac{\text{var}(L_t)}{\text{var}(TH_t)}\right)$	β_{cov} $\left(\frac{2\text{cov}(h_t,L_t)}{\text{var}(TH_t)}\right)$
		1965:1-2007:IV				
CPS data						
	Demeaned	0.15	0.85	0.07	0.78	0.15
	Linear	0.34	0.66	0.15	0.46	0.39
	HP	0.28	0.72	0.12	0.56	0.32
SW data						
	Linear	0.48	0.52	0.39	0.44	0.17
	HP	0.28	0.72	0.14	0.57	0.29
		1965:1-2014:IV				
*CPS data**						
	Demeaned	0.17	0.83	0.09	0.74	0.17
	Linear	0.29	0.71	0.11	0.53	0.37
	HP	0.31	0.69	0.14	0.51	0.35
SW data						
	Linear	0.16	0.84	0.24	0.92	-0.16
	HP	0.27	0.73	0.13	0.58	0.29

*Data from 1965:1-2011:IV.

(Notice that we have used the fact that $\partial w_t^n / \partial h_t = 0$, which stems from equation (9) in the main text.) Moreover, notice that

$$\frac{\partial S_{t+1}^f}{\partial w_t^n} = \phi^w \bar{A}_{t+1} \left(\frac{w_{t+1}^n}{w_t^n} \pi_C^{\iota_w - 1} \pi_{Ct}^{-\iota_w} - \bar{g}_A \right) \frac{w_{t+1}^n \pi_C^{\iota_w - 1} \pi_{Ct}^{-\iota_w}}{(w_t^n)^2}. \tag{A-27}$$

By inserting (A-27) into (A-26), we finally obtain:

$$\frac{\partial S_t^f}{\partial w_t^n} P_t = -h_t - \phi^w \bar{A}_t \left(\pi_{wt} \pi_C^{\iota_w - 1} \pi_{Ct-1}^{-\iota_w} - \bar{g}_A \right) \frac{\pi_C^{\iota_w - 1} \pi_{Ct-1}^{-\iota_w} \pi_{Ct}}{w_{t-1}} \tag{A-28}$$

$$+ \phi^w (1 - \lambda) E_t \left[\beta_{t,t+1} \bar{A}_{t+1} \left(\pi_{wt+1} \pi_C^{\iota_w - 1} \pi_{Ct}^{-\iota_w} - \bar{g}_A \right) \frac{\pi_{wt+1} \pi_C^{\iota_w - 1} \pi_{Ct}^{-\iota_w}}{w_t} \right], \tag{A-29}$$

where $w_t \equiv w_t^n / P_t$.

Finally,let

$$\eta_{wt} = \frac{\bar{\eta}_t \frac{\partial S_t^w}{\partial w_t^n}}{\bar{\eta}_t \frac{\partial S_t^w}{\partial w_t^n} - (1 - \bar{\eta}_t) \frac{\partial S_t^f}{\partial w_t^n}}.$$

The latter means that

$$1-\eta_{wt}=\frac{-(1-\bar{\eta}_t)\frac{\partial S_t^f}{\partial w_t^n}}{\bar{\eta}_t\frac{\partial S_t^w}{\partial w_t^n}-(1-\bar{\eta}_t)\frac{\partial S_t^f}{\partial w_t^n}}.$$

Using the above expression, the sharing rule in (A-25) can be written more compactly as

$$(1-\eta_{wt})S_t^w=\eta_{wt}S_t^f,$$

where η_{wt} measures the effective bargaining power of the worker and $1-\eta_{wt}$ is the effective bargaining power of the firm. Notice that, using equation (A-28), the effective bargaining power of the worker can be written as

$$\eta_{wt}=\frac{\bar{\eta}_t h_t}{\bar{\eta}_t h_t-(1-\bar{\eta}_t)\left[\begin{array}{c}-h_t-\phi^w\bar{A}_t\left(\pi_{wt}\pi_C^{\iota_w-1}\pi_{Ct-1}^{-\iota_w}-\bar{g}_A\right)\frac{\pi_C^{\iota_w-1}\pi_{Ct-1}^{-\iota_w}\pi_{Ct}}{w_{t-1}}\\+\phi^w E_t\beta_{t,t+1}(1-\lambda)\bar{A}_{t+1}\left(\pi_{wt+1}\pi_C^{\iota_w-1}\pi_{Ct}^{-\iota_w}-\bar{g}_A\right)\frac{\pi_{wt+1}\pi_C^{\iota_w-1}\pi_{Ct}^{-\iota_w}}{w_t}\end{array}\right]}.$$

When $\phi^w=0$, the expression above simplifies to $\eta_{wt}=\bar{\eta}_t$.

C Balanced Growth Path and Detrended Variables

All the non-stationary variables are normalized by the level of labor productivity, i.e., $X_t/\bar{A}_t$ (with the exception of the marginal utility of consumption, which is normalized by $u_{Ct}\bar{A}_t$). In order to economize on notation, we do not change notation for those variables. Table A.4 below describes the stationary version of the benchmark model, while Table A.5 presents the stationary version of the equations that appear in Table 3.

D Log-linearized Model

We log-linearize the model around the deterministic balanced growth path. Below, endogenous variables that appear without a time subscript denote constant normalized variables. Notice that, in the deterministic steady state:

$$\bar{Z}=\bar{\beta}=\bar{p}_K=\zeta_K=\pi_C=u_K=1,$$

while $\pi_w = g_A$. The parameter δ_{K1} is calibrated so that $u_K = 1$ at steady state (that is, $rk = \delta_{K1}$). $\delta_K = \delta_{K0}$ at steady state. For future reference, we define the parameter ς such that $\delta_{K2}/\delta_{K1} = \varsigma/(1-\varsigma)$. Finally, let $\hat{x}_t \equiv dx_t/x \simeq \log(x_t) - \log(x)$. Table A.6 presents the log-linearized equations. Finally, notice that, starting from the stationary log-linear system, we recover a given non-stationary variables x_t^L by constructing $x_t^L = \left(e^{\hat{x}_t + x}\right)\bar{A}_t$. The growth rate of the non-stationary variable is then obtained as follows:

$$\Delta x_t^L \equiv \log\left(x_t^L\right) - \log\left(x_{t-1}^L\right) = \hat{x}_t - \hat{x}_{t-1} + \hat{g}_{At} + \log\left(g_A\right).$$

E Parametrized Wealth Effects in Labor Supply: Alternative Specification

We mantain the assumption of complete markets within the household, so that family members pool their income in each period. This assumption implies that the consumption of each household's member does not depend on employement history, but only on the current emloyment status. Let c_{jt}^e and h_{jt}^e denote, respectively, consumption and hours worked by a family member j who is employed at time t. Let c_{jt}^u be the consumption of an unemployed family member j.

The per-period utility function for an employed worker j is given by

$$u_{jt}^e = \log\left(c_{jt}^e - h_C C_{t-1} - \bar{h}_t X_{jt}^e \frac{h_{jt}^{e1+\omega}}{1+\omega}\right),$$

where C_t denotes aggregate consumption and

$$X_{jt}^e = \left(c_{jt}^e - h_C C_{t-1}\right)^\gamma X_{t-1}^{e1-\gamma}. \tag{A-30}$$

where $X_{t-1}^e = \int_0^{L_{t-1}} X_{jt-1}^e dj$. The parameter h_C continues to capture consumption habit, although we now assume external habit formation. The per-period utility of an unemployed worker j is

$$u_{jt}^u = \log\left(c_{jt}^u - h_C C_{t-1}\right).$$

Household's welfare aggregates the welfare of individual members:

$$W_t \equiv E_t \sum_{s=t}^{\infty} \beta^{s-t} \bar{\beta}_s \left[\int_0^{L_s} \log\left(c_{js}^e - h_C C_{s-1} - \bar{h}_s X_{js}^e \frac{h_{js}^{e1+\omega}}{1+\omega}\right) dj + \int_{L_s}^1 \log\left(c_{js}^u - h_C C_{s-1}\right) dj\right]. \tag{A-31}$$

Since family members pool their income in each period, the representative household allocates c_t^e

and c_t^u in order to maximize (A-31) subject to the consolidated resource constraint

$$P_t \int_0^{L_t} c_{jt}^e dj + P_t \int_{L_t}^1 c_{jt}^u dj + P_t I_{Kt} + B_{t+1} \tag{A-32}$$
$$= i_t B_t + w_t^n \int_0^{L_t} h_{jt}^e dj + r_{Kt} P_t K_t + (1 - L_t)\, b \bar{A}_t P_t + P_t \Pi_t^I + P_t \int_0^1 \Pi_t^F (i)\, di + T_t^g,$$

the constraint (A-30), and the law of motion for capital, equation (13) in the main text. As in the benchmark model the househld takes L_t and h_{jt}^e as given when choosing c_{jt}^e and c_{jt}^u.

Let Ω_t and μ_{jt}^e be, respectively, the Lagrange multipliers attached to the constraints (A-32) and (A-30). The first-order condition for c_{jt}^e implies:

$$P_t \Omega_t = u_{c_j^e t}^e, \tag{A-33}$$

where $u_{c_j^e t}^e \equiv \partial u_{jt}^e / \partial c_{jt}^e$ denotes the marginal utlity of consumption for the employed worker j:

$$u_{c_j^e t}^e = \bar{\beta}_t \Psi_{jt}^{e^{-1}} + \gamma \mu_{jt}^e \left(c_{jt}^e - h_C C_{t-1}\right)^{\gamma - 1} X_{t-1}^{e^{1-\gamma}},$$

with

$$\Psi_{jt}^e \equiv c_{jt}^e - h_C C_{t-1} - \bar{h}_t X_{jt}^e \frac{h_{jt}^{1+\omega}}{1+\omega}.$$

Notice that equation (A-33) implies that the marginal utility of consumption across employed workers is equalized: $u_{c_j^e t}^e = u_{c^e t}^e$. The first-order condition for X_{jt}^e implies:

$$\mu_{jt}^e \equiv -\bar{\beta}_t \bar{h}_t \Psi_{jt}^{-1} \frac{h_{jt}^{1+\omega}}{1+\omega}.$$

The first-order condition for c_{jt}^u implies:

$$P_t \Omega_t = u_{c_j^u t}^u, \tag{A-34}$$

where $u_{c_j^u t}^u \equiv \partial u_t^u / \partial c_t^u$ denotes the marginal utlity of consumption for the unemployed worker j:

$$u_{c_j^u t}^u = \frac{\bar{\beta}_t}{c_{jt}^u - h_C C_{t-1}}. \tag{A-35}$$

Equation (A-34) implies that the marginal utility of consumption across unemployed workers is equalized: $u^u_{c^u_j t} = u^u_{c^u t}$. In turn, equation (A-35) implies that the consumption level of unemployed workers is symmetric: $c^u_{jt} = c^u_t$. Finally, equations (A-33) and (A-34) show that the complete markets assumption implies that the marginal utility of consumption for employed and unemployed workers are equalized $u^e_{c^e t} = u^u_{c^u t}$.

Since all employed workers are symmetric, we drop the subsrict j from now on. Aggregate consumption is then given by:

$$C_t = L_t c^e_t + (1 - L_t)\, c^u_t.$$

Hours per Worker

The alternative preference specification affects the marginal rate of substitution between consumption and leisure, since the marginal disutility of hours for a generic worker j is now given by:

$$W_{ht} \equiv \frac{\partial u^e_t}{\partial h_t} = -\Psi_t^{-1} \bar{\beta}_t \bar{h}_t h_t^{\omega} X_{jt},$$

while the marginal utility of consumption is defined by $u^e_{c^e t}$.

Optimality in hours per worker continues to equate the worker's marginal rate of substitution between consumption and leisure to the value marginal value product of an extra hour worked:

$$-\frac{W_{ht}}{u^e_{c^e t}} = (1 - \alpha)\, \varphi_t \left(\frac{k_{jt}}{\bar{A} \tilde{h}_t} \right)^{\alpha} \bar{A}_t \Delta_{\tilde{h} t}.$$

Wage Bargaining

The worker's surplus is now given by

$$S^w_t = \frac{w^n_t}{P_t} h_t - b\bar{A}_t - \frac{\Psi_t^{-1} \bar{\beta}_t \bar{h}_t h_t^{1+\omega} X_t}{(1+\omega)\, u_{Ct}} + E_t \left[\beta_{t,t+1} (1 - \lambda)\, S^w_{t+1} \left(1 - \frac{M_{t+1}}{U_{t+1}} \right) \right].$$

As usual, we rewrite all the model equations in terms of detrended variables. Next, we compute the non-stochastic steady state pf the transformed model, and loglinearly approximate it around the deterministic steady state. Details are available upon requests.

F Variance Decomposition

[TO BE WRITTEN]

TABLE A.2: BENCHMARK MODEL EQUATIONS

(1)	$L_t = (1-\lambda)L_{t-1} + M_t$
(2)	$\frac{\bar{\beta}_t \bar{h}_t h_t^{\omega}}{u_{Ct}} = (1-\alpha)\varphi_t \left(\frac{u_{Kt}\tilde{K}_t}{\bar{A}h_t L_t}\right)^{\alpha} \bar{A}_t$
(3)	$\tilde{K}_{t+1} = (1-\delta_{Kt})\tilde{K}_t + \bar{P}_t^K I_{Kt}\left[1 - \frac{\nu_K}{2}\left(\frac{I_{Kt}}{I_{Kt-1}} - g_A\right)^2\right]$
(4)	$r_{Kt} = \zeta_{Kt}[\delta_{K1} + \delta_{K2}(u_{Kt}-1)]$
(5)	$\zeta_{Kt} = \beta E_t\left\{\frac{u_{Ct+1}}{u_{Ct}}\left[r_{Kt+1}u_{Kt+1} + (1-\delta_{Kt+1})\zeta_{Kt+1}\right]\right\}$
(6)	$1 = \left[\begin{array}{c} \zeta_{Kt}\bar{P}_t^K\left[1 - \frac{\nu_K}{2}\left(\frac{I_{Kt}}{I_{Kt-1}} - g_A\right)^2 - \nu_K\left(\frac{I_{Kt}}{I_{Kt-1}} - g_A\right)\frac{I_{Kt}}{I_{Kt-1}}\right] \\ +\nu_K\beta E_t\left[\frac{u_{Ct+1}}{u_{Ct}}\bar{P}_{t+1}^K\zeta_{Kt+1}\left(\frac{I_{Kt+1}}{I_{Kt}} - g_A\right)\left(\frac{I_{Kt+1}}{I_{Kt}}\right)^2\right]\end{array}\right]$
(7)	$M_t = \bar{\chi}_t U_t^{\varepsilon} V_t^{1-\varepsilon}$
(8)	$\frac{\kappa}{q_t}\bar{A}_t V_t^{\tau} = S_t^f$
(9)	$\eta_{wt}S_t^f = (1-\eta_{wt})S_t^w$
(10)	$\pi_{wt} = \frac{w_t}{w_{t-1}}\pi_{Ct}$
(11)	$1 = i_t\beta E_t\left[\frac{u_{Ct+1}}{u_{Ct}}\frac{1}{\pi_{Ct+1}}\right]$
(12)	$\frac{i_{t+1}}{i} = \left(\frac{i_t}{i}\right)^{\varrho_i}\left[\left(\frac{\pi_{Ct}}{\pi_C}\right)^{\varrho_\pi}\left(\frac{Y_{gt}}{Y_g}\right)^{\varrho_Y}\right]^{1-\varrho_i}\bar{\imath}_{it}$
(13)	$1 = \frac{\bar{\theta}_t}{(\bar{\theta}_t - 1)\Xi_t}\varphi_t$
(14)	$\left(u_{Kt}\tilde{K}_t\right)^{\alpha}\left(\bar{A}_t L_t h_t\right)^{1-\alpha}\left[1 - \frac{\nu}{2}\left(\pi_{Ct}\pi_C^{\iota_p - 1}\pi_{Ct-1}^{-\iota_p} - 1\right)^2\right] = \left[\begin{array}{c} C_t + I_{Kt} + \kappa_t\bar{A}_t V_t + G_t \\ +\frac{\phi^w}{2}\bar{A}_t\left(\pi_{wt}\pi_C^{1-\iota_w}\pi_{Ct-1}^{-\iota_w} - g_A\right)^2 L_t\end{array}\right]$
(15)	$r_{Kt} = \varphi_t\alpha\left(\frac{u_{Kt}\tilde{K}_t}{\bar{A}_t L_t h_t}\right)^{\alpha-1}$
(D.1)	$Y_t^g = \frac{C_t + I_{Kt} + G_t}{\tilde{C}_t + \tilde{I}_{Kt} + G_t}$
(D.2)	$U_t = 1 - (1-\lambda)L_{t-1}$
(D.3)	$\Xi_t \equiv 1 - \frac{\phi^p}{2}\left(\pi_{Ct}\pi_{Ct-1}^{-\iota_p}\pi_C^{\iota_p - 1} - 1\right)^2 + \frac{\phi^p}{\bar{\theta}_t - 1}\left\{\begin{array}{c} \pi_C^{\iota_p - 1}\left(\pi_{Ct}\pi_{Ct-1}^{-\iota_p}\pi_C^{\iota_p - 1} - 1\right)\pi_t(\omega)\pi_{Ct-1}^{-\iota_p} \\ -\beta E_t\left[\frac{u_{Ct+1}}{u_{Ct}}\left(\pi_{Ct+1}\pi_{Ct}^{-\iota_p}\pi_C^{\iota_p - 1} - 1\right)\pi_{Ct+1}\pi_{Ct}^{-\iota_p}\frac{Y_{t+1}^C}{Y_t^C}\right]\end{array}\right\}$
(D.4)	$S_t^f = (1-\alpha)\varphi_t\left(\frac{u_{Kt}\tilde{K}_t}{\bar{A}h_t L_t}\right)^{\alpha}\bar{A}_t h_t - w_t h_t - \frac{\phi^w}{2}\bar{A}_t\left(\pi_{wt}\pi_C^{1-\iota_w}\pi_{Ct-1}^{-\iota_w} - g_A\right)^2 + (1-\lambda)\beta E_t\left(\frac{u_{Ct+1}}{u_{Ct}}S_{t+1}^f\right)$
(D.5)	$S_t^w = w_t h_t - b\bar{A}_t - \frac{\bar{\beta}_t\bar{h}_t h_t^{1+\omega}}{(1+\omega)u_{Ct}} + (1-\lambda)\beta E_t\left[\frac{u_{Ct+1}}{u_{Ct}}S_{t+1}^w\left(1 - \frac{M_{t+1}}{U_{t+1}}\right)\right]$
(D.6)	$u_{Ct} \equiv \frac{\bar{\beta}_t}{(C_t - h_C C_{t-1})} - h_C E_t\left[\frac{\bar{\beta}_{t+1}}{(C_{t+1} - h_C C_t)}\right]$
(D.7)	$q_t = \frac{M_t}{V_t}$
(D.8)	$\delta_{Kt} \equiv \delta_{K0} + \delta_{K1}(u_{Kt} - 1) + (\delta_{K2}/2)(u_{Kt} - 1)^2$
(D.9)	$\kappa_t = \kappa V_t^{\tau}/(1+\tau)$
(D.10)	$\eta_{wt} = \frac{\bar{\eta}_t h_t}{\bar{\eta}_t h_t + (1-\bar{\eta}_t)\left[\begin{array}{c} -h_t - \phi^w\bar{A}_t\left(\pi_{wt}\pi_C^{\iota_w - 1}\pi_{Ct-1}^{-\iota_w} - \bar{g}_A\right)\frac{\pi_C^{\iota_w - 1}\pi_{Ct-1}^{-\iota_w}\pi_{Ct}}{w_{t-1}} \\ +\phi^w(1-\lambda)\beta E_t\left[\frac{u_{Ct+1}}{u_{Ct}}\bar{A}_{t+1}\left(\pi_{wt+1}\pi_C^{\iota_w - 1}\pi_{Ct}^{-\iota_w} - \bar{g}_A\right)\frac{\pi_{wt+1}\pi_C^{\iota_w - 1}\pi_{Ct}^{-\iota_w}}{w_t}\right]\end{array}\right]}$

Note: $\tilde{C}_t$and $\tilde{I}_{Kt}$in equation (D.1) are consumption and investment observed when $\phi^w = \phi^p = \varepsilon_{\bar{\eta}t} = \varepsilon_{\bar{\theta}t} = 0$. Variables without a time subscript denoyes steady-state values.

TABLE A.3: ALTERNATIVE MODEL NEW EQUATIONS

(2')	$\frac{\Psi_t^{-1}\bar{\beta}_t\bar{h}_t h_t^{\omega} X_t}{u_{Ct}} = (1-\alpha)\,\varphi_t \left(\frac{K_t}{\bar{A}\tilde{h}_t L_t}\right)^{\alpha} \bar{A}_t \Delta_{\tilde{h}t}.$
(14')	$\left(u_{Kt}\tilde{K}_t\right)^{\alpha}\left(\bar{A}_t L_t \tilde{h}_t\right)^{1-\alpha}\left[1-\frac{\nu}{2}\left(\pi_{Ct}\pi_C^{\iota_p-1}\pi_{Ct-1}^{-\iota_p}-1\right)^2\right] = \left[\begin{array}{c} C_t + I_{Kt} + \kappa_t \bar{A}_t V_t + G_t \\ +\frac{\phi^w}{2}\bar{A}_t\left(\pi_{wt}\pi_C^{1-\iota_w}\pi_{Ct-1}^{-\iota_w} - g_A\right)^2 L_t \end{array}\right]$
(15')	$r_{Kt} = \varphi_t \alpha \left(\frac{u_{Kt}\tilde{K}_t}{\bar{A}_t L_t \tilde{h}_t}\right)^{\alpha-1}$
(D.4')	$S_t^f = (1-\alpha)\,\varphi_t\left(\frac{u_{Kt}\tilde{K}_t}{\bar{A}\tilde{h}_t L_t}\right)^{\alpha}\bar{A}_t\tilde{h}_t - w_t h_t - \frac{\phi^w}{2}\bar{A}_t\left(\pi_{wt}\pi_C^{1-\iota_w}\pi_{Ct-1}^{-\iota_w} - g_A\right)^2 + (1-\lambda)\,\beta E_t\left(\frac{u_{Ct+1}}{u_{Ct}}S_{t+1}^f\right)$
(D.5')	$S_t^w = w_t h_t - b\bar{A}_t - \frac{\Psi_t^{-1}\bar{\beta}_t\bar{h}_t h_t^{1+\omega}X_t}{(1+\omega)u_{Ct}} + (1-\lambda)\,\beta E_t\left[\frac{u_{Ct+1}}{u_{Ct}}S_{t+1}^w\left(1-\frac{M_{t+1}}{U_{t+1}}\right)\right]$
(D.6')	$u_{Ct} = \bar{\beta}_t\Psi_t^{-1} + \gamma\mu_t\left(C_t - h_C C_{t-1}\right)^{\gamma-1}X_{t-1}^{1-\gamma} - \beta h_C E_t\left(\bar{\beta}_{t+1}\Psi_{t+1}^{-1}\right) - \gamma h_C \beta E_t\left[\mu_{t+1}\left(C_{t+1}-h_C C_t\right)^{\gamma-1}X_t^{1-\gamma}\right]$
(D.12)	$\mu_t \equiv -\Psi_t^{-1}\frac{\bar{\beta}_t\bar{h}_t h_t^{1+\omega}}{1+\omega}L_t + (1-\gamma)\,\beta E_t\left[\mu_{t+1}\left(C_{t+1}-h_C C_t\right)^{\gamma}X_t^{-\gamma}\right]$
(D.13)	$\Psi_t \equiv C_t - h_C C_{t-1} - L_t\frac{\bar{h}_t h_t^{1+\omega}}{1+\omega}X_t$
(D.14)	$X_t = \left(C_t - h_C C_{t-1}\right)^{\gamma}X_{t-1}^{1-\gamma}$
(D.15)	$\tilde{h}_t = h_t^{\bar{\alpha}_{ht}}\left[1-\frac{\phi_h}{2}\left(h_t - h\right)^2\right]$
(D.16)	$\Delta_{\tilde{h}t} = \bar{\alpha}_{ht}\frac{\tilde{h}_t}{h_t} - \phi_h h_t^{\bar{\alpha}_{ht}}\left(h_t - h\right)$

Note: Other equations are unchanged relative to Table 2.

TABLE A.4: MODEL EQUATIONS, STATIONARY DEVIATIONS FROM TREND

(1)	$L_t = (1-\lambda) L_{t-1} + M_t$
(2)	$\frac{\bar{\beta}_t \bar{h}_t h_t^{\omega}}{u_{Ct}} = (1-\alpha)\varphi_t \left(\frac{u_{Kt}\tilde{K}_t}{\bar{g}_{At} L_t h_t}\right)^{\alpha}$
(3)	$\tilde{K}_{t+1} = (1-\delta_{Kt})\frac{\tilde{K}_t}{\bar{g}_{At}} + \bar{p}_t^K I_{Kt}\left[1 - \frac{\nu}{2}\left(\bar{g}_{At}\frac{I_{Kt}}{I_{Kt-1}} - g_A\right)^2\right]$
(4)	$r_{Kt} = \zeta_{Kt}[\delta_{K1} + \delta_{K2}(u_{Kt}-1)]$
(5)	$\zeta_{Kt} = \beta E_t\left\{\frac{u_{Ct+1}}{u_{Ct}}\frac{1}{\bar{g}_{At+1}}\left[r_{Kt+1}u_{Kt+1} + (1-\delta_{Kt+1})\zeta_{Kt+1}\right]\right\}$
(6)	$1 = \zeta_{Kt}\bar{p}_t^K\left[1 - \frac{\nu}{2}\left(\bar{g}_{At}\frac{I_{Kt}}{I_{Kt-1}} - g_A\right)^2 - \nu\left(\bar{g}_{At}\frac{I_{Kt}}{I_{Kt-1}} - g_A\right)\bar{g}_{At}\frac{I_{Kt}}{I_{Kt-1}}\right]$ $+\nu\beta E_t\left[\frac{u_{Ct+1}}{u_{Ct}}\bar{g}_{At+1}\bar{p}_{t+1}^K\zeta_{Kt+1}\left(\bar{g}_{At+1}\frac{I_{Kt+1}}{I_{Kt}} - g_A\right)\left(\frac{I_{Kt+1}}{I_{Kt}}\right)^2\right]$
(7)	$M_t = \bar{\chi}_t U_t^{\varepsilon} V_t^{1-\varepsilon}$
(8)	$\kappa\frac{V_t^{\tau}}{q_t} = S_t^f$
(9)	$\eta_{wt} S_t^f = (1-\eta_{wt}) S_t^w$
(10)	$\pi_{wt} = \bar{g}_{At}\frac{w_t}{w_{t-1}}\pi_{Ct}$
(11)	$1 = \beta R_t E_t\left[\frac{u_{Ct+1}}{u_{Ct}\bar{g}_{At+1}}\frac{1}{\pi_{Ct+1}}\right]$
(12)	$\frac{i_{t+1}}{i} = \left(\frac{i_t}{i}\right)^{\varrho_i}\left[\left(\frac{1+\pi_{Ct}}{1+\pi_C}\right)^{\varrho_\pi}\left(\frac{Y_{gt}}{Y_g}\right)^{\varrho_Y}\right]^{1-\varrho_i}\bar{\imath}_{it}$
(13)	$1 = \frac{\bar{\theta}_t}{(\bar{\theta}_t - 1)\Xi_t}\varphi_t$
(14)	$\left(\frac{u_{Kt}\tilde{K}_t}{\bar{g}_{At}}\right)^{\alpha}(L_t h_t)^{1-\alpha}\left[1 - \frac{\nu}{2}\left(\pi_{Ct}\pi_C^{\iota_p - 1}\pi_{Ct-1}^{-\iota_p} - 1\right)^2\right] = \left[\begin{array}{c} C_t + I_{Kt} + \kappa_t V_t + G_t \\ +\frac{\phi^w}{2}\left(\pi_{wt}\pi_C^{1-\iota_w}\pi_{Ct-1}^{-\iota_w} - g_A\right)^2 L_t \end{array}\right]$
(15)	$r_{Kt} = \varphi_t \alpha\left(\frac{u_{Kt}\tilde{K}_t}{\bar{g}_{At} L_t h_t}\right)^{\alpha - 1}$
(D.1)	$Y_t^g = \frac{C_t + I_{Kt} + G_t}{\tilde{C}_t + \tilde{I}_{Kt} + G_t}$
(D.2)	$U_t = 1 - (1-\lambda) L_{t-1}$
(D.3)	$\Xi_t \equiv 1 - \frac{\phi^p}{2}\left(\pi_{Ct}\pi_{Ct-1}^{-\iota_p}\pi_C^{\iota_p - 1} - 1\right)^2 + \frac{\phi^p}{\bar{\theta}_t - 1}\left\{\begin{array}{c} \pi_C^{\iota_p - 1}\left(\pi_{Ct}\pi_{Ct-1}^{-\iota_p}\pi_C^{\iota_p - 1} - 1\right)\pi_t(\omega)\pi_{Ct-1}^{-\iota_p} \\ -E_t\left[\beta\frac{u_{Ct+1}}{u_{Ct}}\left(\pi_{Ct+1}\pi_{Ct}^{-\iota_p}\pi_C^{\iota_p - 1} - 1\right)\pi_{Ct+1}\pi_{Ct}^{-\iota_p}\frac{Y_{t+1}^C}{Y_t^C}\right] \end{array}\right\}$
(D.4)	$S_t^f = (1-\alpha)\varphi_t\left(\frac{u_{Kt}\tilde{K}_t}{\bar{g}_{At} L_t h_t}\right)^{\alpha} h_t - w_t h_t - \frac{\phi^w}{2}\left(\pi_{wt}\pi_C^{\iota_w - 1}\pi_{Ct-1}^{-\iota_w} - \bar{g}_A\right)^2 + (1-\lambda)\beta E_t\left(\frac{u_{Ct+1}}{u_{Ct}}S_{t+1}^f\right)$
(D.5)	$S_t^w = w_t h_t - b - \frac{\bar{\beta}_t \bar{h}_t h_t^{1+\omega}}{(1+\omega)u_{Ct}} + (1-\lambda)\beta E_t\left[\frac{u_{Ct+1}}{u_{Ct}}S_{t+1}^w\left(1 - \frac{M_{t+1}}{U_{t+1}}\right)\right]$
(D.6)	$u_{Ct} = \bar{\beta}_t\frac{1}{\left(C_t - h_C\frac{C_{t-1}}{\bar{g}_{At}}\right)} - h_C\beta E_t\left[\bar{\beta}_{t+1}\frac{1}{(C_{t+1}\bar{g}_{At+1} - h_C C_t)}\right]$
(D.7)	$q_t = \frac{M_t}{V_t}$
(D.8)	$\delta_{Kt} \equiv \delta_{K0} + \delta_{K1}(u_{Kt} - 1) + (\delta_{K2}/2)(u_{Kt} - 1)^2$
(D.9)	$\kappa_t = \kappa V_t^{\tau}/(1+\tau)$
(D.10)	$\eta_{wt} = \frac{\bar{\eta}_t h_t}{\bar{\eta}_t h_t + (\bar{\eta}_t - 1)\left[\begin{array}{c} -h_t - \phi^w \bar{g}_{At}\left(\pi_{wt}\pi_C^{\iota_w - 1}\pi_{Ct-1}^{-\iota_w} - \bar{g}_A\right)\frac{\pi_C^{\iota_w - 1}\pi_{Ct-1}^{-\iota_w}\pi_{Ct}}{w_{t-1}} \\ +\phi^w(1-\lambda)\beta E_t\left[\frac{u_{Ct+1}}{u_{Ct}}\left(\pi_{wt+1}\pi_C^{\iota_w - 1}\pi_{Ct}^{-\iota_w} - \bar{g}_A\right)\frac{\pi_{wt+1}\pi_C^{\iota_w - 1}\pi_{Ct}^{-\iota_w}}{w_t}\right] \end{array}\right]}$

Note: $\tilde{C}_t$and $\tilde{I}_{Kt}$in equation (D.1) are consumption and investment observed when $\phi^w = \phi^p = \varepsilon_{\bar{\eta}t} = \varepsilon_{\bar{\theta}t} = 0$.
Variables without a time subscript denoyes steady-state values.

TABLE A.5: ALTERNATIVE MODEL, STATIONARY DEVIATIONS FROM TREND, NEW EQUATIONS

(2')	$\frac{\Psi_t^{-1}\bar{\beta}_t\bar{h}_t h_t^{\omega} X_t}{u_{Ct}} = (1-\alpha)\,\varphi_t \left(\frac{K_t}{\bar{g}_{At}\tilde{h}_t L_t}\right)^{\alpha} \Delta_{\tilde{h}t}.$
(14')	$\left(\frac{u_{Kt}\tilde{K}_t}{\bar{g}_{At}}\right)^{\alpha}\left(L_t\tilde{h}_t\right)^{1-\alpha}\left[1-\frac{\nu}{2}\left(\pi_{Ct}\pi_C^{\iota_p-1}\pi_{Ct-1}^{-\iota_p}-1\right)^2\right] = \left[\begin{array}{c} C_t + I_{Kt} + \kappa_t V_t + G_t \\ +\frac{\phi^w}{2}\left(\pi_{wt}\pi_C^{1-\iota_w}\pi_{Ct-1}^{-\iota_w} - g_A\right)^2 L_t \end{array}\right]$
(15')	$r_{Kt} = \varphi_t\alpha\left(\frac{u_{Kt}\tilde{K}_t}{\bar{g}_{At}L_t\tilde{h}_t}\right)^{\alpha-1}$
(D.4')	$S_t^f = (1-\alpha)\,\varphi_t\left(\frac{u_{Kt}\tilde{K}_t}{\bar{g}_{At}\tilde{h}_t L_t}\right)^{\alpha}\tilde{h}_t - w_t h_t - \frac{\phi^w}{2}\left(\pi_{wt}\pi_C^{1-\iota_w}\pi_{Ct-1}^{-\iota_w} - g_A\right)^2 + (1-\lambda)\,\beta E_t\left(\frac{u_{Ct+1}}{u_{Ct}}S_{t+1}^f\right)$
(D.5')	$S_t^w = w_t h_t - b - \frac{\Psi_t^{-1}\bar{\beta}_t\bar{h}_t h_t^{1+\omega} X_t}{(1+\omega)u_{Ct}} + (1-\lambda)\,\beta E_t\left[\frac{u_{Ct+1}}{u_{Ct}}S_{t+1}^w\left(1-\frac{M_{t+1}}{U_{t+1}}\right)\right]$
(D.6')	$u_{Ct} = \left[\begin{array}{c} \bar{\beta}_t\Psi_t^{-1} + \gamma\mu_t\left(C_t - h_C\frac{C_{t-1}}{\bar{g}_{At}}\right)^{\gamma-1}X_{t-1}^{1-\gamma}\bar{g}_{At}^{\gamma-1} - \beta h_C E_t\left[\bar{\beta}_{t+1}\left(\Psi_{t+1}\bar{g}_{At+1}\right)^{-1}\right] \\ -\gamma\beta h_C E_t\left[\frac{\mu_{t+1}}{\bar{g}_{At+1}}\left(C_{t+1}\bar{g}_{At+1} - h_C C_t\right)^{\gamma-1}X_t^{1-\gamma}\right] \end{array}\right]$
(D.13)	$\mu_t = -\bar{\beta}_t\Psi_t^{-1}L_t\bar{h}_{xt}\frac{h_t^{1+\omega}}{1+\omega} + (1-\gamma)\beta E_t\left\{\frac{\mu_{t+1}}{\bar{g}_{At+1}}\left(C_{t+1}\bar{g}_{At+1} - h_C C_t\right)^{\gamma}\tilde{X}_t^{-\gamma}\right\}$
(D.14)	$\Psi_t = C_t - h_C\frac{C_{t-1}}{\bar{g}_{At}} - \frac{\bar{h}_{xt}L_t h_t^{1+\omega}X_t}{1+\omega}$
(D.15)	$X_t = \left(C_t - h_C\frac{C_{t-1}}{\bar{g}_{At}}\right)^{\gamma}\left(\frac{X_{t-1}}{\bar{g}_{At}}\right)^{1-\gamma}$
(D.16)	$\tilde{h}_t = h_t^{\bar{\alpha}_{ht}}\left[1-\frac{\phi_h}{2}\left(h_t - h\right)^2\right]$
(D.17)	$\Delta_{\tilde{h}t} = \bar{\alpha}_{ht}\frac{\tilde{h}_t}{h_t} - \phi_h h_t^{\bar{\alpha}_{ht}}\left(h_t - h\right)$

Note: Other equations are unchanged relative to Table A.4.

TABLE A.6: LOG-LINEARIZED MODEL EQUATIONS

(1)	$L\hat{L}_t = L(1-\lambda)\hat{L}_{t-1} + M\hat{M}_t$
(2)	$\hat{\bar{\beta}}_t + \hat{\bar{h}}_t + \omega\hat{h}_t - \hat{u}_{Ct} = \hat{\varphi}_t + \alpha\left(\hat{u}_{Kt} + \hat{\tilde{K}}_t - \hat{g}_{At} - \hat{L}_t - \hat{h}_t\right)$
(3)	$\hat{\tilde{K}}_{t+1} = (\frac{1-\delta_{K0}}{g_A})(\hat{\tilde{K}}_t - \hat{g}_{At}) - (\frac{\delta_{K0}}{g_A})\hat{\delta}_{Kt} + \left(1 - \frac{1-\delta_{K0}}{g_A}\right)[\widehat{\bar{P}}_t^K + \hat{I}_{Kt}]$
(4)	$\hat{r}_{Kt} = \hat{\zeta}_{Kt} + \frac{\varsigma}{1-\varsigma}\hat{u}_{Kt}$
(5)	$\hat{\zeta}_{Kt} = E_t\hat{u}_{Ct+1} - \hat{u}_{Ct} - E_t\hat{g}_{At+1} + \frac{\beta}{g_A}r_K\left(E_t\hat{r}_{Kt+1} + E_t\hat{u}_{Kt+1}\right) + \frac{\beta}{g_A}(1-\delta_{K0})E_t\hat{\zeta}_{Kt+1} - \frac{\beta}{g_A}\delta_{K1}E_t\hat{u}_{Kt+1}$
(6)	$(1+\beta)\hat{I}_{kt} - \frac{1}{g_A^2\nu}\left(\hat{\zeta}_{Kt} + \hat{\bar{p}}_t^K\right) - \beta E_t\hat{I}_{Kt+1} + \hat{g}_{At} - \beta E_t\hat{g}_{At+1} = \hat{I}_{Kt-1}$
(7)	$\hat{M}_t = \hat{\bar{\chi}}_t + \varepsilon\hat{U}_t + (1-\varepsilon)\hat{V}_t$
(8)	$\tau\hat{V}_t - \hat{q}_t = \hat{S}_t^f$
(9)	$\hat{\eta}_{wt} + \hat{S}_t^f = \frac{\eta_w}{1-\eta_w}\hat{\eta}_{wt} + \hat{S}_t^w$
(10)	$\hat{\pi}_{wt} = \hat{g}_{At} + \hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_{Ct}$
(11)	$\hat{\imath}_t + E_t\hat{u}_{Ct+1} - \hat{u}_{Ct} - E_t\hat{g}_{At+1} - E_t\hat{\pi}_{Ct+1} = 0$
(12)	$\hat{\imath}_t = \phi_R\hat{\imath}_{t-1} + (1-\phi_R)\phi_\pi\hat{\pi}_{Ct} + (1-\phi_R)\phi_Y\hat{Y}_{Dt}^g + \widehat{\bar{\imath}}_t$
(13)	$0 = \frac{1}{\bar{\theta}-1}\widehat{\bar{\theta}}_t - \hat{\Xi}_t + \hat{\varphi}_t$
(14)	$\alpha(\hat{u}_{Kt} + \hat{\tilde{K}}_{t-1} - \hat{g}_{At}) + (1-\alpha)(\hat{\bar{Z}}_t + \hat{L}_t + \hat{h}_t) = C\hat{C}_t + \hat{I}_{Kt} + \kappa\hat{\kappa}_t + \bar{G}\hat{\bar{G}}_t$
(15)	$\hat{r}_{Kt} = \hat{\varphi}_t + (\alpha-1)\left(\hat{u}_{Kt} + \hat{\tilde{K}}_t - \hat{g}_{At} - \hat{L}_t - \hat{h}_t\right)$
(D.1)	$\hat{Y}_{gt} = C\left(\hat{C}_t - \widehat{\tilde{C}}_t\right) + I_K\left(\hat{I}_{Kt} - \widehat{\tilde{I}}_{Kt}\right) + G\hat{G}_t$
(D.2)	$U\hat{U}_t = -(1-\lambda)L\hat{L}_{t-1}$
(D.3)	$\hat{\Xi}_t = -\frac{1}{\bar{\theta}-1}\left[\phi^p\left(\hat{\pi}_{pt} - \iota_p\hat{\bar{\pi}}_{pt-1}\right) - \phi^p\beta\left(E_t\hat{\pi}_{pt+1} - \iota_p\hat{\bar{\pi}}_{pt}\right)\right]$
(D.4)	$S_t^f\hat{S}_t^f = \left[\begin{array}{l}(1-\alpha)\varphi\left(\frac{u_K\tilde{K}}{\bar{g}_A Lh}\right)^\alpha h\left[\alpha\left(\hat{u}_{Kt} + \hat{\tilde{K}}_t - \hat{g}_{At} - \hat{L}_t\right) + (1-\alpha)\hat{h}_t\right] \\ -wh(\hat{w}_t + \hat{h}_t) + \beta(1-\lambda)S^f\left[E_t\hat{u}_{Ct+1} - \hat{u}_{Ct} + E_t\hat{S}_{t+1}^f\right]\end{array}\right]$
(D.5)	$S^w\hat{S}_t^w = \left[\begin{array}{l}wh(\hat{w}_t + \hat{h}_t) - \frac{\bar{h}h^{1+\omega}}{(1+\omega)u_C}[\widehat{\bar{h}}_t + \widehat{\bar{\beta}}_t + (1+\omega)\hat{h}_t - \hat{u}_{Ct}] \\ +(1-\lambda)\beta S^w\left(1-\frac{M}{U}\right)(E_t\hat{u}_{Ct+1} - \hat{u}_{Ct} + E_t\hat{S}_{t+1}^w) - (1-\lambda)\beta S^w\frac{M}{U}(E_t\hat{M}_{t+1} - E_t\hat{U}_{t+1})\end{array}\right]$
(D.6)	$\hat{u}_{Ct} = \left[\begin{array}{l}\frac{g_A\beta h_C}{(g_A-\beta h_C)(g_A-h_C)}E_t\hat{C}_{t+1} - \frac{g_A^2+\beta h_C^2}{(g_A-\beta h_C)(g_A-h_C)}\hat{C}_t \\ +\frac{g_A h_C}{(g_A-\beta h_C)(g_A-h_C)}\hat{C}_{t-1} + \frac{g_A\beta h_C\rho_{g_A} - h_C g_A}{(g_A-\beta h_C)(g_A-h_C)}\hat{g}_{At} + \frac{g_A-\beta h_C\rho_b}{g_A-\beta h_C}\hat{\bar{\beta}}_t\end{array}\right]$
(D.7)	$\hat{q}_t = \hat{M}_t - \hat{V}_t$
(D.8)	$\hat{\delta}_{Kt} \equiv \frac{\delta_{K1}}{\delta_{K0}}\hat{u}_{Kt}$
(D.9)	$\hat{\kappa}_t = \tau\hat{V}_t$
(D.10)	$\hat{\eta}_{wt} = \widehat{\bar{\eta}}_t + \hat{h}_t - \frac{1}{h}\left\{\widehat{\bar{\eta}}_t h\bar{\eta} + \hat{h}_t h\bar{\eta} + (1-\bar{\eta})\left[\begin{array}{c}-h\hat{h}_t - \phi^w g_A\frac{\pi_C}{w}(\hat{\pi}_{wt} - \iota_w\tilde{\pi}_{wt}) \\ +\phi^w(1-\lambda)\beta E_t\left(\frac{\pi_w\pi_C^{-1}}{w}(\hat{\pi}_{wt+1} - \iota_w\tilde{\pi}_{wt+1})\right)\end{array}\right]\right\}$

Note: $\tilde{C}_t$and $\tilde{I}_{Kt}$in equation (D.1) are consumption and investment observed when $\phi^w = \phi^p = \varepsilon_{\bar{\eta}t} = \varepsilon_{\bar{\theta}t} = 0$.
Variables without a time subscript denoyes steady-state values; $\varsigma/(1-\varsigma) = \delta_{K2}/\delta_{K1}$.

TABLE A.7: ALTERNATIVE MODEL, LOG-LINEARIZED EQUATIONS

(2') $$\widehat{\bar{\beta}}_t + \omega \widehat{\bar{h}}_{xt} - \hat{\Psi}_t + \hat{h}_t + \hat{X}_t - \hat{u}_{Ct} = \hat{\varphi}_t + \alpha \left(\hat{u}_{Kt} + \hat{\tilde{K}}_t - \hat{g}_{At} - \hat{L}_t - \hat{\tilde{h}}_t \right) + \hat{\Delta}_{\tilde{h}t}$$

(14') $$\alpha(\hat{u}_{Kt} + \hat{\tilde{K}}_{t-1} - \hat{g}_{At}) + (1-\alpha)(\hat{\tilde{Z}}_t + \hat{L}_t + \hat{\tilde{h}}_t) = C\hat{C}_t + \hat{I}_{Kt} + \kappa \hat{\kappa}_t + \bar{G}\hat{\tilde{G}}_t$$

(15') $$\hat{r}_{Kt} = \hat{\varphi}_t + (\alpha - 1) \left(\hat{u}_{Kt} + \hat{\tilde{K}}_t - \hat{g}_{At} - \hat{L}_t - \hat{\tilde{h}}_t \right)$$

(D.4') $$S_t^f \hat{S}_t^f = \begin{bmatrix} (1-\alpha)\,\varphi \left(\frac{u_K \tilde{K}}{\bar{g}_A L h} \right)^{\alpha} h \left[\alpha \left(\hat{u}_{Kt} + \hat{\tilde{K}}_t - \hat{g}_{At} - \hat{L}_t \right) + (1-\alpha)\,\hat{\tilde{h}}_t + \hat{\Delta}_{\tilde{h}t} \right] \\ -wh(\hat{w}_t + \hat{h}_t) + \beta(1-\lambda) S^f \left[E_t \hat{u}_{Ct+1} - \hat{u}_{Ct} + E_t \hat{S}_{t+1}^f \right] \end{bmatrix}$$

(D.5') $$S^w \hat{S}_t^w = \begin{bmatrix} wh(\hat{w}_t + \hat{\tilde{h}}_t) - \frac{\bar{\beta}\bar{h}h^{1+\omega}}{(1+\omega)u_C} [\widehat{\bar{\beta}}_t - \hat{\Psi}_t + \widehat{\bar{h}}_{xt} + (1+\omega)\hat{h}_t + \hat{X}_t - \hat{u}_{Ct}] \\ +(1-\lambda)\beta S^w \left(1 - \frac{M}{U}\right) (E_t \hat{u}_{Ct+1} - \hat{u}_{Ct} + E_t \hat{S}_{t+1}^w) - (1-\lambda)\beta S^w \frac{M}{U} (E_t \hat{M}_{t+1} - E_t \hat{U}_{t+1}) \end{bmatrix}$$

(D.6') $$\hat{u}_{Ct} u_C = \begin{bmatrix} \Psi^{-1} \left(\widehat{\bar{\beta}}_t - \tilde{\Psi}_t \right) + \gamma\mu \left[C \left(1 - \frac{h_C}{g_A} \right) \right]^{\gamma-1} X^{1-\gamma} g_A^{\gamma-1} \left[\hat{\mu}_t + (1-\gamma)\,\hat{X}_{t-1} + (\gamma-1)\,\hat{g}_{At} \right] \\ +\gamma\,(\gamma-1)\,\mu X^{1-\gamma} g_A^{\gamma-1} \left[C \left(1 - \frac{h_C}{g_A} \right) \right]^{\gamma-2} \left[\hat{C}_t C - h_C \frac{C}{g_A} \left(\hat{C}_{t-1} - \hat{g}_{At} \right) \right] \\ -\beta h_C \,(\Psi g_A)^{-\sigma} E_t \left[\widehat{\bar{\beta}}_{t+1} - \left(\hat{\Psi}_{t+1} + \hat{g}_{At+1} \right) \right] \\ -\gamma\beta h_C \mu g_A^{-1} \left[C\,(g_A - h_C) \right]^{\gamma-1} X^{1-\gamma} \left(E_t \hat{\mu}_{t+1} - E_t g_{At+1} + (1-\gamma)\,\hat{X}_t \right) \\ -(\gamma-1)\,\gamma\beta h_C \tilde{\mu} g_A^{-1} \left[C\,(g_A - h_C) \right]^{\gamma-2} X^{1-\gamma} \left(E_t \hat{C}_{t+1} C g_A + E_t \hat{g}_{At+1} C g_A - h_C \hat{C}_t C \right) \end{bmatrix}$$

(D.12) $$\hat{\mu}_t \mu = \begin{bmatrix} -\Psi^{-1} L \frac{h^{1+\omega}}{1+\omega} \left(\widehat{\bar{\beta}}_t - \hat{\Psi}_t + L_t + \widehat{\bar{h}}_{xt} + (1+\omega)\,\hat{h}_t \right) \\ +(1-\gamma)\beta\mu g_A^{-1} \left[C\,(g_A - h_C) \right]^{\gamma} X^{-\gamma} \left[\hat{\mu}_{t+1} - \hat{g}_{At+1} - \gamma \hat{X}_t \right] \\ +\gamma(1-\gamma)\beta\mu g_A^{-1} \left[C\,(g_A - h_C) \right]^{\gamma-1} X^{-\gamma} \left(E_t \hat{C}_{t+1} C g_A + E_t g_{At+1} C g_A - h_C \hat{C}_t C \right) \end{bmatrix}$$

(D.13) $$\hat{\Psi}_t \Psi = \hat{C}_t C - h_C \frac{C}{g_A} \left(\hat{C}_{t-1} - \hat{g}_{At} \right) - L \frac{h^{1+\omega}}{1+\omega} X \left(\widehat{\bar{h}}_{xt} + \hat{L}_t + (1+\omega)\,\hat{h}_t + \hat{X}_t \right)$$

(D.14) $$\hat{X}_t X = \begin{bmatrix} \gamma \left[C \left(1 - \frac{h_C}{g_A} \right) \right]^{\gamma-1} \left(\frac{X}{g_A} \right)^{1-\gamma} \left[\hat{C}_t C - h_C \frac{C}{g_A} \left(\hat{C}_{t-1} - \hat{g}_{At} \right) \right] \\ +(1-\gamma) \left[C \left(1 - \frac{h_C}{g_{At}} \right) \right]^{\gamma} \left(\frac{X}{g_A} \right)^{1-\gamma} \left(\hat{X}_{t-1} - \hat{g}_{At} \right) \end{bmatrix}$$

(D.15) $$\tilde{h}_t = \hat{h}_t$$

(D.16) $$\Delta_{\tilde{h}} \hat{\Delta}_{\tilde{h}t} = \bar{\alpha}_h \frac{\tilde{h}}{h} \left(\hat{\bar{\alpha}}_{ht} + \hat{\tilde{h}}_t - \hat{h}_t \right) - \phi_h h^{\bar{\alpha}_h + 1} \hat{h}_t$$

Note: Other equations are unchanged relative to Table A.6.