Disentangling adverse selection, moral hazard and supply induced demand: An empirical analysis for the demand for health care services

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Abstract

In the health care sector moral hazard, adverse selection (self-selection) and induced demand are three very important phenomena that affect patient behaviour. Despite there exists a vast theoretical and empirical literature on these phenomena, so far no contribution has been able to approach them jointly. This is mostly due to the difficulty to model the joint determination of health service utilisation and health insurance choice by means of a tractable structural simultaneous equation model. In this paper we provide a solution to this problem and estimate a simultaneous four equation structural model with four latent variables, where the first two equations are meant to deal with the self-selection issue, while the third and fourth equation deal with the induced demand issue. A closed form solution for the Log-likelihood function - which guarantees an exact solution - is maximised by means of FIML, using the BHHH algorithm. The empirical analysis is conducted using cross-sectional data from the Italian health care system. The empirical analysis has confirmed the theoretical predictions of our structural model. In particular, we find that: *1)* the propensity of a person to receive any medical service may not be considered as exogenous in the equation explaining the propensity of the same person to select a supplementary health insurance plan; *2)* insured individuals with supplementary health insurance have a higher probability to access any private health service, given that he has used some medical treatment. *3)* insured individuals with supplementary insurance are also more prone to suffer of supply induced demand by providers. *4)* we do not find evidence of patient moral hazard, while we see an important supply induced demand effect. These results are extremely important from a health policy perspective, given the existing debate on the development of a second pillar in the financing of the health care system in Italy and in Europe.

1 Introduction

Since the seminal paper by Pauly (1968) and Rothschild and Stiglitz (1976), economists have largely investigated how asymmetric information in insurance markets can produce inefficient outcomes, notably overconsumption due to the insurgence of moral hazard (MH) and adverse selection (AD) behaviour by individuals who buy more insurance if they expect to use more medical care (different risk types). The health insurance sector represents a particularly interesting setting where to explore these issues, given the role that information asymmetry play between patients and physicians and the well know phenomenon of rising health care expenditure that characterises this sector. This last aspect is particularly relevant for the economic and financial sustainability of both government and employer-provided insurance, giving rise to a considerable academic and public policy debate.

According to Cutler and Zeckhauser (2000) and, more recently, to Powell and Goldman (2016), the economic literature has extensively studied these two phenomena, arguing that is the relative importance of adverse selection *vs.* moral hazard that should drive the design of optimal policies, given that the policy instruments to address selection are different from those required to address moral hazard: in the first case "risk pooling" may be the best choice, while in the second case "cost-sharing" is more suitable. In fact, while the first effect deals with the presence of additional insurance, the second effects is related to consumption decisions and implies that insured individuals consume more than uninsured.

From a policy maker point of view, disentangling these two effects is extremely important to design effective policies. Complexity in this setup can be further increased if we consider that MH is the sum of two components: patient overconsumption behaviour ("demand-side" moral hazard) and the providers' moral hazard or Supply Induced Demand (SID) behaviour ("supply-side" moral hazard), which is the prescription of services that a well-informed consumer would not want to use.[1]

[1] Bickerdyke, Dolamore, Monday, and Preston (2002) provide a better definition of SID, as "the notion that doctors, in acting as agents for their patients, can use their Ôdiscretionary powerÕ to engage in demand-shifting or inducement activities such that their recommended care differs from that which an informed agent would deem appropriate". According to Zweifel et al. (2000), SID occurs because *i)* providers have an informational advantage over patients, which could be used to alter patient demand functions, whenever it is unlikely that the medical treatment harms the patient; *ii)* patients hold supplementary health insurances which limit patient out-of-pocket expenditure: the lower is patient co-payment, the higher is provider ability to induce demand. In these cases patients could be reluctant to fill the asymmetry information gap (which is a costly activity) and more prone to accept the extra health care services proposed by physicians; *iii)* physician have incentive to create his own demand. This idea is formalized by Evans (1974), who assumes that the physicianÕs utility depends positively on income and

Given that AS and MH are positively correlated with the relationship between ex-post realisation of risk and insurance coverage (Chiappori and Salanie, 2000), separating these three effects it is not an easy task if researchers observe only the relationship between the presence of a supplementary health insurance and the use of medical care (Chiappori and Salanie [2000], Chiappori [2000], Finkelstein and Poterba [2004]). In general, the empirical literature on market failure in the insurance sector has approached the problem leaving self-selection and moral hazard as distinct phenomena. This is a very well known theoretical and empirical problem in the literature and so far we are not aware of any contribution able to jointly approach these problems and then to empirically disentangle the single effects. As recognised by Einav et al (2013), the early seminal theoretical contributions in this field have abstracted from moral hazard and have focused on selection driven by one-dimensional heterogeneity in risk aversion (Finkelstein and McGarry, 2006; Cohen and Einav, 2007; Fang, Keane, and Silverman, 2008; Einav, Finkelstein, and Schrimpf, 2010). Unfortunately, with this approach researchers can loose important pieces of information. On the contrary, as long as data from large randomised experiments were available, control for endogenous insurance selection was possible, and moral hazard effects has been correctly estimated.[2] However, a potential drawback inherited in randomisation is that it removes the endogeneity aspect of the choice and, as noticed, "it thus abstracts, by design, from any selection on moral hazard, which could have important implications for the spending reductions achieved through offering plans with higher consumer cost sharing." (Einav et al., 2013, p.179)

However, as also highlighted by Cronin (2015), obtaining experimental data is not so easy and appropriate quasi-experimental data are rarely available. To circumvent these problems, some researchers have approached the problem using structural modelling, which allows to separately identify moral hazard from adverse selection through different identification hypotheses. Furthermore, this class of model is useful also to simulate and predict behavioural responses induced by new policies.

negatively on the level of inducement. In other words, the physician is only willing to induce demand if this leads to a higher income. Clearly, whenever these situations occur jointly, the probability to observe SID effect is higher.

[2] Very good examples are the RAND HIE (Manning et al. (1987), Newhouse et al. (2008)) or the Oregon HIE (Finkelstein et al. (2012) and Baicker et al. (2013)). The RAND Health Insurance Experiment (HIE) was started in 1971 and was a multi-year, 295 million (in 2011 dollars, (Greenberg and Shroder, 2004)) medical care study that, among other things, randomly distributed health insurance plans to participants in 6 U.S. cities and recorded health and medical care consumption in the years following. The Oregon HIE is an experiment started in 2008 by the Oregon Health Authority, who expanded the stateÕs Medicaid program to 10,000 additional low-income adults using a lottery. In both cases, the random assignment has allowed researchers to study moral hazard without concern for endogenous insurance selection.

As already observed, among others, by Cameron *et al.* (1988), it is difficult to model the joint behaviour of health service utilisation and health insurance choice by means of a tractable structural simultaneous equation model. This partly justify the lack of empirical evidence based on structural model that we have witnessed so far. However, a growing body of empirical literature based on structural modelling is emerging. Apart from the original work by Cameron *et al.* (1988), Gilleskie (1998), Holly *et al.* (1998), Harris and Keane (1999), Cardon and Hendel (2001), Vera-Hernandez (2003), Blau and Gilleskie (2003) and Khwaja (2001, 2005). More recently, new interest has been brought in this field of research thanks to the development of more complex structural models and the possibility to estimate them via simulation techniques. Khwaja (2010), Einav *et al.* (2011), Kowalski (2012), Bajari *et al.* (2014), Handel (2013) and Powell and Goldman (2016) are all such examples.

In this paper we try to improve with respect to existing literature by proposing for the first time a four equation structural model that allows to estimate and disentangle adverse-selection, demand-side moral hazard and Supply Induced Demand. Our model build on previous work by Holly *et al.* (1998), where the authors have firstly provided some tractable structural analysis to deal with self-selection and demand-side moral hazard. However, their model is limited to consider a simultaneous two equation model, which simply related use per person to insurance plan. In their model the first equation is a reduced form equation of insurance choice, while the second equation is a structural equation for the propensity that someone has at least one inpatient stay, given that he has used some medical treatment and conditional on the type of insurance plan he/she has selected. Thus, the second equation contains endogenous latent variables as well as an endogenous dummy variable.

In this paper, we estimate a simultaneous four equation model with four latent variables, where the first two equations are meant to deal with the self-selection issue, while the last two equations deal with the issues related to consumption (MH and SID), similar to the second equation of Holly *et al.* (1998). The empirical analysis is conducted using a large Italian cross-sectional survey from the 1999 Multiscopo Survey (MS) (ISTAT, 2001). We also improve from the econometric methodology point of view as we derive a closed form solution of a Log-likelihood function that guarantees an exact solution, which is maximised by means of a FIML estimator using the BHHH algorithm. Compared to the standard simulation procedures available in the literature, notably the simulated score method suggested by Hajivassiliou and McFadden (1997) (see also Gouriéroux-Monfort (1996)), the use of the numerical method suggested in this paper is quicker and more accurate.

In what follow, the paper is organised in seven section including this introduction. Section 2 presents some background information concerning the Italian health care system. Section 3 introduces the econometric model and discusses the condition for identification. Section 4 describes the data used and presents some descriptive statistics concerning the use of private health insurance and the related use of private health care services. Section 5 presents the empirical results, while in section 6 we add some robustness checks. Finally, Section 7 discusses the policy implications of these findings and draws some conclusions.

2 The health care system in Italy and the private health insurance demand.

Since 1978, the Italian health care system has been regulated and administered through the a National Health Service (Servizo Sanitario Nazionale, referred to henceforth as SSN). The system is based on four main principles: universal coverage, full range of the health services provided, participation of citizens to the management of the SSN and organizational pluralism (State, Regions and Local Health Authorities). The main objective of the system is then to guarantee everyone equal access to uniform levels of health care according to needs. Income, personal or social characteristic or geographical location ought not to influence the level of coverage.

The system is organized in three hierarchic levels: national, regional and local units. The national level is responsible for designing the National Health Plans with the aim of ensuring the general health objectives and interventions. It is then the responsibility of the regional governments to achieve the objectives posed by the National Health Plan. Regions are the ones who deliver the benefit package to the population through a network of population based health care organizations (Local Health Units) and public and accredited private hospitals. Each region plans health care activities and organizes the supply according to population needs. Moreover, they have the responsibility to guarantee the quality, appropriateness and efficiency of the services provided. Local Health Units ought to guarantee equal access, efficacy of preventive, curative and rehabilitation interventions and efficiency in the distribution of services. They are also responsible for the balance between the funding provided by regions and the expenditures for the provision of health care services.

In providing health care services the SSN uses a wide array of providers: hospitals with different forms of ownership (directly managed, public hospital trusts; public and private teaching hospitals; public and private research hospitals; non profit hospitals; for profit hospitals) GPs, public and private ambulatory care facilities (specialist and diagnostic), public and private rehabilitation facilities and private community nurses. Physician services are provided by both General Practitioners (GPs) and by specialists, through a referral system. GPs are paid on a capitation basis. Patients do not pay for visits to GPs, and there is no limit to the number of visits they can have. Concerning specialists, there are public (SSN) and private specialists. Private specialists are, generally, self-employed and usually they work in "single practices" ambulatory while very rarely we find two or more specialists with different specializations. For each visit to a specialist in the public system, patients pay a fee (about 40 Euros, quite low compared to the average fees in the private sector), but access is regulated by GPs. Visits to private specialists are unregulated. Similarly, there is a maximum fee of 40 euros for diagnostic tests done in public facilities, while no charge is requested for hospitalization[3].

Co-payments depend on the income, age and health conditions of patients and are required for drugs, ambulatory treatments, specialists and for some diagnostic and laboratory tests. Public health care system coexisted with private provision of medical services. Private financing plays an increasing and not so marginal role. In this case, about 90 percent of private expenditure is financed by patient out-of-pocket payments, with the remainder 10 percent covered by private health insurance and company health plans. Furthermore, according to 1999 Multiscopo survey by the Italian National Institute for Statistics (ISTAT), the share of the Italian population covered by private health insurance is about 10 percent, well below what is recorded in other EU countries. This low share comes despite the fact that after the health reforms occurred between 1992-1994, the household private health expenditure has reached a level close to 30 percent of the overall health care expenditure, thus posing Italy as one of the industrialized countries with the highest use of the so called out-of-pocket health expenditure.

Presently, the insurance market proposes individual and collective insurance policies. Individual

[3] Co-payment fee and access to specialists may vary across regions due to different co-payment policies adopted

insurance policies are mainly structured as payment of a daily allowance for each sick leave and they are typically subscribed by self-employed workers with a high income, with the aim of reducing economic damage from illness; collective policies grant a substitute coverage (as a matter of fact it is a duplication of the public health care): usually they are restricted to executives or high income workers - who pay twice - in order to obtain health care services with more comfort.

Private utilisation of health care services is strongly influenced by two main factors: waiting lists and perceived low quality of health care services. Recently, some public hospitals have been organised in a way to offer "hotel" services (mainly single rooms with better services) to patients who are willing to pay out-of-the-pocket the service. In this last case, and although hospitalised in a public facility, private insurance can help to afford such costs.

3 The econometric model

As partly discussed in the introduction, health care utilisation and private health insurance are strongly interrelated giving rise to three different effects: adverse selection (AS), moral hazard (MH) and supply induced demand (SID). In particular, those who expect to use more health care services are more likely to buy supplementary insurance, thus causing problems of AD (or self selection). At the same time, MH intervenes each time patients see their *out-of-the-pocket* expenditure reduced due the presence of supplementary private insurance. Finally, patients with supplementary private insurance may experience a higher exposition to "unwanted" medical care - or SID. Therefore, in presence of health insurance patients choices with respect to these three phenomena are not independent, nor it is easy to estimate and disentangle their effects on the demand of health care services based only on knowledge of supplementary insurance subscription and utilisation of health care services. In order to better understand and to measure these relationships, in this section we introduce a theoretical framework making use of a simple four equation structural model.

In a more formal way let y_1^* be an endogenous variable representing the propensity of a person to receive any medical service, y_2^* an endogenous variable representing the propensity of the same person to purchase a supplementary health insurance, while y_3^* and y_4^* the propensity the same person has to receive any type of health care service from private hospitals and any type of "invasive" health care service from private hospitals, respectively.[4] To sketch our model, we start assuming that individuals make predictions about their future health status and, therefore, on their propensity to receive any medical service (y_1^*), conditional on a set of exogenous variables (x_1). We then assume that individuals decide whether to buy or not supplementary insurance (y_2^*) conditional on y_1^* and on a set of exogenous variables (x_2). This is equivalent to state that y_2^* is simultaneously determined by y_1^* and a set of exogenous variables x_2. As such, these two equations are meant to deal with the AS issue. To deal with MH and SID effects we need to further assume that, after making that choice, once the person is ill, his/her propensity to use privately provided health care services (y_3^*, y_4^*) depends on the presence of additional insurance coverage (y_2^*) and on personal characteristics (x_3, x_4). Unfortunately, this assumption alone does not allows to disentangle between MH and SID, as y_2 is responsible for both effects (in a positive way) on the demand for private health care services. One possibility to disentangle these two effects is to identify a subset of health care services, which are affected either by MH or by SID. In our case we decided to look at the subset of services that we define "invasive" (y_4^*), which by their nature should not be subject to "demand induced" MH effect. Therefore, we define a fourth equation where y_4^* is determined by y_2 and by a set of exogenous variables x_4. Therefore, the occurrence of y_1^* is a precondition for y_3^* and the occurrence of y_3^* is precondition for y_4^*. As a result, this model has two selection processes and an endogenous variable and it falls in the category of what Amemiya (1975) called "sequential models". The *structural form* of the model may thus be written as

$$y_1^* = x_1'\beta_1^0 + u_1^0 \quad (1)$$

$$y_2^* = x_2'\beta_2^0 + \alpha_{21}^0 y_1^* + u_2^0 \quad (2)$$

$$y_3^* = x_3'\beta_3^0 + \gamma_{32}^0 y_2 + u_3^0 \quad (3)$$

$$y_4^* = x_4'\beta_4^0 + \gamma_{42}^0 y_2 + u_4^0 \quad (4)$$

[4] We define as "invasive" procedures all sort of treatments that may cause a substantial discomfort to patients in terms of both pain and potential side effects. In particular we consider as invasive treatments surgery interventions, gastroscopies and colonoscopies.

where

$$\mathcal{D}\begin{pmatrix} u_1^0 \\ u_2^0 \\ u_3^0 \\ u_4^0 \end{pmatrix} = \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^{02} & \sigma_{12}^0 & \sigma_{13}^0 & \sigma_{14}^0 \\ \sigma_{12}^0 & \sigma_2^{02} & \sigma_{23}^0 & \sigma_{24}^0 \\ \sigma_{13}^0 & \sigma_{23}^0 & \sigma_3^{02} & \sigma_{34}^0 \\ \sigma_{14}^0 & \sigma_{24}^0 & \sigma_{34}^0 & \sigma_4^{02} \end{pmatrix}\right) \tag{5}$$

$$\begin{aligned}
y_1 &= \begin{cases} 1 & \text{if } y_1^* > 0 \\ 0 & \text{otherwise} \end{cases} \\
y_2 &= \begin{cases} 1 & \text{if } y_2^* > 0 \\ 0 & \text{otherwise} \end{cases} \\
y_3 &= \begin{cases} 1 & \text{if } y_1^* > 0 \text{ and } y_3^* > 0 \text{ (any medical treatment in a private hospital)} \\ 0 & \text{if } y_1^* > 0 \text{ and } y_3^* \leq 0 \text{ (any medical treatment in a private hospital)} \\ \text{n.a.d.} & \text{otherwise} \end{cases} \\
y_4 &= \begin{cases} 1 & \text{if } y_3^* > 0 \text{ and } y_4^* > 0 \text{ (only an ``invasive" treatment in a private hospital)} \\ 0 & \text{if } y_3^* > 0 \text{ and } y_3^* \leq 0 \text{ (only an ``invasive" treatment in a private hospital)} \\ \text{n.a.d.} & \text{otherwise} \end{cases}
\end{aligned}$$

since y_3^* and y_4^* are not appropriately defined if a patient has not received a medical treatment from a private hospital.

Note that the first two equations say that the intentions about y_1 and y_2 are simultaneously determined by the exogenous variables in x_1 and x_2. This formulation is supposed to deal with the AS issue.

To simplify our notation, it is convenient to denote by x the vector formed by all components of the vectors x_1, x_2, x_3 and x_4, and write

$$x_i' = x' S_{bi} \qquad i = 1, 2, 3, 4. \tag{6}$$

where S_{bi} is the $K \times K_i$ selection matrix, selecting the components of x_i from those of x. Also, we define the four-dimensional vectors y^* and y as

$$y^{*\prime} = (y_1^*, y_2^*, y_3^*, y_4^*),$$

and

$$y' = (y_1, y_2, y_3, y_4)$$

respectively.

The simultaneous four-equations model in equations (1) through (4) is a *model with a mixed structure* in the terminology suggested by Maddala (1983). It contains endogenous latent variables ($y^{*\prime}$), as well as an endogenous dummy variable (y_2), and it may be compactly written as

$$A^0 y^* = B^0 x + \Gamma^0 y + u^0, \tag{7}$$

where

$$A^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -\alpha_{21}^0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{8}$$

$$B^0 = \begin{pmatrix} \beta_1^{0\prime} S_{b1}' \\ \beta_2^{0\prime} S_{b2}' \\ \beta_3^{0\prime} S_{b3}' \\ \beta_4^{0\prime} S_{b4}' \end{pmatrix}$$

$$\Gamma^0 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

3.1 Logical consistency

We observe that the matrix $A^{0-1}\Gamma^0$ given by

$$A^{0-1}\Gamma^0 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

is lower-triangular with zeros on the diagonal. If there were no selection in the model under consideration then, applying a result of Schmidt (1982), we could conclude that the model (7) is logically consistent without additional conditions. According to Gourieroux-Laffont-Monfort (1980), this implies that the model has a well defined reduced form which expresses the latent variables in terms of the exogenous variables and the disturbance terms. However, given that the model under consideration is with two endogenous sampling selections, the result by Schmidt (1982) does not seem to be directly applicable.

To examine the logical consistency of the model one could adopt the procedure developed in Gourieroux-Laffont-Monfort (1980). However, a more direct method consists in following the approach presented in Maddala (1983), which considers under which conditions on the parameters the sum of of the different probabilities of the endogenous dichotomous variables is equal to one.

To this end, we first observe that, by using (7) and inverting A^0, the latent variables can be experessed in terms of y_2 and the exogenous variables by the following system of equations :

$$y^* = A^{0-1}B^0x + A^{0-1}\Gamma^0 y + v^0, \tag{9}$$

where

$$v^0 = A^{0-1}u^0$$

and

$$\mathcal{D}\left(v^0\right) = \mathcal{N}\left(0, \Omega^0\right),$$

where

$$\Omega^0 = A^{0-1}\Sigma^0 A^{0-1\prime} \tag{10}$$

and

$$\Sigma^0 = \begin{pmatrix} \sigma_1^{02} & \sigma_{12}^0 & \sigma_{13}^0 & \sigma_{14}^0 \\ \sigma_{12}^0 & \sigma_2^{02} & \sigma_{23}^0 & \sigma_{24}^0 \\ \sigma_{13}^0 & \sigma_{23}^0 & \sigma_3^{02} & \sigma_{34}^0 \\ \sigma_{14}^0 & \sigma_{24}^0 & \sigma_{34}^0 & \sigma_4^{02} \end{pmatrix} \tag{11}$$

Similarly, Ω^0 may be written as

$$\Omega^0 = \begin{pmatrix} \omega_1^{02} & \omega_{12}^0 & \omega_{13}^0 & \omega_{14}^0 \\ \omega_{12}^0 & \omega_2^{02} & \omega_{23}^0 & \omega_{24}^0 \\ \omega_{13}^0 & \omega_{23}^0 & \omega_3^{02} & \omega_{34}^0 \\ \omega_{14}^0 & \omega_{24}^0 & \omega_{34}^0 & \omega_4^{02} \end{pmatrix} \tag{12}$$

Following Heckman (1978) we shall refer to model 9) as *semi-reduced form model.*

It is useful to write equations (9) in detail as follows:

$$y_1^* = x_1'\beta_1^0 + v_1^0 \tag{13}$$

$$y_2^* = x_2'\beta_2^0 + x_1'\beta_1^0\alpha_{21}^0 + v_2^0 \tag{14}$$

$$y_3^* = x_3'\beta_3^0 + \gamma_{32}^0 y_2 + v_3^0 \tag{15}$$

$$y_4^* = x_4'\beta_4^0 + \gamma_{42}^0 y_2 + v_4^0 \tag{16}$$

We now turn to the expression of the different probabilities of the endogenous dichotomous variables.

To this end, we first observe that due to the endogenous sampling selections, the probabilities $P\{y_1 = 0, y_2, y_3\}$ are identically equal to zero, for any values of (y_2, y_3). Next, in order to derive the remaining probabilities, we define

$$\begin{aligned} P_{ij}^0 &= P^0\,(y_1 = i, y_2 = j) \quad (i = 0,1; j = 0,1) \\ P_{ijk}^0 &= P^0\,(y_1 = i, y_2 = j, y_3 = k) \quad (i = 0,1; j = 0,1; k = 0,1) \\ P_{ijkm}^0 &= P^0\,(y_1 = i, y_2 = j, y_3 = k, y_4 = m) \quad (i = 0,1; j = 0,1; k = 0,1; m = 0,1) \end{aligned}$$

The eight different possible sets of values for y_1, y_2, y_3 and y_4 together with their respective probabilities are given in the following table:

y_1	y_2	y_3	y_4	*Probability*
0	0			$P_{00}^0 = P\left\{v_1^0 < -x_1'\beta_1^0, v_2^0 < -(x_2'\beta_2^0 + x_1'\beta_1^0\alpha_{21}^0)\right\}$
0	1			$P_{01}^0 = P\left\{v_1^0 < -x_1'\beta_1^0, v_2^0 \geqq -(x_2'\beta_2^0 + x_1'\beta_1^0\alpha_{21}^0)\right\}$
1	0	0		$P_{100}^0 = P\left\{v_1^0 \geqq -x_1'\beta_1^0, v_2^0 < -(x_2'\beta_2^0 + x_1'\beta_1^0\alpha_{21}^0), v_3^0 < -x_3'\beta_3^0\right\}$
1	0	1	0	$P_{1010}^0 = P\left\{v_1^0 \geqq -x_1'\beta_1^0, v_2^0 < -(x_2'\beta_2^0 + x_1'\beta_1^0\alpha_{21}^0), v_3^0 \geqq -x_3'\beta_3^0, v_4^0 < -x_4'\beta_4^0\right\}$
1	0	1	1	$P_{1011}^0 = P\left\{v_1^0 \geqq -x_1'\beta_1^0, v_2^0 < -(x_2'\beta_2^0 + x_1'\beta_1^0\alpha_{21}^0), v_3^0 \geqq -x_3'\beta_3^0, v_4^0 \geqq -x_4'\beta_4^0\right\}$
1	1	0		$P_{110}^0 = P\left\{v_1^0 \geqq -x_1'\beta_1^0, v_2^0 \geqq -(x_2'\beta_2^0 + x_1'\beta_1^0\alpha_{21}^0), v_3^0 < -(x_3'\beta_3^0 + \gamma_{32}^0)\right\}$
1	1	1	0	$P_{1110}^0 = P\left\{v_1^0 \geqq -x_1'\beta_1^0, v_2^0 \geqq -(x_2'\beta_2^0 + x_1'\beta_1^0\alpha_{21}^0), v_3^0 \geqq -(x_3'\beta_3^0 + \gamma_{32}^0), v_4^0 < -(x_4'\beta_4^0 + \gamma_{42}^0)\right\}$
1	1	1	1	$P_{1111}^0 = P\left\{v_1^0 \geqq -x_1'\beta_1^0, v_2^0 \geqq -(x_2'\beta_2^0 + x_1'\beta_1^0\alpha_{21}^0), v_3^0 \geqq -(x_3'\beta_3^0 + \gamma_{32}^0), v_4^0 \geqq -(x_4'\beta_4^0 + \gamma_{42}^0)\right\}$

(17)

It is easy to verify that

$$P_{00}^0 + P_{01}^0 = P\{v_1^0 < -x_1'\beta_1^0\} \tag{18}$$

$$P_{1010}^0 + P_{1011}^0 = P\{v_1^0 \geq -x_1'\beta_1^0, v_2^0 < -(x_2'\beta_2^0 + x_1'\beta_1^0\alpha_{21}^0), v_3^0 \geq -x_3'\beta_3^0\} \tag{19}$$

$$P_{1110}^0 + P_{1111}^0 = P\{v_1^0 \geq -x_1'\beta_1^0, v_2^0 \geq -(x_2'\beta_2^0 + x_1'\beta_1^0\alpha_{21}^0), v_3^0 \geq -(x_3'\beta_3^0 + \gamma_{32}^0)\}, \tag{20}$$

and thus

$$P_{110}^0 + P_{1110}^0 + P_{1111}^0 = P\{v_1^0 \geq -x_1'\beta_1^0, v_2^0 \geq -(x_2'\beta_2^0 + x_1'\beta_1^0\alpha_{21}^0)\}. \tag{21}$$

Also, we have

$$P_{100}^0 + P_{1010}^0 + P_{1011}^0 = P\{v_1^0 \geq -x_1'\beta_1^0, v_2^0 < -(x_2'\beta_2^0 + x_1'\beta_1^0\alpha_{21}^0)\}. \tag{22}$$

Therefore, we get from (21) and (22) that

$$P_{100}^0 + P_{110}^0 + P_{1010}^0 + P_{1011}^0 + P_{1110}^0 + P_{1111}^0 = P\{v_1^0 \geq -x_1'\beta_1^0\}, \tag{23}$$

and, finally, using (18),

$$P_{00}^0 + P_{01}^0 + P_{100}^0 + P_{110}^0 + P_{1010}^0 + P_{1011}^0 + P_{1110}^0 + P_{1111}^0 = 1 \tag{24}$$

Equation (24) implies that the sum of the different probabilities of the endogenous dichotomous variables is equal to one and, therefore, that the model (7) is logically consistent without imposing any additional restrictions on the parameters.

Once we have shown the existence of the reduced form for the latent variables in terms of the exogenous variables and the disturbance terms, the next step consists in considering the identification of the model. To this end below we explicit the constraints imposed on the parameters of the model.

3.2 Constraints on the parameters

In addition to the constraints on the matrices A^0, B^0 and Γ^0 implied by the exclusion restrictions, there are a number of restrictions resulting from the specification of the model.

To see this, it is useful to write equation (10) in detail as follows:

$$\begin{pmatrix} \omega_1^{02} & \omega_{12}^0 & \omega_{13}^0 & \omega_{14}^0 \\ \omega_{12}^0 & \omega_2^{02} & \omega_{23}^0 & \omega_{24}^0 \\ \omega_{13}^0 & \omega_{23}^0 & \omega_3^{02} & \omega_{34}^0 \\ \omega_{14}^0 & \omega_{24}^0 & \omega_{34}^{02} & \omega_4^0 \end{pmatrix} = \begin{pmatrix} \sigma_1^{02} & \alpha_{21}^0\sigma_1^{02}+\sigma_{12}^0 & \sigma_{13}^0 & \sigma_{14}^0 \\ \alpha_{21}^0\sigma_1^{02}+\sigma_{12}^0 & \alpha_{21}^{02}\sigma_1^{02}+2\alpha_{21}^0\sigma_{12}^0+\sigma_2^{02} & \alpha_{21}^0\sigma_{13}^0+\sigma_{23}^0 & \alpha_{21}^0\sigma_{14}^0+\sigma_{24}^0 \\ \sigma_{13}^0 & \alpha_{21}^0\sigma_{13}^0+\sigma_{23}^0 & \sigma_3^{02} & \sigma_{34}^0 \\ \sigma_{14}^0 & \alpha_{21}^0\sigma_{14}^0+\sigma_{24}^0 & \sigma_{34}^0 & \sigma_4^{02} \end{pmatrix} \tag{25}$$

Because y_1^*, y_2^*, y_3^* and y_4^* are observed as dichotomous variables, we need to impose the conditions

$$\omega_1^{02} = 1; \qquad \omega_2^{02} = 1; \qquad \omega_3^{02} = 1; \qquad \omega_4^{02} = 1$$

Therefore, the first and second order structural form parameters are subject to the additional constraints:

$$\sigma_1^{02} = 1 \tag{26}$$

$$\alpha_{21}^{02} + 2\alpha_{21}^0\sigma_{12}^0 + \sigma_2^{02} = 1 \tag{27}$$

$$\sigma_3^{02} = 1 \tag{28}$$

$$\sigma_4^{02} = 1 \tag{29}$$

such that,

$$\Sigma^0 = \begin{pmatrix} 1 & \sigma_{12}^0 & \sigma_{13}^0 & \sigma_{14}^0 \\ \sigma_{12}^0 & \sigma_2^{02} & \sigma_{32}^0 & \sigma_{42}^0 \\ \sigma_{13}^0 & \sigma_{23}^0 & 1 & \sigma_{43}^0 \\ \sigma_{14}^0 & \sigma_{42}^0 & \sigma_{43}^0 & 1 \end{pmatrix} \tag{30}$$

where σ_{12}^0 and σ_2^{02} are related to α_{21}^0 according to (27), and

$$\Omega^0 = \begin{pmatrix} \omega_1^{02} & \omega_{12}^0 & \omega_{13}^0 & \omega_{14}^0 \\ \omega_{12}^0 & \omega_2^{02} & \omega_{23}^0 & \omega_{24}^0 \\ \omega_{13}^0 & \omega_{23}^0 & \omega_3^{02} & \omega_{34}^0 \\ \omega_{14}^0 & \omega_{24}^0 & \omega_{34}^0 & \omega_4^{02} \end{pmatrix} = \begin{pmatrix} 1 & \alpha_{21}^0 + \sigma_{12}^0 & \sigma_{13}^0 & \sigma_{14}^0 \\ \alpha_{21}^0 + \sigma_{12}^0 & 1 & \alpha_{21}^0\sigma_{13}^0 + \sigma_{23}^0 & \alpha_{21}^0\sigma_{14}^0 + \sigma_{24}^0 \\ \sigma_{13}^0 & \alpha_{21}^0\sigma_{13}^0 + \sigma_{23}^0 & 1 & \sigma_{34}^0 \\ \sigma_{14}^0 & \alpha_{21}^0\sigma_{14}^0 + \sigma_{24}^0 & \sigma_{34}^0 & 1 \end{pmatrix}, \tag{31}$$

3.3 Identification of the full model

The identification of the model is required to be able to estimate the structural parameters involved. It has to be checked once the logical consistency conditions are satisfied. The identifiability conditions in a mixed structure model are somewhat different from those in the usual simultaneous-equations models. Following Huguenin (2004), we shall examine the identifiability of the full model by applying the well-known concept of equivalent structures as first stated in Koopmans (1949) and developed in Koopmans et al. (1950).

Assuming normality of the distribution of the vector u^0, a structure S^0 consists of a set of values of the coefficient matrices A^0, B^0, Γ^0 and the variance-covariance matrix Σ^0, and we write $S^0 = (A^0, B^0, \Gamma^0, \Sigma^0)$.

Let M be a nonsingular matrix and consider the linear transformation of model (7),

$$MA^0 y^* = MB^0 x + M\Gamma^0 y + Mu^0, \tag{32}$$

$$\mathcal{D}\left(Mu^0\right) = \mathcal{N}\left(0, M\Sigma^0 M'\right) \tag{33}$$

Let $S^{\oplus 0}$ be a structure derived by S^0 by the transformation

$$A^{\oplus 0} = MA^0, \quad B^{\oplus 0} = MB^0, \quad \Gamma^{\oplus 0} = M\Gamma^0, \quad \Sigma^{\oplus 0} = M\Sigma^0 M' \tag{34}$$

Clearly, the structures S^0 and $S^{\oplus 0} = (A^{\oplus 0}, B^{\oplus 0}, \Gamma^{\oplus 0}, \Sigma^{\oplus 0})$ share the same semi-reduced form. Therefore, given that the logical consistency conditions are satisfied, they necessarily share the same reduced form which expresses the latent variables in terms of the exogenous variables and the disturbance terms. For this reason, the two structures S^0 and $S^{\oplus 0}$ will be called (observationally) equivalent.

We now consider the set of structures $S^{\oplus 0}$ which satisfy all the *a priori* restrictions on A^0, B^0, Γ^0 and Σ^0. We shall say that the structure S^0 is identifiable by this set of *a priori* restrictions if there is a unique equivalent structure $S^{\oplus 0}$ which satisfies the same set of *a priori* restrictions. Obviously, this uniqueness property implies there is a unique matrix M which produces an equivalent structure, namely I_4 the idendity matrix of order four.

One could apply rank conditions as in Huguenin (2004) to verify whether model (7) is identified. However, a direct verification is quite easy to carry-out given the simple triangular structure and the small number of equations of the model under consideration. Identifiability conditions of the equations can be applied to the full model including some second-order restrictions on the covariances of the disturbance terms.

Consider a nonsingular matrix M given by

$$M = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix} \tag{35}$$

We have

$$A^{\oplus 0} = MA^0 = \begin{pmatrix} m_{11} - m_{12}\alpha_{21}^0 & m_{12} & m_{13} & m_{14} \\ m_{21} - m_{22}\alpha_{21}^0 & m_{22} & m_{23} & m_{24} \\ m_{31} - m_{32}\alpha_{21}^0 & m_{32} & m_{33} & m_{34} \\ m_{41} - m_{42}\alpha_{21}^0 & m_{42} & m_{43} & m_{44} \end{pmatrix}$$

For the matrix $A^{\oplus 0}$ to satisfy the same restrictions as A^0 given by (8), we need to impose the following restrictions on the components of M: all its diagonal components are equal to 1 and all remaining components except possibly m_{21} are equal to 0.

In other words, permissible matrices M are of the form

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ m_{21} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{36}$$

and thus,

$$A^{\oplus 0} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ m_{21} - \alpha_{21}^0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Similarly, we have

$$B^{\oplus 0} = \begin{pmatrix} \beta_1^{0\prime} S_{b1}^{'} \\ m_{21}\beta_1^{0\prime} S_{b1}^{'} + \beta_2^{0\prime} S_{b2}^{'} \\ \beta_3^{0\prime} S_{b3}^{'} \\ \beta_4^{0\prime} S_{b4}^{'} \end{pmatrix},$$

$$\Gamma^{\oplus 0} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix},$$

and

$$\Sigma^{\oplus 0} = \begin{pmatrix} 1 & m_{21}+\sigma^0_{12} & \sigma^0_{13} & \sigma^0_{14} \\ m_{21}+\sigma^0_{12} & m_{21}(m_{21}+\sigma^0_{12})+m_{21}\sigma^0_{12}+\sigma^{02}_2 & m_{21}\sigma^0_{13}+\sigma^0_{32} & m_{21}\sigma^0_{14}+\sigma^0_{42} \\ \sigma^0_{13} & m_{21}\sigma^0_{13}+\sigma^0_{23} & 1 & \sigma^0_{43} \\ \sigma^0_{14} & m_{21}\sigma^0_{14}+\sigma^0_{42} & \sigma^0_{43} & 1 \end{pmatrix}.$$

Without introducing additional information, it follows that the second equation is not identified. However, if we assume *a priori* that $\sigma^0_{12} = 0$, then Σ^0 given by (30) becomes

$$\Sigma^0 = \begin{pmatrix} 1 & 0 & \sigma^0_{13} & \sigma^0_{14} \\ 0 & \sigma^{02}_2 & \sigma^0_{32} & \sigma^0_{42} \\ \sigma^0_{13} & \sigma^0_{23} & 1 & \sigma^0_{43} \\ \sigma^0_{14} & \sigma^0_{42} & \sigma^0_{43} & 1 \end{pmatrix}. \tag{37}$$

Under this assumption, $\Sigma^{\oplus 0}$ is equal to

$$\Sigma^{\oplus 0} = \begin{pmatrix} 1 & m_{21} & \sigma^0_{13} & \sigma^0_{14} \\ m_{21} & m^2_{21}+\sigma^{02}_2 & m_{21}\sigma^0_{13}+\sigma^0_{32} & m_{21}\sigma^0_{14}+\sigma^0_{42} \\ \sigma^0_{13} & m_{21}\sigma^0_{13}+\sigma^0_{23} & 1 & \sigma^0_{43} \\ \sigma^0_{14} & m_{21}\sigma^0_{14}+\sigma^0_{42} & \sigma^0_{43} & 1 \end{pmatrix}.$$

For $\Sigma^{\oplus 0}$ to satisfy the same restrictions as Σ^0 given by (37), we need to impose the additional restriction $m_{21} = 0$. Therefore, under the assumption that $\sigma^0_{12} = 0$, the system of equation (7) is identified without introducing further restrictions on the model. It that case, the matrix Ω^0 given in (31) is equal to

$$\Omega^0 = \begin{pmatrix} \omega^{02}_1 & \omega^0_{12} & \omega^0_{13} & \omega^0_{14} \\ \omega^0_{12} & \omega^{02}_2 & \omega^0_{23} & \omega^0_{24} \\ \omega^0_{13} & \omega^0_{23} & \omega^{02}_3 & \omega^0_{34} \\ \omega^0_{14} & \omega^0_{24} & \omega^0_{34} & \omega^{02}_4 \end{pmatrix} = \begin{pmatrix} 1 & \alpha^0_{21} & \sigma^0_{13} & \sigma^0_{14} \\ \alpha^0_{21} & 1 & \alpha^0_{21}\sigma^0_{13}+\sigma^0_{23} & \alpha^0_{21}\sigma^0_{14}+\sigma^0_{24} \\ \sigma^0_{13} & \alpha^0_{21}\sigma^0_{13}+\sigma^0_{23} & 1 & \sigma^0_{34} \\ \sigma^0_{14} & \alpha^0_{21}\sigma^0_{14}+\sigma^0_{24} & \sigma^0_{34} & 1 \end{pmatrix} \tag{38}$$

In order to avoid multicollinearity problems, we set also $\sigma_{13}^0 = \sigma_{14}^0 = \sigma_{34}^0 = 0$. It then follows that Ω^0 could be conveniently rewritten as:

$$\Omega^0 = \begin{pmatrix} \omega_1^{02} & \omega_{12}^0 & \omega_{13}^0 & \omega_{14}^0 \\ \omega_{12}^0 & \omega_2^{02} & \omega_{23}^0 & \omega_{24}^0 \\ \omega_{13}^0 & \omega_{23}^0 & \omega_3^{02} & \omega_{34}^0 \\ \omega_{14}^0 & \omega_{24}^0 & \omega_{34}^0 & \omega_4^{02} \end{pmatrix} = \begin{pmatrix} 1 & \alpha_{21}^0 & 0 & 0 \\ \alpha_{21}^0 & 1 & \sigma_{23}^0 & \sigma_{24}^0 \\ 0 & \sigma_{23}^0 & 1 & 0 \\ 0 & \sigma_{24}^0 & 0 & 1 \end{pmatrix} \tag{39}$$

3.4 Deriving the FIML function

Estimation of the model (7) by full information maximum likelihood (FIML) method can be carried out based on results presented earlier. The FIML function can be expressed in terms of the eight different possible sets of values for y_1, y_2, y_3 and y_4 together with their respective probabilities given Table (17).

Specifically, taking into account the two endogenous sample selections present in the model, the Log-likelihood function for the complete model can be written compactly as:

$$\mathcal{L}_N = \sum_{n=1}^{N} \sum_{y_2=0}^{1} \sum_{y_4=0}^{1} \ln \left[(1 - y_{n1}) P_{n0y_2} + y_{n1} (1 - y_{n3}) P_{n1y_20} + y_{n1} y_{n3} P_{n1y_21y_4} \right] \tag{40}$$

where, from Table (17),

$$P_{n0y_2} = \begin{cases} P\{v_{n1} < -x'_{n1}\beta_1, v_{n2} < -(x'_{n2}\beta_2 + x'_{n1}\beta_1\alpha_{21})\} \text{ if } y_2 = 0 \\ P\{v_{n1} < -x'_{n1}\beta_1, v_{n2} \geq -(x'_{n2}\beta_2 + x'_{n1}\beta_1\alpha_{21})\} \text{ if } y_2 = 1 \end{cases},$$

$$P_{n1y_20} = \begin{cases} P\{v_{n1} \geq -x'_{n1}\beta_1, v_{n2} < -(x'_{n2}\beta_2 + x'_{n1}\beta_1\alpha_{21}), v_{n3} < -x'_{n3}\beta_3\} & \text{if } y_2 = 0 \\ P\{v_{n1} \geq -x'_{n1}\beta_1, v_{n2} \geq -(x'_{n2}\beta_2 + x'_{n1}\beta_1\alpha_{21}), v_{n3} < -(x'_{n3}\beta_3 + \gamma_{32})\} & \text{if } y_2 = 1 \end{cases},$$

$$P_{n1y_21y_4} = \begin{cases} P\{v_{n1} \geq -x'_{n1}\beta_1, v_{n2} < -(x'_{n2}\beta_2 + x'_{n1}\beta_1\alpha_{21}), v_{n3} \geq -x'_{n3}\beta_3, v_{n4} < -x'_{n4}\beta_4\} & \text{if } y_2 = 0 \text{ and } y_4 = 0 \\ P\{v_{n1} \geq -x'_{n1}\beta_1, v_{n2} < -(x'_{n2}\beta_2 + x'_{n1}\beta_1\alpha_{21}), v_{n3} \geq -x'_{n3}\beta_3, v_{n4} \geq -x'_{n4}\beta_4\} & \text{if } y_2 = 0 \text{ and } y_4 = 1 \\ P\{v_{n1} \geq -x'_{n1}\beta_1, v_{n2} \geq -(x'_{n2}\beta_2 + x'_{n1}\beta_1\alpha_{21}), v_{n3} \geq -(x'_{n3}\beta_3 + \gamma_{32}), v_{n4} < -(x'_{n4}\beta_4 + \gamma_{42})\} & \text{if } y_2 = 1 \text{ and } y_4 = 0 \\ P\{v_{n1} \geq -x'_{n1}\beta_1, v_{n2} \geq -(x'_{n2}\beta_2 + x'_{n1}\beta_1\alpha_{21}), v_{n3} \geq -(x'_{n3}\beta_3 + \gamma_{32}), v_{n4} \geq -(x'_{n4}\beta_4 + \gamma_{42})\} & \text{if } y_2 = 1 \text{ and } y_4 = 1 \end{cases}$$

Computation of these probabilities requires integration of the multivariate normal density over infinite range subsets of $\mathbb{R}^2$, $\mathbb{R}^3$ and $\mathbb{R}^4$.

Extending the work of Lazard-Holly and Holly (2002), Huguenin (2004) showed how to transform multiple infinite range integrals of the multivariate normal density into a sum of lesser order multiple finite range integrals where the domains of integration are bounded and delimited by the correlation coefficients.[5] A further advantage of this analytical decomposition is to allow for the expression of the gradient of the log-likelihood function with respect to any of its parameters in a quite simple way.

The BHHH approach (see Berndt et al., 1974) lends itself naturally well in this setting, as it avoids computing the analytical Hessian function of the log-likelihood.

Numerically, both the log-likelihood function and the gradient function involve small finite-range multiple integrals that can be evaluated by Gauss-Legendre quadrature, and the maximisation can be performed with the help of a numerical Hessian. Specific numerical evaluation and estimation methods have been programmed in Huguenin (2004) under the software GAUSS, using its maximum likelihood library MAXLIK, in the form of a library providing user friendly procedures.

From computational point of view there are few routines that estimate multivariate probit using simulation method. In particular, Roodman (2011, 2014) proposed a new routine (`cmp` Stata command) which gives the possibility to estimate a class of models like the one proposed in this paper using Simulated Maximum Likelihood (SML) techniques. Roodman's procedure is based on the Geweke-Hajivassiliou-Keane (GHK) algorithm to compute higher dimensional cumulative normal distributions. According to Huguenin (2004) and Huguenin et al. (2014), the estimates obtained with this FIML procedure seem to be more accurate than those based on SML methods.[6]

3.5 Testing and disentangling AS, MH and SID: the identification strategy

Given this econometric framework, testing for AS, MH and SID is a rather simple task, while disentangling them remains more complicated.

The presence of adverse selection is related with the endogeneity of y_1^* in Eq.(2). Under the

[5] See also Huguenin et al. (2014) for an exposition of the main results of Huguenin (2004).

[6] Additionally, the procedure proposed by Roodman (2014) gives different results according to different sorting of the dataset, while our FIML procedure does not, being based on an exact solution of the likelhood function. The estimates obtained using SML procedures and using different sorting dataset are available upon request

normality assumption of our model, y_1^* is exogenous in Eq.(2) if and only if $\gamma_{21}^0 + \rho_{12}^0 = 0$. Given that the covariance between the perturbations of Eq.(1) and Eq.(2), (ρ_{12}^0), has been set to zero for identification reasons, it follows that a simple t-test on the significance of the parameter $gamma_{21}^0$ (associated with the variable "use" in Eq.(2)) is a test for the presence of AS.

Unfortunately, testing for the presence of MH and SID, and disentangling the two effects, is more difficult, as both effects work in the direction of increasing health care utilisation. The only chance we have to separate them is to exploit the existence of potential differences in the way patients can see the demand of health care services. Therefore, our identification strategy relies on the possibility to separate health care services which are "invasive", and therefore supposed to be unwanted or avoidable if possible by patients, from those who are not. Clearly, our maintained hypothesis is that the "invasive" category should be less affected (or not affected at all) by MH effect, and if an effect if found, it should be imputed to SID. On the contrary, for "non invasive" procedures we could observe MH and SID at the same time. By simply computing the difference between the two parameters should lead to identify the two effects. This is why in Eq.(3) the variable y_3^* includes all health care services provided in a private setting (the more sensible to private insurance effect), which are theoretically affected by both MH and SID, while in Eq.(4) the dependent variables is restricted to the subset of "invasive" private health care services, which should not be affected by MH. Given this setup, a t-test over γ_{42}^0 could help us to identify SID.

Given the selection structure between Eq.(3) and (4), and under the assumption that $\gamma_{ij} > 0$ (i.e., MH cannot be negative), the following different meaningful alternatives can be obtained when comparing and interpreting the relative importance of "demand side" MH and SID:

1. if $\gamma_{32}^0 > 0$ and $\gamma_{42}^0 = 0$, this implies that we have "demand side" MH but not SID;
2. if $\gamma_{32}^0 > \gamma_{42}^0$ and both are > 0, this implies that SID is lower than "demand side" MH;
3. if $\gamma_{32}^0 \leq \gamma_{42}^0$ and both are > 0, this implies that SID is greater than "demand side" MH, although we cannot exclude that "demand side" MH is null (it is observationally equivalent). This case includes also the sub case $\gamma_{32}^0 = \gamma_{42}^0 > 0$, which implies the existence of both effects with similar magnitude.

Although potentially available, for obvious reasons we rule out situations in which γ_{32}^0 is not statistically significant when γ_{42}^0 is statistically significant. Finally, a simple Wald test can be used to easily test if $\gamma^0_{32} \neq \gamma^0_{42}$.

In this last case, given that we avoid SID by construction, the empirical model can reduce to a simpler tri-variate Probit model.

4 Data used in the empirical analysis

The data used for the empirical analysis are from the ISTAT Multiscopo Survey (MHS), conducted by Italian National Institute of Statistics in 1999-2000. This survey is conducted each five years, with the aim of describing and analyzing the health conditions and the use of the health care services (GPs, specialists - public and private - and hospital care and related heath care services) of the Italian population. A representative sample of households and individuals are interviewed to have a picture of the health conditions and access to health care services of the Italian population. Two more recent waves are available for this survey, however they don't contain any information about additional insurance coverage.

The used sample for the 1999-2000 survey is made of 52,332 households, with an equivalent of 140,011 individuals. After dropping individuals less than 18 years of age and observations with missing values, the final sample used in this analysis consists of 104,422 observations. Children up to the age of 18 years were not considered for two main reasons. First, their access to health care goes mainly through SSN pediatricians, that are considered specialists, but operate as primary care physicians. Second, in general the decision to access health care services for children is mediated by their parents.

The covariates used in our study are typical of those used in previous studies on the determinants of medical care. In particular, the explanatory variables include variables reporting the health status of the individuals, income, gender, age, education, and geographic location. On top of these, we have also used supply side variables to help control the identification of supply induced demand effects.

In aable 1 we report descriptive statistics regarding the variables used in the empirical analysis. Concerning the dependent variables, in a given month, about 36 percent of individuals had a control, a treatment or spent at least one day in the hospital. In addition, 13 percent declared to have a supplementary private health insurance and only 20 percent of the population used private health care services. Although not reported in the table, it is interesting to note that in the Italian health care system public and private specialist use is roughly the same, in spite of the fact that visits to private specialists are considerably more expensive.

Information on health status is provided by three different variables: limitation in daily activities, self-reported health status, and a body mass index. Concerning the first variable, the question asked to individuals is "Are you hampered in your daily activity by any physical or mental health problem, illness or disability?" In this case, the dummy variable is based on three categories: "yes, severely", "yes, to same extent", and "no". In the same way, responders rate their health status by choosing between five categories: "very bad", "bad", "fair", "good" and "very good" health conditions. Finally, the body mass index has been constructed as the ratio of weight to the square of height. Two variables describing "under-weight" and "over-weight" people have then been constructed. They do not add to 1 because they are defined as lower or higher than some medically accepted threshold (18.5 and 25 respectively). The "normal-weight" body mass index is the reference variable. This allows for the non-linearity of the effect of the body mass index on patient health.

The majority of the individuals declared they were in good health. In fact, 12 percent of people considered their health as "very good", 42.6 percent as "good", and 37,3 percent as "fair". Only about 8,0 percent of the population declared a "poor" or "very poor" health status. People who reported in general to be in a "very poor" or "poor" health status were concentrated in the lower income quintiles. It is also interesting to note that from the geographical perspective the share of people in either "good" or "very good" health status was higher in the North compared to the South.

Information on income is not directly available in the our database. However, excellent measures of income are available in the Survey of Household Income and Wealth (SHIW) conducted by the Bank of Italy for the year 2000. We then use an imputation procedure based on cell matching to impute income data from the Bank of Italy survey in the Multiscopo survey (details are reported in Atella and Pollastri (2003))[7].

For the education level responders could choose among nine categories, that have been collapsed in four widely accepted categories: "Up to elementary degree", "Middle school degree", "High

[7] Although we are aware of the fact that the use of imputed income introduces measurement errors, we must also recognize that the MHS is an incredible source of individual-level information on the demand of health care services in Italy that would otherwise be under-utilised. This means also that in commenting the empirical results our conclusions on the income parameter should be very cautious. In any case, Atella and Deb (2004) have shown that the introduction of imputed income does not alter the magnitude and significance of the other variables.

school degree" and "University degree". More than half of the sample (66.1) had up to the middle school degree. Only 6.5 percent of the sample holds a university degree.

The sample is almost equally divided into males (47.0 percent) and females (53.0 percent) and with a large fraction living in the North of Italy (40.0 percent) and in the South (42.0 percent). A small fraction of individuals (7.0 percent) report being in poor bad health, although 51.0 percent report a chronic condition. The average age of the population (after dropping all children below 18 years) under investigation is 47 years.

About 10.7 percent of the population is exempted for illness reasons, while another 10.6 percent is exempted for other reasons (income, unemployment status, invalid, etc.). One third of the population over the last 12 months did not get any diagnostic control (blood pressure, cholesterol test, pap test, etc.), while about 65.0 percent did at least one control. More than 44.4 percent of patients in the sample was a smoker at the time of the interview.

About 19.3 percent of households involved in the survey were childless couples, 59.4 percent couples with children, 9.5 percent singles, while the remaining were mainly single parents households (of which lone mothers - about 7.0 percent - were the majority), and other more complex typologies.

Finally, our set of regressors includes five "supply-side" variables collected at regional level. Supply variables are of particular interest when modelling health care service demanded by patients. Among other things, they should help the identification of the parameters in the empirical analysis. The first two variables are related to the availability of GPs on the territory. We have been able to collect information on GP density (number of GPs per patient) and on the distribution of patient across GPs. In fact, although the distribution of GPs across the Italian region is an interesting phenomenon to capture, the way in which patients are allocated to GP within a region is another important aspect to consider. In other words, we can have the same amount of GPs per patients in two regions, but in one region patients are equally allocated to the GPs available, while in the other region some GPs could concentate most of the patients while some other have very few patients. This difference may cause serious problems in terms of gate-keeping and referral, thus causing pressure problems for health care demand.

The remaining three variables are specialist density (number of specialists per Km^2, number of bed per patient in public hospitals and in private hospitals. In the Italian public system there is no co-payment to pay for hospitalization in public structures. Co-payments are due only in case patients require a better accommodation (single room with more comfort). However, this kind of accommodation is limited by the fact that many hospitals do not have adequate infrastructure to offer such service. A completely different scenario exists for private hospitals. In this case the moral hazard effect may be extremely high. This also explains why in the empirical model we have used private utilization of health care as dependent variable in one of our equations.

With respect to the adverse selection effect we know that it is more relevant in that part of the population that may incur a higher probability of using hospitalization. However, the effect may depend also on the relative availability of public and private hospital structures. In those regions where public hospitals work properly and efficiently the adverse selection effect may be lower. On the contrary, where the public hospitals are not so efficient and of low quality the effect may be larger. In any case, private insurers can refuse the renewal of the insurance plan to those patients with high expenditure records or with specific conditions. This is why in the econometric model it is important to condition on regional differences and on supply variables, such as number of GPs and public specialist per patient.

Tables 2 and 3 present some statistics concerning the utilization of services by patients according to gender, age classes and presence of a private insurance plan.

Table 4 shows the number of insurance plans and type of health service utilization according to age and sex, while table 5 shows the proportions of patients with private health insurance together with the proportion of patients who have utilized health services. A quite interesting result emerges from the analysis of these data. In fact, while the proportion of patients who use public health care services has the common U-shaped form – with a minimum at the age of 35-49 – an almost flat pattern comes out from the private health care service utilization. In any case, the groups of older patients have a lower utilization of private health care services (partially explained by the lower level of income that characterized pensioners).

In table 5 we observe that while the percentage of patients with private insurance who access public health care services is not different from those without private insurance, differences exist between insurance plans for access to private health services. In fact, the percentage of “private services" is higher for patients with “private insurance" compared to those relying only on the public insurance plan (SSN). Of course, this result stems from the peculiarity of the Italian system being an SSN.

5 The econometric results

In this section we comment the econometric results obtained from the estimation of different models. Equations (13) and (14) have been estimated using a sample containing 104,556 observations, while Equations (15) has been estimated conditional on patient use of public health care services, thus using only 38,352 observations, and Equations (16) conditional on patient use of private health care services, thus using only 21,440 observations. Overall, the model is well determined and the parameter estimates are statistically significant and with the expected signs. The few cases in which they are not, they are concentrated in those case where lack of significance is not a problem for our hypotheses.

In table 6 we report the model estimates. The empirical results are strongly and neatly in favour of the presence of endogeneity and cross-equation error correlation. This results suggest that there is a self-selection problem in the Italian health insurance market, which is in line with the theory and with the literature (see Holly et al 1998; Jones et al. (2007); Buchmueller et al (2004); Dardanoni et al. (2012)). In fact, the positive and statistically significant latent variable parameter (α^0_{21}) in the second equation shows that a higher probability to demand medical treatment is positively related to a higher probability to apply for supplementary private insurance (adverse-selection).

Having controlled for AS, the estimates obtained in Equations (15) and (16) will allow to disentangle the reasons leading privately insured patients to "over-consume" health care services (SID *vs.* MH). Focusing on Equation (15), we see that the probability to receive a treatment from private hospitals is higher if patients own a supplementary private insurance ($\gamma^0_{32} = 0.2488$), which implies the presence of MH. However, what we do not know yet is if this "over-consumption" behaviour is caused either by "demand side"' MH or by SID, or by both. In order to disentangle these two effects, we need to focus on the parameter γ^0_{42} in Equation (16), where the dependent variable has been restricted to the subset of "invasive" treatments received in private hospitals, which we assume are difficult to be induced by demand-side MH. In table 6 we see that the parameter γ^0_{42} associated to the private insurance variable in Equations (16) is double compared to the parameter (γ^0_{32}) in Equations (15) and the value of the Wald test at the bottom of the third column in table 6 informs us that the difference between the two parameters is statistically different from zero. This result is equivalent to alternative 3 seen in Section 3.5, which implies that SID is more relevant than "demand induced" MH, although we cannot rule out that this last effect is null.

As already discusses in Section 3.5, the only way to understand the magnitude of the "demand induced" MH effect is to restrict the type of health care services demanded to those services potentially non being affected by SID. In the Italian context, this type of services could be represented by private specialist visits. In fact, specialist visits are performed by "self-employed" professionals working in independent practices, without strong connections with their colleagues. Within this setting we could exclude the possibility that one specialist could stimulate the demand for other specialists, particularly in different specialties (i.e., it is quite unrealistic that a psychiatrist will induce demand for an orthopaedist).[8] Given this setting, we computed an indicator of demand for health services which is equal to one if in the last 4 weeks before the interview patients have been prescribed more than one outpatient specialist visit in different specialties.[9] With this definition of private health care demand we should exclude, by construction, the presence of SID and, therefore, our model reduces to a simpler tri-variate Probit. In this new setting, positive and statistically significant values of $\gamma_{32}^0 > 0$ lead to infer the existence of "demand induced" MH. However, as we can see from Table 7, the parameter for the presence of additional insurance is slightly positive and not statistically significant. Based on these results we may conclude against the presence of "demand induced" MH, while in favour of AD and SID.

A final problem that we need to tackle is the low statistical significance of γ_{42}^0 in Equation (15), which may casts some doubts about the validity of our hypotheses. This result could be easily explained if we consider that this analysis focuses on privately insured patients, and that elderly patients can hardly get a private insurance when they are very old (+80). By including these patients in our sample it is equivalent to dilute the MH effect, given that we include patients who would like to buy an insurance but are refused. Therefore, we have decided to restrict our sample by excluding patients +80 years of age, who will never have the chance to apply for a private health insurance. These new results are presented in Panel *a* of Table 8, from which we can see how the estimates remain almost unchanged, except for the parameter γ_{42}^0 which now is significant at 5%.

[8] This situation was particularly true in Italy at the time when the survey data we use were collected, as specialists were not working in practices where several specialists were working together, which could falsify our claim. Of course, given that we cannot rule out the existence of SID within the same specialist, we exclude them.

[9] As robustness checks we have also used different thresholds: "more than two" and "more than three" specialist visits. Results are similar and are available upon request.)

We conclude this section commenting on the remaining regressors in our model. Concerning socio-economic determinants we find that income has a positive and a highly significant effect only in the equation in which insurance decision is considered. This is not a surprising result. In fact, in the case of the probability of using public health care services the NHS seems to work properly (this result is supported also by evidence obtained by Atella et al. (2004) and Atella and Deb (2004)), while in the case of private health insurance a possible explanation could be the existence of collinearity with the educational attainment variable and exemption reasons (in particular those not linked to health conditions).

Across different specifications, age has a non linear effect on equation (1) and (2), while in equation (3) doesn't have any effect, except in the specification reported in table 6. Highly educated patients tends to use more health care services, and in particular to use private services. However, this is not true when we consider invasive treatments. It clearly shows up that patients with exemptions tend to use more the public system. At the same time, exempted patients have a lower access to private insurance.

Smokers tend to use more both public and private services, as well as to by more private insurance plans, although this is not true when we consider more invasive treatment provided by private hospitals. Patients who decided to take prevention controls over the last year have, obviously, a positive relationship with the dependent variables. In particular, patients with high education tend to have both a higher probability of buying a private insurance plan and a higher probability of using private health care services. Obviously this relation is not longer true in case in which we consider only more invasive treatment. High levels of education are usually associated with high level of income and, therefore, with high opportunity costs for waiting time for public health care services. However it may also indicate quality effects in the provision of health care through private providers. Unfortunately, with the available data this it is not possible to disentangle the two effects. Geographical parameters need to be interpreted as differential effects with respect to the reference region, namely Sardinia. While there are some geographical differences for accessing public health care structures, we cannot detect any special pattern. However, it is worth to mention that for two regions like Trentino and Emilia Romagna, considered as the most performing regions for the provision of public health care services, we observe positive and strong differential effects. At the same time, for Piemonte, Liguria, Puglia, Campania and Lazio, among the less performing, we observe negative values. Finally, the positive and significant effect we observe for Calabria could be explained considering that this is one of the regions with the lowest level of private structures. A more interesting pattern emerges while analyzing the regional effects on the probability of having a private insurance plan. In fact, in this case we note that all Northern regions present positive differential effects with respect to Sardinia, while for almost all Southern regions present negative differential effects.

Concerning the third equation we have not used regional dummies because our supply side variables have been collected at regional level. All these variables are significant and with the expected signs. In particular, as expected, the number of public bed per patient (bed density) have a negative sign on the probability of using private health services. At the same time, the number of specialists per Km^2 have a positive effect on the probability to use private services.

6 Robustness check

In this section we perform some robustness checks that allow to obtain more robust and reliable conclusions on our capacity to disentangling AS, MH and SID. Our first check concerns the possibility to distinguish between patients in terms of discretionary choice. In particular, we know if the choice of a private hospital by a patient is “necessary” or “discretionary”. Intuitively, if the choice is “necessary” we should have less room for SID. For example, if we think of an emergency, it is very likely that patients choose simply the nearest hospital and the probability to be exposed to SID should be low. Conversely, if the choice is “discretionary”, patients may choose where and when receive treatments and in this case the possibility to be exposed to some kind of “pressure” in order to undergo some treatment could be higher. Using the previous model, we exploit this information and use as dependent variable in the third equation a dummy variables that distinguish between “necessary” and “discretionary” choice. As we can see from Table 9, the results seem to confirm our arguments: insured patients seem to consume more health care services provided by private hospitals only when “discretionary” (Panel c) choices are considered.

Additionally, we can distinguish between treatments that should be fully paid or partially paid by patients (because subsidised by NHS). We expect that insured should consume more with respect uninsured, especially when treatments are fully paid by patients. In fact, in this case the gain in terms of “demand induced" MH should be higher. On the other hand, if overconsumption by insured is due because patients are “MH type” we should see this effect whatever is the compensation scheme for the private hospitals. In Panel *a* and *b* of table 9 we investigate these features in the data using the previous econometric approach. Empirical results seem to confirm our expectations: the parameter related to insurance is positive and statistically significant only when we consider health care services not subsidised by NHS.

7 Conclusions

Adverse selection, “demand induced" moral hazard and SID are the three most important factors affecting insured patient decisions to use health care services. Although there exists a vast theoretical and empirical literature that has approached these phenomena, so far no contribution has been able to approach them jointly. As largely discussed in the paper, this is mostly due to the difficulty to model the joint determinants of health care service utilization and health insurance choice by means of a tractable structural simultaneous equation model.

In this paper we have provided a solution to this problem by presenting a four equation structural model with four latent variables, where the first two equations are meant to deal with the self-selection issue, while the last two equations deal with the over-consumption issues (MH/SID). A closed form solution for the Log-likelihood function has been proposed and then maximised by means of FIML using BHHH algorithm. The empirical analysis has been conducted using the Italian cross-sectional data from the 1999 Multiscopo Survey (MS).

The empirical analysis has confirmed the theoretical prediction of our structural model. In particular, we found that: *1)* the propensity of a person to receive any medical service may not be considered as exogenous in the equation explaining the propensity of the same person to buy a supplementary health insurance plan; *2)* the presence of a supplementary health insurance has a positive effect on the probability that he/she will have at least one access to any private health service (due to SID effect), given that he has used some medical treatment; *3)* we don’t find evidence in favour of “demand induced" MH by insured patients.

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Table 1: Descriptive statistics

Variable	**Mean**	**Std. Dev.**	**Min**	**Max**
Dependent variables				
Dummy for use of health service	0,367	0,482	0	1
Dummy for presence of private insurance	0,130	0,337	0	1
Dummy for use of private health service	0,205	0,404	0	1
Dummy for invasive treatments povided by private hospitals	0,006	0,075	0	1
Dummy for surgery povided by private hospitals	0,002	0,050	0	1
Regressors				
Level of patient invalidity	1,107	0,405	1	3
Self assessed health status	3,569	0,841	1	5
Patient under weight	0,023	0,149	0	1
Patient over weight	0,420	0,494	0	1
Number of prevention tests	1,953	1,800	0	6
Patient smoker	0,444	0,497	0	1
Patient exempted for pathology	0,107	0,310	0	1
Patient exempted for other reasons	0,107	0,309	0	1
Household equivalent income	3,107	1,413	1	5
Household below poverty line	0,239	0,426	0	1
Age of patient / 100	0,472	0,181	0,18	0,9
Age squared of patient / 10000	0,255	0,183	0,032	0,81
Patient pensioner	0,137	0,344	0	1
Marital status	0,605	0,489	0	1
Gender	1,531	0,499	1	2
Educational attainment	2,107	0,950	1	4
Dummy for couples	0,193	0,395	0	1
Dummy for couples with children	0,595	0,491	0	1
Dummy for single mother	0,067	0,251	0	1
Dummy for single father	0,012	0,111	0	1
Dummy for single	0,095	0,294	0	1
Number of bed in public hospital per patient	4,609	1,397	2,8	10,4
Number of bed in private hospital per patient	0,771	0,471	0	1,9
Distribution of patients per GP	0,380	0,088	0,341	0,786
Number of specialists per Km2	0,236	0,153	0,023	0,546

Table 2: Number of insurance plans and type of health service utilization according to age and sex

Age	Patients whith only NHS insurance (1)	Patients with NHS + Private insurance (2)	Total (1)+(2)	Patients with hospital visits	Patients with any form of health care service utilization	Patients with any form of private health care service utilization
			Total			
15-34	29089	4335	33424	959	9442	5257
35-49	20379	5281	25660	678	8141	4703
50-64	21789	3312	25101	1020	9843	5658
≥ 65	19656	715	20371	1556	10926	5822
Total	90913	13643	104556	4213	38352	21440
			Men			
15-34	14124	2455	16579	333	3934	1985
35-49	9160	3097	12257	288	3406	1874
50-64	9722	2024	11746	512	4088	2287
≥ 65	8037	408	8445	715	4433	2396
Total	41043	7984	49027	1848	15861	8542
			Women			
15-34	14965	1880	16845	626	5508	3272
35-49	11219	2184	13403	390	4735	2829
50-64	12067	1288	13355	508	5755	3371
≥ 65	11619	307	11926	841	6493	3426
Total	49870	5659	55529	2365	22491	12898

Table 3: Proportion of insurance plans and type of health service utilization according to age and sex

Age	Patients whith only NHS insurance (1)	Patients with NHS + Private insurance (2)	Patients with hospital visits	Patients with any form of health care service utilization	Patients with any form of private health care service utilization
			Total		
15-34	87,0	13,0	2,9	28,2	15,7
35-49	79,4	20,6	2,6	31,7	18,3
50-64	86,8	13,2	4,1	39,2	22,5
≥ 65	96,5	3,5	7,6	53,6	28,6
			Men		
15-34	85,2	14,8	2,0	23,7	12,0
35-49	74,7	25,3	2,3	27,8	15,3
50-64	82,8	17,2	4,4	34,8	19,5
≥ 65	95,2	4,8	8,5	52,5	28,4
			Women		
15-34	88,8	11,2	3,7	32,7	19,4
35-49	83,7	16,3	2,9	35,3	21,1
50-64	90,4	9,6	3,8	43,1	25,2
≥ 65	97,4	2,6	7,1	54,4	28,7

Table 4: Number of patients by sex, type of insurance plan, and health care service

	Patiets	**Patients with hospital visits**	**Patients with any form of health care service utilization**	**Patients with any form of private health care service utilization**
		Total		
Only NHS Insurance	90913	3817	33390	18464
NHS + Private insurance	13643	396	4962	2976
Total	104556	4213	38352	21440
		Men		
Only NHS Insurance	41043	1647	13281	7076
NHS + Private insurance	7984	201	2580	1466
Total	49027	1848	15861	8542
		Women		
Only NHS Insurance	49870	2170	20109	11388
NHS + Private insurance	5659	195	2382	1510
Total	55529	2365	22491	12898

Table 5: Percentage of patients by sex, type of insurance plan, and health care service

	Patiets	**Patients with hospital visits**	**Patients with any form of health care service utilization**	**Patients with any form of private health care service utilization**
		Total		
Only NHS Insurance	87,0	4,2	36,7	20,3
NHS + Private insurance	13,0	2,9	36,4	21,8
Total	100,0	4,0	36,7	20,5
		Men		
Only NHS Insurance	83,7	4,0	32,4	17,2
NHS + Private insurance	16,3	2,5	32,3	18,4
Total	100,0	3,8	32,4	17,4
		Women		
Only NHS Insurance	89,8	4,4	40,3	22,8
NHS + Private insurance	10,2	3,4	42,1	26,7
Total	100,0	4,3	40,5	23,2

Table 6: FIML estimates of Quadri-variate Probit model

Variables	**Total Healtcare Demand**	**Private Insurance**	**Total Private Healtcare Demand**	**Total Private Healtcare Demand**
AS, MH and SID parameters				
Total Healtcare Demand (α_{21})*		0.0556***		
Dummy for private insurance (γ_{32} and γ_{42})			0.2488**	0.5368*
Supply side variable				
Density of GPs			2.49E-06	-0.0015
No. of bed per patient in public hosp.			-0.0521***	0.034082
No. of bed per private in public hosp.			-0.00628	0.0671
Patients per GP			0.645229	6.235357
Specialist density			0.1309**	0.197303
Other covariates				
Costant	0.847***	-2.8076***	-0.36666	-1.9421**
Invalidity status	0.1847***	-0.0762***	0.015436	-0.1017***
Health status (Good=1)	-0.3321***	0.0678***	-0.0496***	-0.1702***
Household equivalent income	-0.00951	0.3696***	0.063957	0.04015
Dummy for pensioner	0.2018***	0.012512	0.0703***	0.04125
Age / 100	-2.517***	6.9981***	0.380115	-0.24762
(Age / 100)^2	1.8181***	-8.1858***	-0.6161**	0.120994
Married	0.0959***	0.2383***	-0.0088	-0.09618
Gender (male=1)	0.005849	-0.316***	0.016156	-0.1871***
Educational attainment	0.0178***	0.2047***	0.1128***	-0.02599
Dummy for couple	-0.0619***	-0.2197***	0.1368***	0.094259
Dummy for couple with children	-0.138***	-0.2144***	0.0806**	0.071974
Single parent mother	-0.0698***	-0.1157***	-0.02689	0.032218
Single parent father	-0.0819**	-0.1652***	0.030417	-0.01176
Household below poverty line	-0.0386***	-0.0092	0.0337**	0.040637
Exempted for pathology	0.324***	-0.1064***	0.0556***	-0.03887
Exempted for other reasons	0.2174***	-0.1815***	0.032286	-0.01363
Patient smoker	0.0727***	0.0441***	0.004087	0.007536
Number of prevention tests	0.1791***	0.0404***	0.0915***	0.0459***
Patient BMI = underweight	0.018655	0.016016	0.024209	-0.01171
Patient BMI = overweight	0.0182**	0.008946	-0.0252*	-0.0691*
Regional dummy (Lombardia is the reference)				
Piemonte	-0.0657***	0.2357***		
Valle d'Aosta	-0.0907***	0.3921***		
Lombardia	0.030025	0.2639***		
Tentino Alto Adige	0.0797***	0.6459***		
Veneto	0.0787***	0.2027***		
Friuli Venezia Giulia	0.0727***	0.2311***		
Liguria	-0.0641**	0.1742***		
Emilia Romagna	0.1321***	0.1998***		
Toscana	0.0496**	0.1972***		
Umbria	-0.0115	0.0881**		
Marche	0.030253	0.0752**		
Lazio	-0.03232	0.1641***		
Abruzzo	0.027336	0.03628		
Molise	0.011441	-0.0231		
Campania	-0.0462**	-0.3592***		
Puglia	-0.0995***	-0.2091***		
Basilicata	-0.0073	-0.0764**		
Calabria	0.068***	-0.3045***		
Sicilia	0.006855	-0.2975***		
Macro-regional dummy (Center is the reference)				
North-West			-0.0486**	0.049448
North-East			-0.03277	0.048083
South			-0.00071	0.1375**
Islands			-0.00456	0.133*
corr2&3	-0.10135			
corr2&4	-0.2732*			
Log-Likelihood	-124361			
Total Obs.	104556		38352	21440
Wald Test H_0**:** $\gamma_{42} - \gamma_{32} = 0$ (Acceptance level ≤ 0.004)		0.1892		

Notes. The model is estimated using an exact FIML with a double selection: y_4 is observed only when y_3 is equal to one, and in turn y_3 is observed only when y_1 is equal to one.

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 7: FIML estimates of Tri-variate Probit model - MH for specialistic visits

Variables	**Total Healtcare Demand**	**Private Insurance**	**Over-Consumption of Specialistic Visits**
AS, MH and SID parameters			
Total Healtcare Demand (α_{21})*		0.0557***	
Dummy for private insurance (γ_{32})			0.244411
Supply side variable			
Density of GPs			-0.00043
No. of bed per patient in public hosp.			0.0613**
No. of bed per patient in private hosp.			0.0649**
Patients per GP			0.834325
Specialist density			0.112454
Other covariates			
Costant	0.8423***	-2.8121***	-1.793***
Invalidity status	0.183***	-0.0783***	0.1118***
Health status (Good=1)	-0.3315***	0.0675***	-0.195***
Household equivalent income	-0.01035	0.3705***	0.037229
Dummy for pensioner	0.2026***	0.012312	0.0932**
Age / 100	-2.5148***	7.0294***	-0.25381
(Age / 100)^2	1.8161***	-8.2226***	-0.09775
Married	0.0958***	0.2382***	0.031855
Gender (male=1)	0.006369	-0.3166***	0.0732**
Educational attainment	0.0177***	0.2051***	0.0748***
Dummy for couple	-0.0616***	-0.2157***	0.035121
Dummy for couple with children	-0.1373***	-0.2115***	0.055926
Single parent mother	-0.0689***	-0.1124***	0.1026**
Single parent father	-0.0796**	-0.1628***	0.01216
Household below poverty line	-0.039***	-0.009	-0.03636
Exempted for pathology	0.3234***	-0.1077***	0.0886***
Exempted for other reasons	0.2164***	-0.18***	-0.00236
Patient smoker	0.0738***	0.0446***	0.0484**
Number of prevention tests	0.1789***	0.0404***	0.0717***
Patient BMI = underweight	0.018126	0.014185	-0.05172
Patient BMI = overweight	0.0182**	0.009212	0.009543
Regional dummy (Lombardia is the reference)			
Piemonte	-0.063***	0.2344***	
Valle d■Aosta	-0.0879***	0.3872***	
Lombardia	0.032485	0.2627***	
Tentino Alto Adige	0.0813***	0.6454***	
Veneto	0.0817***	0.202***	
Friuli Venezia Giulia	0.0745***	0.2282***	
Liguria	-0.0643**	0.1731***	
Emilia Romagna	0.1352***	0.1989***	
Toscana	0.0495**	0.197***	
Umbria	-0.01011	0.0872**	
Marche	0.031277	0.0745**	
Lazio	-0.03096	0.1615***	
Abruzzo	0.029947	0.032526	
Molise	0.012913	-0.0292	
Campania	-0.0461**	-0.3595***	
Puglia	-0.098***	-0.2121***	
Basilicata	-0.0049	-0.0772**	
Calabria	0.0709***	-0.3057***	
Sicilia	0.007278	-0.2995***	
Macro-regional dummy (Center is the reference)			
North-West			-0.03526
North-East			0.004216
South			0.055025
Islands			0.1265***
corr2&3	-0.07732		
Log-Likelihood	-103718		
Total Obs.	104422		38218

Notes. The model is estimated using an exact FIML with a single selection: y_3 is observed only when y_1 is equal to one.

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 8: Comparison of relevant estimates across different model specifications using a Quadri-variate Probit model

Variables	Total Healtcare Demand	Private Insurance	Total Private Healtcare Demand	♣ Private Healtcare Demand
(a) ♣ Total services; limiting patients' age to ≤ 80				
AS, MH and SID parameters				
*Total Healtcare Demand** (α_{21})		0.0558***		
Dummy for private insurance (γ_{32} *and* γ_{42})			0.278**	0.573**
Wald Test H_0**:** $\gamma_{42} - \gamma_{32} = 0$ (Acceptance level ≤ 0.004)			0.2044	
(b) ♣ Surgery interventions				
AS, MH and SID parameters				
*Total Healtcare Demand** (α_{21})		0.0556***		
Dummy for private insurance (γ_{32} *and* γ_{42})			0.2495**	0.7183*
Wald Test H_0**:** $\gamma_{42} - \gamma_{32} = 0$ (Acceptance level ≤ 0.004)			0.1892	
(b) ♣ Surgery interventions; limiting patients' age to ≤ 80				
AS, MH and SID parameters				
*Total Healtcare Demand** (α_{21})		0.0558***		
Dummy for private insurance (γ_{32} and γ_{42})			0.2812**	0.7092*
Wald Test H_0**:** $\gamma_{42} - \gamma_{32} = 0$ (Acceptance level ≤ 0.004)			0.3252	

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 9: FIML estimates of Tri-variate Probit model - Robustness Check

Variables	Total Healtcare Demand	Private Insurance	Total Private Healtcare Demand♠
a	♠Fully paid by patients		
AS, MH and SID parameters			
*Total Healtcare Demand** (α_{21})		0.0562***	
Dummy for private insurance (γ_{32})			0.477***
b	♠Partially subsidized by NHS		
AS, MH and SID parameters			
*Total Healtcare Demand** (α_{21})		0.0559***	
Dummy for private insurance (γ_{32})			-0.12996
c	♠Discretionary choice of Private hospitals		
AS, MH and SID parameters			
*Total Healtcare Demand** (α_{21})		0.0557***	
Dummy for private insurance (γ_{32})			0.2668**
d	♠Necessary choice of Private hospitals		
AS, MH and SID parameters			
*Total Healtcare Demand** (α_{21})		0.0557***	
Dummy for private insurance (γ_{32})			0.058092

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$