

U.S. Wage Growth in the Aftermath of Deep Recessions: The Roles of Inflation and Unemployment*

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Abstract

We propose a new model to study the relationship between U.S. wage growth and unemployment for the 1965-2015 period. In particular, we estimate a threshold vector autoregression with multiple threshold variables and multiple threshold parameters for each threshold variable. Previous nonlinear models of the Wage Phillips curve (WPC) rely on regime-switching driven by changes in unemployment only. However, the fact that wages are indexed to price inflation suggests that there is a role for price inflation in determining the dynamics of the WPC. The empirical approach suggests that the relationship between wage growth and unemployment changes according to the dynamics of both unemployment and price inflation. Specifically, the WPC evolves as the unemployment rate transitions according to estimated thresholds defined by 5.73% and 7.39%. Simultaneously, it also evolves depending on whether inflation acceleration is above or below 0.21%. The results show a strong negative relationship between wage growth and unemployment. The relationship weakens, although remains negative, during periods of deep recessions and low inflation acceleration, when wage growth responds more strongly to price inflation shocks. This suggests that the negligible wage growth observed during 2013-2015 may have been driven by the low inflation environment prevailing during that period and, to a lesser extent, by labor market slack.

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*All errors are our own.

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1 Introduction

In recent years, the relationship between wage growth and unemployment in the U.S. has been at the heart of the debate regarding the stance of monetary policy. Much of this attention stems from the policy implications behind the behavior of these variables after the normalization of policy in December 2015. On the one hand, there are arguments that favor a rapid increase in the Federal Funds rate (FFR) to avoid returning to the zero lower bound, in response to the decline in the rate of unemployment (Bullard, 2015). On the other hand, a different strand of the literature argues for a slow and incremental FFR policy normalization, given the recent low inflationary environment (Blanchflower and Levin, 2015).

This debate has led to a renewed interest in the Wage Phillips Curve (WPC), which describes the relationship between wage inflation and the unemployment rate (Galí, 2011; Galí, Smets, and Wouters, 2012; Blanchflower and Posen, 2014). While recent studies have found overwhelming evidence of nonlinearities in this relationship (Fisher and Koenig, 2014; Kumar and Orrenius, 2015; Donayre and Panovska, forthcoming), the time-varying behavior in the WPC has been motivated by changes in the unemployment rate exclusively. However, a number of studies has found evidence that wages are indexed to price inflation and, thus, that wage growth depends on monetary policy and its effect on the level or the volatility of the growth rate of the price level (Christiano, Eichenbaum, and Evans, 2005; Smets and Wouters, 2007; Hofmann, Peersman, and Straub, 2012; De Schryder, Peersman, and Wauters, 2015). Meanwhile, some studies find little power in wages explaining price inflation, suggesting that unexpectedly low or high inflation could occur regardless of the recent behavior of wages (see, for example, Bidder (2015) and the references therein). To the extent that the effect of labor market slack on wages and wage indexation to inflation are supported by economic theory, it is likely that the behavior of wage growth may change when both unemployment and price inflation change.

To address the dependence of wage growth on price inflation and unemployment, this paper estimates a threshold vector autoregressive model (TVAR) of the WPC using two threshold variables, building on the single-equation models of Chen, Chong, and Bai (2012) and Chong and Yan (2014). Additionally, we consider multiple threshold parameters for the threshold variables. In this context, the contribution of this empirical model to the literature is three-fold. First, this is the first paper that explicitly allows the behavior of wage growth to exhibit threshold-type nonlinearities where the change in regimes depends on the dynamics of both price inflation and the unemployment rate. Second, we extend the models in Chen et al. (2012) and Chong and Yan (2014) to a multivariate environment. To the extent that wage growth, unemployment and price inflation are interdependent, a single-equation approach to model the WPC would not be appropriate. Our multivariate nonlinear threshold TVAR approach better captures the underlying dynamic interrelations among all variables in the system. Third, we also extend the

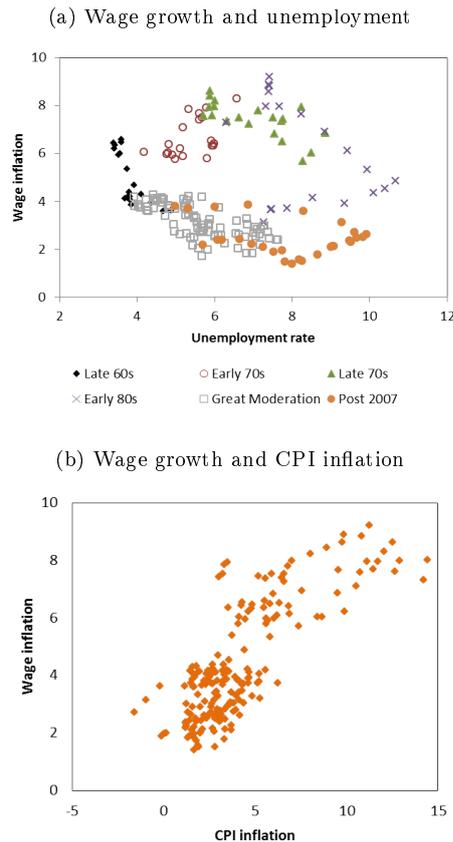
models in Chen et al. (2012) and Chong and Yan (2014) to consider multiple threshold parameters for the multiple threshold variables. This is motivated by the overwhelming evidence of the WPC changing dynamics as the unemployment rate transition between two different threshold parameters (Kumar and Orrenius, 2015; Donayre and Panovska, forthcoming).

Understanding how this relationship depends on the dynamics of both unemployment and price inflation can have important policy implications. On the one hand, a WPC that changes according to the level of unemployment only implies that the negligible wage growth observed in recent years may be driven by labor market slack, thus suggesting the need for more accommodative monetary policy. However, if the nonlinear relationship between wage growth and unemployment depends, in addition, on the dynamics of inflation, then low wage growth could be driven by the recent low price inflation environment, which suggests the need to increase the FFR.

When considering the relationship between wage growth and unemployment in the U.S., there is evidence that the implied WPC exhibits threshold-type nonlinearities and that the regime-switching is better described by changes in the dynamics of both unemployment and the acceleration in price inflation. In particular, the estimated threshold parameters suggest that this relationship changes according to whether the unemployment rate is above, in between or below regimes determined by 5.73% and 7.39%. Simultaneously, the relationship also changes as inflation acceleration is above or below 0.21%. The dynamics of the model is studied by means of impulse-response functions (IRFs) where the shocks are orthogonalized via sign restrictions. The analysis of the IRFs suggests a strong negative relationship between wage growth and unemployment, consistent with the implications of a WPC derived from the standard New Keynesian model with staggered wage setting in Galí (2011). The relationship weakens, although remains negative, during periods of deep recessions and low inflation acceleration, while the response of wage growth to price inflation shocks becomes stronger. This implies that because unemployment has a smaller effect on wage growth after deep recessions, the low wage growth observed during 2013-2015 may have been driven by the low price inflation environment prevailing during that period and, to a lesser extent, by labor market slack. This argument would thus call for policy actions that focus on the inflation side of the dual mandate.

The remainder of the paper is organized as follows. The second section motivates the use of two threshold variables in modeling the nonlinear relationship between wage growth and unemployment. The model and the empirical procedure are described in the third section. In the fourth section, we discuss the results. Some concluding remarks are provided in the last section.

Figure 1: U.S. wage growth, unemployment and CPI inflation: 1965-2015



The top panel shows a scatter plot of U.S. annualized wage growth and unemployment for 1965-2014. Sub-samples are defined as follows: 1965-1969 (late 60s), 1970-1974 (early 70s), 1975-1979 (late 70s), 1980-1983 (early 80s), 1984-2007 (Great Moderation), 2008-2014 (post 2007). The bottom panel shows a scatter plot of wage growth and CPI inflation.

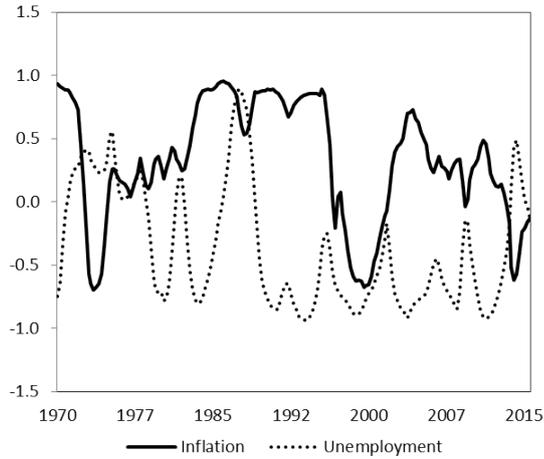
2 Motivation

In an environment in which wage inflation is driven primarily by changes in unemployment, a lack of wage growth is symptomatic of labor market slack. However, if wage growth is also influenced by the dynamics of price inflation, then a scenario of low wage inflation may be explained by wage indexation to price inflation when the latter is low and stable, as occurred during 2014 and 2015. To the extent that the different scenarios call for different policy actions, determining whether the relationship between wage inflation and unemployment depends on one or both variables is important for the future of economic policy.

As a way to motivate how the WPC may depend on unemployment and price inflation, the top (bottom) panel of figure 1 shows a scatter plot of wage growth and unemployment (price inflation) for the 1965-2015 period.¹ In the top panel of figure 1, the evolution of the relationship between wage

¹Wage inflation is computed as the 12-month growth rate of the earnings data from production and non-supervisory workers from the Establishment Survey. The focus on this earnings-based measure of wages is motivated by the substantial

Figure 2: Time-varying correlation: wage growth and unemployment (dotted) and wage growth and CPI inflation (solid)



Five-year correlation coefficients for wage growth and unemployment (dotted line) and wage growth and CPI inflation (solid line) over rolling windows starting with the 1965-1970 window.

growth and unemployment is shown across different periods. A simple visual analysis suggests that such relationship changes over time. In particular, it inverted during the late 1970s, early 1980s and post-2007, but not during other recessions, where it remains strongly negative. This is consistent with the evidence found in studies where the WPC changes regimes according to the level of unemployment (Donayre and Panovska, forthcoming; Kumar and Orrenius, 2015). Meanwhile, the bottom panel of figure 1 illustrates a strong positive correlation between wage growth and price inflation, in line with studies that find high and significant values for the indexation parameter in New Keynesian Dynamic Stochastic General Equilibrium (DSGE) models (Hofmann et al., 2012; De Schryder et al., 2015). It is important to note that the correlation becomes stronger in periods in which price inflation is higher.

The time-varying nature of both relationships is also evident by means of a simple analysis of their correlation coefficients over time. The correlation coefficients between wage growth and unemployment and between wage growth and price inflation are displayed in figure 2 and calculated as 5-year correlation coefficients over rolling quarterly windows starting with the 1965-1970 window. The evolution of these correlation coefficients is in line with the nonlinear dynamics in the WPC previously found, as their values change over time. On the one hand, the correlation between wage growth and unemployment is, overall, negative and tends to intensify during periods of mild recessions. However, it reverts during deep recession periods, like 1973, 1982 and 2007. On the other hand, the correlation between wage

high-frequency variations in the compensation-based measure from the Productivity and Cost publication of the Bureau of Labor Statistics, which suggest the presence of large measurement errors. Furthermore, these two measures of average wages do not seem to produce different empirical results, as argued by Galí (2011).

inflation and price inflation is, overall, positive and strengthens significantly during periods of high inflation, like the late 1970s and early 1980s. In general, the strong correlation coefficients suggest that both unemployment and price inflation have important effects on the WPC.

3 Econometric Procedure

This section describes the empirical model strategy, the estimation and testing procedures and the computation of the simulation-based impulse-response functions.

3.1 Empirical modeling

In an environment in which wage growth may be influenced by unemployment and price inflation, the behavior of all three variables is likely to be interdependent and determined simultaneously. However, most previous studies examine the behavior of the WPC from a univariate perspective (Galí, 2011; Fisher and Koenig, 2014; Kumar and Orrenius, 2015). We seek to emphasize the interdependence of unemployment and wage and price inflation by means of a nonlinear TVAR that allows for the underlying dynamic interrelations among all variables in the system.

Several studies have examined the price Phillips curve in a nonlinear multivariate approach. See, *inter alia*, Wong (2013) and Gross and Semmler (2015). Our nonlinear TVAR departs from existing models that study price and wage Phillip curves in two important directions. First, we allow for regime changes that may be driven by the dynamic of both price inflation and unemployment. Second, we allow for the possibility of multiple threshold parameters. Specifically, we consider a TVAR with two threshold variables, two threshold parameters for the threshold variable associated with unemployment and one threshold parameter for the threshold variable associated with inflation acceleration:

$$\mathbf{y}_t = \begin{cases} \Phi_0^1 + \Phi_1^1 \mathbf{y}_{t-1} + \dots + \Phi_p^1 \mathbf{y}_{t-p} + e_t, & \text{if } u_{t-d_1} \leq c_1 \quad \text{and} \quad \Delta \pi_{t-d_2}^p \leq c_3 \\ \Phi_0^2 + \Phi_1^2 \mathbf{y}_{t-1} + \dots + \Phi_p^2 \mathbf{y}_{t-p} + e_t, & \text{if } u_{t-d_1} \leq c_1 \quad \text{and} \quad \Delta \pi_{t-d_2}^p > c_3 \\ \Phi_0^3 + \Phi_1^3 \mathbf{y}_{t-1} + \dots + \Phi_p^3 \mathbf{y}_{t-p} + e_t, & \text{if } c_1 \leq u_{t-d_1} < c_2 \quad \text{and} \quad \Delta \pi_{t-d_2}^p \leq c_3 \\ \Phi_0^4 + \Phi_1^4 \mathbf{y}_{t-1} + \dots + \Phi_p^4 \mathbf{y}_{t-p} + e_t, & \text{if } c_1 \leq u_{t-d_1} < c_2 \quad \text{and} \quad \Delta \pi_{t-d_2}^p > c_3 \\ \Phi_0^5 + \Phi_1^5 \mathbf{y}_{t-1} + \dots + \Phi_p^5 \mathbf{y}_{t-p} + e_t, & \text{if } u_{t-d_1} > c_2 \quad \text{and} \quad \Delta \pi_{t-d_2}^p \leq c_3 \\ \Phi_0^6 + \Phi_1^6 \mathbf{y}_{t-1} + \dots + \Phi_p^6 \mathbf{y}_{t-p} + e_t, & \text{if } u_{t-d_1} > c_2 \quad \text{and} \quad \Delta \pi_{t-d_2}^p > c_3 \end{cases} \quad (1)$$

where $\mathbf{y}_t = (\mu_t \ \pi_t^p \ \pi_t^w)'$ is a (3×1) vector of observations at time t with μ_t , π_t^p and π_t^w being a measure of joblessness, price inflation and wage growth, respectively; p is a lag length; $\mathbf{q}_t \equiv (u_{t-d_1}, \Delta \pi_{t-d_2}^p)$ are

the threshold variables; $\mathbf{d} \equiv (d_1, d_2)$ are delay lags; $\mathbf{c} \equiv (c_1, c_2, c_3)$ are the threshold parameters; Φ_0^i for $i = 1, \dots, 6$ are vectors of intercepts for regime i ; Φ_j^i for $i = 1, \dots, 6$ and $j = 1, \dots, p$ are matrices of coefficients; and \mathbf{e}_t is a vector of disturbances with mean zero and covariance matrix Ω .

The model in equation (1) can be rewritten in the following way

$$\mathbf{y}_t = \sum_{i=1}^6 (\Phi_0^i + \Phi_1^i(L)) \Psi_t^{(i)}(\mathbf{c}) + e_t \quad (2)$$

where $\Phi_1^i(L)$ are lag polynomial matrices of coefficients and $\Psi_t^{(i)}(\mathbf{c})$ equals one when the threshold conditions are satisfied in regime i . In particular,

$$\begin{aligned} \Psi_t^{(1)}(\mathbf{c}) &= \mathbb{1}(u_{t-d_1} \leq c_1, \Delta\pi_{t-d_2}^p \leq c_3) \\ \Psi_t^{(2)}(\mathbf{c}) &= \mathbb{1}(u_{t-d_1} \leq c_1, \Delta\pi_{t-d_2}^p > c_3) \\ \Psi_t^{(3)}(\mathbf{c}) &= \mathbb{1}(c_1 < u_{t-d_1} \leq c_2, \Delta\pi_{t-d_2}^p \leq c_3) \\ \Psi_t^{(4)}(\mathbf{c}) &= \mathbb{1}(c_1 < u_{t-d_1} \leq c_2, \Delta\pi_{t-d_2}^p > c_3) \\ \Psi_t^{(5)}(\mathbf{c}) &= \mathbb{1}(u_{t-d_1} > c_2, \Delta\pi_{t-d_2}^p \leq c_3) \\ \Psi_t^{(6)}(\mathbf{c}) &= \mathbb{1}(u_{t-d_1} > c_2, \Delta\pi_{t-d_2}^p > c_3) \end{aligned}$$

where $\mathbb{1}(\cdot)$ is an indicator function, for $t = 1, 2, \dots, T$. Let $\Phi^i = (\Phi_0^i, \Phi_1^i(L))$ be the set of coefficients for regime i . Therefore, the dynamics of \mathbf{y}_t in regime i are described by Φ^i in the regime determined by $\Psi_t^{(i)}(\mathbf{c})$. In this way, the TVAR model in equation (2) splits the sample in six different regimes, depending on whether the level of unemployment is above, in between or below the threshold values c_1 and c_2 , and on whether inflation acceleration surpasses or not the c_3 threshold parameter.

3.2 Estimation and testing

The observed sample is $\{\pi_t^w, \pi_t^p, u_t\}_{t=1}^n$. The model in equation (2) can be expressed in matrix form according to:

$$\mathbf{Y} = \sum_{i=1}^6 \Psi^{(i)} \mathbf{X} \Phi^i + \mathbf{U} \quad (3)$$

where $\mathbf{Y} = (\mathbf{y}'_p, \mathbf{y}'_{p+1}, \dots, \mathbf{y}'_n)$ is the $(n - p + 1) \times 3$ matrix of independent variables; $\Psi^{(i)}(\mathbf{c}) = \text{diag}\{\Psi_p^{(i)}(\mathbf{c}), \Psi_{p+1}^{(i)}(\mathbf{c}), \dots, \Psi_n^{(i)}(\mathbf{c})\}$ is a $(n - p + 1)$ diagonal matrix; $\mathbf{X} = (\mathbf{x}'_{p-1}, \mathbf{x}'_p, \dots, \mathbf{x}'_{n-1})$ is the $(n - p + 1) \times (3p + 1)$ matrix of regressors with $\mathbf{x}_t = (1 \quad \mathbf{y}'_t \quad \mathbf{y}'_{t-1} \quad \mathbf{y}'_0)'$; and $\mathbf{U} = (\mathbf{e}'_p, \mathbf{e}'_{p+1}, \dots, \mathbf{e}'_n)$ is a $(n - p + 1) \times 3$ matrix of disturbances.

Letting $\Phi = (\Phi^{(1)} \dots \Phi^{(6)})$, the parameters (Φ, \mathbf{c}) can be estimated by least squares (LS). By

definition, the LS estimates $(\hat{\Phi}, \hat{c})$ minimize the sum of squared residuals function, for which \mathbf{c} is assumed to be restricted to a bounded set $\Gamma = [c_1, \bar{c}_1] \times [c_2, \bar{c}_2] \times [c_3, \bar{c}_3]$.²

Computationally, the LS estimates can be obtained through concentration. Conditional on \mathbf{c} , equation (3) is linear in Φ , yielding the conditional LS estimator $\hat{\Phi}(\mathbf{c})$ defined as:

$$\hat{\Phi}(\mathbf{c}) = \left(\mathbf{X}'\Psi^{(i)}(\mathbf{c})\mathbf{X} \right)^{-1} \mathbf{X}'\Psi^{(i)}(\mathbf{c})\mathbf{Y}$$

The concentrated sum of squared residuals is then given by:

$$S_n(\mathbf{c}) = \left\| \mathbf{Y} - \sum_{i=1}^6 \Psi^{(i)}(\mathbf{c})\mathbf{X}\hat{\Phi}^i(\mathbf{c}) \right\|^2$$

and, therefore, $\hat{\mathbf{c}}$ is the value that minimizes $S_n(\mathbf{c})$. Formally:

$$\hat{\mathbf{c}} = \underset{\mathbf{c} \in \Gamma_n}{\operatorname{argmin}} S_n(\mathbf{c}) \quad (4)$$

where $\Gamma_n = \Gamma \cap \{u_1, \dots, u_n\} \cap \{\Delta\pi_1^p, \dots, \Delta\pi_n^p\}$. The LS estimates are thus given by $\hat{\Phi} = \hat{\Phi}(\hat{\mathbf{c}})$. The grid-search over possible values for \mathbf{c} is also used to obtain $\hat{\mathbf{d}}$.

In practice, implementing grid search procedure over two threshold variables, three threshold parameters and two delay lags increases the estimation and testing time exponentially. Therefore, we estimate the threshold parameters for each threshold variable independently and one threshold value at a time. Chong and Yan (2014) shows that when the threshold variables are independent, we can search for the critical threshold value on one threshold variable by assigning an arbitrary value to another threshold estimate. Hansen (1999), similarly, shows that in the case of a threshold variable with two threshold parameters, we can first estimate one of the threshold parameters, obtain the delay lag and conditional on both, estimate the second threshold parameter.³

With respect to the estimation of the threshold parameters (c_1, c_2, c_3) , it is important to ensure that all six regimes have, at least, $n\tau$ observations, for some $\tau \in (0, 1)$.⁴ Specifically, the grid searches are required to contain at least $n\tau$ observations in the regime determined by $\Psi^{(i)}$. For expositional purposes, the error term is assumed to be i.i.d. $\mathcal{N}(0, \sigma^2)$, but the estimation and testing can be easily extended to the case of heteroskedastic errors.⁵

To determine the optimal number of regimes, a general-to-specific approach is adopted to determine

²This is standard in the literature to avoid end-of-sample distortions. In particular, Γ is set to contain the middle 70% of the vector of ordered threshold values, for each threshold variable.

³Within the context of the u_{t-d_1} threshold variable and $\{c_1, c_2\}$ threshold parameters from equation (1), Hansen (1999) shows that the LS estimates of d_1 obtained after searching for one threshold is consistent for d_1 and that the LS estimate for the unique threshold will be consistent for one element of the pair (c_1, c_2) . Then, the second stage estimate \hat{c}_2 will be consistent for the remaining element of (c_1, c_2) .

⁴Following the empirical literature, we set $\tau = 0.15$ for all models estimated.

⁵We allow for heteroskedasticity in all specifications that we consider in the following subsections.

the optimal number of regimes. First, a five-regime model is tested against a six-regime model. In particular, each of the following null hypotheses (which imply the presence of five-regimes)

$$\begin{aligned}
H_0 : \Phi^1 &= \Phi_2 & H_0 : \Phi^2 &= \Phi_3 & H_0 : \Phi^3 &= \Phi_5 \\
H_0 : \Phi^1 &= \Phi_3 & H_0 : \Phi^2 &= \Phi_4 & H_0 : \Phi^3 &= \Phi_6 \\
H_0 : \Phi^1 &= \Phi_4 & H_0 : \Phi^2 &= \Phi_5 & H_0 : \Phi^4 &= \Phi_5 \\
H_0 : \Phi^1 &= \Phi_5 & H_0 : \Phi^2 &= \Phi_6 & H_0 : \Phi^4 &= \Phi_6 \\
H_0 : \Phi^1 &= \Phi_6 & H_0 : \Phi^3 &= \Phi_4 & H_0 : \Phi^5 &= \Phi_6
\end{aligned} \tag{5}$$

is tested against the alternative hypothesis, H_1 , of six regimes. Thus, an F_{56} statistic can be defined according to

$$F_{56} = \sup_{\mathbf{c} \in \Gamma_n} F_{56}(\mathbf{c}) \tag{6}$$

where $F_{56}(\mathbf{c}) = n \frac{S_5(\mathbf{c}) - S_6(\mathbf{c})}{S_6(\mathbf{c})}$ is the standard F -statistic against the alternative hypothesis for a given \mathbf{c} and $S_k = \hat{\epsilon}'_k \hat{\epsilon}_k$ and $\hat{\epsilon}_k$ is the vector of estimated residuals of the k -regime threshold regression. Since one of the elements in \mathbf{c} is not identified under the null hypothesis, the asymptotic distribution of F_{56} is non-standard. Hansen (1996) shows that the asymptotic distribution may be approximated by a bootstrap procedure for the case of threshold autoregressive processes with a single threshold variable, and Chong and Yan (2014) extend it to the case of general threshold regressions with multiple threshold variables. The algorithm involves the following steps: first, a random sample of error terms e_t^* is generated by sampling with replacement from the estimated residuals, $\hat{\epsilon}_t$. Then, a sample π_t^{w*} is simulated by feeding the model with the random residuals, $\hat{\epsilon}_t^*$, the estimated coefficients, $\hat{\theta}^i$, fixed initial conditions, \hat{u}_{t-1} and π_{t-1}^p and the observed data, $\{\pi_t^p, u_t\}_{t=1}^n$. Based on the simulated series π_t^{w*} , a F_{56}^* statistic can be obtained as in equation (6). The bootstrapped p -value is computed as the number of times the simulated F_{56}^* exceeds the observed F_{56} statistic across 1,000 simulations.

If at least one of the null hypotheses in (5) cannot be rejected, then there are less than six regimes. Therefore, a four-regime model is tested, in a similar way, against the five-regime model associated with the null hypothesis in (5) that was not rejected. This approach is followed until all of the null hypotheses in a given test are rejected. Specifically, a similar F -statistic, $F_{ij} = n \left(\frac{S_i - S_j}{S_j} \right)$, for $i = 1, 2, 3, 4, 5$, $j = 2, 3, 4, 5, 6$, and $j > i$ can be defined, given the values of c_1 , c_2 and c_3 . Notice that the null hypothesis of no threshold effect is given by $H_0 : \Phi^1 = \Phi^2 = \Phi^3 = \Phi^4 = \Phi^5 = \Phi^6$.

3.3 Simulation-based Impulse-Response Functions

The rejection of the restricted models implied by the different F_{ij} test statistics necessarily implies that at least one of the responses to at least one of the structural shocks is different across regimes. However, the degree of this nonlinearity and the dynamic impact of the shocks in the system on the variables of interest can only be evaluated by looking at impulse-response functions (IRFs). Given the nonlinear nature of the model described in equation (3), we follow Koop, Pesaran, and Potter (1996) and Teräsvirta and Yang (forthcoming) and construct simulation-based IRFs defined as the change in the conditional expectation of Y_{t+k} as a result of a shock at time t :

$$IRF[h, \varepsilon_t, F_{t-1}] = E[Y_{t+h}|\varepsilon_t, F_{t-1}] - E[Y_{t+h}|F_{t-1}] \quad (7)$$

where ε_t is the shock at time t and F_{t-1} is the information set at time $t-1$. Calculating the IRFs requires specifying the nature of the shock and the initial conditions F_{t-1} . Then, the conditional expectations $E[Y_{t+k}|\varepsilon_t, F_{t-1}]$ and $E[Y_{t+k}|F_{t-1}]$ are computed by simulating the model.

In practice, we follow the TVAR literature and construct the set of IRFs by assuming that the system remains in a given regime for several periods [see, among others, Auerbach and Gorodnichenko (2012)]. The advantage of this approach is that, once a regime is fixed, the model is linear and the IRFs are not a function of the history of the system. While this assumption is somewhat restrictive, fixed-state responses are routinely reported in the TVAR literature because they can be used as a useful benchmark to test for evidence of nonlinearity. It is important to note that the entire distribution of IRFs is evaluated to obtain fixed responses.

To identify the shocks, we consider sign restrictions to pin down a particular orthogonal decomposition of the vector of reduced-form residuals, following Uhlig (2005), Mountford and Uhlig (2009) and Fry and Pagan (2011). There are two important advantages to the use of sign restrictions as a method of identification relative to other approaches. First, the identification of the shocks does not rely on the imposition of zero constraints on impact responses, which have been shown to be problematic (Faust, 1998). Second, the orthogonalization of the shocks through sign restrictions rely on economic theory and are, thus, economically interpretable. Therefore, we identify the shocks by relying on the following set of sign restrictions, summarized in table 1.

Based on these sign restrictions, the procedure to obtain simulation-based IRFs is as follows. For each regime, we generate an orthonormal matrix \mathbf{Q} using a QR decomposition of $\mathbf{W}'\mathbf{W}$, where \mathbf{W} is a random normal matrix.⁶ Then, shocks are obtained from the orthogonalization $\mathbf{A} = \mathbf{Q}\mathbf{Q}^{-1}$ and fed into

⁶The QR decomposition of any matrix \mathbf{A} implies that $\mathbf{A} = \mathbf{Q}\mathbf{R}$, where \mathbf{Q} is an orthogonal matrix and \mathbf{R} is an upper triangular matrix.

Table 1: Sign restrictions for identification of shocks

Responses	Shock		
	Unemployment	Inflation	Wage growth
Unemployment	(+) 2 quarters	Unrestricted	Unrestricted
Inflation	(-) 2 quarters	(+) 2 quarters	Unrestricted
Wage growth	Unrestricted	Unrestricted	(+) 2 quarters

Sign restrictions for unemployment, price inflation and wage inflation used for the orthogonalization of the shocks and computation of simulation-based IRFs as described in appendix A.

the system through the estimated model to produce a simulated data series for 20 periods. The result is a baseline forecast of the variables, conditional on the initial values and a particular sequence of shocks. The same procedure is carried out, given the same initial values and shocks, except that the shock in period zero is set to a particular value (e.g., 1 percent or 1 standard deviation). A forecast is then produced and compared to the baseline forecast. The difference between the two is the IRF. From the set of IRFs derived in this way, we only select those that display the effects on the endogenous variables associated with the structural shock we seek to identify (i.e., those that satisfy the sign restrictions in table 1). All others are discarded.

Given the strong evidence in favor of nonlinearity based on the fixed-state responses, we follow the conventional approach in the literature when constructing the IRFs and abstract from any model uncertainty. The reader is referred to the appendix for details on the computation of the simulation-based IRFs.

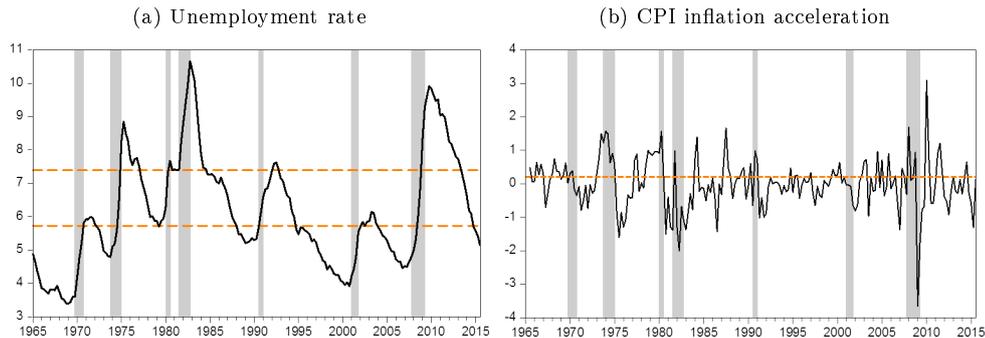
4 Empirical Evidence

In this section, we use the formal econometric approach described in section 3 to study the relationship between wage inflation and unemployment in the U.S., and we examine to what extent a TVAR model of wage inflation with two threshold variables and multiple threshold parameters can account for their joint behavior. The following subsections describe the data and present the results of the model and of the testing procedures.

4.1 Data description

The empirical analysis relies on quarterly U.S. data drawn from the Federal Reserve Economic Data (FRED) website. Unemployment is measured as one hundred times the ratio of unemployed people relative to the civilian labor force. The natural rate of unemployment used here is the Congressional

Figure 3: Threshold variables (solid) and estimated threshold parameters (dashed)



The left panel shows the unemployment rate (solid line) and the two associated estimated threshold values (dashed line). The right panel shows CPI inflation acceleration (solid line) and the associated estimated threshold value (dashed line). In both cases, NBER-dated recession dates are shown in shaded regions.

Budget Office (CBO) estimate, which is obtained directly from the FRED database.⁷ Price inflation is calculated based on the consumer price index (CPI). The measure of wages is based on earnings data for production and non-supervisory workers from the Establishment Survey. Both measures of inflation are constructed as the 4-quarter percentage change. The data is obtained for the period between 1964:Q1 and 2015:Q3, and the effective sample goes from 1965:Q1 through 2015:Q3, corresponding to 203 observations.

Standard unit-root tests, such as the augmented Dickey-Fuller, suggest that all variables are stationary with the exception of price inflation. Consequently, the threshold variables considered are the level of the unemployment rate and inflation acceleration for d_2 periods, defined as $\Delta\pi_{t-1}^p \equiv \pi_{t-1}^p - \pi_{t-d_2}^p$.

4.2 Results

Standard lag selection tests suggest that the optimal number of lags is $p = 2$. The minimization of the sum of squared residuals determined the optimal optimal delay vector, $\hat{\mathbf{d}} = (1, 2)$. The threshold regression model of equation (3) is estimated for the entire sample period, 1965:Q1 through 2015:Q3. The estimated threshold parameters for the unemployment rate, $\hat{c}_1 = 5.73$ and $\hat{c}_2 = 7.39$, are displayed in the left panel of figure 3 (dashed orange line), along with the threshold variable (solid line). Similarly, the estimated threshold parameter for inflation acceleration, $\hat{c}_3 = 0.21$, is shown in the right panel of figure 3. NBER-dated recession dates are shown in shaded regions. The estimated threshold values split the sample into six regimes which roughly correspond to periods of relatively deep recessions, with low and high inflation acceleration; periods of mild recessions, with low and high inflation acceleration; and periods of expansions, with low and high inflation acceleration. The first regime is active when $u_{t-1} \leq 5.73$ and $\Delta\pi_{t-1}^p \leq 0.21$ and captures the dynamics between wage inflation and unemployment

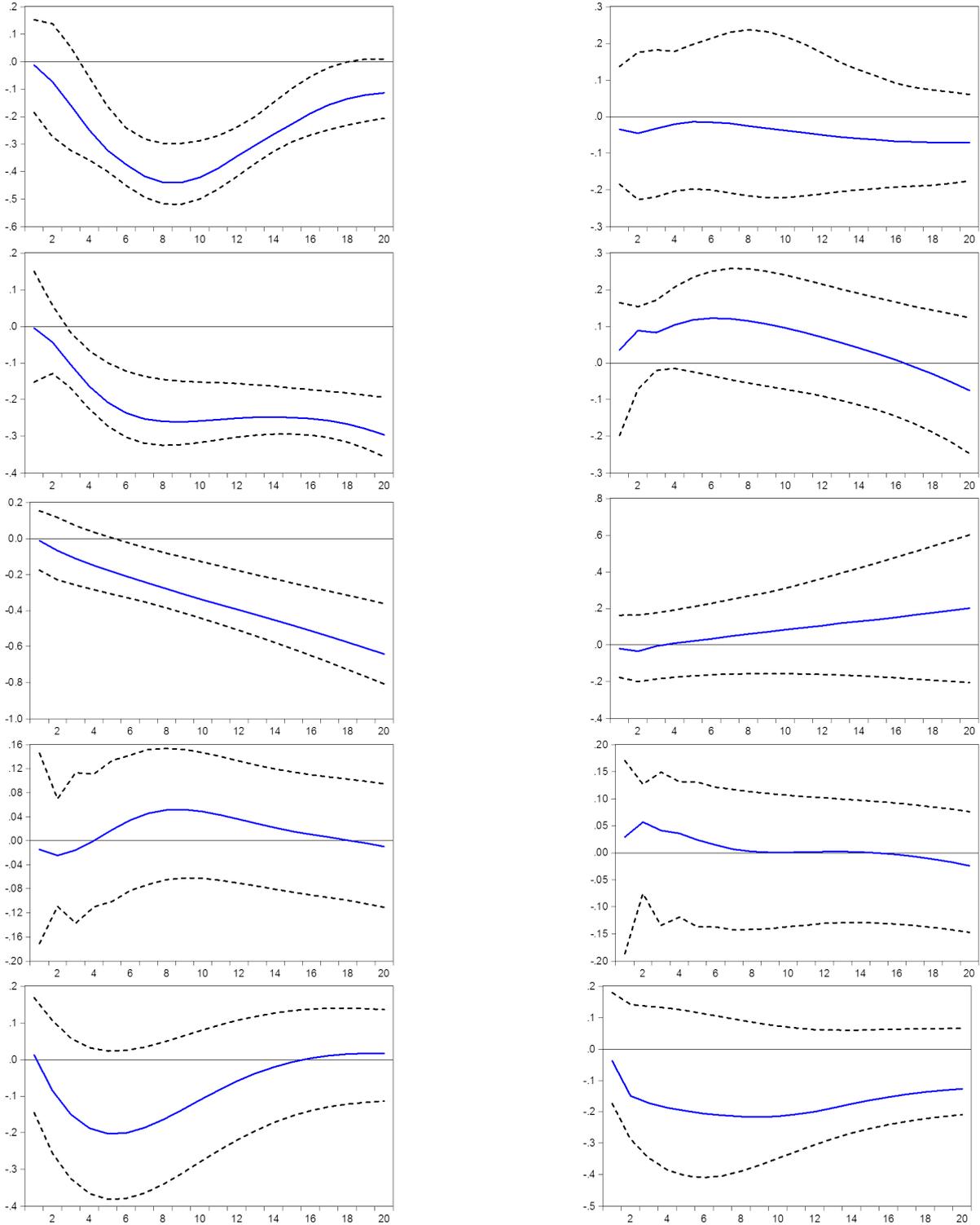
⁷That is, the rate of unemployment arising from all sources except fluctuations in aggregate demand.

when the economy expands at moderate rates and inflation acceleration is low. Figure 3 shows that the nonlinear NKWPC can be described according to the dynamics of the first regime during the recovery periods in the aftermath of mild recessions (1970, 1991 and 2001). In turn, the second regime includes periods during which the economy grows moderately and inflation acceleration is high, consistent with $u_{t-1} \leq 5.73$ and $\Delta\pi_{t-1}^p > 0.21$. This regime corresponds closely to the expansionary periods prior to relatively mild U.S. postwar recessions (*e.g.*, the expansionary periods leading to the recessions of 1970, 1991 and 2001). The third (fourth) regime activates during mild recessions with low (high) inflation acceleration, corresponding to the recovery period following (expansionary periods prior to) the recessions of 1980 and 1991. The fifth regime activates during periods of relatively deep recessions and low inflation acceleration, when $u_{t-1} > 7.39$ and $\Delta\pi_{t-1}^p \leq 0.21$. That is, regime 5 captures the recovery periods after the deep recessions of 1973-75, 1982 and 2007-09. Finally, the sixth regime captures the dynamics between wage inflation and unemployment when $u_{t-1} > 7.39$ and $\Delta\pi_{t-1}^p > 0.21$. This regime corresponds, mostly, to the recovery following the deep recession of 1982.

Even though the estimated coefficients might be different across regimes, it is important to directly evaluate these responses over time since our primary focus is to study the behavior of wage growth over time in response to shocks in unemployment and price inflation. Figure 4 reports the IRFs for the estimated model. The response of wage inflation to a shock to cyclical unemployment is shown in the left panel of figure 4, while the right panel shows the response of wage inflation to a shock in price inflation. The rows correspond to the different responses across the regimes identified by $\Psi_t^{(i)}(\mathbf{c})$ for $i = 1, \dots, 5$. The responses for the sixth regime are highly non-stationary (*e.g.*, the highest eigenvalue is 1.4) and the rejection probability for the orthogonal draws is 99.43%. Intuitively, the explosive dynamics of the IRFs in the sixth regime are explained by the fact that it corresponds to periods deep recessions with high inflation acceleration, associated with high unemployment / high inflation spirals and stagflation. To the extent that a stagflation spirals are unlikely to be repeated, we omit the responses from the sixth regime.

The graphs in figure 4 report median responses in a solid line, together with 16% and 84% bands in dashed lines. In regime 1, when unemployment and inflation acceleration are low with ($u_{t-1} \leq 5.73$, $\Delta\pi_{t-1}^p \leq 0.21$), the response of wage growth is driven, primarily, by shocks to cyclical unemployment. In particular, a one percentage point shock in cyclical unemployment reduces wage growth by 0.4 percentage points after 8 quarters. This response is statistically significant and persistent, even after 20 quarters. Meanwhile, the response of wage growth to a one percentage point shock in price inflation does not generate a response in wage growth that is very different from zero. In an environment of low unemployment and low inflation acceleration, such as periods of expansion leading to mild recessions, the impact of price inflation on wage growth is only marginal. This is consistent with the wage Phillips

Figure 4: Responses of wage growth to unemployment and inflation shocks



The left panel shows the median response of wage growth to a 1% shock in cyclical unemployment. The right panel shows the median response of wage growth to a 1% shock in CPI inflation acceleration. The rows indicate the responses for each regime. In all cases, 16% and 84% confidence bands are displayed in dashed lines.

curve derived by Galí (2011), where price inflation does not affect wage growth on impact.

In the second regime, when unemployment is low but inflation acceleration is high with ($u_{t-1} \leq 5.73$ and $\Delta\pi_{t-1}^p > 0.21$), the response of wage growth is still driven by cyclical unemployment, although price inflation has a higher role relative to the first regime. A one percentage point shock in cyclical unemployment reduces wage growth with a lag, as wage growth does not significantly decrease until the first two quarters. After the second quarter, the total reduction in wage growth is statistically significant and equal to 0.26 percentage points by the quarter 8, and it persists until it reaches a maximum fall of 0.30 percentage points after 20 quarters. Meanwhile, the response of wage growth to a one percentage point shock in price inflation is larger in magnitude relative to the response in regime 1, although only marginally significant. For example, an increase in price inflation of 1 percentage point increases wages growth to a maximum of 0.12 percentage points by quarter 5 and then slowly dies out. This is consistent with a more relevant role of CPI inflation in the dynamics of wages when inflation acceleration is relatively high, as determined by the estimated threshold $\hat{c}_3 = 0.21$.

When the economy is in periods of mild recessions and low inflation acceleration ($5.73 \leq u_{t-1} \leq 7.39$ and $\Delta\pi_{t-1}^p \leq 0.21$) in regime 3, the relevance of both, cyclical unemployment and price inflation, in explaining the dynamics of wage growth increase relative to the first two regimes. On the one hand, a one percentage point shock in cyclical unemployment triggers a large and statistically significant reduction in wage growth after the first 4 quarters. Consistent with the dynamics of wage inflation in previous regimes, the response is highly persistent and continues to fall, reaching a maximum decrease of 0.64 percentage points by quarter 20. On the other hand, an increase in price inflation of one percentage point increases wage growth with a 4-quarter lag. After the fourth quarter, wage growth increases steadily and reaches a maximum of 0.20 percentage points by quarter 20. While the response is large when compared to the responses in the previous two regimes, it is still insignificant.

The response of wage growth in the fourth regime, associated with mild recessions and high inflation acceleration ($5.73 \leq u_{t-1} \leq 7.39$ and $\Delta\pi_{t-1}^p > 0.21$), is small and insignificant in general. While wage growth weakly decreases in response to a shock of one percentage point in cyclical unemployment, the response is small, -0.04 percentage points, and statistically insignificant. Similarly, wage growth increases mildly by 0.05 percentage points in response to a shock in price inflation of one percentage point, although the confidence bands are wide. This period mostly corresponds to the few quarters preceding the 1980 and 1991 recessions, when the dynamics of wage growth were relatively stable.

Finally, the behavior of wage growth during the fifth regime, associated with deep recessions and low inflation acceleration ($u_{t-1} > 7.39$ and $\Delta\pi_{t-1}^p \leq 0.21$), exhibits a stronger influence of price inflation relative to other regimes. After a shock to cyclical unemployment of one percentage point, wage growth falls and reaches a maximum decrease of 0.20 percentage points by the fifth quarter, and then slowly dies

out by quarter 16. The dynamics of wage growth are significant in the first few quarters only. When compared to the response of wage inflation during periods associated with expansions or mild recessions with low inflation acceleration, the influence of joblessness is also smaller in magnitude. At the same time, the response of wage growth to a price inflation shock of one percentage point is larger magnitude than any of the previous five regimes, suggesting a larger role of price inflation during deep recessions with low inflation acceleration. Moreover, the response of wage growth is negative with a maximum fall of 0.21 percentage points by quarter 6, and then slowly dies out, which could explain, in part, the negligible wage growth observed during the 2013-2015 period.

Overall, the analysis of IRFs suggests that shocks to cyclical unemployment play a large role in explaining the dynamics of wage growth, especially during expansions and mild recessions with low inflation acceleration. Meanwhile, neither cyclical unemployment nor price inflation shocks significantly affect wage growth during periods of mildly high unemployment and high inflation acceleration (regime 4). During deep recessions with low inflation acceleration, however, the relative role of cyclical unemployment shocks decreases and the relative role of price inflation increases in explaining the dynamics of wage growth.

5 Conclusions

We estimate a threshold vector autoregressive model of U.S. wage inflation, price inflation and cyclical unemployment for the 1965-2015 period, where the regime-switching is driven by the dynamics of both, the unemployment rate and inflation acceleration. The multivariate model is motivated by the interlinkages between wage growth and unemployment, as described in the wage Phillips curve derived from the New Keynesian model with staggered wage setting in Galí (2011), as well as the interdependence of wage growth and price indexation, as suggested in the wage indexation literature (Christiano et al., 2005; Smets and Wouters, 2007; Hofmann et al., 2012; De Schryder et al., 2015). We find changes in the dynamics of wage growth and unemployment, which depend on whether the unemployment rate is inside or outside [5.73%, 7.32%] and on whether inflation acceleration increases beyond 0.21% or not. Consistent with previous findings in the literature (Galí, 2011; Kumar and Orrenius, 2015; Donayre and Panovska, forthcoming), the analysis of impulse-response functions suggests a negative relationship between wage inflation and unemployment, although the negative relationship is weaker during periods of deep recessions and low inflation acceleration.

In general, the behavior of wage growth changes across the six different regimes identified by the empirical model. Labor market slack, as measured by shocks to cyclical unemployment, largely explains the dynamics of wage inflation, especially during periods of expansions (regimes 1 and 2) and periods

of mild recessions with low inflation acceleration (regime 3). The effect of shocks to price inflation on wage growth is small and statistically insignificant, especially during periods of low inflation acceleration. However, the effect of price inflation shocks on wage growth during periods associated with deep recessions and low inflation acceleration (regime 5) increases, while that of cyclical unemployment shocks falls. These results suggest that the negligible wage growth observed during 2013-2015, in the aftermath of the 'Great Recession', may be driven by the prevailing low inflationary environment and, to a lesser extent, labor market slack. This finding thus calls for policy actions that aggressively target the inflation goal of the Fed's dual mandate.

Appendix A: Simulation-Based Impulse-Response Functions

The procedure for computing the simulation-based impulse-response functions (IRFs) follows Koop et al. (1996) and Teräsvirta and Yang (forthcoming), with the modification of considering an orthogonal structural shock identified through the sign restrictions specified in table 1. The simulation-based IRFs are defined as the effect of a one-time shock on the forecast of the level of the variables in the model, and the response is compared to a baseline ‘no shock’ scenario:

$$IRF_y(h, \varepsilon_t, F_{t-1}) = E[Y_{t+h}|\varepsilon_t, F_{t-1}] - E[Y_{t+h}|F_{t-1}] \quad (\text{A-1})$$

where h is the forecasting horizon, F_{t-1} denotes the initial values of the variables in the model, and ε_t is the one-period shock. The response is then computed by simulating the model. The shock is normalized to be equal to 1 percent (at the time the shock occurs). Thus, for (Φ, \mathbf{c}) , the response IRF_y for $H = 20$ periods is generated using the following steps⁸:

1. Pick a history F_{t-1} . The history is the actual value of the lagged endogenous variable at a particular date.
2. Simulate 3-dimensional errors ε_{t+h} , $h = 0, 1, \dots, H$. The errors are simulated assuming an independent Gaussian process with mean zero and variance-covariance matrix equal to Ω .
3. Using Φ and ε_{t+h} , simulate the evolution of Y_{t+h} over $H + 1$ periods. Denote the resulting path $Y_{t+h}(\varepsilon_{t+h}, F_{t-1})$ for $h = 0, 1, \dots, H$.
4. Generate an orthonormal matrix \mathbf{Q} using the QR decomposition of $\mathbf{W}'\mathbf{W}$, for \mathbf{W} a random normal matrix, to orthogonalize the shocks: $\mathbf{A} = \mathbf{Q}\mathbf{Q}^{-1}$. With the orthogonalized shocks, reconstruct the implied vector of errors and denote it as ε_t^0 , and the resulting simulated evolution of Y_{t+h} over $H + 1$ periods as $Y_{t+h}(\varepsilon_{t+k}^0, F_{t-1})$ for $h = 0, 1, \dots, H$.
5. Construct a draw of a sequence of impulse-responses as $Y_{t+h}(\varepsilon_{t+h}^0, F_{t-1}) - Y_{t+h}(\varepsilon_{t+h}, F_{t-1})$ for $h = 0, 1, \dots, H$.
6. Repeat steps 2 to 5 for B times, with $B = 1,000$, and average the sequences of responses to obtain a consistent estimate of the IRF conditional on the history.
7. To obtain the average response for a subset of histories, repeat steps 1-6 for the subset of histories of interest.
8. From the set of IRFs derived in this way, select those satisfy the sign restrictions in table 1.

⁸Note that for the fixed-regime IRFs, the response from steps 1-8 is simply the standard IRF rotated by the matrix \mathbf{Q} .

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