Extracting the structure of the shocks of information perceived by the Bank of England

VERY PRELIMINARY. DO NOT QUOTE

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Abstract

A new measure of inflation uncertainty is derived in this paper. By observing the revision of the inflation density forecasts that the Bank of England produces each quarter, it is possible to extract the density of the shock of new information perceived each quarter. This allows to perform an evaluation of how the change in risk assessment has evolved during time, and the efficiency in the incorporation of new information to subsequent revisions of density forecasts.

Keywords: Inflation, probability forecast, revisions, convolution.

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1 Introduction

Density forecasts have become a useful tool for researchers and policymakers as a way to evaluate the uncertainty surrounding key economic and financial variables. As such, Alessi et al. (2014) stress the importance of assessing the accuracy of this method in the light of the recent financial crisis. In particular, these authors focus on the performance of the Federal Reserve Bank of New York and the ECB showing that, although those Central Banks did not perform worse than private forecasters, there is room for improvement. Regarding the Bank of England, regular density inflation forecasts in the form of fan charts have been produced since 1993, following the introduction of inflation targeting for monetary policy in the UK. Wallis (2004), Clements (2004) or Dowd (2007) evaluate the forecasting ability of these density forecasts showing that, although they are more accurate for short term forecasting, the uncertainty is generally overestimated. Internal assessment of the fan charts within the Bank of England has been performed by Britton et al. (1998), Elderetal (2005) or more recently Fawcett et al. (2015), who evaluate the forecasting ability of the baseline COMPASS model. More recently Gneiting and Ranjan (2011) or Galbraith and van Norden (2012) re evaluate these forecasts pointing again to the overestimation of risks and the lack of resolution.

Density forecast evaluation is based on the idea of the probability integral transform described in Diebold, Gunther and Tay (1998). A model produces good density forecasts if the implied probability of the realizations is distributed uniformly over time. Berkowitz (2001), Amisano and Giacomini (2007), Bao, Lee and Saltoglu (2007) or Mitchell and Hall (2005) contribute to the literature proposing new measures of density forecast (see e.g. Corradi and Swanson, 2006, for a review). All these methods can be used to evaluate how accurate a density forecast is ex post, i.e., when the realized value of inflation is observed. In this paper a new way of evaluating a density forecast is proposed.

When the Bank of England produces its density forecasts, it does so for a given number of quarters ahead (12 since 2013). Therefore, each quarter the density forecasts made the previous quarter are revised. The difference between the predictive density of inflation for a given point in the future made at to consecutive quarters should be interpreted as the density of the shock of new information perceived by the Bank during the last quarter. This shock may change the balance of risks and, therefore, it is an important source of information to understand how the monetary authority perceives and processes information. Economic agents could anticipate monetary policy reactions knowing these densities, although the link between these shocks and policy outcomes is yet to explore. Finally, it can be assessed how this information is incorporated to new forecasts.

The paper is structured as follows: section 2 outlines the way of extracting the density of the socks of information via convolutions, section 3 applies this technique to the Bank of England fan charts, and section 4 concludes.

2 Recovering the densities of shocks via convolutions

Let z_{t+h} be the forecasted random variable with unknown density function $f_{t+h}(\cdot)$. At moment t a density forecast, $\hat{f}_{t|h}(\cdot)$ is produced for this random variable with all the information available up to that point in time. At time $t+1$, a new density forecast for the same random variable is produced, say $\hat{f}_{t+1|h-1}(\cdot)$. This new forecast density should incorporate only information released between t and $t+1$. Hence, the forecast made at time $t+1$ for z_{t+h}, say $z_{t+1}(h-1)$, which density function is $\hat{f}_{t+1|h-1}(\cdot)$, can be decomposed as $z_{t+1}(h-1) = z_t(h) + \epsilon_{t+1}$. This last random variable can be interpreted as the shock occurred between t and $t+1$, and its density function would inform of the change in the assessment of risk between both periods. If $z_{t+1}(h-1)$ and $z_t(h)$ are uncorrelated, the density function of the shock can be obtained as the convolution of the known predictive densities as

$$\tilde{f}_{\epsilon}(z) = \int_{-\infty}^{\infty} \hat{f}_{t+1|h-1}(x)\hat{f}_{t|h}(x-z)dx \qquad (1)$$

However, it is unreasonable to assume that $z_{t+1}(h-1)$ is not somehow correlated with $z_t(h)$. In that case, the density of the shock should be obtained as

$$\hat{f}_\epsilon(z) = \int_{-\infty}^{\infty} f_{z_{t+1}(h-1)z_t(h)}(x, x-z)dx \tag{2}$$

where $f_{z_{t+1}(h-1)z_t(h)}(\cdot,\cdot)$ is the joint density of $z_{t+1}(h-1)$ and $z_t(h)$. This joint density function is usually unknown but it can be approximated using the notion of copulas. The joint cumulative distribution function $F_{z_{t+1}(h-1)z_t(h)}(x_1, x_2)$ is a function such that

$$F(x_1, x_2) = C\left[F_1(x_1), F_2(x_2)\theta\right]. \tag{3}$$

From Sklar (1959) if the marginals are continuous, the copula function $C(\cdot,\cdot)$ is unique. A complete description of copula modeling can be found in Joe (1997). Therefore, we can define a copula as a function $C : [0,1]^2 \to [0,1]$ which measures the dependence of both random variables. Once defined the copula used in analysis, the probability density function and conditional distributions can be easily obtained as

$$f(x_1, x_2) = c\left[F_1(x_1), F_2(x_2)|\theta\right] f_1(x_1)f_2(x_2), \tag{4}$$

where $c(\cdot,\cdot)$ denotes the density of the copula. Notice that the resulting density for the shock would depend on the copula parameter θ which, again, is unknown. In order to solve this problem, notice that, given that ϵ_{t+1} and $z_t(h)$ are independent by construction, an estimate of $\hat{f}_{t+1|h-1}(\cdot)$ can be constructed by the convolution of the density of the forecast at time t and the estimate of the density of the shock in a way similar to (1). Notice that this new density, say $\hat{g}_{t+1|h-1}(\cdot|\theta)$ will depend on the unknown parameter θ, which can be obtain by minimizing the Kullback-Leibler information criterion distance measure between the 'true' density $\hat{f}_{t+1|h-1}(\cdot)$ and the approximated one. Thus, the proposed estimation method for θ is

$$\arg\min_\theta \int_{-\infty}^{\infty} \hat{f}_{t+1|h-1}(x) \ln\left[\frac{\hat{f}_{t+1|h-1}(x)}{\hat{g}_{t+1|h-1}(x|\theta)}\right] dx \tag{5}$$

This estimator falls into the so called minimum distance estimators. Properties of these estimators can be found in Cressie and Read (1984). See also Basu et al. (2011) for a general review of the methodology. Finally, with an estimate of the copula parameter an estimate of the Pearson and Kendall rank correlation coefficient can be obtain given the copula. This could be interpreted as a measure of how dependent revisions are with respect to previous forecasts.

A vast number of copulas is available in the literature (see e.g. Weiss, 2010, for a review). In this paper the Frank copula (Frank, 1979; Genest, 1987) is used to construct the joint density (4). The Frank copula density function is defined as

$$c(u_1, u_2) = \theta\eta e^{-\theta(u_1+u_2)}/[\eta - (1-e^{-\theta u_1})(1-e^{-\theta u_2})]^2 \qquad \eta = \theta - 1$$

with $\theta > 0$. This copula allows for negative dependence between the marginals, and dependency is symmetric. Also, this copula does not allow for tail dependence. Therefore, it is useful to model strong positive or negative dependence where this dependence is centered in the middle of the distribution.

3 Evaluating the Bank of England fan charts

Since 1993, and following the introduction of inflation targeting for UK monetary policy in 1992, the Bank of England produces its inflation forecasts[1] in the form of density forecasts. Each quarter the Bank releases its forecasts for the next 12 quarters (8 until the second quarter of 2013) under two assumptions: constant official interest rates and the assumption that interest rates will follow market interest rates. The graphical representation of these forecasts is known as the *fan chart* and it depicts the Bank's balance of risks around the central tendency of inflation. The Bank reports the Monetary Policy Committee judgement on the future evolution of the central tendency (mode, μ), uncertainty (σ) and skewness[2] (η) of inflation. The value of these parameters and a report containing all the extra assumptions underlying the construction of the fan charts are

[1]RPIX inflation until December 2003 and CPI inflation since.

[2]Skewness is measured as the difference between the mean and the mode.

publicly available on the Bank of England website[3]. Once these parameters are known, the fan charts are constructed using the Two Piece Normal (TPN) distribution

$$f(x) = \begin{cases} A \exp\left[-\frac{(x-\mu)^2}{2\sigma_1^2}\right] & \text{if } x < \mu \\ A \exp\left[-\frac{(x-\mu)^2}{2\sigma_2^2}\right] & \text{otherwise} \end{cases} \tag{6}$$

The parameters of the distribution (μ, σ_1 and σ_2) are chosen so that it replicates the evolution of mode, uncertainty and skewness of inflation projected by the Bank using the following relations that can be derived from Britton et al. (1998) and Wallis (2004)

$$b = \frac{\pi\eta^2}{2\sigma^2}, \quad g = \text{sign}(\eta) \times \left[1 - \left(\frac{(\sqrt{1+2b}-1)^2}{b}\right)\right], \quad \sigma_1 = \frac{\sigma}{\sqrt{1+g}}, \quad \sigma_2 = \sqrt{\frac{(1+g)}{(1-g)}}\sigma_1$$

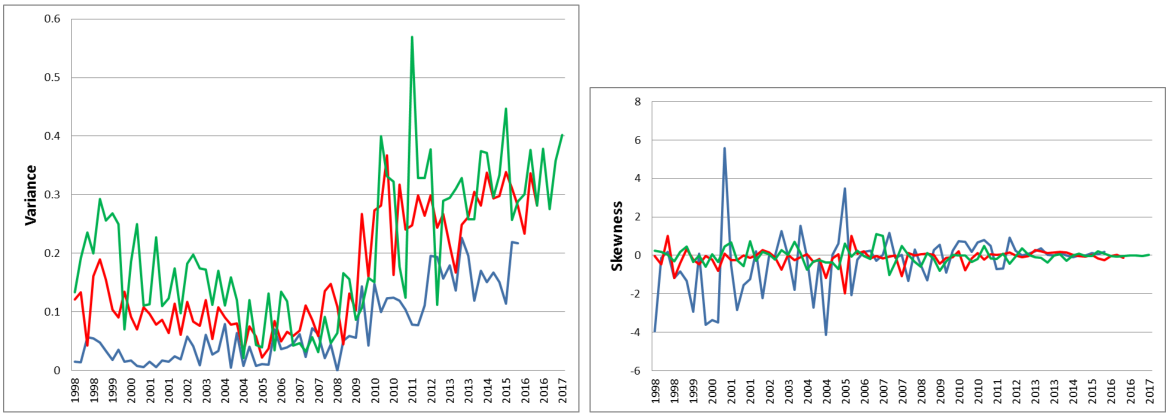
with μ equal to the mode projected by the MPC.

Every quarter, the Bank of England releases a new set of density forecasts which imply a revision of the previous ones. This forecasts update the previous ones incorporating the new information available to the Bank during the previous quarter. This new information may or may not the balance of risk assessment made by the bank, which would affect the shape of the updated density forecasts of inflation. The analysis starts in January 1998, since this is the year in which the Bank of England was granted independence.

Figure 3 plot the variance (a) and skewness coefficient (b) of the errors extracted from one quarter ahead forecasts (blue line), one year ahead (red line) and two years ahead (green line). Panel (a) is quite informative about the shocks observed by the Bank and the efficiency in using this information. The picture shows a decrease in the variance of the shocks of information from the end of the 90s to 2006. Also, the variances of the shock for a given quarted in this period decreases with the forecast horizon. This can be interpreted as a prove that the Bank is efficiently incorporating information to its forecasts. For the very nature of the forecast, the shock expected for inflation in two years time should have a high variance, reflecting the uncertainty surrounding the evolution of the economy. However, the incorporation o information in subsequent quarters

[3] `http://www.bankofengland.co.uk/publications/Pages/inflationreport/irprobab.aspx`

Figure 1: Moments of errors

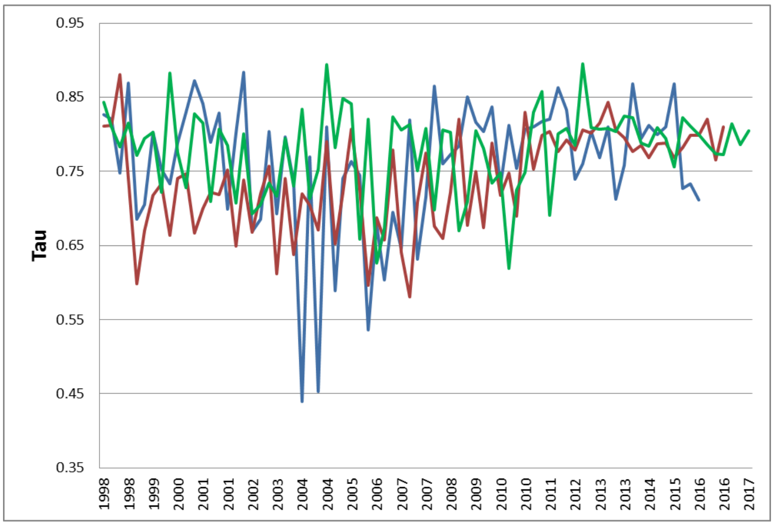


(a) variance (b) skewness

should decrease uncertainty, and thus the variance of the errors extracted from the fan charts. This is what can be seen in the graph until the beggining of the financial crisis. During the period 2008-2010 this mechanishm of information incorporation seems to break. The variance of the errors increase sharply, reflecting the increase of economic uncertainty, and there is no clear evidence of the decrease of this variance with forecast horizon. This can mean tha the Bank is not able to incorporate the new information it is observing into the density forecasts. The variances of the errors, although still quite high with respect to its previous values, seem to have stabilized since 2010. Although there is no clear the variance for one and two years ahead are distinct of each other, there seem to be an efficient incorporation of information in one step ahead forecasts.

With respect to the skewness coefficient, the distribution of uncertainty shocks of inflation forecast for one and two years ahead seem to be quite symmetric, meaning that the Bank does not have a strong beleif about the likelihood of extreme events. However, the MPC seem to revise its risk assessment when producing short term forecasts. The skewess coefficent of the information shock fluctuates widely until the beggining of the financial crisis. However, since then, the distribution of the shocks perceived by the Bank seem to have a more symmetric distribution. There seem to be no clear learning mechanism with respect to skewness as there is with variance. This result, together with the result of incresing variance of information shocks, means that the Bank is perceiving a more uncertain environment in which large positive shocks

Figure 2: Kendall's tau



are as likely as large negative ones.

Finally, graph 2 shows the rank correlation coefficient between the density forecast for one date and the one made one quarter earlier for each forecast horizon. This is interpreted as the dependence of current forecasts on previous ones. Although this dependence is quite high during all the period, it seem to have decreased in the quarters previous to the financial crisis for one step ahead forecasts. This means that the Bank seem to be paying more attention to incoming information than in previous quarters. Since the, the correlation returns to values similar to the ones observed at the beggining of the sample. If the Bank is incorporating new information into the fan charts, one would expect this rank correlation coefficient, which measures the inertia of current forecasts on the previous ones, to decrease during the financial crisis. This can mean that, either the Bank is not able to incorporate information efficiently into its forecasts, or that it is not confident of the accuracy of the relevance of the perceived shocks. Notice that similar results can be observed if we do the same exercise with the fan charts constructed using the assumption that the interest rates follow the market (see appendix).

4 Conclusion

This paper proposes a way to extract the density of information shocks in inflation perceived by the Bank of England between two consecutive quarters. Studying this density gives information about the change in the balance of risks and it could help understand the mechanism of information incorporation to subsequent density forecasts. The variance of these shocks at different forecast horizons can be interpreted as a new measure of inflation uncertainty. The results show that until the beginning of the financial crisis, the Bank was perceiving a period of decreasing uncertainty manifested in decreasing variances of shocks. Also, information seemed to have been incorporated efficiently, as these variances decreased with the forecast horizon. However, the financial crisis increased the uncertainty and no clear learning mechanism seem to have worked until 2011 in which perceived uncertainty seems to have stabilized. Finally, the rank correlation coefficient between the density forecast for the same event made with two quarters of difference seem to be quite high. This may be interpreted either as an indication of an inability to incorporate new information to new forecasts, or an active decision of not introducing too much noise to the forecasts. This result opens interesting areas of research: it should help improve density forecast evaluation tests, as it can be used to eliminate the dependence present in multi-step forecasts. Also, it will be studied if these shock have affected monetary policy outcomes and how.

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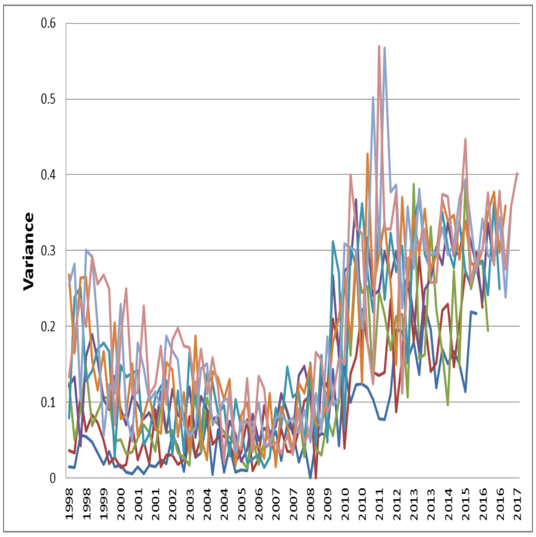
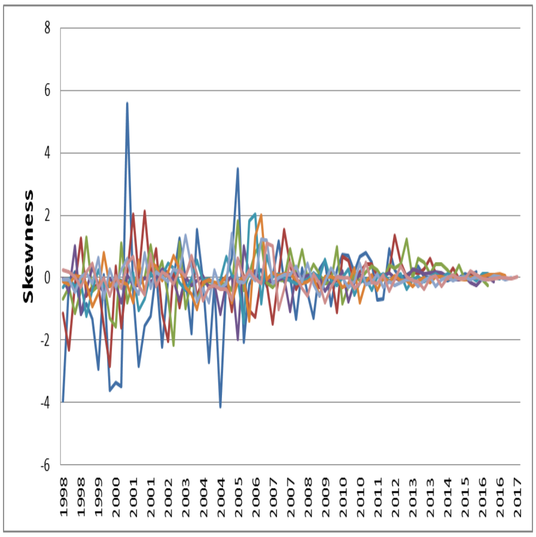
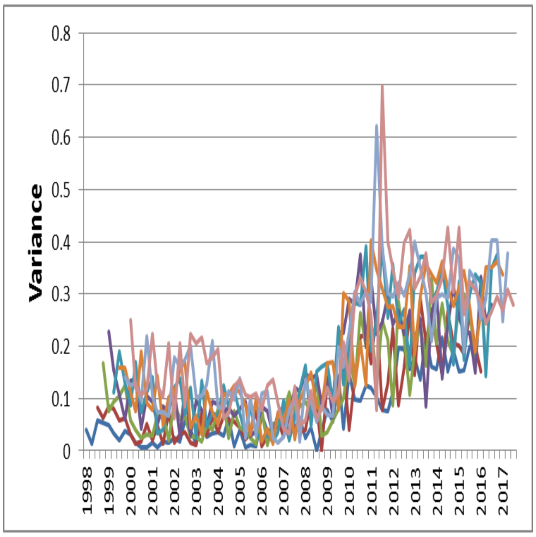
Figure 3: Extra graphs



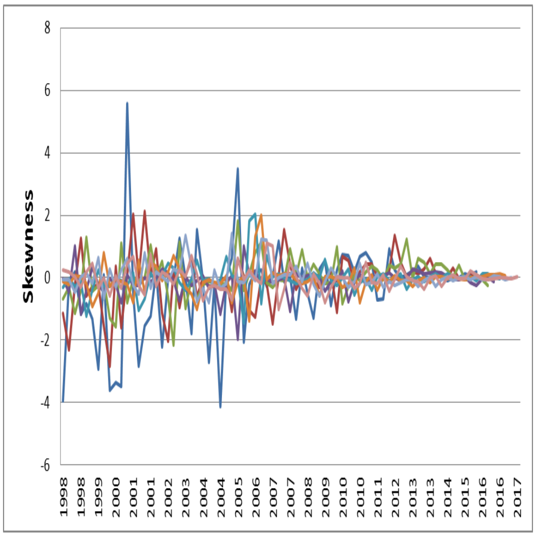
(a) Variance (constant interest rates)



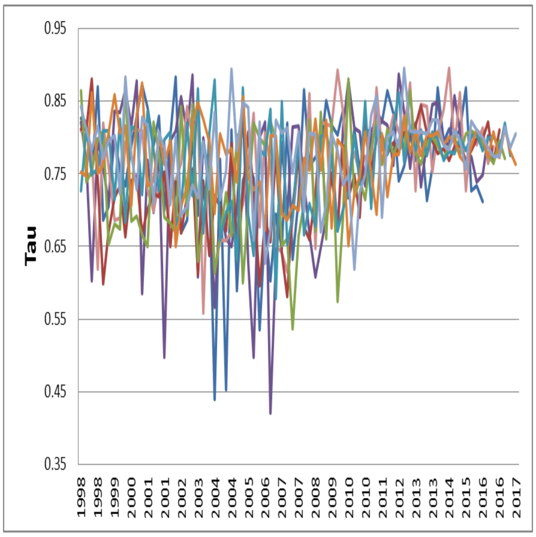
(b) Skewness (constant interest rates)



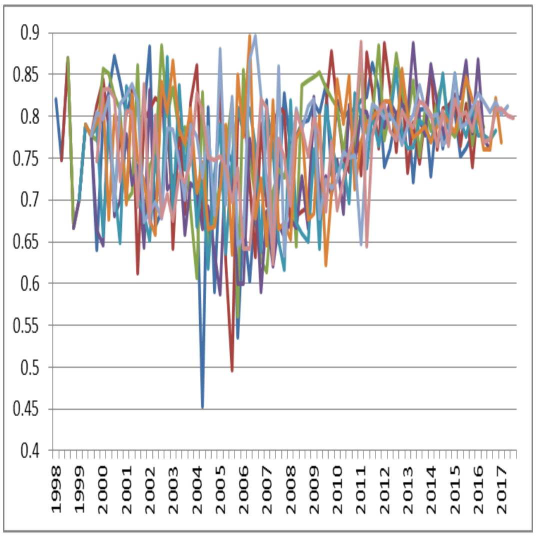
(c) Variance (market interest rates)



(d) Skewness (market interest rates)



(e) Tau (constant interest rates)



(f) Tau (market interest rates)