

# Unemployment Flows, Participation and the Natural Rate of Unemployment: Evidence from Turkey\*

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## Abstract

The level of the unemployment rate the economy converges to in the long-run has long been a major concern for policy makers. This question is traditionally addressed by exploiting the co-movement between inflation (nominal variables) and unemployment in the context of a Phillips curve. In developing countries and emerging markets, which go through structural changes and major transitions, relying on such a framework becomes more challenging. Moreover, labor market size in developing countries also changes, adding to the challenge of using traditional measures. This paper uses labor market flow rates, taking participation into account explicitly, to estimate the unemployment rate trend, using Turkish economy as a laboratory. The paper finds that natural rate of unemployment estimated using unemployment flows gives a different picture than the estimates using nominal variables. Moreover, taking participation rate explicitly into account changes the value of natural rate estimate, while preserving its pattern.

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# 1 Introduction

The long-run rate of unemployment (the underlying trend) has attracted a lot of attention since the Great Recession. In an environment where many developed countries and emerging market economies face exceptionally high levels of unemployment, policy makers and economists focused on identifying the unemployment rate that is feasible in the long run, i.e, the "natural" rate, to gauge the extent of labor market slack (see, for instance, Daly et al. (2012) and Weidner and Williams (2011)). This question is traditionally addressed by exploiting the co-movement between inflation (nominal variables) and unemployment in the context of a Phillips curve. In developing countries and emerging markets, which go through structural changes and major transitions, relying on such a framework becomes more challenging. Even for developed market economies, a stable relationship between inflation and unemployment might be somewhat tenuous.<sup>1</sup>

Our main contribution in this paper is to provide a framework that could be useful even for economies where we cannot rely on traditional Phillips-curve based approaches. We think Turkey provides a unique laboratory in this respect. The country has gone through significant changes in its monetary policy environment, followed by sharply declining inflation in the early 2000s.<sup>2</sup> The traditional approach of estimating a natural rate by focusing on the relationship between labor market variables and the price level (NAIRU) will not necessarily inform us about the underlying dynamics of the Turkish labor market.<sup>3</sup> We show that natural rate estimates extracted using the NAIRU method imply an almost invariant level of unemployment, in spite of substantial movements in the data and significant changes in the labor market in the sample period. Essentially, price level movements do not seem to be informative about an aggregate unemployment rate that could be sustained in the long run, in the absence of high-frequency shocks.

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<sup>1</sup>See for instance Knotek and Zaman (2014).

<sup>2</sup>The Central Bank of Turkey implemented implicit inflation targeting from 2002 to 2006, and has been officially targeting inflation since then. Please see Kara (2006) and Kara and Orak (2008), among others, for more information regarding Turkey's monetary policy.

<sup>3</sup>For a recent discussion regarding NAIRU see Fitzgerald and Nicolini (2013).

Turkish case provides another important challenge to evaluate the usefulness of an approach for estimating the natural rate. In addition to major structural changes in Central Bank objectives, labor force participation increased sharply from 46 percent to 51 percent within a 5-year span in our sample, mostly led by rising female labor force participation. Hence, any attempt to understand the long-run potential of the labor market cannot ignore the participation margin for Turkey. Even though this margin has been relatively stable (and hence ignored) in developed economies, there is now some evidence that the Great Recession might have recently changed this as well (Elsby et al. (2013) and Erceg and Levin (2013)).

This paper confronts both challenges and develops a method to estimate the rate of unemployment in the long run without relying on variations in the general price level and taking into account changes in the labor force participation rate. We estimate our model using data on Turkey, where the labor force participation rate has increased sharply since 2003 and is three times more volatile than the US participation rate (see Sengul (2014)).

To face these challenges, we need a framework that relies exclusively on labor market features. A recent example of such a framework is Tasci (2012) which approaches the problem by estimating the unemployment rate trend using the underlying flow rates. Tasci (2012) builds an unobserved-components method and uses data on flows between employment and unemployment for the US. and argues, in the context of US labor markets, that this method provides an estimate of the natural rate that has several desirable statistical features and comes very close to the language of the modern theory of unemployment. In this paper, we adopt his methodology to incorporate labor force changes and to estimate the natural rate of unemployment for Turkey.

Extending this work, we estimate the unemployment rate trend for Turkey from 2001 to 2013, taking into account flows from inactivity. In doing so, we also draw on the work of Sengul (2014), which estimates Turkey's monthly flow rates from 2005 to 2011, including flows from nonparticipation to unemployment. We first estimate quarterly flow rates from

2001 to 2013. Then, using a parsimonious unobserved-components method, as in Tasci (2012), we decompose the flow rates into their trend and cyclical components. Once we infer the trend components, we provide an estimate of the unemployment rate trend, i.e., the natural rate, implied by the steady-state description of the unemployment rate in a standard labor-market search model.

To assess the effects of participation on the unemployment rate, we also estimate the trend unemployment rate while restricting flows to those between employment and unemployment. Our results show a distinct pattern for trend unemployment: The trend unemployment increases during the first two-thirds of the sample period and starts to decline after the 2008-09 recession. This pattern holds, whether or not one explicitly allows for time-varying labor force participation. However, if an explicit role is given to labor force participation, the estimated unemployment trend varies somewhat relative to the level implied by the model where we assume a constant participation rate over time. The largest discrepancy occurs in the aftermath of the last recession, which coincides with the beginning of the secular rise in participation. Moreover, we find that this pattern for the natural rate is led by a similar pattern for the inflow rate into unemployment, first increasing and then declining in 2008-09, and a secular rise in the rate of outflow from unemployment over the whole sample. Taken together, these findings imply that Turkish labor markets looked far more dynamic at the end of 2013 than in 2001. We also highlight another potentially useful feature of our framework. It improves the accuracy of the unemployment forecast in the short term-even though it is not designed for this purpose. This is an important additional benefit of the framework discussed here, in a country where unemployment data releases have a two-month lag.

Our paper is the first study that relates general output dynamics to the underlying labor market flows for Turkey in order to construct an empirically useful natural rate measure. We believe that this exercise is valuable, not only in its own right but also because it allows us to highlight the usefulness of our approach in the face of the interesting

challenges posed by various structural issues that many economies face. As we argued above, for instance, many developing countries, Turkey included, have a very limited span of data that covers substantial changes in the aggregate economy. The framework we provide in the paper is flexible enough to incorporate labor force participation-in spite of data limitations- for a relatively less dynamic labor market that goes through major structural changes, which in turn makes traditional Phillips curve-based measures rather uninformative.

The rest of the paper proceeds as follows: In the next section, we lay out the unobserved-components model with the assumption that labor force participation moves over time. Since the model with a constant labor force assumption is nested in our model, we describe that model in the appendix. After describing the methodology for measuring the flow rates and estimating the trends, we present the data and the estimation results for both constant and time-varying labor force models in section 3. A more detailed discussion of the natural rate concept we develop here in conjunction with the more conventional measures of the natural rate used in the literature, including a NAIRU, is in Section 4. We also address the robustness of the estimation in this section. Section 5 presents the forecasting performance of the model. The last section concludes.

## **2 Model with Participation**

We now describe our approach for identifying an unemployment trend for Turkey under the assumption that workers can move between three labor market states; employment, unemployment, and inactivity. To estimate the long run unemployment rate while allowing for variations in the labor force participation rate, we extend the unobserved components model described in Tasci (2012). This extended model nests two-state model of Tasci (2012) as a special case. When we discuss the results of our model, we compare the model with participation to its alternative two-state model.

We compute the long run unemployment rate as the steady state description of un-

employment based on trend inflow and outflow rates. Hence, we need labor market flow rates to estimate the unemployment rate trend. We begin with describing the estimation procedure for the flow rates.

## 2.1 Measurement of Flow Rates

To measure flow rates, we rely on the aggregate data. This is because we use quarterly data to estimate unemployment rate trend to have a larger sample size and household level data for Turkey is only available annually. There is now an extensive literature on measuring the flow rates using aggregate data. Most of this literature uses a simple unemployment duration based measurement to infer these rates (i.e., Shimer (2012), Elsby et al. (2009), Fujita and Ramey (2009), Elsby et al. (2013), Petrongolo and Pissarides (2008)). We follow the method used in Sengul (2014), which extends the method used by Elsby et al. (2013), Shimer (2012) and Elsby et al. (2009), to allow for changes in labor force between two consecutive periods.

In what follows, we assume that time is continuous, and the data is available at discrete months  $t$ . Hence, “period  $t$ ” refers to the interval  $[t, t + 1)$ . Let  $N_{t+\tau}$  be the number of population and let the population grow at a rate  $\rho_t$  and the participation rate (the ratio of the labor force to the population) grow at a rate  $G_t$ . Laws of motion for the population and the participation rate are  $\dot{N}_{t+\tau} = \rho_t N_{t+\tau}$  and  $\dot{P}_{t+\tau} = G_t P_{t+\tau}$ , respectively.  $P_{t+\tau}$  is the participation rate and it is computed as the ratio of the labor force to the population ( $P_{t+\tau} = L_{t+\tau}/N_{t+\tau}$ ), where  $L_{t+\tau}$  is the number of labor force.

Furthermore, let  $U_{t+\tau}$ , and  $U_t^{<1}(\tau)$  be the number of unemployed and the number of unemployed for less than 5 weeks at time  $t + \tau$ , respectively, while  $A_t$  is the fraction of the inactive population ( $N_{t+\tau} - L_{t+\tau}$ ) that decide to look for a job.

People become unemployed because they separate from their employment or they enter labor force as unemployed, and they leave unemployment because they either find a job or leave the labor force. Let  $S_t$  and  $F_t$  be the job-separation and unemployment exit (outflow) rates during period  $t$ . We can write the law of motion for unemployment as

follows:

$$\dot{U}_{t+\tau} = (L_{t+\tau} - U_{t+\tau})S_t - U_{t+\tau}F_t + (N_{t+\tau} - L_{t+\tau})A_t. \quad (1)$$

We are limited in our ability to distinguish between exits from unemployment into employment or into inactivity in the data due to lack of data availability at a high frequency. Hence,  $F_t$  absorbs exits from unemployment, regardless of their destination. Since some of the outflow may be due to the inactivity,  $F_t$  is the unemployment exit rate, not necessarily the job-finding rate. This way of modeling does not affect our analysis as the focus of the paper is to measure and estimate the flows into and out of unemployment. We do not need to have specific information about the nature of the exit from unemployment *per se*. Note that if the labor force is constant, then we have  $\rho_t = 0$  and  $G_t = 0$  and  $A_t = 0$ . In this case, the equation above is the same as the equation of Elsby et al. (2013), which assumes constant labor force between two consecutive periods.<sup>4</sup>

We solve equation (1) and iterate it three months to get the evolution of the unemployment rate based on observed data in discrete intervals as:<sup>5</sup>

$$u_t = u_{t-3}(1 - \lambda_t) + \frac{\lambda_t S_t}{\theta_t + G_t} + \frac{\lambda_t A_t}{\theta_t + G_t} \left( \frac{(\theta_t + G_t)(1 - e^{-3\theta_t})}{P_t \theta_t \lambda_t} - 1 \right), \quad (2)$$

where  $\lambda_t = (1 - e^{-3(S_t + F_t + \rho_t + G_t)})$  is the quarterly convergence rate and  $\theta_t = S_t + F_t + \rho_t$  for notational convenience. One can use equation (2) and write the steady state unemployment rate as

$$u_t^{ss} = \frac{S_t}{\theta_t + G_t} + \frac{A_t}{\theta_t + G_t} \left( \frac{(\theta_t + G_t)(1 - e^{-3\theta_t})}{P_t \theta_t \lambda_t} - 1 \right). \quad (3)$$

Note that introducing variable labor force affects the steady state unemployment through multiple channels. First we need to account for the fraction of the inactive population who start looking for a job and become unemployed, i.e.  $A_t/(\theta_t + G_t)$ . Second we have to take into account the fact that the size of the labor force changes with participation, as well as the population growth ( $\theta_t + G_t$ ). Lastly, change in participation also changes the size

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<sup>4</sup>We derive flow equations under this assumption in the appendix.

<sup>5</sup>We use quarterly data.

of the inactive pool, which affects the flows from inactivity to unemployment. To see this last point, suppose  $G_t = 0$ , then the coefficient of  $A_t$  becomes coefficient of  $A_t/(\theta_t + G_t)$  in the equation above becomes  $(N_t/L_t - 1)$ .

To compute the flow rates, we also need the law of motion for short-term unemployed, i.e. unemployed for less than five weeks:

$$\dot{U}_t^{<1}(\tau) = (L_{t+\tau} - U_{t+\tau})S_t - U_t^{<1}(\tau)F_t + (N_{t+\tau} - L_{t+\tau})A_t. \quad (4)$$

The change in the number of short-term unemployed consists of workers entering unemployment, workers separating from their jobs and workers who became unemployed after the last time data was available and did not leave unemployment, respectively.

Subtracting equation (4) from equation (1) results in

$$\dot{U}_{t+\tau} = \dot{U}_t^{<1}(\tau) - (U_{t+\tau} - U_t^{<1}(\tau))F_t. \quad (5)$$

We do not observe  $A_t$  in (5) explicitly. This is not because participation does not affect the law of motion for unemployment, but because the fraction of inactive population entering to unemployment is already present in  $U_t^{<1}$  and  $F_t$  also covers unemployed workers leaving the labor force.

Solving the differential equation above and the laws of motion for the population and the participation rate (and rewriting the equation in terms of rates) yields:

$$u_t = e^{-F_t - \rho_t - G_t} u_{t-1} + u_t^{<1}. \quad (6)$$

where  $u_t$  denotes the unemployment *rate* in period  $t$ .

Assuming unemployment exit occurs with a Poisson process with parameter  $F_t$ , the probability of exiting unemployment within a month is  $\hat{F}_t = 1 - e^{-F_t}$ . Therefore, equation (6) can be rewritten as

$$\hat{F}_t = 1 - \frac{u_t - u_t^{<1}}{e^{-G_t - \rho_t} u_{t-1}}. \quad (7)$$

The intuition behind (7) is that we infer the average outflow probability by measuring the size of the decline in the unemployment pool who are not short-term unemployed.

Notice that  $\rho_t + G_t$  is the labor force growth rate, as labor force varies due to changes in population and the participation decisions. The equation above takes into account the change in the size of the labor force to get the average outflow probability.

The monthly outflow probability relates to associated monthly outflow hazard rate for the short-term unemployed,  $F_t^{<1}$ , through the equation  $F_t^{<1} = -\ln(1 - \hat{F}_t^{<1})$ .

Equation (7) works well to estimate the outflow probability in labor markets for which the flow rate out of unemployment is high (duration of unemployment is low). For countries with longer durations, like Turkey, there are relatively few people in  $u_t^{<1}$  at any time since exit rates are low. Hence, the variance of the estimate will be higher ( $\hat{F}$  will be noisy). We follow Elsby et al. (2013) and Sengul (2014) and use additional duration data to increase the precision of the estimate of  $\hat{F}_t$ . Based on the unemployment data by duration, we can calculate the probability that an unemployed worker exits unemployment within  $d$  months as

$$\hat{F}_t^d = 1 - \frac{u_t - u_t^{<d}}{e^{-\sum_{j=0}^{d-1}(G_{t-j} + \rho_{t-j})} u_{t-d}}. \quad (8)$$

Subsequently, we can calculate the outflow rates as  $F_t^{<d} = -\ln(1 - \hat{F}_t^d)/d$  for different durations,  $d = 1, 3, 6, 9, 12$ . This rate is interpreted as the rate at which an unemployed worker exits unemployment within the subsequent  $d$  months.

If the exit rate from unemployment is independent of the duration of unemployment, then  $F_t^{<d}$  for different values of  $d$  would not be much different from each other, and we have the monthly outflow hazard rate as  $F_t^{<1}$ . However, if the exit rate from unemployment depends on the duration of unemployment, then the  $F_t^{<1}$  rate would not be a consistent estimate of the average outflow rate. We formally test the duration dependence by testing the hypothesis that  $F_t^{<1} = F_t^{<3} = F_t^{<6} = F_t^{<9} = F_t^{<12}$ .<sup>6</sup> The approach in general is to derive the asymptotic distribution of unemployment rates and unemployment rates for different durations, and then to apply the Delta method to compute the joint asymptotic

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<sup>6</sup>Formal details of the test can be found in Elsby et al. (2013) with the only difference being that this paper has an extra term, the duration  $d < 9$ .

distribution of the outflow rate estimates. For Turkey, the hypothesis that there is no duration dependence (i.e., the hypothesis that  $F_t^{<d}$  is the same for all  $d$ ) can be rejected at 95 percent confidence level. We use the asymptotic distribution to compute an optimally weighted estimate of outflow rate that minimizes the mean squared error of the estimate. We discuss computation of  $A_t$  series below when we describe the data, as we infer the series directly from the data.

Given  $F_t$ ,  $u_t$  and  $A_t$  series, the equation (2) gives us the separation rate data we need to proceed with our estimation.

## 2.2 Estimating Trend Unemployment Rate

After discussing the measurement of flow rates, we now present our approach for identifying an unemployment trend in the presence of varying labor force participation. In doing so, we extend the model used in Tasci (2012) to include participation. Tasci (2012) uses a simple reduced form unobserved components model that incorporates the comovement of flows between employment and unemployment into previous attempts at estimating the natural rate, such as Clark (1987, 1989) and Kim and Nelson (1999).

Our reduced form model assumes that real GDP - the measure of the aggregate business cycle we use- has both a stochastic trend and a stationary cyclical component, where only real GDP is observed by the econometrician. The stochastic trend follows a random walk while the cyclical component is an autoregressive process. More specifically, let  $Y_t$  be log real GDP,  $\bar{y}_t$  be a stochastic trend component, and  $y_t$  be the stationary cyclical component. Then we consider an unobserved components model starting with the process for the GDP as,

$$\begin{aligned}
 Y_t &= \bar{y}_t + y_t, \\
 y_t &= \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t^{yc}, \\
 \bar{y}_t &= r_{t-1} + \bar{y}_{t-1} + \varepsilon_t^{yn}, \\
 r_t &= r_{t-1} + \varepsilon_t^r,
 \end{aligned} \tag{9}$$

where  $r_t$  is a drift term in stochastic trend component of output, which is also a random walk, and the cyclical component of output follows an AR(2) process, as in Ozbek and Ozlale (2005).

The model also assumes that both unemployment exit and job separation rates ( $F_t$  and  $S_t$ ) have a stochastic trend as well as a stationary cyclical component. Furthermore, the stochastic trend in these flow rates follows a random walk while their cyclical component depends on the cyclical component of real GDP. Let  $\bar{f}_t$  and  $\bar{s}_t$  be the stochastic trend components, and  $f_t$  and  $s_t$  be stationary cyclical components of  $F_t$  and  $S_t$ , respectively. The time series behavior of these flow rates take the following form:

$$\begin{aligned} F_t &= \bar{f}_t + f_t, & \bar{f}_t &= \bar{f}_{t-1} + \varepsilon_t^{fn}, \\ f_t &= \tau_1 y_t + \tau_2 y_{t-1} + \tau_3 y_{t-2} + \varepsilon_t^{fc}, \end{aligned} \tag{10}$$

and

$$\begin{aligned} S_t &= \bar{s}_t + s_t, & \bar{s}_t &= \bar{s}_{t-1} + \varepsilon_t^{sn}, \\ s_t &= \theta_1 y_t + \theta_2 y_{t-1} + \theta_3 y_{t-2} + \varepsilon_t^{sc}. \end{aligned} \tag{11}$$

We assume that all the error terms are independent white noise processes.

As equations (10) and (11) show, we also assume that the cyclical component of the outflow and separation rates move with the aggregate cycle. This idea captures the empirical pattern that recessions are times when a substantial number of matches dissolve because they cease to be productive enough and significantly fewer new matches are formed because firms do not demand as much labor anymore. Hence, a priori, we expect a negative co-movement between the cyclical components of the flow rates,  $s_t$  and  $f_t$  and cyclical component of real GDP.<sup>7</sup> This basic description of the comovement between flow rates and the aggregate cycle can be easily reconciled with the extensions of the basic labor market search model with endogenous separations, as in Mortensen and Pissarides (1994). Tasci

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<sup>7</sup>We are agnostic about the existence of any co-movement among the trends of the flow rates, if any, as long as they are not correlated with the aggregate output. Even though such interaction is possible, we abstract from it. Given the short sample we are working with, any more complication in the form of another latent variable will substantially reduce the precision of the estimates we get in this unobserved components model.

(2012) argues that the low-frequency movements in the trends,  $\bar{f}_t$  and  $\bar{s}_t$ , will capture the effects of institutions, demographics, tax structure, labor market rigidities, and the long-run matching efficiency of the labor markets, which will be more important in determining the steady state of unemployment.

After modeling unemployment exit and separation rates, we are left with the time series behavior of participation rate, inactivity-to-unemployment flow rate and population. Due to the length of our sample and the additional number of parameters that arise with an additional variable in our unobserved components model, we cannot fully model all the flow rates that determine the steady state unemployment rate. Hence, we need to make some assumptions. We begin by assuming that the population growth  $\rho_t$  has a trend and a cycle that are independent of the GDP, and we identify these components using HP filter.<sup>8</sup> We also subject  $A_t$  series to the same procedure. Even though one expects the cyclical component of flows from inactivity to unemployment to depend on the overall cycle (in GDP), we cannot model it together with the participation rate and its growth as we run out of degrees of freedom. Since  $A_t$  is measured indirectly, we think including  $P_t$  and  $G_t$  in our model can be more informative.

Given these assumptions, we then model participation as having a cyclical component and potentially a stochastic growth component in its trend:

$$\begin{aligned}
 P_t &= \bar{p}_t + p_t \\
 p_t &= \mu_1 y_t + \mu_2 y_{t-1} + \mu_3 y_{t-2} + \varepsilon_t^{pc} \\
 \bar{p}_t &= \bar{p}_{t-1} + g_{t-1} + \varepsilon_t^{pn} \\
 g_t &= g_{t-1} + \varepsilon_t^g
 \end{aligned} \tag{12}$$

The model described by equations (9) through (12) can be represented in a state-space form in the following way:

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<sup>8</sup>We also fit an AR process to the population growth and see that trend we extract does not change much.

$$\begin{bmatrix} Y_t \\ F_t \\ S_t \\ P_t \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tau_1 & \tau_2 & \tau_3 & 0 & 0 & 0 & 1 & 0 \\ 0 & \theta_1 & \theta_2 & \theta_3 & 0 & 0 & 0 & 0 & 1 \\ 0 & \mu_1 & \mu_2 & \mu_3 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{y}_t \\ y_t \\ y_{t-1} \\ y_{t-2} \\ r_t \\ g_t \\ \bar{p}_t \\ \bar{f}_t \\ \bar{s}_t \end{bmatrix} + \begin{bmatrix} 0 \\ \varepsilon_t^{fc} \\ \varepsilon_t^{sc} \\ \varepsilon_t^{pc} \end{bmatrix}, \quad (13)$$

$$\begin{bmatrix} \bar{y}_t \\ y_t \\ y_{t-1} \\ y_{t-2} \\ r_t \\ g_t \\ \bar{p}_t \\ \bar{f}_t \\ \bar{s}_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \phi_1 & \phi_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{y}_{t-1} \\ y_{t-1} \\ y_{t-2} \\ y_{t-3} \\ r_{t-1} \\ g_{t-1} \\ \bar{p}_{t-1} \\ \bar{f}_{t-1} \\ \bar{s}_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{yn} \\ \varepsilon_t^{yc} \\ 0 \\ 0 \\ \varepsilon_t^r \\ \varepsilon_t^g \\ \varepsilon_t^{pn} \\ \varepsilon_t^{fn} \\ \varepsilon_t^{sn} \end{bmatrix}. \quad (14)$$

where all error terms come from an i.i.d. normal distribution with zero mean and variance  $\sigma_i$ , such that  $i = \{yn, yc, r, g, fn, fc, sn, sc, pn, pc\}$ .

We use the Kalman filter to filter the unobserved components and write the log-likelihood function to estimate the model via maximum likelihood. Since we are interested in the unobserved stochastic trend and cyclical components, once we estimate the model, we use the Kalman smoother to infer them over time. Then, we obtain the unemployment rate trend using the flow steady state equation and evaluate at the current trend levels of the variables:

$$\bar{u}_t = \frac{(\bar{s}_t - \bar{a}_t)}{\bar{s}_t + \bar{f}_t + \bar{\rho}_t + g_t} + \frac{\bar{a}_t(1 - e^{-3(\bar{s}_t + \bar{f}_t + \bar{\rho}_t)})}{\bar{p}_t(\bar{s}_t + \bar{f}_t + \bar{\rho}_t)\bar{\lambda}_t}, \quad (15)$$

where  $\bar{\lambda}_t = 1 - e^{-3(\bar{s}_t + \bar{f}_t + \bar{\rho}_t + g_t)}$ . Recall that  $\bar{\rho}_t$  and  $\bar{a}_t$  are not estimated through the model, but computed separately as the trend implied by the HP filter.

Tasci (2012) interprets the unemployment rate trend expressed in (15) as the *steady state* unemployment rate that is implied by the current *trend* estimates of the flow rates. Note that, since trend flow rates are random walks, current trend estimates are also the best estimates for future trend values. Hence, we interpret this rate as the rate of

unemployment in the long run, to which the actual unemployment rate would converge in the limit.

### 3 Data and Estimation

Before proceeding to the estimation, we describe our data sources and the treatments we have to implement to address some concerns. Then we present the estimation results for the flow rates and the long-run unemployment rate implied by these rates for Turkey. To better understand the contribution of incorporating changes in the labor force to the unemployment rate trend, we also estimate the model with the assumption that the labor force does not change between two consecutive quarters. Since our model already nests the constant labor force model, we leave the explicit discussion of the model without variable labor force to the appendix.

The data used in estimating the flow rates is from the Household Labor Force Survey conducted by Turkish Statistical Agency (TurkStat).<sup>9</sup> We have quarterly data from 2000:Q1 to 2013:Q4 on the population, the number of workers in the labor force, and unemployed persons for less than  $d$  months, where  $d \in \{1, 3, 6, 9, 12\}$ .<sup>10</sup>

The raw data requires some adjustments due to breaks prior to construction of the flow hazard rates,  $F_t$  and  $S_t$ . First, there is a break in the 2005:Q1 data. This break is due to a change in population projection methods that took place in 2007.<sup>11</sup> TurkStat updated quarterly data prior to the break until 2005:Q1 and yearly data until 2004. To correct the data prior to 2005, we make use of the availability of unadjusted quarterly and adjusted

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<sup>9</sup>For more information go to <http://www.tuik.gov.tr>. Household labor force survey is the main data source for labor force statistics in Turkey. The survey began in 1988 and conducted at biannual frequency until 2000. In 2000, the survey was extended to include a larger sample and more questions. Also its frequency increased allowing for quarterly aggregate results. In 2004, the sample size increased further to allow for statistics for 26 regions, compared to 9 regions of previous surveys. Microdata is available at annual frequency since 2000.

<sup>10</sup> $d = 1$  corresponds to the number of workers unemployed for less than five weeks and this data is provided by TurkStat upon request.

<sup>11</sup>In 2007, Turkey implemented an address-based population registration system (ADNKYS), which allows yearly data for population. Turkstat was using population numbers based on projections from census data prior to this change, and it realized a discrepancy between the projections and the actual numbers delivered by ADNKYS.

annual values for 2004. As such, we update the unadjusted quarterly values for 2004 such that quarterly growth rates within 2004 are the same for adjusted and unadjusted series and the average of the new quarterly data for 2004 is the same as the adjusted annual value reported by TurkStat. Once we adjust the quarterly series of 2004, we also update the data prior to 2004 such that the quarterly growth rates are the same as in the unadjusted series.

In addition, there is a break in 2004 in the data for unemployed with different durations.<sup>12</sup> To correct for this, we assume that the growth rate of the share of unemployed with a duration of  $d$  months among all unemployed from 2003:Q4 to 2004:Q1 is the average of the growth rate of the same quarter of the two previous and the following years' shares. Then, we back up the new shares for periods prior to 2003:Q4 from the new growth rates, and readjust all duration data so that the shares add up to 1. We adjust the number of unemployed for less than one month such that their share among unemployed for less than three months (in unadjusted series) remains the same.<sup>13</sup> All these treatments are unfortunately dictated by the concerns we have due to data breaks, survey redesign, and methodological changes. However, the fact that there was no major aggregate economic shock hitting the economy around this time reassures us that the impact of our treatments on the estimation results will be nonsubstantial.<sup>14</sup> Finally, we also use the aggregate real GDP data from the TurkStat.<sup>15</sup>

Figure 1 shows the unemployment and the participation rate throughout the sample. The Turkish unemployment rate hovered around 10 percent during the sample period, with the exceptions of the rise during the global crisis. Participation rate was around 47

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<sup>12</sup>In 2004, the sample size of Household labor force survey increased. Regional results were available for 9 regions prior to 2004. With the increased sample, TurkStat began to release results for 26 regions. This increase in sample size might have allowed better estimates of unemployed with different durations.

<sup>13</sup>There was also an anomaly in the unemployed for 6-7 months data for 2003:Q2 and 2003:Q3, which generated a level shift in the seasonally adjusted data. We replace the growth rates of shares from 2003:Q1 to 2003:Q2 and from 2003:Q2 to 2003:Q3 with the average of the growth rate of the same quarter of the two previous and the following years' shares.

<sup>14</sup>We also used some other imputation methods for this break. The results show no significant change.

<sup>15</sup>Expenditure based, in 1998 prices.

percent until 2007 and it increased to around 50 percent by 2013. In addition to the data described above, we make use of the data on unemployment by reason to construct  $A_t$  series. Figure 2 displays the series. Ideal computation would require data on labor market transitions of entrants who will be unemployed for less than one month. The ratio of this pool to the inactive population would be  $A_t$ . However, data on the number of unemployed for less than one month by reason of unemployment is not available. Thus, we use data on unemployment by reason for a duration less than three months and assume that the fraction of entrants among unemployed for less than three months (the shortest duration for which we have data) is the same as the fraction of entrants among unemployed for less than one month. The assumption implies that  $\frac{U_t^{e,<1}}{U_t^{<1}} \approx \frac{U_t^{e,<3}}{U_t^{<3}}$ , where  $U_t^{e,<d}$  denotes labor market entrants who are unemployed for less than  $d$  months. Note that  $U_t^{e,<1} \approx U_t^{<1} \frac{U_t^{e,<3}}{U_t^{<3}}$  and we have data for the right-hand side of this approximation. Hence, we compute  $A_t$  as  $U_t^{<1} \frac{U_t^{e,<3}}{U_t^{<3}} / (N_t - L_t)$ .

Before proceeding to the estimation and results, we would like to discuss an issue that needs to be tackled in estimating the model. The model, as spelled out in equations (13)-(14), has four observable series and ten shock parameters that needs estimating, and hence is subject to a potential identification problem. The solution involves normalizing the standard deviation of the cyclical component of a variable relative to its trend component, thereby reducing the number of parameters to estimate. We address this issue in more detail and describe the process in Section 3.2.

### 3.1 Results for Constant Labor Force Model

Once we make necessary adjustments to the data, we compute the aggregate flow rates following our discussion in the preceding section. We start with the restricted model which assumes that the labor force is constant, which is effectively equivalent to restricting assumption that we have  $\rho_t = 0$  and  $G_t = 0$  and  $A_t = 0$  in our model. We start with this simplified case to present an easier benchmark. Table 1 presents the basic moments of the data. Average unemployment in Turkey was about 10.5 percent over our sample

period, rising from around 7.5 percent to more than 14 percent in the middle of the last recession. We are in a sense fortunate to have unemployment move around this much, as it helps to identify the movements in the trend and cycle components in the flow rates even within a short sample. Observed flow rate levels in Table 1 show that the Turkish labor market also features very low rates of turnover, similar to some OECD countries. As such, our approach to use more duration data to compute the average flow hazards is clearly warranted. Similar to the pattern we observe in other countries, outflow hazard,  $F_t$ , is at least six times more volatile than the inflow hazard,  $S_t$ .

We estimate the unobserved components model with constant labor force via maximum likelihood using the flow rates described above.<sup>16</sup> The potential identification issue discussed above appears not to be a major one for the data at hand. The log-likelihood function turns out to be well behaved and quite variable, such that we can avoid the normalization for the GDP components that Tasci (2012) relies on for the U.S. data. The same is not true for the flow rates, which implies that we estimate the process for both  $\varepsilon_t^{yn}$  and  $\varepsilon_t^{yc}$ , but we resort to normalization for the flow rates. Our estimation results suggest that the drift term for the trend output for this time-period in Turkey was constant, that is  $\sigma_r = std(\varepsilon_t^r) = 0$ . Hence, we impose this restriction in our estimation, obtaining  $r = 0.012$  for the sample period. This rate translates into a 4.9 percent average annualized quarterly growth rate for the trend output. The normalization we find to be optimal for the flow rates in this restricted model estimation implies that  $\gamma_f = \frac{\sigma_{fn}}{\sigma_{fc}} = 0.75$  and  $\gamma_s = \frac{\sigma_{sn}}{\sigma_{sc}} = 0.75$ . The procedure to choose parameter values for  $\gamma_s$  and  $\gamma_f$  is explained in detail in Section 4.

In our estimation, we rely on the Kalman filter to generate the log-likelihood function and to obtain the smoothed unobserved states. Because we have several variables following a random walk, initializing the Kalman filter requires starting with a diffuse prior, which requires us to exclude some of the quarters at the beginning of the sample. We exclude

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<sup>16</sup>This version of the model is expressed in (A.25)-(A.26) in the appendix.

the first eight quarters of the data in our estimation. We discuss the potential effects of this exclusion restriction in Section 4.

In Figure 3, we plot the estimated unobserved trend components as well as the data on the flow rates, unemployment rate, and the rate of convergence,  $\lambda_t$ . The upper panel of Figure 3 shows interesting changes in the underlying trends for the flow rates. In particular, the outflow rate, rate at which an average unemployed would find a job in a given month, has increased over the course of the decade by essentially doubling from 0.06 to 0.12, implying a monthly probability of roughly 11 percent by the end of the sample. In a somewhat similar fashion, the inflow rate also trended up over the sample period, tripling from its 0.005 level to 0.015. Since the end of the last recession, the trend changed course and has started to decline towards a level of 0.012.

These trend changes together imply a relatively stable pattern for the unemployment rate trend early on in the sample period, with the exception of the first recessionary episode. Then, trend unemployment gradually declines from its recession era highs of 12 percent to around 9 percent at the end of the sample. In the first part of the sample, trend changes in  $F$  and  $S$  offset each other to some extent as they push trend unemployment in opposing directions. However, since the end of the last recession, changes in direction of the trend behavior of  $S$  reinforced the decline in the unemployment rate trend that is implied by the gradual increase in the outflow rate,  $F$ , over time.

A more important observation is that overall reallocation in the labor markets have experienced a steady increase in Turkey. The picture on the lower-right panel plots the reallocation measure we look at,  $\lambda_t$ , which governs the rate at which unemployment approaches its *flow* steady state. The magnitude of the changes over time implies that the half-life of a cyclical gap in the unemployment rate declined from more than five quarters in early 2000s to around three quarters by the end of the sample. Hence, our results not only suggest a declining trend for the unemployment rate, but also more churning in the labor market implying faster adjustments in response to cyclical changes in the

unemployment rate.

### 3.2 Results for Variable Labor Force Model

Next we turn to the case with variable labor force participation, which is our main focus. We begin with the descriptive statistics for flow rates under the assumption that measurement takes into account variation in the labor force participation over time. Table 1 shows the average levels of flow rates for both cases; with constant and varying labor force assumptions. We observe that relaxing constant labor force assumption affects both the levels and the standard deviations of flow rates.

Results for the estimation of the extended model with participation are displayed in Table 3. Some of the individual parameter estimates lose significance, however, overall the model is preferable to the one with these parameters excluded and to the model with no participation, as the improvement in log-likelihood is significant. Contrary to the stochastic growth rate for the output trend, labor force participation indeed has a time-varying growth rate in its trend. Consistent with the cyclical behavior of  $F$  and  $S$ , we observe that  $\tau_1$  is positive while  $\theta_1$  is negative. We see that  $\tau_3$  is not independently significant, but the model is preferable to the one without  $\tau_3$ .

Our estimates of the model with varying labor force suggest that the impact on the unemployment rate could be substantial. Figure 4 plots the unemployment rate trend from the restricted model together with the estimated trend from the extended model of this section. According to our estimates, for most of the early part of the sample, the difference between two models are quite substantial, and the difference is smaller towards the end of the sample. For instance, we observe as much as a 2.5 percentage point difference between two trend estimates in the middle of the sample and 0.5 percentage point difference at the end of the sample period.

To better understand the effect of including variable labor force, first notice through equation 3 that including flows from inactivity has positive effect on the steady state unemployment rate. However, the increase in unemployment rate due to flows from in-

activity per se is small (see Figure A.11). As flow rate from inactivity to unemployment is high at the earlier episodes of the sample, we observe a larger contribution from the second term of equation 3 compared to the latter part.<sup>17</sup> The major difference across the unemployment rates with constant vs. variable labor force stems from the fact that including participation in the model affects the estimated trend values of the flow rates. As Figure A.12 shows, although modeling the flows from inactivity does not affect the estimated flow rates much, it affects the trend estimates of flow rates.

Figure 5 displays all of the important unobserved components for the extended model with variable labor force participation rate. Even though the implied trend estimates for  $F$  and  $S$  change somewhat, results confirm the secular trends we obtained from the restricted model. More importantly, the participation rate trend implied by the estimation (right figure in middle panel) shows that there has been an important trend growth change. The participation rate has been growing in Turkey over this period, and our model identifies part of this as a trend increase. This is not unlike the behavior in the U.S. where participation hardly responds to the business cycle, if at all. Taken together, the convergence rate now reflects the added impact of an increasing growth rate in the labor force participation, which is pictured in the lower panel.

## 4 Discussion and Robustness

We have proposed and estimated a natural rate for Turkey using a relatively parsimonious model purely relying on the flow rates in and out of unemployment. We view this concept in line with Tasci (2012) and perceive it as the steady state unemployment rate that is implied by the current trend estimates of the flow rates. Practically, this means that it is the rate of unemployment in the long-run, to which the actual unemployment rate would converge.

This view comes as a stark contrast to the alternatives that the literature focuses on,

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<sup>17</sup>Our measurement of  $A_t$  implies a level of 0.0016 at the beginning of the sample, tumbling later by more than 75 percent over the next 12 years, most of which happened in the first five quarters.

such as Gordon (1997) and Staiger et al. (1997, 2001). These studies are concerned with a natural rate concept that relates price pressures to a level of unemployment that is consistent with constant inflation rate. As we argued in the introduction, there were some structural changes in the case of Turkey, that renders such a concept uninformative. In this section, we address this issue and compare our estimates to some alternatives, including a NAIRU. Furthermore, we address some of the robustness issues of the underlying estimation we employed, such as the normalization implied by  $\gamma_s$  and  $\gamma_f$ , as well as the exclusion restrictions for the early part of the sample in the maximum likelihood estimation.

## 4.1 Alternative Natural Rates and Filters

In this section, we present a basic comparison between our measures of the natural rate and some alternatives proposed in the literature. One of these alternatives is a NAIRU. One can also take a different approach and use an unobserved components method without using the flow rates, but instead focusing on the unemployment rate. We will refer to this alternative as the bivariate unobserved components model with unemployment rate (UC-UR). Finally, we will also address whether purely statistical filters could be good substitutes for our proposed natural rate. We view our approach as an alternative that solely relies on the data from labor markets and the real economy in determining long-run trend for the unemployment rate. There is a widespread use of similar terminology in the literature, as nicely discussed in Rogerson (1997). Getting into the details of this discussion will be out of the scope of this paper.

The NAIRU estimation takes a simple form, relating the current inflation to lagged inflation and the “unemployment gap” (Gordon (1997)), using quarterly changes in headline CPI at an annualized rate for the measure of inflation.<sup>18</sup> The bivariate model we have in mind is similar to the flow model, but only uses data on the actual unemployment rate and

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<sup>18</sup>More specifically, we assume that,  $\pi_t = \beta_\pi \pi_{t-1} + \beta_u [u_t - \bar{u}_t] + \varepsilon_\pi$ , where  $\pi_t$  and  $u_t$  denote actual inflation and unemployment rate, respectively. The natural rate,  $\bar{u}_t$ , follows a random walk, whereas the “unemployment gap”,  $u_t^c = u_t - \bar{u}_t$ , is assumed to follow an AR (2) process;  $u_t^c = \theta_1 u_{t-1}^c + \theta_2 u_{t-2}^c + \varepsilon_u$ .

real output as in Clark (1987, 1989) and Kim and Nelson (1999).<sup>19</sup> In both frameworks, one can use the Kalman filter to infer the unobserved trends in the unemployment rate much like we do for the unobserved trends in the flow rates. Our comparison relies on these unobserved trends, which are interpreted as alternative natural rates.<sup>20</sup>

Figure 6 presents these alternatives along with the flow-based estimates of the natural rate from the restricted and the extended models. Both estimated NAIRU and UC-UR are almost constant at around 10.5 percent over the entire sample period.<sup>21</sup> There is virtually no variation at all.<sup>22</sup> For NAIRU, it is very easy to understand why this is the case. Turkey experienced a sharp drop in the consumer inflation over the early part of the sample period, caused by the aggressive efforts by the newly independent central bank that effectively instituted an inflation target. This will undoubtedly affect the statistical relationship between inflation and the unemployment rate, that any NAIRU estimate will rely on. Inflation tumbled from levels of more than 60 percent per year to single digits in a relatively short period, while unemployment only increased modestly and stayed at those levels for some time. This, in turn, renders the relative variation in inflation with respect to unemployment uninformative. Thus, we obtain a flat NAIRU. It is important to keep in mind that we do not restrict the unobserved NAIRU to be constant in our empirical estimation.

The bivariate model, UC-UR, also implies an almost constant natural rate over our sample period. This model exploits the variation in the observed unemployment relative to the cyclical changes in the real GDP to identify the natural rate. First, we observe that there are two major episodes of business cycle contractions in our sample; the first one

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<sup>19</sup>Output is modeled as in equation (9). The observed unemployment has cyclical and trend components such that the trend component follows a random walk and the cyclical component depends on the cyclical component of the real output, much like the flow rates.

<sup>20</sup>Both alternative models are estimated using maximum likelihood estimation and results are available upon request.

<sup>21</sup>Though our NAIRU estimation assumes time-invariant parameters, we do not restrict the natural rate itself to be constant over time.

<sup>22</sup>Us (2014) estimates NAIRU for Turkey using time-variant parameters, her findings is inline with our estimations under the constant labor force assumption.

within the first year of the sample by 6 percent and the second one coinciding with the global recession by about 15 percent.<sup>23</sup> Even though the output contractions were significantly different, unemployment rate increases were almost identical, by about 70 percent, in both episodes. Moreover, the unemployment rate did not decline at all following the first recession, demonstrating a lot of persistence.<sup>24</sup> These factors imply a constant natural rate in the UC-UR case. Our method, on the other hand, can address the persistence in the unemployment rate without implying a constant natural rate since we focus on the underlying flow rates, thereby easily accommodating the non-linearities.<sup>25</sup>

One might argue that if our objective is to derive an empirically useful unemployment rate trend, a pure statistical unemployment rate trend might be more practical, if unemployment flows do not seem to provide us with any additional information. In order to address this issue, we focus on different statistical filtering methods with and without unemployment flows to distinguish the role they play. For the sake of exposition, we focus on the restricted model.

Taking an HP-filter of the unemployment rate itself has been one approach used in the literature to identify a trend for the unemployment rate in the context of the natural rate debate (see Rogerson (1997)). We compare our estimate of the long-run trend for the unemployment rate with those that could be obtained using an HP or a bandpass filter. Figure 7 presents the results of this exercise. When we omit the information on unemployment flows and filter the quarterly unemployment rate (top panel), we find a lot of variation in the trend and significant diversion across different filters. For instance, applying an HP-filter with a high smoothing parameter (1600) gives a relatively smooth trend that moves closely with the preferred trend from the flow model. However, a band-

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<sup>23</sup>Note that the first recession actually started right before the beginning of our sample, in 2000:Q4, with an overall peak-to-trough decline of 10 percent in real GDP.

<sup>24</sup>Please see Ceritoğlu et al. (2012) for more on the comparison of the unemployment in two recessions.

<sup>25</sup>Tasci (2012) also compares a variant of our baseline model with flows to these alternatives on some other dimensions, such as the precision of estimates, required retrospective revisions with additional data, and prediction accuracy for inflation and concludes that the flow-based approach has several desirable properties along those dimensions as well.

pass filter or an HP-filter with a smaller smoothing parameter (98) produces much more variation in the trend. The top panel also shows the well-known problem related to the end points of the sample in one-sided filters.

A relatively different picture emerges if we include information on unemployment flows and impute an unemployment rate trend, as we did in the paper, based on the trends of these underlying flows. As the lower panel of Figure 7 shows, unemployment trends imputed this way do not vary much across different filters and are much smoother than the trend estimates based solely on unemployment rate information. Moreover, the flow model, which puts a lot more structure on the comovement of flows and real output, produces a trend that moves closely with these other filters. We interpret this result as evidence of the importance of unemployment flows in understanding the unemployment rate trend over the long run. The obvious discrepancy between various estimates of the trend with different filters when flows data are ignored makes it harder to get an empirically consistent, and otherwise useful measure.

## 4.2 Robustness of the Estimation

When computing our estimate for the trend unemployment rate, we rely on equation (15) where we substitute the HP filter of the variables  $A_t$  and  $\rho_t$ . We resort to this solution because of the data availability, but we are also mindful of its potential impact on our results. Therefore, we conduct a robustness check where we model the process that governs  $A_t$  and  $\rho_t$  in a linear AR process and analyze the effect on the trend unemployment. Note that this exercise still confines to the same model with participation but the process that determines the trend components of  $A_t$  and  $\rho_t$  are assumed to be a product of a process different from a basic HP filter. The actual estimate of the trend we back out assuming AR processes yields virtually the same result. We do not report them separately to save space here<sup>26</sup>.

In principle, the results of our estimation could be sensitive to the exact values of  $\gamma_f$

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<sup>26</sup>Results are available upon request.

and  $\gamma_s$  that we use. In the benchmark estimation, we use values of 0.75 for both. These parameters control the relative variation in the cyclical components of the flow rates with respect to their estimated trends. Hence, it is reasonable to have different implied unemployment rate trends with different values. To pin down the exact numbers, we follow the approach proposed in Tasci (2012). This essentially means that we re-estimate the model over a fine grid for both  $\gamma_f$ , and  $\gamma_s$ ;  $\gamma_f = \{0.25, 0.375, 0.5, \dots, 3.375, 3.5\}$  and  $\gamma_s = \{0.5, 0.625, 0.75, \dots, 3.875, 4\}$ . We target two moments to match: one is the maximum log-likelihood over this combination of points, the other is the maximum correlation between the implied natural rate from the estimation and the trend of the observed unemployment rate, calculated using a bandpass filter. Since we do not use the actual unemployment rate in the estimation, we are trying to impose some discipline on the estimation by not letting it diverge too much from the data.<sup>27</sup> The objective here is to maximize the likelihood of the model without getting an implied unemployment trend that is far from a statistical trend obtained by the bandpass filter. We implement this robustness check exercise using the restricted model with constant labor force. The effects on the log-likelihood and the overall objective function responds the same way in the extended model.

Figure 8 shows how these two moments change across  $\gamma_f$  and  $\gamma_s$ . The preferred benchmark values maximize the objective of high log-likelihood and high correlation, which is clear from Figure 8. For instance, we do not improve the likelihood of the model for higher values of  $\gamma_f$ , whereas smaller values do not result in any reduction. The likelihood value seems more concave in  $\gamma_f$ , and the preferred value of 0.75 is close to its global maximum. As  $\gamma_s$  declines, the trend of the separation converges to a straight line; hence, the natural rate will be determined more by the trend of the job-finding rate. The opposite is true when  $\gamma_f$  is small and its trend is close to a straight line. Hence, when one flow has a constant trend imposed (low  $\gamma_i$ ), and the other flow has a very small cyclical variation (high  $\gamma_j, j \neq i$ ), we miss the low-frequency movements in the observed unemployment rate by

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<sup>27</sup>Note that with the flow rates themselves, the unemployment rate does not give any more information for our model, hence, it is not part of it.

a significant margin. Any increase in  $\gamma_s$  sharply reduces the correlation of the statistical filter with the trend estimate to the extent that the correlation potentially changes sign. The objective function determines the optimal trade-off between these two dimensions by putting more weight on the more informative moment, that is, by using the inverse of the covariance matrix as the weighting matrix. Finally, for almost all of the values of  $\gamma_f$  and  $\gamma_s$ , the natural rate implied by the model varies between 9.5 percent and 11 percent at the end of the sample.

Another robustness issue arises with respect to the exclusion restrictions. Recall that, since we model most of the trend variables as random walks, we had to start with a diffuse prior for the Kalman filter. The impact of the diffuse prior sometimes can be substantial for the first few periods, as the Kalman filter does not converge on a reasonable unconditional variance for the unobserved states. This is usually handled by ignoring the initial several periods in the actual estimation - by not considering its contribution to the log-likelihood. Since we have a very short sample, this might be somewhat tricky and we are concerned about potentially losing useful information that the Kalman filter can infer from the likelihood function for the initial data points, which in this case coincide with a recession. The tradeoff is between losing valuable information from the first several quarters versus getting potentially noisy estimates for the unconditional variance due to the diffuse prior.

In order to address this, we have re-estimated the model several times, each time excluding a larger number of quarters from the initial part of the sample. Our results suggest that after 8 quarters, the estimates for the unconditional variance behave well. Figure 9 plots the estimated natural rates corresponding to each exclusion case and shows that with the exception of the excluded part of the sample, our results do not change much. Estimated parameters reported in Table 3 correspond to the case where the likelihood function ignores the first 8 quarters. Note that this does not mean that the smoothed unobserved variables we present do not include them. They include the first 8 data points,

but the parameter estimates are only estimated using the rest of the data.

## 5 Forecasting Performance

Using flow rates provides us with a measure of the natural rate for the Turkish economy, which in turn can help policymakers gauge the extent of the labor market slack. Beyond providing a simple way to measure the unemployment rate trend in a theoretically meaningful way, another useful feature of this framework has recently been highlighted by Meyer and Tasci (2013): its forecasting accuracy. Meyer and Tasci (2013) argue that by essentially disciplining the long-run trends with the unobserved components method, this modeling framework does a remarkable job in forecasting the evolution of the unemployment rate in the short- and medium-run. Since the framework heavily relies on the flow rates more than the unemployment rate itself, it is especially very flexible in capturing the non-linearities around the turning points in the business cycle. We suspect that this is even more of a concern for Turkey, where reallocation rates are much lower relative to U.S. levels. Moreover, the absence of high frequency, timely information about the unemployment rate provides the necessary motivation to come up with a good forecasting framework for Turkey<sup>28</sup>.

To evaluate the forecast performance of the framework, we estimate both the baseline model and the extended version with the participation rate over time starting from 2007 fourth quarter and repeating the exercise for every quarter until the end of 2013. For every estimation sample, we produce two-period ahead forecasts for the unemployment rate using the predicted flows and the observed initial condition for the unemployment rate. Note that the models produce forecasts of the flow rates internally. However, we rely on the respective equation of motion for the unemployment rate, that is equations (A.23) and (2). In order to gauge forecasting performance of the framework, we report one-period and two-period ahead root mean squared forecast errors (RMSFE) relative to

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<sup>28</sup>Turkish Statistical Institute only releases unemployment rate data with more than two months of lag.

those generated from a simple time series process for the measured unemployment rate. In particular, we choose an AR(2) process.<sup>29</sup> It is important to remember that we are not running this numerical exercise with real-time data. Given the changes in the data collection and methodology over the sample period and the sheer length of the data span (or lack thereof), repeating this experiment in real time seems like a futile effort.

Table 4 reports RMSFEs for one and two-quarter ahead forecasts from the two models we used in the paper and the AR process that does not rely on flow rates at all. As forecast errors suggest, both models produce more accurate unemployment rate forecasts relative to the time series model for the forecast sample period we considered, especially at one-quarter ahead forecast horizon. This relative improvement in forecast accuracy over the near-term could provide a useful tool for policymakers in Turkey.

Having established a relative improvement in forecasting the unemployment rate with the unobserved components models we used in the paper, we finally provide the predictions of them conditional on the data we have for the whole sample; 2001:Q1-2013:Q4. Figure 10 presents the forecast paths for the extended model as well as the restricted model with constant labor force. Regardless of the model we use, we predict a gradual decline in the unemployment rate beyond 2013. Recall that the model with participation implies a lower natural rate in the long-run, therefore yielding a lower path beyond 2015 relative to the restricted model. Unfortunately, Turkish Statistical Institute implemented a methodological change beginning in 2014, which made the data prior incomparable. Nevertheless, the first quarter data which is available confirms the forecast qualitatively, pointing a decline in the aggregate unemployment rate.

## 6 Conclusion

We use a parsimonious unobserved components model with unemployment flow rates, similar to the one used by Tasci (2012) for the U.S., to estimate a time-varying unem-

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<sup>29</sup>The AR process we assume takes the form  $u_t = \kappa_1 u_{t-1} + \kappa_2 u_{t-2} + \epsilon_t^u$ , where data is quarterly.

ployment rate trend for Turkey that is grounded in the modern theory of labor market search. We believe that the specific challenges presented by the Turkish data makes it a compelling case to consider. One of these challenges was the importance of the participation rate behavior, which we handled by extending the basic model to incorporate time-varying labor force participation.

Our results suggest that by the end of 2013, the natural rate for unemployment, or the underlying trend, is hovering around 9 percent for Turkey. Models with and without the participation margin imply substantially different estimates at the earlier parts of the sample period and the gap narrows over time, with the extended model featuring the participation rate predicting a level slightly below 9.5 percent. This is due to a slow down in the rate of flows from inactivity to unemployment. More importantly, we find that the reallocation rate, the sum of the inflow and outflow rates, has been gradually trending up for Turkey suggesting an increasingly dynamic labor market. Finally, we argue that the modeling framework we provide here can be used for near-term forecasting of the unemployment rate with relative ease and accuracy.

We are mindful of the main caveat of our paper: the sample size. Our data covers only 13 years at quarterly frequency. However, the fact that we have quite a bit of variation in the variables of interest over the sample period reassures us that the lack of longer time-series data does not undermine the usefulness of our approach. In future work, it would be interesting to focus on understanding the secular increase in the reallocation rate over time.

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Table 1: Flow Rates

	u	F	S	A
Changing Labor Force	0.105 (0.014)	0.087 (0.022)	0.010 (0.004)	0.001 (0.0003)
Constant Labor Force		0.089 (0.022)	0.011 (0.003)	- -

Note: Standard deviations are in parentheses.

Table 2: Estimation Results: 2001:Q1-2013:Q4

	Estimate	Std		Estimate	Std
$\phi_1$	1.2542	(0.2026)	$\sigma_{yn}$	0.0169	(0.0022)
$\phi_1$	-0.5112	(0.1735)	$\sigma_{yc}$	0.0117	(0.0030)
$\tau_1$	0.2023	(0.0935)	$\sigma_{fn}$	0.0037	(0.0005)
$\tau_2$	0.1129	(0.0911)	$\sigma_{sn}$	0.0007	(0.0002)
$\tau_3$	0.0531	(0.0775)	$r$	0.0120	(0.75 * 10 <sup>-5</sup> )
$\theta_1$	-0.1346	(0.0338)			
$\theta_2$	0.1028	(0.0415)			
$\theta_3$	-0.0264	(0.0224)			

Note: Standard deviations are in parentheses. Log-likelihood is 492.1310,  $\gamma_f = 0.75$ , and  $\gamma_s = 0.75$ .

Table 3: Estimation Results: 2001:Q1-2013:Q4

	Estimate	Std		Estimate	Std
$\phi_1$	1.6115	(0.0960)	$\mu_1$	-0.8547	(0.4787)
$\phi_1$	-0.7934	(0.0982)	$\mu_2$	0.4211	(0.4415)
$\tau_1$	1.7549	(0.9046)	$\mu_3$	0.1300	(0.2814)
$\tau_2$	-0.8706	(0.7672)	$\sigma_{yn}$	0.0214	(0.0023)
$\tau_3$	-0.2784	(0.4064)	$\sigma_{yc}$	0.0019	(0.0010)
$\theta_1$	-0.8100	(0.3994)	$\sigma_g$	0.0004	(0.0010)
$\theta_2$	1.2852	(0.6756)	$\sigma_{pn}$	0.0041	(0.0005)
$\theta_3$	-0.5556	(0.3266)	$\sigma_{fn}$	0.0033	(0.0004)
			$\sigma_{sn}$	0.0008	(0.0001)

Note: Log likelihood is 659.17. Standard deviations are in parentheses.  $\gamma_f = 0.75$ , and  $\gamma_s = 0.75$

Table 4: Forecast Performance: RMSFEs for 2009:Q1-2013:Q4

	AR (2) in UR	Restricted Model	Extended Model
$t + 1$	0.6447	0.5089	0.5040
$t + 2$	1.1001	0.9477	0.9784

Figure 1: Unemployment and Participation Rates (in %)

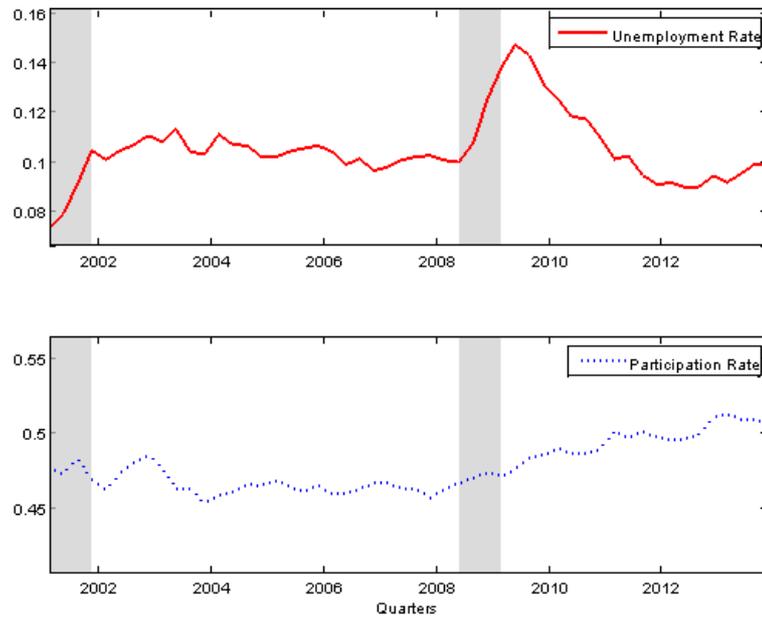
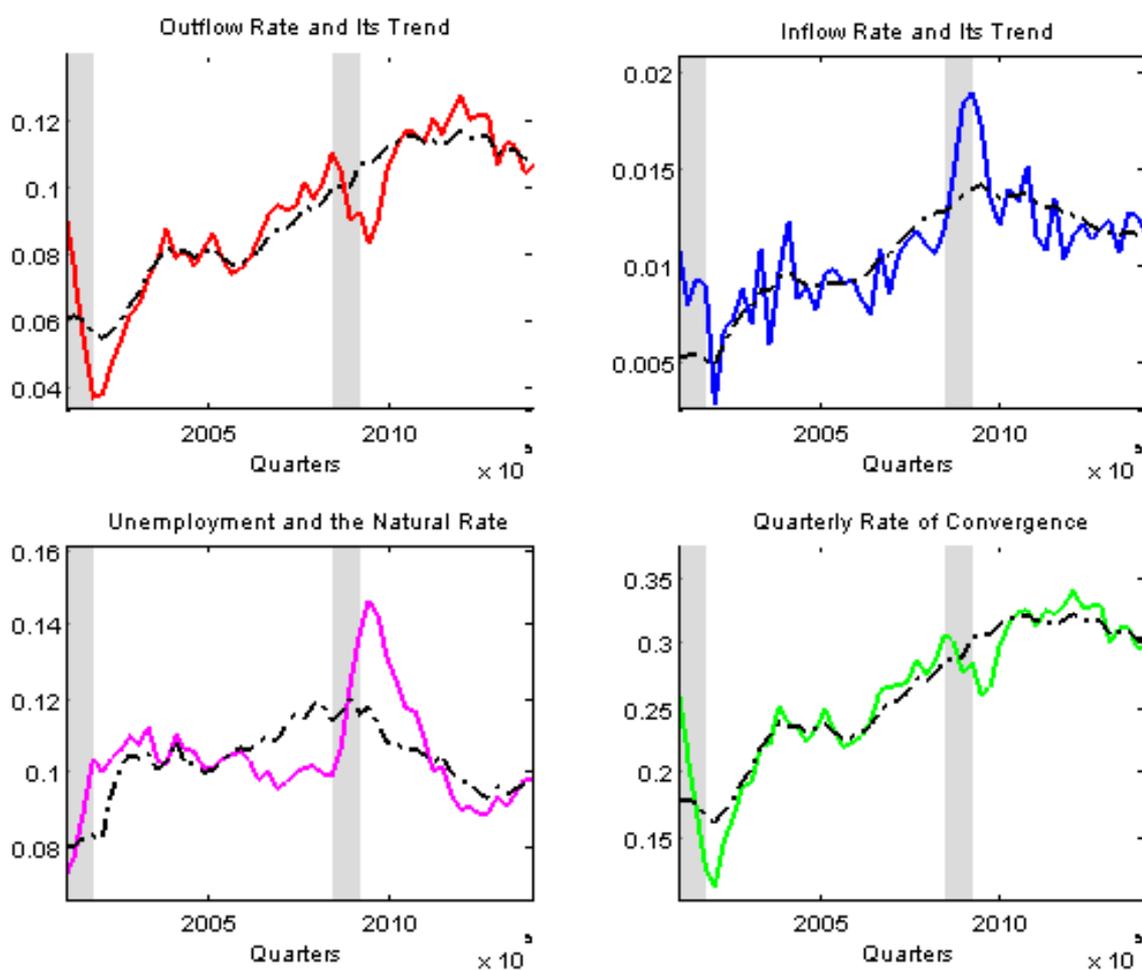


Figure 2: Unemployment Entry from Inactivity and Participation Growth Rates (in %)



Figure 3: Estimation Results (Constant Labor Force)



Note: Dashed lines are trend and solid lines are actual data.

Figure 4: Unemployment Rate Trends - Impact of the Variable Participation

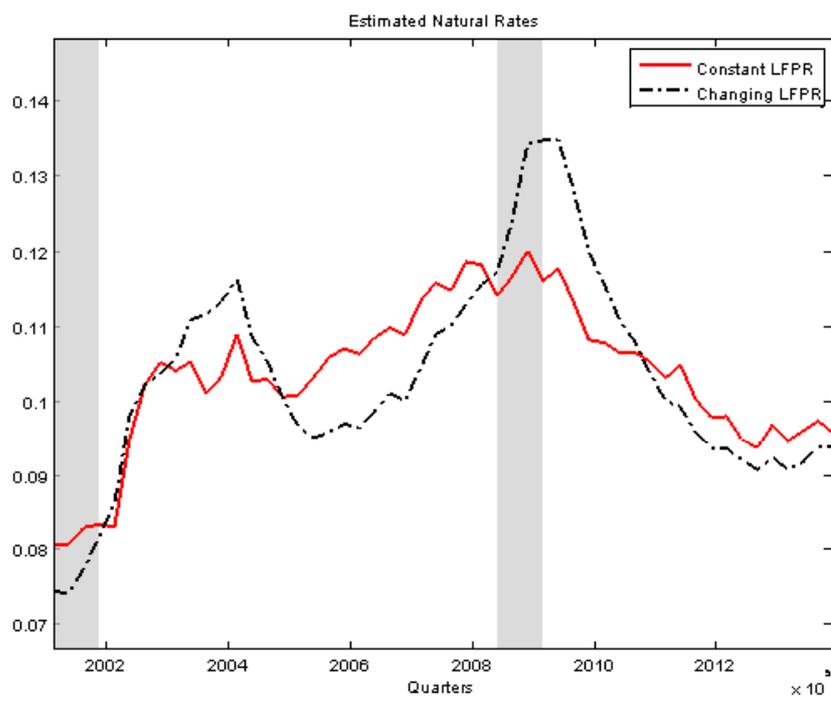
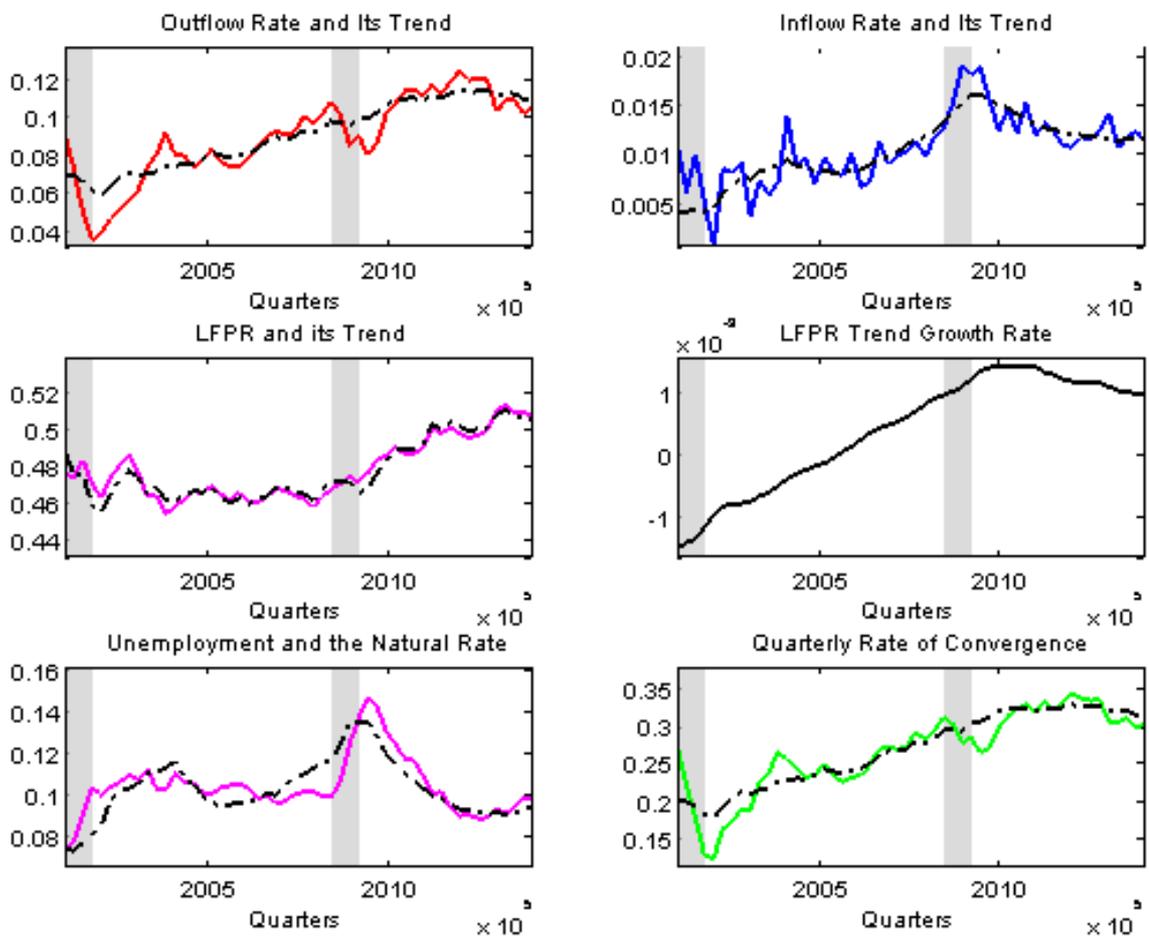


Figure 5: Estimation Results (Variable Labor Force)



Note: Dashed lines are trend and solid lines are original series.

Figure 6: Alternative Natural Rates

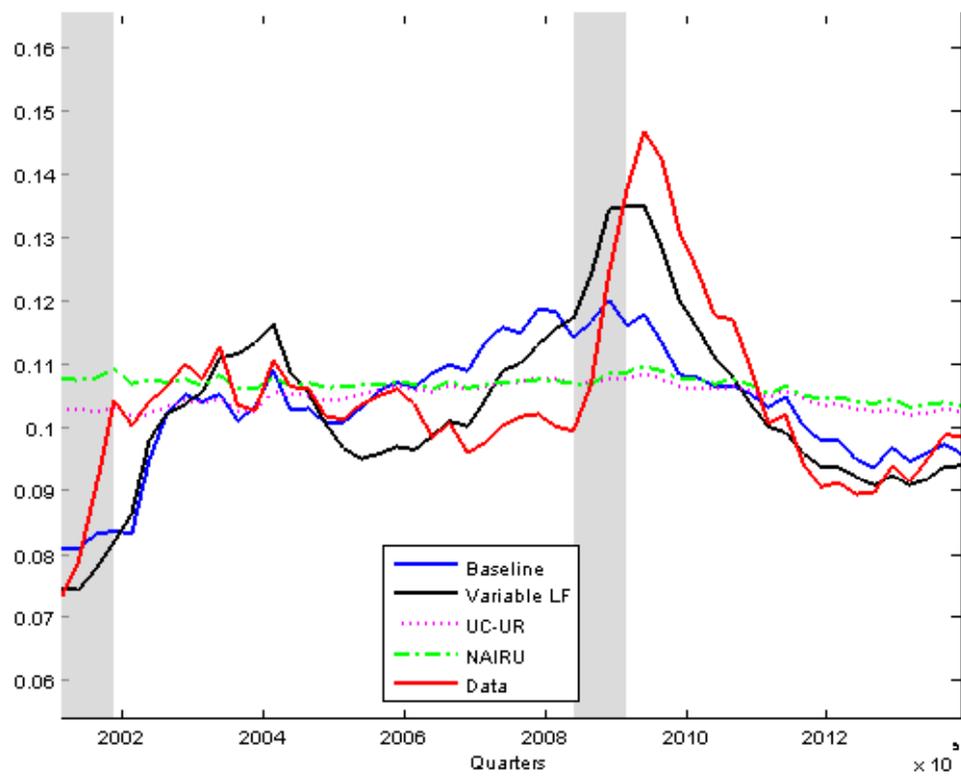


Figure 7: Alternative Filters - The Role of Flows

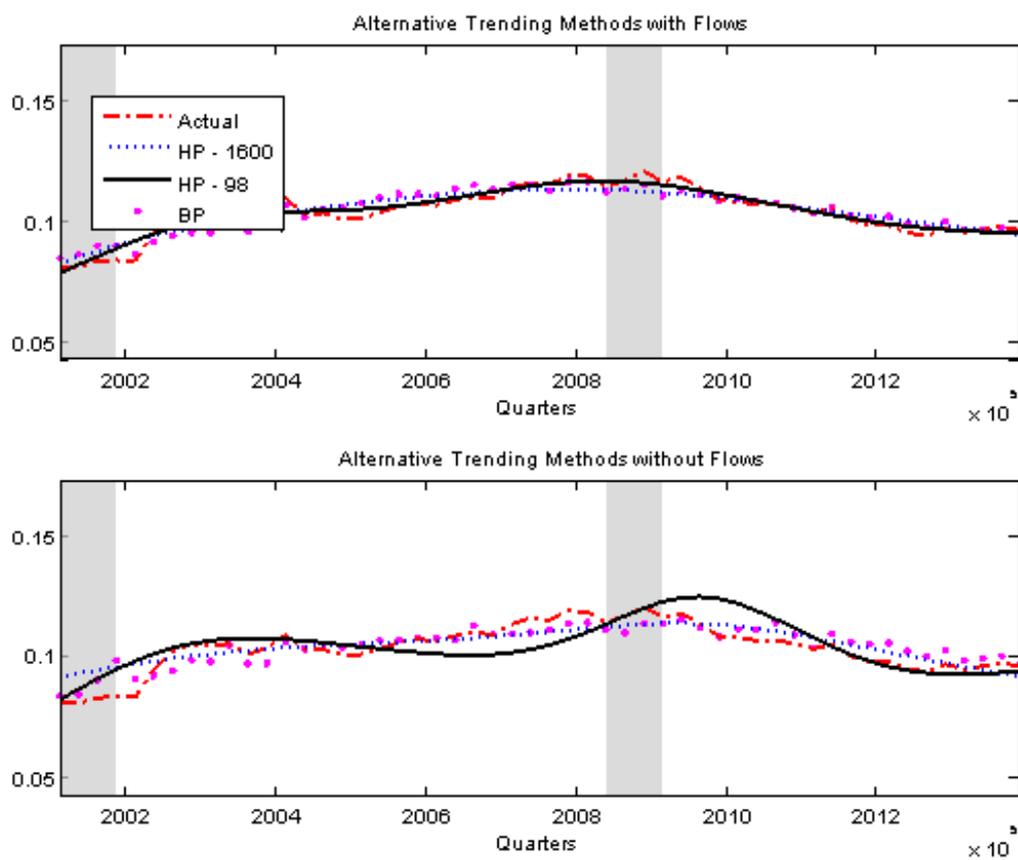


Figure 8: Robustness for  $\gamma_f$ , and  $\gamma_s$

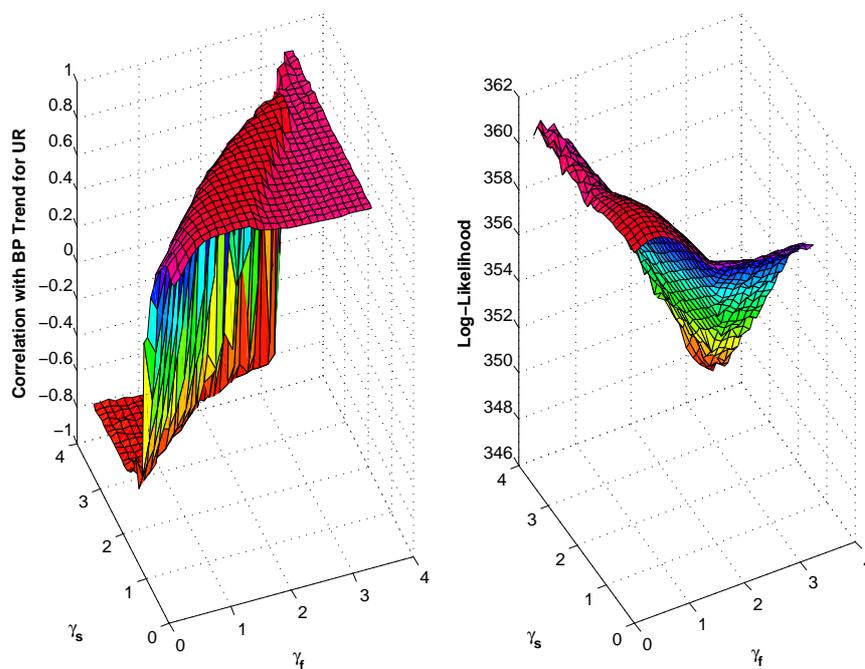


Figure 9: Robustness for Exclusion Restrictions

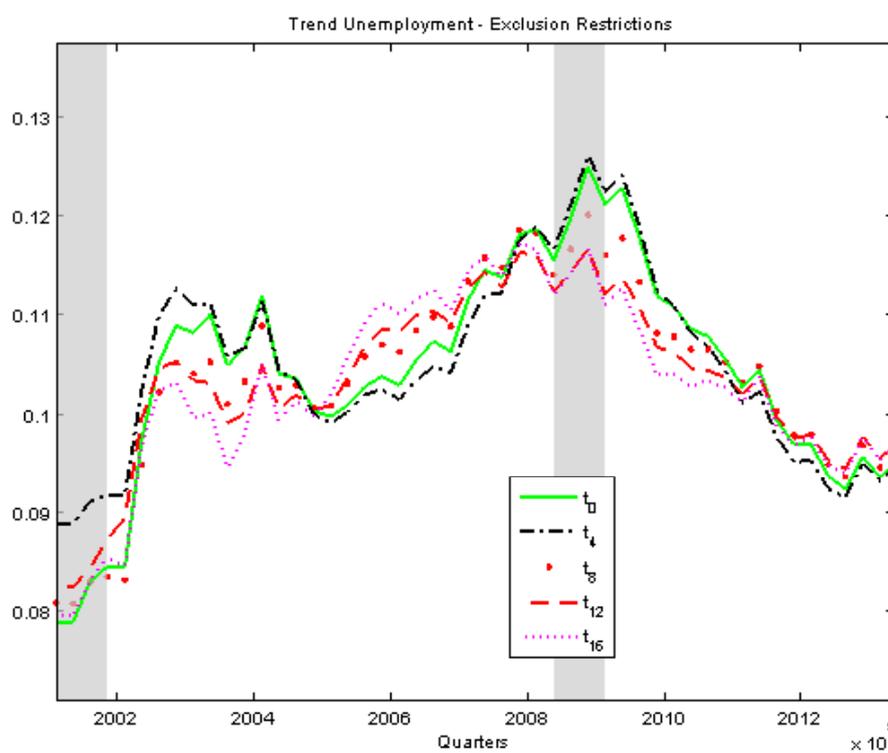
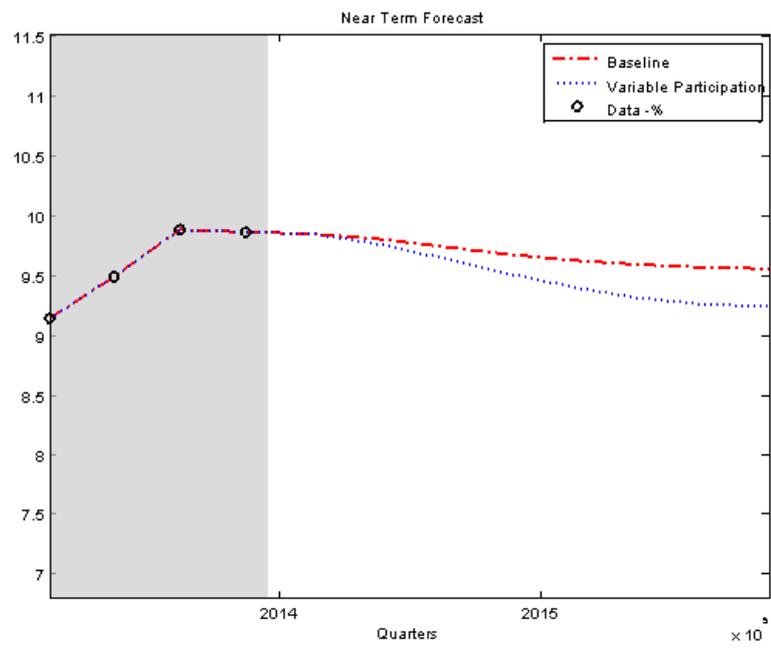


Figure 10: Forecasting Performance of Both Models



## A Appendix

In this section we lay out the model which abstracts from the variation in the labor force in detail. This is essentially nested in the benchmark model we present in the main text with the restriction that  $\rho_t = 0$ ,  $G_t = 0$  and  $A_t = 0$ .

**Flow Rates in Restricted Model:** Let us start with the flow rates. Since there is no change in the labor force, all flows are between unemployment and employment in this restricted model. Recall that we need the law of motion for unemployment and the short term unemployment to compute the flow rates. The law of motion for unemployment becomes:

$$\dot{U}_{t+\tau} = (L_{t+\tau} - U_{t+\tau})S_t - U_{t+\tau}F_t. \quad (\text{A.16})$$

Note that the only difference between the equation above and equation 1 is that the former lacks the term with  $A_t$ . The law of motion for short-term unemployed, unemployed for less than five weeks is:

$$\dot{U}_t^{<1}(\tau) = (L_{t+\tau} - U_{t+\tau})S_t - U_t^{<1}(\tau)F_t. \quad (\text{A.17})$$

As mentioned earlier, this equation is not affected by the assumption regarding labor force directly, it is the same as equation 4. However, here  $F$  is the job finding rate.

Subtracting equation (A.17) from equation (A.16) yields:

$$\dot{U}_{t+\tau} = \dot{U}_t^{<1}(\tau) - (U_{t+\tau} - U_t^{<1}(\tau))F_t. \quad (\text{A.18})$$

Solving the differential equation above provides us with a simple measurement equation for the outflow hazard:

$$u_t = e^{-F_t}u_{t-1} + u_t^{<1}, \quad (\text{A.19})$$

where  $u_t$  denotes the unemployment *rate* in period  $t$ . This equation is the same as 6 with  $\rho_t = 0$  and  $G_t = 0$ .

Regardless of the assumption on labor force, if unemployment exit occurs with a Poisson process with parameter  $F_t$ , then the probability of exiting unemployment within a month is  $\hat{F}_t = 1 - e^{-F_t}$ . Therefore, equation (A.19) can be rewritten as

$$\hat{F}_t = 1 - \frac{u_t - u_t^{<1}}{u_{t-1}}. \quad (\text{A.20})$$

The monthly outflow probability relates to associated monthly outflow hazard rate,  $F_t^{<1}$ , through the following equation:  $F_t^{<1} = -\ln(1 - \hat{F}_t)$ .

We rely on additional duration data to estimate  $\hat{F}_t$ . Based on the unemployment data by duration, we can calculate the probability that an unemployed worker exits unemployment within  $d$  months as

$$\hat{F}_t^d = 1 - \frac{u_t - u_t^{<d}}{u_{t-d}}. \quad (\text{A.21})$$

As before, we can calculate the outflow rates as

$$F_t^{<d} = -\ln(1 - \hat{F}_t^d)/d, \quad (\text{A.22})$$

for different durations,  $d = 1, 3, 6, 9, 12$ . We follow the procedure described in section 2.1 to estimate  $\hat{F}_t$ .

Solving equation (A.16) and iterating it three months, we get the evolution of unemployment rate in the data, observed in discrete intervals, as:

$$u_t = u_{t-3}(1 - \lambda_t) + \lambda_t \frac{S_t}{S_t + F_t}, \quad (\text{A.23})$$

where  $\lambda_t = (1 - e^{-3(S_t + F_t)})$  is the quarterly convergence rate. Note that this is the original equation of Elsby et al. (2013). Solving this equation for the steady state leads to the definition of the *flow* steady state unemployment as follows

$$u_t^{ss} = \frac{S_t}{S_t + F_t}. \quad (\text{A.24})$$

**Unobserved Components in Restricted Model:** Having the flow rates, we now turn to unobserved components model. Note that modeling labor force does not affect the process the real output follows. It is given by equation 9. Furthermore, the time series behavior of  $F$  and  $S$  are also not directly affected from the assumption regarding the labor force, and hence are given by 10 and 11, respectively. However, note that now interpretation of  $F$  is different.  $F$  is the unemployment exit rate (which could be exit to employment or to inactivity) in the benchmark model while  $F$  in restricted model is job finding rate as all exits from unemployment must go to employment. Since we have  $\rho_0$ ,  $G_t = 0$ , and  $A_t = 0$  we are left with no other flow rate.

We can express the empirical in a convenient state-space representation as

$$\begin{bmatrix} Y_t \\ F_t \\ S_t \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tau_1 & \tau_2 & \tau_3 & 0 & 1 & 0 \\ 0 & \theta_1 & \theta_2 & \theta_3 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{y}_t \\ y_t \\ y_{t-1} \\ y_{t-2} \\ r_t \\ \bar{f}_t \\ \bar{s}_t \end{bmatrix} + \begin{bmatrix} 0 \\ \varepsilon_t^{fc} \\ \varepsilon_t^{sc} \end{bmatrix}, \quad (\text{A.25})$$

$$\begin{bmatrix} \bar{y}_t \\ y_t \\ y_{t-1} \\ y_{t-2} \\ r_t \\ \bar{f}_t \\ \bar{s}_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \phi_1 & \phi_2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{y}_{t-1} \\ y_{t-1} \\ y_{t-2} \\ y_{t-3} \\ r_{t-1} \\ \bar{f}_{t-1} \\ \bar{s}_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{yn} \\ \varepsilon_t^{yc} \\ 0 \\ 0 \\ \varepsilon_t^r \\ \varepsilon_t^{fn} \\ \varepsilon_t^{sn} \end{bmatrix}, \quad (\text{A.26})$$

where all error terms come from an i.i.d. normal distribution with zero mean and variance  $\sigma_i$ , such that  $i = \{yn, yc, r, fn, fc, sn, sc\}$ .

As in the extended model with variable participation, we use the Kalman filter to filter the unobserved components and write the log-likelihood function to estimate the model via maximum likelihood. Once we estimate the model, we use the Kalman smoother to infer unobserved stochastic trend and cyclical components over time. These time-varying trend

estimates for the flow rates,  $\bar{f}_t$  and  $\bar{s}_t$ , determine the unobserved unemployment rate trend over time. More specifically, our definition of the long-run trend for the unemployment rate is given by

$$\bar{u}_t = \frac{\bar{s}_t}{\bar{s}_t + \bar{f}_t}, \quad (\text{A.27})$$

which is consistent with the search theory of the labor market.

Figure A.11: Unemployment Rate with Variable Labor Force

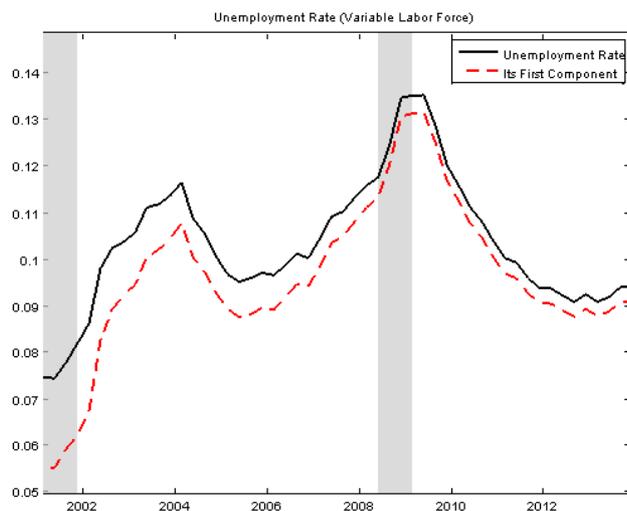


Figure A.12: Flow Rates of Different Models

