Global Currency Hedging with Dynamic Copulas*

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Abstract

We perform out-of-sample evaluations of portfolio risk measure forecasts calculated using several copula models that allow for time-varying dependence, assymetric dependence and tail dependence. The copulas we consider are those by Oh and Patton (2013b), and Christoffersen et al. (2012), and some benchmark copulas. We assess Value-at-Risk (VaR) and expected shortfall (ES) forecasts of several portfolios of a global equity investor who can hedge currency exposure, namely the unhedged, fully hedged, and minimal-ES portfolios. We find strong evidence that including time-varying dependence improves portfolio risk measure forecasts. Moreover, we find some evidence that asymmetric dependence, and tail dependence improve portfolio risk measure forecasts.

Keywords: financial risk measures, dependence, correlation, copula, forecast performance, dynamic models.

JEL-Classification: C32, C58, G01, G11, G15, G17.

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1 Introduction

The financial crisis has shown that dependence amongst financial asset returns is not adequately captured by normally distributed models. Dependence features that normal models cannot account for include tail dependence between assets, and asymmetric dependence. During the crisis, extreme co-movements provide evidence of tail dependence between financial assets, whereas evidence for asymmetric dependence is given by greater dependence between assets in times of economic distress. Many models also assume time-invariant dependence structures, whereas we observe persistently time-varying dependence in financial asset returns.

Copulas are compelling tools to incorporate these dependence features into an econometric model of returns. They allow us to model the dependence amongst returns seperately from their marginal behavior. A wide array of marginal models for financial returns series is available in the literature, such as many variants of the generalized auto-regressive conditional heteroskedasticity (GARCH) model (Bollerslev, 1986), which are conveniently summarized in Hansen and Lunde (2005). Therefore, we can focus our effort primarily on modelling the dependence structure. Moreover, copula models allow different amounts of tail-fatness for each marginal returns series, such that they are easily applied to returns datasets in which we observe large differences in tail-fatness amongst returns.

Recently, several multivariate copula models have been developed that can incorporate time-varying, asymmetric, and tail dependence, and can be applied to high dimensional financial data sets of more than 100 assets. In-sample evidence shows that including these features improves model fit significantly. Christoffersen et al. (2012) propose a cDCC copula in which they make the copula dispersion matrix time-varying using the cDCC framework of Aielli (2013). The authors apply their copula model to a large dataset of international equity returns, and find evidence of time-variation, fat tails and negative skewness in the copula model. Oh and Patton (2013a,b) propose a copula model based on a simple factor model, which also allows for time-variation, fat tails, and skewness in the dependence. It has a more parsimonious parameterization than the cDCC copula, and may thus be less exposed to estimation error. Time-variation is introduced using the GAS framework (Creal et al., 2013). In an application to 100 US credit default swap (cds) spreads they find strong evidence of time-variation and fat tails, some evidence for asymmetry, and no evidence for tail dependence. Using Bayesian estimation techniques, Creal and Tsay (2015) propose a time-varying copulas based on the state space framework with generalized hyperbolically distributed errors, which nest normal, t, and skew t errors, amongst others. Their application considers a dataset of 200 cds spreads and equity returns, for which they find evidence of fat-tails, and time-variation in the copula. In this research we will use the skew t, t, and Gaussian cDCC copula models of Christoffersen et al. (2012), the factor skew t copula of Oh and Patton (2013a,b), and the implicit Gaussian, and t copulas as benchmarks. We propose a small change to the parameterization of the cDCC skew t copula to allow for consistent estimation of all parameters.

The main purpose of this research is to evaluate out-of-sample performance of the previously mentioned copula models. As copula modelling, and dependence modelling in general, is mostly exploited for risk management, we assess out-of-sample performance based on the portfolio risk measure forecasts of the copula models. It is then the question which dependence features are of importance when forecasting portfolio risk measures. The two commonly used risk measures in finance are Value-at-Risk (VaR), and expected shortfall (ES). Consequently, we test for correct Value-at-Risk (VaR) and expected shortfall (ES) forecasts. Moreover, we test for pairwise equal performance of VaR forecasts.

VaR is a tail quantile of a returns distribution, and ES is the conditional mean for all returns below VaR. VaR thus denotes the loss which we do not expect to violate with a certain probability α, and ES is defined as the mean loss when we violate VaR. In an important paper, Artzner et al. (1999) show that VaR is not a coherent risk measure, with as a most serious consequence that it is not subadditive. This means that the VaR of a portfolio can be less extreme than the sum of the VaRs of the individual assets. Obviously, this is an an u unappealing feature. ES is a coherent risk measure and therefore does not have this pitfall. In light of this, in 2014 the Basel Committee on Banking Supervision stipulates that in the near future banks will be judged on their ES forecasts, at the $\alpha = 2.5\%$ level, instead of VaR forecasts (BCBS, 2014). Still, VaR and ES forecasts at different levels are meaningful statistics to banks to assess the risk profile of their asset holdings.

Backtesting of VaR has attracted much attention in the literature. Among the first papers to test correct coverage conditionally and unconditionally are Kupiec (1995) and Christoffersen (1998), whereas newer tests have for instance looked at encompassing VaR forecasts Giacomini and Komunjer (2005). An overview of VaR backtesting can be found in Komunjer (2013). We will utilise the Christoffersen tests, and use the Giacomini and White (2006) testing framework to compare VaR forecasts conditionally and unconditionally.

Expected shortfall backtests are less numerous, but examples are the unconditional tests by McNeil and Frey (2000), Berkowitz (2001), Kerkhof and Melenberg (2004), and Wong (2008). A recent paper by Du and Escanciano (2015) present both conditional and unconditional tests for ES. We will use these last two tests.

In our setup we consider the portfolio of a global, US-based investor who can hedge his currency exposure. He can invest in the stock markets of Australia, Canada, Germany, Japan, Switzerland, the UK, and the US. We follow Campbell et al. (2010) in that the currency hedge is obtained by taking long and short positions in foreign and domestic short rate instruments, which provides an accurate approximation to a hedge with currency forwards. We consider an unhedged portfolio, a fully hedged portfolio, and an optimally hedged portfolio, in which the portfolio ES is minimized. Therefore, to forecast VaR and ES, and to calculate optimal hedging weights, the investor needs to jointly model 7 equity index returns, and the 6 corresponding currency returns[1]. He will use the previously mentioned copula models to do so.

Our data sample contains weekly data from 30 May 1996 to 25 September 2014 (957 observations). Using a moving window, our investor re-estimates his econometric models each four weeks, and uses the models to compute VaR and ES forecasts for the different portfolios. He can then evaluate the 300 VaR and ES forecasts he obtained with the VaR and ES tests described previously.

We find strong evidence for the importance of time-varying dependence in both VaR and ES forecasts. In many cases the time-invariant implicit copulas are all strongly outperformed in a pairwise comparison with time-varying copulas. The time-invariant factor skew t model of Oh and Patton (2013a) is also beaten by time-varying copulas, but less often, and less strongly. Moreover, the cDCC skew t copula beats the cDCC t and Gaussian copulas in all cases, suggesting the importance of including skewness in the dependence structure. The tests on the accuracy of ES forecasts show that the time-invariant implicit copulas are strongly rejected, both conditionally and unconditionally. The time-varying t copula has poor performance for ES at the 2.5% confidence level, both unconditionally and conditionally. At this level there is also weak evidence of unconditional inaccuracy of the other time-varying cDCC copulas. The static and dynamic factor skew t copulas pass all tests. In fully hedged portfolios and constrained minimum-ES portfolios we cannot distinguish performance of VaR and ES forecasts. As these

[1]We only have 6 currency returns are the base currency for the exchange rates is the US, which is also one of the developed countries in our sample. Moreover, the short rates are assumed to be risk-free and thus do not need to be modelled.

portfolios are less exposed to currency returns, this may be due to the nature of the dependence amongst currency returns, and the dependence between currency and equity returns. Moreover, the small power of VaR and ES tests, due to few informative observations[2], may cause the these tests to be inable to distinguish between models.

The rest of the paper is as follows. We discuss our data in Section 2. In Section [Methods] we discuss the concept of currency hedging, the marginal models, the copula models, and their estimation, the risk measures, the construction of the minimum-ES portfolio, and the VaR and ES backtests. Results are given in Section 5, and we conclude in Section 6.

2 Data

Our dataset consists of weekly country equity market price indices, exchange rates, and short-term interest rates, for seven countries: Australia, Canada, Germany, Japan, Switzerland, the United Kingdom, and the United States. We use weekly observations to reduce the problem of non-synchronous trading hours in the equity markets across the different countries. All exchange rates concern the local currency against the US dollar, because we analyze currency hedging decisions from the perspective of a US-based investor. Concerning tbe exchange rate of the German currency with respect to the US dollar, we use the Deutsche Mark before the introduction of the euro (2002), and the euro thereafter.[3] We collect exchange rates from WM/Reuters. The equity market price indices are MSCI price indices. Short rates are either interbank rates or (secondary market) T-bill rates, depending on the availability of weekly data. Both types of rates are for instruments with a 3-month horizon.[4] All data are obtained from Datastream for a sample period covering 30 May 1996 to 25 September 2014 (957 observations). We work with weekly equity and exchange rate log returns by taking the first difference of the log transformations of these series.

Table 1 presents descriptive statistics of equity and exchange rate log returns. We detect serial correlation for half of our returns series, and find serial correlation in all squared returns series, which indicates heteroskedastic returns.

Table 2 shows the sample cross-correlations of the equity and exchange rate returns. We find

[2] Informative observations would be those in violation of the Value-at-Risk.

[3] To be more precise, in our case an exchange rate of 100 means 100 units of local currency equal the value of one dollar. We will thus denote the exchange rate between, for example, the Japanese yen and the US dollar as as JPY/USD.

[4] We use 3-month interbank rates for Germany, Japan, UK, and Switzerland. We use 3-month treasury bill rates for Australia, and Canada. We use the secondary market 3-month T-bill rates for the US.

the usual pattern of moderately high, positive correlation amongst equity returns. Moreover, we find positive correlation of different magnitudes amongst exchange rate returns. The EUR/USD, CHF/USD, and the GBP/USD are highly correlated, which can be explained by the European market. The AUD/USD and CAD/USD are also highly correlated, perhaps explained by their dependence on the US market. The JPY/USD is least correlated with other exchange rates.

Between exchange rates and equity returns we observe positive and negative correlation. We should thus allow for positive and negative dependence structures in our statistical models in later sections.

[Table 1 about here.]

[Table 2 about here.]

3 Global currency hedging

The dollar denominated return of a global equity portfolio of a US-based investor depends on the foreign equity returns which are denominated foreign currency, and the exchange rate returns of those foreign currencies against the US dollar. A positive return on a foreign equity position can for instance be offset by an appreciation of the US dollar against the foreign currency. An investor who wants to decrease her exposure to the exchange rate return, e.g. because she is a skillful stockpicker in foreign markets, but knows little of exchange rate movements, can set up a currency hedge.

Here we first describe the currency hedging problem of this global equity investor, and show why she needs a joint model of equity returns and currency returns to solve this.

To hedge her currency exposure, the investor can use currency forwards. Alternatively, he can approximate the currency hedge by taking long and short positions in foreign and domestic short rate instruments. Campbell et al. (2010) show that in continuous time, with lognormal returns, the two approaches are equivalent, and that at the monthly or higher frequencies the approximation is accurate.

The return on a portfolio of equities in M countries, at time t, denoted by $R_{p,t}$, and denoted in the domestic currency, can be written as

$$R_{p,t} = \sum_{i=1}^{M} w_{i,t} \left[R_{i,t} \frac{S_{i,t}}{S_{i,t-1}} \right] - k_{i,t} \left[(1 + I_{i,t}) \frac{S_{i,t}}{S_{i,t-1}} - (1 + I_{1,t}) \right], \qquad (1)$$

where $R_{i,t}$, $S_{i,t}$, and $I_{i,t}$ denote the locally denominated country equity return, the exchange rate defined as the number of dollars one can by for one unit of foreign currency, and the short rate at time t of country i, respectively. The home country of the investor is indexed by $i = 1$. The parameters $w_{i,t}$, and $k_{i,t}$, denote the equity portfolio weight, and the currency hedging weight of the investor specific to country i. In our application we set $\sum_i w_{i,t} = 1$. Essentially, $k_{i,t} > 0$ defines a self-financing portfolio consisting of a long position in the domestic short rate instrument, and short position of equivalent value in the short rate instrument of country i, and vice versa for $k_{i,t} < 0$. An appreciation of the foreign currency, i.e. $S_{i,t}/S_{i,t-1} < 1$, will decrease the dollar denominated return on the foreign equity position, but this will be offset by a similar decrease in the dollar denominated value of the repayment of the foreign short rate instrument. In similar fashion, a depreciation of the foreign currency, i.e. $S_{i,t}/S_{i,t-1} > 1$, will have an opposite effect on the investments.

When $k_{i,t} = w_{i,t}$ the portfolio is fully hedged with regard to currency i, whereas $k_{i,t} = 0$ represents an unhedged position. We thus define *net* currency exposures $m_{i,t} = w_{i,t} - k_{i,t}$ and use this quantity in what follows to be consistent with Campbell et al. (2010). The global currency hedge portfolio return can then be rewritten as

$$\begin{aligned} R_{p,t} = \sum_{i=1}^{M} w_{i,t} & \left[R_{i,t} \frac{S_{i,t}}{S_{i,t-1}} - (1 + I_{i,t}) \frac{S_{i,t}}{S_{i,t-1}} + (1 + I_{1,t}) \right] \\ & + m_{i,t} \left[(1 + I_{i,t}) \frac{S_{i,t}}{S_{i,t-1}} - (1 + I_{1,t}) \right], \end{aligned} \tag{2}$$

As an example we consider an investor who is fully invested in stock market i, that is $w_{i,t} = 1$. Then, $m_{i,t} = 0.30$ indicates that the investor decreases her exposure to the currency of country i. As such, the investor shorts 0.70 units of domestic bills for every unit invested in stock market i, and finances this using a long position in bills denominated in the currency of country i.

Our investor has an equally-weighted equity portfolio, i.e. $w_{i,t} = 1/M$, and considers unhedged, fully hedged, and optimally hedged currency exposures. We define optimal hedging based on the idea that the investor measures the risk of his portfolio using the risk measures Value-at-Risk (VaR) and expected shortfall (ES). Therefore, from a risk management perspective, he considers optimal-ES currency hedges. To forecast the VaR and ES of his portfolio, and to find optimal-ES hedging weights, the investor uses a joint model for the exchange rate and equity returns. This way he utilizes important dependence relations between the different asset returns. We do not need to model the short rates, as they observed one period in advance.

The multivariate models the investor considers are multivariate copula models that allow for skewness, extreme dependence, and time-variation in the dependence structure. These models are discussed in Section 4.

3.1 Value-at-Risk & Expected shortfall

The VaR and ES of portfolio return $R_{p,t+1}$, conditional on past information, are defined as:[5]

$$\mathrm{VaR}^{\alpha}_{t+1|t} = \inf\{c \in \mathbb{R} : P_t[R_{p,t+1} \leq c] \geq \alpha\}, \tag{3}$$

$$\mathrm{ES}^{\alpha}_{t+1|t} = \mathrm{E}_t[R_{p,t+1} \mid R_{p,t+1} < \mathrm{VaR}^{\alpha}_{t+1|t}], \tag{4}$$

where P_t denotes the true conditional cdf with respect to some information set, and $\alpha \in [0,1]$ denotes the level.

Note that $\mathrm{VaR}^{\alpha}_{t+1|t}$ denotes the return we are $(1-\alpha)\cdot 100\%$ likely to not fall short of, and $\mathrm{ES}^{\alpha}_{t+1|t}$ denotes the mean of the possible return outcomes that do fall short of $\mathrm{VaR}^{\alpha}_{t+1|t}$.[6]

Our copula models provide a joint model of the returns of the portfolio assets conditional on some information set of past observations. Given the portfolio weights we can then obtain the implied conditional distribution of $R_{p,t+1}$ through simulation, which we denote $F^{j}_{p,t}$ for copula model j. In similar fashion we can then obtain $\mathrm{VaR}^{\alpha}_{t+1|t}$, and $\mathrm{ES}^{\alpha}_{t+1|t}$, which are pure functions of the distribution of $R_{p,t}$. We denote these forecasts based on $F^{j}_{p,t}$ as $\widehat{\mathrm{VaR}}^{\alpha,j}_{t+1|t}$, and $\widehat{\mathrm{ES}}^{\alpha,j}_{t+1|t}$.

3.2 Optimal currency hedge exposures

Campbell et al. (2010) suggest investors use currency hedge exposures that minimize the variance of the portfolio return. We propose optimal currency hedge weights that minimize the $\mathrm{ES}^{\alpha}_{t+1|t}$, as it is a risk measure of particular importance to financial institutions. The Basel Committee on Banking Supervision has expressed and reiterated the intention to move from Value-at-Risk to ES as the basis of financial risk management for banks in the near future (BCBS, 2012, 2013b,a, 2014).

Rockafellar and Uryasev (2000) are the first to consider portfolio optimization with a minimal ES criterion.[7] The authors show that optimization is an easy exercise using simulation

[5] We assume here that $R_{p,t}$ has a continuous distribution, which holds for all joint distributions we propose for the portfolio asset returns.

[6] Value-at-Risk has several definitions. For instance, practitioners often consider losses instead of returns, such that by changing the sign, Value-at-Risk is stated as a positive number. We opt for our approach as it remains closest to the idea of quantiles.

[7] Rockafellar and Uryasev (2000) use the slightly different risk measure Conditional VaR (CVaR). This risk

methods and linear programming. Moreover, Kountzakis (2011, 2013) shows that optimization of Expected Shortfall over a convex set has at least one solution. Our problem can be stated as finding the optimal net currency exposure vector $\mathbf{m}_{t+1} = (m_{1,t+1}, \dots, m_{M,t+1})'$ that

$$\min_{\mathbf{m}_{t+1}} \mathrm{ES}^{\alpha}_{t+1|t}(\mathbf{m}_{t+1}) \text{ s.t.} \tag{5}$$

$$\sum_{i=1}^{M} m_{i,t+1} = 0, \;\; l_i \leq m_{i,t+1} \leq u_i, \;\; \forall i = 1, \dots, M, \tag{6}$$

where u_i and l_i denote some upper and lower bound. This problem satisfies the conditions of Kountzakis (2013) such that there exists at least one optimal $\mathbf{m}_{t+1}$.

We cannot perform a similar optimization with $\mathrm{VaR}^{\alpha}_{t+1|t}$, because it is not a *coherent* risk measure, as opposed to $\mathrm{ES}^{\alpha}_{t+1|t}$ (Acerbi and Tasche, 2002). Coherence as a property of risk measures is defined in Artzner et al. (1997, 1999), and is important in the context of portfolio optimization with risk measures as objectives, as it implies convexity of these risk measures.

3.3 Backtesting Value-at-Risk

For the unhedged and fully hedged portfolios we assess the accuracy and pairwise performance of the VaR forecasts of the different copula models. We do not consider optimal-VaR portfolios, as we cannot use VaR to optimize portfolios. This is because VaR is not a coherent risk measure. We backtest the accuracy of individual forecasts using the unconditional coverage, independence, and conditional coverage tests of Christoffersen (1998), which are commonly used in the literature. To test for equal performance of different models, we use the conditional and unconditional Giacomini-White tests of equal predictive ability in combination with the tick-loss function (Giacomini and White, 2006). The unconditional test is equivalent to the Diebold-Mariano test of equal predicitive ability (Diebold and Mariano, 1995).

In the Giacomini-White test we use the tick-loss function, which is a proper scoring function for quantiles, and is discussed in Komunjer (2013). Let $e^{j}_{t+1|t} = R^{p}_{t+1} - \widehat{\mathrm{VaR}}^{\alpha,j}_{t+1|t}$. Then the tick-loss function is given as

$$T_{\alpha}(e^{j}_{t+1|t}) = \left[\alpha - \mathbb{1}(e^{j}_{t+1|t} < 0)\right] e^{j}_{t+1|t}, \tag{7}$$

where $\mathbb{1}\{A\}$ denotes the indicator function which takes value one when event A is realized and

measure is equivalent to ES in case of continuous distributions, which is the case we investigate.

zero otherwise.

$T_\alpha(e^j_{t+1|t})$ is optimal for the true $\text{VaR}^\alpha_{t+1|t}(R_{p,t+1})$, meaning that $\text{E}[T_\alpha(e^j_{t+1|t})]$ is smallest in that case.

We can then pairwise test equivalent performance of models k, and l, i.e. $E\left[T_\alpha(e^k_{t+1|t}) - T_\alpha(e^l_{t+1|t})\right] = 0$. In the unconditional case this reduces to a Diebold-Mariano test (Diebold and Mariano, 1995) on the difference between the sample averages of those tick-losses, denoted $\Delta L_{k,l} = n^{-1}\sum_{t=1}^n\left[T_\alpha(e^k_{t+1|t}) - T_\alpha(e^l_{t+1|t})\right]$. The test statistic is given by

$$t^{(k,l)}_{VaR} = n^{1/2}\frac{\Delta L_{k,l}}{\hat{V}[\Delta L_{k,l}]}, \tag{8}$$

which is asymptotically standard normal under the null hypothesis, and $\hat{V}[\Delta L_{i,j}]$ denotes a HAC estimator of the variance of $\Delta L_{i,j}$. We use the HAC estimator of Newey and West (1987).

3.4 Backtesting Expected Shortfall

Du and Escanciano (2015) propose an unconditional and conditional backtest for Expected Shortfall based on the cumulative violations process. We define the violations process of $\text{VaR}^\alpha_{t+1|t}$ as

$$h_{t+1|t}(\alpha) = \mathbb{1}\{R_{p,t+1} \leq \text{VaR}^\alpha_{t+1|t}\}. \tag{9}$$

Unconditional and conditional VaR tests, such as those by Kupiec (1995) and Christoffersen (1998) are based on the martingale difference sequence (mds) property of the violations process, that is: $\text{E}_t[h_{t+1|t}(\alpha) - \alpha] = 0$, for all $\alpha \in [0,1]$.

Du and Escanciano (2015) note that $\text{ES}^\alpha_{t+1|t}$ can be rewritten as $\text{ES}^\alpha_{t+1|t} = 1/\alpha\int_0^\alpha \text{VaR}^u_{t+1|t}du$, such that, intuitively, a test for ES can be based on the cumulative violations process

$$H_{t+1|t}(\alpha) = \frac{1}{\alpha}\int_0^\alpha h_{t+1|t}(u)du = \frac{1}{\alpha}(\alpha - u_{t+1|t})\mathbb{1}\{u_{t+1|t} \leq \alpha\}, \tag{10}$$

which is also a mds by Fubini's theorem, and where $u_{t+1|t} = P_t(R_{p,t+1})$.[8] Under the true distribution P_t, u_t is uniformly distributed on $[0,1]$, such that $\text{E}[H_{t+1|t}(\alpha)] = \alpha/2$, and $\text{var}[H_{t+1|t}(\alpha)] = \alpha/3 - \alpha^2/4$.

[8]In order to apply asymptotic theory, it must hold that true distribution of $R_{p,t+1}$ at time t, P_t, is some parametric model.

We can then test correct $\text{ES}^{\alpha}_{t+1|t}$ specification of our models using the model specific estimated counterparts to $u_{t+1|t}$ and $H_{t+1|t}$, being $\hat{u}^j_{t+1|t} = F^j_{p,t}(R_{p,t+1})$, and $\widehat{H}^j_{t+1|t} = 1/\alpha(\alpha - \hat{u}^j_{t+1|t})\mathbb{1}\{\hat{u}^j_{t+1|t} \leq \alpha\}$, for our different models j.

The unconditional test is a standard t-test with corresponding test statistic

$$t^j_{\text{ES}} = \frac{\sqrt{n}(\bar{H}^j_n(\alpha) - \alpha/2)}{\sqrt{\alpha/3 - \alpha^2/4}}, \tag{11}$$

which is standard normally distributed under the null hypothesis that $\text{E}[H_t] = \alpha/2$, and where $\bar{H}^j_n(\alpha) = \frac{1}{n}\sum_{t=1}^n \hat{H}^j_{t+1|t}(\alpha)$.

The conditional backtest is a Box-Pierce portmanteau test (Box and Pierce, 1970) which can be expressed in terms of the sample autocorrelations

$$\hat{\rho}^j_{nj} = \frac{\hat{\gamma}^j_{nj}}{\hat{\gamma}^j_{n0}}, \tag{12}$$

$$\hat{\gamma}^j_{nj} = \frac{1}{n-j}\sum_{t=1+j}^{n}(\hat{H}^j_t(\alpha) - \alpha/2)(\hat{H}^j_{t-j}(\alpha) - \alpha/2). \tag{13}$$

Then under assumptions as outlined in Du and Escanciano (2015) it holds that

$$\text{BP}^j_{\text{ES}}(m) := n\sum_{j=1}^{m}\hat{\rho}^j_{nj} \xrightarrow{d} \chi^2_m, \tag{14}$$

where m denotes the number of autocorrelations considered, and χ^2_m denotes the chi-squared distribution with m degrees of freedom. An important assumption of this asymptotic result is that as $T, n \to \infty$, $n/T \to 0$, which in practice means the forecasting sample should be small relative to the estimation sample.

4 Copula models

We now concern ourselves with the multivariate copula models of the equity and exchange rate returns, which we need to forecast VaR and ES. In what follows we collect the equity and exchange rate returns in a vector $\mathbf{Y}_t = [Y_{1,t}, \ldots, Y_{2M-1,t}]'$. A copula model refers to a multivariate model of $\mathbf{Y}_t$ in which we model the marginal behaviors of $Y_{i,t}$ separately from the dependence amongst elements of $\mathbf{Y}_t$, where the latter is modelled by a copula.

We refer to the joint conditional cdf of random vector $\mathbf{Y}_{t+1}$ as $F_t(\mathbf{Y}_{t+1};\boldsymbol{\theta})$, where $\boldsymbol{\theta}$ denotes the parameter vector. We use the convention that a subscript t denotes conditioning on the information set $\mathcal{F}_t$. In our setting the information set contains all realizations observed until and including time t, such that $\mathcal{F}_t$ is the σ-field $\mathcal{F}_t = \sigma\{\mathbf{Y}_t, \mathbf{Y}_{t-1}, \ldots\}$. Let $F_{i,t}(Y_{i,t+1}; \boldsymbol{\theta_i})$ denote the conditional marginal distribution of $Y_{i,t+1}$ at time t, and $\boldsymbol{\theta}_i$ its parameter vector.

The separation of the joint distribution of a random vector into marginal distribution functions and a copula distribution function is justified by Sklar's theorem (see Nelsen (2006)). This theorem is extended by Patton (2006) to include time-varying copulas, and states that every joint conditional distribution of $\mathbf{Y}_{t+1}$ can be written as

$$F_t(\mathbf{Y}_{t+1}; \boldsymbol{\theta}) = C_t(\mathbf{U}_{t+1}; \boldsymbol{\theta}_c), \tag{15}$$

where $C_t(\mathbf{U}_{t+1})$ is the copula distribution function at time t, with $\mathbf{U}_{t+1} = (U_{1,t+1}, \ldots, U_{N,t+1})'$; $U_{i,t+1} = F_{i,t}(Y_{i,t+1}; \boldsymbol{\theta}_i)$ being the probability integral transform (PIT) of $Y_{i,t+1}$. The copula is parametrized by a vector $\boldsymbol{\theta}_c$. For an overview of copula methods in time series analysis we refer to Patton (2013).

A consequence of (15) is that the joint pdf can also be written in terms of the copula density function c_t and the marginal pdfs $f_{i,t}$ as

$$f_t(\mathbf{Y}_{t+1}; \boldsymbol{\theta}) = c_t(\mathbf{U}_{t+1}; \boldsymbol{\theta}_c) \cdot f_{1,t}(Y_{1,t+1}; \boldsymbol{\theta}_1) \cdots f_{N,t}(Y_{N,t+1}; \boldsymbol{\theta}_N). \tag{16}$$

4.1 Marginal models

We find evidence of autocorrelation and heteroskedasticity in the equity and exchange rate returns, which coincides with findings in Patton (2006) and Jondeau and Rockinger (2006). Therefore, for the marginal model of both equity and currency returns we assume we a AR(1) conditional mean model,[9] and a GJR-GARCH(1,1) conditional variance model (Glosten et al., 1993), which allows for different propagation of negative and positive shocks. We assume that the errors have a Hansen skew t distribution (Hansen, 1994). We will refer this process as the AR(1)-GJR-GARCH(1,1) model.

[9] We have also estimated a model with the lagged interest rate differential included as regressor in the mean equation of the exchange rate returns. However, the coefficient of the interest rate differential was generally insignificant, such that we exclude it in our further analysis

The AR(1)-GJR(1,1) model for $Y_{i,t+1}$, for all $i = 1, \dots, N$, is given by

$$Y_{i,t+1} = a_{i,0} + a_{i,1} Y_{i,t} + \sqrt{h_{i,t+1}} \eta_{i,t+1}, \tag{17}$$

$$h_{i,t+1} = b_{i,0} + b_{i,1} h_{i,t} \eta_{i,t}^2 + b_{i,2} h_{i,t} + b_{i,3} h_{i,t} \eta_{i,t}^2 \mathbb{1}\{\eta_{i,t} < 0\}, \tag{18}$$

$$\eta_{i,t} \sim st(\nu_i, \lambda_i), \tag{19}$$

where $h_{i,t}$ denotes the latent conditional volatility at time t for assets $i = 1, \dots, N$, and $st(\nu, \lambda)$ denotes the Hansen skew t distribution with ν degrees of freedom and skewness parameter $\lambda \in (-1, 1)$. This skew t distribution is standardized such that the variance of $\eta_{i,t}$ equals one. We collect the parameters in the marginal model for $Y_{i,t}$ in parameter vector $\boldsymbol{\theta}_i$.

4.2 Implicit copulas

The first copulas we consider are time-invariant implicit copulas. From Sklar's theorem we obtain a simple method to construct a copula distribution function from a joint distribution function, denoted by G_t, and its respective marginal distribution functions, $G_{i,t}$, which in a time-varying context can be stated as

$$C_t(\mathbf{U}_{t+1}; \boldsymbol{\theta}_c) = G_t(G_{1,t}^{-1}(U_{1,t}; \boldsymbol{\theta}_c), \dots, G_{N,t}^{-1}(U_{N,t}; \boldsymbol{\theta}_c); \boldsymbol{\theta}_c), \tag{20}$$

where $G_{i,t}^{-1}$ denotes the generalized inverse of $G_{i,t}$, for each $i = 1, \dots, N$, and $\boldsymbol{\theta}_c$ denotes the copula parameter vector.

The copulas constructed by this method are called implicit copulas. Some commonly used examples are the Gaussian, t, and skew t copulas, which are constructed from the multivariate normal, t and skew t cdfs, respectively.[10]

Following Christoffersen et al. (2012) we will use the multivariate normal, t, and skew t distribution of McNeil et al. (2015, p. 80) to construct copulas. The multivariate t and normal distributions are nested by the skew t distribution. Therefore, we will elaborate on the skew t copula in what follows, and discuss the special cases of the Gaussian and t copulas in Appendix A.2. The skew t copula is parametrized by a dispersion matrix Ψ, a skewness vector $\boldsymbol{\lambda}$, and a degrees of freedom parameter ν. Moreover, Ψ is restricted to have ones on the diagonal.

[10] Note that the implicit copula is invariant to affine transformations, such that changes in the mean or variance of the marginals does not affect the copula function. As such, one always uses the multivariate distribution of demeaned and standardized variables.

Copulas are invariant under monotonic transformations, such that Proposition 3.13 of McNeil et al. (2015) shows that this is just an identification restriction of thecopula. Moreover, the unit-diagonal restriction implies that marginals also have unit dispersion, ν degrees of freedom, and skewness parameter λ_i. We follow Christoffersen et al. (2012) by restricting all elements in $\boldsymbol{\lambda}$ to be equivalent. The copula density functions of the Gaussian copula, the t copula and the skew t copula are presented in Appendix A.2.

We follow Christoffersen et al. (2012) and introduce time-variation in the implicit copula through the cDCC model (Aielli, 2013) to the copula dispersion matrix. We alter the dynamic specification of Christoffersen et al. (2012) to correct for a biased variance target of the unconditional mean dispersion matrix of the skew t copula.

First, we obtain the *pseudo* observations $\tilde{Y}_{i,t+1} = G_{i,t}^{-1}(U_{i,t+1};\boldsymbol{\theta}_i)$ for all $i = 1,\dots,N$. Under the multivariate skew t copula, the vector $\tilde{\mathbf{Y}}_{t+1} = (\tilde{Y}_{1,t+1},\dots,\tilde{Y}_{N,t+1})'$ follows a multivariate skew t distribution conditional on $\mathcal{F}_t$, and parameterized by $(\Psi_{t+1},\nu,\boldsymbol{\lambda})$, where the dispersion matrix Ψ_{t+1} is now a time-varying cDCC process.

The cDCC dispersion matrix Ψ_{t+1} can be factorized as $\Psi_{t+1} = Q_{t+1}^{*-1/2} Q_{t+1} Q_{t+1}^{*-1/2}$, where $Q_{t+1}^{*} = \text{diag}(Q_{t+1})$. The cDCC updating equation of matrix Q_{t+1} is given by

$$Q_{t+1} = (1-\alpha_\Psi-\beta_\Psi)\Omega + \alpha_\Psi f(\tilde{\mathbf{Y}}_t) + \beta_\Psi Q_t, \tag{21}$$

where Ω is a constant matrix representing the unconditional mean of Q_{t+1}. The matrix function $f(\tilde{\mathbf{Y}}_t)$ represents the updating term. For the skew t copula it equals

$$f(\tilde{\mathbf{Y}}_t) = Q_t^{*1/2}\left[\frac{\nu-2}{\nu}\left[(\tilde{\mathbf{Y}}_t - \frac{\nu}{\nu-2}\boldsymbol{\lambda})(\tilde{\mathbf{Y}}_t - \frac{\nu}{\nu-2}\boldsymbol{\lambda})' - \frac{2\nu^2\boldsymbol{\lambda}\boldsymbol{\lambda}'}{(\nu-2)^2(\nu-4)}\right]\right]Q_t^{*1/2}, \tag{22}$$

whereas for the t and Gaussian copulas it has the usual form

$$f(\tilde{\mathbf{Y}}_t) = Q_t^{*1/2}\tilde{\mathbf{Y}}_t\tilde{\mathbf{Y}}_t' Q_t^{*1/2}. \tag{23}$$

The derivation of $f(\tilde{\mathbf{Y}}_t)$, and the estimation method are further explained in Appendix A.2.

4.3 Factor copulas

Oh and Patton (2013a,b) introduce time-invariant and time-varying factor copula models that can be applied to high dimensional datasets. They consider copulas implied by a random vector that follows a linear latent factor structure, where the factors and idiosyncratic shocks may have different distributions. The difference of their method with the implicit copulas of the previous section is that the distribution of the random vector they consider may not be known explicitly (e.g. the distribution implied by a t distributed latent factor with normally distributed errors).

However, in the one factor case we can numerically approximate the copula density function. This facilitates the introduction of time-variation in the factor copula through the generalized autoregressive score (GAS) framework of Creal et al. (2013), which relies on the score of the likelihood.

The one factor copula is the copula implied by the random vector $\mathbf{X}_t = [X_{1,t}, \ldots, X_{N,t}]'$, which follows a linear latent factor structure

$$\begin{aligned} X_{i,t} &= \lambda_{i,t} Z_t + \varepsilon_{i,t}, \\ Z_t &\sim F_{Z,t}, \\ \varepsilon_{i,t} &\sim \text{i.i.d. } F_{\varepsilon,t}, \\ \varepsilon_{k,t} &\perp\!\!\!\perp \varepsilon_{l,t} \ \forall \ k, l, \end{aligned} \tag{24}$$

where $\lambda_{i,t}$ is the factor loading, $F_{Z,t}$ is the distribution of the common latent factor Z_t, and $F_{\varepsilon,t}$ is the univariate distribution of the errors $\epsilon_{i,t}$, which is equivalent for all $i = 1, \ldots, N$. Oh and Patton (2013b) propose to use the univariate skew t distribution of Hansen (1994) as the factor distribution $F_{Z,t}$, and the t distribution as the error distribution $F_{\varepsilon,t}$.

In the time-invariant case, we set $\lambda_{i,t} = \lambda_i$. To introduce time-variation in the factor copula framework, Oh and Patton (2013b) apply the GAS framework to $\lambda_{i,t}$. The number of time-varying parameters in the factor skew t copula is thus N, whereas it is of $N(N-1)/2$ for the cDCC copulas. Under some stationarity assumptions the GAS evolution equation of $\lambda_{i,t}$ can be stated as

$$\lambda_{i,t} = \mathrm{E}[\lambda_{i,t}](1-\beta) + \beta\lambda_{i,t-1} + \alpha s_{i,t-1}, \tag{25}$$

where $s_{i,t-1} = \frac{\partial \log c_t(\mathbf{u_t};\boldsymbol{\theta}_c)}{\partial \lambda_{i,t}}$ denotes the score function of the copula likelihood with respect to $\lambda_{i,t}$. Moreover, α and β are scalars. Note that we do not use the $\log \lambda_{i,t}$ specification of Oh and Patton (2013b), as we observe negative correlations between some equity and exchange rate returns. Equation (25) allows for a separate-step GMM estimator of $\mathrm{E}[\lambda_{i,t}]$, analogous to a variance targeting estimator of the unconditional correlation matrix in the DCC model. The other model parameters can be estimated by numerically approximated maximum likelihood estimation.

4.4 Estimation

We estimate marginal and copula parameters in a separate stages. This procedure is referred to as inference function for margins (IFM) and is commonly used to estimate copula models (Joe, 2005). The likelihoods of our models, denoted $\mathcal{L}(\boldsymbol{\theta})$, can be separated into marginal likelihoods, denoted $\mathcal{L}_i(\boldsymbol{\theta}_i)$ and the copula likelihood, denoted $\mathcal{L}_c(\boldsymbol{\theta}_c)$, as

$$\mathcal{L}(\boldsymbol{\theta}) = \sum_{t=1}^{T} \log f_t(\mathbf{Y}_t; \boldsymbol{\theta}), \tag{26}$$

$$= \sum_{t=1}^{T} \log c_t(\mathbf{U}_t; \boldsymbol{\theta}_c) + \sum_{t=1}^{T} \sum_{i=1}^{N} \log f_{i,t}(Y_{i,t}; \boldsymbol{\theta}_i), \tag{27}$$

$$= \mathcal{L}_c(\boldsymbol{\theta}_c) + \sum_{i=1}^{N} \mathcal{L}_i(\boldsymbol{\theta}_i), \tag{28}$$

where f_t denotes the joint pdf, c_t the copula density function, and $f_{i,t}$ the marginal pdf for all $i = 1, \ldots, N$. Firstly, we estimate the marginal parameters $\boldsymbol{\theta}_i$ using maximum likelihood estimation for each marginal model seperately. Using parameter estimates $\hat{\boldsymbol{\theta}}_i$ we construct the probability integral transforms $\widehat{U}_{i,t} = F_{i,t}(Y_{i,t}; \hat{\boldsymbol{\theta}}_i)$. Then we estimate the copula parameter vector $\boldsymbol{\theta}_c$ by maximum likelihood estimation with the copula likelihood with the $\widehat{\mathbf{U}}_t$ plugged in, where in case of the factor copula we first estimate by GMM the unconditional mean loadings, and in case of the cDCC copula we estimate the unconditional mean dispersion matrix within each iteration of the copula maximum likelihood estimation.

5 Results

5.1 Marginal models

We first present results for the marginal models. We estimate the parameters of the AR(1)-GJR-GARCH(1,1) models for the equity and exchange rate returns in a moving window scheme. We re-estimate the model each four weeks, which means we estimate the copula model at the first week of our estimation sample, keep those parameter estimates fixed for the next three subsamples, and re-estimate the models in the fifth week on our fifth moving-window subsample, and so on. We choose a moving window length of 657 weeks, such that our first estimation sample is 30 May 1996 to 25 December 2008. Thus, a total of 300 VaR and ES forecasts are computed per copula model for the period 1 January 2009 to 25 September 2014.

Table 3 presents descriptive statistics of the marginal parameter estimates over the sample of parameter estimates. With regard to the mean model we find small but generally positive intercepts for the equity returns, and small but generally negative exchange rate return intercepts. The AR(1) term is generally negative for equity returns, whereas the AR(1) terms of the exchange rate return models are both positive and negative.

Concerning the conditional variance model, we observe mostly small ARCH effects for the equity returns, and find larger ARCH effects for exchange rate returns. The GARCH effect is large for both asset classes. We find mostly small negative and positive leverage effects for exchange rate returns, whereas the leverage effects are larger for equity returns. This result agrees with Hansen and Lunde (2005), who find that a leverage effect improves the forecastibility of the conditional mean for equity returns, but not for currency returns. With regard to the equity returns, the combination of small ARCH effects and large positive leverage effects indicates that volatility is mainly driven by negative shocks.

The estimates of the parameters of the error distribution show that the equity return errors are negatively skewed, with estimates ranging between -0.36 to -0.04. Exchange rate return errors show less skewness, and are both positive and negative. As exchange rates can be defined in two ways, and are influenced by shocks from at least two international markets, the presence of both positive and negative skewness parameter estimates is expected. We find that the sample distribution of degrees of freedom estimates is highly skewed, with much higher sample means, than sample medians. This can be partly explained by large outliers (i.e. $\nu_i > 250$), where the model moves towards the limit of no fat-tailedness. This large range of the estimates of the degrees of freedom parameters, and skewness parameters provides an argument for the copula approach, as it allows for different parameters in each marginal model.

We find that our models correct for autocorrelation in the returns series, as all error series pass Ljung-Box tests. The models also adequately capture heteroskedasticity in the returns series, as all equity return residual series pass Ljung-Box tests, and only 8% of the exchange rate error series fail.

[Table 3 about here.]

A requirement for the use of copulas is that the marginal models fit the data accurately.

We therefore test the marginal models for goodness-of-fit using the Kolmogorov-Smirnov (KS), Anderson-Darling (AD), and Cramer-Von Mises (CM) tests. Table 4 shows the results for these goodness-of-fit tests on the marginal models estimated for the full sample. We find that out of 13 time series only 1 series is rejected at the 5% level for 1 series per test. The AD and CM tests reject the marginal model for the USD/AUD exchange rate return, whereas the KS test rejects the model for the USD/JPY exchange rate return. We note that our models do capture the heteroskedasticity and autocorrelation in the data well, and therefore do not look into more advanced methods to model the univariate series.

[Table 4 about here.]

5.2 Copula models

We also estimate the copulas every four weeks in the same moving window scheme as the marginal models. The copulas we consider are the time-invariant Gaussian, t, skew t, and factor skew t copulas, and the time-varying counterparts, where the Gaussian, t, and skew t copulas are made time-varying using the cDCC (Aielli, 2013) framework, and the factor skew t copula using the GAS (Creal et al., 2013) framework.

The copulas under consideration differ in whether they allow the dependence structure to be time-varying, fat-tailed, or skewed, or a combination of those. Here we discuss for which of those features we find in-sample evidence.

Table 5 presents descriptive statistics for the sample of copula parameter estimates that are estimated in our moving-window scheme. More specifically, it shows estimates of the time-variation parameters and the skewness and degrees of freedom parameters. Results for the 'unconditional correlation' type parameters estimates, which are obtained in a seperate step using GMM, are not presented, but can be provided upon request.

The implicit t and skew t copula both have median dof estimates of approximately 20, with small deviations over the sample. A t distribution with 20 dof is already approximated quite well by a normal distribution, such that we find only little evidence of fat-tailed dependence here. The skewness parameter estimates of the skew t copula have a median value of -0.15, and are consistently negative over the cross section. We thus find evidence for negative skewness, and fairly thin tails for both time-varying approaches.

The time-invariant factor skew t copula models the dependence structure using a simple factor structure with a common, latent skew t factor, and an idiosyncratic t error. We find that the common factor has a median degrees of freedom (dof) parameter of 129. In fact, this large value is mainly due to a large increase in the later part of the sample, where it becomes greater than 300, which effectively means that it moves to a skew normal limit. This upward move is also reflected in the large standard deviation in the estimation sample of this parameter. The median dof of the idiosyncratic errors is 11, and also increases over the sample, but much less so than the factor dof. Oh and Patton (2013b) note that when the factor dof is smaller than the error dof, this means that tail dependence, the probabililty of two variables both being below their q-quantile, when $q \to 0$, is zero. This is counter-intuitive as we propose using fat-tailed copulas to incorporate non-zero tail dependence. Nevertheless, quantile dependence for $q > 0$ can still be notable. The median skewness parameter estimate of the time-invariant skew t copula is -0.34, which is large in comparison to findings in Oh and Patton (2013b) on CDS spreads, and considering its domain is $(-1, 1)$.

[Table 5 about here.]

The time-varying implicit Gaussian, t, and skew t copulas are made time-varying using the cDCC framework. It thus introduces time-variation in the dispersion/correlation matrix of the implicit copulas. The persistence of the time-varying parameters is given by $\alpha_{cDCC} + \beta_{cDCC}$. The mean value of this sum is approximately 0.97 for the all three cDCC copula, and all have small standard deviation. The persistence of these copulas is thus higher than that of the time-varying factor skew t, and similar to most marginal models.

We find similar dof and skewness parameter estimates of the cDCC t and skew t copula with regard to their time-invariant counterparts, although the skewness parameter of the time-varying model has slightly larger range and standard deviation.

The time-varying factor skew t copula uses the GAS framework to introduce time-variation in the latent factor loadings. The β_{GAS} parameter models the persistence in the time-varying parameters, and has a median value 0.898, which does not differ much over the sample. The time-variation in the copula is thus generally less persistent than what we find for the marginal models. The updating parameter α_{GAS} has a median of 0.017, and also has a low standard deviation. As the score is not standardized in the factor skew t model, this parameter has no intuitive interpretation.

The skewness and dof parameter estimates of the factor skew t copula largely agree with those of its time-invariant counterpart, although the factor dof is less large here. We thus find that factor skew t copula distribution parameters are robust to time-variation.

We thus find in-sample evidence of negatively skewed and slightly fat-tailed dependence across the different estimation samples of the moving window scheme. Concerning time-variation, we consistently find persistence, which is smaller than the persistence commonly found in conditional variance models in case of the factor skew t copula, and similar in magnitude in case of the cDCC copulas.

5.3 Value-at-Risk tests

We construct VaR forecasts at the levels $\alpha_{VaR} = 2.5\%$ and $\alpha_{VaR} = 5\%$ for fully hedged and unhedged global portfolios using our copula models. We test for correct coverage using the tests provided by Christoffersen (1998). We also compare VaR forecasts pairwise using the tick-loss function in the Giacomini-White (GW) (Giacomini and White, 2006) framework, which conditionally and unconditionally tests for equal forecasting performance of any pair of forecasts. The unconditional GW test is equivalent to the Diebold-Mariano (DM) test of equal forecasting performance.

Table 6 shows p-values of the unconditional coverage, independence, and conditional coverage test. We find that all hedged portfolio VaR forecasts and unhedged VaR forecasts at the $\alpha_V aR = 2.5\%$ level pass the Christoffersen tests. In the unhedged case at the $\alpha_V aR = 5\%$ level, we find that the time-invariant Gaussian copula fails the unconditional coverage test at the 10% significance level, but passes the independence and conditional coverage tests. The time-invariant skew t and t copulas fail both unconditional and conditional coverage tests at the 5% level. The time-invariant factor skew t copula passes all tests. The time-varying Gaussian copula, and time-varying skew t and t copulas fail the unconditional coverage test, but pass independence and conditional coverage tests. The time-varying factor skew t copula passes all tests. This indicates that for those series the joint hypothesis of independence and correct unconditional coverage cannot be rejected.

Table 7 show results for the pairwise DM tests on the VaR forecasts of the unhedged and fully hedged portfolios, for $\alpha_{VaR} = 2.5\%$ and $\alpha_{VaR} = 5\%$. The top panel of the table presents the results for the unhedged case, whereas the bottom panel shows the results for the fully hedged case. Left of the diagonal are results for the $\alpha_{VaR} = 2.5\%$ level, and right of the diagonal are those for $\alpha_{VaR} = 5\%$. A negative number indicates that the row model pairwise outperforms the column model.

We find significant evidence in favor of copula time-variation and skewness with respect to the VaR forecasts for the unhedged portfolios. At the 5% level we find that the cDCC copula forecasts beat all time-invariant implicit copula forecasts. Moreover, the time-varying factor skew t copula significantly beats the time-invariant skew t and t copulas.

At the α_{VaR} = 2.5% level we observe similar pairwise performance as at the 5% level, as we again find that many time-invariant copula forecasts are beaten by time-varying ones. We find that the cDCC skew t copula significantly beats the time-invariant skew t copula, and that the other cDCC copulas beat all time-invariant implicit copulas. The time-varying factor skew t beats its time-invariant counterparts. Moreover, the time-invariant skew t copula beats the time-invariant t and Gaussian copulas.

With regard to the fully hedged portfolio we find no significant performance distinction at the α_{VaR} = 5% level. At the α_{VaR} = 2.5% level we find that the time-invariant skew t copula significantly underperforms the other time-invariant implicit copulas.

[Table 6 about here.]

[Table 7 about here.]

Table 8 show p-values for the conditional pairwise GW tests on the VaR forecasts of the unhedged and fully hedged portfolios, for α_{VaR} = 2.5% and α_{VaR} = 5%. Again, the top panel of the table presents the results for the unhedged case, whereas the bottom panel shows the results for the fully hedged case. Left of the diagonal are results for the α_{VaR} = 2.5% level, and right of the diagonal are those for α_{VaR} = 5%. Using the decision rule proposed by Giacomini and White (2006), we include a subscript *'r'* (*'c'*) when the *row* (*column*) model has beter performance.

We find convincing evidence for the inclusion of time-variation in copula models, whereas evidence of skewness and fat-tailedness is inconclusive. For the unhedged portfolios, we find that at the α_{VaR} = 5% level the cDCC copulas again significantly beat all time-invariant implicit copulas. Moreover, the time-varying factor skew t copula is outperformed by the cDCC t and Gaussian copulas, but not by the cDCC skew t copula. With regard to the time-invariant copulas we find that the factor skew t copula outperforms the other copulas.

At the α_{VaR} = 2.5% level we find that the cDCC skew t copula is outperformed by the other time-varying copulas. All time-invariant implicit copulas are beaten by the cDCC copulas. The time-varying factor skew t copula beats its time-invariant counterpart.

As regards the hedged portfolio we again find less significant performance differences, although the time-varying skew t copula is beaten by the cDCC skew t and normal copulas.

Generally, in unhedged portfolios, where there is still plenty exposure to exchange rate returns we find strong evidence for the inclusion of time-variation in the copulas. Evidence for the inclusion of fat tails and asymmetry is less clear cut, and we find several pair-wise comparisons that are not in agreement.

The small numbers of significant distinctions between forecasts in the hedged portfolio may be due to the reduced variability of this portfolio, as the influence of exchange rate return shocks is greatly reduced. Indeed we find that all copula models pass the Christoffersen tests when forecasting the hedged portfolio VaR. Moreover, we find less benefits for the inclusion of time-variation in the hedged case as observed in pairwise combinations, than in the unhedged case.

Therefore, the results in this section suggest that including time-variation is important in modelling dependence between equity and exchange rate returns, and among exchange rate returns. Conversely, including time-variation and fat tails is of less importance in modelling the dependence amongst equity returns.

[Table 8 about here.]

5.4 Expected shortfall portfolios

We use the copula models to construct ES forecasts of unhedged and hedged portfolios. With regard to ES, we can construct optimal-ES portfolios that aim to minimize the ES of next period's portfolio return. We construct two types of optimal-ES portfolio. Firstly, we consider *constrained* portfolios, where we allow the currency hedge weights to take values between the unhedged, and fully hedged case. As such, the investor cannot take speculative hedging positions in this portfolio. Secondly, we consider a portfolio where we do allow for speculative hedges. We call this a emphunconstrained hedging portfolio, and allow the investor to take positions betwen negative and positive the value of the equity share of the portfolio.

Figures 1 to 6 show *net* currency exposures to the different currencies for the constrained hedging portfolio. Figures 7 to 12 show net currency exposures for the unconstrained hedging portfolio. Note that a net currency exposure less than zero corresponds to the overhedged case, an exposure of zero to the fully hedged case, an exposure between zero and $1/M$ to the partially hedged case, an exposure of $1/M$ to the unhedged case, and an exposure greater than

$1/M$ to the underhedged case. We only show results for the factor skew t and skew t copulas, as the differences between net currency exposures of implicit copulas are small, in both the time-invariant and time-varying case.

We find that the investor is almost completely unhedged with regard to the Canadian and Australian currencies in the constrained case, and overexposed to these currencies in the unconstrained case. The investor is almost always fully hedged to the Japanese yen in the constrained case, whereas he is underexposed in the unconstrained case, with similar patterns for all copulas.

Concerning the European currencies, i.e. the German mark/euro, the British pound, and the Swiss franc, the pattern is not as clear, with the investor switching between unhedged and fully hedged positions over time. In the constrained case, we find that the time-varying factor skew t copula more often favors unhedged positions in these currencies than the cDCC copulas. The cDCC copulas favor even mostly fully hedged positions to the Swiss franc in the constrained case. This pattern is amplified in the unconstrained portfolio for the Swiss franc, and the British pound, with mostly overexposed positions suggested by the factor skew t copula, and underexposured positions suggested by the cDCC copulas. However, in the German case we find that the cDCC copulas allow for more overexposure than the factor copula, thus showing a contrary pattern to the constrained case.

As concerns the time-invariant copulas, the factor skew t copula suggests a move towards unhedged European currency positions, whereas the implicit copulas favor fully hedged positions, with only partly hedged positions from 2012 onwards for the German and British currencies. Again, we find an amplification of this pattern for the British pound and Swiss francs. In the German case, we find a move from underexposure to overexposure over time as suggested by the factor skew t copula, though it only catches up with the the stable overexposure suggested by the cDCC copulas from mid 2012 onwards.

5.5 Expected shortfall tests

We compute ES forecasts based on each copula model for the unhedged and fully hedged portfolios. Moreover, we compute ES forecasts for the ES-optimally hedged portfolios, using the copula model which was also used to obtain the optimal weights. We consider the α_{ES} = 2.5% and 5% levels, and test for correct specification of the forecasts using the unconditional and conditional ES backtests of Du and Escanciano (2015).

Table 9 shows results for the unhedged portfolio. The upper part of the table shows results for the level $\alpha_{ES} = 2.5\%$, and the lower part for $\alpha_{ES} = 5\%$. Each part has results for the time-varying and time-invariant copulas. The table includes the sample average of the cumulative hits, denoted $\bar{H}(\alpha_{ES})$, which should equal $\alpha_{ES}/2$ under the null hypothesis. The case $\bar{H}(\alpha_{ES}) > \alpha_{ES}/2$ indicates our ES forecast is too extreme on average, whereas $\bar{H}(\alpha_{ES}) < \alpha_{ES}/2$ indicates our ES forecast is too lenient on average. The table also includes p-values of the t-test, and the conditional Box-Pierce (BP) tests, at lags 1, 3, and 5.

We find correct unconditional and conditional specification for all copula models. Only the time-invariant skew t copula fails the unconditional specification test at the 10% level, and significantly underestimates ES.

Table 10 shows results for the fully hedged portfolio. Here we find good performance of copula models at the $\alpha_{ES} = 5\%$ level. At the $\alpha_{ES} = 2.5\%$ level all copula models fail the conditional specification tests at lag 3 at the 5% significance level, and at lag 5 at the 10% significance level.

Table 11 shows the results of the unconditional and conditional ES tests for the unconstrained optimal-ES portfolio. We find that the unconditional test cannot reject correct mean specification for all time-varying and time-invariant copulas at the $\alpha_{ES} = 5\%$ level, thus we cannot reject correct ES specification based on the unconditional test. The time-invariant implicit copulas fail one or more conditional ES tests at this level, whereas the time-varying, and factor skew t copulas do not, such that we can discard the time-invariant implicit copulas, when considering ES forecasts.

At the $\alpha_{ES} = 2.5\%$ level we find evidence that the time-invariant implicit copulas fail the unconditional test at a significance level of 5%. We also find some evidence that the time-varying Gaussian and skew t copulas fail the unconditional test for $\alpha_{ES} = 2.5\%$ at a significance level of 10%. The cDCC t copula can be rejected at the 5% level. All the models that fail the test propose ES forecasts that are too lenient.

With regard to the conditional tests, we find strong evidence that the time-invariant implicit copulas fail the conditional test at the 1% significance level. Moreover, the cDCC t copula also fails the conditional tests at the 1% level. The other cDCC copulas perform wel. Again, the time-invariant and time-varying factor skew t copula performs well.

Table 12 shows results for the constrained optimal-ES portfolios. We find that for this portfolio it is impossible to reject the accuracy of the forecasts of any copulas. This may be due to the reduced exposure to currency return shocks, such that all copulas can capture the dependence easily, or it could be due to the small power of the test, as only few observations, i.e. those below VaR, are influential.

We thus find that choosing the right copula model matters for speculative investors, as time-varying copulas provide better risk management of unconstrained optimal-ES portfolios. For the other portfolios we cannot distinguish the performance of ES forecasts, as either all copula models pass or fail the tests in the same scenarios.

[Table 9 about here.]

[Table 10 about here.]

[Table 11 about here.]

[Table 12 about here.]

6 Concluding remarks

In our comparison of multivariate copula models applied to the returns of two asset classes, equities and currencies, we find that strong evidence time-variation is important to include in both Value-at-Risk (VaR) and expected shortfall (ES) forecasts. In many cases the time-invariant implicit copulas are all strongly outperformed in a pairwise comparison with time-varying copulas. The time-invariant factor skew t model of Oh and Patton (2013a) is also beaten by time-varying copulas, but less often, and less strongly. Moreover, the cDCC skew t copula beats the cDCC t and Gaussian copulas in all cases, suggesting the importance of including skewness in the dependence structure. The tests on the accuracy of ES forecasts show that the time-invariant implicit copulas are strongly rejected, both conditionally and unconditionally. The time-varying t copula has poor performance for ES at the 2.5% confidence level, both unconditionally and conditionally. At this level there is also weak evidence of unconditional inaccuracy of the other time-varying implicit copulas. The static and dynamic factor skew t copulas pass all tests. This result may be due to the restricted estimation of the unconditional correlation matrix of the copula, whereas this is estimated by unrestricted GMM for the time-varying and time-invariant implicit copulas. Taking into account both VaR and ES tests, we must conclude that the cDCC skew t and time-varying factor skew t copulas perform best.

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A Appendix

A.1 A multivariate skew t distribution

McNeil et al. (2015) construct a multivariate t distribution based on a simple stochastic representation. It is this multivariate skew t distribution that they and Christoffersen et al. (2012) use to construct a skew t copula. According to their definition, a demeaned skew t distributed random vector $\mathbf{Y}$ of dimension N has the stochastic representation

$$\mathbf{Y} = \sqrt{W}\mathbf{Z} + \boldsymbol{\lambda} W, \tag{29}$$

where $W \sim IG(\nu/2, \nu/2)$ is an inverse gamma distributed scalar random variable, $\mathbf{Z} \sim N(0, \Psi)$ is a normally distributed random vector of size N with zero mean and covariance matrix Ψ and independent of W, and $\boldsymbol{\lambda}$ is a skewness parameter vector of size N.

When we set $\boldsymbol{\lambda} = \mathbf{0}$, the random vector $\mathbf{Y}$ is multivariate t distributed with dispersion matrix Ψ and ν degrees of freedom. When we set $\boldsymbol{\lambda} = \mathbf{0}$, and let $\nu \to \infty$, the random vector $\mathbf{Y}$ is multivariate normally distributed with covariance matrix Ψ.

The joint probability density function of the multivariate skew t is given by

$$f_{ST,Y}(\mathbf{y}) = c \frac{K_{(\nu+N)/2}\left(\sqrt{(\nu + Q(\mathbf{y}))\boldsymbol{\lambda}'\Psi^{-1}\boldsymbol{\lambda}}\right) \exp\left(\mathbf{y}'\Psi^{-1}\boldsymbol{\lambda}\right)}{\left(\sqrt{(\nu + Q(\mathbf{y}))\boldsymbol{\lambda}'\Psi^{-1}\boldsymbol{\lambda}}\right)^{-(\nu+N)/2}(1 + Q(\mathbf{y})/\nu)^{(\nu+N)/2}}, \tag{30}$$

where the normalizing constant equals

$$c = \frac{2^{1-(\nu+N)/2}}{\Gamma(\nu/2)(\pi\nu)^{N/2}|\Psi|^{1/2}}, \tag{31}$$

$K_p(\cdot)$ denotes the modified Bessel function of the third kind[11] with index p, and $Q(\mathbf{y}) = \mathbf{y}'\Psi^{-1}\mathbf{y}$.

Using the law of total expectation with the property that $\mathrm{E}[W^i] = \frac{i}{\prod_{j-1}^{i}(\nu=2j)}$, Christoffersen et al. (2012) obtain the first two moments of random vector $\mathbf{Y}$ as

$$\mathrm{E}[\mathbf{Y}] = \frac{\nu}{\nu - 2}\boldsymbol{\lambda}, \tag{32}$$

$$\mathrm{Cov}[\mathbf{Y}] = \frac{\nu}{\nu - 2}\Psi + \frac{2\nu^2\boldsymbol{\lambda}\boldsymbol{\lambda}'}{(\nu - 2)^2(\nu - 4)}. \tag{33}$$

[11] See McNeil et al. (2015, p. 497) for more details.

A.2 Skew t, t and Gaussian copulas

[NOTE: Old time-varying implicit copula section]

We follow Christoffersen et al. (2012) and introduce time-variation in the implicit copula through the cDCC model (Aielli, 2013) to the copula dispersion matrix, but alter their dynamic specification of Christoffersen et al. (2012) to correct for their biased variance target of the unconditional mean dispersion matrix of the skew t copula.

First, we obtain the *pseudo* observations $\tilde{Y}_{i,t+1} = G_{i,t}^{-1}(U_{i,t+1};\boldsymbol{\theta}_i)$ for all $i = 1,\dots,N$. Under the multivariate skew t copula, the vector $\tilde{\mathbf{Y}}_{t+1} = (\tilde{Y}_{1,t+1},\dots,\tilde{Y}_{N,t+1})'$ follows a multivariate skew t distribution conditional on $\mathcal{F}_t$, and parameterized by $(\Psi_{t+1},\nu,\boldsymbol{\lambda})$, where the dispersion matrix Ψ_{t+1} is now a time-varying cDCC process.

The cDCC dispersion matrix Ψ_{t+1} can be factorized as $\Psi_{t+1} = Q_{t+1}^{*-1/2} Q_{t+1} Q_{t+1}^{*-1/2}$, where $Q_{t+1}^{*} = \text{diag}(Q_{t+1})$. The cDCC updating equation of matrix Q_{t+1} is given by

$$Q_{t+1} = (1-\alpha_\Psi-\beta_\Psi)\Omega + \alpha_\Psi f(\tilde{\mathbf{Y}}_t) + \beta_\Psi Q_t, \tag{34}$$

where Ω is a constant matrix representing the unconditional mean of Q_{t+1}, and $f(\tilde{\mathbf{Y}}_t)$ is a matrix functional of $\tilde{\mathbf{Y}}_t$, and represents the updating term. To allow for consistent estimation of Ω it must hold that $\text{E}_t[f(\tilde{\mathbf{Y}}_{t+1})] = Q_{t+1}$, such that the desired property $\Omega = E[Q_{t+1}]$ holds[12].

In that case we can use *variance targeting* to estimate Ω when Q_{t+1} is a stationary process. Using Equation (33) we find that this holds for

$$f(\tilde{\mathbf{Y}}_t) = Q_t^{*1/2}\left[\frac{\nu-2}{\nu}\left[(\tilde{\mathbf{Y}}_t - \frac{\nu}{\nu-2}\boldsymbol{\lambda})(\tilde{\mathbf{Y}}_t - \frac{\nu}{\nu-2}\boldsymbol{\lambda})' - \frac{2\nu^2\boldsymbol{\lambda}\boldsymbol{\lambda}'}{(\nu-2)^2(\nu-4)}\right]\right]Q_t^{*1/2}. \tag{35}$$

Subsequently, we can define the cDCC conditional estimator for the skew t copula as

$$\hat{\Omega}_{\boldsymbol{\theta}_c} = \varphi\left(T^{-1}\sum_{t=1}^{T} f(\tilde{\mathbf{Y}}_t)\right), \tag{36}$$

with $\varphi(\cdot)$ denoting the function that standardizes a matrix. The diagonal elements of diagonal matrix $\tilde{Q}_t^*$, which are necessary to compute $\hat{\Omega}_{\boldsymbol{\theta}_c}$, are easily obtained as

$$\tilde{q}_{ii,t} = (1-\alpha_\Psi-\beta_\Psi) + \alpha_\Psi \tilde{q}_{ii,t-1}\left(\frac{\nu-2}{\nu}(\tilde{Y}_{i,t-1} - \frac{\nu}{\nu-2}\lambda_i)^2 - \frac{2\nu^2\lambda_i^2}{(\nu-2)^2(\nu-4)}\right) + \beta_\Psi \tilde{q}_{ii,t-1}. \tag{37}$$

[12] This can be observed by taking expectations $\text{E}[Q_t] = (1-\alpha_\Psi-\beta_\Psi)\Omega + \alpha_\Psi\text{E}[f(\tilde{\mathbf{Y}}_{t-1})] + \beta_\Psi\text{E}[Q_{t-1}] = (1-\alpha_\Psi-\beta_\Psi)\Omega + \alpha_\Psi\text{E}[\text{E}_{t-2}[f(\tilde{\mathbf{Y}}_{t-1})]] + \beta_\Psi\text{E}[Q_{t-1}] = (1-\alpha_\Psi-\beta_\Psi)\Omega + \alpha_\Psi\text{E}[Q_{t-1}] + \beta_\Psi\text{E}[Q_{t-1}]$.

The estimation is than carried out similar to Definition 3.4 in Aielli (2013), with the quasi-log-likelihood function replaced by the copula log-likelihood function. The estimation of the normal and t copula are special cases of this estimation procedure by letting $(\boldsymbol{\lambda} = \mathbf{0}, \nu \to \infty)$ and $\boldsymbol{\lambda} = \mathbf{0}$, respectively. In those cases, the conventional $f(\tilde{\mathbf{Y}}_t) = Q_t^{*1/2}\tilde{\mathbf{Y}}_t\tilde{\mathbf{Y}}_t' Q_t^{*1/2}$ and $\hat{\Omega}_{\boldsymbol{\theta}_c} = \varphi\left(T^{-1}\sum_{t=1}^{T}\tilde{Q}_t^{*1/2}\tilde{\mathbf{Y}}_t\tilde{\mathbf{Y}}_t'\tilde{Q}_t^{*1/2}\right)$ apply.

A.3 Net currency exposures over time for non-speculative currency weights.

[Figure 1 about here.]

[Figure 2 about here.]

[Figure 3 about here.]

[Figure 4 about here.]

[Figure 5 about here.]

[Figure 6 about here.]

A.4 Net currency exposures over time for speculative currency weights.

[Figure 7 about here.]

[Figure 8 about here.]

[Figure 9 about here.]

[Figure 10 about here.]

[Figure 11 about here.]

[Figure 12 about here.]

A.5 Cumulative hit process plots

[Figure 13 about here.]

[Figure 14 about here.]

Table 1: Descriptive statistics of log returns

	Mean	Median	St.Dev.	Min.	Max.	ADF	LB	LB^2
				Equities				
Germany	0.135	0.378	3.100	-13.871	15.907	0.001	0.677	0.000
Australia	0.111	0.271	2.114	-11.567	8.178	0.001	0.065	0.000
Canada	0.157	0.340	2.529	-17.976	14.658	0.001	0.035	0.000
Japan	0.016	0.207	2.862	-16.165	11.119	0.001	0.002	0.000
Switzerland	0.126	0.311	2.438	-13.835	13.650	0.001	0.751	0.000
UK	0.082	0.201	2.343	-11.306	10.290	0.001	0.005	0.000
US	0.143	0.316	2.504	-18.360	18.177	0.001	0.000	0.000
				Exchange rates				
Germany	0.010	0.017	1.357	-8.615	6.277	0.001	0.581	0.000
Australia	0.005	-0.074	1.740	-6.535	10.241	0.001	0.295	0.000
Canada	-0.014	-0.052	1.235	-4.531	6.833	0.001	0.002	0.000
Japan	0.013	0.045	1.468	-12.404	5.813	0.001	0.468	0.000
Switzerland	-0.018	-0.026	1.467	-10.925	9.259	0.001	0.825	0.000
UK	0.001	-0.070	1.254	-5.396	7.322	0.001	0.000	0.000

Note: This table presents descriptive statistics of the weekly log returns of the equity markets of the countries in the leftmost column, and of the exchange rates of the currencies of those countries with respect to the US dollar. All log returns are in percentages and annualized. The mean, median, standard deviation, and the minimum and maximum values of the log returns are presented over the full sample period: 30 May 1996 - 25 September 2014.
The table also presents p-values of the Ljung-Box test for autocorrelation, applied to the log returns (LB), and to the squared demeaned log returns (LB^2). We use 15 lags in the LB tests.

Table 2: Correlation of equity and exchange rate returns

		Equity						*Exchange rates*					
		Aus.	Can.	Jap.	Swi.	UK	US	Ger.	Aus.	Can.	Jap.	Swi.	UK
Equity	Germany	0.59	0.66	0.49	0.77	0.80	0.75	0.01	-0.34	-0.38	0.19	0.14	-0.07
	Australia		0.55	0.56	0.58	0.63	0.57	-0.13	-0.41	-0.42	0.16	0.00	-0.17
	Canada			0.43	0.57	0.67	0.78	-0.13	-0.41	-0.40	0.10	-0.03	-0.17
	Japan				0.44	0.48	0.46	-0.05	-0.33	-0.32	0.21	0.04	-0.11
	Switzerland					0.78	0.69	0.06	-0.32	-0.29	0.20	0.22	-0.01
	UK						0.74	-0.04	-0.39	-0.39	0.20	0.10	-0.02
	US							-0.10	-0.36	-0.42	0.14	0.03	-0.14
Exchange rates	Germany								0.52	0.43	0.26	0.86	0.65
	Australia									0.64	0.12	0.39	0.49
	Canada										0.02	0.31	0.45
	Japan											0.34	0.14
	Switzerland												0.56

Note: This table presents sample correlations between country equity market returns and exchange rate returns of Australia, Canada, Germany, Japan, Switzerland, the UK, and the US. The sample ranges from 30 May 1996 to 25 September 2014.

Table 3: Moving window estimates of marginal models

	Mean	Median	St. Dev.	Skew.	Kurt.	Min.	Max.
				Equities			
$a_{i,0}$	0.12	0.13	0.06	-0.41	2.73	-0.04	0.25
$a_{i,1}$	-0.08	-0.08	0.03	0.59	3.09	-0.14	0.00
$b_{i,0}$	0.28	0.25	0.13	0.96	3.45	0.09	0.72
$b_{i,1}$	0.03	0.01	0.03	0.68	2.10	0.00	0.12
$b_{i,2}$	0.84	0.86	0.05	-0.49	2.03	0.72	0.92
$b_{i,3}$	0.18	0.18	0.08	0.07	2.98	0.02	0.38
λ_i	-0.26	-0.28	0.08	1.37	4.06	-0.36	-0.04
ν_i	37.83	10.45	80.94	2.81	9.11	6.18	299.97
				Exchange rates			
$a_{i,0}$	-0.04	-0.05	0.03	0.14	2.07	-0.11	0.03
$a_{i,1}$	0.01	0.03	0.04	-1.41	4.06	-0.10	0.07
$b_{i,0}$	0.10	0.07	0.09	1.88	5.97	0.01	0.45
$b_{i,1}$	0.08	0.09	0.03	-0.58	2.63	0.00	0.13
$b_{i,2}$	0.88	0.89	0.05	-1.87	6.49	0.68	0.95
$b_{i,3}$	-0.03	-0.04	0.05	0.31	1.84	-0.11	0.08
λ_i	0.09	0.10	0.14	-0.09	1.77	-0.16	0.34
ν_i	40.96	15.89	70.34	2.94	10.34	6.54	299.84
				Test rejections			
			ADF	*LB*	LB^2		
	Equities		100%	0%	0%		
	Exchange rates		100%	0%	7.78%		

Note: This table presents descriptive statistics of the marginal model parameter estimates. Descriptive statistics are given for the pooled estimates of all equity returns, and those of all exchange rate returns. Each parameter is estimated 75 times in the moving window procedure. The first estimation window is 30 May 1996 to 25 December 2008 (657 observations), and the last estimation window is 21 February 2002 to 18 September 2014. There are four weeks between consecutive windows.
The parameters belong to the AR(1)-GJR-GARCH(1,1) model with conditional mean equation: $Y_{i,t+1} = a_{i,0}+a_{i,1}Y_{i,t}+\sqrt{h_{i,t+1}}\eta_{t+1}$, and conditional variance equation: $h_{i,t+1} = b_{i,0} + b_{i,1}\eta_t^2 + b_{i,2}h_t + b_{i,3}\eta_t^2 \mathbb{1}\{\eta_t < 0\}$. The error η_{t+1} is Hansen (1994) skew t distributed with ν_i degrees of freedom, and skewness parameter λ_i. The table also presents the percentage of rejections of the null for the augmented Dickey-Fuller (ADF) test, which has a null hypothesis of unit root, and the Ljung-Box test for autocorrelation, applied to the standardized residuals (LB), and to the squared standardized residuals (LB^2).

Table 4: Univariate goodness-of-fit tests

	KS	AD	CM
	Equity returns		
Germany	0.23	0.20	0.18
Australia	0.66	0.72	0.73
Canada	0.17	0.31	0.19
Japan	0.73	0.46	0.68
Switzerland	0.79	0.73	0.65
UK	0.50	0.66	0.58
US	0.63	0.85	0.78
	Currency returns		
Germany	0.85	0.53	0.60
Australia	0.10	0.02	0.04
Canada	0.64	0.47	0.49
Japan	0.02	0.07	0.07
Switzerland	0.45	0.53	0.66
UK	0.62	0.93	0.90

Note: This table reports p-values of univariate goodness-of-fit tests of the AR(1)-X-GJR(1,1) models with skew t distributed innovations for the full sample: 30 May 1996 to 25 September 2014. Results are presented for the Kolmogorov-Smirnov ($K - S$), Anderson-Darling ($A - D$), and the Cramer-von Mises ($C - vS$) test statistic. Critical values are obtained by Monte-Carlo method using 1,000 simulations.

Table 5: Descriptive statistics of copula parameter estimates

	Mean	Median	St.Dev.	Low	High
		Dynamic factor skew t			
α_{GAS}	0.017	0.018	0.002	0.008	0.021
β_{GAS}	0.888	0.898	0.038	0.794	0.934
factor dof	45.62	36.38	27.06	14.82	90.01
skewness	-0.28	-0.29	0.05	-0.34	-0.19
error dof	8.13	8.17	0.66	6.58	8.91
		cDCC skew t			
$\alpha_{c\text{DCC}}$	0.015	0.015	0.001	0.011	0.017
$\beta_{c\text{DCC}}$	0.954	0.955	0.008	0.939	0.970
skewness	-0.13	-0.12	0.05	-0.27	-0.03
dof	22.92	22.91	3.64	17.05	30.54
		cDCC t			
$\alpha_{c\text{DCC}}$	0.015	0.015	0.001	0.011	0.017
$\beta_{c\text{DCC}}$	0.956	0.957	0.008	0.938	0.972
dof	24.65	25.98	3.44	18.27	30.44
		cDCC Gaussian			
$\alpha_{c\text{DCC}}$	0.016	0.017	0.002	0.012	0.020
$\beta_{c\text{DCC}}$	0.954	0.955	0.009	0.935	0.970
		Static factor skew t			
factor dof	181.13	128.55	129.75	16.13	341.89
skewness	-0.31	-0.34	0.05	-0.37	-0.25
error dof	10.64	11.07	1.66	6.91	12.23
		Skew t			
skewness	20.14	21.51	2.66	16.68	24.15
dof	-0.15	-0.15	0.04	-0.22	-0.07
		t			
dof	20.59	20.82	2.12	16.46	24.42

Note: This table presents descriptive statistics of the parameter estimates of the different copulas over the estimation period. Each parameter is estimated 75 times in the moving window procedure. The first estimation window is 30 May 1996 to 25 December 2008 (657 observations), and the last estimation window is 21 February 2002 to 18 September 2014. There are four weeks between consecutive windows.

Table 6: Christoffersen tests on VaR forecasts

		UC	*Ind.*	*CC*	*UC*	*Ind.*	*CC*
		$\alpha_{VaR} = 5\%$			$\alpha_{VaR} = 2.5\%$		
	Time-varying						
	skew t	0.04	0.51	0.11	0.16	0.74	0.35
	t	0.04	0.51	0.11	0.33	0.68	0.57
	Gaussian	0.09	0.45	0.18	0.33	0.68	0.57
	factor skew t	0.60	0.30	0.50	0.33	0.68	0.57
Unhedged							
	Time-invariant						
	skew t	0.02	0.56	0.05	0.16	0.74	0.35
	t	0.02	0.56	0.05	0.16	0.74	0.35
	Gaussian	0.04	0.51	0.11	0.16	0.74	0.35
	factor skew t	0.42	0.49	0.57	0.16	0.74	0.35
	Time-varying						
	skew t	0.78	0.82	0.94	0.85	0.20	0.42
	t	0.78	0.82	0.94	0.85	0.20	0.42
	Gaussian	0.59	0.92	0.86	0.85	0.20	0.42
	factor skew t	0.59	0.92	0.86	0.58	0.26	0.45
Hedged							
	Time-invariant						
	skew t	0.59	0.92	0.86	0.58	0.26	0.45
	t	0.59	0.92	0.86	0.58	0.26	0.45
	Gaussian	0.59	0.92	0.86	0.58	0.26	0.45
	factor skew t	0.80	0.63	0.86	0.85	0.20	0.42

Note: This table shows p-values for the unconditional coverage (UC), independence ($Ind.$), and conditional coverage (CC) tests of Christoffersen (1998) on the unhedged portfolio Value-at-Risk (VaR) forecasts of the copula models. The weekly forecasts are calculated for the period

Table 7: Value-at-Risk Diebold-Mariano tests

		1	2	3	4	5	6	7	8
	Unhedged					α_{VaR} = 5%			
1			0.38	-1.22	0.20	**-4.16**	**-3.79**	**-3.75**	-1.01
2		-1.76		-1.45	0.13	**-3.97**	**-3.68**	**-3.51**	-1.26
3		-0.78	0.49		0.53	**-3.51**	**-3.71**	**-3.65**	-0.76
4		-0.89	-0.58	-0.73		**-1.98**	**-1.99**	-1.81	-1.31
5	α_{VaR} = 2.5%	**2.62**	**3.47**	**3.00**	1.89		0.69	1.38	1.70
6		1.58	**2.39**	**2.31**	1.53	**-2.49**		1.39	1.53
7		1.63	**2.52**	**2.32**	1.45	**-3.25**	-0.14		1.30
8		0.51	0.93	0.87	**2.55**	-0.95	-0.42	-0.39	
	Hedged					α_{VaR} = 5%			
1			-0.49	1.21	-0.41	0.02	-0.40	-0.07	1.38
2		-0.61		1.39	-0.06	0.27	-0.10	0.26	1.48
3		-1.48	-1.18		-0.91	-0.42	-0.83	-0.56	0.80
4		-0.30	0.43	0.92		0.42	-0.11	0.62	1.69
5	α_{VaR} = 2.5%	0.28	0.63	1.04	0.62		-0.65	-0.13	1.10
6		-0.90	-0.30	0.16	-0.80	**-2.92**		0.91	1.59
7		-1.08	-0.24	0.23	-0.64	**-2.30**	0.14		1.35
8		-0.81	-0.39	0.20	-0.72	-0.98	0.00	-0.07	

Note: This table presents pairwise Diebold-Mariano (DM) test statistics on Value-at-Risk (VaR) forecasts of the unhedged, and fully hedged portfolio returns. The test statics $t_{i,j}$, where i indicates the row, and j indicates the column, are calculated for VaR forecasts at the $\alpha_{VaR} = 2.5\%$, and $\alpha_{VaR} = 5\%$ level. The numbers above the diagonal are DM-statistics for VaR forecasts at $\alpha_{VaR} = 5\%$, and the numbers below the diagonal are those for $\alpha_{VaR} = 5\%$. *Positive* (*negative*) numbers imply that the row model performs *worse* (*better*) than the column model. The null hypothesis of equal performance is rejected at the 10, 5, or 1% when $|t_{i,j}| > c_\alpha$, for $c_\alpha = 1.65, 1.96$, or 2.58, respectively.
The models are numbered as follows. 1: cDCC skew t, 2: cDCC t, 3: cDCC Gaussian, 4: Dynamic factor skew t, 5: Static skew t, 6: Static t, 7: Static Gaussian, and 8: Static factor skew t.

Table 8: Conditional Giacomini-White test on Value-at-Risk forecasts

		1	2	3	4	5	6	7	8
	Unhedged				α_{VaR} = 5%				
1			0.18	0.39	**0.00**^{r}	**0.00**^{r}	**0.00**^{r}	**0.00**^{r}	0.60
2		**0.02**^{r}		0.27	**0.08**^{r}	**0.00**^{r}	**0.00**^{r}	**0.00**^{r}	0.40
3		**0.05**^{r}	0.75		**0.00**^{r}	**0.00**^{r}	**0.00**^{r}	**0.00**^{r}	0.74
4		**0.04**^{r}	0.72	0.27		**0.00**^{r}	**0.00**^{r}	**0.00**^{r}	0.38
5	α_{VaR} = 2.5%	**0.02**^{c}	**0.00**^{c}	**0.01**^{c}	**0.00**^{c}		0.13	0.29	**0.00**^{c}
6		**0.07**^{c}	**0.04**^{c}	**0.04**^{c}	0.18	**0.00**^{r}		0.30	**0.00**^{c}
7		**0.00**^{c}	**0.01**^{c}	**0.01**^{c}	0.12	**0.00**^{r}	0.26		**0.01**^{c}
8		0.88	0.53	0.60	**0.02**^{c}	**0.00**^{r}	0.35	0.26	
	Hedged				α_{VaR} = 5%				
1			0.34	0.38	**0.08**^{r}	0.19	0.14	0.17	0.36
2		0.70		0.25	0.38	0.35	0.25	0.29	0.22
3		0.29	0.42		**0.02**^{r}	0.18	**0.05**^{r}	0.15	**0.07**^{c}
4		0.56	0.57	0.59		0.53	0.52	0.27	0.21
5	α_{VaR} = 2.5%	0.44	0.70	0.34	0.53		0.73	0.61	0.53
6		0.13	0.61	0.34	0.70	**0.01**^{r}		0.20	0.21
7		0.43	0.88	0.76	0.45	**0.05**^{r}	0.67		0.38
8		0.64	0.92	0.12	0.62	0.55	0.74	0.63	

Note: This table presents p-values of pairwise conditional Giacomini-White tests on Value-at-Risk (VaR) forecasts of the unhedged, and fully hedged portfolio returns. The test has a null of equal forecasting performance conditional on past information. We use the previous pairwise loss differential as information. The numbers above the diagonal are DM-statistics for VaR forecasts at α_{VaR} = 5%, and the numbers below the diagonal are those for α_{VaR} = 5%. For p-values below 10% we include the preferred model using the decision rule for the conditional test of Giacomini and Komunjer (2005). An 'r' ('l') denotes that in more than 50% of the periods in the forecast sample the test prefers the *row* (*column*) model.
The models are numbered as follows. 1: cDCC skew t, 2: cDCC t, 3: cDCC Gaussian, 4: Dynamic factor skew t, 5: Static skew t, 6: Static t, 7: Static Gaussian, and 8: Static factor skew t.

Table 9: Unconditional and conditional ES test results for unhedged portfolios

		Skew t	t	Gaussian	Factor skew t
α_{ES} = 0.05			*Dynamic copulas*		
	$\bar{H}(\alpha_{ES})$	0.013	0.013	0.014	0.020
	t-test	-1.67	-1.60	-1.55	-0.67
	BP(1)-test p-value	0.97	0.93	0.91	0.60
	BP(3)-test p-value	0.60	0.65	0.52	0.59
	BP(5)-test p-value	0.87	0.89	0.81	0.77
			Static copulas		
	$\bar{H}(\alpha_{ES})$	0.012	0.013	0.012	0.017
	t-test	-1.82	-1.69	-1.73	-1.09
	BP(1)-test p-value	0.94	0.98	0.99	0.77
	BP(3)-test p-value	0.47	0.23	0.38	0.90
	BP(5)-test p-value	0.77	0.50	0.69	0.97
α_{ES} = 0.025			*Dynamic copulas*		
	$\bar{H}(\alpha_{ES})$	0.007	0.007	0.007	0.009
	t-test	-1.06	-1.07	-1.03	-0.65
	BP(1)-test p-value	0.94	0.94	0.93	0.84
	BP(3)-test p-value	1.00	1.00	1.00	0.99
	BP(5)-test p-value	1.00	1.00	1.00	1.00
			Static copulas		
	$\bar{H}(\alpha_{ES})$	0.006	0.007	0.006	0.008
	t-test	-1.23	-1.15	-1.25	-0.88
	BP(1)-test p-value	0.99	0.97	0.98	0.89
	BP(3)-test p-value	1.00	1.00	1.00	1.00
	BP(5)-test p-value	1.00	1.00	1.00	1.00

Note: This table presents results for the unconditional and conditional ES tests of Du and Escanciano (2015) on the unhedged portfolios. The unconditional test is a t-test on the mean of the cumulative hit process, denoted $\bar{H}(\alpha_{ES})$, which should equal $\alpha_{ES}/2$ under the null hypothesis. The conditional BP(l)-test is a portmanteau Box-Pierce test for l lags on the autocorrelations of the cumulative hit process.

Table 10: Unconditional and conditional ES test results for fully hedged portfolios

		Skew t	t	Gaussian	Factor skew t
α_{ES} = 0.05			*Dynamic copulas*		
	$\bar{H}(\alpha_{ES})$	0.025	0.026	0.025	0.026
	t-test	0.02	0.10	-0.01	0.16
	BP(1)-test p-value	0.48	0.52	0.55	0.46
	BP(3)-test p-value	0.27	0.31	0.29	0.33
	BP(5)-test p-value	0.42	0.46	0.45	0.48
			Static copulas		
	$\bar{H}(\alpha_{ES})$	0.026	0.028	0.027	0.022
	t-test	0.17	0.37	0.21	-0.45
	BP(1)-test p-value	0.51	0.53	0.48	0.42
	BP(3)-test p-value	0.23	0.29	0.23	0.11
	BP(5)-test p-value	0.38	0.42	0.37	0.24
α_{ES} = 0.025			*Dynamic copulas*		
	$\bar{H}(\alpha_{ES})$	0.011	0.011	0.011	0.011
	t-test	-0.21	-0.34	-0.38	-0.26
	BP(1)-test p-value	0.95	0.99	0.89	0.76
	BP(3)-test p-value	0.01	0.01	0.01	0.03
	BP(5)-test p-value	0.05	0.05	0.05	0.09
			Static copulas		
	$\bar{H}(\alpha_{ES})$	0.013	0.013	0.012	0.008
	t-test	0.07	0.08	-0.08	-0.79
	BP(1)-test p-value	0.99	0.89	0.84	0.98
	BP(3)-test p-value	0.00	0.01	0.00	0.00
	BP(5)-test p-value	0.01	0.03	0.01	0.00

Note: This table presents results for the unconditional and conditional ES tests of Du and Escanciano (2015) on the fully hedged portfolios. The unconditional test is a t-test on the mean of the cumulative hit process, denoted $\bar{H}(\alpha_{ES})$, which should equal $\alpha_{ES}/2$ under the null hypothesis. The conditional BP(l)-test is a portmanteau Box-Pierce test for l lags on the autocorrelations of the cumulative hit process.

Table 11: Unconditional and conditional ES test results for unconstrained optimally-hedged portfolios

		Skew t	t	Gaussian	Factor skew t
$\alpha_{ES} = 0.05$			*Dynamic copulas*		
	$\bar{H}(\alpha_{ES})$	0.014	0.016	0.014	0.018
	t-test p-value	-1.53	-1.29	-1.47	-1.01
	BP(1)-test p-value	0.81	0.95	0.67	0.49
	BP(3)-test p-value	1.00	0.97	0.98	0.86
	BP(5)-test p-value	0.96	0.70	0.96	0.94
			Static copulas		
	$\bar{H}(\alpha_{ES})$	0.014	0.014	0.014	0.016
	t-test p-value	-1.56	-1.52	-1.53	-1.17
	BP(1)-test p-value	0.00	0.03	0.00	0.93
	BP(3)-test p-value	0.03	0.20	0.04	0.99
	BP(5)-test p-value	0.12	0.46	0.13	0.96
$\alpha_{ES} = 0.025$			*Dynamic copulas*		
	$\bar{H}(\alpha_{ES})$	0.003	0.002	0.003	0.005
	t-test p-value	-1.83	-2.10	-1.73	-1.38
	BP(1)-test p-value	0.21	0.00	0.30	0.90
	BP(3)-test p-value	0.19	0.00	0.36	1.00
	BP(5)-test p-value	0.16	0.00	0.38	1.00
			Static copulas		
	$\bar{H}(\alpha_{ES})$	0.002	0.001	0.002	0.008
	t-test p-value	-1.98	-2.25	-2.04	-0.96
	BP(1)-test p-value	0.02	0.00	0.00	0.91
	BP(3)-test p-value	0.00	0.00	0.00	1.00
	BP(5)-test p-value	0.00	0.00	0.00	1.00

Note: This table presents results for the unconditional and conditional ES tests of Du and Escanciano (2015) on the constrained currency-hedged portfolios. The unconditional test is a t-test on the mean of the cumulative hit process, denoted $\bar{H}(\alpha_{ES})$, which should equal $\alpha_{ES}/2$ under the null hypothesis. The conditional BP(l)-test is a portmanteau Box-Pierce test for l lags on the autocorrelations of the cumulative hit process.

Table 12: Unconditional and conditional ES test results for constrained optimally-hedged portfolios

		Skew t	t	Gaussian	Factor skew t
$\alpha_{ES} = 0.05$			*Dynamic copulas*		
	$\bar{H}(\alpha_{ES})$	0.018	0.019	0.018	0.018
	t-test p-value	-0.92	-0.84	-1.01	-0.96
	BP(1)-test p-value	0.83	0.78	0.88	0.68
	BP(3)-test p-value	0.96	0.93	0.96	0.95
	BP(5)-test p-value	0.98	0.97	0.98	0.98
			Static copulas		
	$\bar{H}(\alpha_{ES})$	0.019	0.019	0.020	0.017
	t-test p-value	-0.77	-0.76	-0.74	-1.11
	BP(1)-test p-value	0.52	0.57	0.49	0.69
	BP(3)-test p-value	0.84	0.81	0.82	0.95
	BP(5)-test p-value	0.92	0.91	0.90	0.98
$\alpha_{ES} = 0.025$			*Dynamic copulas*		
	$\bar{H}(\alpha_{ES})$	0.006	0.006	0.006	0.007
	t-test p-value	-1.30	-1.16	-1.20	-1.02
	BP(1)-test p-value	0.93	0.97	1.00	0.92
	BP(3)-test p-value	1.00	1.00	1.00	1.00
	BP(5)-test p-value	1.00	1.00	1.00	1.00
			Static copulas		
	$\bar{H}(\alpha_{ES})$	0.007	0.007	0.006	0.006
	t-test p-value	-1.14	-1.15	-1.17	-1.29
	BP(1)-test p-value	0.96	0.97	0.98	0.96
	BP(3)-test p-value	1.00	1.00	1.00	1.00
	BP(5)-test p-value	1.00	1.00	1.00	1.00

Note: This table presents results for the unconditional and conditional ES tests of Du and Escanciano (2015) on the constrained currency-hedged portfolios. The unconditional test is a t-test on the mean of the cumulative hit process, denoted $\bar{H}(\alpha_{ES})$, which should equal $\alpha_{ES}/2$ under the null hypothesis. The conditional BP(l)-test is a portmanteau Box-Pierce test for l lags on the autocorrelations of the cumulative hit process.

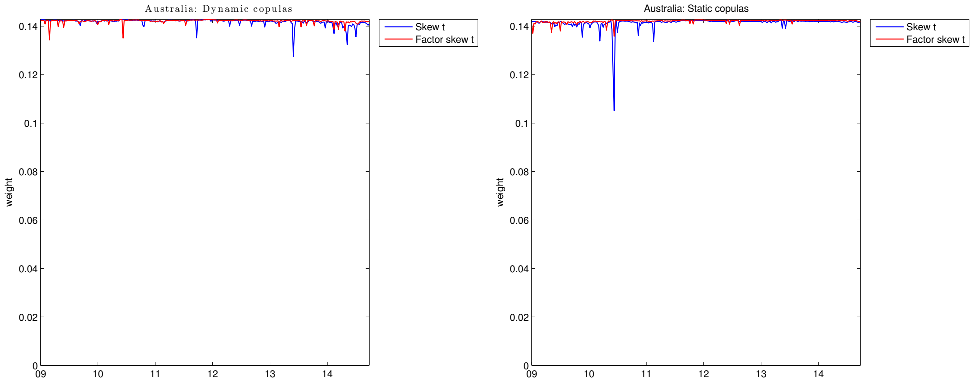


Figure 1: Optimal net currency exposure of a US-based investor to the Australian market, in a constrained portfolio.

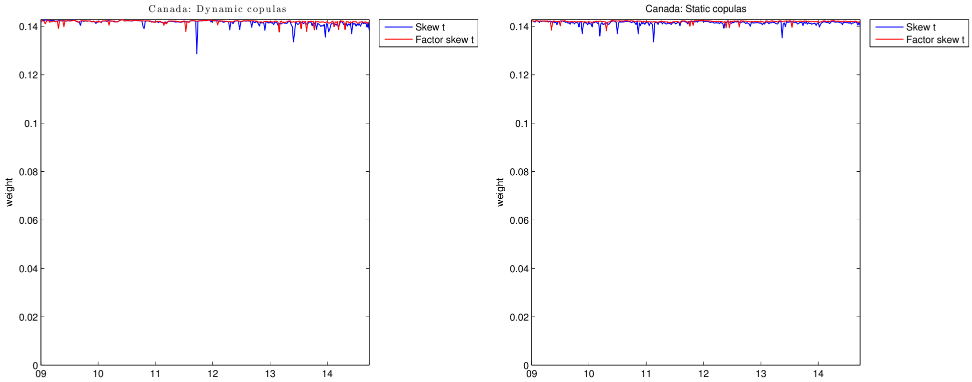


Figure 2: Optimal net currency exposure of a US-based investor to the Canadian market, in a constrained portfolio.

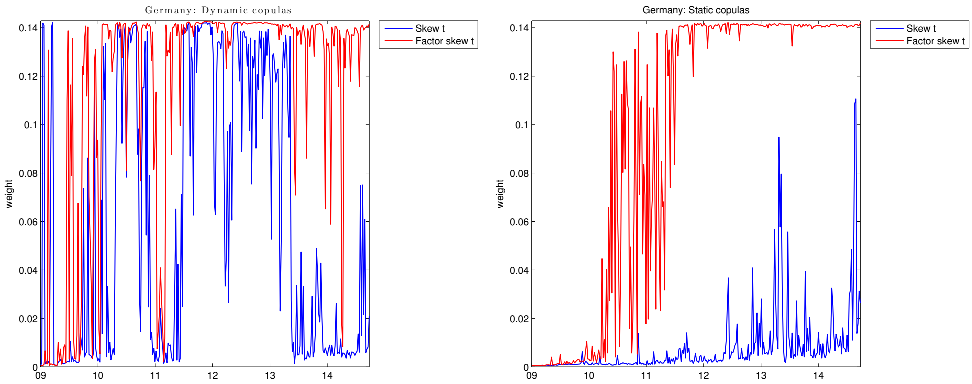


Figure 3: Optimal net currency exposure of a US-based investor to the German market, in a constrained portfolio.

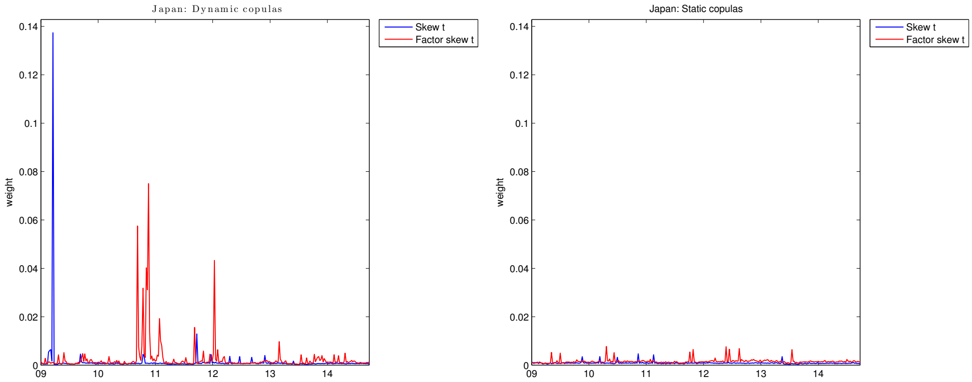


Figure 4: Optimal net currency exposure of a US-based investor to the Japanese market, in a constrained portfolio.

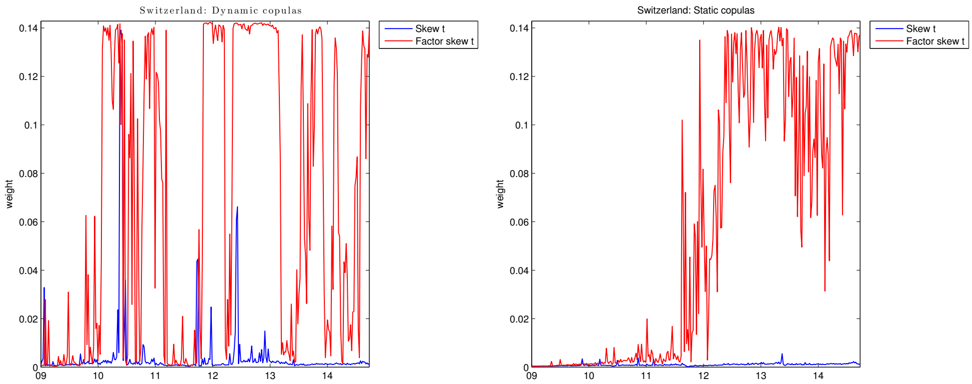


Figure 5: Optimal net currency exposure of a US-based investor to the Swiss market, in a constrained portfolio.

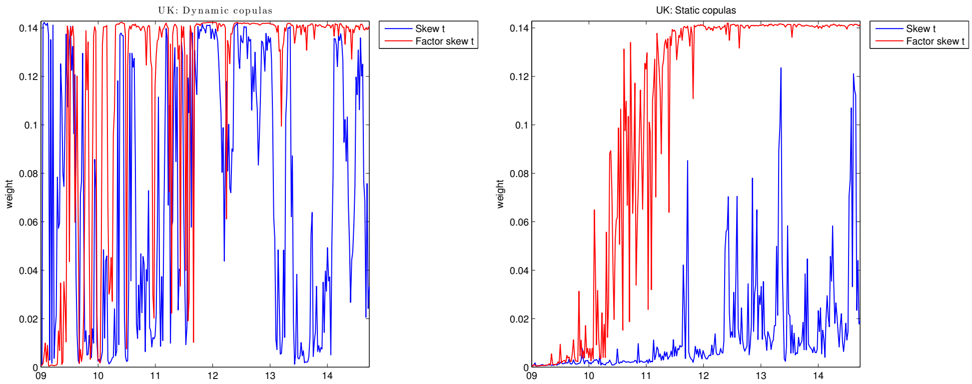


Figure 6: Optimal net currency exposure of a US-based investor to the UK market, in a constrained portfolio.

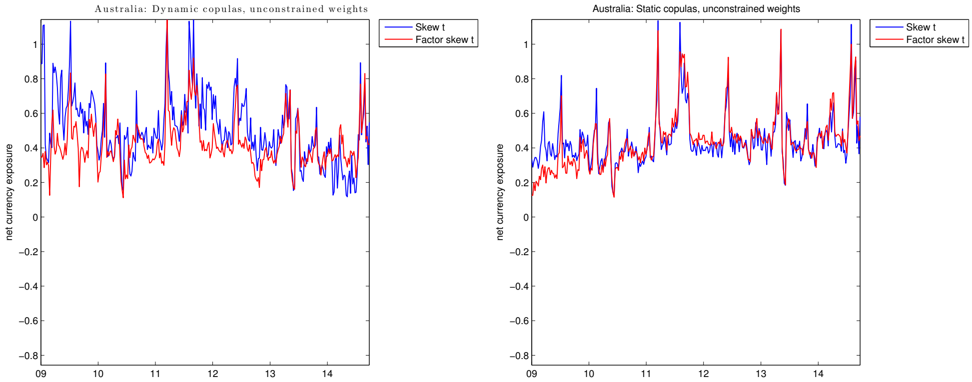


Figure 7: Optimal net currency exposure of a US-based investor to the Australian market, in a unconstrained portfolio.

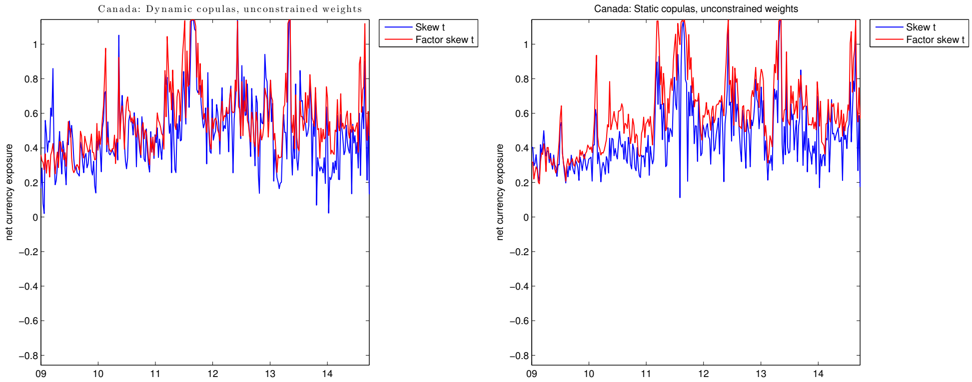


Figure 8: Optimal net currency exposure of a US-based investor to the Canadian market, in a unconstrained portfolio.

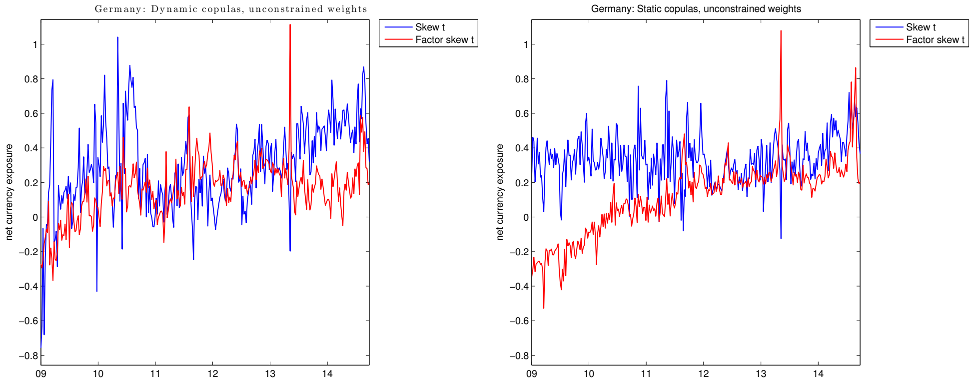


Figure 9: Optimal net currency exposure of a US-based investor to the German market, in a unconstrained portfolio.

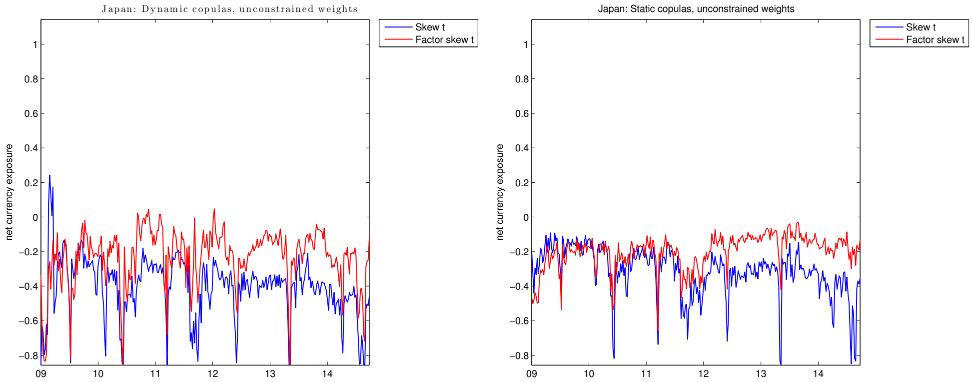


Figure 10: Optimal net currency exposure of a US-based investor to the Japanese market, in a unconstrained portfolio.

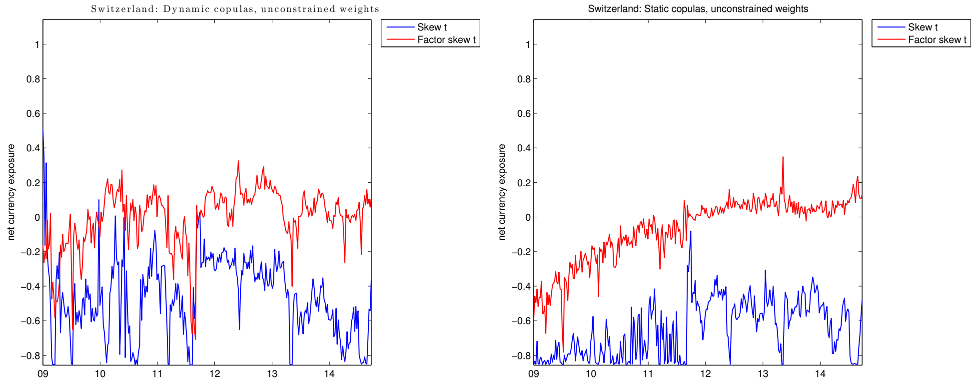


Figure 11: Optimal net currency exposure of a US-based investor to the Swiss market, in a unconstrained portfolio.

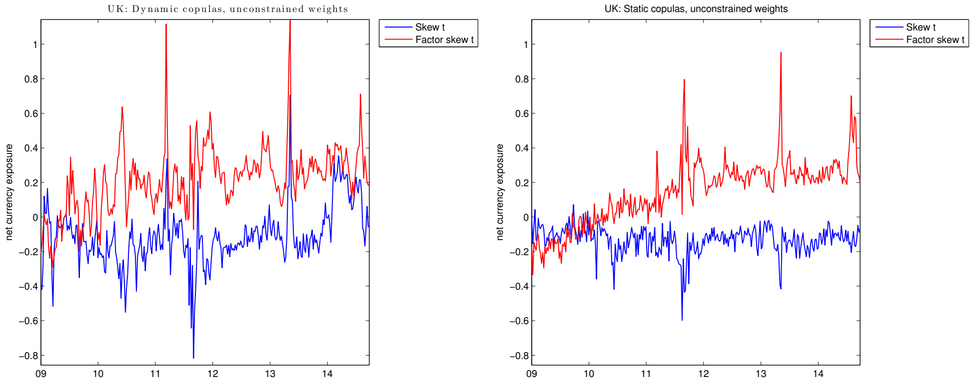


Figure 12: Optimal net currency exposure of a US-based investor to the UK market, in a unconstrained portfolio.

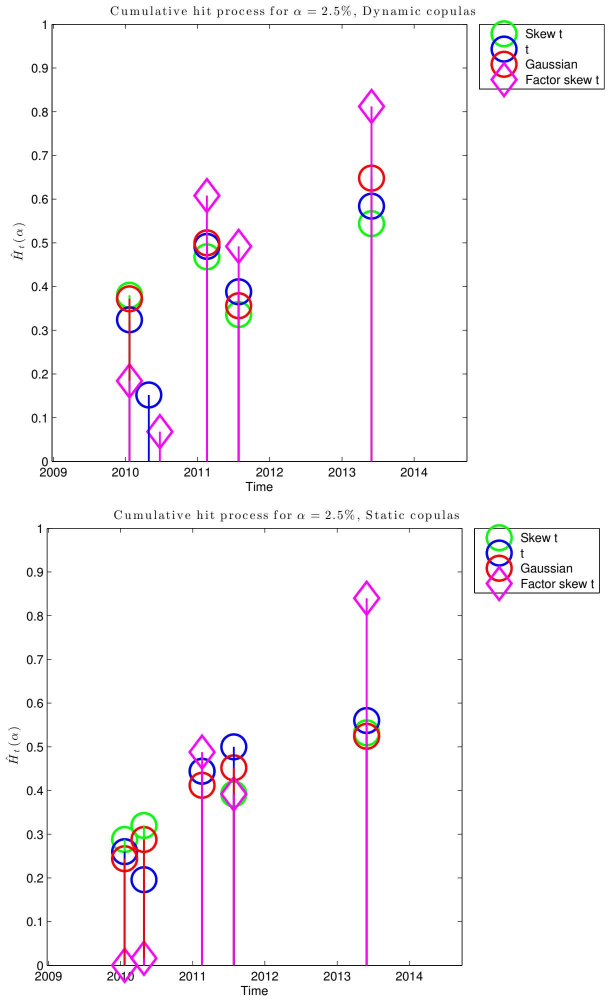
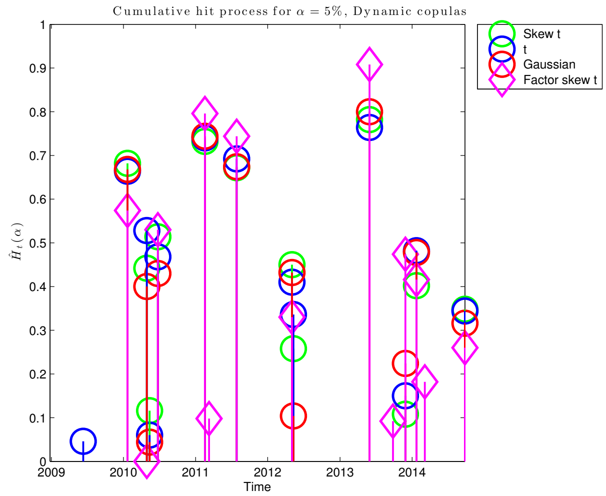


Figure 13: The cumulative hit process $\hat{H}_t(\alpha)$ of Du and Escanciano (2015) for the nonspeculative currency-hedged portfolios based on dynamic and static copula models for level $\alpha = 0.025$.



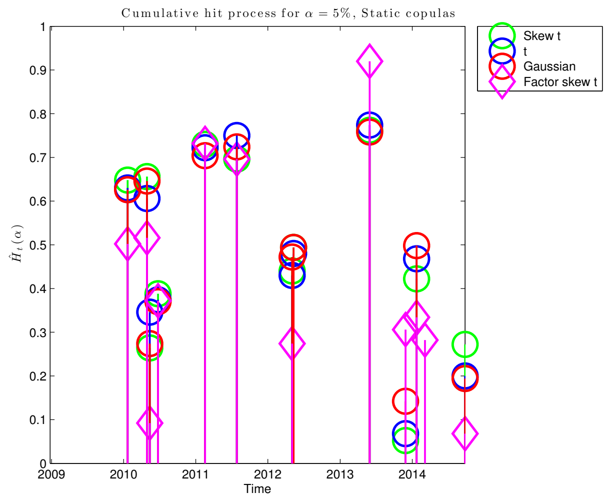


Figure 14: The cumulative hit process $\hat{H}_t(\alpha)$ of Du and Escanciano (2015) for the nonspeculative currency-hedged portfolios based on dynamic and static copula models for level $\alpha = 0.05$.