The Competition Effect in a Public Procurement Model
An error-in-variables approach

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Abstract

Auction theory suggests that as the number of bidders (competition) increases, the sizes of the participants' bids decrease. An issue in the empirical literature on auctions is which measurement(s) of competition to use. Utilizing a dataset on public procurements containing measurements on both the actual and potential number of bidders I find that a workhorse model of public procurements is best fitted to data using only actual bidders as measurement for competition. Acknowledging that all measurements of competition may be erroneous, I propose an instrumental variable estimator that (given my data) brings about a competition effect bounded by those generated from models using the actual and potential number of bidders, respectively. Also, some asymptotic results are provided for non-linear least squares estimators obtained from a dependent variable transformation model.

Keywords: dependent variable transformation model, instrumental variable, measurement error, non-linear least squares

JEL classification: C26, C51, C57, D22, D44

1 Introduction

Auctions are used to extract consumers or producers surplus in favor of the auctioneer. Many firms and governments take advantage of this mechanism to lower sellers' prices and increase their own or the public's benefit by using (descending first price sealed bid) auctions as the foundation for procurements. Mainstream economic theory (e.g., Riley and Samuelson, 1981; McAfee and McMillan, 1987; Milgrom, 1989) implies that an increased number of bidders leads to lower bids in such auctions where bidders compete for contracts.

To stylize the intuition, denote a firm's bid by b, its cost by c and the corresponding markup by $m = b - c \geq 0$. The currently dominating one of the economic theories suggests that

$$b = c + m\left(N\right) \tag{1}$$

where $m\left(\cdot\right)$ is a decreasing function in the number of bidders, N. So, as N grows, the distance $b - c$ decreases; this is the competition effect. In this paper, I will use the words *firm* and *bidder* interchangeably as firms cast the bids in the procurement auctions under study.

The workhorse model in empirical studies of auctions shares the general structure with (1). A common objective of those studies is the estimation of firms' costs[1] (as in, e.g., Krasnokutskaya and Seim, 2011; Marion, 2007; Flambard et al., 2007; Shneyerov, 2006; Henderson et al., 2012).[2] The usual line of action to motivate estimators in such studies is to see $m\left(\cdot\right)$ as a function implicitly determined by the observed b and then to calculate $\hat{m}\left(\cdot\right)$ (Guerre et al., 2000). This estimator $\hat{m}\left(\cdot\right)$ in conjunction with b then provides estimates of c, as $\hat{c} = b - \hat{m}\left(\cdot\right)$. However, using an incorrectly measured N may make the estimator $\hat{c}$ biased and inconsistent which is detrimental to the quality of the conclusions provided by those studies.

The objective of the present paper is to study how estimates of the auction model in (1) differ when using different but plausible measures of competition, i.e. different manifest variables for the latent variable N; this explains the subtitle of the paper. I will consider how the different measurements bring about the competition effect, i.e. $\widehat{\partial b/\partial N}$.[3] This exercise is enabled by a detailed dataset on Swedish public procurements for internal cleaning services. Consequently, this paper sheds light on some measurement issues.

[1] Or valuations if the bidders are buyers, as in, e.g., Laffont et al. (1995) and Li (2010). Then the corresponing version of (1) is $b = v - e\left(N\right)$ where v denotes the buyer's valuation and $e\left(\cdot\right)$ is a markdown.

[2] For a broader discussion on the topic, see Reiss and Wolak (2007) and Einav and Levin (2010).

[3] Note that I will use the notation $\partial b/\partial N$ for the competition effect, even though N is an integer.

In (1), N is the number of firms that the bidders see as competition (including themselves). This is understood to be the *potential* number of bidders (Li, 2010)[4] which I here denote by n_p. However, it is seldom the case that all potential bidders cast bids in every auction: most often there are only $n_a < n_p$ *actual* bidders participating. As a consequence of a lack of measurements on n_p many empirical applications of (1) use n_a to proxy for N (see, e.g., Guerre et al. (2000) and Li (2010)). This implies that these studies do not distinguish between potential and actual bidders. Most likely reality does, however. So, while theory sets $N \equiv n_p$ (see footnote 5), some empirical studies set $N \equiv n_a$ or that N is a function of n_a (Guerre et al., 2000; Li, 2010). Yet, useful empirical implementation of theory to generate sound policy guidance necessitates that the notion of competition is made practically operational. If not, one can not provide econometrically sound measures of the competition effect. That being said, theory is not clear on how to define a potential bidder, although some definitions have emerged in specific applications.[5] In this paper, I will make use of novel data enabling me to consider a firm as potential bidder if the following two conditions are fulfilled: 1) the firm is registered in the same geographic area as where the procurement is being held and 2) the firm is registered as a producer of the service under consideration by the procurer. All these firms are eligible to place a bid in a public procurement auction and are therefore potential bidders in the general meaning of the word.

Of course, it may be the case that neither n_a nor n_p are ideal measurements on how firms perceive competition. In this case including either n_a or n_p for N may lead to biased and inconsistent estimators due to measurement error. We will see (in Section 2) that the measure of competition will enter the theoretical as well as the empirical model in a non-linear fashion. Inconsistent estimators resulting from measurement error in non-linear models with more than one covariate is a problematic task to correct without more information or assumptions on the measurement error (Carroll et al., 1995). To remedy this situation I propose an instrumental variable that makes consistent estimation of the competition effect possible.

The primary contributions I give in this paper are the following. First, I provide some conjectures on how the most commonly employed approach to estimate firms' costs in public procurements behaves under different measurements of competition. Second,

[4]In particular, see the discussion surrounding footnote 18, p. 126.

[5]One such application is the commonly studied institution of the procurement of U.S. highway works (e.g., Li and Zheng, 2009; Somaini, 2011). In these auctions firms can collect blueprints and then a subset of the collectors choose to cast bids or not. Here it is natural to define all collectors as the potential set of bidders, as the number of collectors are public information. In most other cases though, n_a is used instead of n_p as the measure of competition. The case where researchers use n_a to proxy for n_p is equivalent to seeing the number of actual bidders as the number of potential bidders (as in, e.g., Guerre et al. (2000) and Li (2010)). More specifically, those studies take the number of potential bidders to be the largest n_a across auctions. Another way to sidestep the problem is to assume that the number of bidders is exogenously given and common knowledge (as in, e.g., Rezende (2008)).

I propose an instrumental variable estimator for the competition effect. Third, I give some asymptotic results for a non-linear least squares estimator in a dependent variable transformation framework.

The paper proceeds as follows. In the next section I discuss the theoretical underpinnings of the workhorse economic analysis of procurement auctions. In Section 3, I discuss my choice of empirical model along with estimation issues. The section also provides some results on the asymptotic behavior of a dependent variable transformation model. Some discussions on measurement error issues and an instrumental variable estimator are given here as well. Section 4 illustrates the finite sample properties of the estimator in the chosen modeling approach. In Sections 5 and 6 I discuss the data I utilize and the results from the estimation exercises, respectively. In Section 7 I make some concluding remarks.

2 Theoretical Underpinnings

Public procurements are in general arranged as first price sealed-bid auctions where the firm that casts the lowest bid wins the contract. I emphasize that I want to study measurement issues related to the empirical side of the mainstream approach to model such auctions and how they affect the empirical results in such studies, and not the theoretical model itself. Yet, I briefly describe the workhorse theoretical model as follows. First, assume that bidders are risk-neutral, symmetric and independent. Here, symmetric means that the firms' costs are allowed to differ but only as draws from the same distribution. Moreover, this distribution is assumed to be common knowledge.

Following Guerre et al. (2000), let $b = f(c)$ be a bid function on which a Bayesian Nash equilibrum assumption is invoked. This means that all firms use the same strategy (i.e. bid function) when determining their bids. The function $f(\cdot)$ is assumed to be strictly increasing, i.e. one-to-one. Based on these assumptions the expected profit from firm 1's point of view can be written as

$$(b_1 - c_1)\, P\left(b_1 < b_i \mid c_1\right), \quad \forall i \in I_{-1}$$

where b_1 and c_1 is the bid and cost of firm 1, respectively. The I_{-1} denotes the set of all bidders except bidder 1. Note that c_1 and hence b_1 is known (i.e. fixed) to firm 1, while all other firms' costs (and bids) are stochastic to firm 1 following the distribution discussed above. Also, one can write $P\left(b_1 < b_i \mid c_1\right) = \left[1 - G\left(b_1\right)\right]^{N-1}$ under the assumptions stated above.[6] Now, assuming firm i to choose bid as to maximize expected

[6]The symmetry assumption implies that the bidders' know the distribution of the costs, and the Bayesian Nash assumption that all bidders choose the same bid function, i.e. the same function $b_i = f(c_i)$. Note that there is no subscript on $f(\cdot)$. Further, the independence assumption allows for multiplying the $N -$

profits leads to the following bid

$$b_i = c_i + \frac{[1 - G(b_i)]}{g(b_i)} \frac{1}{N-1} \tag{2}$$

where $G(\cdot)$ and $g(\cdot)$ denote the cumulative distribution function (cdf) and probability density function (pdf) of the bids, respectively (Guerre et al., 2000). Note the similarity to (1): the second term of the right hand side in (2) is the markup, i.e. $m(\cdot)$ in (1). So, theory (e.g., Riley and Samuelson, 1981; Klemperer, 1999; Krishna, 2009) shows us that a larger number of bidders lowers markups and hence the sizes of the bids, ceteris paribus; this is seen in (2). This competition effect is defined as how the bidder's optimal decision (2) changes when the number of competitors change. Mathematically, this can be displayed as

$$\frac{\partial b}{\partial N} = -\frac{[1 - G(b_i)]}{g(b_i)} \frac{1}{(N-1)^2} < 0 \tag{3}$$

Considering (2) and (3), it is evident that the competition effect only affects the size of the bid through the markup. This is particular to first price sealed bid auctions (Krishna, 2009). Of course, the intuition is that a larger N makes it more likely that the firm will not win the auction i.e. that someone will underbid it, so the firm adjusts its bid downward, closer to its cost. Note that the size of (3) can be calculated only once the model (2) is estimated.

3 Empirical Modeling and Estimation

Most studies using a setup of the type in (2) aim at the estimation of c_i directly. The endeavor here is different as I want to study the consequences of the choice of measurement of competition (e.g., n_a, n_p or some combination thereof) to use for N in the term $1/(N-1)$ of the agent's optimal decision (2). Different measures of N are likely to lead to different conjectures on the effect of competition, (3). As a consequence, I take the cost in (2) to be generated by an economic model. This allows me to focus on the choice of manifest variable for N. As is common in economic theory, I assume that firms minimize their costs given their commitment to supply a certain quantity, q, of a good or service. Further, I assume that the process that transforms their input factors capital and labor into output can be described by a Cobb-Douglas type of production function (Cobb and Douglas, 1928). Assuming that firms are minimizing costs to deliver the quantity q specified in the contract, I write the firms' cost function as

1 probabilities together, i.e. $[1 - G(b_1)]^{N-1}$. Taken together these three conditions allow one to write $P(b_1 < b_i \mid c_1) = [1 - G(b_1)]^{N-1}$.

$$c\left(w,r,q\right)=Aw^{a_w/\left(a_r+a_w\right)}r^{a_r/\left(a_r+a_w\right)}q^{1/\left(a_r+a_w\right)} \tag{4}$$

Here w and r are prices on capital and labor, respectively, while the a_w and a_r are output elasticities in the Cobb-Douglas production function. The A is a constant composed of a_w and a_r.[7] I have measures for w, r and q in the dataset.

Now, adopting (2) to a regression framework we observe that the dependent variable, the bid, is transformed in a non-linear way. To be more explicit, the dependent variable is written as

$$b_i-\frac{\left[1-G\left(b_i\right)\right]}{g\left(b_i\right)}\frac{1}{N-1}$$

Note that the transformation depends on the unknown parameters that determine $G\left(\cdot\right)$ and $g\left(\cdot\right)$. This is reminiscent of Zellner and Revankar's (1969) generalized production function where the dependent variable, output, is written in a non-linear fashion and estimation is by maximum likelihood. Abrevaya and Hausman (2004) consider dependent variable transformation models in a setting where the dependent variable is contaminated by measurement error. They suggest some transformations in a Box and Cox (1964) sense and also take a Taylor-approximation approach. I do not choose either of the Zellner and Revankar or Abrevaya and Hausman approaches. If I, in the fashion of Zellner and Revankar, would go for maximum likelihood and bring a distributional assumption to the model, while considering the structure of the functional form in (2) with $G\left(\cdot\right)$ and $g\left(\cdot\right)$, the distribution of the dependent variable would be very complicated. Further, the intractable form of the expansions of the tranformation of the dependent variable in Abrevaya and Hausmans' approach is a drawback in the setup of this current paper. That being said, b_i is implicitly defined and the properties of a non-linear least squares estimator should not be affected as such.

Consequently, and as (2) is a highly non-linear function in the parameters I suggest the following semi-parametric non-linear least squares criterion function

$$Q\left(\boldsymbol{\theta}\right)=\frac{1}{K}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{i=1}^{I}\left\{b_{lji}-c_{lji}-\frac{\left[1-G\left(b_{lji}\right)\right]}{g\left(b_{lji}\right)}h_{lj}\right\}^2 \tag{5}$$

to estimate the parameters in (2). The $l=1,\ldots,L$ indicates geographic area, $j=1,\ldots,J$ indicates auctions, $i=1,\ldots,I$ indicates firms, and K is the sample size.[8] Here, $\{\cdot\}$ is the

[7]The Cobb-Douglas assumption is not innocent; (4) implies that firms do not vary in their capital to labor ratio (i.e. that firms technologies are Hicks-neutral) and also that the elasticity of substitution between labor and capital to be one (Mas-Colell et al., 1995). Hence, the choice of cost function (equivalently, technology) must be made carefully, taking the nature of the firms generating the data into consideration. I will discuss how the Cobb-Douglas assumption is appropriate to my data in Section 5.

[8]This notation implies that there is at least one measurement of each firm. It is common that datasets

error term ν_{lji} whose sum of squares is to be minimized by choosing the appropriate $\boldsymbol{\theta}$. I denote the $\boldsymbol{\theta}$ that minimizes (5) by $\hat{\boldsymbol{\theta}}_{NLS}$ and this is the non-linear least squares estimator. For future reference I denote $c_{lji} + \left\{ \left[1 - G\left(b_{lji}\right)\right] / g\left(b_{lji}\right) \right\} h_{lj}$ by f_{lji}. That is, $b_{lji} - \hat{f}_{lji} = \hat{\nu}_{lji}$ is the residual. I do not put any distributional restrictions on the error term ν_{lji}, i.e. I assume a semiparametric setting. The h_{lj} is shorthand for the multiplicative factor $1/\left(N_{lj} - 1\right)$ and is to be modeled in different ways as will be described below. The subscripts l and j on h indicates that the measure of competition possibly varies over geographic areas and auctions. Moreover, $G(\cdot)$ and $g(\cdot)$ is the cdf and pdf of some family of distributions, both parameterized by fixed but unknown parameters such as the mean μ and the standard deviation σ.

From (2) and (5) it is clear that costs enter the agents' optimization problem. Considering (4), I propose the following form of the cost function to be estimated

$$c_{lji} = \alpha w_l^{\beta_1} r_l^{\beta_2} q_j^{\beta_3} - \nu_{lji} \tag{6}$$

In my model, (6) is substituted for c_{lji} in (5). In (6), w_l is the price of labor in the geographical area of auction j at the time of the letting process and r_l is the price of capital at the time of the letting process. The q_j specifies the size of contract j. The error term ν_{lji} defined in (5) allows for the individual-auction specific unobserved heterogeneity to vary over firms.

Finally, I recognize the possiblity to employ some additional observed auction-specific heterogeneity in the model. First, I make assumptions on how some specific observed heterogeneity affects specific input factors and the quantity of output to be supplied. Then, I let the heterogeneity enter the model through the parameters β_1, β_2 and β_3. This allows me to write β_1, β_2 and β_3 as β_{1j}, β_{2j} and β_{3j}, respectively. More explicitly $\beta_{1j} = \boldsymbol{x}'_{1j}\boldsymbol{\pi}$, $\beta_{2j} = \boldsymbol{x}'_{2j}\boldsymbol{\phi}$ and $\beta_{3j} = \boldsymbol{x}'_{3j}\delta$. Here $\left(\boldsymbol{x}'_{1j}, \boldsymbol{x}'_{2j}, \boldsymbol{x}'_{3j}\right)'$ represents auction-specific observed heterogeneity assumed to influence the cost of the input factors labor and capital as well as the quantity to be supplied, respectively. The $\left(\boldsymbol{\pi}', \boldsymbol{\phi}', \delta'\right)$ consists of the parameters corresponding to $\left(\boldsymbol{x}'_{1j}, \boldsymbol{x}'_{2j}, \boldsymbol{x}'_{3j}\right)'$. The parameter vector to be estimated is $\boldsymbol{\theta}' = \left(\mu, \sigma, \alpha, \boldsymbol{\pi}', \boldsymbol{\phi}', \delta'\right)$ and its estimator is $\hat{\boldsymbol{\theta}}_{NLS}$.

Summing up, the model to be estimated is stated as follows.

$$b_{lji} = \alpha w_l^{\boldsymbol{x}'_{1j}\boldsymbol{\pi}} r_l^{\boldsymbol{x}'_{2j}\boldsymbol{\phi}} q_j^{\boldsymbol{x}'_{3j}\delta} + \nu_{lji} + \frac{\left[1 - G\left(b_{lji}; \mu, \sigma\right)\right]}{g\left(b_{lji}; \mu, \sigma\right)} h_{lj} \tag{7}$$

Note the resemblance to (1) and (2). When comparing (5) (or (7)) to the firm's optimal

of auctions assume this repeated measurements-form where firms are observed participating in several auctions. The sample size is $K = \sum_{l=1}^{L} \sum_{j=1}^{J} \sum_{i=1}^{I} \mathbb{1}\left(i \in j\right)$ where $\mathbb{1}\left(\cdot\right)$ is the indicator function assuming the value 1 if firm i participates in auction j, denoted as $i \in j$. See Section 5 for a descripton of the data I utilize to get a feel for the different levels as indicated by l, j and i, respectively.

decision (2) we see that h_{lj} is equivalent to $1/\left(N_{lj}-1\right)$. The main object of interest in this paper is the competition effect, i.e. (3) and the endeavour is to estimate it soundly. However, estimation of (7) is required to enable calculating (3).

Considering $G(\cdot)$ and $g(\cdot)$, I choose them to represent the cdf and pdf of the log-normal distribution, respectively. The motivation for choosing this particular distribution is that I, elsewhere (Sundström, 2014) cannot reject the hypothesis that the bids (and costs) are log-normally distributed in the current dataset.[9]

As I have information on both n_a and n_p I estimate (7) using both $h_{lj}=1/\left(n_{a,j}-1\right)$ (Model 1) and $h_{lj}=1/\left(n_{p,l}-1\right)$ (Model 2). Estimation of (7) as Model 1 and Model 2 allows for a comparison of estimates and the corresponding goodness of fit measures to be discussed in Section 7. Note that, in general, n_a is measured on the auction level j while n_p is measured on the level of the geographic area, l.

In addition, I consider some linear combinations of n_a and n_p as measures of the latent variable N in terms of the goodness of fit of the model to data. That is, I consider $h_{lj}=1/\left\{\psi(n_{a,j}-1)+(1-\psi)\left(n_{p,l}-1\right)\right\}$ for fixed $\psi\in[0,1]$. This makes it possible to study the fit of the model over different values on ψ, i.e. for different linear combinations of the measure of competition.

3.1 Consistency and Asymptotic Normality

To provide arguments that $\hat{\boldsymbol{\theta}}_{NLS}$ is consistent and asymptotically normal, I utilize Newey and McFadden's (1994) theorems on the asymptotic behavior of extremum estimators. The estimator $\hat{\boldsymbol{\theta}}$ is said to be an extremum estimator if there is an objective function $\hat{Q}_K(\boldsymbol{\theta})$, where K is the sample size, such that

$$\hat{\boldsymbol{\theta}} \text{ maximizes } \hat{Q}_K(\boldsymbol{\theta}) \text{ subject to } \boldsymbol{\theta}\in\boldsymbol{\Theta} \tag{8}$$

Here, $\boldsymbol{\Theta}$ denotes the parameter space. Further, define $Q_0(\boldsymbol{\theta})$ to be the limit of $\hat{Q}_K(\boldsymbol{\theta})$ as $K\to\infty$, i.e. $Q_0(\boldsymbol{\theta})=E\left[\left(b-c-\{[1-G(b)]/g(b)\}h\right)^2\right]$ in this setup. That is, $Q_0(\boldsymbol{\theta})$ can be interpreted as the (true) objective function derived from the population regression function. Newey and McFaddens consistency and asymptotic normality theorems are stated as follows.

Theorem (Newey and McFadden (1994, Theorem 2.1)) If there is a function $Q_0(\boldsymbol{\theta})$ such that (i) $Q_0(\boldsymbol{\theta})$ is uniquely minimized at the true parameter value $\boldsymbol{\theta}_0$, (ii) the parameter space $\boldsymbol{\Theta}$ is compact, (iii) $Q_0(\boldsymbol{\theta})$ is continuous and (iv) $\hat{Q}_K(\boldsymbol{\theta})$ converges uniformly in probability to $Q_0(\boldsymbol{\theta})$, then $\hat{\boldsymbol{\theta}}\xrightarrow{p}\boldsymbol{\theta}_0$.

[9] In buying auctions, valuations (the equivalent to cost in buying auctions) is often assumed to be log-normally distributed, as in, e.g., Laffont et al. (1995).

Proof See Newey and McFadden (1994, p. 2121-2122).

Theorem (Newey and McFadden (1994, Theorem 3.1)) Suppose that $\hat{\boldsymbol{\theta}}$ satisfies (8), $\hat{\boldsymbol{\theta}} \xrightarrow{P} \boldsymbol{\theta}_0$ and (i) $\boldsymbol{\theta}_0$ is contained in the interior of $\boldsymbol{\Theta}$, (ii) $\hat{Q}_K(\boldsymbol{\theta})$ is twice continuously differentiable in a neighborhood $\mathcal{N}$ of $\boldsymbol{\theta}_0$, (iii) $\sqrt{K}\nabla_0\hat{Q}_K(\boldsymbol{\theta}_0) \xrightarrow{d} N(\mathbf{0}, \boldsymbol{\Sigma})$, (iv) there is a Hessian $H(\boldsymbol{\theta})$ that is continuous at $\boldsymbol{\theta}_0$ and $\sup_{\boldsymbol{\theta}\in\mathcal{N}} \parallel \nabla_{\boldsymbol{\theta}\boldsymbol{\theta}}\hat{Q}_K(\boldsymbol{\theta}) - H(\boldsymbol{\theta}) \parallel \xrightarrow{P} 0$ and (v) $H = H(\boldsymbol{\theta}_0)$ is nonsingular. Then $\sqrt{K}\left(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0\right) \xrightarrow{d} N\left(\mathbf{0}, \boldsymbol{H}^{-1}\boldsymbol{\Sigma}\boldsymbol{H}^{-1}\right)$.

Proof See Newey and McFadden (1994, p. 2143).

Utilization of the above theorems allow for the following results.

Theorem (Consistency and asymptotic normality of $\hat{\boldsymbol{\theta}}_{NLS}$) The estimator $\hat{\boldsymbol{\theta}}_{NLS}$ generated by the non-linear least squares criterion function in (5) is consistent and asymptotically normal.

Proof See Appendix A.

In the current context , the $\boldsymbol{H}$ and $\boldsymbol{\Sigma}$ of $\sqrt{K}\left(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0\right) \xrightarrow{d} N\left(\mathbf{0}, \boldsymbol{H}^{-1}\boldsymbol{\Sigma}\boldsymbol{H}^{-1}\right)$ (see the last line of the asymptotic normality theorem of Newey and McFadden) are defined as

$$\boldsymbol{H} = \text{plim}\, \frac{1}{K}\boldsymbol{D}'\boldsymbol{D}\Big|_{\boldsymbol{\theta}_0} \qquad \text{and} \qquad \boldsymbol{\Sigma} = \text{plim}\, \frac{1}{K}\boldsymbol{D}'\boldsymbol{\Omega}_0\boldsymbol{D}\Big|_{\boldsymbol{\theta}_0}$$

Here, $\boldsymbol{D} = \partial\boldsymbol{\nu}/\partial\boldsymbol{\theta}'$ and $\boldsymbol{\nu} = \boldsymbol{b} - \boldsymbol{f}$.[10] The sample counterparts to $\boldsymbol{H}$ and Σ are

$$\hat{\boldsymbol{H}} = \frac{1}{K}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{i=1}^{I} \frac{\partial \nu_{lji}}{\partial\boldsymbol{\theta}}\Big|_{\hat{\boldsymbol{\theta}}} \frac{\partial \nu_{lji}}{\partial\boldsymbol{\theta}'}\Big|_{\hat{\boldsymbol{\theta}}} \qquad \text{and} \qquad \hat{\boldsymbol{\Sigma}} = \frac{1}{K}\hat{\boldsymbol{D}}'\hat{\boldsymbol{\Omega}}\hat{\boldsymbol{D}}$$

where $\hat{\boldsymbol{D}} = \partial\boldsymbol{\nu}/\partial\boldsymbol{\theta}' \mid_{\hat{\boldsymbol{\theta}}}$ and $\hat{\boldsymbol{\Omega}} = \text{diag}\left(\hat{\nu}_{lji}^2\right)$. This leads to the following way of estimating the asympotic covariance matrix in a fashion that is consistent under heteroskedasticity:

$$\hat{\boldsymbol{V}}\left(\hat{\boldsymbol{\theta}}\right) = \left(\hat{\boldsymbol{D}}'\hat{\boldsymbol{D}}\right)^{-1}\hat{\boldsymbol{D}}'\hat{\boldsymbol{\Omega}}\hat{\boldsymbol{D}}\left(\hat{\boldsymbol{D}}'\hat{\boldsymbol{D}}\right)^{-1}$$

In the case of homoskedasticity, $\boldsymbol{\Sigma} = \sigma^2\boldsymbol{H}$ which yields $\hat{\boldsymbol{V}}\left(\hat{\boldsymbol{\theta}}\right) = \sigma^2\boldsymbol{H}^{-1}$ and an estimator that is asymptotically efficient among least squares estimators (Cameron and Trivedi, 2005, p. 154).

[10] Here, $\boldsymbol{b}$ is a column vector of all the b_{lji} and $\boldsymbol{f}$ is a column vector consisting of all the f_{lji}. So, $\boldsymbol{\nu}$ is a column vector of all the residuals. Also, see footnote 8 for further information on how the typical dataset is arranged in these settings.

3.2 Measurement Error and an Instrumental Variable Estimator

If firms perceive of neither n_a nor n_p as N, using either of them will bring measurement errors to the model. Such measurement errors make the non-linear least squares criterion function in Section 3 inconsistent. This is described in the following lemma.

Lemma (Inconsistency of $\hat{\boldsymbol{\theta}}_{NLS}$ under measurement error of N) The estimator $\hat{\boldsymbol{\theta}}_{NLS}$ generated by the non-linear least squares setup proposed in (5) is inconsistent when N is measured with error.

Proof Consider the left hand side of expression (A.2) in Appendix A. If the h_{lj} that corresponds to the left (and true) term differs from the h_{lj} that corresponds to the right term, then h_{lj} cannot be factored out of the parenthesis as written in (A.2). This implies that the expression is equal to zero only when either $\mu_0 \neq \mu$ or $\sigma_0 \neq \sigma$. Violating condition (i) of the consistency theorem; consistency of $\hat{\boldsymbol{\theta}}_{NLS}$ no longer holds. ■

In the next section I provide some illustrative evidence of this inconsistency.

In general, the consequences of measurement errors in non-linear models are more involved than in linear models. Chesher (1991) provides some conjectures on the consequences of data that is contamined by measurement error in some general cases. Further discussions on the topic are found in Carroll et al. (1995). The general message of these studies is that measurement error in non-linear models is an intricate task to remedy without additional information.

Consequently, to provide a consistent estimator for the competition effect I utilize an instrumental variables approach suitable for a non-linear context. In this case the GMM-type criterion function is

$$T(\boldsymbol{\theta}) = \left[(\boldsymbol{b} - \boldsymbol{f})' \boldsymbol{Z}\right] \hat{\boldsymbol{S}}^{-1} \left[\boldsymbol{Z}' (\boldsymbol{b} - \boldsymbol{f})\right] \tag{9}$$

and $\hat{\boldsymbol{\theta}}_{IV}$ is the value of $\boldsymbol{\theta}$ that minimizes $T(\cdot)$. In (9) $\hat{\boldsymbol{S}} = K^{-1} \sum_l \sum_j \sum_i \hat{v}_{lji} \boldsymbol{z}_{lji} \boldsymbol{z}'_{lji}$ and $\boldsymbol{Z}$ is a matrix of instruments and regressors and $\boldsymbol{b}$ and $\boldsymbol{f}$ are defined as in Section 3 (also, see footnote 10). I obtain $\hat{\boldsymbol{S}}$ by minimizing (9) in a first step utilizing the identity matrix for $\hat{\boldsymbol{S}}$.

The heteroskedasticity-consistent asymptotic covariance matrix is calculated as

$$\hat{\boldsymbol{V}}(\hat{\boldsymbol{\theta}}) = K \left[\left(\hat{\boldsymbol{E}}' \boldsymbol{Z}\right) \hat{\boldsymbol{S}}^{-1} \left(\boldsymbol{Z}' \hat{\boldsymbol{E}}\right)\right]^{-1} \tag{10}$$

where $\hat{\boldsymbol{E}} = \partial \boldsymbol{v} / \partial \boldsymbol{\theta}' \mid_{\hat{\boldsymbol{\theta}}}$ (Cameron and Trivedi, 2005, p. 195).

So, all in all I consider three models: I estimate (7) with both $h_{lj} = 1/\left(n_{a,j} - 1\right)$ (Model 1) and $h_{lj} = 1/\left(n_{p,l} - 1\right)$ (Model 2) and also Model 3 where $1/\left(n_{a,j} - 1\right)$ is instrumented by $1/\left(n_{p,l} - 1\right)$ and the municipality population size. The estimation results from these models are discussed in Section 6.

4 Monte Carlo Illustration

To illustrate the proposed estimators' properties in finite samples I provide a small scale simulation study. The first part studies bias issues with respect to different choices of h_{lj}, that is, correctly and incorrectly chosen measurements of competition. The second part displays some indications of the consistency of the estimator. The third part discusses the non-linear instrumental variables estimator. Some results discussed here are found in Appendix B.

For this Monte Carlo illustration, I assume that bids b_{lji} are generated according to the model

$$b_{lji} = \alpha w_l^{\pi_0 + \pi_1 x_{1j}^1} r_l^{\phi_0 + \phi_1 x_{2j}^1} + \nu_{lji} + \frac{\left[1 - G\left(b; \mu, \sigma\right)\right]}{g\left(b; \mu, \sigma\right)} h_{lj} \tag{11}$$

with some slight variations discussed below. In (11), x_{1j}^1 and x_{2j}^1 are observed heterogeneities associated with w_l and r_l, respectively. I set $\boldsymbol{\theta}' = \left(\mu, \sigma, \alpha, \pi_0, \pi_1, \phi_0, \phi_1\right) = (1, 1, 1, 0.1, 0.1, 0.1, 0.1)$. Further, I take w_l, x_{1j}^1, r_l and x_{2j}^1 to be uniformly distributed as $U\left(2, 8\right)$ random variables. I use different notions for h_{lj} to mimic the different measurements of the number of actual and potential bidders as found in data, respectively (see Section 5). As for the number of actual bidders, n_a, I generate 100 auctions with 5 bidders in each, and 50 auctions with 10 bidders in each. This amounts to 1000 bids, in a total of 150 auctions, i.e. the sample size is 1000. In this setup, and for the given $\boldsymbol{\theta}$, I solve the implicit equation (11) for every bid, b_{lji}. When I have each of the 1000 bids I add a disturbance term ν_{lji} to it. Here, ν_{lji} is taken to be normally distributed with location and dispersion parameters set to $\mu = 0$ and $\sigma = 1/2$, respectively.

First, I study the estimator's properties under correct and incorrect choices of h_{lj}. When I take h_{lj} to be composed of the number of potential bidders, n_p, I scale up n_a by a factor of five, i.e. n_p is either 25 or 50. The number of replicates is 100. I provide Monte Carlo results for three models, see Table B1 in Appendix B. In the first model (Model A), h_{lj} is taken to be $1/\left(n_{a,j} - 1\right)$ in the data generating process (DGP) so that a correct assumption of the DGP is made. In Model B, the DGP once again follows $h_{lj} = 1/\left(n_{a,j} - 1\right)$ but the researcher incorrectly selects $1/\left(n_{p,l} - 1\right)$ to be the measurement of h_{lj}. In Model C, $h_{lj} = 1/\left(n_{p,l} - 1\right)$ in the DGP but this time the researcher erroneously selects $1/\left(n_{a,j} - 1\right)$ as the measurement of h_{lj}.

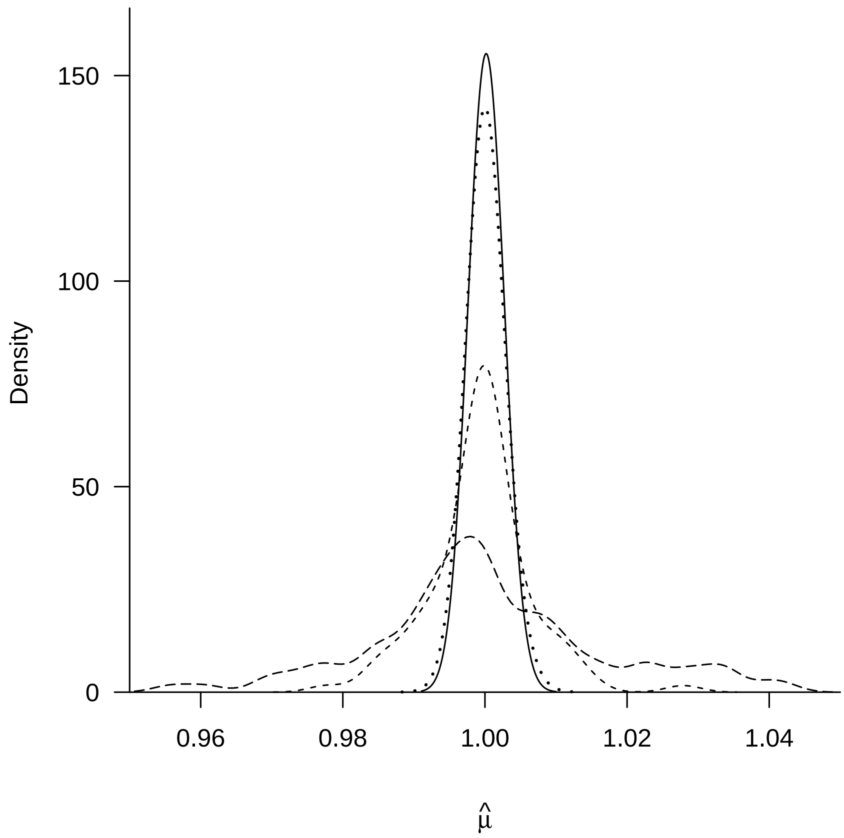

Figure 1: Kernel estimated sampling distributions of $\hat{\mu}$ using Model A under different sample sizes, K. Sample sizes 50, 200, 1000 and 3000 correspond to long dashes, short dashes, dots and solid line, respectively.

As for the results reported in Table B1, the estimator seems to be unbiased under Model A. In Model B (Model C) the estimators of parameters μ, σ and α, respectively, seem to be positively (negatively) biased.

To give indications whether the NLS estimator based on the criterion function in (5) is consistent, I vary the sample size as $K = (50, 200, 1000, 3000)$. Here, I consider Model A only, i.e., the model is correctly specified according to the DGP. Some visual examples are given for the parameters μ and σ in Figures 1 and 2, respectively. In Table B2 in Appendix B, I provide the corresponding results for all the parameters in (11). In Figure 1 we observe that the finite sample sampling distributions are centered on the true value $\mu = 1$. The figure also suggests consistency of $\hat{\mu}$ as more probability mass is concentrated close to μ as the sample size increases.

When it comes to σ (Figure 2) we observe that that the sampling distributions are not centered equally close to the true parameter value of σ. Inspecting Figure 2 we see that

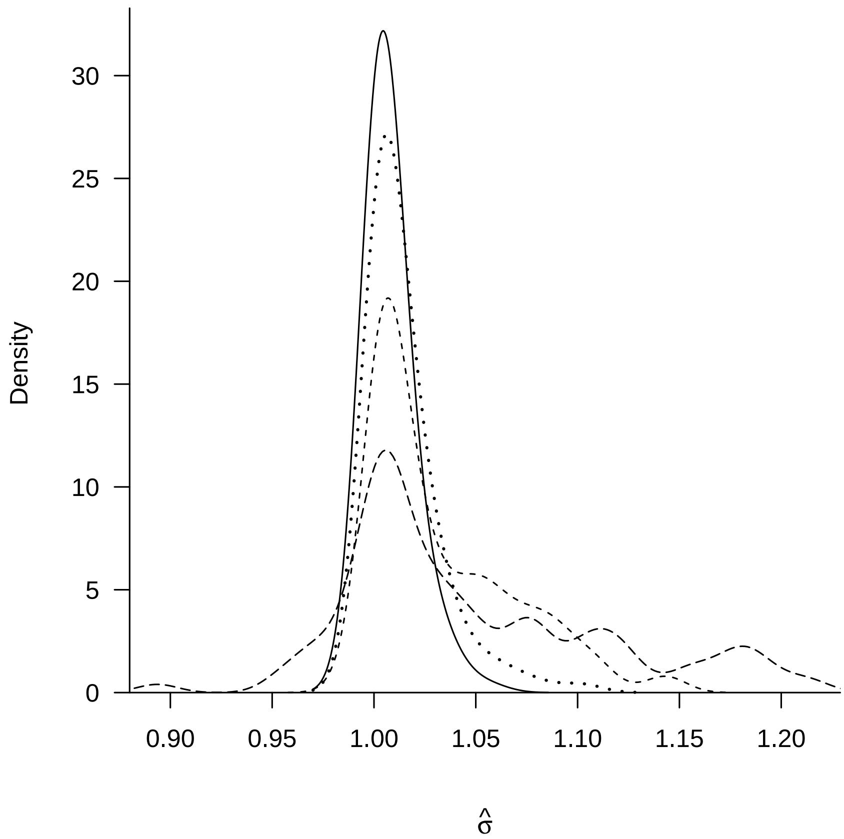

Figure 2: Kernel estimated sampling distributions of $\hat{\sigma}$ using Model B under different sample sizes, K. Sample sizes 50, 200, 1000 and 3000 correspond to long dashes, short dashes, dots and whole line, respectively.

the estimator for σ seems to be skewed to the right in finite samples. This is not very surprising however, as, e.g., $(n-k)\, s^2\sigma^{-2}$ is χ^2-distributed in ordinary linear regressions under normality. And surely, the mass of the sampling distributions seems to gather at the true parameter value as the sample size increases, which is a necessary condition for the estimator to be consistent. In my opinion, these indications provide arguments that the asymptotic results given in Section 3.1 are reasonable approximations for samples of finite sizes.

Now, I turn to the instrumental variable estimator. As above, I set n_p to be 25 and 50. As discussed in Section 3, n_p is the instrument (see also Section 5 for a more thorough discussion). I set n_a to be $0.2n_p + \zeta$ where ζ is normally distributed with location and dispersion parameters set as $\mu = 0$ and $\sigma = 1/2$, respectively. That is, the number of actual bidders are (on average) one fifth of the number of potential bidders. In the DGP I set the true number of bidders to be either 20 or 40. So, the DGP corresponds to neither

n_p nor n_a. When I estimate the model I use n_a as the endogenous (or, "problematic") variable and n_p as instrument. Results are given in Table B3 in Appendix B, for the instrumental variable estimator, but also for the problematic, endogenous, model. For this endogenous model we see that there is bias that goes in the same direction as in Model C above. This is to be expected as the actual measurement of competition is smaller than the competition in the DGP. A visual example is found in Appendix B; in Figure B1, sampling distributions of $\hat{\alpha}$ is plotted, both for the endogenous model and for the IV model. We observe here that the instrumental variable estimator is clearly superior in measurement error settings. Further indications like these are found in Table B3. That is, even though I did not use the true variable for competition, the instrumental variable approach yields estimates close to the parameters. All in all I do not reject the claim that the instrumental variable estimator works well.

To study the consequences of measurement error for the sizes of markups and the competition effect I estimate 100 replications of the endogenous model and the IV model, respectively, as discussed in the preceding paragraph. Two versions of the markup are reported in Table 1, the mean of the predicted values and the fitted value at the mean of the included variables, respectively, i.e. $K^{-1} \sum \left\{ \left[1 - \widehat{G}(b) \right] / \widehat{g}(b) \right\} h$ and $\left\{ \left[1 - \widehat{G}(b) \right] / \widehat{g}(b) \right\} h \Big|_{K^{-1} \sum b}$. Also in Table 2, the competition effect $\widehat{\partial b / \partial N} = - \left\{ \left[1 - \widehat{G}(\cdot) \right] / \widehat{g}(\cdot) \right\} h_{lj}^{-2}$ is reported in means of the predicted values, i.e., $K^{-1} \sum \widehat{\partial b / \partial N}$ as well as at the mean of the variables, $\widehat{\partial b / \partial N}|_{K^{-1} \sum b}$. Note that the numbers in both Table 1 and 2 are written in fractions of the bids. For example (Table 1), the IV model predicts the markup to be of the size of approximately 12 percent of the bids. Considering Table 2, the IV model predicts bids to decrease by 2.5 percent of the bid size when one more competitor enters the auction.

From the tables we see that the IV model predicts markups and competition effects quite close to the truth. Also, if the researcher chooses a measurement of the competition lower than the one that generates the data, both markups and competition effects will be underestimated. This has implications for empirical studies in the auction literature, such as the ones discussed in the introduction.

Table 1: Markup (as a fraction of the bids)

	Endogenous model	IV	Truth
Mean of markup	0.052	0.119	0.119
Markup at means of variables	0.043	0.101	0.101

Table 2: Competition effect (as a fraction of the bids)

	Endogenous model	IV	Truth
Mean of competition effect	0.011	0.025	0.025
Competition effect at means of variables	0.007	0.015	0.015

5 Data

The data consist of information on first price sealed bid auctions of Swedish public procurements for internal cleaning services conducted between December 2008 and December 2010. Internal cleaning services are chosen as they constitute a homogenous service keeping the service-specific heterogeneity to a minimum. The data is novel in the sense that they contain much observed heterogeneity, especially when it comes to the characteristics of the auctions. Descriptives are found in Table 3.

Table 3: Descriptive statistics (sample size 1916 observations)

Variable	Mean	s.d.	Min	Max
1. Bid, b (in $SEK \times m^{-2}$)	127.43	89.13	27.80	1676.4
2. Wage, w (in $SEK \times 1000^{-1}$)	19.94	0.29	19.20	20.74
3. Price on capital, r	0.41	0.28	0.15	1.82
4. Size of contract (in m^2), q	11230.93	26724.76	67	220000
5. Number of actual bidders, n_a	7.48	4.20	2	23
6. Number of potential bidders, n_p	462.30	573.37	2	1353
7. Number of wage criteria, x_1^1	3.38	1.21	0	6
8. Number of capital criteria, x_2^1	3.40	1.35	0	7
9. Number of environmental criteria, x_2^2	4.62	2.94	0	12
10. Office (YES=1, NO=0), x_3^1	0.69	0.46	0	1
11. Municipality population size	324894.40	340778	3161	847073

Notes: Variable 1 is further standardized such that it can be interpreted as the bid a firm requests to provide cleaning services during the time period of a year. m^2 denotes square meters. The x_1^1, x_2^1, x_2^2 and x_3^1 relate to (12) in Section 6 below.

The bids are standardized to be interpretable as the payment a firm asks for to conduct internal cleanings services over the area of one square meter during the time period of one year, adjusted for inflation. Wage is mean wage in different regions[11] of Sweden in 2008. The measure is then adjusted by a monthly wage-index from December 2008 to December 2010 as collected by Statistics Sweden. The price on capital is the quarterly rate of return on Swedish treasury bills collected from the Swedish Central Bank and it varies by month but not by geographical location. The number of actual bidders is the number of bidders that cast a bid in the particular auction and was not disqualified

[11] These regions are the so-called NUTS-regions. This classification is used for the reporting of statistics within the European Union.

by the procurer. The number of potential bidders represents all firms registered in the internal cleaning services industry. This variable is measured on the municipality level.

In the call for tenders, the procuring authorities are free to state some criteria on the services, e.g., that the firm must use environmentally friendly fuel in their vehicles, that the employees must have a certain experience, and so on. These criteria are counted and aggregated into three categories: wage, capital and environmental criteria, respectively. I assume that the environmental and capital criteria influence the cost of capital while the wage criteria influence the wage paid by the firms to their employees. I let the first two enter the empirical model through β_1 and the latter through β_2 in the cost function (6) as described in the discussion surrounding expression (6). I list all the criteria making up x_1^1, x_2^1 and x_2^2 in Appendix C. The variable office is a dummy variable indicating whether the auctioned object is an office or not. I hypothesize that a square meter of an office is cheaper to clean than, e.g., a square meter of a hospital or a square meter of a kindergarten as the sanitary demands on office cleaning are not as rigorous as for other types of objects. I specify this variable to enter (6) through β_3.

As discussed in Section 3, the Cobb-Douglas assumptions impose that firms are Hicks-neutral and the elasticities of substitution between the input factors are equal to one (see footnote 7). Depending on the industry under study, the Hicks-neutral assumption could be too restrictive. The data under studied here, however, are generated by firms in the internal cleaning industry. Such firms are quite homogenous with respect to technology and consequently unlikely to differ much across firms with respect to capital-labor ratio. Hence, the Hicks-neutral condition is reasonable. The unit elasticity of subsitution is not as strong assumption, and I take it to be a reasonable one considering the industry that I study.

I will use two instruments in the instrumental variables model: the number of potential bidders and also the population size of the municipality where the auction is held. That is, I assume that the instrument n_p is uncorrelated with the bids. As for the the second instrument, the municipality population size, there is nothing in economic theory that implies that it is correlated with the bid size.[12]

Now, some descriptives and discussion relating to the instrumental variable estimator follow. I assume that the instrument n_p is uncorrelated with the bids, b, except through N. The linear correlation is as small as $corr\left[b, 1/(n_{p,l}-1)\right] = -0.001$ which corroborates the assumption. Further, the correlation between $1/\left(n_{a,j}-1\right)$ and

[12]One may argue that $1/\left(n_{a,j,t-1}-1\right)$ would be a good candidate to instrument for $1/\left(n_{a,j,t}-1\right)$. Here t denotes time. I have measurements over 25 months. A problem here if I want to use $1/\left(n_{a,j,t-1}-1\right)$ to instrument for $1/\left(n_{a,j,t}-1\right)$ is that as $n_{a,j,t}$ is the measurement of the actual amount of bidders in an auction, an auction that is held in a particular municipality. In data, I most often have only one measurement per municipality over the 25 months in data, which rules out the calculation of $n_{a,j,t-1}$.

$1/\left(n_{p,l}-1\right)$ is as large as 0.330. Also, $corr\left(b, n_{p,l}\right) = -0.012$ and $corr\left(n_{a,l}, n_{p,l}\right) = 0.421$. As mentioned, regarding the second instrument, the municipality population size, there is nothing in economic theory that implies that it is correlated with the bid size. Intuition, however suggests that the population size is correlated with the number of participating bidders in a municipality. Bivariate correlations confirm these conjectures as municipality population size have empirical correlation coefficients of sizes -0.014 and 0.425 to bid size and the number of actual bidders, respectively. This provides motivation for the non-linear instrumental variable exercise.

6 Results

Estimation results from the non-linear least squares regression models and the instrumental variable model, i.e. Models 1, 2 and 3 are presented in Table 4.[13]

To facilitate interpretation I state the model I estimate as follows.

$$b_{lji} = \alpha w_l^{\pi_0+\pi_1 x_{1j}^1} r^{\phi_0+\phi_1 x_{2j}^1+\phi_2 x_{2j}^2} q_j^{\delta_0+\delta_1 x_{3j}^1} + \nu_{lji} + \frac{\left[1-G\left(b;\mu,\sigma\right)\right]}{g\left(b;\mu,\sigma\right)} h_{lj} \tag{12}$$

Here, w_l is the cost of labor, x_{1j}^1 contains observed heterogeneities of auctions assumed to influence the cost of labor, r is the cost of capital and x_{2j}^1 contains observed auction criteria assumed to influence capital costs, while x_{2j}^2 contains environmental criteria also assumed to influence capital costs.[14] Further, q_j is the size of the contract in terms of area to be cleaned and x_{3j}^1 is a dummy indicating whether the premise to be cleaned is an office or not. See Table 3 for descriptive statistics of these observed heterogeneities and the preceding section for a discussion.

In Table 4, we see that Model 1 is best fitted to data with respect to $\hat{s}^2$, followed by Model 2 and Model 3 in that order. Considering the standard errors, we observe that some of the parameter estimates on the observed heterogeneities in the cost function are precisely estimated, sometimes implying significant effects in a statistical sense. The estimates of δ_1 are statistically significant on the one percent level across all models. So, if the premises to be cleaned consist of offices, the firms' average bids (and costs) are lower than the bids (and costs) for objects that are not offices, e.g., hospitals, schools and so forth. For an object of size 1000 square meters, the multiplicative factor $q_j^{\delta_0+\delta_1 x_{3j}^1}$ in the firms' cost function would on average be 0.168 SEK lower if the object was an office as compared to if it was not an office, holding all else constant. In such a scenario, it would have decreased from 0.743 (not office) to 0.575 (office).

[13] I carried out the estimation in R and optimized (5) and (9) utilizing the conjugate gradients method.

[14] Note here that r is not taken to vary by geographical location as my data does not allow for it. Implicitly, however, r varies over time. See Section 5 for a description of the data.

Table 4: Estimation results (sample size 1916 observations)

	Model 1 n_a		Model 2 n_p		Model 3 IV	
Parameter	Estimate	s.e.	Estimate	s.e.	Estimate	s.e.
μ	1.859	6.858	3.993	4.933	4.034	0.076
σ	2.343	1.846	0.538	2.120	0.645	0.245
α	1.053	0.027	1.058	0.067	1.053	0.002
π_0	0.529	0.018	0.590	0.020	0.528	0.014
π_1	-0.005	0.003	3×10^{-4}	0.002	-6×10^{-4}	0.003
ϕ_0	0.263	0.052	0.296	0.039	0.201	0.080
ϕ_1	-0.032	0.014	-0.045	0.010	-0.044	0.013
ϕ_2	-0.010	0.005	0.003	0.005	-0.009	0.009
δ_0	-0.051	0.014	-0.082	0.010	-0.043	0.010
δ_1	-0.034	0.007	-0.045	0.006	-0.037	0.005
$\hat{s}^2$	4653		6868		7943	

Note: The $\hat{s}^2$ is calculated as $\hat{s}^2 = K^{-1} \sum_l \sum_j \sum_i \hat{v}_{lji}^2$.

Further, the estimates of ϕ_0 are positive and statistically significant indicating that increases in the general level of capital costs lead to increases of the firms' bids and costs. Regarding the estimates $\hat{\phi}_1$ I find them statistically significant and negative, indicating that more capital criteria actually reduce firms' bids. A potential explanation here could be that more criteria make the firms more certain of the procurers' intentions, i.e. it reduces uncertainty. Environmental criteria ($\hat{\phi}_2$) however, have no statistically significant effect on firms' costs (in Model 2 and Model 3).[15]

The number of wage criteria ($\hat{\pi}_1$) does not seem to have a statistically significant effect on bids, but the intercept term ($\hat{\pi}_0$) seems to do. That is, increasing wages leads generally to higher costs for the firms, but not specifically through the wage criteria in public procurements. More specifically, that $\hat{\pi}_0 = 0.528$ implies that if the wage in the industry would increase from 20 to 21 in thousands of SEK, and holding everything else constant, the wage's mulitplicative effect on costs (to clean one square meter during a yearlong period) would increase by 0.13 SEK on average, from 4.86 to 4.99.

Using the fitted models, I report predicted markup in Table 5. Two versions are reported, the mean of the predicted values and the fitted value at the mean of the included variables, respectively, i.e. $K^{-1} \sum \left\{ \left[1 - \widehat{G}(b) \right] / \widehat{g}(b) \right\} h$ and $\left\{ \left[1 - \widehat{G}(b) \right] / \widehat{g}(b) \right\} h \Big|_{K^{-1} \sum b}$. Note that the markups are given in fraction of the average bid which if of of size 127 SEK (see Table 3). The two versions of the markup calculated by Model 3 are both contained in the interval of the ones calculated using Model 1 and 2, respectively. The IV model predicts the markup to be 9 percent of the bid if the model is evaluated at the means of

[15]This finding is consistent with other studies using this dataset (Lundberg et al., 2015; Sundström, 2015).

Table 5: Markup (as a fraction of the bids)

	Model 1	Model 2	Model 3
	n_a	n_p	IV
Mean of markup	0.339	0.005	0.094
Markup at means of variables	0.207	5.5×10^{-4}	0.058

Table 6: Competition effect (as a fraction of the bids)

	Model 1	Model 2	Model 3
	n_a	n_p	IV
Mean of competition effect	0.147	5.6×10^{-4}	0.039
Competition effect at means of variables	0.032	1.3×10^{-6}	0.009

all predicted markups, or 6 percent of the bid if the markup is calculated at the means of the variables.

Further, using the estimated parameters I calculate the competition effect given by (3) explicitly, as $\widehat{\partial b/\partial N} = -\left\{\left[1-\widehat{G}(\cdot)\right]/\widehat{g}(\cdot)\right\} h_{lj}^{-2}$. As above, I report the mean of the predicted values, i.e., $K^{-1}\sum\widehat{\partial b/\partial N}$ and the effect at the mean of the variables, $\widehat{\partial b/\partial N}|_{K^{-1}\sum b}$, respectively. As in Table 5, the competition effects are reported in fractions of the mean bid. The calculated competition effects under the three models are given in Table 6. Here we see that the competition effect differs with respect to the different models. The effect in the instrumental variable model is in the interval between the effects of the other two models. The IV model says that, if competition increases by one firm, the competition effect lower the bids by 4 (or 1 percent, depending on row in Table 6) as a fraction of the mean bid.

Note that Table 5 and 6 are comparable to Tables 1 and 2 in the Monte Carlo section.

Now, consider (12) but set $h_{lj} = 1/\left\{\psi(n_{a,j}-1) + (1-\psi)\left(n_{p,l}-1\right)\right\}$ for fixed $\psi \in [0,1]$ as was described in Section 3. Here $\psi = (0, 0.02, 0.04, \ldots, 0.98, 1)$. I find that the fit in terms of $\hat{s}^2$ for the 51 estimated models improve monotonically in ψ.

Finally, I conducted Hausman tests of Model 3 against Model 1 and and Model 2, respectively. The test statistics were calculated to be respectively 116 and 103 in these two cases. This indicates that the instrumental variables estimator is the consistent one as in comparison to Model 1 and Model 2. Note that this conjecture is valid only under homoskedastic standard errors, as only then is the non-linear least squares estimator the most efficient one among least squares estimators (Newey and McFadden, 1994).

7 Conclusions

A few studies use the number of actual bidders (n_a) rather than the number of potential bidders (n_p) when estimating models such as (2). This is potentially problematic as theory specifies that n_p would be the proper measurement of N in (2) (see the references in the introduction). In addition, if firms perceive of neither n_p nor n_a as competition when making decisions, an econometrician using either n_p or n_a as N brings measurement error to the model. With the aim of predicting the competition effect $\partial b/\partial N$ soundly, I have proposed an instrumental variable estimator of (5) as well as conducted a study on estimation issues using both n_a and n_p as N, and how $\partial b/\partial N$ is brought about in these different setups.

Starting out with estimation of the model utilizing n_a and n_p, I gave (2) a non-linear least squares interpretation in a dependent variable transformation model setup. In comparison to most other studies within this research field, the main object of inquiry here was the choice of measurement of competition and its implications for calculating the competition effect as discussed above, hence not the estimation of the costs *per se*. Therefore, I modeled the costs as generated by Cobb-Douglas technology which was argued to be reasonable considering the characteristics of the industry under study. This is novel to the field and found to provide plausible estimates of the parameters of the cost function, as interpreted in Section 6. Also, I provided some asymptotic results for the non-linear least squares estimator corresponding to the dependent variable transformation model.

As a consequence of the possibility of both n_a and n_p being erroneous measurements of N, I proposed an instrumental variable approach in an attempt to remedy this error-in-variables problem. Using the standard motivations for instrumental variables, I interpreted n_p and the municipality population size as instruments for N. As was the case with the (non-linear least squares) estimators based on n_a and n_p, the instrumental variables estimator generated reasonable estimates on the cost function and corresponding markups as well, in the sense that the bids were larger than the estimated costs. Further, the instrumental variables estimator generated predicted markups and competition effects in the interval between those calculated from the non-linear least squares estimator utilizing n_a and n_p as measurements for N (see Tables 5 and 6). Based on this information it is hard to tell which approach that provided the most reasonable estimates in the sense of being correct. To assist in drawing conclusions regarding this matter, Hausman tests corroborated the belief that the instrumental variable estimator was consistent as compared to the other two estimators based on n_a and n_p. Hence the IV columns of Tables 5 and 6, respectively, are the ones that should be consulted when one seeks to find markups and competition effects predicted by soundly estimated models.

Further, regarding the non-linear least squares estimator, I found the one using n_a as the measurement of competition to be better fitted to reality, in comparison with the one having n_p as N. Fitting the model for some linear combinations of n_a and n_p corroborated this finding. As for the measurement feature of the paper, an implication of this result is that n_a is a better choice for N than n_p when specifying this type of model; e.g., when calculating costs using a setup like the one in (2).

Moreover, the fact that I found models using only n_a to best correspond to reality allows for the following policy conclusion. If the government wants to reduce prices on the goods and services that they procure, i.e. to transfer wealth from the private to the public sector in terms of lowering firms' markups, they should conduct policy as to increase the *actual* participation in public procurement auctions. The redistribution of wealth could be quite substantial as public procurements turn over approximately 16 percent of the GDP in the EU countries (European Commission, 2008). Note that this policy conclusion should be taken with caution as it is based on inconsistent estimators.

As some useful results on the asymptotic behavior of the non-linear least squares estimator derived from the dependent variable transformation model was established here, further studies are encouraged utilizing this approach. However, as I have shown in this paper, care should be taken to ensure the quality of the measurements of competition brought to the model.

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Appendix A: Proofs

A1 Proof of Consistency

To establish consistency of the estimaton procedures considered in this paper, I need to check that the criterion function (5) fulfills conditions (i)-(iv) in the consistency theorem stated in Section 3.1. I check these conditions and provide some surrounding discussion in this section of this appendix. Note that this proof only applies to the special case of the log-normal distribution, as will be evident under (i) below.

(i) Unique minimum

Lemma $Q_0(\boldsymbol{\theta})$ is uniquely minimized at the true parameter values $\boldsymbol{\theta}_0$

Proof If the lemma holds, then the difference between the parts of (7) is zero at the true parameter values. To save some notation we divide the parameter vector $\boldsymbol{\theta}' = (\mu, \sigma, \alpha, \boldsymbol{\pi}, \boldsymbol{\phi}, \boldsymbol{\delta})$ into two parts, $\boldsymbol{\theta} = \left(\boldsymbol{\theta}^1, \boldsymbol{\theta}^2\right)'$ where $\boldsymbol{\theta}^1 = (\mu, \sigma)'$ and $\boldsymbol{\theta}^2 = (\alpha, \boldsymbol{\pi}, \boldsymbol{\phi}, \boldsymbol{\delta})'$. A subindex of 0 on the parameter denotes true parameter values. I also assume that the covariates are fixed. Further, let $c\left(\boldsymbol{\theta}^2\right) = \alpha w_l^{x'_{1j}\boldsymbol{\pi}} r_l^{x'_{2j}\boldsymbol{\phi}} q_j^{x'_{3j}\boldsymbol{\delta}}$ be the cost function. I want to show that

$$E\left[\left\{b - c\left(\boldsymbol{\theta}^2\right) - \frac{[1 - G(b; \mu, \sigma)]}{g(b; \mu, \sigma)} h_{lj}\right\}^2\right] \tag{A.1}$$

is uniquely minimized at the true parameter values $\boldsymbol{\theta}_0$. Consequently, it is required that $c\left(\boldsymbol{\theta}_0^2\right) - c\left(\boldsymbol{\theta}^2\right) = 0$ and that

$$\left\{\frac{[1 - G(b; \mu_0, \sigma_0)]}{g(b; \mu_0, \sigma_0)} - \frac{[1 - G(b; \mu, \sigma)]}{g(b; \mu, \sigma)}\right\} h_{lj} = 0 \tag{A.2}$$

That is, I want to establish that $\mu = \mu_0$ and $\sigma = \sigma_0$ holds in the population. It is trivial that $c\left(\boldsymbol{\theta}_0^2\right) - c\left(\boldsymbol{\theta}^2\right) = 0$ holds at $\boldsymbol{\theta}_0^2 = \boldsymbol{\theta}^2$ for fixed values of the covariates, assuming $\alpha \neq 0$. However, that (A.2) holds is not as easily seen. Therefore I will provide an argument that (A.2) holds in the following. First, assume that $h_{lj} > 0$. We see that (A.2) implies that

$$\frac{g(b, \mu_0, \sigma_0)}{[1 - G(b, \mu_0, \sigma_0)]} = \frac{g(b, \mu, \sigma)}{[1 - G(b, \mu, \sigma)]}$$

Integrating both sides as

$$\int_0^{\bar{b}} \frac{g(b, \mu_0, \sigma_0)}{[1 - G(b, \mu_0, \sigma_0)]} db = \int_0^{\bar{b}} \frac{g(b, \mu, \sigma)}{[1 - G(b, \mu, \sigma)]} db$$

yields

$$\ln \left[1 - G\left(\bar{b}, \mu_0, \sigma_0\right)\right] = \ln \left[1 - G\left(\bar{b}, \mu, \sigma\right)\right]$$

Using the expontential function this can be written as

$$G\left(\bar{b}, \mu_0, \sigma_0\right) = G\left(\bar{b}, \mu, \sigma\right)$$

Using the definition of the (log-normal) cdf we see that

$$G\left(b, \mu_0, \sigma_0\right) = \int_0^{\bar{b}} \frac{1}{\sqrt{2\pi}\sigma_0} \exp\left[-\frac{\left(\ln b - \mu_0\right)^2}{2\sigma_0^2}\right] db$$

and

$$G\left(b, \mu, \sigma\right) = \int_0^{\bar{b}} \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{\left(\ln b - \mu\right)^2}{2\sigma^2}\right] db$$

Hence,

$$\int_0^{\bar{b}} \frac{1}{\sqrt{2\pi}\sigma_0} \exp\left[-\frac{\left(\ln b - \mu_0\right)^2}{2\sigma_0^2}\right] db = \int_0^{\bar{b}} \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{\left(\ln b - \mu\right)^2}{2\sigma^2}\right] db \tag{A.3}$$

Now, define two different bid values as b' and b''. Considering the expontents in (A.3) we see that

$$\frac{\left(\ln b' - \mu_0\right)^2}{2\sigma_0^2} = \frac{\left(\ln b' - \mu\right)^2}{2\sigma^2}$$

and

$$\frac{\left(\ln b'' - \mu_0\right)^2}{2\sigma_0^2} = \frac{\left(\ln b'' - \mu\right)^2}{2\sigma^2}$$

hold simultaneously only when $\mu = \mu_0$ and $\sigma = \sigma_0$ hold. ■

(ii) Compact parameter space

We assume that the parameter space is compact, which is a standard assumption to make.

(iii) Continuity

Lemma $Q_0\left(\boldsymbol{\theta}\right)$ is continuous.

Proof By inspecting the population regression function version of the criterion function (5), i.e. $Q_0(\boldsymbol{\theta}) = E\left[\left(b - c\left(\boldsymbol{\theta}^2\right) - \{[1-G(b;\mu,\sigma)]/g(b;\mu,\sigma)\}h\right)^2\right]$ we see that the cost function $c\left(\boldsymbol{\theta}^2\right) = \alpha w_l^{x'_{1j}\pi} r_l^{x'_{2j}\phi} q_j^{x'_{3j}\delta}$ is continuous. Further, $[1-G(b;\mu,\sigma)]$ and $g(b;\mu,\sigma)$ are continuous by the definition of the cdf and pdf, respectively. Also the difference and quotient of continuous functions are themselves continous functions, establishing the continuity of $\left(b - c\left(\boldsymbol{\theta}^2\right) - \{[1-G(b;\mu,\sigma)]/g(b;\mu,\sigma)\}h\right)$. Further, a continuous transformation of a continuous function is also continuous and consequently $\left(b - c\left(\boldsymbol{\theta}^2\right) - \{[1-G(b;\mu,\sigma)]/g(b;\mu,\sigma)\}h\right)^2$ is continuous. Also, $E[\cdot]$ is a continous (linear) operator which establishes the result in the lemma. ■

(iv) Uniform convergence

Lemma $\hat{Q}_K(\boldsymbol{\theta})$ converges uniformly in probability to $Q_0(\boldsymbol{\theta})$.

Proof Let $a(\boldsymbol{z},\boldsymbol{\theta})$ be a matrix of functions of observations $\boldsymbol{z}$ and the parameter $\boldsymbol{\theta}$. Here $\boldsymbol{z}$ contains all variables, both dependent variable and covariates. Also, define $\| a(\boldsymbol{z},\boldsymbol{\theta}) \| = \left(\sum_{j,k} a_{jk}^2\right)^{\frac{1}{2}}$. Then

Lemma (2.4 of Newey and McFadden, (1994, p. 2129)) If data are iid, the parameter space is compact, $a(\boldsymbol{z},\boldsymbol{\theta})$ is continuous at each $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ with probability one and there is a function $d(\boldsymbol{z})$ such that $\| a(\boldsymbol{z},\boldsymbol{\theta}) \| \leq d(\boldsymbol{z})$ for all $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ and $E[d(\boldsymbol{z})] < \infty$, then $E[a(\boldsymbol{z},\boldsymbol{\theta})]$ is continuous and $\sup_{\theta\in\Theta} \| K^{-1}\sum_{i=1}^{K} a(z_i,\theta) - E[a(\boldsymbol{z},\boldsymbol{\theta})] \| \xrightarrow{P} 0$.

That is, $a(\boldsymbol{z},\boldsymbol{\theta})$ converges in probability to $E[a(\boldsymbol{z},\boldsymbol{\theta})]$. I have already assumed that the parameter space is compact and checked continuity above. It remains to be checked that the dominance condition $\| a(\boldsymbol{z},\boldsymbol{\theta}) \| \leq d(\boldsymbol{z})$ holds while $E[d(\boldsymbol{z})] < \infty$. In my setup, $a(\boldsymbol{z},\boldsymbol{\theta}) = \left(b - \alpha w_l^{x'_{1j}\pi} r_l^{x'_{2j}\phi} q_j^{x'_{3j}\delta} - \{[1-G(b;\mu,\sigma)]/g(b;\mu,\sigma)\}h\right)^2$. Expanding $a(\boldsymbol{z},\boldsymbol{\theta})$ yields

$$
\begin{aligned}
a(\boldsymbol{z},\boldsymbol{\theta}) = {} & b^2 - 2\alpha w_l^{x'_{1j}\pi} r_l^{x'_{2j}\phi} q_j^{x'_{3j}\delta} + 2\alpha w_l^{x'_{1j}\pi} r_l^{x'_{2j}\phi} q_j^{x'_{3j}\delta} \frac{[1-G(b;\mu,\sigma)]}{g(b;\mu,\sigma)} \\
& - \left(\alpha w_l^{x'_{1j}\pi} r_l^{x'_{2j}\phi} q_j^{x'_{3j}\delta}\right)^2 - 2b\frac{[1-G(b;\mu,\sigma)]}{g(b;\mu,\sigma)} + \left\{\frac{[1-G(b;\mu,\sigma)]}{g(b;\mu,\sigma)}\right\}^2
\end{aligned}
$$

Now, say that all individual parameter spaces are bounded by the constant C, respectively. Then $\| a(\boldsymbol{z},\boldsymbol{\theta}) \| \leq d(\boldsymbol{z})$ for

$$d(\boldsymbol{z}) = 2b^2 + 3C\left(\| \boldsymbol{w}_l \| \vee 1\right)^{\left(\|\boldsymbol{x}_{1j}\|\vee 1\right)2C} \left(\| \boldsymbol{r}_l \| \vee 1\right)^{\left(\|\boldsymbol{x}_{2j}\|\vee 1\right)2C} \left(\| \boldsymbol{q}_j \| \vee 1\right)^{\left(\|\boldsymbol{x}_{3j}\|\vee 1\right)2C}$$
$$\times \frac{[1 - G(b;\mu,\sigma)]}{g(b;\mu,\sigma)} + 2\left\{\frac{[1 - G(b;\mu,\sigma)]}{g(b;\mu,\sigma)}\right\}^2$$

where $\vee$ denotes element-wise maximum. We assume that b has a finite second moment and that $\| \boldsymbol{w}_l \|$, $\| \boldsymbol{r}_l \|$ and $\| \boldsymbol{q}_j \|$ have finite $\left(\| \boldsymbol{x}_{1j} \| \vee 1\right) 2C$, $\left(\| \boldsymbol{x}_{2j} \| \vee 1\right) 2C$ and $\left(\| \boldsymbol{x}_{3j} \| \vee 1\right) 2C$ moments, respectively. ■

Now it is checked that the criterion function fulfills conditions (i)-(vi) in Newey and McFadden's (1994) theorem on the consistency of extremum estimators discussed in Section 3.1 and hence it can be concluded that the proposed estimator is consistent.

A2 Proof of Asymptotic Normality

Here, I check that the the conditions (i)-(v) of the asymptotic normality theorem given in Section 3.1 hold.

(i) Parameter in the interior of the parameter space

This condition requires that $\boldsymbol{\theta}_0' = (\mu_0, \sigma_0, \alpha_0, \boldsymbol{\pi}_0, \boldsymbol{\phi}_0, \delta_0)$ are interior points of the allowed intervals. These intervals are $[-\infty, \infty]$ for all parameters except for $\sigma_0 \in (0, \infty]$.

(ii) Differentiability

By inspection, we can conclude that $\hat{Q}_K(\boldsymbol{\theta})$ is twice continuously differentiable in a neighborhood $\mathcal{N}$ of $\boldsymbol{\theta}_0$.

(iii) Convergence of the gradient

Lemma $\sqrt{K}\nabla_0 \hat{Q}_K(\boldsymbol{\theta}_0) \xrightarrow{d} N(\boldsymbol{0}, \boldsymbol{\Sigma})$ applies to (5).

Proof The gradient of $\hat{Q}_K(\boldsymbol{\theta}_0)$ is

$$\nabla_0 \hat{Q}_K(\boldsymbol{\theta}_0) = \begin{bmatrix} -\frac{2}{K}\sum\left[b - c(\cdot) - \frac{1-G(\cdot)}{g(\cdot)}h\right]\left\{\frac{G_{\boldsymbol{\theta}^1}(\cdot)g(\cdot)+[1-G(\cdot)]g_{\boldsymbol{\theta}^1}(\cdot)}{[g(\cdot)]^2}h\right\} \\ -\frac{2}{K}\sum\left[b - c(\cdot) - \frac{1-G(\cdot)}{g(\cdot)}h\right]c_{\boldsymbol{\theta}^2}(\cdot) \end{bmatrix}$$

where the two elements are the derivatives taken with respect to $\boldsymbol{\theta}^1 = (\mu, \sigma)'$ and $\boldsymbol{\theta}^2 = (\alpha, \boldsymbol{\pi}, \boldsymbol{\phi}, \delta)'$, respectively, in order to increase readability. Subscripts denote the corresponding partial derivatives, e.g., $g_{\boldsymbol{\theta}^1}$ is the derivative of $g(\cdot)$ with respect to $\boldsymbol{\theta}^1$. Hence we see that $\sqrt{K}\nabla_0\hat{Q}_K(\boldsymbol{\theta}_0)$ consists of averages. Hence, e.g., the Lindeberg-Lévy Central Limit Theorem and evaluating $\sqrt{K}\nabla_0\hat{Q}_K(\boldsymbol{\theta}_0)$ at the true parameters, $\boldsymbol{\theta}_0 = \left(\boldsymbol{\theta}_0^1, \boldsymbol{\theta}_0^2\right)'$ establishes $\sqrt{K}\nabla_0\hat{Q}_K(\boldsymbol{\theta}_0) \xrightarrow{d} N(\mathbf{0}, \boldsymbol{\Sigma})$, i.e. the lemma above. ■

(iv) Uniform convergence

The Hessian is

$$H = \begin{bmatrix} \frac{2}{K}\sum\left\{\frac{\left[G_{\boldsymbol{\theta}^1}(\cdot)g(\cdot)+[1-G(\cdot)]g_{\boldsymbol{\theta}^1}(\cdot)\right]A}{[g(\cdot)]^6}h\right\} & \frac{2}{K}\sum\left\{\frac{G_{\boldsymbol{\theta}^1}(\cdot)g(\cdot)+[1-G(\cdot)]g_{\boldsymbol{\theta}^1}(\cdot)}{[g(\cdot)]^2}h\right\}c_{\boldsymbol{\theta}^2}(\cdot) \\ \frac{2}{K}\sum c_{\boldsymbol{\theta}^2}(\cdot)\left\{\frac{G_{\boldsymbol{\theta}^1}(\cdot)g(\cdot)+[1-G(\cdot)]g_{\boldsymbol{\theta}^1}(\cdot)}{[g(\cdot)]^2}h\right\} & \frac{2}{K}\sum\left\{\left[c_{\boldsymbol{\theta}^2}(\cdot)\right]^2 - Bc_{\boldsymbol{\theta}^2\boldsymbol{\theta}^2}(\cdot)\right\} \end{bmatrix}$$

where

$$\begin{aligned} A = {} & \left[G_{\boldsymbol{\theta}^1\boldsymbol{\theta}^1}(\cdot)\, g(\cdot) + g_{\boldsymbol{\theta}^1\boldsymbol{\theta}^1}(\cdot) - G(\cdot)\, g_{\boldsymbol{\theta}^1\boldsymbol{\theta}^1}(\cdot)\right][g(\cdot)]^2 \\ & - \left\{G_{\boldsymbol{\theta}^1}(\cdot)\, g(\cdot) + [1 - G(\cdot)]\, g_{\boldsymbol{\theta}^1}(\cdot)\right\} 2g(\cdot)\, g_{\boldsymbol{\theta}^1}(\cdot) \end{aligned}$$

and

$$B = \left[b - c(\cdot) - \frac{1 - G(\cdot)}{g(\cdot)}h\right]$$

Here, as above derivatives are taken with respect to $\boldsymbol{\theta}^1$ and $\boldsymbol{\theta}^2$ to increase readability. Now, the Lemma under (iv) uniform convergence in the consistency section above can be invoked, in particular when the Hessian depends on averages (Newey and McFadden, 1994), as the case is here.

(v) Nonsingularity of the Hessian

It follows by inspection of the Hessian shown in (iv) above that $\det H \neq 0$ which implies that H is nonsingular.

Appendix B: Tables and Figure from Monte Carlo Illustration

Table B1: Monte Carlo results, Models A, B and C (sample size: 1000)

Model A	Parameter	True value	Mean	s.d.
	μ	1	1.000	0.002
	σ	1	1.016	0.015
	α	1	0.998	0.006
	π_0	0.1	0.099	0.006
	π_1	0.1	0.100	0.001
	ϕ_0	0.1	0.098	0.006
	ϕ_1	0.1	0.100	0.001
Model B				
	μ	1	1.029	0.009
	σ	1	1.086	0.027
	α	1	1.196	0.014
	π_0	0.1	0.098	0.005
	π_1	0.1	0.095	0.001
	ϕ_0	0.1	0.097	0.005
	ϕ_1	0.1	0.095	0.001
Model C				
	μ	1	0.842	0.016
	σ	1	0.548	0.051
	α	1	0.943	0.016
	π_0	0.1	0.105	0.002
	π_1	0.1	0.101	3×10^{-4}
	ϕ_0	0.1	0.105	0.002
	ϕ_1	0.1	0.101	3×10^{-4}

Table B2: Monte Carlo results, Model A estimated from different sample sizes

Sample size 50	Parameter	True value	Mean	s.d.
	μ	1	0.997	0.029
	σ	1	1.046	0.078
	α	1	0.992	0.060
	π_0	0.1	0.100	0.031
	π_1	0.1	0.100	0.003
	ϕ_0	0.1	0.097	0.030
	ϕ_1	0.1	0.100	0.003
Sample size 200				
	μ	1	0.999	0.007
	σ	1	1.032	0.035
	α	1	0.993	0.027
	π_0	0.1	0.100	0.015
	π_1	0.1	0.100	0.001
	ϕ_0	0.1	0.099	0.015
	ϕ_1	0.1	0.100	0.002
Sample size 1000				
	μ	1	1.000	0.001
	σ	1	1.014	0.018
	α	1	0.999	0.007
	π_0	0.1	0.100	0.007
	π_1	0.1	0.100	6×10^{-4}
	ϕ_0	0.1	0.099	0.006
	ϕ_1	0.1	0.099	7×10^{-4}
Sample size 3000				
	μ	1	1.000	6×10^{-4}
	σ	1	1.007	0.010
	α	1	0.999	0.003
	π_0	0.1	0.099	0.003
	π_1	0.1	0.099	4×10^{-4}
	ϕ_0	0.1	0.100	0.003
	ϕ_1	0.1	0.099	4×10^{-5}

Table B3: Monte Carlo results, instrumental variables estimator

Endogenous model, sample size 1000	Parameter	True value	Mean	s.d.
	μ	1	0.800	0.031
	σ	1	0.620	0.045
	α	1	0.931	0.020
	π_0	0.1	0.104	0.006
	π_1	0.1	0.101	0.001
	ϕ_0	0.1	0.104	0.007
	ϕ_1	0.1	0.101	0.001
IV model, sample size 50				
	μ	1	0.999	2.3×10^{-5}
	σ	1	0.999	8.1×10^{-5}
	α	1	1.000	6.8×10^{-4}
	π_0	0.1	0.099	8.6×10^{-4}
	π_1	0.1	0.094	0.003
	ϕ_0	0.1	0.100	0.001
	ϕ_1	0.1	0.095	0.003
IV model, sample size 200				
	μ	1	0.999	5.3×10^{-6}
	σ	1	0.999	1.1×10^{-5}
	α	1	0.999	9.1×10^{-5}
	π_0	0.1	0.099	1.3×10^{-4}
	π_1	0.1	0.095	4.1×10^{-4}
	ϕ_0	0.1	0.099	1.2×10^{-4}
	ϕ_1	0.1	0.095	4.2×10^{-4}
IV model, sample size 1000				
	μ	1	0.999	1.9×10^{-6}
	σ	1	0.999	4.6×10^{-6}
	α	1	0.999	2.4×10^{-5}
	π_0	0.1	0.099	3.7×10^{-5}
	π_1	0.1	0.095	1.7×10^{-4}
	ϕ_0	0.1	0.099	3.7×10^{-5}
	ϕ_1	0.1	0.095	1.9×10^{-4}
IV model, sample size 3000				
	μ	1	0.999	9.8×10^{-7}
	σ	1	0.999	2.4×10^{-6}
	α	1	0.999	1.3×10^{-5}
	π_0	0.1	0.099	2.0×10^{-5}
	π_1	0.1	0.095	9.8×10^{-5}
	ϕ_0	0.1	0.099	2.0×10^{-5}
	ϕ_1	0.1	0.095	9.9×10^{-5}

Note: Results for the endogenous model corresponding to sample sizes $T = (50, 200, 3000)$ are omitted here but are available upon request.

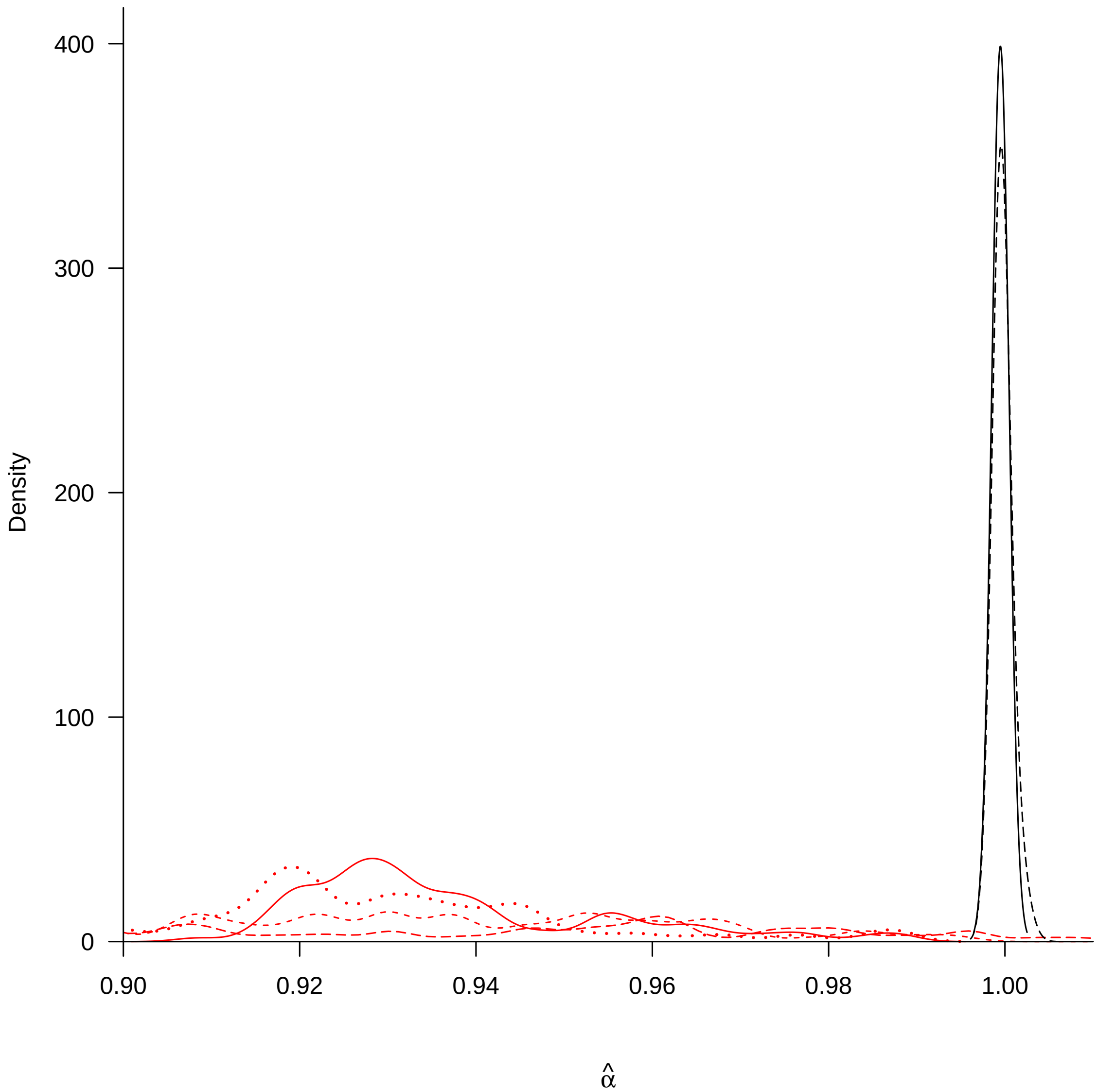

Figure B1: The horizontal axis shows $\hat{\alpha}$. Red (black) sampling distributions of $\hat{\alpha}$ corresponding to the endogenous (instrumental variable) model. Sample sizes 50, 200, 1000 and 3000 correspond to long dashes, short dashes, dots and whole lines, respectively. Only sample sizes 50 and 3000 included for the instrumental variable estimator. Note that $\alpha = 1$ and that some distributions are truncated to increase readability of the figure. The pattern in Table B3 is evident here.

Appendix C: Variables making up x_1^1, x_2^1 and x_2^2

In the data section (Section 5) I specify x_1^1, x_2^1 and x_2^2 to be the number of criteria (stated in the calls for tenders) relating to wage, capital and environmental costs, respectively. In the tables below, I list these criteria.

Table C1: Wage criteria x_1^1

Criteria	Meaning
1. Janitor	The firm has to provide a janitor
2. Experience	The firm's employees must have certain experience
3. References	References are required
4. Prylsry	Firm's employees must have gone through a certain education
5. CV	CV of foreman is required
6. Swedish	Firm's employees must be Swedish speakers
7. Union	Firm must be connected to a workers' union

Table C2: Capital criteria x_2^1

Criteria	Meaning
1. Floor	Firm must provide special floor cleaning
2. Windows	Firm must polish windows
3. Insurance	Firm needs insurance
4. Rating	Firm needs a credit rating
5. Annual report	Firm needs to supply annual report
6. Turnover	Firm must state information on turnover
7. Quality	Firm needs a quality certificate

Table C3: Environmental criteria x_2^2

Criteria	Meaning
1. Revision	Firm must actively plan for environmental revision
2. Products	Firm required to use green cleaning products
3. MSR	Firm must comply to Swedish Environmental Management council
4. Chemicals	Firm must provide lists of chemicals in use
5. Environmental code	Firm must comply to Swedish Environmental code
6. REACH	Firm must follow some environmental guidelines
7. Chemical statues	Firm must follow some chemical statutes
8. Chemical bookkeeping	Firm must bookkeep their chemical use
9. Intended chemicals	Firm must give information on chemicals they intend to use
10. Vehicles	Firm's vehicles must be environmentally friendly
11. Management system	Firm must have an environmental management system
12. ISO	Firm must be ISO certified