The Cross-Section of Expected Stock Returns in Brazil ℵ

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Abstract

This article replicates Fama and French (1992) "The Cross-Section of Expected Stock Returns" as close as possible, using a broad and carefully checked sample of the Brazilian stock market from 1999 to 2015. Similar to Fama and French, but with weaker significance, the book-to-market and size of individual firms capture some of the cross-sectional variation in average stock returns, while market β does not play a convincing role. The positive relation with book-to-market is more evident than the negative relation with size. However, because none of these characteristics show explanatory power for the subperiod July 2007-June 2015, we are not as convinced that they capture fundamental risk factors.

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Keywords: size and book-to-market characteristics, emerging markets.

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Este artigo replica Fama and French (1992) "The Cross-Section of Expected Stock Returns" da forma mais próxima possível, utilizando uma amostra ampla e cuidadosamente conferida do mercado acionário brasileiro entre 1999 e 2015. Como em Fama e French, porém com menos significância, as características de valor contábil/valor de mercado e tamanho das firmas capturam parte da variação seccional dos retornos acionários, enquanto o β de mercado não tem papel convincente. A relação positiva com a razão valor contábil/valor de mercado é mais evidente que a relação negativa com o tamanho das firmas. Entretanto, porque nenhuma destas caraterísticas apresenta poder explicativo para o subperíodo Julho de 2007-Junho de 2015, não somos convincentemente convencidos que elas capturam fatores de risco fundamental.

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1. Introduction

Fama and French (1992) – hereafter F&F – convincingly summarized the evidence that "diversifiable" individual characteristics could capture important cross-sectional variation in average stock returns. Whether their combination of size and book-to-market capture some common risk factors as in Fama and French (1993) three factor model, or some inefficiency as in Daniel and Titman (1997) is still being debated until today (see Campbell, 2014). If stocks with these characteristics are fundamentally riskier, we might expect them to continue to pay return premiums in the future. Investors who are less sensitive to these sources of risk may continue to enjoy their high returns. If, on the other hand, return premiums arose because of mispricing, we might expect the excess returns to fade away along time as investors learn about them.

Inspired by F&F, Costa Jr. and Neves (2000) study the Brazilian stock market for the period from March 1988 to February 1996 and find that not only the size and the book-to-market, but the portfolios' β and earnings-to-price ratios were individually and jointly significant. The results of Braga and Leal (2003) are little different, confirming that high book-to-market firms porfolios have greater average returns, but not that small size firms porfolios have greater average returns for the period July1991 - June1998.[1]

The short time span of data available in the early 2000s bounded deeper analyses. Qualitative and quantitative differences in this pioneering literature seem due the inclusion or exclusion of a couple of years, given the influential weight of each included observations (1/N), what is specially an issue in a period of reduction and stabilization of inflation that affected nominal returns (i.e., likely changes in the data generating process).

[1] See Braga and Leal's (2003) list of references and their different conclusions for the Brazilian market in the 1990s.

In this article, we review the explanatory power of the size and book-to-market characteristics in Brazil. Using a reliable sample of stock returns and fundamentals ratios, we study the monthly returns of individual stocks from July 1999 to June 2015 to show that the market beta does not explain cross-section average returns in Brazil, as in F&F. Also like F&F, we find that the size and book-to-market of individual firms capture some of the cross-sectional variation in average stock returns, while the market β does not play a convincing role. In general, the patterns documented by F&F hold for the Brazilian stock market, with the qualification of less significance. The book-to-market pattern is more evident than the size pattern. But, when the sample is split into two subperiods, none of the variables show explanatory power for the later period. This last result is similar to Schwert (2002), who found that the abnormal return for characteristics such as size effect, value and others documented seem to have weakened or simply disappeared after the papers that highlighted them were published. Thus, through practitioners' exploration of these uncovered premiums, the research findings caused the market to become more efficient.

To more convincingly dismiss the market β and size as factor premiums, like F&F in the Appendix, we look for the continuation of their premiums into the longer 27-year period July 1988-June 2015 available at Economatica. We surprisingly find a positive and economically import premium for the market β, as well as a negative and economically import premium for size, when they are the only factor controlled for in this broader sample. These factors' significance for the overall July 1988-June 2015 period is pointed to come entirely from the strong individual significance of the market β and size for the subperiod from 7/1988 - 6/2007; thus not contradicting our main results and confirming the early market β's significance found by Costa Jr and Neves (2000). However, given the high correlation between the market β and size, neither variable is individually significant for the overall sample (i.e., collinearity problem); with the market β dominating in 7/1988-6/1999 and size dominating in 7/1999-6/2007.

Finally, we speculate on the issues of liquidity and state-controlled firms. Liquidity and state governance risk are characteristics of special concern to investors in Brazil. For example, if growth stocks are more liquid (or less state-controlled) than value stocks, the value premium may in part reflect a compensation for the lower liquidity (or for greater management inefficiency) of value firms.

We reveal traces that liquidity lightly reduces the value premium, supporting the covariance hypothesis with liquidity risk. Regarding state-control, we find no evidence of a covariance explanation. Because state-controlled firms – which are larger in size and book-to-market – rewarded stockholder below the market average in the sample studied, their exclusion increases the importance of the size effect and decreases the importance of the value effect.

The article proceeds as follows: the Fama and French's (1992) methodology used in this study is reviewed in section 2. We describe our sample in section 3 and discuss our results in section 4. In section 5, we present some preliminary conclusions and the planned next steps.

2. Methodology

According to the market efficiency hypothesis, all expected return in excess to the risk-free rate should be the "fair" reward to the investment's exposition to non-diversifiable systemic risk. In the CAPM, the most popular market efficient asset pricing model, all systemic risk is summarized in the market portfolio, thus implying that:

$$E(\tilde{R}_{i,t} - R_{f,t}) = \beta_i E(\tilde{R}_{M,t} - R_{f,t}), \tag{1}$$

where: $E(\tilde{R}_{i,t} - R_{f,t})$ is asset's i excess return over the risk-free rate $R_{f,t}$ during period t and $E(\tilde{R}_{M,t} - R_{f,t})$ is the market portfolio excess return over the risk-free rate. In words, the CAPM states that the asset's premium comes from its covariance with the market portfolio – $\beta_i = \frac{cov(\tilde{R}_{i,t}, \tilde{R}_{M,t})}{var(\tilde{R}_{M,t})}$ and not from its own variance – and is linear in the systemic risk incurred.

Since Black, Jensen and Scholes (1972) and Fama and MacBeth (1973), many tests of the CAPM have been proposed. Intuitively, they all boil down in estimating regressions like:

$$(\tilde{R}_{i,t} - R_{f,t}) = \alpha_i + \beta_i(\tilde{R}_{M,t} - R_{f,t}) + \tilde{e}_i, \tag{2}$$

and testing whether the intercept α_i equals zero, with varying degrees of econometric sophistication and power (see also Gibbons Ross and Shanken 1989).

Alternatively, given that only non-diversifiable risks should be priced in efficient markets, non-arbitrage in the CAPM implies that:

$$E\left(\tilde{R}_{i,t} - R_{f,t}\right) = \lambda^M \beta_i \,, \qquad (3)$$

where: λ^M is the market price of the stock market risk (or, the market reward per unit of the stock market risk factor incurred).

One can thus run the cross-section regression:

$$\left(R_{i,t} - R_{f,t}\right) = \lambda_t^0 + \lambda_t^M \beta_i + v_i, \qquad \forall\, i \qquad (4)$$

where $\left(R_{i,t} - R_{f,t}\right)$ is the proxy for stock i average excess return, and, given β_i is the stock i sensitivity to the only systemic risk premium, test if $\lambda_t^0 = 0$ and $\lambda_t^M > 0$ on average; i.e., $\overline{\lambda^0} = (T^{-1} \sum_{t=1}^{T} \lambda_t^0) = 0$ and $\overline{\lambda^M} = (T^{-1} \sum_{t=1}^{T} \lambda_t^M) > 0$. A significant $\overline{\lambda^0} \neq 0$ means that the portfolios returned more than justified by their sensitivities to the non-diversifiable risks, i.e., there are arbitrage opportunities if this same pattern of average return can be extrapolated into the future.

Alternatively, one can rewrite (4) as:

$$R_{i,t} = \lambda_t^{rf} + \lambda_t^M \beta_i + v_i, \qquad \forall\, i \qquad (4')$$

and test if $\overline{\lambda^{rf}} = \left(T^{-1} \sum_{t=1}^{T} R_{f,t}\right)$ and $\overline{\lambda^M} > 0$.

Other factors hypothesized to explain expected returns can also be included in (4) and allowed to vary over time, an approach called the "Fama-MacBeth regressions", since Fama-MacBeth (1973). For example, F&F regress the month-by-month cross-section of returns on size and book-to-market, in addition to the market β :

$$R_{i,t} = \lambda_t^0 + \lambda_t^M \beta_{it} + \lambda_t^{ME} ln(ME_{it}) + \lambda_t^{BM} ln(BM_{it}) + v_i, \quad \forall\ i \tag{5}$$

and test if λ_t^M , λ_t^{ME} , λ_t^{BM} are significantly different from zero, where ME_{it} is the market value of equity and $BM_{it} = (BE_{it}/ME_{it})$ is the book value of equity to its market value. They thus examine whether the market, size and book-to-market factors are common among stocks, i.e., are systematic and non-diversifiable. Insignificant $\overline{\lambda^M}$, $\overline{\lambda^{ME}}$ or $\overline{\lambda^{BM}}$ mean that these risks are not priced in the market, i.e., are diversifiable. That is the main approach we follow below.

As explained in Fama (1976), because $R_{f,t}$ is not subtracted from the stock return in equation (5), and the factors do not have the interpretation of risk premiums, λ^0 is not expected to be zero, but it is the return on a portfolio in which the weighted averages of the explanatory variables are 0, i.e. $ln(ME_i) = 0$ and $ln(BM_i) = 0$.[2]

Along the way, we also follow F&F nowadays standard approach of sorting stocks based on some ex-ante visible characteristics "to separate skill from luck", in Cochrane (2008) words. We present univariate descriptive statistics for the portfolios built to characterize the size and book-to-market effects, as well as the CAPM's β systemic risk, that help to describe the cross-section of expected stock returns in Brazil.

[2] For an econometric overview of the Fama and MacBeth approach, see also Shanken (1992).

3. Data

Our sample comes from Quantum [3] and initially comprises the 419 stocks traded in Brazil from July 1997 to June 2015, which fulfill the requirement of at least one trade in the first ten days of the month in all the twelve months from July to June. Because we need at least two years of monthly returns to estimate betas before the first year of actual test, we exclude the 87 stocks in the sample that have less than 36 observations. We are thus left with an unbalanced panel of 332 stocks from July 1999 to June 2015, comprising 192 months.

There are two potential problems in the accounting data. The change in the accounting standards from the Brazilian GAAP to the IFRS, that took place in 2010, and the lack of consolidated balance sheet for few companies in few years. The first problem does not affect the book value, and in the second, the alternative was to use the individual balance sheet.

The monthly return is calculated as:

$$R_{i,t} = \frac{P_{i,t} + D_{i,t}}{P_{i,t-1}} - 1$$

where: $P_{i,t-1}$ is the first closing price of the month *t-1*, $P_{i,t}$ is the first closing price of month t and $D_{i,t}$ are the dividends and other payments to stockholders (like "interest on equity"). Liquidity in the Brazilian market is definitely a problem. As anticipated, if there is no price at the first business day t, then we get the first closing price in the next 10 business days. The return interval mismatched can have ten days at maximum. If there is no price

[3] Quantum is a large provider of Brazilian financial market data. The data used here was downloaded in July, 2015 at www.quantumfundos.com.br<http://www.quantumfundos.com.br> .

within those days, then there is no month return. The stocks that miss the return for one month are excluded from the portfolio formed in that year, from July to June.

Below, following F&F, in June of each year, we form equally weighted portfolios of stocks based on their fundamentals, like market value of equity (*ME*), book-to-market ratio (*BM=(BE/ME)*), β_is , leverage ratios and earnings-to-price ratio. Only stocks with returns for every month in the year following June entered the portfolios. For example, sorting on the size dimension (*ME*), we form five quintile portfolios. Within each size quintile, by sorting on the book-to-market dimension (*BM*), we form five book-to-market portfolios, totaling 25 portfolios. To reduce multicollinearity problems, we observe the size values as of June and the book-to-market values as of December of the previous year. Other variables like the market βs, leverage and earnings-to-price are also observed in December of the year preceding June.

Again according to F&F, because estimates of market βs are more precise for portfolios than for stocks, based on the *ME* and the pre-β distributions together, we also form twenty-five portfolios and assign their estimated βs to the individual stocks within each portfolio in the tests below.

4. Results

As stated before, for comparison purposes, we follow Fama and French's (1992) presentation closely.

Like in F&F, Table 1 shows that portfolios formed on size and pre-ranking βs , rather than just on size, widen the range of post-ranking βs. Across all 25 size-β portfolios, the post-ranking βs in panel 1.B range from 0.49 to 1.37, an interval almost 3 times wider than the interval of 0.33 (=1.05-0.72) obtained with size portfolios alone.[4] While panel 1.B suggests there is an inverse relation between post-ranking β and size, panel 1.C does not relate size to pre-ranking β.

Table 1 - Average Returns, Post-Ranking βs and Average Size For Portfolios Formed on Size and then β (July 1999 to June 2015)

	All	Low-β	β-2	β-3	β-4	High-β
Panel 1.A: Average Monthly Returns (in Percent)						
All		1.77	1.95	1.82	1.71	1.36
Small-ME	1.99	2.41	2.90	1.66	1.61	1.21
ME-2	1.63	1.65	1.89	1.01	2.18	1.48
ME-3	2.03	2.11	1.89	2.41	1.49	2.18
ME-4	1.77	1.48	1.76	2.43	1.77	1.40
Large-ME	1.23	1.24	1.32	1.58	1.52	0.50
Panel 1.B: Post-Ranking βs						
All		0.72	0.85	0.94	1.06	1.12
Small-ME	1.05	0.78	1.02	1.02	1.12	1.37
ME-2	0.93	0.79	0.85	0.92	1.17	0.94
ME-3	1.10	0.76	0.96	1.20	1.18	1.40
ME-4	0.85	0.77	0.70	0.86	1.02	0.94
Large-ME	0.72	0.49	0.71	0.67	0.82	0.95
Panel 1.C: Average Size (ln(ME))						
All		7.10	7.14	7.05	7.13	6.98
Small-ME	4.46	4.42	4.66	4.37	4.54	4.25
ME-2	6.20	6.19	6.06	6.18	6.31	6.24
ME-3	7.21	7.19	7.25	7.19	7.20	7.23
ME-4	8.11	8.14	8.09	8.13	8.15	8.06
Large-ME	9.49	9.63	9.70	9.48	9.49	9.13

Table 2 shows post-ranking average returns for July 1999 to June 2015 for portfolios formed from one-dimensional sorts of stocks on size (in 2.A) or βs (in 2.B). In panel 2.A,

[4] We choose to ignore the portfolio ME-3 & High-β while describing the big picture of Panel 1.B, given it is out of line with its neighbor cells.

with portfolios formed on just size, we observe a negative relation between size and average return, and a positive (though not strict) relation between average return and β. Thus, a simple size sort is not enough to reject the CAPM prediction of a positive relation between βs and average returns. Unlike the size portfolios, the pre β-sorted portfolios are less supportive of the CAPM in panel 2.B. There is no positive relation between βs and average returns, along the β-sorted portfolios.

Table 2 - Properties of Portfolios Formed on Size or Pre-Ranking β (July 1999 to June 2015)

	1	2	3	4	5
Panel 2.A: Portfolios Formed on Size					
Return	1.99	1.63	2.03	1.77	1.23
Post-βs	1.05	0.93	1.10	0.85	0.72
ln(ME)	4.46	6.20	7.21	8.11	9.49
ln(BM)	0.34	-0.07	-0.30	-0.46	-0.62
ln(A/ME)	1.51	0.85	0.66	0.50	0.27
ln(A/BE)	1.15	0.92	0.96	0.96	0.89
E/P dummy	0.41	0.20	0.15	0.13	0.10
E(+)/P	0.11	0.13	0.10	0.09	0.08
Panel 2.B: Portfolios Formed on Pre-Ranking β					
Return	1.73	2.03	1.78	1.66	1.44
Post-βs	0.71	0.81	0.96	1.08	1.11
ln(ME)	7.53	7.20	7.10	6.97	6.60
ln(BM)	-0.38	-0.21	-0.27	-0.11	-0.11
ln(A/ME)	0.57	0.66	0.72	0.88	0.98
ln(A/BE)	0.95	0.87	0.98	0.98	1.10
E/P dummy	0.13	0.13	0.19	0.17	0.37
E(+)/P	0.10	0.11	0.10	0.12	0.08

The puzzlingly evidence in Table 2 on the relation between β and average return for portfolios formed on size or β alone is detailed in Table 1. The second-pass β sort does not produce the variation in average returns predicted by the CAPM. Although the post-ranking βs in Table 1.B increase along the row of each size quintile, average returns do not get higher. Whilst, within the columns of Table 1, the average returns and βs decrease with increasing size.

In sum, like in F&F, Table 1 says that there is an inverse relation between size and average return, but controlling for size, there is no positive relation between β and average return. In Table 1 and 2, we show that there is no reliable relation between β and average returns. Variation in beta does not seem compensated in the average returns of stocks in Brazil during July 1999-June 2015.

Table 3 - Average Slopes (t-statistics) from Month-by-Month Regressions of Stock Returns (July 1999 to June 2015)

Row	β	ln(ME)	ln(BM)	ln(A/ME)	ln(A/BE)	E/P Dummy	E(+)/P
1	0.45 (0.65)						
2		-0.14 (-1.47)					
3	-0.15 (-0.21)	-0.14 (-1.42)					
4			0.44 (2.54)				
5	0.08 (0.10)		0.39 (2.09)				
6		-0.07 (-0.70)	0.36 (2.07)				
7				0.44 (2.37)	-0.29 (-1.14)		
8		-0.04 (-0.45)		0.38 (2.06)	-0.27 (-1.07)		
9						-0.02 (-0.04)	2.26 (1.54)
10		-0.14 (-1.51)				-0.35 (-0.95)	1.53 (1.07)
11			0.33 (1.96)			-0.24 (0.60)	1.36 (1.01)

As F&F, we present our main results in Table 3. Using the Fama and MacBeth (1973) approach presented in section 2, each month, the cross-section of stock returns is regressed on candidate explanatory variables. Then, the means of the slopes of the monthly regressions provide the tests of whether these explanatory variables were on average priced by the market during July 1999 - June 2015.

The first row of Table 3 shows that the market β does not explain the average stock return for 1999-2015. The average slope from the regressions of returns on β alone is 0.45 and insignificant.

Like the average returns in Tables 1 and 2, the regressions estimates in Table 3 suggest that size helps explain the cross-section of average stock returns, however lacking convincing significance. The average slope from the regressions of returns on size alone is -0.14, with a t-statistic of -1.47. This result is between Costa Jr. and Neves (2000) – who found size to be significant – and Braga and Leal (2003) or Antunes et al. (2006) – who have not found important size effect in the Brazilian market.[5]

In the regressions of return on size and β (in row 3), the size keeps its magnitude and significance, while the β becomes negative, though still insignificant. The negative relation between size and average stock return persists with other explanatory variables in the regressions, but loose significance when variables related to BM are included, in the following rows of Table 3.

There is a strong cross-sectional relation between average returns and book-to-market equity. The average slope of the monthly regressions of returns on $ln(BM)$ alone is 0.44, with a t-statistic of 2.54 (in row4); and such a significance is not due to the omission of market beta

[5] Our time window overlaps with Antunes et al. (2006), and not with Costa Jr and Neves (2000) or Braga and Leal (2003). In Table 6 below, we discuss the stability of factors' premium estimates for subsamples of the period studied. For the period July 1999 – June 2007, closer to Antunes et al. (2006), our estimate of the size effect is more significant.

(in row5). In Table 4, panel 4.A, we see the positive relation between average return and book-to-market equity from another perspective. Average returns rise from 1.26% for the lowest *BM* portfolio to 2.25% for the highest *BM* portfolio. This amplitude is greater than the observed for the size portfolios in Table 2, and the positive BM-return is strictly monotonic. These results are in line with Santos and Montezano (2011), who find that value stock over-perform growth stocks in Brazil in the 1989-2009 period.

As in F&F, the book-to-market relation seems stronger than the size effect, alone and jointly. In row 6 of Table 3, when both *ME* and *BM* are included in the regressions, they become individually less significant because of their correlation. Both coefficients suffer meaningful reductions in magnitude and significance, with the size effect diminishing more.

In row 7 of Table 3, we see that market leverage (*A/ME*) is significant in explaining average returns, although book leverage (*A/BE*) is not. F&F get significant coefficients for both measures, of approximately the same magnitude and opposite signs, supporting the claim that it is not leverage that matters, but the *BM* ($ln(BM) = ln(BE/ME) = ln(A/ME)-ln(A/BE)$), which captures a "relative-distress effect. The results in row 7 do not confirm such claim, but do not reject it either. The fact that market leverage increases expected return, while book leverage does not decrease it, means that leverage matters and it is better measured by market imposed leverage.

In row 8, we see that if size and leverages are used together, size recedes and market leverage becomes smaller, but still significant, in a pattern similar to when size and book-to-market are employed jointly (in row 6).

In row 9 of Table 3, we see that average returns increase with positive *E/P* , but the coefficient for negative *E/P* (i.e., the $E/P\ Dummy = 1$ if $E/P<0$) is almost zero. The fact that Brazilian firms with negative earning did not offered higher average returns is also shown in Table 4, panel 4.B, where we do not find the clear U-shape relation described by F&F for the US stocks. Although average returns decline from the negative *E/P* portfolio to the second lowest *E/P* portfolio, this decrease is from 1.49% to 1.23% only, and average returns then increase monotonically, reaching 2.27% per month for the highest *E/P* portfolio. Like F&F, we find that the higher returns of negative *E/P* are better captured by their size, given the negative *E/P* group is composed of the smallest firms on average, in panel 4.B.

Finally, rows 10 and 11 show that the average slopes for *ln(ME)* or *ln(BM)* in the regressions that include leverage or E/P are similar to those in the regressions of average returns on size and book-to-market equity only, while the E/P effect lessens considerable. As pointed by F&F, these suggest that the relation between positive *E/P* and average return, or market leverage and average return, is due to the positive correlation between *E/P*, *(A/ME)* and *BM*, illustrated in Table 4.[6] Those three are all measures of market imposed discount for relative-distress risk.

[6] When looking at panel 4.B, ignore the first column of negative E/P firms.

Table 4 - Properties of Portfolios Formed on Book-to-Market Equity (BM) and Earnings-Price Ratio (E/P) (July 1999 to June 2015)

	1	2	3	4	5
Panel 4.A: Stocks Sorted on Book-to-Market Equity (BM)					
Return	1.26	1.54	1.75	1.87	2.25
Post-βs	0.90	0.97	0.88	0.88	1.02
ln(ME)	7.96	7.66	7.34	6.59	5.83
ln(BM)	-1.40	-0.59	-0.18	0.21	0.91
ln(A/ME)	-0.17	0.35	0.76	1.11	1.77
ln(A/BE)	1.22	0.94	0.94	0.91	0.86
E/P dummy	0.16	0.14	0.17	0.23	0.29
E(+)/P	0.06	0.09	0.11	0.12	0.13
Panel 4.B: Stocks Sorted on Earnings-Price Ratio (E/P)					
Return	1.49	1.23	1.75	1.77	2.27
Post-βs	1.17	0.98	0.81	0.79	0.85
ln(ME)	6.15	7.65	7.76	7.37	6.67
ln(BM)	-0.05	-0.55	-0.48	-0.25	0.17
ln(A/ME)	1.19	0.35	0.35	0.64	1.10
ln(A/BE)	1.24	0.90	0.83	0.89	0.93
E/P dummy	0.81	0.00	0.00	0.00	0.00
E(+)/P	0.00	0.04	0.08	0.13	0.27

Table 5 illustrates the two-dimensional variation in average returns that results when the 5 size quintiles of firms are each subdivided into 5 portfolios according to *BM* . Within a size quintile, average returns tend to increase with *BM* (with the exception of ME-2). Similarly, within a *BM* quintile, average returns tend to decrease with the increase in size. This average return matrix clarifies the conclusion from Table 3 that, controlling for size, book-to-market equity captures variation in average returns; and controlling for book-to-market equity, size captures variation in average returns (though weakly than in F&F).

The correlation between *ln(ME)* and *ln(BM)* for individual stocks is -0.41, almost twice the measure documented by F&F. The correlation between size and book-to-market equity affects the regressions in Table 3. The inclusion of *ln(BM)* moves the average slope on *ln(ME)* from -0.14 (t = -1.42) in the univariate regressions to -0.07 (t = -0.70) in the bivariate regressions. Similarly, by including *ln(ME)* in the regressions lowers the average slope on *ln(BM)* from 0.44 to 0.36 (still a 2.07 standard errors from 0). Although the -0.41 correlation between *ln(ME)* and *ln(BM)* is high, and the average slopes in the bivariate regressions in Table 3 show that *ln(ME)* significance is questionable in explaining the cross-section of average returns, the 5x5 matrix in Table 5 farther documents that, (a) book-to-market equity captures important variation in the cross-section of average returns after controlling for size,, and (b) average returns are still related to size after controlling for *BM* .

Table 5 - Average Monthly Returns on Portfolios Formed on Size and Book-to-Market Equity (July 1999 to June 2015)

	Book-to-Market (BM) Portfolios					
	All	Low	2	3	4	High
All		1.55	1.26	1.66	1.92	2.31
Small-ME	1.99	1.48	1.76	1.38	2.14	3.42
ME-2	1.63	1.73	1.51	1.51	1.78	1.64
ME-3	2.03	1.57	1.25	2.64	1.95	2.74
ME-4	1.77	1.63	1.11	1.72	2.24	2.21
Large-ME	1.23	1.36	0.64	1.03	1.54	1.57

Summing up in order of evidence strenght, on average, book-to-market equity had a positive premium, size showed vestiges of a negative premium, and the market β did not seem to explain the cross-section of stock returns for the period from July 1999 to June 2015. As outlined in the introduction, if these characteristics are proxies for fundamental risk, we might expect them to continually pay premiums. If, on the other hand, return premiums arose because of mispricing, we might expect these excess returns not to generalize along time as investors arbitrate them.

To check if these patterns holds in general, Table 6 presents the average FM slopes for two equal subperiods (July 1999-June 2007 and July 2007- June 2015) from five cross-section regressions of stock returns on: (a) β only, (b) $ln(ME)$ only, (c) $ln(ME)$ only, (d) $ln(ME)$ and $ln(BM)$, and (e) β, $ln(ME)$ and $ln(BM)$. The average returns on the equal-weighted index (E.W.I.), Ibovespa (the BOVESPA index) and the interbank deposit rate (C.D.I.) are also shown for reference.

Recalling equation (5) in section 2, the intercept is the return on a portfolio in which the weighted averages of the explanatory variables are 0. In our sample, the intercept is weighted toward firms with no sensitivity to the market portfolio ($\beta=0$), small in size (ME is in millions of Brazilian Reais, so $ln(ME) = 0$ implies $ME = R\$1million$), and with medium-high book-to-market ratios (Table 4 indicates that $ln(BM) = 0$ is between the 3rd and the 4rd quintiles). Thus, for example, if small stocks had higher average returns in the whole sample, and $ln(ME) = 0$ is for small firms, the average intercept is above the returns on the E.W.I and Ibovespa, and vice-versa.[7]

Our results in Table 6 differ from F&F, who found that size and book-to-market premiuns kept the negative and positive signals respectively in both subperiods. We find that the size premium, which was negative and significant in the earlier subperiod, becames positive and insignificant in the later subperiod. The significant positive book-to-market premium in the earlier subperiod is also turned into an insignificant coefficient in the later subperiod. For the market β, the small positive and insignificant β's average premium for the overall period is revealed to be the combination of a positive premium for the earlier subperiod and a negative premium in the later subperiod.[8]

With the caveat that inferences for such short subperiods lack power, the weaker effects of size and book-to-market in the second period, and the change in signal of the market

[7] Ibovespa is a liquidity based index that tends to overweight large firms.
[8] The fact that the β's market price of risk seems insignificant, and negative, has already been documented in Brazil, by Hagler and Brito (2006) and Murakoshi and Brito (2009) among others.

premium, undermine the hyphotesis that those variables consistently explain the cross-section of average stock returns.

Table 6 - Subperiod Average Monthly Returns on the Ibovespa and Subperiod Means of the Intercepts and Slopes from the Monthly FM Cross-Sectional Regressions

	7/1999 - 6/2015 (192 Mos.)			7/1999 - 6/2007 (96 Mos.)			7/2007 - 6/2015 (96 Mos.)		
Variable	Mean	Std. Dev.	t(Mn)	Mean	Std. Dev.	t(Mn)	Mean	Std. Dev.	t(Mn)
Benchmark Returns									
E.W.I.	1.84	6.64	3.85	3.37	6.77	4.88	0.31	6.17	0.50
Ibovespa	1.09	7.84	1.93	1.98	8.37	2.31	0.21	7.21	0.28
C.D.I.	1.10	0.34	44.55	1.38	0.24	56.52	0.82	0.15	54.73
$R_{it} = \lambda^0_t + \lambda^\beta_t \beta_{it} + e_{it}$									
λ^0	1.31	7.59	2.39	1.23	9.00	1.34	1.39	5.90	2.31
λ^β	0.45	9.60	0.65	1.96	10.94	1.75	-1.06	7.81	-1.33
$R_{it} = \lambda^0_t + \lambda^{ME}_t \ln(ME_{it}) + e_{it}$									
λ^0	2.65	11.64	3.16	5.15	12.59	4.01	0.15	10.06	0.15
λ^{ME}	-0.14	1.28	-1.47	-0.30	1.44	-2.07	0.03	1.09	0.29
$R_{it} = \lambda^0_t + \lambda^{BM}_t \ln(BM_{it}) + e_{it}$									
λ^0	1.72	6.36	3.75	3.01	7.06	4.17	0.43	5.29	0.79
λ^{BM}	0.44	2.37	2.54	0.88	2.78	3.09	-0.01	1.79	-0.04
$R_{it} = \lambda^0_t + \lambda^{ME}_t \ln(ME_{it}) + \lambda^{BM}_t \ln(BM_{it}) + e_{it}$									
λ^0	2.18	12.03	2.51	4.32	13.42	3.15	0.04	10.10	0.04
λ^{ME}	-0.07	1.34	-0.70	-0.19	1.54	-1.19	0.05	1.10	0.47
λ^{BM}	0.36	2.40	2.07	0.66	2.90	2.23	0.05	1.71	0.30
$R_{it} = \lambda^0_t + \lambda^\beta_t \beta_{it} + \lambda^{ME}_t \ln(ME_{it}) + \lambda^{BM}_t \ln(BM_{it}) + e_{it}$									
λ^0	2.43	13.77	2.44	3.36	15.66	2.10	1.50	11.59	1.26
λ^β	-0.16	10.66	-0.21	0.79	12.68	0.61	-1.12	8.10	-1.35
λ^{ME}	-0.07	1.38	-0.75	-0.15	1.57	-0.93	0.00	1.17	0.00
λ^{BM}	0.34	2.56	1.83	0.61	3.21	1.86	0.07	1.66	0.39

A comparison of the Brazilian literature also seem to indicate some instability of these premiums in Brazil. For the period from March 1988 to February 1996, not included in our main sample, Costa Jr. and Neves (2000) use a Seemingly Unrelated Regression of eight portfolios' returns on the portfolios' β, *E/P*, size and *BM*. They find that the three fundamental varibles are individually and jointly significant at the 5% level with the same signs of F&F Table 3 (the equivalente of our Table 3). However, they also get that the market β price of risk (λ^{β}) is the most important factor in their sample.[9]

Braga and Leal (2003) study 16 portfolios of Brazilian stocks sorted by size and *BM* (4x4 portfolios) for the period July1991 - June1998 and confirm that high *BM* stocks have greater average returns and Sharpe ratios than low *BM* ones, but do not find any size effect.

Alternatively, to more convincingly dismiss the market β and size as factor premiums, and to allow the reader fully compare the Brazilian case with F&F, we look for the continuation of these premiums into the longer 27-year period July 1988-June 2015 available at Economatica, in the Appendix. Basically, the results from Tables A.1 to A.4 confirm that the size is more powerful than β to explain the cross-section of expected returns in Brazil, but even size does not seem to consistent risk-factor explanation , given its subperiod variability.

Finally, we attempt a first glance at the issues of liquidity and state-controlled firms. Liquidity and state governance risk are characteristics of concern to investors in Brazil. For example, if growth stocks are more liquid (or less state-controlled) than value stocks, the value premium may in part reflect a compensation for the lower liquidity (or for greater management inefficiency) of value firms. Are the return factors size and book-to-market cross-sectionally correlated with liquidity and state-control?

[9] In the Appendix, below, we also find significant λ^{β} for the overall July 1988 - June 2015.

Table 7 - Average Monthly Returns on Portfolios Formed on Size and Book-to-Market Equity (July 1999 to June 2015) for Subsamples

	Book-to-Market (BM) Portfolios					
	All	Low	2	3	4	High
			7.1. Full sample			
All		1.55	1.26	1.66	1.92	2.31
Small-ME	1.99	1.48	1.76	1.38	2.14	3.42
ME-2	1.63	1.73	1.51	1.51	1.78	1.64
ME-3	2.03	1.57	1.25	2.64	1.95	2.74
ME-4	1.77	1.63	1.11	1.72	2.24	2.21
Large-ME	1.23	1.36	0.64	1.03	1.54	1.57
			7.2. Only companies with more than 5 trades in the month			
All		1.61	1.00	1.77	2.21	1.84
Small-ME	1.98	2.19	1.08	1.45	3.14	2.31
ME-2	1.60	1.74	0.99	1.70	2.10	1.54
ME-3	2.00	1.58	1.30	2.77	2.07	2.56
ME-4	1.51	1.56	0.11	1.62	2.67	1.51
Large-ME	1.18	0.99	1.42	1.48	0.99	1.06
			7.3. Only private controlled companies			
All		1.61	1.33	1.50	1.98	2.38
Small-ME	1.96	1.65	1.68	1.53	1.80	3.14
ME-2	1.59	1.85	1.53	1.19	1.55	1.84
ME-3	2.10	1.69	1.40	2.09	2.60	2.91
ME-4	1.84	1.54	1.47	1.65	2.28	2.29
Large-ME	1.26	1.36	0.58	0.97	1.66	1.79
			7.4. Only private controlled companies with more than 5 trades in the month			
All		1.65	1.01	1.59	1.90	2.47
Small-ME	1.98	2.42	1.23	1.39	2.59	2.38
ME-2	1.71	1.46	1.08	1.85	1.61	2.67
ME-3	1.83	1.29	1.50	2.17	1.93	2.67
ME-4	1.62	1.18	0.31	1.70	2.30	2.62
Large-ME	1.28	1.77	0.85	0.82	1.07	1.97

The Tables 7 and 8 show that liquidity does not seem to change the pattern of returns regarding size. But Table 7 lightly indicates that liquidity reduces the value premium – average returns of high BM are lower and of low BM are higher –, supporting the covariance hypothesis with liquidity risk. Regarding the relation of state-control and average returns,

Table 7 documents little increases in most of the portfolios average returns with the exclusion of state-controlled firms, instead of the decrease in premium predicted by the covariance hypothesis with state mismanagement risk. Basically this result reveals that state-controlled companies rewarded stockholder below the average of firms with similar size and book-to-market. Because state-controlled firms usually have larger size and higher book-to-market, their exclusion decreases the importance of the size premium and increases the importance of the value premium in Table 8.

Table 8 - Subperiod Means of the Intercept and Slopes from the Monthly FM Cross-Sectional Regression $R_{it} = \lambda^0_t + \lambda^{ME}_t \ln(ME_{it}) + \lambda^{BM}_t \ln(BM_{it}) + e_{it}$

	7/1999 - 6/2015 (192 Mos.)			7/1999 - 6/2007 (96 Mos.)			7/2007 - 6/2015 (96 Mos.)		
Variable	Mean	Std. Dev.	t(Mn)	Mean	Std. Dev.	t(Mn)	Mean	Std. Dev.	t(Mn)
8.1. Full sample									
λ^0	2.18	12.03	2.51	4.32	13.42	3.15	0.04	10.10	0.04
λ^{ME}	-0.07	1.34	-0.70	-0.19	1.54	-1.19	0.05	1.10	0.47
λ^{BM}	0.36	2.40	2.07	0.66	2.90	2.23	0.05	1.71	0.30
8.2. Only companies with more than 5 trades in the month									
λ^0	2.30	14.09	2.26	4.68	15.13	3.03	-0.07	12.61	-0.06
λ^{ME}	-0.10	1.55	-0.93	-0.27	1.78	-1.47	0.06	1.27	0.44
λ^{BM}	0.28	2.64	1.48	0.63	3.21	1.92	-0.07	1.84	-0.36
8.3. Only private controlled companies									
λ^0	1.82	12.30	2.05	3.67	13.70	2.63	-0.04	10.47	-0.03
λ^{ME}	-0.01	1.38	-0.06	-0.08	1.57	-0.48	0.06	1.16	0.55
λ^{BM}	0.45	2.64	2.37	0.83	3.23	2.51	0.08	1.82	0.42
8.4. Only private controlled companies with more than 5 trades in the month									
λ^0	1.94	15.17	1.77	4.00	16.48	2.38	-0.13	13.49	-0.10
λ^{ME}	-0.05	1.74	-0.41	-0.17	2.04	-0.80	0.07	1.38	0.46
λ^{BM}	0.39	2.91	1.83	0.83	3.56	2.27	-0.05	2.00	-0.26

5. Conclusion

In this article, we have followed Fama and French (1992) closely to make sense of how (dis)similar are the Brazilian expected stock returns from the U.S. reference. Size and book-to-market effects were measured for the individual stocks and studied as individual observable characteristics that explain the cross section of returns.

Similar to the U.S., where F&F find that book-to-market is the main factor explaining returns, we confirm that this factor matters in Brazil. Although the book-to-market shows some power alone, or together with other variables, in the more recent period, it fades away. However insignificant in most of the FM regressions, there is some hope for size. The market β does not play a convincing role in explaining the cross section of expected returns either.

Given our studied sample only has 16 years, we are aware that inferences about the average size slopes may lack power. In a future study, we intend to work the size and book-to-market effects as common risk factors of portfolios, like in Fama and French (1993). Whether it is the characteristics or the covariance structure of returns that explains the cross-sectional variation in stock returns, this is something to be debated in Brazil, as suggested in Daniel and Titman 1997.

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Appendix – Size versus β with the Economatica sample from July 1988 to June 2015

Given the instability of the estimated premiums, noted differences with the previous Brazilian literature, and because inferences about the average size slopes may lack power with only 16 years of data, we perform further test on a Economatica sample from July 1986 to June 2015.

Because those years comprise very different nominal returns due to changes in the inflation regime, we work with real variables. And in this sample, to provide more identifying variation – that would bias the result in favor of finding characteristics effects – we include every firm that presented at least 36 months of returns, without necessarily being contiguous (as we required in the main text tests). We choose not to include the accounting ratios in the 1988-2015 tests because of the baffling noise in these series caused by the high inflation until 1995.

Table A.1 presents tests when the βs of size portfolios are allocated to individual stocks (i.e., "one-pass size portfolios). It indicates that, along the size quintiles, the positive relation between average returns and βs of size portfolios also hold from 1988 to 2015. It also shows how the estimation of β as the sum of the slopes of current and prior month E.W.I. returns affects the β. The regressions of portfolios produce similar results than the regressions of individual stock, taming concerns that the β is at a disadvantage in regressions that allocate portfolio βs to individual stocks. Besides, the average residuals from the simple regression of returns on βs of size portfolios exhibit no relation to size. However, when the residuals from the regression of returns on βs of size portfolios are grouped using pre-ranking βs of individual stocks, the average residuals tend to decrease with the increase in

βs in the regression of returns on β alone, contradicting the CAPM prediction of no pattern after the β-risk extraction.

Because the βs of size portfolios leave unexplained some variation in returns, Table A.2 subdivide size quintiles into pre-ranking βs of individual stocks (i.e., "tow-pass size-β portfolios) wondering if the variation in β lost in the size-sorting dimension could returns. As described in panel A.2.A and A.2.B, average returns do not seem to monotonically increase with βs within a size quintile. But the sample has not provided much post-β variation either. This little variation is the reason why, different from F&F, by allowing for variation in βs that is unrelated to size does not kill the relation between returns and βs in Table A.3, nor does it improve the average residuals tendency to decrease with the increase in βs in the regression of returns on β alone.

The correlation between size and β is -0.71 for βs from size portfolios and -0.47 for portfolios from size-β portfolios. That said, the multicollinearity in the bivariate regressions on β and size is obvious in Table A.1, while Table A.2 is more suitable to disentangle these two factors. These estimates in Table A.2 point that the size effect is stronger than the β.

Our β and size effects significance when used alone for the period 1988-2015 are at odds with evidence from Table 3 in the main text. To clarify that this qualitative difference is primarily due to the different sample periods, in Table A.4 we divide the 1988-2015 period into three subperiods: 7/1988 - 6/1999 (132 months), 7/1999 - 6/2007 (96 months) and 7/2007 - 6/2015 (96 months).

In Table A.4, we surprisingly find a positive and economically import premium for the market β, as well as a negative and economically import premium for size, when they are the only factor controlled for in this broader sample. However, these factors' significance for the overall July 1988-June 2015 period come entirely from the strong individual significance of the market β and size for the subperiod from 7/1988 - 6/2007; thus not contradicting our main results. And, given the high correlation between the market β and size, neither variable is individually significant for the overall sample; with the market β dominating in 7/1988-6/1999 and size dominating in 7/1999-6/2007.

Table A.1 - Average returns, post-ranking βs and Fama-MacBeth Slopes for Size Portfolios of Economatica July 1988 - June 2015

	Portfolios Formed on Size				
	1	2	3	4	5
Ave. return	3.67	2.15	2.19	1.84	1.91
Ave. firms	27	27	26	26	26
Simple β	0.35	0.34	0.33	0.29	0.26
Std. error	(0.04)	(0.03)	(0.03)	(0.03)	(0.03)
Sum β	0.39	0.36	0.34	0.27	0.19
Std. error	(0.05)	(0.04)	(0.04)	(0.04)	(0.05)

	Portfolio regressions				Individual stock regressions			
	(1) β	(2) ln(ME)	(3) β	and ln(ME)	(4) β	(5) ln(ME)	(6) β	and ln(ME)
Slope	6.30	-0.25	-11.25	-0.58	6.36	-0.26	-9.87	-0.57
Std. error	(3.42)	(0.14)	(9.40)	(0.42)	(3.42)	(0.13)	(5.50)	(0.23)

	Average Residuals of Stocks Grouped on on Size				
	1	2	3	4	5
Regression (4)	0.84	-0.53	-0.37	-0.25	0.32
Std. error	(0.29)	(0.23)	(0.27)	(0.19)	(0.13)
Regression (5)	0.44	-0.52	-0.14	-0.14	0.39
Std. error	(0.22)	(0.24)	(0.25)	(0.18)	(0.20)
Regression (6)	0.30	-0.35	0.13	-0.18	0.13
Std. error	(0.19)	(0.20)	(0.21)	(0.19)	(0.12)

	Average Residuals of Stocks Grouped on on Pre-Ranking β				
	1	2	3	4	5
Regression (4)	0.32	0.27	-0.47	0.07	-0.20
Std. error	(0.25)	(0.23)	(0.21)	(0.25)	(0.33)
Regression (5)	0.35	0.25	-0.43	0.03	-0.24
Std. error	(0.27)	(0.23)	(0.21)	(0.25)	(0.33)
Regression (6)	0.29	0.18	-0.35	0.07	-0.23
Std. error	(0.25)	(0.23)	(0.21)	(0.25)	(0.32)

Table A.2 - Properties of Portfolios Formed on Size and Pre-Ranking βs : Economatica Sample Sorted by Size then pre-rankng β (July 1988 to June 2015)

	All	Low-β	β-2	β-3	β-4	High-β
		Panel A.2.A: Average Monthly Returns (in Percent)				
All		3.31	2.56	2.43	2.80	2.87
Small-ME	3.67	5.43	2.92	2.87	3.25	4.12
ME-2	2.15	2.18	1.50	1.61	2.71	2.58
ME-3	2.19	2.64	2.61	2.30	1.86	1.45
ME-4	1.84	2.16	2.30	2.03	1.32	1.39
Large-ME	1.91	1.99	1.83	2.15	1.77	1.58
		Panel A.2.B: Post-Ranking βs				
All		0.32	0.31	0.32	0.33	0.40
Small-ME	0.39	0.48	0.39	0.33	0.36	0.37
ME-2	0.36	0.35	0.31	0.37	0.41	0.38
ME-3	0.34	0.38	0.43	0.26	0.32	0.32
ME-4	0.27	0.29	0.21	0.30	0.21	0.36
Large-ME	0.19	0.15	0.14	0.21	0.18	0.29
		Panel A.2.C: Average Size (ln(ME))				
All		6.55	6.55	6.60	6.54	6.45
Small-ME	3.77	3.73	3.85	3.82	3.77	3.70
ME-2	5.53	5.58	5.55	5.53	5.50	5.48
ME-3	6.60	6.63	6.62	6.60	6.58	6.56
ME-4	7.62	7.62	7.66	7.65	7.60	7.59
Large-ME	9.27	9.27	9.22	9.44	9.42	8.98

Table A.3 - Average Slopes, Their Standard Errors (SE), and Average Residuals from Monthly FM Regressions for Individual Economatica Stocks and Portfolio Formed on Size and Pre-Ranking bs : July 1988 - June 2015

	Portfolio regressions				Individual stock regressions			
	(1) β	(2) ln(ME)	(3) β and ln(ME)		(4) β	(5) ln(ME)	(6) β and ln(ME)	
Slope	5.89	-0.25	2.41	-0.18	5.80	-0.26	1.65	-0.21
Std. error	(2.45)	(0.14)	(2.35)	(0.17)	(2.41)	(0.13)	(2.24)	(0.16)

	Average Residuals of Stocks Grouped on on Size				
	1	2	3	4	5
Regression (4)	0.87	-0.50	-0.36	-0.27	0.26
Std. error	(0.33)	(0.24)	(0.26)	(0.19)	(0.22)
Regression (5)	0.44	-0.52	-0.14	-0.14	0.39
Std. error	(0.22)	(0.24)	(0.25)	(0.18)	(0.20)
Regression (6)	0.44	-0.52	-0.18	-0.13	0.41
Std. error	(0.21)	(0.23)	(0.23)	(0.19)	(0.19)

	Average Residuals of Stocks Grouped on on Pre-Ranking β				
	1	2	3	4	5
Regression (4)	0.36	0.25	-0.47	0.13	-0.32
Std. error	(0.29)	(0.23)	(0.22)	(0.27)	(0.33)
Regression (5)	0.35	0.25	-0.43	0.03	-0.24
Std. error	(0.27)	(0.23)	(0.21)	(0.25)	(0.33)
Regression (6)	0.45	0.18	-0.42	0.01	-0.27
Std. error	(0.26)	(0.24)	(0.21)	(0.26)	(0.34)

Table A.4 - Subperiod Average Monthly Returns on the Indices and Average Values of the Intercepts and Slopes for the FM Cross-Sectional Regressions of Individual Stock Returns on β and Size

	7/1988 - 6/2015 (324 Mos.)			7/1988 - 6/1999 (132 Mos.)			7/1999 - 6/2007 (96 Mos.)			7/2007 - 6/2015 (96 Mos.)		
Variable	Mean	Std. Dev.	t(Mn)	Mean	Std. Dev.	t(Mn)	Mean	Std. Dev.	Std. Dev.	Mean	Std. Dev.	Std. Dev.
Benchmark Retunrs												
E.W.I.	4.22	20.26	3.75	7.14	30.51	2.69	4.40	7.17	6.01	0.03	5.61	0.05
Ibovespa	1.76	14.72	2.15	3.55	21.17	1.92	1.35	8.18	1.62	-0.30	6.58	-0.45
$R_{it} = a + b_{1t}\beta_{it} + e_{it}$												
a	0.54	15.68	0.62	0.30	22.66	0.15	1.41	9.20	1.51	-0.02	6.51	-0.03
b_1	5.80	43.43	2.40	10.68	63.64	1.93	5.04	23.06	2.14	-0.16	15.08	-0.10
$R_{it} = a + b_{2t}\ln(ME_{it}) + e_{it}$												
a	3.94	18.58	3.82	5.54	25.97	2.45	5.85	12.02	4.77	-0.16	8.62	-0.18
b_2	-0.26	2.40	-1.97	-0.36	3.48	-1.19	-0.40	1.38	-2.85	0.01	0.92	0.12
$R_{it} = a + b_{1t}\beta_{it} + b_{2t}\ln(ME_{it}) + e_{it}$												
a	3.28	26.25	2.25	2.18	35.95	0.70	8.26	19.53	4.15	-0.18	11.82	-0.15
b_1	1.65	40.31	0.74	7.32	58.38	1.44	-4.49	22.36	-1.97	-0.01	15.48	0.00
b_2	-0.21	2.86	-1.33	-0.13	4.13	-0.37	-0.54	1.71	-3.12	0.01	1.11	0.11