The State Level Impact of Uncertainty Shocks

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Abstract

This paper uses a FAVAR model with stochastic volatility to estimate the impact of uncertainty shocks on real income growth in US states. The results suggest that there is a large degree of heterogeneity in the magnitude and the persistence of the response to uncertainty shocks across states. The response is largest in Michigan, Indiana and Arkansas while the real income in New York, Alaska and New Mexico seems least sensitive to uncertainty. We relate the cross section of responses to state-level characteristics such as industry composition, financial debth and fiscal conditions.

[**Preliminary Version**]

JEL Codes: C15,C32, E32

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1 Introduction

The recent financial crisis and ensuing recession have led to a renewed interest in the impact of uncertainty shocks. A number of proxies for uncertainty have been proposed in the recent literature and several papers use VAR based analyses to estimate the impact of uncertainty shocks (see for example Bloom (2009) and Jurado *et al.* (2013)). In addition, a growing DSGE based literature has documented the transmission mechanism of these shocks from a theoretical point of view (see for example FernÃąndez-Villaverde *et al.* (2015)).

Overall, the empirical literature on this subject provides strong evidence that uncertainty shocks can have a significant adverse impact on the economy. For example, the analysis in Bloom (2009) suggests that a unit increase in uncertainty leads to a 1% decline in US industrial production and similar results are reported in related papers.

However, the estimates reported in these papers typically focus on the impact on aggregate data for the US. To our knowledge, the impact of uncertainty shocks at the level of US states has remained unexplored in the literature. This is surprising given the strong evidence that points to structural differences across US states. For example, the literature on the regional impact of monetary policy shocks (see **?** amongst others), shows that states differ substantially in terms of the type and concentration of industry, the banking sector and the degree of credit frictions. These differences make it likely that their response to US-wide uncertainty shocks may also differ.

This paper attempts to fill this gap in the literature. We propose an extended factor augmented VAR (FAVAR) model that allows the estimation of a measure of uncertainty that encompasses volatility from the real and financial sectors of the economy and is a proxy for macroeconomic uncertainty. The proposed FAVAR model allows this measure of uncertainty to affect state-level real income while accounting for the possible impact of idiosyncratic uncertainty shocks. We

find that the impact of uncertainty shocks differs markedly across states. The magnitude and persistence of the response is estimated to be the largest in Michigan, Indiana and Arizona, with income declining by about 0.3% and the impact persisting for more than three years. In contrast, the uncertainty shock has a smaller impact on states such as New York, Alaska and New Mexico.

The analysis in the paper adds to the literature on uncertainty by systematically investigating how the impact of uncertainty differs across US states. The empirical model proposed in the paper builds upon exisiting VAR and FAVAR models by simultaneously allowing the estimation of time-varying volatility and the impact of this volatility on a large number of state-level and aggregate variables.

The paper is organised as follows: Sections 2 and 3 introduce the empirical model and discuss the estimation method. The results from the empirical model are presented in Section 4.

2 Empirical model

The core of the empirical model is the following vector autoregression:

$$F_t = c + \sum_{j=1}^{P} \beta_j F_{t-j} + \sum_{j=1}^{J} \gamma_j \ln \lambda_{t-j} + \Omega_t^{1/2} e_t \tag{1}$$

where F_t is a set of unobserved factors that we describe below. As in Cogley and Sargent (2005), the covariance matrix of the residuals is defined as:

$$\Omega_t = A^{-1} H_t A^{-1'}$$

where A is lower triangular.

Following Carriero *et al.* (0), the volatility of the shocks e_t is given by:

$$H_t = \lambda_t S \tag{2}$$

$$S = diag(s_1, .., s_N)$$

The overall volatility evolves as an AR(1) process:

$$\ln \lambda_t = \alpha + \beta \ln \lambda_{t-1} + Q^{1/2}\eta_t \tag{3}$$

and the diagonal elements of S are scaling factors.

The factors F_t are linked to the observed data via the following observation equation

$$X_{it} = B_i F_t + \sum_{k=1}^{K} \rho_{k,i} \ln h_{it-k} + v_{it} \tag{4}$$

where the idiosyncratic shock is assumed to be heteroscedastic

$$v_{it} = R_t^{1/2} \varepsilon_{it} \tag{5}$$

with $R_t = diag(h_{1t}, ..h_{Nt})$ and

$$\ln h_{it} = a_i + b_i \ln h_{it-1} + q_i^{1/2} n_{it} \tag{6}$$

The model described by equations 1 to 6 has a number of distinctive features. First, the model allows for time-variation in the volatility of idiosyncratic errors v_{it} and shocks to the common components. The time-varying volatility of v_{it} possibly captures changes in the variance of idio-

syncratic shocks and accounts for variable specific features such as measurement error. On the other hand, λ_t is the average volatility of shocks to the common components that summarise the macroeconomic and financial data included in X_{it}. We interpret this variance of the unpredictable component of F_t as a measure of economy-wide uncertainty and η_t as the uncertainty shock. As we show below, this specification produces estimates of uncertainty that are plausible from a historical perspective and compares favourably to semi-parametric measures of uncertainty recently suggested in the literature.

The volatility in mean formulation of equation 1 implies that shocks to λ_t have an impact on the variables included in X_{it}. As discussed below, X_{it} includes state-specific variables as well as aggregate variables and the specification allows us to calculate the response of these to uncertainty shocks. Note that this estimated response is net of the impact of idiosyncratic volatility as the observation equation allows $\ln h_{it}$ to enter as a regressor. For the state-specific variables of interest, this implies that we account for state level uncertainty as captured by h_{it}.

The formulation presented in equations 2 and 3 is related to a number of recent empirical contributions. For example, the structure of the stochastic volatility model used above closely resembles the formulations used in time-varying VAR models (see Cogley and Sargent (2005) and Primiceri (2005)). Our model differs from these studies in that it allows a direct impact of the volatilities on the level of the endogenous variables. The model proposed above can be thought of as a multivariate extension of the stochastic volatility in mean model proposed in Koopman and Uspensky (2000) and applied in Berument *et al.* (2009), Kwiatkowski (2010) and Lemoine and Mougin (2010). In addition, our model has similarities with the stochastic volatility models with leverage studied in Asai and McAleer (2009) and the non-linear model proposed in Aruoba *et al.* (2011). Finally, the model is based on the VAR with stochastic volatility introduced in Mumtaz

and Zanetti (2013), Mumtaz and Theodoridis (n.d.) and Mumtaz and Surico (2013). Models with a common volatility specification are used in Mumtaz and Theodoridis (2014) and Alessandri and Mumtaz (2014).

3 Estimation and model specification

The model defined in equation 1 and 4 is estimated using an MCMC algorithm. In this section we summarise the key steps of the algorithm and provide the details in the technical appendix.[1] The appendix also presents the details on the prior distributions which are standard.

As noted in Bernanke *et al.* (2005), the FAVAR model is subject to rotational indeterminancy of the factors and factor loadings. Following Bernanke *et al.* (2005), we impose a normalisation under which the first $K \times K$ block of the factor loadings is fixed to an identity matrix.

The MCMC algorithm consists of the following steps:

1. Conditional on a draw for the stochastic volatility λ_t, the factors F_t the matrix A, and the variances S, equation (1) represents a VAR model with a known form of heteroscedasticity. The algorithm of Carter and Kohn (2004) is used to draw the VAR coefficients from their conditional posterior density while accounting for the time-varying variance via the Kalman filter.

2. Conditional on a draw for the stochastic volatility F_t, λ_t and S the non-unity and non-zero elements of A are drawn using methods for linear regressions with heteroscedasticity.

3. Given A and λ_t, The elements of S have an inverse Gamma posterior and these parameters can be easily simulated from this distribution.

[1] The appendix presents a small Monte-Carlo experiment that shows that the algorithm displays a satisfactory performance.

4. Conditional on λ_t, the constant α, autoregressive parameter F and variance Q can be drawn using standard results for linear regressions.

5. Conditional on a draw for the factors $F_t$and the volatilities h_{it} methods for heteroscedastic linear regressions can be used to draw the factor loadings B_i and the coefficients $\rho_{k,i}$.

6. Conditional on a draw for the factors $F_t$and the factor loadings B_i, the coefficients $\rho_{k,i}$ and the parameters of the transition equations 6, the stochastic volatilities h_{it} can be drawn using a date by date independence Metropolis step as described in Cogley and Sargent (2005) and Jacquier *et al.* (1994) (see also Carlin *et al.* (1992)). The same algorithm is used to draw the common volatility λ_t.

7. Given the parameters of the observation equation 4 and the transition equation 1, the Carter and Kohn (2004) algorithm is used to draw from the conditional posterior distribution of the factors F_t.

In the benchmark specifications, we use 200,000 replications and base our inference on the last 5,000 replications. The recursive means of the retained draws (see technical appendix) show little fluctuation providing support for convergence of the algorithm.

3.1 Model specification

We consider models with 3 to 7 factors and select the model which minimises the Bayesian Deviance Information Criterion (DIC). Introduced in Spiegelhalter *et al.* (2002), the DIC is a generalisation of the Akaike information criterion – it penalises model complexity while rewarding fit to the data. As shown in the appendix, the DIC can be calculated as $DIC = \bar{D} + p_D$ where $\bar{D}$ measures goodness of fit and p_D approximates model complexity. A model with a lower DIC is preferred.

Table 1 shows that the DIC is minimised for the model with 3 factors. Therefore, we select 3 factors in our benchmark model.

	DIC
3 factors	36652.03
4 factors	38096.85
5 factors	41663.51
6 factors	65216.41
7 factors	72648.66

Table 1: Model Comparison via DIC. Best fit indicated by lowest DIC

In the benchmark model, the lag length P and J is set to 4. We show in the technical appendix that the results do not change substantially for alternative specifications of the model.

3.2 Data

The dataset is quarterly and runs from 1950Q1 to 2014Q1. The panel contains 92 aggregate variables and the real income for each of the 51 US states. Non-stationary variables are log-differenced. Finally, the series are de-meaned. The 92 aggregate variables cover real activity, prices, short-term and long term interest rates, various corporate bond spreads and series on money and credit growth. Data on stock market variables, commodity prices and exchange rates is included. In summary, the aggregate dataset covers the key sectors of the US economy and incorporates a wide range of information. The technical appendix provides a list of the series included in the panel.

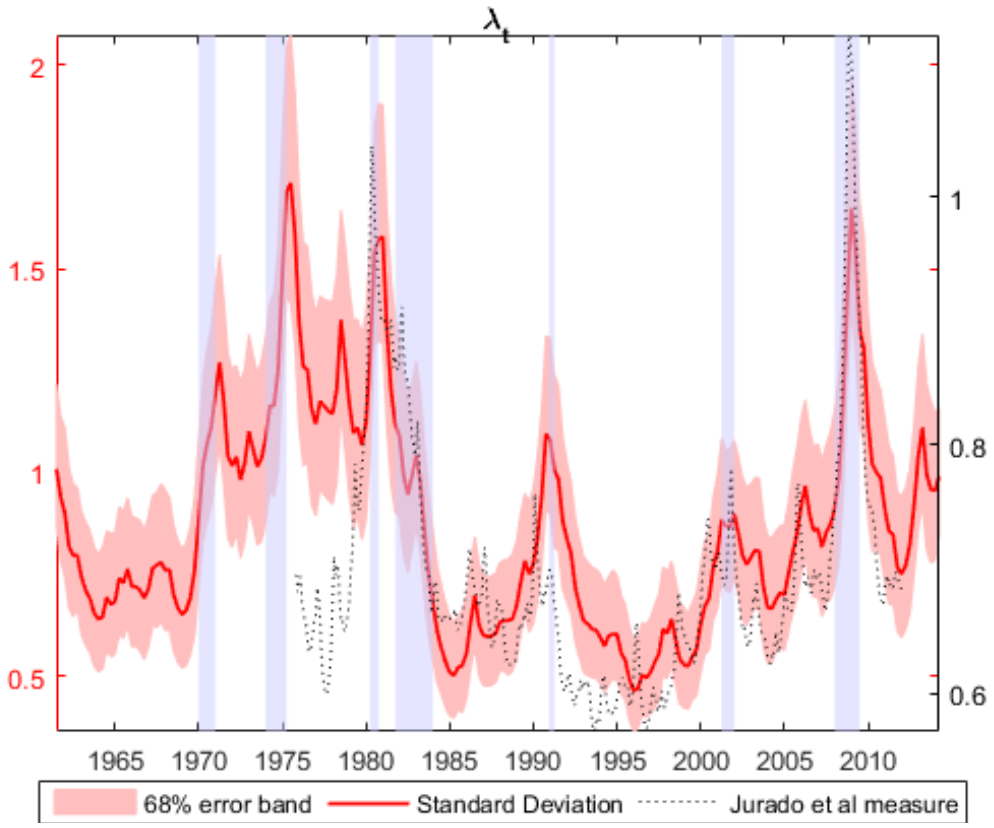
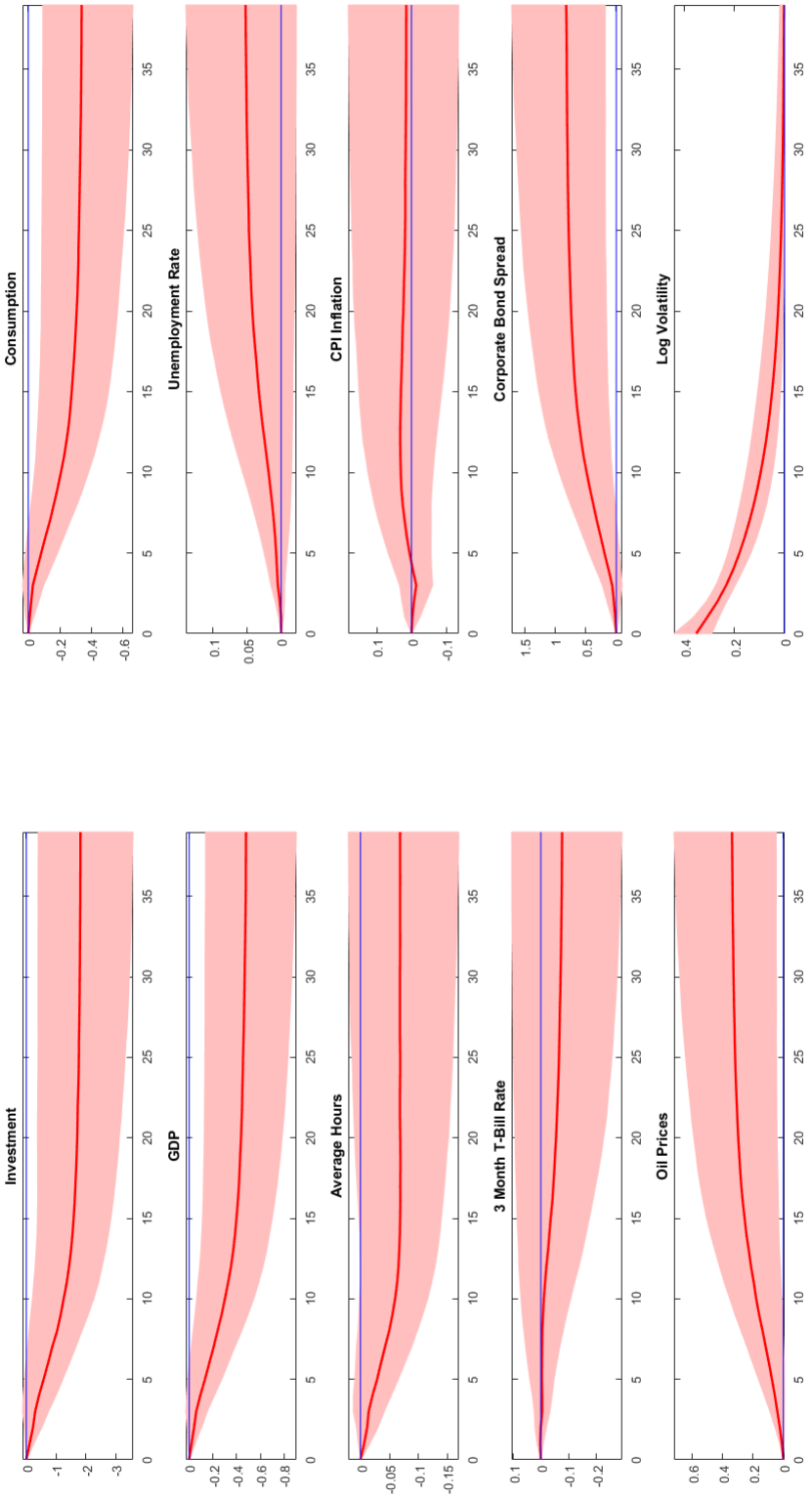


Figure 1: Estimated Volatility λ_t. The posterior estimates of λ_t are compared with the measure of uncertainty reported in Jurado *et al.* (2013). The shaded areas represent the NBER recession dates.

4 Empirical results

4.1 The Measure of Uncertainty

Figure 1 plots the posterior median and the 68% bands for λ_t. The figure also plots the uncertainty measure recently proposed in Jurado *et al.* (2013) for comparison. The measure of uncertainty is high during the early and the mid-1970s reaching a large peak during the early 1980s. The mid-1980s saw the onset of the great moderation and λ_t declined and remained low until the recession during the early 1990s and then during the early 2000s. The recent financial crisis saw a substantial increase in uncertainty with the level of λ_t during 2008/2009 matching the volatility seen during the 1970s and the early 1980s.


Consumption
Unemployment Rate
CPI Inflation
Corporate Bond Spread
Log Volatility
Investment
GDP
Average Hours
3 Month T-Bill Rate
Oil Prices

It is interesting to note that the estimate of λ_t is highly correlated with the measure of uncertainty proposed in Jurado *et al.* (2013). This reflects the fact that the underlying method of capturing uncertainty has a number of similarities with the calculation in Jurado *et al.* (2013). The uncertainty measure in Jurado *et al.* (2013) is the average time-varying variance in the unpredictable component of a large set of real and financial time-series. The volatility specification in equations 2 and 3 has a similar intepretation as it attempts to capture the average volatility in the *shocks* to Z_t where the factors summarise real and financial conditions. However, as discussed above, the model used in this paper offers a distinct advantage for the purpose of estimating the impact of uncertainty shocks – it allows one to recover the responses to overall uncertainty shocks while 'filtering out' the effects of idiosyncratic uncertainty and measurement error captured by h_{it}. In figure 4.1 we consider how innovations to λ_t affect aggregate variables. A one standard deviation uncertainty shock results in a decline in real activity with a fall in hours, investment, consumption and GDP and an increase in the unemployment rate. These estimates are consistent with impulse responses reported in Bloom (2009) and Leduc and Liu (2012) and confirm the conventional view on the impact of uncertainty shocks on real activity. The response of inflation is imprecisely estimated but suggests that the total impact is positive at the two year horizon providing some support to the 'pricing bias' channel postulated in FernÃąndez-Villaverde *et al.* (2015)–in other words, when the economy is characterised by price and wage rigidity, inflation rises in the face of uncertainty because forward looking agents bias their pricing decision upwards in order to avoid supplying goods when demand and costs are high. The uncertainty shock is associated with a deterioration in financial conditions and the corporate bond spread increases. In addition, oil prices rise by about 0.2% in response to an increase in uncertainty.

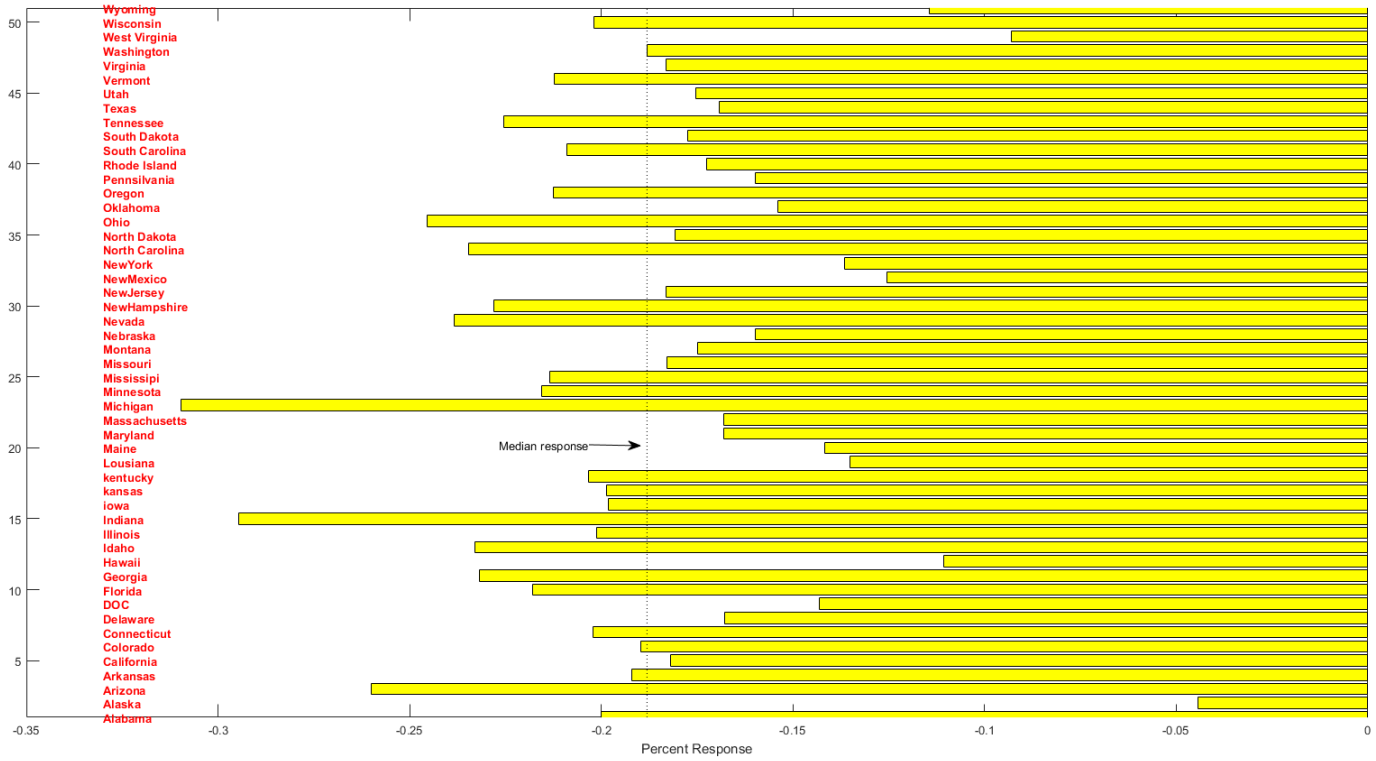
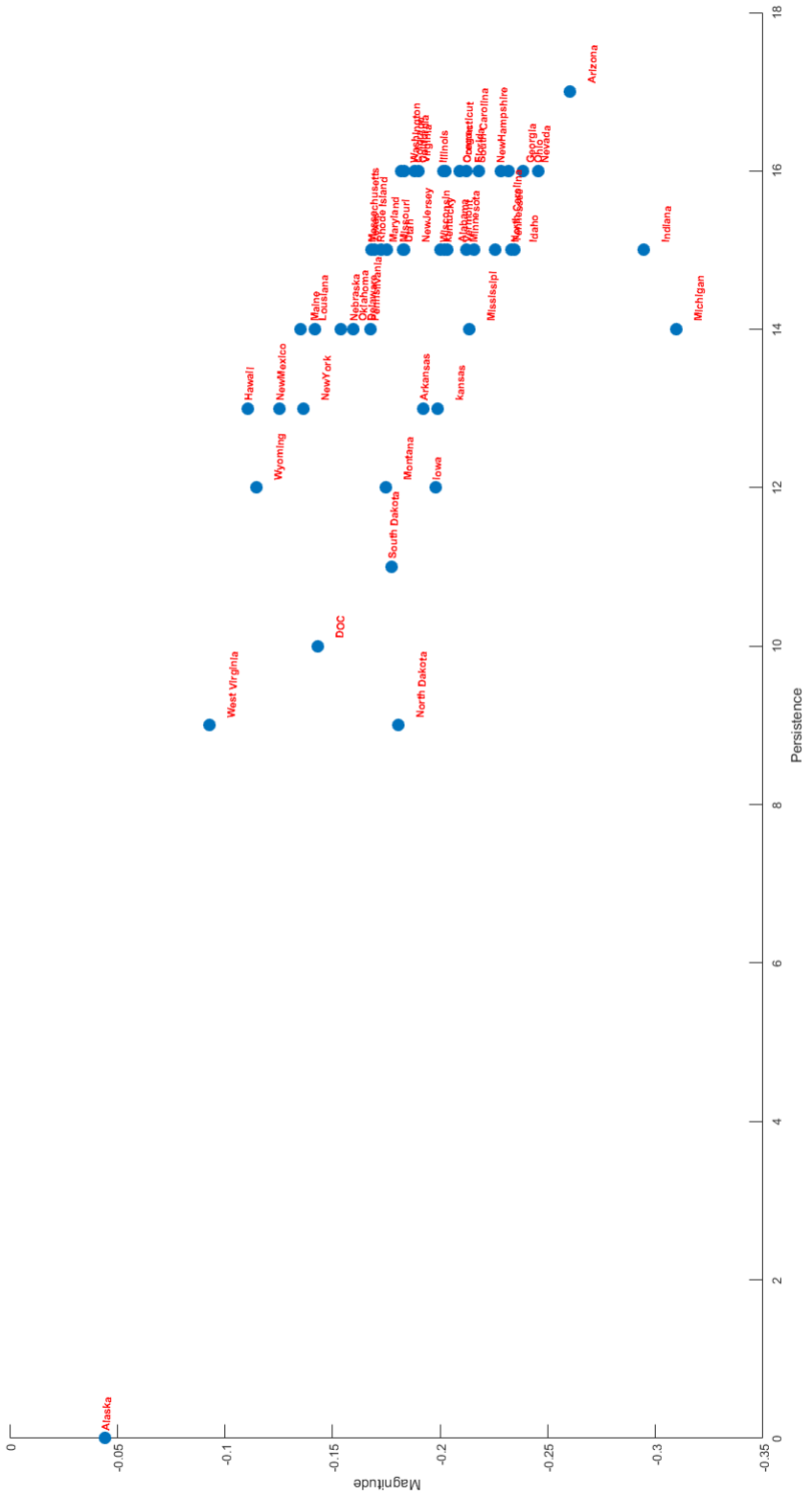


Figure 2: Cumulated response of real income growth to uncertainty shocks at the two year horizon


Persistence
Magnitude
0
2
4
6
8
10
12
14
16
18
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
Alaska
Arizona
Indiana
Michigan
Nevada
Georgia
Ohio
NewHampshire
Idaho
Mississipi
Arkansas
kansas
Montana
Iowa
South Dakota
North Dakota
DOC
West Virginia
Hawaii
NewMexico
NewYork
Wyoming
Maine
Louisiana
Nebraska
Oklahoma
Illinois
NewJersey
Maryland

4.2 State-Level response of uncertainty shocks

In this section we consider the response the real income growth to uncertainty shocks across US states and investigate the presence of heterogeniety at the state level. Figure 2 presents the median estimate of the cumulated response of state-level real income to a one standard deviation uncertainty shock at the two year horizon. Real income declines in all states in response to an increase in US wide uncertainty, with the median decline estimated at 0.18%. The magnitude of the decline is largest in Michigan, Indiana and Arkansas with income falling by about 0.26% to 0.3%. The response in the majority of the states lies within the range -0.15% and -0.25%. States with the lowest response (i.e. larger than -0.15%) include New York, Alaska and New Mexico.

Figure 4.1 shows that the persisitence of the response to uncertainty shocks is larger in states where the impact is estimated to be of a higher magnitude. We measure the persistence by the number of quarters before the hypothesis of a zero response cannot be rejected.[2] In states such as Michigan, Arizona and Indiana, the effects of the uncertainty shock persist for longer than three years. In contrast, there is scant statistical evidence that the impact of the shock is persistent in states such as Alaska.

5 Explaining the heterogeneity of state-level responses

In progress

[2]The 68% posterior error bands are used to calculate the persistence. For states where the error bands include a zero response at more than one point on the horizon, the last instance is used as the persistence measure.

6 Conclusions

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1 Model

The FAVAR model is defined as

$$
\begin{aligned}
X_{it} &= B_i F_t + \sum_{k=1}^{K} \rho_{k,i} \ln h_{it-k} + v_{it} && (1)\\
F_t &= c + \sum_{j=1}^{P} \beta_j F_{t-j} + \sum_{j=1}^{J} \gamma_j \ln \lambda_{t-j} + \Omega_t^{1/2} e_t && (2)\\
R_t &= diag(h_{1t}, ..h_{Nt}) && (3)\\
\Omega_t &= A^{-1} H_t A^{-1'} && (4)\\
H_t &= diag(S_k \lambda_t), k = 1, 2, ..N && (5)\\
\ln \lambda_t &= \alpha + \beta \ln \lambda_{t-1} + Q^{1/2} \eta_t && (6)\\
\ln h_{it} &= a_i + b_i \ln h_{it-1} + q_i^{1/2} n_{it} && (7)\\
& \quad \varepsilon_{it}, e_t, \eta_t, n_{it} \tilde{} N(0,1) && (8)
\end{aligned}
$$

2 Estimation

2.1 Priors

2.1.1 Factor loadings

The prior on $\tilde{B}_i = [B_i; \rho_i]$ is normal and is assumed to be $N(B_{i,0}, V_B)$ where $B_{i,0}$ is set equal to the loadings obtained using a principal component estimate of F_t. The variance V_B is assumed to be equal to 1. The initial estimate of the factors F_t^{PC} provides the initial value of the factors $F_{0\backslash 0}$ with the initial variance set equal to the identity matrix.

2.1.2 VAR Coefficients

Following Banbura *et al.* (2010) we introduce a natural conjugate prior for the VAR parameters $\tilde{b} = \{c, b, \delta\}$ via dummy observations. In our application, the prior means are chosen as the OLS estimates of the coefficients of an AR(1) regression estimated for each endogenous variable using a training sample. As is standard for US data, we set the overall prior tightness $\tau = 0.1$.

2.1.3 Elements of S, A and the parameters of the common volatility transition equation

The elements of S have an inverse Gamma prior: $P(s_i) \tilde{} IG(S_{0,i}, V_0)$. The degrees of freedom V_0 are set equal to 1. The prior scale parameters are set by estimating the following regression: $\bar{\lambda}_{it} = S_{0,i} \bar{\lambda}_t + \varepsilon_t$ where $\bar{\lambda}_t$ is the first principal component of the stochastic volatilities $\bar{\lambda}_{it}$ obtained using a univariate stochastic volatility model for the residuals of each equation of the VAR in equation 2 estimated via OLS using the principal components F_t^{PC}.

The prior for the off-diagonal elements A is $A_0 \sim N\left(\hat{a}^{ols}, V\left(\hat{a}^{ols}\right)\right)$ where $\hat{a}^{ols}$ are the off-diagonal elements of the inverse of the Cholesky decomposition of $\hat{v}^{ols}$, with each row scaled by the corresponding element on the diagonal. These OLS estimates are obtained using the initial VAR model described above. $V\left(\hat{a}^{ols}\right)$ is assumed to be diagonal with the elements set equal to 10 times the absolute value of the corresponding element of $\hat{a}^{ols}$.

We set a normal prior for the unconditional mean $\mu = \frac{\alpha}{1-\beta}$. This prior is $N(\mu_0, Z_0)$ where $\mu_0 = 0$ and $Z_0 = 10$.The prior for Q is $IG(Q_0, V_{Q0})$ where Q_0 is the average of the variances of the transition equations of the initial univariate stochastic volatility estimates and $V_{Q0} = 5$. The prior for β is $N(F_0, L_0)$ where $F_0 = 0.8$ and $L_0 = 1$.

2.1.4 Parameters of the idiosyncratic shock volatility transition equation

We set a normal prior for the unconditional mean $\tilde{\mu} = \frac{a}{1-b}$. This prior is $N(\mu_0, Z_0)$ where $\mu_0 = 0$ and $Z_0 = 10$.The prior for q_i is $IG(q_0, V_{q0})$ where$q_0 = 0.01$ and $V_{q0} = 5$. The prior for b is $N(F_0, L_0)$ where $F_0 = 0.8$ and $L_0 = 1$.

2.2 Gibbs algorithm

The Gibbs algorithm cycles through the following steps:

1. $G\left(F_t \backslash \Xi\right)$: Given a draw for all other parameters (denoted by Ξ), the algorithm of Carter and Kohn (2004) is used to sample from the conditional posterior distribution of F_t. The conditional posterior is: $F_t \backslash X_{it}, \Xi \sim N\left(F_{T\backslash T}, P_{T\backslash T}\right)$ and $F_t \backslash F_{t+1}, X_{it}, \Xi \sim N\left(F_{t\backslash t+1, F_{t+1}}, P_{t\backslash t+1, B_{t+1}}\right)$ where $t = T-1, ..1$. As shown by Carter and Kohn (2004) the simulation proceeds as follows. First we use the Kalman filter to draw $F_{T\backslash T}$ and $P_{T\backslash T}$ and then proceed backwards in time using $F_{t|t+1} = F_{t|t} + P_{t|t} f' P_{t+1|t}^{-1}\left(F_{t+1} - f F_{t\backslash t} - \mu_t\right)$ and $P_{t|t+1} = P_{t|t} - P_{t|t} f' P_{t+1|t}^{-1} f P_{t|t}$. Here f denotes the autoregressive coefficients of the transition equation 2 b in companion form, while μ_t denotes the pre-determined regressors in that equation in companion form.

2. $G\left(\tilde{B}_i \backslash \Xi\right)$: Given a draw for the factors and the variance of the idiosyncratic component, a seperate heteroscedastic linear regression model applies to each X_{it} and the standard formulae for linear regressions apply. In particular, the model for each i is

$$X_{it} = \tilde{B}_i \tilde{F}_t + h_{it}^{1/2} \varepsilon_{it}$$

where $\tilde{F}_t = [F_t, \ln h_{it-1}, \ln h_{it-2}, ...]$. The model can be transformed to remove heteroscedasticity by creating $X_{it}^* = \frac{X_{it}}{\sqrt{h_{it}}}, \tilde{F}_t^* = \frac{\tilde{F}_t}{\sqrt{h_{it}}}$ The conditional posterior is: $N\left(B_i^*, \Lambda_B\right)$

$$\begin{aligned} B_i^* &= \left(V_B^{-1} + \tilde{F}_t^{*\prime} \tilde{F}_t^*\right)^{-1} \left(V_B^{-1} B_{i,0} + \tilde{F}_t^{*\prime} X_{it}^*\right) \\ \Lambda_B &= \left(V_B^{-1} + \tilde{F}_t^{*\prime} \tilde{F}_t^*\right)^{-1} \end{aligned}$$

3. $G\left(h_{it} \backslash \Xi\right)$: Given a draw for the factors, the parameters of the transition equation 7 and the factor loadings $\tilde{B}_i$, a univariate stochastic volatility in mean model applies for each i:

$$\begin{aligned} X_{it} &= B_i F_t + \sum_{k=1}^{K} \rho_{i,l} \ln h_{it-l} + h_{it}^{1/2} \varepsilon_{it} \\ \ln h_{it} &= a_i + b_i \ln h_{it-1} + q_i^{1/2} n_{it} \end{aligned}$$

The algorithm of Jacquier *et al.* (1994) (described below) is used to draw h_{it}.

4. $G(\tilde{b} \backslash \Xi)$.Given a draw of λ_t, the left and the right hand side variables of the VAR: $y_t = F_t$ and $x_t = [c, F_{t-1}, F_{t-2}, ..F_{t-j}, \lambda_t, \lambda_{t-1}, ..\lambda_{t-j}]$ can be transformed to remove the heteroscedasticity in the following manner

$$\tilde{y}_t = \frac{y_t}{\lambda_t^{1/2}}, \tilde{x}_t = \frac{x_t}{\lambda_t^{1/2}}$$

Then the conditional posterior distribution for the VAR coefficients is standard and given by

$$N(\tilde{b}^*, \bar{\Omega} \otimes \left(X^{*\prime} X^*\right)^{-1})$$

where $\tilde{b}^* = \left(X^{*\prime} X^*\right)^{-1}\left(X^{*\prime} Y^*\right), \bar{\Omega} = A^{-1} diag(S) A^{-1\prime}$ and Y^* and X^* denote the transformed data appended with the dummy observations.

5. $G(A \backslash \Xi)$. Given a draw for the VAR parameters the model can be written as $A'\left(v_t\right) = \tilde{e}_t$ where $v_t = F_t - \left(c + \sum_{j=1}^{P} \beta_j F_{t-j} + \sum_{j=1}^{J} \gamma_j \ln \lambda_{t-j}\right)$ and $VAR\left(\tilde{e}_t\right) = H_t$. This is a system of linear equations with a known form of heteroscedasticity. The conditional distributions for a linear regression apply to each equation of this system after a simple GLS transformation to make the errors homoscedastic. The jth equation of this system is given as $v_{jt} = -\alpha v_{-jt} + \tilde{e}_{jt}$ where the subscript j denotes the jth column while $-j$ denotes columns 1 to $j-1$. Note that the variance of $\tilde{e}_{jt}$ is time-varying and given by $\lambda_t S_j$. A GLS transformation involves dividing both sides of the equation by $\sqrt{\lambda_t S_j}$ to produce $v_{jt}^* = -\alpha v_{-jt}^* + \tilde{e}_{jt}^*$ where * denotes the transformed variables and $var\left(\tilde{e}_{jt}^*\right) = 1$. The conditional posterior for α is normal with mean and variance given by M^* and V^* :

$$\begin{aligned} M^* &= \left(V\left(\hat{a}^{ols}\right)^{-1} + v_{-jt}^{*\prime} v_{-jt}^*\right)^{-1} \left(V\left(\hat{a}^{ols}\right)^{-1} \hat{a}^{ols} + v_{-jt}^{*\prime} v_{jt}^*\right) \\ V^* &= \left(V\left(\hat{a}^{ols}\right)^{-1} + v_{-jt}^{*\prime} v_{-jt}^*\right)^{-1} \end{aligned}$$

6. $G(S\backslash\Xi)$. Given a draw for the VAR parameters $A'(v_t) = \tilde{e}_t$. The jth equation of this system is given by $v_{jt} = -\alpha v_{-jt} + \tilde{e}_{jt}$ where the variance of e_{jt} is time-varying and given by $\lambda_t S_j$. Given a draw for λ_t this equation can be re-written as $\bar{v}_{jt} = -\alpha \bar{v}_{-jt} + \bar{e}_{jt}$ where $\bar{v}_{jt} = \frac{v_{jt}}{\lambda_t^{1/2}}$ and the variance of $\bar{e}_{jt}$ is S_j. The conditional posterior is for this variance is inverse Gamma with scale parameter $\bar{e}'_{jt}\bar{e}_{jt} + S_{0,j}$ and degrees of freedom $V_0 + T$.

7. Elements of λ_t. Conditional on the VAR coefficients, and the parameters of the volatility transition equation, the model has a multivariate non-linear state-space representation. Carlin *et al.* (1992) show that the conditional distribution of the state variables in a general state-space model can be written as the product of three terms:

$$\tilde{h}_t\backslash Z_t, \Xi \propto f\left(\tilde{h}_t\backslash\tilde{h}_{t-1}\right) \times f\left(\tilde{h}_{t+1}\backslash\tilde{h}_t\right) \times f\left(Z_t\backslash\tilde{h}_t, \Xi\right) \tag{9}$$

where Ξ denotes all other parameters, Z_t denotes the endogenous variables in equation 2 and $\tilde{h}_t = \ln \lambda_t$. In the context of stochastic volatility models, Jacquier *et al.* (1994) show that this density is a product of log normal densities for λ_t and λ_{t+1} and a normal density for Z_t.Carlin *et al.* (1992) derive the general form of the mean and variance of the underlying normal density for $f\left(\tilde{h}_t\backslash\tilde{h}_{t-1}, \tilde{h}_{t+1}, \Xi\right) \propto f\left(\tilde{h}_t\backslash\tilde{h}_{t-1}\right) \times f\left(\tilde{h}_{t+1}\backslash\tilde{h}_t\right)$ and show that this is given as

$$f\left(\tilde{h}_t\backslash\tilde{h}_{t-1}, \tilde{h}_{t+1}, \Xi\right) \sim N\left(B_{2t}b_{2t}, B_{2t}\right) \tag{10}$$

where $B_{2t}^{-1} = Q^{-1} + F'Q^{-1}F$ and $b_{2t} = \tilde{h}_{t-1}F'Q^{-1} + \tilde{h}_{t+1}Q^{-1}F$. Note that due to the non-linearity of the observation equation of the model an analytical expression for the complete conditional $\tilde{h}_t\backslash Z_t, \Xi$ is unavailable and a metropolis step is required. Following Jacquier *et al.* (1994) we draw from 9 using a date-by-date independence metropolis step using the density in 10 as the candidate generating density. This choice implies that the acceptance probability is given by the ratio of the conditional likelihood $f\left(Z_t\backslash\tilde{h}_t, \Xi\right)$ at the old and the new draw. To implement the algorithm we begin with an initial estimate of $\tilde{h} = \ln \bar{\lambda}_t$ We set the matrix $\tilde{h}^{old}$ equal to the initial volatility estimate. Then at each date the following two steps are implemented:

   (a) Draw a candidate for the volatility $\tilde{h}_t^{new}$ using the density 9 where $b_{2t} = \tilde{h}_{t-1}^{new}F'Q^{-1} + \tilde{h}_{t+1}^{old}Q^{-1}F$ and $B_{2t}^{-1} = Q^{-1} + F'Q^{-1}F$

   (b) Update $\tilde{h}_t^{old} = \tilde{h}_t^{new}$ with acceptance probability $\frac{f\left(Z_t\backslash\tilde{h}_t^{new}, \Xi\right)}{f\left(Z_t\backslash\tilde{h}_t^{old}, \Xi\right)}$ where $f\left(Z_t\backslash\tilde{h}_t, \Xi\right)$ is the likelihood of the VAR for observation t and defined as $|\Omega_t|^{-0.5} - 0.5 \exp\left(\tilde{e}_t\Omega_t^{-1}\tilde{e}'_t\right)$ where $\tilde{e}_t = F_t - \Big(c + \sum_{j=1}^{P}\beta_j F_{t-j} + \sum_{j=1}^{J}\gamma_j \ln \lambda_{t-j} +$ and $\Omega_t = A^{-1}\left(\exp(\tilde{h}_t)S\right)A^{-1'}$

Repeating these steps for the entire time series delivers a draw of the stochastic volatilties.[1]

7. $G(\alpha, \beta, Q\backslash\Xi)$.We re-write the transition equation in deviations from the mean

$$\tilde{h}_t - \mu = \beta\left(\tilde{h}_{t-1} - \mu\right) + \eta_t \tag{11}$$

where the elements of the mean vector μ are defined as $\frac{\alpha}{1-\beta}$. Conditional on a draw for $\tilde{h}_t$ and μ the transition equation 11 is a simply a linear regression and the standard normal and inverse Gamma conditional posteriors apply. Consider $\tilde{h}_t^* = \beta\tilde{h}_{t-1}^* + \eta_t, VAR(\eta_t) = Q$ and $\tilde{h}_t^* = \tilde{h}_t - \mu, \tilde{h}_{t-1}^* = \tilde{h}_{t-1} - \mu$. The conditional posterior of β is $N(\theta^*, L^*)$ where

$$\begin{aligned}
\theta^* &= \left(L_0^{-1} + \frac{1}{Q}\tilde{h}_{t-1}^{*\prime}\tilde{h}_{t-1}^*\right)^{-1}\left(L_0^{-1}F_0 + \frac{1}{Q}\tilde{h}_{t-1}^{*\prime}\tilde{h}_t^*\right) \\
L^* &= \left(L_0^{-1} + \frac{1}{Q}\tilde{h}_{t-1}^{*\prime}\tilde{h}_{t-1}^*\right)^{-1}
\end{aligned}$$

[1] In order to take endpoints into account, the algorithm is modified slightly for the initial condition and the last observation. Details of these changes can be found in Jacquier *et al.* (1994).

The conditional posterior of Q is inverse Gamma with scale parameter $\eta_t'\eta_t + Q_0$ and degrees of freedom $T + V_{Q0}$.

Given a draw for β, equation 11 can be expressed as $\bar{\Delta}\tilde{h}_t = C\mu + \eta_t$ where $\bar{\Delta}\tilde{h}_t = \tilde{h}_t - \beta\tilde{h}_{t-1}$ and $C = 1 - \beta$. The conditional posterior of μ is $N(\mu^*, Z^*)$ where

$$\begin{aligned} \mu^* &= \left(Z_0^{-1} + \frac{1}{Q}C'C\right)^{-1}\left(Z_0^{-1}\mu_0 + \frac{1}{Q}C'\bar{\Delta}\tilde{h}_t\right) \\ Z^* &= \left(Z_0^{-1} + \frac{1}{Q}C'C\right)^{-1} \end{aligned}$$

Note that α can be recovered as $\mu(1-\beta)$

8. $G(a_i, b_i, q_i \backslash \Xi)$. Given a draw for h_{it}, the conditional posterior distributions for the parameters of the transition equations 7 are as described in step 7.

2.3 A Monte-Carlo experiment

In order to examine the performance of this algorithm, we consider a small Monte-Carlo experiment

2.3.1 Data Generating Process

We generate data from the following FAVAR model with 2 factors:

$$X_{it} = B_i F_t + R^{1/2}\varepsilon_{it}$$

where $R = 0.1$, the factor loadings B_i are drawn from $N(0, 0.1)$ and $i = 1, 2, ...100$.

The dynamics of the factors are defined as

$$\begin{pmatrix} F_{1t} \\ F_{2t} \end{pmatrix} = \begin{pmatrix} 0.7 & 0.1 \\ -0.1 & 0.5 \end{pmatrix}\begin{pmatrix} F_{1t-1} \\ F_{2t-1} \end{pmatrix} + \begin{pmatrix} -0.5 \\ 0.5 \end{pmatrix}\ln\lambda_t + \begin{pmatrix} v_{1t} \\ v_{2t} \end{pmatrix}, var\begin{pmatrix} v_{1t} \\ v_{2t} \end{pmatrix} = \Omega_t$$

The variance process is defined as

$$\begin{aligned} \Omega_t &= A^{-1}(S\lambda_t)A^{-1'} \\ A &= \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \\ S &= \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \\ \ln\lambda_t &= -0.1 + 0.75\ln\lambda_{t-1} + (0.5)^{\frac{1}{2}}v_t \end{aligned}$$

We generate 500 observations for X_{it} and drop the first 100 observations to reduce the influenceof initial conditions. The experiment is repeated 500 times. At each iteration, the FAVAR model is estimated using the MCMC algorithm described above using 5000 iterations with a burn-in of 4000 observations. The retained draws are used to calculate the impulse response of X_{it} to a 1 standard deviation shock to $\ln\lambda_t$ for a horizon of 20 periods. In the figures below we report the difference between the cumulated response at various horizons estimated via the MCMC algorithm and the response using the true parameter values for each of the N X_{it}. The figure below shows that, on average, the difference in the estimated responses and the true responses is zero across the panel and across the different horizons considered. This provides evidence that the MCMC algorithm performs well.

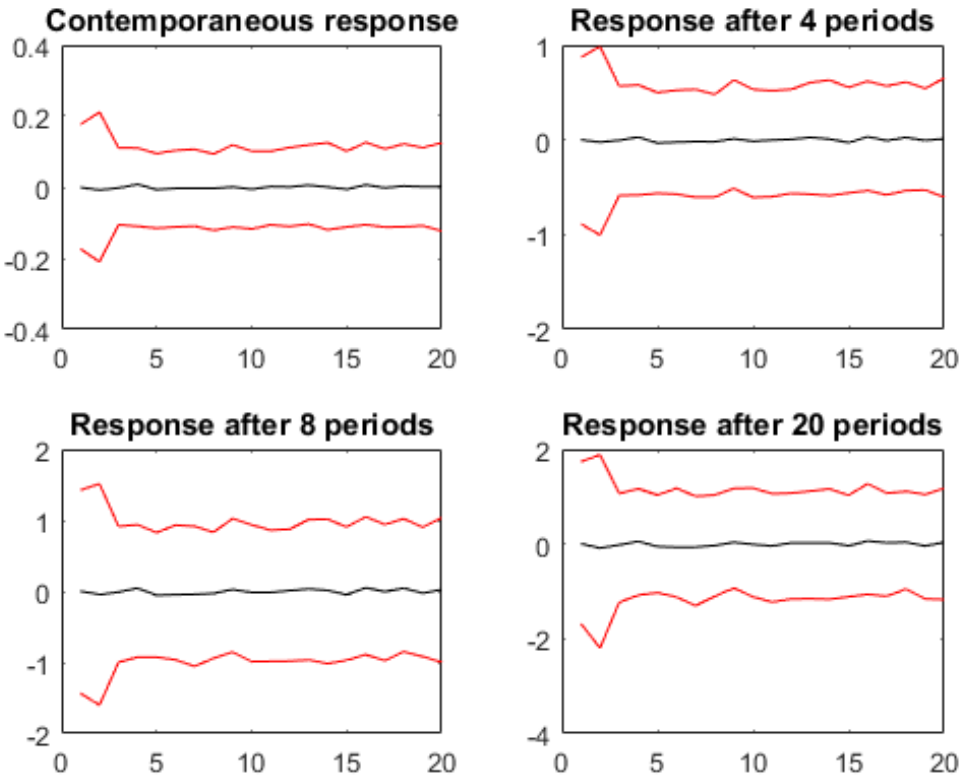

Figure 1: Monte-Carlo experiment

3 Sensitivity Analysis

3.1 Number of factors

We re-estimate the model and set the number of factors to 5. Figure 2 shows the correlation between the cumulated response of state-level income at the 2 year horizon obtained from the five-factor model and the benchmark model. The scatter plot in the figure shows that the pattern of state-level responses in this model is very similar to the benchmark case– in fact the cross-sectional correlation between the two sets of responses at this horizon is 0.8.

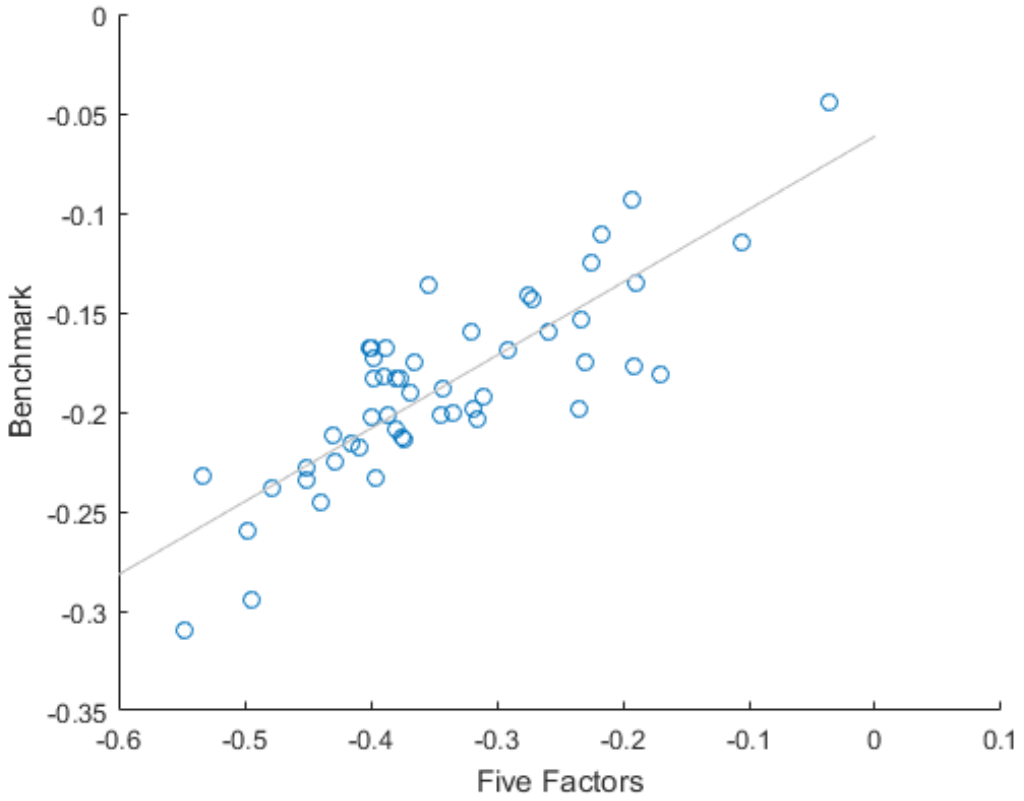

Figure 2: using five factors

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