Restrictions Search for Panel VARs

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Preliminary Version - 2016

Abstract

Panel vector autoregressive (PVAR) models can include several countries and variables in one system and thus are well suited for global spillover analyses. However, PVARs require restrictions to ensure the feasibility of the estimation. The present paper uses a selection prior for a data-based restriction search. It introduces the stochastic search variable selection for PVAR models (SSVSP) as an alternative estimation procedure for PVARs. This extends Koop's and Korobilis's stochastic search specification selection (S^4) to a restriction search on single elements. The SSVSP allows to incorporate dynamic and static interdependencies as well as cross-country heterogeneities. It uses a hierarchical prior to search for data-supported restrictions. The prior differentiates between domestic and foreign variables, thereby allowing a less restrictive panel structure. Absent a matrix structure for restrictions, a Monte Carlo simulation shows that SSVSP outperforms S^4. Furthermore, this is validated by performing a forecast exercise for G7 countries.

Keywords: stochastic search variable selection, PVAR

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I would like to thank my supervisor Helmut Lütkepohl and the seminar participants at the Workshop Empirical Macroeconomics at Freie Universität Berlin, at the Graduate Center Summer Workshop, and at the research seminar for Macroeconomics at DIW Berlin for comments and suggestions.

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1 Introduction

Intensified international good and knowledge flows as well as trade agreements show the importance of international interdependencies among economies. With these inter-linkages spillovers in real and financial variables across countries are essential. Shocks are likely to propagate internationally with asymmetric effects across various economies. Global spillover analyses require taking both the interdependencies and heterogeneities across countries into account. Analyses disregarding country specific information and global dependencies could end up with biased results on spillover effects and transmission channels.[1]

A tool which is able to consider dynamic and static global interdependencies as well as cross-section heterogeneities is the unrestricted Panel vector autoregressive (PVAR) model. In PVAR models lagged foreign variables can impact domestic variables, meaning that dynamic interdependencies exist. Static interdependencies between two variables of two countries occur if the covariance between the two is unequal zero. Finally, the PVAR accounts for heterogeneity across countries since the coefficient matrices can vary across economies. This strength of PVARs comes at the cost of a large number of parameters to estimate. To overcome this problem the researcher has to set restrictions on the PVAR.[2] Papers implementing PVAR models often use assumptions on homogeneity and no dependencies to ensure the feasibility of the estimation. Others follow the cross sectional shrinkage approach proposed by Canova and Ciccarelli (2009) which factorize the coefficients. A third and straightforward way of setting restrictions is to use the panel structure in the data by assuming that there only exist interdependencies between and heterogeneities across countries for specific country and variable combinations. The aim of this paper is to do a data-based restriction search by using a selection prior.

The paper specifies a selection prior for PVAR models which differentiates between domestic and foreign variables for each country. The algorithm based on the selection prior will search for dynamic interdependencies by checking whether the impact of lagged foreign variables is zero. Further, it will assess static interdependencies and use the restrictions as additional zero restrictions on a recursive identified

[1] Compare to Canova and Ciccarelli (2009), Canova and Ciccarelli (2013), Luetkepohl (2014) or Georgiadis (2015).

[2] PVAR models are easily estimable in the case where independence and homogeneity across the panel units are assumed. Estimation procedures are described in Canova and Ciccarelli (2013) and Breitung (2015).

structural PVAR model. This will be done by searching for zero restrictions on the upper triangular decomposition matrix of the covariance matrix. Finally, the algorithm searches for homogeneity between coefficients of domestic variables of different countries. It follows closely the selection prior for PVAR models of Koop and Korobilis (2015), which is called stochastic search specification selection (S^4), but extends the approach from a matrix wide search to single elements as George et al. (2008) do it in their stochastic search variable selection (SSVS) for VAR models. In order to distinguish my algorithm from S^4 and SSVS I call the algorithm stochastic search variable selection for PVAR models (SSVSP).

The SSVSP extends the estimation procedure for PVAR models and thus contributes to the existing literature on PVARs. The paper adds to the selection prior literature, in particular by the extension of the S^4 algorithm. By implementing their prior on country matrices Koop and Korobilis assume a specific panel structure. Namely, that all variables of one country are treated in the similar way being either restricted or not. My prior allows for a less restrictive panel structure. It does not restrict variables on a country basis but searches for dynamic and static interdependencies for each foreign variable as well as for homogeneity for each domestic variable. Thus, the underlying panel structure separates variables in domestic and foreign but not the foreign variables on a country basis.

This less restrictive panel structure has the advantages that, firstly, the SSVSP prior has a wider range for empirical application than has the more rigid S^4. Especially, applications including financial and real variables can benefit from a less restrictive form since the SSVSP can incorporate variable specific restrictions. The prior allows, for example, for the possibility that only foreign financial variables have a dynamic impact on a domestic variable while real variables have no impact. Secondly, the SSVSP is able to provide a clear ranking of posterior probabilities which variables to include in the model and which coefficients are homogeneous for each equation. Doing the restriction search for matrices has the problem that the decision for excluding a single variable depends on the results for a matrix-wide search. Thirdly, compared to the commonly used Litterman prior for Large Bayesian VAR models which assumes a specific shrinkage depending on the lag number the SSVSP differentiates between domestic and foreign variables and thus takes a panel structure into account.[3]

[3]Besides PVARs large Bayesian VAR and Global VAR models are also potential tools to analyze international spillovers. Detailed descriptions of the two models can be find in Banbura et al.

These advantages are reflected by the results of a Monte Carlo simulation and a forecasting exercise. Firstly, the results of the Monte Carlo studies show that when a more flexible panel structure is present, the SSVSP outperforms the S^4. Furthermore, the SSVSP is accurate in the selection of the restrictions displayed in the posterior probabilities for no interdependencies and homogeneity. Secondly, the results of the empirical application demonstrates that the SSVPS gives improved forecasting results compared to the S^4. Dynamic and static interdependencies are found between countries' interest rates. Especially variables of the United States and Japan have no static interdependencies with the remaining G7 countries and coefficients of the domestic variables are heteogeneous compared to other economies. In addition, the impulse responses to a shock in the US interest rate show reliable results. Overall, the results are encouraging about the use of the SSVSP for PVAR models.

2 Literature

So far, the literature uses basically two ways to overcome the curse-of-dimensionality problem in PVAR models. One strand of the literature using PVAR models make the assumptions of homogeneity, no dynamic or static interdependencies.[4] These assumptions should be based on a solid theory. One common restriction is block exogeneity based on the small-open-economy assumption. The second strand of literature follows the cross sectional shrinkage approach proposed by Canova and Ciccarelli (2009).[5] The authors factorize the coefficients into at least a common, a country-specific, and a variable-specific factor whereby they reduce the number of coefficients to be estimated.

For global spillover analyses exogeneity, homogeneity, or no dependence assumptions are hard to justify. However, using the estimation strategy of Canova and

(2010), Pesaran et al. (2004) or Dees et al. (2007). However, these two kinds of models come with some limitations. Large Bayesian VAR models are limited in terms of neglecting the existence of a panel dimension in the data. BVAR models usually assume identical priors for each country. Thus, large BVAR models are especially applicable for analyzing intra-country spillovers including a large number of variables. Global VARs, however, are restrictive in the way that they impose a particular structure on interdependencies by the chosen weights for aggregating the foreign component. GVAR models are especially useful for studies focusing on aggregated impacts or on spillovers from one large economy.

[4]Examples are Love and Zicchino (2006) assuming homogeneity and no dynamic interdependencies or Ciccarelli et al. (2013) restricting for no dynamic interdependencies.

[5]Examples are Canova et al. (2012) or Ciccarelli et al. (2012).

Ciccarelli (2009) complicates the structural shock identification since their model has two potential types of impulses. The first type is an impulse to the factors, the other one to the variables. These two types come from the estimated evolution of the factors and from the regression in which the coefficients depend on a number of factors. To be able to focus only on impulses to the variables, the impulse response analysis has to be done conditional on shocks to the factors and vice versa. This paper will follow a different approach to estimate the PVAR model by using a selection prior. An advantage of this prior is that it can easily account for the panel structure in the data and can handle an over-parameterized unrestricted model as well as a large number of restricted models.

The selection prior literature started with the paper of George and McCulloch (1993) who developed the prior for multiple regression models. The procedure, which the authors called stochastic search variable selection (SSVS), selects the variables which should be included in the regression model. This is done by using a hierarchical prior for the coefficients of the right hand side variables. The variables which should be included in the model occur more frequently while sampling from the conditional posterior distributions in the Gibbs sampler. George et al. (2008) developed the SSVS further and extend it to the use for VAR models. They set a hierarchical prior on the autoregressive coefficients and find the elements which equal zero. Additionally, the authors use the prior for structural identification. They decompose the covariance matrix into two upper triangular matrices and let the SSVS algorithm find additional zero restrictions by searching for the elements of the decomposition matrix which are zero. Korobilis (2008) and Jochmann et al. (2010) show that forecast performance is improved for VAR models when using SSVS. The first paper uses SSVS in a factor model including a large number of macroeconomic variables for the United States. The second paper allows for structural breaks. Using data for the United States the authors show that forecasts improve mainly due to the usage of SSVS and not due to the consideration of structural breaks. Korobilis (2013) extend the selection priors further to nonlinear set-ups.

Koop and Korobilis (2015) are the first ones to develop a selection prior for PVAR models. Their stochastic search specification selection (S^4) builds closely on George et al. (2008) but adds a restriction search for homogeneity of domestic autoregressive coefficients across countries. Further, in contrast to SSVS they do the restriction search on whole matrices including all variables of one country and thus assume a specific matrix panel structure. Therefore, the authors called their procedure specification search. Koop and Korobilis show with their Monte Carlo simulation that S^4 performs better than the OLS estimates. On average, the S^4 estimates are closer to the true vales than the OLS estimates. Using data for sovereign bond yield, industrial production, and bid-ask spread for euro area countries from January 1999 to December 2012, they show that the model fit improves when taking the characteristics of a panel model into account compared to a BVAR model without restriction search. Thus, the results of Koop and Korobilis (2015) show clearly that a prior for the PVAR model has to account for the panel dimension in the data. Korobilis (2015) comes to the same conclusion. He compares different prior specifications for PVAR models. For larger PVAR models priors taking the panel dimension into account deviate less from the true values than other VAR priors. In addition, these priors with a panel dimension improve the forecasting performance which he shows for the same empirical application as in Koop and Korobilis (2015). For small samples, however, the Bayesian shrinkage priors cannot outperform the OLS estimates.

One main drawback of S^4 is that the results loose detailedness since the S^4 algorithm is done for matrices. Koop and Korobilis assume a specific country grouping of the restrictions. The authors can only make statements about interdependencies and heterogeneities between the countries but not which variables are the drivers behind the linkages and country specific coefficients. But this detailedness is essential for further interpretation of results. Doing the restriction search for matrices can also lead to the exclusion of potentially important variables since decisions can only be made for whole matrices. The SSVSP makes instead a restriction search for each variable and can thus give evidence for exclusion of a single lag of a variable. In addition, Korobilis (2015) shows that the absolute deviation from the true value is lower for SSVS than for S^4 in a set-up where country grouping for restrictions does not hold. This result adds to the argument for restriction search on single elements.

One problematic issue is that the SSVS requires the SUR form of a VAR model leading to the inversion of large matrices. This leads to a computationally demanding algorithm for medium and large size VARs.[6] To overcome this problem Koop (2013) develops a natural conjugate selection prior for VARs. Here, no MCMC methods must be used. However, the natural conjugate selection prior comes with two disadvantages.[7] Firstly, each variable can only be either included or excluded

[6]Koop (2013) and Korobilis (2013) elaborate further on this issue.
[7]Koop (2013) explains the disadvantages of the natural conjugate prior in detail.

in the whole VAR system. Secondly, the natural conjugate specification requires a specific covariance prior. Thus, no restriction search is possible for the covariance elements of the VAR. Hence, for my purpose being able to include static interdependencies and to allow for dynamic interdependencies which are not homogeneous across countries the natural conjugate SSVS prior is no alternative. Instead, I accept the computational burden for having a differentiated prior which is able to account for the characteristics of a PVAR model, which should be less of a problem with increasing computational capacities.

3 PVAR Restrictions

A PVAR model for country i at time t with $i = 1, ..., N$ and $t = 1, ..., T$ is given by

$$y_{it} = A_{i1}Y_{t-1} + A_{i2}Y_{t-2} + ... + A_{ip}Y_{t-p} + u_{it}, \tag{1}$$

where $Y_{t-1} = (y'_{1t-1}, ..., y'_{Nt-1})'$ and y_{it} denotes a vector of dimension $[G \times 1]$.[8] The number of variables is defined as G. All A_{ip} have dimension $[G \times NG]$ for lag $p = 1, ..., P$. The index i denotes that the matrices are country specific for country i. The u_{it} are uncorrelated over time and normally distributed with mean zero and covariance matrix Σ_{ii}. The covariance matrix between errors of different countries is defined as $E(u_{it}u'_{jt}) = \Sigma_{ij} \;\; \forall\, i \neq j$ with dimension $[G \times G]$.

The PVAR model for all N countries can then be written as

$$Y_t = A_1Y_{t-1} + A_2Y_{t-2} + ... + A_pY_{t-p} + U_t. \tag{2}$$

The Y_t and U_t are $[NG \times 1]$-vectors. The U_t is normally distributed with mean zero and covariance matrix Σ which is of dimension $[NG \times NG]$. The $[NG \times NG]$-matrix A_p is defined as

$$A_p = \begin{pmatrix} \alpha_{11}^{11} & \cdots & \alpha_{1j}^{1k} & \cdots & \alpha_{1N}^{1G} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \alpha_{i1}^{l1} & \cdots & \alpha_{ij}^{lk} & \cdots & \alpha_{iN}^{lG} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \alpha_{N1}^{G1} & \cdots & \alpha_{Nj}^{Gk} & \cdots & \alpha_{NN}^{GG} \end{pmatrix}.$$

[8] This specification does not include a constant but can be extended to include it.

The element α_{ij}^{lk} refers to the coefficient of variable k of country j in the equation of variable l of country i. Thus, it measures the impact of variable k of country j on variable l of country i.

A simple example will make the notation of α clear. Assume we have a PVAR with one lag including 3 countries and 2 variables ($N = 3$, $G = 2$). The A matrix will have the following form:

$$A_1 = \begin{pmatrix} \alpha_{11}^{11} & \alpha_{11}^{12} & \alpha_{12}^{11} & \alpha_{12}^{12} & \alpha_{13}^{11} & \alpha_{13}^{12} \\ \alpha_{11}^{21} & \alpha_{11}^{22} & \alpha_{12}^{21} & \alpha_{12}^{22} & \alpha_{13}^{21} & \alpha_{13}^{22} \\ \alpha_{21}^{11} & \alpha_{21}^{12} & \alpha_{22}^{11} & \alpha_{22}^{12} & \alpha_{23}^{11} & \alpha_{23}^{12} \\ \alpha_{21}^{21} & \alpha_{21}^{22} & \alpha_{22}^{21} & \alpha_{22}^{22} & \alpha_{23}^{21} & \alpha_{23}^{22} \\ \alpha_{31}^{11} & \alpha_{31}^{12} & \alpha_{32}^{11} & \alpha_{32}^{12} & \alpha_{33}^{11} & \alpha_{33}^{12} \\ \alpha_{31}^{21} & \alpha_{31}^{22} & \alpha_{32}^{21} & \alpha_{32}^{22} & \alpha_{33}^{21} & \alpha_{33}^{22} \end{pmatrix},$$

where the first two rows are the equations for country 1, row 3 and 4 are the equations for country 2, and the last two rows belong to country 3. Then α_{13}^{21}, for example, measures the impact of variable 1 of country 3 on variable 2 of country 1.

A structural form of the PVAR model is derived by decomposing the covariance matrix Σ into $\Sigma = \Psi^{-1'}\Psi^{-1}$ where Ψ is a upper triangular matrix. Therefore, the structural identification is based on a recursive order. An element ψ_{ij}^{lk} of the upper triangular matrix Ψ defines the static relation between variable l of country i and variable k of country j.

This structural PVAR model can account for dynamic interdependencies (DI), static interdependencies (SI), and cross-section heterogeneities (CSH).[9] Firstly, the model allows lagged variables of foreign countries to have an impact on domestic variables. Secondly, there are static interdependencies between two variables of two countries if the element of the upper triangular decomposition matrix of the covariance matrix is equal to zero. Thus, the search for static interdependencies allows for a data-based structural identification of a PVAR model using additional zero restrictions on top of a recursive order. Thirdly, the PVAR accounts for heterogeneity across countries since the A_{ip} matrices can vary across countries.

This strength of PVARs to account for interdependencies and heterogeneities comes at the costs of many parameters to estimate. The unrestricted PVAR model has $(NG)^2P$ parameters of the A-matrix and $\frac{NG(NG+1)}{2}$ parameters of Σ to esti-

[9]Canova and Ciccarelli (2013) provide a survey of the PVAR restrictions.

mate. To overcome this problem the researcher has to set restrictions on the PVAR. A straightforward way of setting restrictions is to use the panel structure in the data. Thus, to expect that there only exist interdependencies between and heterogeneities across countries for specific country and variable combinations. We would for example expect that the short term interest rate of the United States has an impact on the interest rate of the Eurozone or that a strong GDP growth in France has also an impact on German exports. On the other hand, we would assume that the Canadian GDP growth does not influence the short term interest rate of the Eurozone and that Japanese GDP growth is independent of changes in Italian's GDP growth. We would also expect that the sign and magnitude of an impact of Portugal's and Spain's GDP growth on their domestic GDP growth is fairly similar while it would differ from the impact of United States's GDP growth.

Therefore, for some coefficients the following restrictions can be found in the data:

1. **No dynamic interdependencies (DI)**: no lagged impact from variable l of country i to variable k of country j if $\alpha^{lk}_{1,ij} = ... = \alpha^{lk}_{p,ij} = 0$ for $j \neq i$
2. **No static interdependencies (SI)**: no static relation between variable l of country i to variable k of country j if $\psi^{lk}_{ij} = 0$ for $j \neq i$
3. **No cross-section heterogeneities (CSH)**: homogeneous coefficient across the economies if $\alpha^{lk}_{p,jj} = \alpha^{lk}_{p,ii}$ for $j \neq i$ and $\forall p = 1, ..., P$

We can define $[(NG-G)NG]$ DI, $[(N(N-1)/2)G^2]$ SI, and $[(N(N-1)/2)G^2]$ CSH restrictions.[10] The essential part is now to find out for which country and variable combinations these restrictions hold. The SSVSP algorithm is able to search for the PVAR restrictions which are supported by the data. The SSVSP of this paper follows closely Koop and Korobilis (2015).

4 Selection Prior for PVAR

The stochastic search variable selection algorithm for PVARs works with the unrestricted PVAR model. The full unrestricted model with one lag can be rewritten as

$$Y_t = Z_{t-1}\alpha + U_t, \tag{3}$$

[10]Note that while SI restrictions are symmetric this must not be the case for DI restrictions.

where α is the vectorized matrix A_1 and $Z_{t-1} = (I_{NG} \otimes Y_{t-1})$. In the following, the PVAR model is simplified to a model including only one lag. If the researcher includes several lags, the restriction search for dynamic interdependencies would give a guidance which lags should be included in the model. Thus, the DI restriction search can be used as a lag length selection criterion.

The basic idea of a selection prior is that the selection of a variable is done by a hierarchical prior. Each element of α is drawn from a mixture of two Normal distributions centering around the restriction either with a small or large variance. Depending on a hyperparameter, γ, which is Bernoulli distributed the coefficient shrinks to the restriction (small variance case) or is estimated with a looser prior (larger variance case). Thus, the algorithm imposes soft restrictions by allowing for a small variance. In contrast to Koop and Korobilis (2015) the restriction search is done for each single element and not on the whole matrices including all variables of one country. A Gibbs sampler is used to obtain the posterior distributions.

The SSVSP algorithm has now specific priors for the parameters of A_1 and of the covariance matrix building in the DI, SI, and CSH restrictions. The DI restrictions require restrictions on the coefficients of the lagged foreign endogenous variables. The DI prior is given by

$$\begin{aligned} \alpha_{ij}^{lk} \mid \gamma_{DI,ij}^{lk} &\sim (1-\gamma_{DI,ij}^{lk})\mathcal{N}(0,\tau_1^2) + \gamma_{DI,ij}^{lk}\mathcal{N}(0,\tau_2^2) \\ \gamma_{DI,ij}^{lk} &\sim Bernoulli(\pi_{DI,ij}^{lk}). \end{aligned}$$

The prior distribution of α_{ij}^{lk} is conditional on the hyperparameter $\gamma_{DI,ij}^{lk}$. This hyperparameter has a distribution itself. That is why the prior is called a hierarchical prior. $\gamma_{DI,ij}^{lk}$ is Bernoulli distributed.[11] Thus, it takes either the value one or zero. If $\gamma_{DI,ij}^{lk}$ is equal to zero, α_{ij}^{lk} is drawn from the first part of the Normal distribution with mean zero and variance τ_1^2. If $\gamma_{DI,ij}^{lk}$ is equal to one, α_{ij}^{lk} is drawn from the second part of the Normal distribution with mean zero and variance τ_2^2. The values of τ_1^2 and τ_2^2 have to be chosen such that τ_1^2 is smaller than τ_2^2. Thus, if $\gamma_{DI,ij}^{lk} = 0$, the prior is tight in the sense that the parameter is shrunk to zero. Whereas the prior is loose for $\gamma_{DI,ij}^{lk} = 1$ since the prior variance is larger. Hence, if $\gamma_{DI,ij}^{lk} = 0$, no dynamic interdependency is supported by the data and the coefficient will be estimated with a small variance around zero. Going back to the simple 3-country-2-variable exam-

[11] How $\pi_{DI,ij}^{lk}$ is set is described in detail in the Appendix. This holds also for the CSH and SI prior.

ple the coefficients of A which are checked for dynamic interdependencies are now marked with DI:

$$A_1 = \begin{pmatrix} & & DI & DI & DI & DI \\ & & DI & DI & DI & DI \\ DI & DI & & & DI & DI \\ DI & DI & & & DI & DI \\ DI & DI & DI & DI & & \\ DI & DI & DI & DI & & \end{pmatrix}.$$

The covariance matrix of the PVAR model is decomposed into two upper triangular matrices Ψ, $\Sigma = \Psi^{-1'}\Psi^{-1}$. This ensures a recursive order to identify the structural shocks of the PVAR model. The SI prior is set on the elements of the upper triangular matrix. If SI restrictions are found, the structural PVAR is overidentified since additional zero restrictions can be set on top of the recursive ordering. The SSVSP prior is a data-based method to structurally identify the system. It has the drawback that the identification is limited since it only allows a recursive structure. A clear advantage of this decomposition is that it assures that by construction every simulated Σ is positive definite.[12]

The prior for SI restrictions follows the same logic as the DI prior:

$$\begin{aligned} \psi_{ij}^{lk} \mid \gamma_{SI,ij}^{lk} &\sim (1-\gamma_{SI,ij}^{lk})\mathcal{N}(0,\kappa_1^2) + \gamma_{SI,ij}^{lk}\mathcal{N}(0,\kappa_2^2) \\ \gamma_{SI,ij}^{lk} &\sim Bernoulli(\pi_{SI,ij}^{lk}). \end{aligned}$$

The prior is for all $j \neq i$. To assure positive variance elements, the $(\psi_{ii}^{kk})^2$ are gamma distributed, $(\psi_{ii}^{kk})^2 \sim \mathcal{G}(a,b)$. The elements for the same country, ψ_{ii}^{lk} for $l \neq k$, are normally distributed with mean zero and variance κ_2^2. All ψ_{ij}^{lk} elements $(j \neq i)$ are drawn from the specified hierarchical prior. Thus, the parameters are drawn from a weighted Normal distribution with weights $\gamma_{SI,ij}^{lk} \in \{0,1\}$. The parameter κ_1^2 is smaller than κ_2^2. Thus, if $\gamma_{SI,ij}^{lk}$ is equal to zero, the parameter shrinks to zero showing that the data support no static interdependency. The SI restrictions are symmetric. Back to the example, the following elements of the covariance matrix

[12]Compare also to Koop and Korobilis (2015).

are checked for static interdependencies:

$$\Psi = \begin{pmatrix} & & & SI & SI & SI & SI \\ & & & SI & SI & SI & SI \\ & & & & & SI & SI \\ & & & & & SI & SI \\ \\ \\ \end{pmatrix}.$$

The selection prior can also easily be used to estimate only the reduced form of a PVAR model by not defining the Ψ-matrix and not searching for static interdependency restrictions but by assuming a standard distribution for the PVAR variance, e.g. a Inverted Wishart distribution.

Searching for homogeneity across countries is not as straight forward as the zero restrictions for dynamic and static interdependencies. The main contribution of Koop and Korobilis (2015) is exactly the development of a procedure how to search for CSH restrictions. The CSH prior is given by

$$\alpha_{jj}^{lk} \mid \gamma_{CSH}^{w} \sim (1-\gamma_{CSH}^{w})\mathcal{N}(\alpha_{ii}^{lk}, \xi_1^2) + \gamma_{CSH}^{w}\mathcal{N}(0, \xi_2^2)$$
$$\gamma_{CSH}^{w} \sim Bernoulli(\pi_{CSH}^{w}).$$

The prior is for all $j \neq i$. There are $(N(N-1)/2)G^2 = K$ combinations of coefficient which are checked for homogeneity. The index $w = 1, ..., K$ refers to a specific combination. Again, ξ_1^2 is smaller than ξ_2^2. The main difference to the DI and SI prior is that instead of shrinking the parameter to zero in the first part of the Normal distribution the mean is equal to the coefficient for which homogeneity is checked. Going back to my 3-country-2-variables example the coefficients which are marked with CSH are checked for homogeneity:

$$A_1 = \begin{pmatrix} CSH & CSH & & & & \\ CSH & CSH & & & & \\ & & CSH & CSH & & \\ & & CSH & CSH & & \\ & & & & CSH & CSH \\ & & & & CSH & CSH \end{pmatrix}.$$

The algorithm checks for example whether the coefficient of the first variable of country 1 (in the equation of the first variable for country 1) is equal to the coefficient of the first variable of country 2 (in the equation of the first variable for country 2), same between country 1 and 3 as well as 3 and 2. Thus, for the first variable in the equation of the same variable three combinations are checked for homogeneity. The restrictions are $\alpha_{11}^{11} = \alpha_{22}^{11}$, $\alpha_{11}^{11} = \alpha_{33}^{11}$, and $\alpha_{22}^{11} = \alpha_{33}^{11}$.

To be able to check all possible combinations, I follow Koop and Korobilis (2015) who define a selection matrix

$$\Gamma = \prod_{w=1}^{K} \Gamma_w.$$

The number of Γ_w matrices equals the number of possible combinations to check for homogeneity, $w = 1, ..., K$. Each Γ_w has the dimension $[NG \times NG]$. The matrix Γ_w is an identity matrix with two exceptions. The diagonal element at the position α_{jj}^{lk} is set equal to γ_{CSH}^{w} and the off-diagonal element referring to the element α_{ii}^{lk} is set equal to $(1 - \gamma_{CSH}^{w})$. Back to my example, if we check $\alpha_{11}^{11} = \alpha_{22}^{11}$, the restriction matrix is an identity matrix of dimension $[NG \times NG] = [36 \times 36]$. The element in the first row and first column is replaced by γ_{CSH}^{1} and the element in the 15th row and frist column, referring to the position of α_{22}^{11} in the vectorized A matrix, by $(1 - \gamma_{CSH}^{1})$. If γ_{CSH}^{1} equals zero, α_{11}^{11} and α_{22}^{11} are homogeneous. If all coefficients are heterogeneous, all Γ_w are identity matrices. To impose the CSH restrictions the posterior mean of α is multiplied by the selection matrix Γ.

The outcome of the algorithm can be interpreted in two ways.[13] Based on the results of the algorithm the researcher can select one specific restricted PVAR model. Hence, the algorithm is used as a model selection criterion. The posterior probabilities $\gamma_{DI}, \gamma_{SI}, \gamma_{CSH}$ give the information whether a variable is included in the model or not and whether it is homogeneous or not. These probabilities are calculated as the proportion of γ_{DI}, γ_{SI}, or γ_{CSH} draws which equal one over all draws. Based on the estimated $\gamma_{DI}, \gamma_{SI}, \gamma_{CSH}$ values it is possible to provide a ranking for DI, SI and CSH restrictions. The posterior probabilities $\gamma_{DI}, \gamma_{SI}, \gamma_{CSH}$ can be sorted in descending order. The researcher can set the restrictions successively starting with the variable for which the posterior probability of γ_{DI}, γ_{SI}, or γ_{CSH} being zero is highest or for which the probability being one is lowest. The researcher can set the restrictions successively until the model with the best fit is found.

[13] Compare to the general survey in Koop and Korobilis (2010) or the specific explanation for the S^4 in Koop and Korobilis (2015).

Another way to do the selection is via a threshold value. The selection prior literature often uses 0.5 as a threshold value to determine whether a restriction is set. Using the results as a model selection criterion shows particularly well the strong advantages of the SSVSP prior for PVAR compared to the S^4. While Koop and Korobilis (2015) can only make statements about including or excluding a whole country, based on the SSVSP it is possible to make clear decisions on exclusion for every single variable. Using the SSVS of George et al. (2008) would also allow the researcher to make clear statements about single variables, but it neglects the possibility of cross-sectional homogeneities as an important characteristic of PVARs. Alternatively, the outcome of the algorithm can be used as a Bayesian model averaging (BMA) result. Thus, the posterior means averaged over all draws are taken as coefficient estimates. Since each draw leads to a specific restricted model, the BMA results average over all possible restricted models.

5 Monte Carlo Simulation

5.1 Simulation Set-up

In order to evaluate the prior I have conducted two Monte Carlo simulations. I will compare the results using SSVSP with the S^4 algorithm and OLS estimates.[14] Both Monte Carlo simulations include 3 countries, 2 variables, and 1 lag. Assume for an international spillover analysis that dynamic and static interdependencies and cross-sectional heterogeneities exist for specific variable and country combinations. Firstly, assume that country 2 has a dynamic impact on country 1 and country 1 on country 3. Country 3 does not impact the other two countries dynamically. Coefficients are homogeneous between country 2 and 3. Static interdependencies exist between country 1 and 2. This example has a clear country grouping structure. Hence, all variables of one country have either an impact on all variables of a second country or not. The same holds for homogeneity across countries. A scenario like this is given by the first Monte Carlo simulation where the following parameter values are

[14] I simulated 100 samples, each with a length of 100. The Gibbs sampler is done with 55000 draws of which 5000 draws are disregarded as draws of the burn-in-phase. For each of the 100 samples the OLS estimates are calculated. The OLS estimates which are used in the further analysis are the mean values of the OLS estimates over the 100 simulated samples. The prior hyperparameters which are used in the Monte Carlo simulations are given in the Appendix. The calculation is based on a further development of the MATLAB code provided by Koop and Korobilis (https://sites.google.com/site/dimitriskorobilis/matlab/panel_var_restrictions).

set:

$$A_1^{true} = \begin{pmatrix} 0.8 & 0 & 0.2 & 0.2 & 0 & 0 \\ 0 & 0.7 & 0.3 & 0.3 & 0 & 0 \\ 0 & 0 & 0.6 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 \\ 0.3 & -0.4 & 0 & 0 & 0.6 & 0.5 \\ 0.2 & 0.4 & 0 & 0 & 0 & 0.5 \end{pmatrix}, \Psi^{true} = \begin{pmatrix} 1 & 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 1 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Separating A_1 and Ψ into $[2 \times 2]$ matrices which include only variables of one country shows the clear country grouping structure.

Secondly, assume that the interdependency and homogeneity structure is not automatically similar for all variables of one country. Hence, a less restrictive panel structure exists. Thus, the first variable of country 2 and 3 has a dynamic impact on country 1's variables, but not the second variable of the foreign countries. Assume that variable 1 of country 3 is dynamically influenced by both variables of country 1 while there exists no such interdependency structure for variable 2. Static interdependencies and homogeneity across coefficients also only exist for special country and variable pairs. The second Monte Carlo simulation builds in these properties and has the following true parameters:

$$A_1^{true} = \begin{pmatrix} 0.8 & 0 & 0.2 & 0 & 0.2 & 0 \\ 0 & 0.7 & 0.2 & 0 & 0.2 & 0 \\ 0 & 0 & 0.6 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.3 & 0 & 0 \\ 0.3 & -0.4 & 0 & 0 & 0.6 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0.5 \end{pmatrix}, \Psi^{true} = \begin{pmatrix} 1 & 0 & 0.5 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Here, the interdependency structure between the countries and homogeneity across parameters varies across variables. There is no clear country grouping.

The performance of each estimator is checked via the Absolute Percentage De-

Table 1: Absolute Percentage Deviation

	Simulation 1		Simulation 2	
	A	Ψ	**A**	Ψ
SSVSP	0.038	0.0873	0.036	0.0523
S^4	0.037	0.1346	0.045	0.0865
OLS	0.095	0.1037	0.092	0.0822

viation (APD) statistic[15]:

$$APD = \frac{1}{(NG)^2} \sum_{i=1}^{(NG)^2} \mid \alpha_i - \alpha_i^{true} \mid .$$

The statistic measures the absolute deviation of the estimated coefficient α_i from the true value α_i^{true}. The estimated coefficient is a BMA result. Furthermore, the accuracy of the SSVSP to find the restrictions is checked. This is done by comparing the restrictions' probabilities to the true values. Thus, the probabilities that $\alpha_{ij}^{lk} = 0$, $\psi_{ij}^{lk} = 0$, and $\alpha_{jj}^{lk} = \alpha_{ii}^{lk}$ are compared among themselves and in relation to the true values. These posterior probabilities are calculated as the proportion of $\gamma_{DI,ij}^{lk}, \gamma_{SI,ij}^{lk}$, and γ_{CSH}^{w} draws which equal zero averaged over all Gibbs sampler draws and all simulated samples. The higher the proportion of γ draws which equal zero is, the higher the probability is that no dynamic and no static interdependencies exist and coefficients are homogeneous.

5.2 Results

The results of the Monte Carlo study demonstrate that, firstly, when a particular matrix panel structure exists in the data, the SSVSP and S^4 perform equally good. Secondly, when a less restrictive panel structure is present, the SSVSP outperforms the S^4. Thirdly, the SSVSP accurately selects the restrictions. This is validated by the higher posterior probabilities for no interdependencies and homogeneity for parameters which are truly zero or homogeneous compared to the probabilities for non-zero and heterogeneous parameters.

As table 1 shows, the estimated coefficients from S^4 are on average slightly

[15]Koop and Korobilis (2015) and Korobilis (2015) use both mean deviation statistics to evaluate the performance of estimators in Monte Carlo simulations.

closer to the true values compared to SSVSP for the first simulation.[16] The APD for S^4 takes a lower value, $APD_{S^4} = 0.037$, compared to the value for the SSVSP, $APD_{SSVSP} = 0.038$. This result was expected since the underlying panel structure of simulation one is the structure the S^4 is designed for. For the second simulation, where a less restrictive panel structure is present, the results from SSVSP outperform the results from S^4. The estimated coefficients from SSVSP deviate less from the true values, $APD_{SSVSP} = 0.036$, than the estimates of S^4, $APD_{S^4} = 0.045$. For the estimated Ψ matrix the deviations of SSVSP estimates from the true values are lower than for the S^4 estimates. This holds for simulation one and two. In most cases the estimated coefficients from the selection priors are closer to the true parameter than the OLS coefficients for both simulations. This indicates that the use of a prior which incorporates the panel structure in the data is beneficial.

Furthermore, the SSVSP algorithm is accurate in selecting the restrictions. This is true because posterior probabilities that no interdependencies exist are higher for true zero values compared to the probabilities for true non-zero values. The posterior probabilities for $\alpha_{ij}^{lk} = 0$ and $\psi_{ij}^{lk} = 0$ are shown in table 2. The true values of the simulations are presented in bolt, probabilities for the restrictions in italic. Results for simulation one are shown in the left column, results for simulation two in the right column. Looking at simulation one, probabilities that $\alpha_{ij}^{lk} = 0$ are considerably higher for true zero parameters than for true non-zero values. The first are in a range between 0.68 and 0.90 while the latter one are between 0.03 and 0.35. For example, the probability for no dynamic impact of country 2 on country 1 for variable 2, shown in the second row of the table, is 0.08 for variable 1 and 0.16 for variable 2. The true values, each 0.3, show that dynamic interdependencies exist. The variable 2 of country 3, however, has no dynamic impact on country 1, shown by the zero values. The algorithm finds here a substantially higher probability for no dynamic interdependencies with values of 0.86 for variable 1 and 0.76 for variable 2. Turning to simulation two, if no dynamic interdependencies occur in truth, shown by zero values for the parameters, the probabilities that $\alpha_{ij}^{lk} = 0$ are between 0.60 and 0.87. These values are all higher than the probabilities for the parameters which dynamically affect the dependent variables. These probability values are between 0.01 and 0.37.

The SSVPS also selects accurately the SI restrictions in both simulations. This is true since, for both simulations the probabilities that $\psi_{ij}^{lk} = 0$ are higher for true

[16] SSVSP estimates for A, Ψ, and Σ for both simulations are given in the Appendix.

Table 2: Accuracy - DI and SI restrictions

Simulation 1						Simulation 2					
DI restricions						DI restricions					
-	-	**0.2**	**0.2**	**0**	**0**	-	-	**0.2**	**0**	**0.2**	**0**
		0.29	*0.35*	*0.87*	*0.81*			*0.37*	*0.60*	*0.31*	*0.64*
-	-	**0.3**	**0.3**	**0**	**0**	-	-	**0.2**	**0**	**0.2**	**0**
		0.08	*0.16*	*0.86*	*0.76*			*0.33*	*0.68*	*0.29*	*0.69*
0	**0**	-	-	**0**	**0**	**0**	**0**	-	-	**0**	**0**
0.71	*0.74*			*0.87*	*0.75*	*0.82*	*0.76*			*0.81*	*0.70*
0	**0**	-	-	**0**	**0**	**0**	**0**	-	-	**0**	**0**
0.75	*0.78*			*0.90*	*0.79*	*0.82*	*0.75*			*0.82*	*0.69*
0.3	**-0.4**	**0**	**0**	-	-	**0.3**	**-0.4**	**0**	**0**	-	-
0.15	*0.03*	*0.79*	*0.68*			*0.18*	*0.01*	*0.79*	*0.67*		
0.2	**0.4**	**0**	**0**	-	-	**0**	**0**	**0**	**0**	-	-
0.27	*0.05*	*0.77*	*0.69*			*0.87*	*0.75*	*0.85*	*0.72*		
SI restrictions						SI restrictions					
-	-	**0.50**	**0.50**	**0**	**0**	-	-	**-0.5**	**0**	**0**	**0**
		0.59	*0.55*	*0.75*	*0.81*			*0.51*	*0.95*	*0.79*	*0.95*
-	-	**-0.50**	**-0.50**	**0**	**0**	-	-	**0**	**-0.5**	**0**	**0**
		0.57	*0.52*	*0.76*	*0.75*			*0.91*	*0.20*	*0.86*	*0.91*
-	-	-	-	**0**	**0**	-	-	-	-	**0**	**0**
				0.89	*0.89*					*0.91*	*0.91*
-	-	-	-	**0**	**0**	-	-	-	-	**0**	**0**
				0.91	*0.91*					*0.92*	*0.92*

True values are in bolt, probabilities for restrictions, $p(\alpha_{ij}^{lk} = 0)$ and $p(\psi_{ij}^{lk} = 0)$, are in italic.

zero compared to non-zero parameters. The results for simulation one show that probabilities are in a range of 0.75 and 0.91 for zero values while for the existing static interdependencies between country 1 and 2 for both variables probabilities are between 0.52 and 0.59. For simulation two the probabilities for no static interdependencies, between 0.79 and 0.95, are clearly higher for the true zero values compared to the probabilities for non-zero values, 0.51 and 0.20.

Moreover, the SSVSP is mostly accurate in the selection of the cross-section heterogeneity restrictions. The results for $p(\alpha_{jj}^{lk} = \alpha_{ii}^{lk})$ are presented in table 3. For both simulations probabilities that the coefficients are homogeneous are higher for true homogeneous coefficients. However, especially for true values which are close to each other but not equal probabilities for homogeneity are relatively high with values around 0.5. For example, $\alpha_{22}^{11} = \alpha_{33}^{11}$, true values 0.6 and 0.6, has a higher posterior

Table 3: Accuracy - CSH restrictions

	Simulation 1			Simulation 2		
coefficients	**true** α_{jj}^{lk}	**true** α_{ii}^{lk}	$p(\alpha_{jj}^{lk}=\alpha_{ii}^{lk})$	**true** α_{jj}^{lk}	**true** α_{ii}^{lk}	$p(\alpha_{jj}^{lk}=\alpha_{ii}^{lk})$
$\alpha_{11}^{11}=\alpha_{22}^{11}$	0.8	0.6	*0.60*	0.8	0.6	*0.66*
$\alpha_{11}^{21}=\alpha_{22}^{21}$	0	0	*0.71*	0	0	*0.78*
$\alpha_{11}^{12}=\alpha_{22}^{12}$	0	0.5	*0.16*	0	0.5	*0.24*
$\alpha_{11}^{22}=\alpha_{22}^{22}$	0.7	0.5	*0.50*	0.7	0.3	*0.29*
$\alpha_{11}^{11}=\alpha_{33}^{11}$	0.8	0.6	*0.61*	0.8	0.6	*0.66*
$\alpha_{11}^{21}=\alpha_{33}^{21}$	0	0	*0.73*	0	0	*0.80*
$\alpha_{11}^{12}=\alpha_{33}^{12}$	0	0.5	*0.16*	0	0.5	*0.25*
$\alpha_{11}^{22}=\alpha_{33}^{22}$	0.7	0.5	*0.55*	0.7	0.5	*0.39*
$\alpha_{22}^{11}=\alpha_{33}^{11}$	0.6	0.6	*0.79*	0.6	0.6	*0.77*
$\alpha_{22}^{21}=\alpha_{33}^{21}$	0	0	*0.79*	0	0	*0.79*
$\alpha_{22}^{12}=\alpha_{33}^{12}$	0.5	0.5	*0.65*	0.5	0.5	*0.56*
$\alpha_{22}^{22}=\alpha_{33}^{22}$	0.5	0.5	*0.64*	0.3	0.5	*0.49*

Probabilities for CSH restrictions, $p(\alpha_{jj}^{lk}=\alpha_{ii}^{lk})$, are in italic.

probability for homogeneity, 0.79 for simulation one and 0.77 for simulation two, than the clearly heterogeneous coefficients α_{11}^{12} and α_{33}^{12}, 0 and 0.5, with probabilities of 0.16 for simulation one and 0.25 for simulation two. However, the coefficients α_{11}^{11} and α_{33}^{11}, with true values 0.8 and 0.6, which differ but are close to each other, have a relatively high posterior probability for homogeneity, 0.61 for simulation one and 0.66 for simulation two.

6 Empirical Application

6.1 Data and Procedure

I now apply the SSVSP to a simple empirical application. The analysis consists of three key macroeconomic variables: the growth rate of industrial production (IP), a CPI growth rate (CPI), and a short term interest rate (IR). The model includes the G7 countries. The application can be used to study cross-country spillovers in macroeconomic variables. The variables can show synchronized business cycles or spillovers from monetary policy. The data are from the OECD and have monthly frequency from 1990:1 to 2015:2. The PVAR model includes one lag.[17]

The variables are ordered in a recursive way. Thus, the upper triangular matrix Ψ has the following simplified form focusing on the country order:

$$\begin{matrix} CA \\ I \\ UK \\ F \\ J \\ D \\ US \end{matrix} \begin{pmatrix} \times & \times & \times & \times & \times & \times & \times \\ & \times & \times & \times & \times & \times & \times \\ & & \times & \times & \times & \times & \times \\ & 0 & & \times & \times & \times & \times \\ & & & & \times & \times & \times \\ & & & & & \times & \times \\ & & & & & & \times \end{pmatrix}.$$

For each country the three macroeconomic variables are included. The industrial production growth rate is ordered first, CPI growth rate second, and the short term interest rate third. The recursive country ordering is based on the openness of a country. Openness is measured based on yearly import and export data for the economies. The higher the trade of a country is, the more open it is. The countries are ranged in ascending order meaning that the most open country, the United States, is ordered last. Thus, US variables can influence all other countries contemporaneously but are not affected by the variables of the remaining G7 countries.

Using the empirical application as an example, the SSVSP is validated based on it's forecasting performance, on a ranking of restriction probabilities, and on an impulse response analysis. At first, forecasts are provided for 12 horizons for the period beginning from January 2005 to the end of the sample.[18] I use the reduced from of the PVAR model to conduct the forecasts. Therefore, no SI restriction search is done and the covariance matrix is drawn from an Inverted Wishart distribution. The forecasts are evaluated using the mean squared forecast error (MSFE). The error is calculated as the difference between the estimated forecast and the true value given by the data. Furthermore, the posterior probabilities for a restriction are ranked and the lowest and highest probabilities are presented. Finally, a impulse response analysis is conducted based on the recursive identification system.

[17]The hyperparameters of the prior distributions are set like in the Monte Carlo simulations. Detailed information is given in the Appendix.

[18]The forecasts for the included 21 variables are generated iteratively. Forecasts start conditional on the data from January 1990 to December 2004.

Table 4: MSFEs for SSVSP relative to S^4

horizon	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
IP_CA	**0.98**	**0.98**	**0.95**	**1.00**	**0.95**	**0.92**	**0.94**	**0.93**	**0.92**	**0.92**	**0.93**	**0.95**
CPI_CA	**0.87**	**0.95**	**0.86**	**0.90**	**0.83**	**0.82**	**0.83**	**0.81**	**0.82**	**0.86**	**0.84**	**0.96**
IR_CA	1.00	**0.82**	**0.91**	**0.93**	**0.94**	**0.89**	**0.91**	**0.87**	**0.88**	**0.86**	**0.84**	**0.86**
IP_I	**0.99**	1.01	**0.98**	**0.99**	**0.99**	**0.97**	**0.99**	**0.96**	**0.98**	**0.97**	**0.95**	**0.94**
CPI_I	1.03	**0.78**	**0.82**	**0.82**	**0.84**	**0.80**	**0.78**	**0.77**	**0.76**	**0.71**	**0.79**	**0.82**
IR_I	1.03	1.03	1.07	1.03	1.03	**1.00**	1.01	1.01	1.02	**0.99**	**0.98**	**0.97**
IP_UK	**0.89**	**0.97**	**0.94**	**0.95**	**0.95**	**0.94**	**0.90**	**0.96**	**0.90**	**0.92**	**0.88**	**0.96**
CPI_UK	**0.71**	**0.85**	**0.93**	**0.84**	**0.83**	**0.89**	**0.96**	**0.80**	**0.82**	**0.83**	**0.99**	**0.86**
IR_UK	1.00	**0.99**	**0.97**	**0.99**	1.01	1.05	**1.00**	1.03	1.03	1.04	1.01	1.06
IP_F	**0.76**	**0.97**	**0.93**	**0.95**	**0.90**	**0.86**	**0.90**	**0.86**	**0.86**	**0.85**	**0.85**	**0.84**
CPI_F	**0.65**	**0.84**	**0.89**	**0.81**	**0.79**	**0.86**	**0.82**	**0.75**	**0.84**	**0.81**	**0.91**	1.04
IR_F	1.03	**0.87**	**0.94**	**0.99**	**0.97**	**0.95**	**0.95**	**0.95**	**0.95**	**0.94**	**0.94**	**0.95**
IP_J	**0.99**	1.01	1.00	**0.99**	1.02	**0.99**	**0.99**	1.00	1.01	**0.96**	**0.97**	**0.96**
CPI_J	**0.91**	**0.97**	**0.93**	**0.98**	1.01	1.01	1.01	**0.94**	**0.95**	**0.99**	**0.97**	1.00
IR_J	**0.97**	**0.78**	**0.76**	**0.79**	**0.78**	**0.77**	**0.76**	**0.77**	**0.77**	**0.79**	**0.78**	**0.77**
IP_D	**0.91**	1.07	1.01	**0.94**	1.08	**0.90**	**0.95**	**0.98**	**0.92**	**0.89**	**0.94**	**0.93**
CPI_D	**0.60**	**0.88**	**0.94**	**0.88**	1.12	**0.86**	**0.98**	**0.95**	**0.82**	1.03	**0.93**	**0.93**
IR_D	**0.94**	**0.91**	**0.96**	**0.97**	**0.99**	**0.96**	**1.00**	1.03	**1.00**	1.01	**0.99**	**0.98**
IP_US	1.00	**0.99**	1.01	1.03	1.00	1.01	1.02	1.00	**1.00**	1.03	1.00	1.00
CPI_US	1.00	1.00	1.00	1.01	**0.98**	1.00	1.00	**0.97**	**0.97**	**0.98**	**1.00**	**0.98**
IR_US	1.00	**0.98**	**0.96**	**0.98**	**0.98**	1.03	**0.97**	**0.99**	**0.98**	1.00	**0.99**	1.01

MSFEs are given relative to S^4, MSFE<1 are in bold. Σ drawn from Inverted Wishart distribution.

6.2 Results

The results of the empirical application demonstrate three key findings. Firstly, the SSVSP gives improved forecasting results compared to the S^4. Secondly, the ranking of the posterior probabilities for the restrictions indicates that domestic interest rates evolve unaffected by lagged foreign industrial production growth rates, validated by high posterior probabilities for no dynamic interdependencies. The interest rate of a country depends likely on foreign interest rates dynamically and statically. No static interdependencies with and heterogeneity compared to the remaining G7 countries are in particular found for variables of the United States and Japan. Thirdly, the impulse response analysis supports the reliability of the results. In the following the key findings are explained in more detail.

The forecasts of SSVSP outperform the forecasts of S^4. Taking all 12 forecast

Table 5: Ranking of 10 highest restriction probabilities

DI		**SI**		**CSH**	
α	$\mathbf{p}(\alpha_{ij}^{lk}=0)$	ψ	$\mathbf{p}(\psi_{ij}^{lk}=0)$	α	$\mathbf{p}(\alpha_{jj}^{lk}=\alpha_{ii}^{lk})$
$\alpha_{I,F}^{CPI,IP}$	1.00	$\psi_{J,US}^{IP,IP}$	0.99	$\alpha_{F,F}^{CPI,IP}=\alpha_{J,J}^{CPI,IP}$	1.00
$\alpha_{J,D}^{IR,IP}$	1.00	$\psi_{J,US}^{IP,CPI}$	0.99	$\alpha_{J,J}^{IR,IP}=\alpha_{D,D}^{IR,IP}$	1.00
$\alpha_{I,J}^{IR,IP}$	1.00	$\psi_{J,US}^{IP,IR}$	0.99	$\alpha_{F,F}^{IR,IP}=\alpha_{J,J}^{IR,IP}$	1.00
$\alpha_{F,I}^{IR,IP}$	1.00	$\psi_{J,D}^{IP,IP}$	0.99	$\alpha_{CA,CA}^{IR,IP}=\alpha_{J,J}^{IR,IP}$	1.00
$\alpha_{J,F}^{IR,IP}$	1.00	$\psi_{J,D}^{IP,CPI}$	0.99	$\alpha_{CA,CA}^{IR,IP}=\alpha_{D,D}^{IR,IP}$	1.00
$\alpha_{D,F}^{IR,IP}$	1.00	$\psi_{J,D}^{IP,IR}$	0.99	$\alpha_{D,D}^{IR,IP}=\alpha_{US,US}^{IR,IP}$	1.00
$\alpha_{US,J}^{IR,IP}$	1.00	$\psi_{D,US}^{IP,IR}$	0.99	$\alpha_{I,I}^{CPI,IP}=\alpha_{F,F}^{CPI,IP}$	1.00
$\alpha_{US,D}^{IR,IP}$	1.00	$\psi_{I,D}^{IP,CPI}$	0.99	$\alpha_{UK,UK}^{IR,IP}=\alpha_{J,J}^{IR,IP}$	1.00
$\alpha_{UK,F}^{IR,IP}$	1.00	$\psi_{D,US}^{CPI,IP}$	0.99	$\alpha_{F,F}^{CPI,IP}=\alpha_{D,D}^{CPI,IP}$	1.00
$\alpha_{F,J}^{CPI,IP}$	1.00	$\psi_{I,F}^{IP,IR}$	0.99	$\alpha_{J,J}^{CPI,IP}=\alpha_{D,D}^{CPI,IP}$	1.00

10 highest probabilities are presented for DI, SI, and CSH restrictions.

horizons into account, in 77.3% of the cases the SSVSP performs better than S^4, meaning that the MSFEs relative to S^4 are below one, marked in bold in table 4. The MSFEs are considerably below one with notable many values below or around 0.8. This means that improvements of the forecast performance using SSVSP are in many cases above 20%. Moreover, values above one are only slightly above one, with a largest MSFE of 1.08. These findings emphasize that forecasts of SSVSP clearly outperform the forecasts of S^4. However, it is noteworthy that the forecast performance relative to OLS is weak for both selection priors.[19] The SSVSP performs in 23.41 % of the cases better than OLS, indicated by 59 MSFEs below one relative to OLS. For the S^4 this is only the case in 27 out of 252, thus in 10.71%. Hence, the forecast performance of the SSVSP still outperforms the performance of S^4. However, OLS has its limitations in large systems. The feasability of OLS is problematic while the selection priors are able to handle larger systems due to their shrinkage property.

[19]Results are given in the Appendix.

Table 6: Ranking of 10 lowest restriction probabilities

DI		**SI**		**CSH**	
α	$\mathbf{p}(\alpha_{ij}^{lk} = 0)$	ψ	$\mathbf{p}(\psi_{ij}^{lk} = 0)$	α	$\mathbf{p}(\alpha_{jj}^{lk} = \alpha_{ii}^{lk})$
$\alpha_{CA,I}^{IR,IR}$	0.00	$\psi_{CA,F}^{IR,IR}$	0.00	$\alpha_{D,D}^{CPI,CPI} = \alpha_{US,US}^{CPI,CPI}$	0.00
$\alpha_{F,D}^{IR,IR}$	0.00	$\psi_{CA,US}^{CPI,CPI}$	0.00	$\alpha_{F,F}^{IP,IR} = \alpha_{J,J}^{IP,IR}$	0.06
$\alpha_{I,D}^{IR,IR}$	0.01	$\psi_{UK,F}^{CPI,CPI}$	0.00	$\alpha_{D,D}^{IP,IP} = \alpha_{US,US}^{IP,IP}$	0.06
$\alpha_{D,UK}^{IP,IR}$	0.01	$\psi_{UK,D}^{IR,IR}$	0.00	$\alpha_{I,I}^{IP,IR} = \alpha_{J,J}^{IP,IR}$	0.07
$\alpha_{F,UK}^{IP,IR}$	0.03	$\psi_{D,US}^{IR,IR}$	0.00	$\alpha_{CA,CA}^{IP,IR} = \alpha_{J,J}^{IP,IR}$	0.07
$\alpha_{F,US}^{CPI,CPI}$	0.04	$\psi_{UK,US}^{IR,IR}$	0.00	$\alpha_{F,F}^{CPI,CPI} = \alpha_{US,US}^{CPI,CPI}$	0.07
$\alpha_{CA,US}^{IP,IR}$	0.04	$\psi_{CA,US}^{IR,IR}$	0.00	$\alpha_{UK,UK}^{IP,IR} = \alpha_{J,J}^{IP,IR}$	0.08
$\alpha_{F,US}^{IP,IR}$	0.04	$\psi_{CA,F}^{CPI,CPI}$	0.00	$\alpha_{J,J}^{IP,IR} = \alpha_{D,D}^{IP,IR}$	0.09
$\alpha_{F,US}^{IP,IP}$	0.05	$\psi_{UK,J}^{CPI,CPI}$	0.00	$\alpha_{J,J}^{IP,IR} = \alpha_{US,US}^{IP,IR}$	0.09
$\alpha_{CA,J}^{IR,IR}$	0.05	$\psi_{F,D}^{CPI,CPI}$	0.00	$\alpha_{J,J}^{CPI,CPI} = \alpha_{D,D}^{CPI,CPI}$	0.10

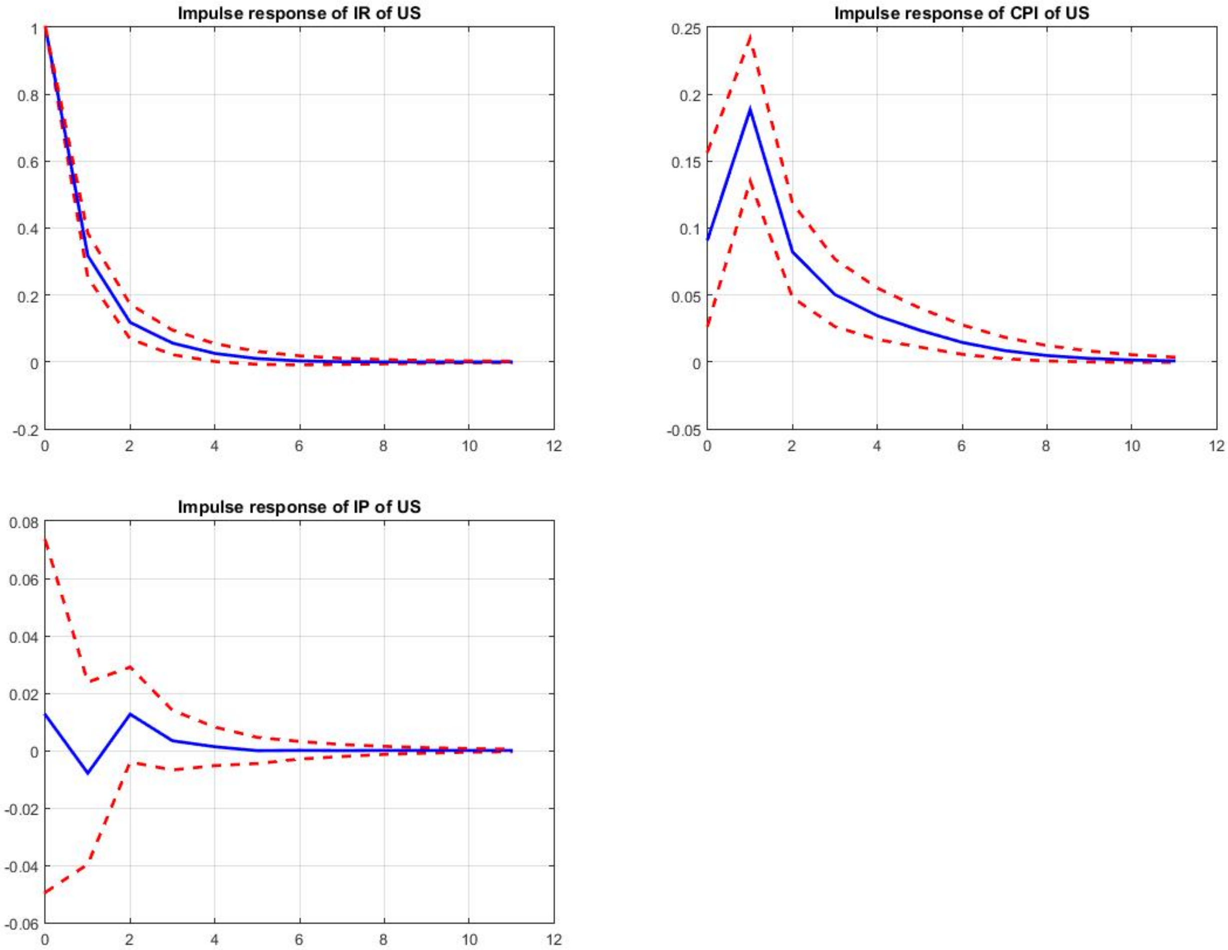
10 lowest probabilities are presented for DI, SI, and CSH restrictions.

The ranking of restriction probabilities shows that restrictions are especially supported for the industrial production variable and for variables of the United States and Japan.[20] Table 5 provides the ten highest posterior probabilities for no DI, no SI and homogeneity. In detail, the results show that the probabilities are high that no dynamic impacts of foreign lagged IP on interest rates exist. Thus, interest rates movements are not influenced by lagged foreign industrial production. Furthermore, the probabilities are high that no static interdependencies exist between the United States' as well as Japan's variables and the remaining G7 countries. Additionally, industrial production seems to be fairly independent from other variables, shown by the high probabilities for no static interdependencies between IP and other variables. Finally, the probabilities for homogeneity of coefficients are especially high for the industrial production variables in other equations.

In comparison, the lowest probabilities for the restrictions are found for combinations of the same variable. Results are shown in table 6. Lagged foreign interest

[20]Detailed results for all parameters are given in the Appendix.

Figure 1: Responses of US variables to a shock to US interest rate

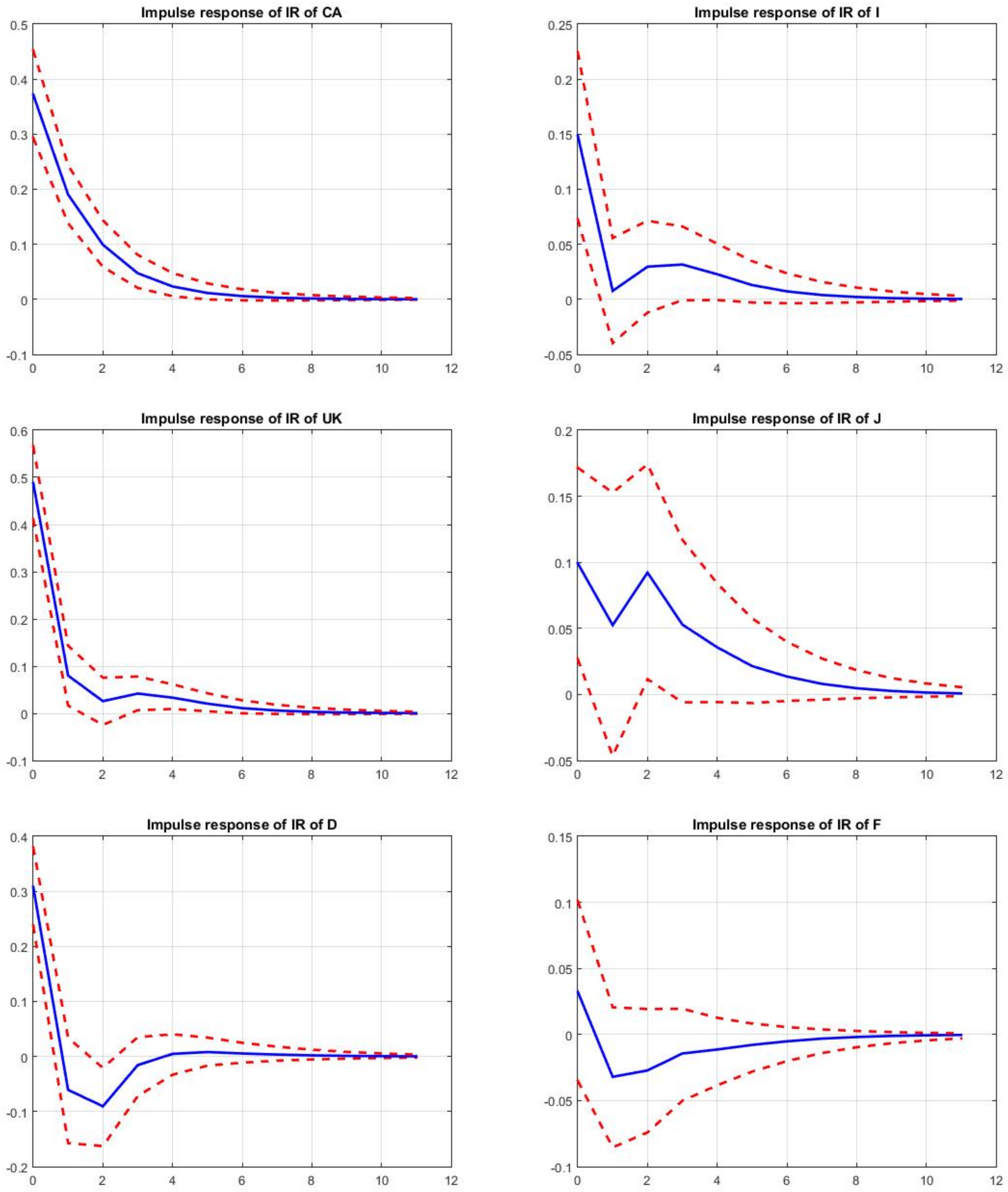


Solid line shows response, dotted lines present upper 84% and lower 16%.

rates seem to affect domestic interest rates. Furthermore, US variables have a dynamic impact on other countries' variables. Both findings are supported by a low probabilities for no dynamic interdependencies. The evidence is high that static interdependencies exist between countries' CPIs and between interest rates. Low probabilities for homogeneity are found for variables of the United States and of Japan. Thus, the two countries seem to behave differently compared to the remaining countries.

The impulse response analysis sheds light on the reliability of the findings. Exemplary, I will take a closer look at the responses to a shock to the US interest rate, presented in figure 1 for US variables and in figure 2 for foreign interest rates. A contractionary US monetary policy, shown by an increase in the US interest rate,

Figure 2: Responses of foreign interest rates to a shock to US interest rate



Solid line shows response, dotted lines present upper 84% and lower 16%.

leads to a rise in US CPI. The response of industrial production growth is insignificant. The increase of inflation in response to a tightening in the monetary policy is in line with the price puzzle. The price puzzle - first mentioned by Sims (1992) - refers to this result contradicting theoretical models and empirical findings which would claim that a rise in the interest rate leads to a decline in inflation. The puzzle is expected for VAR models which just include industrial production growth, inflation, and a short term interest rate and have a structural identification based on a recursive system. The foreign interest rates immediately raise in response to a tightening in the US monetary policy. The increases in the interest rates are lower, below 0.5, than the initial raise in the US interest rate, which is normalized to one. The UK interest rate is initially affected most, followed by the Canadian and German interest rate responses. After around two horizons the effect of the US shock is insignificant for the interest rate of the United Kingdom, Germany, and Italy. The responses of the interest rates of Japan and France are lowest. For Japan the response is insignificant after the first horizon while for France the response is insignificant for all horizons. The raise in the Canadian interest rate lasts longest and comes to zero after six horizons. To sum up, the impulse repose functions show that the results based on SSVSP are reliable.

7 Concluding Remarks

This paper introduces the SSVSP as an extension of Koop's and Korobilis's S^4. The SSVSP is an alternative estimation procedure for PVARs which is able to fully incorporate dynamic and static interdependencies as well as cross-country heterogeneities. It allows for a flexible panel structure since it distinguishes only between domestic and foreign variables. Using a hierarchical prior the SSVSP searches for the restrictions which are supported by the data.

The results of the Monte Carlo simulations show that when a less restrictive panel structure is present, the SSVSP outperforms the S^4. The average deviation of the estimated parameters from the true values is less for the SSVSP, $APD_{SSVSP} = 0.036$, than for S^4, $APD_{S^4} = 0.045$. In most cases, OLS estimates deviate more from the ture values than the estimates of the selection priors. Thus, using a prior which is able to account for a panel structure is beneficial. Furthermore, the accuracy of the SSVSP in selecting the restrictions is proofed by the posterior probabilities for no interdependencies and homogeneity.

The results of the empirical application are summarized in three main findings. Firstly, compared to the S^4 the SSVPS gives improved forecasting results. Secondly, posterior probabilities for DI and SI restrictions show that interest rates likely depend on foreign interest rates. Variables of the United States and Japan have no static interdependencies with the remaining G7 countries and are heterogeneous. Thirdly, responses to a shock in the US interest rate are in line with expected response functions.

The SSVSP prior can be further developed. The SI restriction search is a first way for structural identification in a data based way but it comes with the limitation of building on a recursive system. For just identified systems the BMA result of the reduced form can be used combined with the clear mapping between reduced form covariance matrix and a short run restriction matrix to obtain the structural form. For overidentified systems, however, the draws of the coefficient matrices have to be directly from the structural form. This means a selection prior for A conditional on a restriction matrix A_0 has to be stated and a valid Gibbs sampler has to be derived.

One critical issue is the selection of hyperparameters. In my specification the hyperparameters are fixed for all parameters which are estimated. George et al. (2008) propose a default semi-automatic approach to select the hyperparameters. The values are not fixed but vary for each coefficient. For example $\tau_{1,i} = c_1\sqrt{var(\alpha_i)}$ and $\tau_{2,i} = c_2\sqrt{var(\alpha_i)}$ whereby $c_1 = 0.1$ and $c_2 = 10$. The $var(\alpha_i)$ is the estimated variance of the OLS estimate for α_i in a model without restriction search. The κ and ξ are set in an equal manner. Trying this approach leads to hyperparameters which tend to be so small, that the majority of values are drawn from the loose part of the prior. Koop and Korobilis (2015) specify distributions for the hyperparameters. This allows them to have varying hyperparameters and a less subjective choice of hyperparameters.

To sum up, the findings of the conducted Monte Carlo simulations and the exemplary empirical application encourage the use of the SSVSP to estimate PVAR models. However, the recursive structural identification as well as the specified hyperparameters leave room for further improvements.

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Gibbs Sampler Algorithm

Step 1:
Sample α from a Normal posterior conditional on Σ, γ_{DI}, γ_{CSH}.

$$\alpha \mid \Sigma, \gamma_{DI}, \gamma_{CSH} \sim \mathcal{N}(\Gamma\mu_\alpha, V_\alpha),$$

where $V_\alpha = ((D'D)^{-1} + \Sigma^{-1} \otimes X'X)^{-1}$ with $X = Y_{t-1}$ and $\mu_\alpha = V_\alpha((\Sigma^{-1} \otimes X'X)\alpha_{OLS})$. D is a diagonal matrix with $D = diag(h_{11}^{11}, ..., h_{NN}^{GG})$. The value of h depends on γ_{DI} and γ_{CSH}: $h_{ij}^{lk} = \begin{cases} \tau_1, & if\ \gamma_{DI,ij}^{lk} = 0 \\ \tau_2, & if\ \gamma_{DI,ij}^{lk} = 1 \end{cases}$ for the parameters where DI restriction search is done $(i \neq j)$ and $h_{jj}^{lk} = \begin{cases} \xi_1, & if\ \gamma_{CSH}^{w} = 0 \\ \xi_2, & if\ \gamma_{CSH}^{w} = 1 \end{cases}$ for the block diagonal parameters where CSH restriction search is done. α_{OLS} is the OLS estimate of α. The posterior mean is restricted with the selection matrix Γ.

Step 2:
Update γ_{DI} and γ_{CSH} from Bernoulli distribution:

$$\begin{aligned} \gamma_{DI,ij}^{lk} &\sim Bernoulli(\pi_{DI,ij}^{lk}) \\ \pi_{DI,ij}^{lk} &= \frac{u2_{DI,ij}^{lk}}{u1_{DI,ij}^{lk} + u2_{DI,ij}^{lk}} \\ \gamma_{CSH}^{w} &\sim Bernoulli(\pi_{CSH}^{w}) \\ \pi_{CSH}^{w} &= \frac{v2_{CSH}^{w}}{v1_{CSH}^{w} + v2_{CSH}^{w}}. \end{aligned}$$

Hereby, $u1_{DI,ij}^{lk} = F(\alpha_{ij}^{lk} \mid 0, \tau_1^2)prob_{DI}$ and $u2_{DI,ij}^{lk} = F(\alpha_{ij}^{lk} \mid 0, \tau_2^2)(1 - prob_{DI})$. $F()$ denotes the p.d.f. of the Normal distribution with mean zero and variance τ_1^2 or τ_2^2 evaluated at α_{ij}^{lk}. The parameter $prob_{DI}$ is set equal to 0.5. This shows that a priori the researcher assumes that it is equally likely that a dynamic interdependency between two variables of country i and j exists or does not exist. $v1_{CSH}^{w} = F(\alpha_{jj}^{lk} \mid \alpha_{ii}^{lk}, \xi_1^2)prob_{CSH}$ and $v2_{CSH}^{w} = F(\alpha_{jj}^{lk} \mid 0, \xi_2^2)(1 - prob_{CSH})$. Again, $prob_{CSH}$ is set equal 0.5. Depending on γ_{CSH}^{w} the elements in Γ_w are updated.

Step 3:
Update $\Sigma = \Psi^{-1'}\Psi^{-1}$ and γ_{SI}. The variance elements, ψ_{ii}^{kk}, are drawn from a

Gamma distribution:

$$(\psi_{ii}^{kk})^2 \sim \mathcal{G}(a + 0.5 * T, B_n),$$

where $n = 1, ..., NG$ and

$$B_n = \begin{cases} b + 0.5SSE_{nn} & n = 1 \\ b + 0.5(SSE_{nn} - s_n'(S_{n-1} + (R'R)^{-1})^{-1} s_n) & n = 2, ..., NG \end{cases}.$$

Note that ψ_{11}^{11} is assigned to B_2, ψ_{11}^{22} to B_2, ..., and ψ_{NN}^{GG} to B_{NG}. T is defined as the length of the time series and SSE as the sum of squared residuals. S_n is the upper-left $n \times n$ submatrix of SSE, and $s_n = (s_{1n}, ..., s_{n-1,n})'$ contains the upper diagonal elements of SSE. R is a diagonal matrix with $R = diag(r_{11}^{11}, ..., r_{NN}^{GG})$. The value of r depend on γ_{SI}: $r_{ij}^{lk} = \begin{cases} \kappa_1, & if\ \gamma_{SI,ij}^{lk} = 0 \\ \kappa_2, & if\ \gamma_{SI,ij}^{lk} = 1 \end{cases}.$

Define the vector $\psi = (\psi_{12}^{11}, ..., \psi_{N-1,N}^{GG})'$. Thus, ψ contains the covariance elements, ψ_{ij}^{lk} for all $i \neq j$ and has the dimension $n_{SI} \times 1$, where $n_{SI} = 1, .., N_{SI}$ and N_{SI} is the length equal to the number of SI restrictions. The elements of ψ are updated from a Normal distribution:

$$\psi_{n_{SI}} \mid \alpha, \psi, \gamma_{SI} \sim \mathcal{N}(\mu_{n_{SI}}, V_{n_{SI}}).$$

Hereby, $\mu_{n_{SI}} = -\psi_{ii}^{kk}(S_{n_{SI}-1} + (R'R)^{-1})^{-1} s_{n_{SI}}$ and $V_{n_{SI}} = (S_{n_{SI}-1} + (R'R)^{-1})^{-1}$. The element ψ_{ii}^{kk} is the variance element in the same row of Psi as $\psi_{ij}^{lk} = \psi_{n_{SI}}$ for all $i \neq j$. The off-diagonal elements of the covariance matrix which belong to one country are drawn from a Normal distribution with mean zero and variance κ_2.

Hyperparameter

Table 7: Hyperparameters

τ_1	τ_2	ξ_1	ξ_2	κ_1	κ_2	a	b
0.2	4	0.2	4	0.3	4	0.01	0.01

The comparison of the different prior is also based on these specific hyperparameters. A value of $\tau_1 = 0.2$ and $\tau_2 = 4$ means that the variance of the tight prior equals 0.04 and 16 for the loose prior. The criterion that the variance of the first part of the Normal distribution is smaller than the second part is clearly fulfilled. Several other specification are also checked. The accuracy of the algorithm in selecting the restrictions varies with the specification of the hyperparameters. If τ_1, κ_1, and ξ_1 are chosen too small, the majority of values is drawn from the second part of the Normal distribution (γ equals one with a very high probability). Still, γ equals more often one in the cases no restriction is set in the true specification of the Monte Carlo simulation. Values for hyperparameters smaller or equal 0.1 prove to be too small resulting in the mentioned difficulties. George et al. (2008) propose a default semi-automatic approach to selecting the hyperparameters. The values are not fixed but vary for each coefficient. For example $\tau_{1,i} = c_1\sqrt{var(\alpha_i)}$ and $\tau_{2,i} = c_2\sqrt{var(\alpha_i)}$ whereby $c_1 = 0.1$ and $c_2 = 10$. $var(\alpha_i)$ is a OLS estimated of the variance of the coefficient in an unrestricted model. κ and ξ are set in an equal manner. Trying this approach also leads to hyperparameters smaller than 0.1.

Estimates - Monte Carlo Simulation

The simulation is done with 100 samples, each with a length of 100. The Gibbs sampler has 55000 draws of which 5000 draws are disregarded as draws in the burn-in-phase. The estimated parameters for the first Monte Carlo Simulation based on the SSVSP prior are the following:

$$A_1^{SSVSP} = \begin{pmatrix} 0.61 & 0.09 & 0.20 & 0.19 & -0.03 & 0.01 \\ 0.01 & 0.50 & 0.31 & 0.29 & -0.03 & -0.01 \\ 0.00 & 0.01 & 0.52 & 0.46 & -0.01 & -0.04 \\ -0.01 & -0.01 & -0.02 & 0.39 & 0.00 & -0.01 \\ 0.28 & -0.39 & -0.03 & -0.02 & 0.52 & 0.46 \\ 0.22 & 0.38 & -0.03 & -0.03 & -0.03 & 0.41 \end{pmatrix}$$

$$\Psi^{SSVSP} = \begin{pmatrix} 0.70 & -0.03 & 0.03 & 0.23 & 0.10 & -0.10 \\ 0.00 & 0.76 & 0.31 & 0.46 & 0.02 & -0.02 \\ 0.00 & 0.00 & 0.91 & 0.01 & 0.08 & 0.01 \\ 0.00 & 0.00 & 0.00 & 1.01 & -0.22 & -0.25 \\ 0.00 & 0.00 & 0.00 & 0.00 & 1.27 & 0.27 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.15 \end{pmatrix}$$

$$\Sigma^{SSVSP} = \begin{pmatrix} 1.42 & 0.16 & -0.39 & -0.48 & 0.21 & 0.19 \\ 0.16 & 1.50 & -0.41 & -0.52 & 0.14 & 0.25 \\ -0.39 & -0.41 & 1.70 & 0.54 & 0.09 & 0.11 \\ -0.48 & -0.52 & 0.54 & 1.61 & 0.00 & -0.01 \\ 0.21 & 0.14 & 0.09 & 0.00 & 1.35 & 0.17 \\ 0.19 & 0.25 & 0.11 & -0.01 & 0.17 & 1.32 \end{pmatrix}$$

The estimated values for the second Monte Carlo Simulation are given by:

$$A_1^{SSVSP} = \begin{pmatrix} 0.55 & 0.12 & 0.18 & 0.00 & 0.20 & -0.02 \\ 0.00 & 0.54 & 0.18 & -0.02 & 0.19 & -0.01 \\ 0.00 & -0.03 & 0.51 & 0.43 & 0.00 & -0.01 \\ -0.02 & 0.00 & -0.01 & 0.27 & 0.01 & -0.01 \\ 0.26 & -0.43 & -0.01 & 0.00 & 0.51 & 0.44 \\ -0.01 & -0.01 & 0.00 & 0.00 & 0.00 & 0.39 \end{pmatrix}$$

$$
\Psi^{SSVSP} = \begin{pmatrix} 0.78 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.72 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.89 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.83 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 1.01 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.10 \end{pmatrix}
$$

$$
\Sigma^{SSVSP} = \begin{pmatrix} 1.77 & 0.23 & -0.45 & -0.01 & 0.30 & 0.01 \\ 0.23 & 1.27 & 0.01 & -0.50 & 0.13 & 0.02 \\ -0.45 & 0.01 & 1.36 & 0.03 & 0.00 & 0.00 \\ -0.01 & -0.50 & 0.03 & 1.31 & 0.00 & 0.01 \\ 0.30 & 0.13 & 0.00 & 0.00 & 1.30 & 0.02 \\ 0.01 & 0.02 & 0.00 & 0.01 & 0.02 & 1.07 \end{pmatrix}
$$

Empirical Application

The Gibbs sampler has 55000 draws of which 5000 draws are disregarded as draws in the burn-in-phase. Posterior probabilities of restrictions are the γ draws equal to zero over all draws. MSFEs of SSVSP and SSSS relative to OLS. Values below one indicate a performance of the estimator better than OLS.

Table 8: MSFEs for SSVSP relative to OLS

horizon	1	2	3	4	5	6	7	8	9	10	11	12
IP_CA	1.00	**0.98**	1.02	**0.95**	**0.98**	1.06	1.09	1.05	1.10	1.13	1.13	1.14
CPI_CA	1.05	1.02	1.05	1.06	1.03	1.01	1.02	1.04	1.04	1.03	**0.95**	**0.97**
IR_CA	1.10	1.15	1.15	1.25	1.32	1.39	1.39	1.45	1.41	1.47	1.40	1.38
IP_I	1.12	**0.98**	1.03	1.04	1.10	1.10	1.07	1.15	1.12	1.12	1.09	1.10
CPI_I	1.25	1.03	1.00	1.03	1.01	**0.99**	1.03	**1.00**	1.00	**0.99**	**0.98**	**0.99**
IR_I	1.41	1.28	1.28	1.41	1.39	1.41	1.41	1.38	1.40	1.32	1.35	1.39
IP_UK	1.07	**0.99**	**0.98**	1.04	1.06	1.05	1.04	1.06	1.02	1.03	1.03	1.02
CPI_UK	**0.99**	**0.98**	1.04	1.02	**0.99**	**0.99**	1.05	**0.99**	1.02	1.00	1.04	**0.95**
IR_UK	1.06	1.03	**0.98**	1.01	1.03	1.09	1.09	1.07	1.06	1.09	1.03	1.01
IP_F	1.02	**0.99**	1.01	**0.98**	1.03	1.06	1.00	1.07	1.03	1.01	1.01	**0.97**
CPI_F	1.04	1.01	1.01	1.02	1.03	1.01	1.02	1.01	1.02	**1.00**	**1.00**	1.01
IR_F	1.59	1.16	1.12	1.21	1.28	1.42	1.42	1.43	1.46	1.42	1.42	1.41
IP_J	**0.99**	**1.00**	1.01	1.01	1.02	1.03	1.01	1.01	**1.00**	**0.98**	**0.98**	1.01
CPI_J	1.04	**0.99**	**1.00**	**1.00**	1.01	1.02	1.01	1.01	1.01	1.00	1.03	1.01
IR_J	3.59	3.08	3.00	2.93	3.07	3.20	3.00	3.33	3.07	2.93	2.71	2.73
IP_D	**0.99**	1.04	**0.98**	1.01	1.02	1.03	1.04	1.05	1.02	1.01	1.03	**0.99**
CPI_D	1.00	1.02	**0.97**	1.01	1.00	1.01	**1.00**	**1.00**	1.00	**1.00**	1.04	**0.97**
IR_D	1.38	1.13	1.02	**0.97**	**0.97**	**1.00**	**0.99**	**0.98**	**0.96**	1.00	**0.99**	**0.98**
IP_US	**1.00**	**0.97**	**0.98**	**0.97**	1.02	1.00	1.02	1.02	1.02	1.04	1.04	1.02
CPI_US	**1.00**	**1.00**	1.01	1.04	1.03	1.01	1.04	1.05	1.03	1.01	**0.99**	**0.99**
IR_US	1.00	1.00	**0.99**	**1.00**	1.01	1.00	1.03	1.02	1.03	1.03	1.03	1.02

MSFEs are given relative to OLS, MSFE<1 are in bold. Σ drawn from Inverted Wishart distribution.

Table 9: MSFEs for S^4 relative to OLS

horizon	1	2	3	4	5	6	7	8	9	10	11	12
IP_CA	1.03	**1.00**	1.07	**0.95**	1.03	1.14	1.16	1.13	1.20	1.23	1.21	1.20
CPI_CA	1.20	1.07	1.23	1.17	1.23	1.23	1.23	1.28	1.26	1.20	1.13	1.01
IR_CA	1.10	1.40	1.26	1.34	1.41	1.56	1.53	1.66	1.60	1.70	1.66	1.61
IP_I	1.13	**0.98**	1.05	1.05	1.10	1.14	1.08	1.20	1.15	1.15	1.14	1.17
CPI_I	1.21	1.31	1.22	1.26	1.20	1.24	1.33	1.30	1.32	1.40	1.24	1.21
IR_I	1.37	1.25	1.19	1.37	1.35	1.41	1.40	1.36	1.37	1.33	1.38	1.43
IP_UK	1.21	1.02	1.04	1.10	1.12	1.12	1.15	1.10	1.13	1.12	1.18	1.06
CPI_UK	1.39	1.15	1.13	1.22	1.20	1.11	1.09	1.23	1.25	1.21	1.05	1.11
IR_UK	1.06	1.04	1.01	1.02	1.03	1.04	1.09	1.03	1.02	1.05	1.02	**0.95**
IP_F	1.34	1.02	1.09	1.04	1.14	1.24	1.12	1.25	1.20	1.19	1.19	1.15
CPI_F	1.59	1.21	1.13	1.26	1.31	1.17	1.24	1.34	1.21	1.23	1.10	**0.97**
IR_F	1.54	1.34	1.20	1.23	1.32	1.50	1.50	1.50	1.54	1.51	1.50	1.48
IP_J	**1.00**	**0.98**	1.01	1.01	1.00	1.04	1.02	1.00	**0.99**	1.03	1.01	1.06
CPI_J	1.14	1.03	1.07	1.02	1.01	1.01	1.00	1.07	1.06	1.02	1.06	1.01
IR_J	3.71	3.92	3.92	3.73	3.93	4.13	3.93	4.33	4.00	3.71	3.50	3.53
IP_D	1.10	**0.97**	**0.97**	1.08	**0.94**	1.14	1.10	1.07	1.11	1.14	1.10	1.06
CPI_D	1.67	1.15	1.03	1.14	**0.89**	1.18	1.02	1.05	1.23	**0.97**	1.12	1.04
IR_D	1.46	1.25	1.06	1.00	**0.97**	1.04	**0.99**	**0.95**	**0.96**	**0.99**	1.00	1.00
IP_US	**1.00**	**0.98**	**0.98**	**0.94**	1.02	**0.99**	1.00	1.02	1.02	1.01	1.03	1.02
CPI_US	**1.00**	**1.00**	1.01	1.03	1.04	1.01	1.03	1.07	1.06	1.03	**0.99**	1.01
IR_US	1.00	1.02	1.03	1.02	1.03	**0.97**	1.06	1.03	1.05	1.03	1.04	1.02

MSFEs are given relative to OLS, MSFE<1 are in bold. Σ drawn from Inverted Wishart distribution.

Table 10: Probabilities($\alpha_{ij}^{lk} = 0$) - DI restrictions

		CA			I			UK			F			J			D			US		
		IP	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**
	IP				0.81	0.34	0.69	0.95	0.62	0.45	0.98	0.49	0.29	0.99	0.68	0.34	0.99	0.17	0.35	0.09	0.45	0.04
CA	**CPI**				0.99	0.79	0.97	1.00	0.96	0.77	1.00	0.69	0.79	1.00	0.95	0.08	1.00	0.91	0.21	0.99	0.06	0.57
	IR				1.00	0.90	0.00	1.00	0.99	0.87	1.00	0.90	0.31	1.00	0.99	0.05	1.00	0.98	0.15	0.98	0.95	0.11
	IP	0.91	0.18	0.10				0.83	0.20	0.37	0.96	0.20	0.47	0.69	0.56	0.28	0.43	0.55	0.15	0.09	0.40	0.39
I	**CPI**	1.00	0.99	0.79				1.00	0.99	0.95	1.00	0.83	0.98	1.00	0.99	0.51	1.00	0.82	0.83	1.00	0.66	0.64
	IR	1.00	0.83	0.75				1.00	0.91	0.44	1.00	0.75	0.87	1.00	0.82	0.11	1.00	0.87	0.01	0.91	0.70	0.57
	IP	0.83	0.64	0.63	0.89	0.53	0.39				0.88	0.50	0.64	0.98	0.60	0.35	0.97	0.65	0.31	0.11	0.60	0.48
UK	**CPI**	1.00	0.49	0.56	1.00	0.77	0.83				1.00	0.72	0.82	1.00	0.57	0.35	1.00	0.30	0.42	0.99	0.12	0.73
	IR	1.00	0.98	0.99	1.00	0.96	0.99				1.00	0.78	0.66	1.00	0.87	0.72	1.00	0.99	0.33	1.00	0.33	0.94
	IP	0.56	0.31	0.35	0.95	0.43	0.60	0.73	0.58	0.03				0.74	0.53	0.07	0.96	0.52	0.15	0.05	0.45	0.04
F	**CPI**	1.00	0.54	0.97	1.00	0.63	0.95	1.00	0.97	0.85				1.00	0.95	0.71	1.00	0.98	0.75	0.99	0.04	0.49
	IR	1.00	0.92	0.92	1.00	0.84	0.95	1.00	0.95	0.82				1.00	0.96	0.68	1.00	0.90	0.00	0.99	0.90	0.83
	IP	0.65	0.24	0.37	0.27	0.26	0.17	0.48	0.39	0.08	0.47	0.28	0.32				0.41	0.32	0.08	0.39	0.23	0.20
J	**CPI**	0.99	0.89	0.86	1.00	0.20	0.96	1.00	0.71	0.78	1.00	0.82	0.93				1.00	0.87	0.46	0.94	0.06	0.80
	IR	1.00	0.99	0.99	1.00	0.99	0.99	1.00	1.00	0.99	1.00	0.99	0.74				1.00	0.99	0.17	1.00	1.00	0.99
	IP	0.82	0.44	0.47	0.92	0.39	0.52	0.63	0.46	0.01	0.60	0.20	0.46	0.07	0.53	0.12				0.23	0.27	0.20
D	**CPI**	1.00	0.46	0.94	1.00	0.39	0.40	1.00	0.98	0.86	1.00	0.39	0.42	1.00	0.51	0.57				1.00	0.86	0.31
	IR	1.00	0.99	1.00	1.00	0.99	1.00	1.00	1.00	0.99	1.00	0.99	0.96	1.00	1.00	0.72				1.00	0.62	0.98
	IP	0.53	0.17	0.78	0.98	0.62	0.38	0.99	0.83	0.22	0.99	0.62	0.79	0.98	0.77	0.40	1.00	0.75	0.50			
US	**CPI**	1.00	0.95	0.89	1.00	0.29	0.97	1.00	0.97	0.87	1.00	0.79	0.93	1.00	0.98	0.54	1.00	0.29	0.69			
	IR	1.00	0.99	0.90	1.00	0.84	0.99	1.00	0.98	0.95	1.00	0.95	0.98	1.00	0.98	0.83	1.00	0.93	0.48			

Table 11: Probabilities($\psi_{ij}^{lk} = 0$) - SI restrictions

		CA			I			UK			F			J			D			US		
		IP	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**
CA	**IP**				0.80	0.99	0.99	0.91	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.99	0.99	0.99	0.99	0.38	0.99	0.90
	CPI				0.87	0.04	0.85	0.85	0.79	0.80	0.73	0.00	0.82	0.84	0.84	0.81	0.21	0.75	0.82	0.62	0.00	0.28
	IR				0.68	0.35	0.46	0.75	0.26	0.72	0.55	0.47	0.00	0.73	0.70	0.32	0.72	0.73	0.69	0.71	0.68	0.00
I	**IP**							0.97	0.99	0.99	0.61	0.99	0.99	0.97	0.99	0.99	0.87	0.99	0.99	0.97	0.99	0.99
	CPI							0.59	0.06	0.27	0.48	0.01	0.34	0.46	0.54	0.38	0.56	0.05	0.27	0.37	0.05	0.46
	IR							0.89	0.79	0.00	0.87	0.79	0.00	0.30	0.68	0.66	0.87	0.73	0.47	0.72	0.85	0.77
UK	**IP**										0.49	0.98	0.99	0.84	0.98	0.98	0.98	0.97	0.93	0.99	0.99	0.89
	CPI										0.72	0.00	0.42	0.86	0.00	0.57	0.63	0.84	0.74	0.68	0.47	0.77
	IR										0.37	0.25	0.08	0.59	0.21	0.43	0.21	0.42	0.00	0.53	0.26	0.00
F	**IP**													0.98	0.99	0.99	0.93	0.99	0.99	0.99	0.99	0.98
	CPI													0.14	0.24	0.44	0.35	0.00	0.54	0.58	0.05	0.54
	IR													0.75	0.76	0.73	0.75	0.74	0.11	0.58	0.73	0.73
J	**IP**																0.99	0.99	0.99	0.99	0.99	0.99
	CPI																0.84	0.60	0.75	0.65	0.19	0.78
	IR																0.21	0.09	0.27	0.40	0.23	0.13
D	**IP**																			0.99	0.99	0.99
	CPI																			0.69	0.77	0.77
	IR																			0.42	0.13	0.00
US	**IP**																					
	CPI																					
	IR																					

Table 12: Probabilities($\alpha^{lk}_{jj} = \alpha^{lk}_{ii}$) - CSH restrictions

variable in equation	**CA, I**	**CA, UK**	**CA, F**	**CA, J**	**CA, D**	**CA, US**	**I, UK**	**I, F**	**I, J**	**I, D**	**I, US**
IP in IP	0.73	0.72	0.66	0.42	0.52	0.35	0.75	0.69	0.37	0.57	0.30
IP in CPI	1.00	0.99	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00
IP in IR	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
CPI in IP	0.28	0.33	0.27	0.26	0.29	0.32	0.28	0.27	0.25	0.29	0.33
CPI in CPI	0.70	0.67	0.61	0.29	0.59	0.12	0.67	0.58	0.35	0.54	0.17
CPI in IR	0.73	0.93	0.74	0.93	0.92	0.84	0.75	0.63	0.76	0.75	0.71
IR in IP	0.29	0.25	0.26	0.07	0.16	0.24	0.24	0.25	0.07	0.15	0.23
IR in CPI	0.86	0.58	0.75	0.44	0.48	0.39	0.63	0.85	0.46	0.50	0.41
IR in IR	0.82	0.84	0.82	0.81	0.75	0.71	0.77	0.77	0.76	0.69	0.68

	UK, F	**UK, J**	**UK, D**	**UK, US**	**F, J**	**F, D**	**F, US**	**J, D**	**J, US**	**D, US**
IP in IP	0.71	0.35	0.59	0.28	0.35	0.58	0.27	0.15	0.80	0.06
IP in CPI	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00
IP in IR	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
CPI in IP	0.26	0.26	0.27	0.30	0.25	0.29	0.35	0.26	0.29	0.41
CPI in CPI	0.62	0.27	0.62	0.11	0.21	0.66	0.07	0.10	0.45	0.00
CPI in IR	0.76	0.96	0.95	0.90	0.77	0.75	0.71	0.99	0.94	0.93
IR in IP	0.21	0.08	0.17	0.29	0.06	0.13	0.18	0.09	0.09	0.21
IR in CPI	0.58	0.40	0.43	0.41	0.46	0.50	0.37	0.38	0.32	0.32
IR in IR	0.81	0.79	0.73	0.70	0.79	0.73	0.71	0.74	0.71	0.69