Asymmetric Jump Beta Estimation with Implications for Portfolio Risk Management

This version: Monday 15th February, 2016

(PRELIMINARY VERSION. PLEASE DO NOT CIRCULATE WITHOUT PERMISSION)

Vitali Alexeev[a,b], Wenying Yao[a], Giovanni Urga[c,d]

[a] *Tasmanian School of Business and Economics, University of Tasmania, Hobart, Tasmania 7001, Australia*
[b] *Department of Economics and Finance, University of Guelph, 50 Stone Rd E, Guelph, ON N1G 2W1, Canada*
[c] *Centre for Econometric Analysis, Faculty of Finance, Cass Business School, 106 Bunhill Row, EC1Y 8TZ, London, UK*
[d] *Department of Management, Economics and Quantitative Methods, University of Bergamo, Bergamo, 24127, Italy*

Abstract

In this paper we study jump dependence of two processes using high-frequency data concentrating only on segments around a few outlying observations that are informative for the jump inference. Assuming that investors care differently about downside losses as opposed to upside gains, we estimate jump sensitivities for the negative and positive market shifts. We investigate the implications of the difference in negative and positive sensitivities to market jumps for portfolio risk management by contrasting the results for individual stocks with the results for portfolios with varying number of holdings. In the context of a portfolio, we investigate to what extend the downside and upside jump risks can be diversified away. This can have a direct impact on the pricing of jump risks and subsequently, investors' decision-making. Varying the jump identification threshold, we show that the asymmetry is more prominent for more extreme events and that the number of holdings required to diversify portfolios' sensitivities to negative jumps is higher than that required for positive jump diversification. We found that ignoring the asymmetry in sensitivities to negative versus positive market jumps may result in under-diversification of portfolios and increased exposure to extreme negative market shifts.

Keywords: Systematic risk, jumps, high frequency, downside beta
JEL: C58, G11, C61

1. Introduction

In this paper, we contribute to the literature on portfolio diversification by evaluating the impact of extreme market shifts on equity portfolios. An important feature explored in our study is the asymmetry in portfolios' behaviour during extreme negative market downturns versus extreme positive uprises. In studying jump dependence of two processes, we use high-frequency observations focusing on segments of data on the fringes of return distributions. Thus, we only consider a few outlying observations that, at the time, are informative for the jump inference. In particular, we study the relationship between jumps of a process for a portfolio of assets and an aggregate market factor, and we analyse the co-movement of the jumps in these two processes. Given the predominance of factor models in asset pricing applications, we focus on a linear relationship between the jumps and portfolio of assets and we assess its sensitivity to jumps in the market. We find that ignoring the asymmetry in sensitivities to negative versus positive market jumps results in under-diversification of portfolios and increases exposure to extreme negative market shifts. We show that investors care differently about extreme downside losses as opposed to extreme upside gains demanding additional compensation for holding stocks with high sensitivities to these movements.

Since Rietz (1988), researchers have modeled the possibility of rare disasters, such as economic depressions or wars, to resolve the equity premium puzzle and related puzzles (e.g. Barro, 2006; Gabaix, 2012). Rietz (1988) asserts that inability of asset pricing models to explain high equity risk premia is due to not capturing the effects of possible market crashes. It has been shown that large risk premia can be obtained in equilibrium when the representative investor treats jump and diffusive risks differently. Barro (2006) shows that rare economic disasters have the potential to explain the high equity premium. Bates (2008) considers investors who are both risk and crash averse. Jump and diffusive risks are both priced even in the absence of crash aversion, but introducing crash aversion allows for greater divergence between the two risk premia. An important feature of the model in Bates (2008) is a representative investor who treats jump and diffusive risks differently, which formalizes the intuition that investors can treat extreme events differently than they treat more common and frequent ones.

Could it be that the markets treat rare events somewhat differently from common, more frequent events? In practice, bearing non-diversifiable jump risk is significantly rewarded. This is evident, for example, from the expensiveness of short-maturity options written on the market index with strikes that are far from its current level. Bates (2008) proposes crash aversion to explain the observed tendency of stock index options to overpredict realized volatility. Guo et al. (2015) show theoretically and empirically that jump risk is also an important determinant of conditional equity premium even when controlled for commonly used stock market return predictors.

Liu et al. (2005) study the asset pricing implication of imprecise knowledge about rare events. The equilibrium equity premium has three components: the diffusive- and jump-risk premia, both driven by risk aversion; and the "rare-event premium", driven exclusively by uncertainty aversion. In financial markets, we see daily fluctuations and rare events of extreme magnitudes. In dealing with the first type of risks, one might have reasonable faith in the model built by financial economists. For the second type of risks, however, one cannot help but feel a tremendous amount of uncertainty about the model. And if market participants are uncertainty averse, then the uncertainty about rare events will eventually find its way into financial prices in the form of a premium. Liu et al. (2005) examine the equilibrium when stock market jumps can occur and investors are both risk averse and averse to model uncertainty with respect to jumps; they obtain similar pricing implications for jump and diffusive risk.

While Santa-Clara and Yan (2010) study the time-series relation between systematic jump risk and expected stock market returns allowing both the volatility of the diffusion shocks, Cremers et al. (2015) examine the pricing of jump risk in the cross-section of stock returns. They find that stocks with high sensitivities to jump risk have low expected returns. The results are significant and economically important.

It has long been recognised that investors care differently about downside losses versus upside gains. Ang et al. (2006a) show that the cross section of stock returns reflects a downside risk premium of 6% per annum. The reward for bearing downside risk is not simply compensation for regular market beta, nor it is explained by common stock market return predictors. Ang et al. (2006b) find a premium for bearing volatility and jump risk.

Lettau et al. (2014) follow Ang et al. (2006a) in allowing a differentiation in unconditional and downside risk. This captures the idea that assets that have a higher beta with market returns conditional on low realization of the market return are particularly risky. The economic intuition underlying downside risk is simple: Agents require a premium not only for securities the more their returns covary with the market return, but also, and even more so, when securities co-vary more with market returns conditional on low market returns.

However, neither Lettau et al. (2014) nor Ang et al. (2006a) investigate the pricing of jump risk in the cross-section of stock returns. Thus, their analysis does not separate jump risk from diffusion risk. Thus, the effects documented by these authors could be related to volatility risk, jump risk, or a combination of both.

Cremers et al. (2015) employ separate measures for jump and volatility risk to disentangle the corresponding asset pricing effects. They find that stocks with high sensitivities to volatility and jump risk have low expected returns, that is, volatility and jump risk both carry negative market prices of risk.

Guo et al. (2015) document asymmetric effects of physical jump risk measures on conditional equity premium and show that signed jump risk measures have statistically significant forecasting power for excess market return. Further, they conclude that negative jump risk has significant effect on conditional equity premium but positive jump risk does not. The fact that the predictive power is negligible when when total realized jump volatility is used further strengthens the importance of this asymmetry effect.

The remainder of the paper is organised as follows. Section 2 sets up the model framework. The data used in our empirical investigation are detailed in Section 3. We discuss our pricing implication for cross section of individual assets in Section 4 and investigate the behaviours of systematic negative and positive jump risk factors in portfolios of assets in Section 5. We draw our conclusions in Section 6.

2. Model Setup

We start with a panel of N assets over a fixed time interval $[0, T]$. Following the convention in high frequency financial econometrics literature, we assume the log-price $p_{i,t}$ of the i^{th} asset follows a semi-martingale plus jumps process in continuous time. In turn, the log-return of any asset, $r_{i,t}$, has the following representation:

$$r_{i,t} \equiv \mathrm{d}p_{i,t} = b_{i,t}\,\mathrm{d}t + \sigma_{i,t}\,\mathrm{d}W_{i,t} + \kappa_{i,t}\,\mathrm{d}\mu_{i,t}, \qquad t \in [0, T], \quad i = 1, 2, \ldots, N, \tag{1}$$

where $b_{i,t}$ is a locally bounded drift term, $\sigma_{i,t}$ denotes the non-zero spot volatility, $W_{i,t}$ is a standard Brownian motion for asset i. The last part in (1) represents the jump component. The jump measure $\mathrm{d}\mu_{i,t}$ is such that $\mathrm{d}\mu_{i,t} = 1$ if there is a jump in $r_{i,t}$ at time t, and $\mathrm{d}\mu_{i,t} = 0$ otherwise. The size of jump at time t is represented by $\kappa_{i,t}$. In fact, $\kappa_{i,t}$ can be defined as $\kappa_{i,t} = p_{i,t} - p_{i,t-}$ in general, where $p_{i,t-} = \lim_{s\uparrow t} p_{i,s}$. It follows immediately that $\kappa_{i,t} = 0$ for $t \in \{t : \mathrm{d}\mu_{i,t} = 0\}$ under this definition.

Return on the market portfolio $r_{0,t}$ can be decomposed in a way similar to (1) as:

$$r_{0,t} = b_{0,t}\,\mathrm{d}t + \sigma_{0,t}\,\mathrm{d}W_{0,t} + \kappa_{0,t}\,\mathrm{d}\mu_{0,t}. \tag{2}$$

In order to simplify the analysis, we assume that the jumps in processes (1) and (2) have only finite activity.[1]

2.1. *Jump Regression*

We focus on the dependence between the jump components of individual assets (or portfolios) and that of the market return, utilising the methodology proposed by Li et al. (2015a) and Li et al. (2015b). Jumps are rare events but have substantially higher market impacts than the diffusive price movements (Patton and Verardo, 2012). The unpredictable nature of jump

[1] Finite activity jumps, as opposed to the infinite activity, means that there is only a finite number of jumps in the given process over $[0, T]$. We make this assumption in the paper since we only focus on "big" jumps with sizes bounded away from zero. For a detailed discussion on finite versus infinite activity in jumps see Aït-Sahalia and Jacod (2012) among others.

makes its risk harder to be diversified away. Starting with Rietz (1988), researchers have modeled the possibility of rare disasters, such as economic depressions or wars, to resolve the equity premium puzzle and related puzzles (e.g., Barro (2006) and Gabaix (2012)). The disasters in this literature are similar to the jumps that we are investigating, but there are some differences. Disasters are extremely rare and they do not match well the short-dated options often used in disentangling the jump and volatility risk. We investigate extreme events using high frequency data. These events or jumps are rare spatially but they occur rather frequently in calendar time when compared to disasters defined in the literature cited above. For instance, we present in Figure 1 the frequency of positive and negative jump occurrence for each year from 2003 to 2011. For more liberal truncation thresholds, majority of the years have more than 100 jumps occurring, while disasters wouldn't happen this often.

Let $\mathcal{T}$ be the collection of jump times for the market portfolio $r_{0,t}$, i.e. $\mathcal{T} = \{\tau : d\mu_{0,\tau} = 1, \tau \in [0, T]\}$. The set $\mathcal{T}$ has finite elements almost surely given the assumption of finite activity jumps. As an analog to the classical one-factor market model, we set the linear factor model for jumps in the following form

$$\kappa_{i,\tau} = \beta_i^d \kappa_{0,\tau} + \epsilon_{i,\tau}, \qquad \tau \in \mathcal{T}, \quad i = 1, 2, \ldots, N, \tag{3}$$

where the superscript d stands for discontinuous (or jump) beta, and $\epsilon_{i,\tau}$ is the residual series. We only consider the jump times of the market portfolio $\mathcal{T}$ because β_i^d is not identified elsewhere. Therefore, β_i^d only exists if there is at least one jump in $r_{0,t}$ in $[0, T]$. Model (3) implicitly assumes that β_i^d is constant over the interval $[0, T]$.

The jump beta β_i^d in (3) has a similar interpretation as the market beta in the CAPM model. It allows us to assess the sensitivity of an asset (or a portfolio of assets) to extreme market fluctuations. Lower β_i^d would signify a resistance of an asset to move as much as a market during extreme event (jump defensive assets), while higher β_i^d values represent high sensitivity of an asset exacerbating the effect of the market moves during the extreme event (jump cyclical assets).

2.2. Isolating Jumps from the Brownian Component

Under discrete-time sampling, neither the jump times $\mathcal{T}$ nor jump sizes $\kappa_{i,\tau}$ are directly observable from the data. Suppose the price and return series are observed every Δ interval, i.e. we obtain return series $r_{i,\Delta}, r_{i,2\Delta}, \ldots, r_{i,m\Delta}$, where $m = \lfloor T/\Delta \rfloor$, for $i = 0, 1, \ldots, N$.

Our first step in constructing the jump regression model (3) is to identify the discrete-time returns on the market portfolio $r_{0,j\Delta} = p_{0,j\Delta} - p_{0,(j-1)\Delta}$ that contain jumps, $j = 1, 2, \ldots, m$. We use the truncation threshold proposed by Mancini (2001) for this purpose (see also Mancini, 2009; Mancini and Renò, 2011). The threshold, denoted by $u_{0,m}$, is a function of the sampling interval Δ, and hence the sampling frequency m. The most widely used threshold is

$$u_{0,m} = \alpha\Delta^{\varpi}, \qquad \text{with} \quad \alpha > 0 \quad \text{and} \quad \varpi \in (0, 1/2). \tag{4}$$

Taking into account the time-varying spot volatility of the return series, the constant α is usually different for different assets, and could vary over time (see, for example, Jacod, 2008). One example is to set α to be dependent on the estimated continuous volatility of the given asset.

As $\Delta \to 0$ and $m \to \infty$, the condition $|r_{0,j\Delta}| > u_{0,m} = \alpha\Delta^{\varpi}$ eliminates the continuous diffusive returns on the market portfolio asymptotically, and hence only keeps returns that contain jumps. We collect the indices of these discrete-time intervals where the market return exceeds the truncation level, and denote this set as

$$\mathcal{J}_m = \{j : 1 \leq j \leq m, |r_{0,j\Delta}| > u_{0,m}\}. \tag{5}$$

We denote the collection of interval returns for $\mathcal{J}_m$ by $\{r_{0,j\Delta}\}_{j \in \mathcal{J}_m}$. Correspondingly, in the continuous-time data generating process for market return (2), for each jump time $\tau \in \mathcal{T}$, we also find the index j such that the jump $\kappa_{0,\tau}$ occurs in the interval $((j-1)\Delta, j\Delta]$,

$$\mathcal{J} = \{j : 1 \leq j \leq m, \tau \in ((j-1)\Delta, j\Delta] \ \text{ for } \ \tau \in \mathcal{T}\}. \tag{6}$$

An important result from Li et al. (2015a) is that, under some general regularity conditions, the probability that the set $\mathcal{J}_m$ coincides with $\mathcal{J}$ converges to one as $\Delta \to 0$. This is formally stated in Proposition 1 in Li et al. (2015a).[2]

Proposition 1. *Under certain regularity assumptions, as $\Delta \to 0$, we have*

(a) $\mathbb{P}(\mathcal{J}_m = \mathcal{J}) \to 1$;

(b) $((j-1)\Delta, r_{0,j\Delta})_{j \in \mathcal{J}_m} \xrightarrow{\mathbb{P}} (\tau, \kappa_{0,\tau})_{\tau \in \mathcal{T}}$.

Note that part (a) of Proposition 1 implies that the the number of elements in the set $\mathcal{J}_m$ consistently estimates the number of jumps in the process $r_{0,t}$. Furthermore, part (b) of Proposition 1 states that as $\Delta \to 0$, the starting point of the interval $(j-1)\Delta$, consistently estimates the jump time τ, and the interval return $r_{0,j\Delta}$ consistently estimates the jump size $\kappa_{0,\tau}$. The asymptotic results in Proposition 1 provides a powerful tool of linking the discrete-time return observations to the unobservable jumps and jump times in the continuous time. We could use the discrete-time return observations to estimate the jump regression (3) and obtain consistent estimator of the jump beta β_i^d.

[2]Please refer to Li et al. (2015a) for more detailed assumptions.

2.3. *Estimating Jump Beta*

In accordance with model (3), the discrete-time jump regression model has the form

$$r_{i,j\Delta} = \beta_i^d r_{0,j\Delta} + \epsilon_{i,\tau}, \qquad j \in \mathcal{J}_m, \quad i = 1, 2, \ldots, N. \tag{7}$$

Hence a naive consistent estimator of β_i^d is the analog of the OLS estimator,

$$\tilde{\beta}_i^d = \frac{\sum_{j\in\mathcal{J}_m} r_{i,j\Delta} \cdot r_{0,j\Delta}}{\sum_{j\in\mathcal{J}_m} (r_{0,j\Delta})^2}, \qquad i = 1, 2, \ldots, N. \tag{8}$$

Li et al. (2015a) propose an efficient estimator for β_i^d. It is an optimal weighted estimator in the sense that it minimizes the conditional asymptotic variance among all weighting schemes. The optimal weight w_j^* is a function of the preliminary consistent estimator $\tilde{\beta}_i^d$, and the approximated pre-jump and post-jump spot covariance matrices $\hat{C}_{j-}$ and $\hat{C}_{j+}$:

$$w_j^* = \frac{2}{(-\tilde{\beta}_i^d, 1)(\hat{C}_{j-} + \hat{C}_{j+})(-\tilde{\beta}_i^d, 1)'}, \qquad \text{for} \quad j \in \bar{\mathcal{J}}_m, \tag{9}$$

where $\bar{\mathcal{J}}_m = \{j \in \mathcal{J}_m : k_m + 1 \le j \le m - k_m\}$, and k_m is an integer such that $k_m \to \infty$ and $k_m \cdot \Delta \to 0$ as $\Delta \to 0$. The spot covariance matrices are estimated in the following manner. We construct the truncation threshold for the vector $\boldsymbol{r}_{j\Delta} \equiv (r_{0,j\Delta}, r_{i,j\Delta})$ jointly in the same way as in (4), and denote it as $\boldsymbol{u}_m \equiv (u_{0,m}, u_{i,m})$. For any $j \in \bar{\mathcal{J}}_m$, we have

$$\hat{C}_{j-} = \frac{1}{k_m \cdot \Delta} \sum_{l=1}^{k_m} \boldsymbol{r}'_{(j+l-k_m-1)\Delta} \cdot \boldsymbol{r}_{(j+l-k_m-1)\Delta} \cdot \mathbb{1}_{\{|\boldsymbol{r}_{(j+l-k_m-1)\Delta}| \le \boldsymbol{u}_m\}}, \tag{10}$$

$$\hat{C}_{j+} = \frac{1}{k_m \cdot \Delta} \sum_{l=1}^{k_m} \boldsymbol{r}'_{(j+l)\Delta} \cdot \boldsymbol{r}_{(j+l)\Delta} \cdot \mathbb{1}_{\{|\boldsymbol{r}_{(j+l)\Delta}| \le \boldsymbol{u}_m\}}, \tag{11}$$

as the approximated pre-jump and post-jump spot covariance matrices, respectively.

Given any weighting function w_j, the class of weighted estimators $\hat{\beta}_j^d$ can be represented as

$$\hat{\beta}_i^d = \frac{\sum_{j\in\bar{\mathcal{J}}_m} w_j \cdot r_{i,j\Delta} \cdot r_{0,j\Delta}}{\sum_{j\in\bar{\mathcal{J}}_m} w_j \cdot (r_{0,j\Delta})^2}, \qquad i = 1, 2, \ldots, N. \tag{12}$$

Theorem 2 in Li et al. (2015a) show that the weighting function in (9) combined with the estimator (12) produces the most efficient estimate of the jump beta β_i^d. The standard errors and subsequently the confidence intervals of the estimators can be constructed using the bootstrap procedure outlined in Li et al. (2015b).

2.4. Asymmetric Jump Beta

In developing Modern Portfolio Theory in 1959, Henry Markowitz recognized that since only downside deviation is relevant to investors, using downside deviation to measure risk would be more appropriate than using standard deviation (Markowitz, 1971). Ang et al. (2006a) explore the asset pricing implications of the downside risk without, however, separately considering extreme events or jumps.[3] In this section, we focus on jump risk and separate the positive and negative jumps. Instead of pooling jumps at both positive and negative ends together, we examine the jump covariation between individual asset (or portfolio) and the equally weighted market index at times of positive market jumps and negative market jumps separately. In this way we could accommodate separate risk premia for these two components.

Although we focus on the negative jump in the market portfolio and the negative jump beta associated with it, our modelling approach naturally gives rise to a similar definition of the positive jump beta. The naive estimators of the two asymmetric betas $\tilde{\beta}_i^{d+}$ and $\tilde{\beta}_i^{d-}$ are as follows:

$$\tilde{\beta}_i^{d-} = \frac{\sum_{j\in\mathcal{J}_m} r_{i,j\Delta}\cdot r_{0,j\Delta}\cdot \mathbb{1}_{\{r_{0,j\Delta}<0\}}}{\sum_{j\in\mathcal{J}_m} (r_{0,j\Delta})^2\cdot \mathbb{1}_{\{r_{0,j\Delta}<0\}}}, \tag{13}$$

$$\tilde{\beta}_i^{d+} = \frac{\sum_{j\in\mathcal{J}_m} r_{i,j\Delta}\cdot r_{0,j\Delta}\cdot \mathbb{1}_{\{r_{0,j\Delta}>0\}}}{\sum_{j\in\mathcal{J}_m} (r_{0,j\Delta})^2\cdot \mathbb{1}_{\{r_{0,j\Delta}>0\}}}, \tag{14}$$

for $i = 1, 2, \ldots, N$. When calculating the weighted estimators, the weighting function (9) would differ for the positive and negative jump betas:

$$w_j^- = \frac{2}{(-\tilde{\beta}_i^{d-},1)(\hat{C}_{j-}+\hat{C}_{j+})(-\tilde{\beta}_i^{d-},1)'}, \quad \text{for} \quad j \in \bar{\mathcal{J}}_m \quad \text{and} \quad r_{0,j\Delta} < 0, \tag{15}$$

$$w_j^+ = \frac{2}{(-\tilde{\beta}_i^{d+},1)(\hat{C}_{j-}+\hat{C}_{j+})(-\tilde{\beta}_i^{d+},1)'}, \quad \text{for} \quad j \in \bar{\mathcal{J}}_m \quad \text{and} \quad r_{0,j\Delta} > 0. \tag{16}$$

Here we assume that before and after the jumps, the spot covariance matrices are the same for positive and negative jumps. These lead to the formation of the weighted estimators of the asymmetric betas:

$$\hat{\beta}_j^{d-} = \frac{\sum_{j\in\bar{\mathcal{J}}_m} w_j^-\cdot r_{i,j\Delta}\cdot r_{0,j\Delta}\cdot \mathbb{1}_{\{r_{0,j\Delta}<0\}}}{\sum_{j\in\bar{\mathcal{J}}_m} w_j^-\cdot (r_{0,j\Delta})^2\cdot \mathbb{1}_{\{r_{0,j\Delta}<0\}}}, \tag{17}$$

$$\hat{\beta}_j^{d+} = \frac{\sum_{j\in\bar{\mathcal{J}}_m} w_j^+\cdot r_{i,j\Delta}\cdot r_{0,j\Delta}\cdot \mathbb{1}_{\{r_{0,j\Delta}>0\}}}{\sum_{j\in\bar{\mathcal{J}}_m} w_j^+\cdot (r_{0,j\Delta})^2\cdot \mathbb{1}_{\{r_{0,j\Delta}>0\}}}, \tag{18}$$

[3] We define the downside market movements in a different way from Ang et al. (2006a). They look at market returns that are lower than its mean, but we simply look at negative returns that contain jumps. It shouldn't affect the results as most jumps are in the tail distribution, which would be far from the mean of the return.

for $i = 1, 2, \ldots, N$. In what follows, we will use an empirical dataset to demonstrate the similarities and differences between the positive and negative jump betas and its implications for portfolio risk management.[4]

3. Data

We investigate the behavior of the β_i^{d+} and β_i^{d-} estimates over a nine year sample period, from January 2, 2003 to December 30, 2011, which includes the period of the financial crisis associated with the bankruptcy of Lehman Brothers in September 2008 and the subsequent period of turmoil in US and international financial markets. The underlying data are 5-minute observations on prices for 501 stocks drawn from the constituent list of the S&P500 index during the sample period, obtained from SIRCA Thomson Reuters Tick History. This dataset was constructed by Dungey et al. (2012) and does not purport to be all the stocks listed on the S&P500 index, but includes those with sufficient coverage and data availability for high frequency time series analysis of this type.

3.1. Data Processing

The original dataset consists of over 900 stocks taken from the 0#.SPX mnemonic code provided by SIRCA for the S&P500 Index historical constituents list. This included a number of stocks which trade OTC and on alternative exchanges, as well as some which altered currency of trade during the period; these stocks were excluded. We adjusted the dataset for changes in Reuters Identification Code (RIC) code during the period through mergers and acquisitions, stock splits, and trading halts. We also removed some stocks with insufficient observations during the sample period. The data handling process is fully documented in the web-appendix to Dungey et al. (2012). The final data set contains 501 individual stocks, hence $N = 501$.

The intra-day returns and prices data start at 9:30 am and end at 4:00 pm, observations with time stamps outside this window and overnight returns are removed. Missing 5-minute price observations are filled with the previous observation, corresponding to zero inter-interval returns. In the case where the first observations of the day are missing, we use the first non-zero price observation on that day to fill backwards. Approximately 20 price observations which are orders of magnitude away from their neighbouring observations are also removed. Thus, we have 77 intra-day observations for 2262 active trading days.

The 5-minute sampling frequency is chosen as relatively conventional in the high frequency literature, especially for univariate estimation, see, for example, Andersen et al. (2007), Lahaye et al. (2011), and for some sensitivity to alternatives, see Dungey et al. (2009). Optimal sampling frequency is an area of ongoing research, and despite the univariate work by Bandi and Russell (2006), this issue is outstanding for analyzing multiple series with varying degrees of liquidity.

[4] Li et al. (2015a) also consider the positive and negative jump betas separately in their empirical application.

The 5-minute frequency is much finer than those employed in Todorov and Bollerslev (2010) and Bollerslev et al. (2008), both of which use 22.5-minute data. Lower sampling frequencies are generally employed due to concerns over the Epps effect (Epps, 1979); however, as the quality of high frequency data and market liquidity have improved in many ways, finer sampling does not threaten the robustness of our results.

Estimates of β_i^{d+} and β_i^{d-} are computed on an annual basis. High frequency data permits the use of 1-year non-overlapping windows to analyse the dynamics of our systematic risk estimates. Li et al. (2015a) also finds in their empirical application using US equity market data that the positive or negative jump beta remains constant over a year most of the time. For each year, only liquid stocks are considered. Given the 5-minute sampling frequency, we define liquid stocks as ones that have at least 75% of the entire 1-year window as non-zero return data, which indicates that these stocks are heavily traded most of the time. We construct an equally weighted portfolio of all investible stocks in each estimation window as the benchmark market portfolio. We use equally weighted portfolios rather than value weighted ones to avoid situations where the weight on one stock is disproportionately large relative to other portfolio constituents.[5]

3.2. *Parameter Values*

In our empirical application we normalize each trading day to be one unit in time. Given the number of observations in each day $m = 77$, the sampling frequency is $\Delta = 1/77$.

Parameters in the truncation threshold (4) are chosen as follows. The constant $\varpi = 0.49$. Taking into account the time-varying volatility $\sigma_{i,t}$, we set α to be a function of the estimated daily continuous volatility component for each individual asset. In finite sampling, the continuous volatility is consistently estimated by the bipower variation (Barndorff-Nielsen and Shephard, 2004, 2006):

$$BV_i = \left(\frac{\pi}{2}\right) \cdot \sum_{j=1}^{m-1} |r_{i,j\Delta}|\, |r_{i,(j+1)\Delta}| \xrightarrow{\mathbb{P}} \int_0^T \sigma_{i,t}^2 \,\mathrm{d}t \quad \text{as} \quad \Delta \to 0, \qquad i = 0, 1, \ldots, N. \tag{19}$$

We set $\alpha_i = 5\sqrt{BV_i}$, which leads to the threshold

$$u_{i,m} = 5\sqrt{BV_i} \cdot (1/77)^{0.49}. \tag{20}$$

The choice of using a multiple of the estimated continuous volatility is relatively standard in the literature for disentangling jumps from the continuous price movements. It could serve the purpose of controlling for the possibly time-varying spot volatility automatically in jump de-

[5] See Fisher (1966) for the discussion of "Fisher's Arithmetic Index", an equally weighted average of the returns on all listed stocks.

tection. In our empirical application, BV_i is calculated on a daily basis and hence the threshold is different for each trading day. It keeps a good balance such that we could find both positive and negative jumps in each estimation window (year) in the market portfolio, and hence β_i^{d+} and β_i^{d-} can be estimated for each year.

4. Empirical Analysis

In this section we present some statistical properties of betas estimated based on the overall market returns, as well as based on the upside and downside market returns separately. In Section 4.3 we use the two-stage regression framework proposed by Fama and MacBeth (1973) to estimate the risk premia on the risk factors of interest.

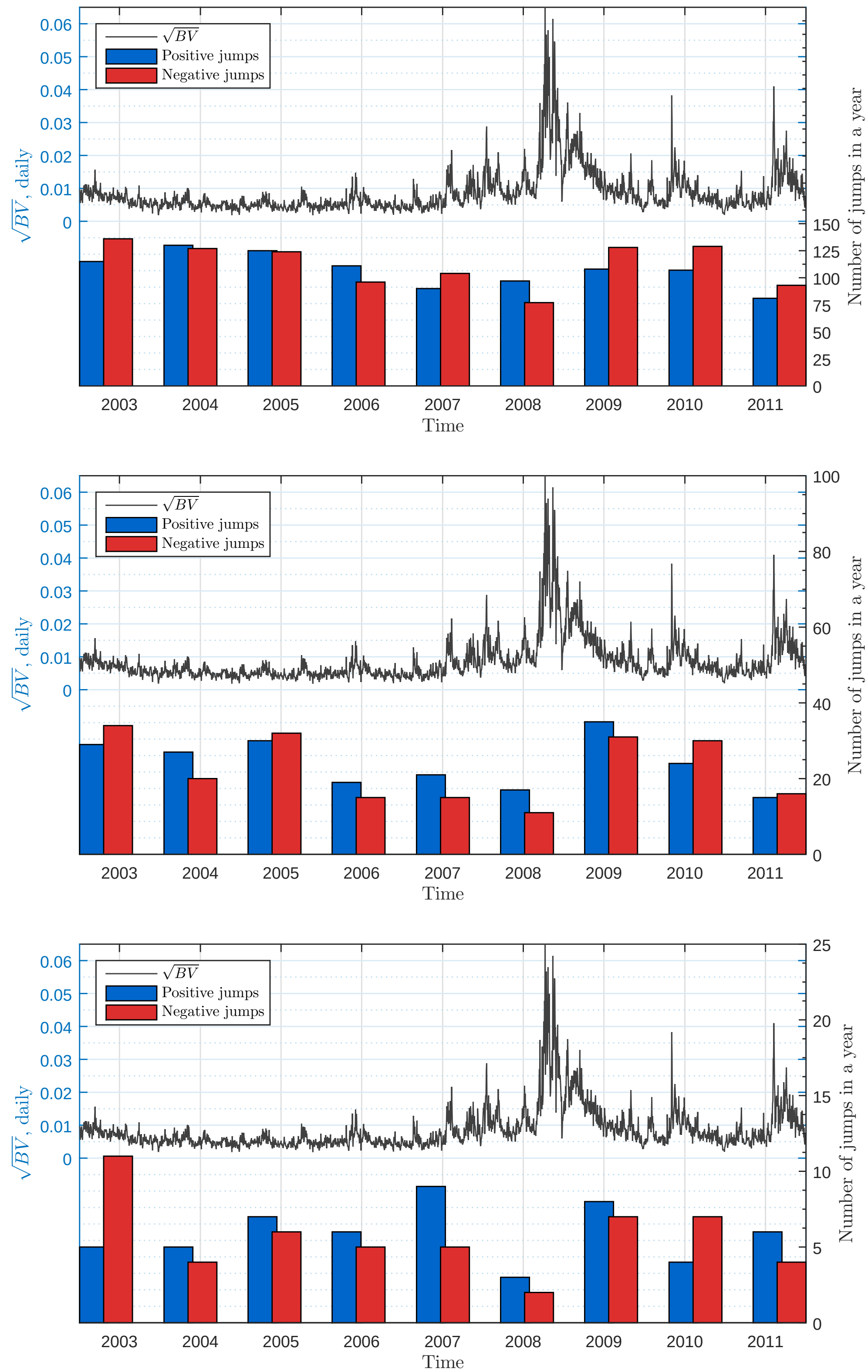
4.1. Market Volatility and Jumps

Figure 1 plots the square root of the daily bipower variation of the equally weighted market index towards the left axis, and the number of positive (blue) and negative (red) jumps for each year from 2003-2011 toward the right axis. The subsample before mid-2007 is much less volatile than the second half of the sample which includes the global financial crisis (GFC). Evidently, market volatility has increased considerably since mid-2007, which is usually regarded as the initial emergence of the GFC, and peaked in late 2008 during the few months after the bankruptcy of Lehman Brothers, the bailout of AIG and the announcement of the TARP (Troubled Asset Relief Program). Other highly volatile periods include mid-2010 during the Greek debt crisis, and late 2011 during the European sovereign debt crisis with the deterioration of economic conditions in the Eurozone as a whole. The two peak values of market volatility after the GFC correspond to the May 6, 2010 flash crash and August 9, 2011. On August 5, 2011 Standard & Poor's downgraded America's credit rating for the first time in history, followed with short-selling ban by Greece on August 8, 2011, and other 4 EU countries on August 11, 2011.

We identify jumps using three different levels of threshold. Using lower thresholds results in identification of too many jumps. For example, for threshold $\alpha_i = 3\sqrt{BV_i}$, the number of negative *and* positive jumps identified is often in excess of 100 jumps a year, meaning that the jumps occur almost every other day. Jumps are rare events and should not happen that often. The number of jumps identified using thresholds $\alpha_i = 4\sqrt{BV_i}$ and $\alpha_i = 5\sqrt{BV_i}$ appear more realistic.[6] Notably, Figure 1 shows that the volatile period during the GFC corresponds to fewer jumps in both directions. In particular, in 2008, using the most conservative threshold, we do not observe more than 5 jumps per year (positive or negative). This result is expected as the market volatility is generally higher during crisis than during calmer period, the threshold of detecting jump observations will be elevated accordingly. Black et al. (2012) also observe that

[6] When using $\alpha_i = 6\sqrt{BV_i}$ as our threshold, we failed to identified any jumps in a number of years.

Figure 1: Daily bipower variation of equally weighted market index versus the identified number of positive (blue) and negative (red) jumps. Jumps where identified using the following thresholds: $\alpha_i = 3\sqrt{BV_i}$ (top panel), $\alpha_i = 4\sqrt{BV_i}$ (middle) and $\alpha_i = 5\sqrt{BV_i}$ (bottom panel).



the stock market has fewer jumps during crisis periods. Since only the jump observations are taken to estimate the non-weighted asymmetric beta in (13) and (14), low number of observations could certainly affect the quality of the estimates. Hence, it is necessary to use the weighted estimators, (17) and (18), in order to reduce the small sample size effect.

4.2. *Estimation Results*

In addition to the weighted estimators of the overall jump beta (12) and the asymmetric betas (17) and (18), we also calculate the high-frequency CAPM beta that is obtained using the OLS regression in the spirit of Andersen et al. (2006)'s realized beta

$$\hat{\beta}_i^{OLS} = \frac{\sum_{j=1}^{m} r_{i,j\Delta} \cdot r_{0,j\Delta}}{\sum_{j=1}^{m} (r_{0,j\Delta})^2}, \qquad i = 1, 2, \ldots, N, \tag{21}$$

where all 5-minute return observations within a year are used to construct $\hat{\beta}_i^{OLS}$. The OLS beta would be able to incorporate the impact of co-movements in the continuous component of individual asset (or portfolio) and the market index.

We calculate the descriptive statistics and the correlations between any pairs of the beta estimates as well as the average monthly returns of each asset, and present them in Table 1. The means of all four estimated beta measures are very close to one, and they are all positively skewed. The three jump beta estimates are statistically insignificant, possibly due to the high heterogeneity between different assets and in different time periods. The monthly return of all stocks are even more dispersed, and negatively skewed.

Following the work of Ang et al. (2006a), we sort stocks in terms of the estimated jump betas, and divide them into quintile portfolios. Table 2 tabulates the annualized realized monthly return on the equally weighted quintile portfolio and all averaged beta estimates for these quintile portfolios. Although our result does not agree with the funding by Ang et al. (2006a) that higher downside beta is associated with higher return, we stress that Ang et al. (2006a) do not separate jump and diffusive components, and that their result could be attributed either to diffusive component of risk, jump component, or both. We separate positive and negative jumps and investigate whether positive and negative jump risk are priced risk factors. We do so through portfolio sorts. We consider contemporaneous relationship between returns and factor risks. The general finding of Table 2 is that stocks with higher sensitivities to aggregate (market) jumps earn lower returns. This is consistent with the result in Cremers et al. (2015) for symmetric jump risk, and applies to our case for the negative jump betas. In fact, our results, show a negative market price of "negative jump" risk. This implies that stocks with high sensitivities to negative market jumps should earn low returns. Cremers et al. (2015) argue that this makes sense economically, as such stocks provide useful hedging opportunities for risk-averse investors, who dislike high systematic negative jump risk. On the contrary, with a positive risk premia for "positive jump" risk, stocks earn higher return.

Table 1: Summary Statistics[a]

	$\hat{\beta}_i^{OLS}$	$\hat{\beta}_i^{d}$	$\hat{\beta}_i^{d+}$	$\hat{\beta}_i^{d-}$	$\bar{r}_i$
Panel A: Descriptive Statistics					
Mean	1.0000	0.9795	0.9852	0.9870	0.0459
Std.Dev.	0.4009	0.7064	1.4190	1.0859	1.0358
Skew.	1.0198	0.8574	4.3356	1.0246	-0.6197
Panel B: Correlation Table					
	$\hat{\beta}_i^{OLS}$	$\hat{\beta}_i^{d}$	$\hat{\beta}_i^{d+}$	$\hat{\beta}_i^{d-}$	$\bar{r}_i$
$\hat{\beta}_i^{OLS}$	1.0000				
$\hat{\beta}_i^{d}$	0.5301**	1.0000			
$\hat{\beta}_i^{d+}$	0.2693**	0.5065**	1.0000		
$\hat{\beta}_i^{d-}$	0.3468**	0.5949**	0.0949**	1.0000	
$\bar{r}_i$	-0.0450**	-0.0205**	-0.0004	-0.0199**	1.0000

[a] This table shows the descriptive statistics (Panel A) and time-series means of pairwise correlations (Panel B) for individual firm continuous betas, β^{OLS}, jump beta, β^{d}, positive and negative jump betas, β^{d+} and β^{d-}, respectively, as well as the (annualized) realized monthly returns, $\bar{r}$. All estimates are obtained using 5-minute data within each calendar month (without overlap). The estimates for all individual stocks and all calendar months are pooled together in calculating the statistics in this table.

[b] Significance levels: † : 10%, $*$: 5%, $**$: 1%.

4.3. *Risk Premia of the Asymmetric Jump Betas*

We use the two-stage regression by Fama and MacBeth (1973) to estimate the risk premia of the asymmetric jump betas. The beta estimates obtained above are taken as the explanatory variables in the cross-sectional regression

$$\bar{r}_{i,s} = \gamma_s \hat{\boldsymbol{\beta}}_{i,s-1} + \epsilon_{i,s}, \qquad i = 1, 2, \ldots, N, \tag{22}$$

and the time index $s = 2, 3, \ldots, 9$ indicates that we estimate (22) for each year separately. The vector of coefficients γ_{s-1} represent the estimated risk premium awarded to each risk factor in $\hat{\boldsymbol{\beta}}_{i,s-1}$. Also note that the realized annual returns of each individual stock on the left hand-side of (22) are from the year after the beta estimates. Hence equation (22) is estimated from 2004 onwards.

Table 3 displays the estimated risk premia γ for each year from 2004 to 2011, as well as their t-statistics in parentheses. We also take the time-series average of γ_s over the 8 years and present it in the bottom panel of Table 3. There are two different model specifications under consideration: model (1) decomposes the overall systematic risk into the continuous and discontinuous factors without taking into account asymmetry, while model (2) separate the positive and negative discontinuous betas. Contrasting these two models helps us detect the existence and degree of asymmetry in the jump risk. The t-statistics in Table 3 underneath the

Table 2: Average returns of sorted quintile portfolios[a]

Portfolio	$\bar{r}_0$	$\hat{\beta}_i^{OLS}$	$\hat{\beta}_i^{d}$	$\hat{\beta}_i^{d+}$	$\hat{\beta}_i^{d-}$
Panel A: Stocks sorted by $\hat{\beta}^d$					
1 - Low	0.0870	0.7700	0.1166	0.1020	0.1977
2	0.0662	0.8323	0.6316	0.6551	0.6578
3	0.0377	0.9515	0.9173	0.9219	0.9233
4	0.0070	1.0842	1.2387	1.2559	1.2280
5 - High	0.0315	1.3618	1.9931	1.9910	1.9279
High-Low	-0.0556	0.5918	1.8764	1.8890	1.7302
Panel B: Stocks sorted by $\hat{\beta}^{d+}$					
1 - Low	0.3427	0.8577	0.3746	-0.4224	0.8414
2	0.0698	0.8470	0.6784	0.5445	0.8565
3	0.0588	0.9434	0.9194	0.9180	0.9304
4	0.0482	1.0743	1.1926	1.3313	1.0509
5 - High	0.0182	1.2775	1.7323	2.5646	1.2558
High-Low	-0.0160	0.4198	1.3577	2.9970	0.4144
Panel C: Stocks sorted by $\hat{\beta}^{d-}$					
1 - Low	0.0925	0.8490	0.4236	0.8265	-0.2896
2	0.0737	0.8502	0.6900	0.8480	0.5441
3	0.0286	0.9487	0.9208	0.9468	0.9148
4	0.0089	1.0664	1.1812	1.0608	1.3231
5 - High	0.0257	1.2858	1.6815	1.2440	2.4423
High-Low	-0.0669	0.4368	1.2579	0.4175	2.7319
Panel D: Stocks sorted by $\hat{\beta}^{d+} - \hat{\beta}^d$					
1 - Low	-0.0045	1.0637	1.0316	-0.0322	1.7415
2	0.0598	0.9679	0.9045	0.7166	1.3591
3	0.0876	0.9255	0.9034	0.9059	0.8970
4	0.0662	0.9077	0.9863	1.1775	0.5420
5 - High	0.0204	1.0656	1.0714	2.1579	0.3954
High-Low	0.0249	0.0018	0.0398	2.1901	-1.3461
Panel E: Stocks sorted by $\hat{\beta}^{d-} - \hat{\beta}^d$					
1 - Low	0.0828	1.0743	1.1327	1.7635	0.1162
2	0.0468	0.9608	0.9469	1.3707	0.7325
3	0.0044	0.9366	0.8986	0.9082	0.8982
4	0.0371	0.9633	0.9426	0.5212	1.1620
5 - High	0.0589	1.0649	0.9764	0.3621	2.0259
High-Low	-0.0234	-0.0094	-0.1563	-1.4014	1.9098

[a] We pool the beta estimates and annualized realized monthly returns for all investable stocks from all months together, and create equally weighted quintile portfolios by sorting them based on the corresponding realized betas: $\hat{\beta}^d$ (Panel A), $\hat{\beta}^{d+}$ (Panel B), $\hat{\beta}^{d-}$ (Panel C), $\hat{\beta}^{d+} - \hat{\beta}^d$ (Panel D), and $\hat{\beta}^{d-} - \hat{\beta}^d$ (Panel E). All reported portfolio characteristics are contemporaneous with the betas used to construct the portfolio sorts.

[b] Significance levels: $\dagger$: 10%, $*$: 5%, $**$: 1%.

estimated risk premia are calculated using heteroskedasticity and auto-correlation consistent standard errors.

Table 3: Fama-MacBeth regressions and risk premia (γ) associated with the estimated risk factors (betas)[a]

Model	(1)		(2)		
	$\hat{\beta}^c$	$\hat{\beta}^d$	$\hat{\beta}^c$	$\hat{\beta}^{d+}$	$\hat{\beta}^{d-}$
2004	-0.2496 (6.6478)	0.0028 (0.1635)	-0.2489 (6.6221)	-0.0143 (1.7649)	0.0108 (0.8146)
2005	-0.1617 (5.7792)	0.0006 (0.0598)	-0.1574 (5.5499)	0.0046 (0.4578)	-0.0023 (1.3969)
2006	-0.2588 (6.4133)	0.0569 (5.2799)	-0.2530 (6.2249)	0.0079 (2.0819)	0.0448 (4.3569)
2007	0.0144 (0.3645)	0.0278 (1.5598)	0.0126 (0.3215)	0.0331 (2.0272)	-0.0010 (0.1770)
2008	-0.2535 (3.1671)	0.0536 (1.5894)	-0.2463 (3.0701)	0.0129 (0.4768)	0.0361 (1.8461)
2009	-0.0157 (0.2810)	0.0164 (1.0018)	-0.0271 (0.4903)	0.0112 (0.8668)	0.0149 (1.5761)
2010	-0.1639 (6.2132)	0.0235 (1.1540)	-0.1488 (5.3502)	0.0185 (1.6087)	-0.0156 (0.8345)
2011	-0.3413 (9.5325)	0.0064 (0.2233)	-0.3389 (9.3413)	0.0049 (0.2451)	-0.0012 (0.0536)
Average	-0.1788 (5.8674)	0.0235 (3.0441)	-0.1760 (6.1028)	0.0098 (1.9633)	0.0108 (1.6983)

[a] This table investigates the cross-sectional pricing of the continuous and jump risks (both symmetric and asymmetric). The sample period is from January 2003 to December 2011. We run Fama-MacBeth regressions of annual returns on lagged one-period realized betas. The t-statistics are given in parentheses underneath the coefficient estimates.

Table 3 shows that in both model specifications, the continuous beta receives a negative risk premium, which is highly significant in most years except 2007 and 2009. This result is consistent with the finding of Alexeev et al. (2015). On the other hand, the jump beta always carries a positive risk premium. The average γ over the entire sample period for all beta estimates are statistically significant. In model (2) both the positive and negative jump betas have positive risk premia pointing to the importance of distinguishing between these two risk factors. As expected, the negative jump beta receive slightly higher premium than the positive jump beta on average.

In this section we showed that individual stock betas can vary greatly. Experienced investors will always consider allocating wealth in a number of assets in favour of investing all wealth in a single security. As the number of holdings increase, the range of portfolios betas will become more limited compared to betas of individual securities, eventually converging to unity for an equiweighted market benchmark. Given that investing in all securities listed on a abroad market benchmark may not be informationally or cost effective, finding the optimal number of holdings in a portfolio to mitigate most jump risk (positive or negative) is of significant importance. We discuss this in our next section.

5. Portfolio Simulation

Ever since the development of Modern Portfolio Theory by Harry Markowitz[7] the quest for optimal portfolio size that reduces diversifiable risk has been an integral part of portfolio choice literature. The concept of diversification is simple: the level of return variability falls as the number of holdings in a portfolio increases.[8]

The availability of high-frequency data allowed new insights into portfolio diversification. For example, Silvapulle and Granger (2001) investigate asset correlations at the tails of return distributions and discusses the implications for portfolio diversification. Bollerslev et al. (2013) examine the relationship between jumps in individual stocks and jumps in a market index. The authors find that jumps occur more than three times as often at the individual stock level compared to jump occurrence in an aggregate equiweighted index constructed from the same stocks. This may point to the fact that jumps are diversifiable. In fact, Pukthuanthong and Roll (2014) do consider implications of jumps for international diversification. They find that cross-country diversification is less effective if jumps are frequent, unpredictable, and strongly correlated. Studies directly investigating optimal portfolio size using high-frequency data only recently started to emerge (e.g., Alexeev and Dungey (2014)).

In this section, using extensive portfolio simulation techniques, we evaluate the variability reduction of portfolio jump betas as the number of holdings in these portfolios increase.[9] We analyse the spread of estimated betas in equiweighted randomly constructed portfolios of different sizes focusing on the difference in convergence between negative and positive jump betas as the number of holdings in portfolios increases. For investors, the knowledge that individual stocks respond differently to the positive and negative extreme events is likely to be a valuable addition to in developing portfolio risk management strategies. However, investors who hold several S&P500 stocks may be rightfully concerned with the overall exposure of their portfolios to systematic jump risk. Moreover, investors exhibit different attitudes towards extreme gains and extreme losses. We assert that if an asset tends to move downwards in a declining market more than it moves upward in a rising market, such asset is unattractive to hold, especially during market downturns when wealth of investors is low.

Using a 12-month estimation window, for each year from 2003 to 2011 we construct 1,000 random equally weighted portfolios with a number of holdings ranging from 1 to 200. For

[7] See Markowitz (1952) and Markowitz (1959).

[8] This is true for any coherent measure of risk (e.g., Artzner et al. (1997) and Artzner et al. (1999)).

[9] Detailed description of the techniques employed is provided in the appendix. See Algorithm 1 on page 26.

each of these portfolios we estimate several systematic discontinuous risks. We estimate β^d without taking into account the asymmetry, as well as β^{d+} and β^{d-}. We assess the stability of the systematic portfolio risks by analyzing the inter-quartile ranges of the beta distributions as the number of stocks in portfolio increases. Defined as the difference between two percentiles, 75% and 25%, the inter-quartile range (IQR) is a stable measure that is robust to outliers. That is,

$$IQR_{(n)} \equiv EDF^{-1}_{(n)}(.75) - EDF^{-1}_{(n)}(.25), \tag{23}$$

where $EDF_{(n)}$ is the empirical distribution function of the estimated betas (β^d, β^{d+} or β^{d-}) for randomly drawn n-stock portfolios.

Figure 2: Distribution of β^{d-} (top panel) and β^{d+} (bottom panel) across portfolio sizes. Red points represent maxima and minima, black lines represent interpercentaile range from 2.5% to 97.5%, blue lines denote interguantile range and the black circles are the midians of the distributions. We use 2008 data in estimating the results in this figure.

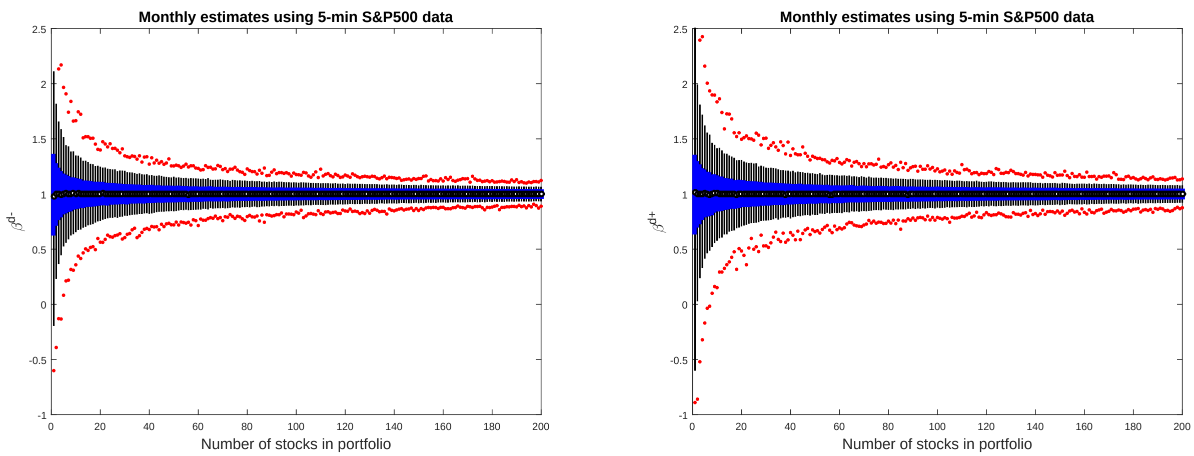


Figure 2 depicts the typical distributions of β^{d-} and β^{d+} for equally weighted randomly drawn portfolios of $n = 1, \ldots, 200$ stocks. Since these central ranges are dependent on the particular time period analyzed, For each year, we normalize the IQR for the n-stock portfolios and represent it as a fraction of the IQR of the single-stock portfolio. The normalized IQRs, or $IQR_{(n)}/IQR_{(1)}$, of $\hat{\beta}^d$, β^{d+} and β^{d-} for year 2008 are depicted in Figure 3 for $n = 1, \ldots, 200$. Since the market index is an equally weighted portfolio consisting of all investible stocks, and is thus unique, it has $IQR_{(N)} = 0$. As a result, the normalized IQRs in Figure 3 are bounded between 0 and 1. We find that the difference among the normalized IQRs for the three different betas are more pronounced during periods of high volatility[10] and for more extreme events (consider the top vs the bottom panel in Figure 3).

Figure 3 shows that the IQR of portfolio jump betas decrease substantially as the num-

[10] Results for years other than 2008 are omitted for brevity.

Figure 3: Normalized IQR of betas across portfolio sizes. Both panels display results based on year 2008. The top panel displays results based on a threshold $\alpha_i = 3\sqrt{BV_i}$ (in Equation 20) and the bottom panel is based on $\alpha_i = 5\sqrt{BV_i}$. As can be observed from the figures below, the asymmetry in signed betas is more pronounced when more extreme events are considered. The optimal number of holdings in a portfolio is determined at the intersection of the normalised IQR curve (red, blue and black) with the desired level of variability reduction (in this case 0.2 denoted by horizonal purple line).

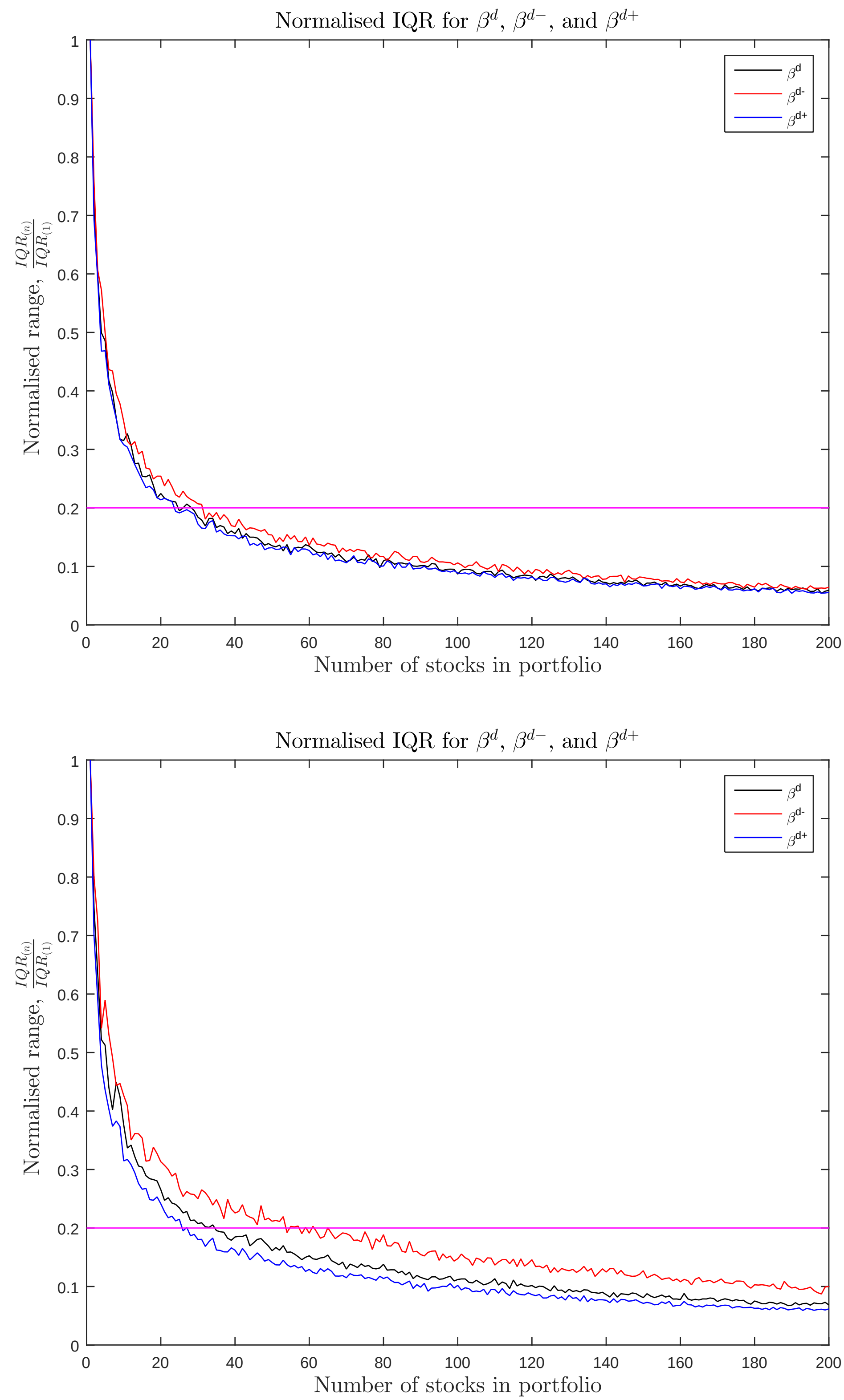


Table 4: Portfolio sizes, n, required to reduce normalised IQR, $IQR_{(n)}/IQR_{(1)}$ to 0.2.[a]

Year(s)	$\alpha_i = 3\sqrt{BV_i}$			$\alpha_i = 4\sqrt{BV_i}$			$\alpha_i = 5\sqrt{BV_i}$		
	β^d	β^{d-}	β^{d+}	β^d	β^{d-}	β^{d+}	β^d	β^{d-}	β^{d+}
2003	30	31	30	36	38	35	36	39	35
2004	31	33	31	36	37	36	34	40	33
2005	30	31	29	35	36	36	35	40	33
2006	30	30	29	34	35	34	33	39	32
2007	28	29	28	32	33	32	32	39	30
2008	25	32	24	29	36	25	35	54	26
2009	30	31	28	34	36	33	34	41	31
2010	31	31	31	35	37	34	35	42	32
2011	24	30	23	29	40	28	34	51	27
2003-2011	29	31	28	34	36	32	35	41	29

[a] This table shows the number of holding in a portfolio required to stabilize portfolio betas, that is β^d, β^{d-} or β^{d+}. We include the results for a number of severities of extreme events (threshold α used to identify jumps). We consider level of $IQR_{(n)}/IQR_{(1)} = 0.2$ as appropriate: the IQR of of individual stock betas can be reduced fivefold if a portfolio is constructed with at least a number of stocks outlined in the table. For example, in 2003, for $\alpha_i = 5\sqrt{BV_i}$, 39 stocks are required to reduce portfolio sensitivity to negative jumps by a factor of 5 compared to when a single stock is held. To get the same reduction in sensitivity to positive shocks, 35 stocks will suffice. This is in contrast with year 2008, where as many as 54 stocks are required to reduce sensitivity to negative events with only 26 stocks needed to get the same reduction in sensitivity of portfolio to positive events. Note that the asymmetry in results is more pronounced for more extreme events (e.g., larger threshold level α).

ber of stocks, n, in the portfolio increases. Using the normalized IQRs enables us to contrast the required portfolio sizes at different periods of time, in order to achieve the same proportional reduction in IQRs of beta for these portfolios relative to the beta spreads of individual securities. Table 4 outlines the required portfolio sizes to reduce the normalized IQR five fold, respectively. It is evident that during periods of market distressed characterised by high volatility, the number of stocks required to reduce IQR and stabilise negative jump beta is considerably higher than in the less volatile periods. A reduction in a spread of positive jump risk component in majority of portfolios by a factor of 5 relative to the positive jump beta spread of individual securities requires roughly 1/4 less stocks during these volatile periods with no substantial difference during normal market periods. During the periods of normal market activity, the market seems to be indifferent to the distinction between negative and positive jump risk. The difference in the number of stocks required in order to achieve the same proportionate reduction in beta spreads for negative vs positive jumps only differs substantially during 2008 and 2011. Ignoring the asymmetry in sensitivities to negative vs positive market jumps may result in under-diversification of portfolios and increased exposure to extreme negative market shifts. For example, consider portfolio sizes optimized for the most extreme jumps identified with $\alpha_i = 5\sqrt{BV_i}$ threshold (last three columns in Table 4). If we ignore the asymmetry, optimal portfolio sizes are 35 and 34 in 2008 and 2011 respectively. However, if investor is concerned with extreme negative shifts in the market, it would be advisable to hold 54 and 51 stocks instead, to reduce the sensitivity of portfolio returns to extreme negative market shifts compared to a single stock portfolio.

6. Conclusion

In this paper we studied jump dependence of two processes using high-frequency observations concentrating only on segments of data around a few outlying observations that are informative for the jump inference. In particular, we studied the relationship between jumps of a process for an asset (or portfolio of assets) and an aggregate market factor and analysed the co-movement of the jumps in these two processes. Given the predominance of factor models in asset pricing applications, we focused on a linear relationship between the jumps and assessed the sensitivity of jumps in (portfolios of) assets to jumps in the market.

We show that investors care differently about downside losses as opposed to upside gains and demand additional compensation for holding stocks with high sensitivities to downside market movements. We estimated jump betas for the negative and positive market shifts and investigated the implications for portfolio risk management using upside and downside jump betas. We assert that if an asset tends to move downwards in a declining market more than it moves upward in a rising market, such asset is unattractive to hold, especially during market downturns when wealth of investors is low.

In the context of portfolio of assets, we investigated to what extend the downside and upside jump risk can be diversified. This has important implications for pricing of jump risk and can have a direct impact on investors' decision-making. We found that ignoring the asymmetry in sensitivities to negative versus positive market jumps results in under-diversification of portfolios and increases exposure to extreme negative market shifts. These results are more pronounced for the most extreme events and during periods of very high market volatility.

References

Aït-Sahalia, Yacine and Jean Jacod, "Analyzing the Spectrum of Asset Returns: Jump and Volatility Components in High Frequency Data," *Journal of Economic Literature*, 2012, *50* (4), 1007–50.

Alexeev, Vitali and Mardi Dungey, "Equity portfolio diversification with high frequency data," *Quantitative Finance*, nov 2014, *15* (7), 1205–1215.

_, _, and Wenying Yao, "Time-varying continuous and jump betas: The role of firm characteristics and periods of stress," Technical Report, University of Tasmania Working Paper series 2015.

Andersen, Torben G., Tim Bollerslev, and Francis X. Diebold, "Roughing It Up: Including Jump Components in the Measurement, Modeling, and Forecasting of Return Volatility," *Review of Economics and Statistics*, nov 2007, *89* (4), 701–720.

_, _, _, and Ginger Wu, *Realized Beta: Persistence and Predictability*, Vol. 20 of *Econometric Analysis of Financial and Economic Time Series (Advances in Econometrics)*, Emerald Group Publishing Ltd,

Ang, Andrew, Joseph Chen, and Yuhang Xing, "Downside Risk," *Review of Financial Studies*, 2006, *19* (4), 1191–1239.

_, Robert J. Hodrick, Yuhang Xing, and Xiaoyan Zhang, "The Cross-Section of Volatility and Expected Returns," *The Journal of Finance*, jan 2006, *61* (1), 259–299.

Artzner, Philippe, Freddy Delbaen, Jean-Marc Eber, and David Heath, "Thinking Coherently," *Risk*, 1997, *10*, 68–71.

_, _, _, and _, "Coherent Measures of Risk," *Mathematical Finance*, 1999, *9* (3), 203–228.

Bandi, R.M and J.R. Russell, "Separating Microstructure Noise from Volatility," *Journal of Financial Economics*, 2006, *79*, 655–692.

Barndorff-Nielsen, Ole E. and Neil Shephard, "Power and Bipower Variation with Stochastic Volatility and Jumps," *Journal of Financial Econometrics*, 2004, *2* (1), 1–37.

_ and _ , "Econometrics of Testing for Jumps in Financial Economics Using Bipower Variation," *Journal of Financial Econometrics*, 2006, *4* (1), 1–30.

Barro, Robert J., "Rare Disasters and Asset Markets in the Twentieth Century," *The Quarterly Journal of Economics*, 2006, *121* (3), 823–866.

Bates, David S., "The market for crash risk," *Journal of Economic Dynamics and Control*, jul 2008, *32* (7), 2291–2321.

Black, Angela, Jing Chen, Oleg Gustap, and Julian M Williams, "The Importance of Jumps in Modelling Volatility during the 2008 Financial Crisis," Technical Report July 2012.

Bollerslev, Tim, Tzuo Hann Law, and George Tauchen, "Risk, Jumps, and Diversification," *Journal of Econometrics*, May 2008, *144* (1), 234–256.

_ , Viktor Todorov, and Sophia Zhengzi Li, "Jump Tails, Extreme Dependencies, and the Distribution of Stock Returns," *Journal of Econometrics*, 2013, *172* (2), 307 – 324. <ce:title>Latest Developments on Heavy-Tailed Distributions</ce:title>.

Cremers, Martijn, Michael Halling, and David Weinbaum, "Aggregate Jump and Volatility Risk in the Cross-Section of Stock Returns," *The Journal of Finance*, mar 2015, *70* (2), 577–614.

Dungey, Mardi, Matteo Luciani, and David Veredas, "Ranking Systemically Important Financial Institutions," Working Papers ECARES 2013/130530, ULB – Universite Libre de Bruxelles 2012.

_ , Michael McKenzie, and Vanessa Smith, "Empirical Evidence on Jumps in the Term Structure of the US Treasury Market," *Journal of Empirical Finance*, 2009, *16*, 430–445.

Epps, Thomas W., "Comovements in Stock Prices in the Very Short Run," *Journal of the American Statistical Association*, 1979, *74* (366), 291–298.

Fama, Eugene F. and James D. MacBeth, "Risk, Return, and Equilibrium: Empirical Tests," *Journal of Political Economy*, 1973, *81* (3), 607–636.

Fisher, Lawrence, "Some New Stock-Market Indexes," *The Journal of Business*, 1966, *39* (1), 191–225.

Gabaix, Xavier, "Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance," *The Quarterly Journal of Economics*, 2012, *127* (2), 645–700.

Guo, Hui, Kent Wang, and Hao Zhou, "Good Jumps, Bad Jumps, and Conditional Equity Premium," *Working Paper*, February 2015.

Jacod, Jean, "Asymptotic properties of realized power variations and related functionals of semimartingales," *Stochastic Processes and their Applications*, 2008, *118* (4), 517 – 559.

Lahaye, Jérôme, Sébastien Laurent, and Christopher J. Neely, "Jumps, cojumps and macro announcements," *Journal of Applied Econometrics*, 2011, *26* (6), 893–921.

Lettau, Martin, Matteo Maggiori, and Michael Weber, "Conditional risk premia in currency markets and other asset classes," *Journal of Financial Economics*, 2014, *114* (2), 197 – 225.

Li, Jia, Viktor Todorov, and George Tauchen, "Jump Regressions," *Working Paper*, 2015.

_ , _ , _ , and Rui Chen, "Mixed-scale Jump Regressions with Bootstrap Inference," Working Paper, Duke University March 2015.

Liu, Jun, Jun Pan, and Tan Wang, "An Equilibrium Model of Rare-Event Premia and Its Implication for Option Smirks," *Review of Financial Studies*, 2005, *18* (1), 131–164.

Mancini, Cecilia, "Disentangling the Jumps of the Diffusion in a Geometric Jumping Brownian Motion," *Giornale dell'Istituto Italiano degi Attuari*, 2001, *64*, 19–47.

_ , "Non-parametric Threshold Estimation for Models with Stochastic Diffusion Coefficient and Jumps," *Scandinavian Journal of Statistics*, jun 2009, *36* (2), 270–296.

_ **and Roberto Renò**, "Threshold estimation of Markov models with jumps and interest rate modeling," *Journal of Econometrics*, jan 2011, *160* (1), 77–92.

Markowitz, Harry, "Portfolio Selection," *The Journal of Finance*, mar 1952, *7* (1), 77.

Markowitz, H.M., *Portfolio Selection: Efficient Diversification of Investments*, New York: John Wiley & Sons, 1959.

_ , *Portfolio Selection: Efficient Diversification of Investments* Cowles Commission for Research in Economics: Monographs, Yale University Press, 1971.

Patton, Andrew J. and Michela Verardo, "Does Beta Move with News? Firm-specific Information Flows and Learning About Profitability," *Review of Financial Studies*, 2012, *25*, 2789–2839.

Pukthuanthong, K. and R. Roll, "Internationally Correlated Jumps," *Review of Asset Pricing Studies*, nov 2014, *5* (1), 92–111.

Rietz, Thomas A., "The equity risk premium a solution," *Journal of Monetary Economics*, 1988, *22* (1), 117 – 131.

Santa-Clara, Pedro and Shu Yan, "Crashes, Volatility, and the Equity Premium: Lessons from S&P500 Options," *Review of Economics and Statistics*, may 2010, *92* (2), 435–451.

Silvapulle, P. and C.W.J. Granger, "Large returns, conditional correlation and portfolio diversification: a value-at-risk approach," *Quantitative Finance*, 2001, *1* (5), 542–551.

Todorov, Viktor and Tim Bollerslev, "Jumps and betas: A new framework for disentangling and estimating systematic risks," *Journal of Econometrics*, 2010, *157* (2), 220–235.

Algorithm 1 CONSTRUCTING SIMULATED PORTFOLIOS AND OBTAINING RESULTS.

1. For a year τ, using data with sampling frequency identified from a signature plot of $BV_i - IV_i$, for each constituent of the S&P500 Index estimate:
   - symmetric jump beta, β^d
   - upside jump beta, β^{d+}
   - downside jump beta, β^{d-}
   - calculate *relative* upside and downside jump betas, $\beta^{d+} - \beta^d$ and $\beta^{d+} - \beta^d$ respectively. This is similar to Ang et al. (2006a) for continuous βs. Relative values are helpful when comparing jump βs among multiple assets since jump βs have different average magnitudes. Relative jump βs allow to focus on relative deviations of upside and downside jump betas from the symmetric jump beta values.
2. Given the distribution of estimated values of β^d, β^{d+}, and β^{d-} as well as relative βs in Step 1 above, calculate:
   - interquartile range, $IQR_{(1)} \equiv PR_{(1),75\%-25\%} = EDF^{-1}_{(1)}(.75) - EDF^{-1}_{(1)}(.25)$ where $EDF_{(1)}$ is the empirical distribution function of the estimated values in Step 1 and subscript $_{(1)}$ denoting that these values are estimated for individual stocks (or, equivalently, single-stock portfolios).
3. Randomly select n stocks out of all available stocks, N, without replacement;
4. Calculate equally weighted portfolio returns based on the selection in Step 3 above;
5. Estimate β^d, β^{d+}, β^{d-} and calculate relative βs using portfolio returns in Step 4;
6. Repeat Steps 3-5 1,000 times
7. Based on results of Step 6 (distribution of estimated jump β values for 1,000 randomly constructed n-stock portfolios, find
   - mean and median values
   - $IQR_{(n)}$ and $IPR_{(n),95\%-5\%}$, defined as above with subscript $_{(n)}$ denoting that these values are estimated for n-stock portfolios.
8. Repeat Steps 3-7 for each $n = 2, \ldots, N$ (where $N = 500$ is the maximum number of stocks in S&P500 constituent set)
9. Calculate normalised interquartile and interpercentile ranges, $IQR_{(n)}/IQR_{(1)}$, for each $n = 2, , N$
10. Determine the optimal size of portfolio, n^*, such that $IQR_{(n)}/IQR_{(1)} = c$, where the threshold level is set at $c = 0.2$
11. Repeat Steps 1-10 for each year τ in the period 2003-2011.