# A dynamic component model for forecasting high-dimensional realized covariance matrices

L.Bauwens, M.Braione, G.Storti

*Université catholique de Louvain, CORE, luc.bauwens@uclouvain.be*
*Université catholique de Louvain, CORE, manuela.braione@uclouvain.be*
*University of Salerno, DISES, storti@unisa.it*

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**Abstract**

The Multiplicative MIDAS Realized DCC (MMReDCC) model of Bauwens et al. [5] decomposes the dynamics of the realized covariance matrix of returns into short-run transitory and long-run secular components where the latter reflects the effect of the continuously changing economic conditions. The model allows to obtain positive-definite forecasts of the realized covariance matrices but, due to the high number of parameters involved, estimation becomes unfeasible for large cross-sectional dimensions. Our contribution in this paper is twofold. First, in order to obtain a computationally feasible estimation procedure, we propose an algorithm that relies on the maximization of an iteratively re-computed moment-based profile likelihood function. We assess the finite sample properties of the proposed algorithm via a comprehensive simulation study. Second, we propose a bootstrap procedure for generating multi-step ahead forecasts from the MMReDCC model. In an empirical application on realized covariance matrices from 50 equities over the period 1997-2008, we find that the MMReDCC not only statistically outperforms the selected benchmarks in-sample, but also substantially improves the out-of-sample ability to generate accurate forecasts of the realized covariances over longer horizons.

*Keywords:* Realized covariance, dynamic component models, MIDAS, targeting, Model Confidence Set.

## 1. Introduction

Building models for predicting the volatility of high dimensional portfolios is important in risk management and asset allocation. Considerable work on time-varying covariances in large dimensions includes the constant conditional correlation (CCC) model of Bollerslev [8], where the volatilities of each asset are allowed to vary through time but the correlations are time invariant, the RiskMetrics by [27] and the DECO model by Engle and Kelly [18], who allow correlations to change over time and can be easily applied in vast dimensions. Recently, Andersen et al. [2], Barndorff-Nielsen and Shephard [4] and Barndorff-Nielsen et al. [3], among others, opened up a new channel for increasing the precision of covariance matrix estimates and forecasts by exploiting the information of high frequency asset returns. This development has motivated many researchers to investigate models directly fitted to series of realized covariance matrices (see Gouriéroux et al. [22], Jin and Maheu [25] and Chiriac and Voev [11], among others).

Despite the superiority of these models, illustrated for example by Hautsch et al. [24], there still remain technical and practical challenges one needs to deal with when constructing covariance matrix forecasts for realistic high-dimensional systems. First and foremost, the well-known "curse of dimensionality" problem, implying that the number of parameters grows as a power function of the cross-sectional model dimension. In order to save parameters, a simple solution is represented by the so called covariance (or correlation) targeting approach of Engle [15], which consists in pre-estimating the constant intercept matrix in the model specification by linking it to the unconditional covariance matrix of returns. This method can be applied under the stationarity assumption of the model and is one of the most widely employed techniques to simplifying parameter estimation and reducing the computational burden when the numerical maximization of the likelihood function becomes difficult.

Recently, Bauwens et al. [5] investigated a wide class of multivariate models that simultaneously account for short and long-term dynamics in the conditional (co)volatilities and correlations of asset returns, in line with the empirical evidence suggesting that their level is changing over time as a function of the economic conditions (see, among others, Engle et al. [16]). Herein we focus on the Multiplicative MIDAS Realized DCC (MMReDCC) model, whose main ingredients are a multiplicative component structure, a Mixed Data Sampling (MIDAS) filter to modeling the secular dynamics and a DCC-type parameterization for the short term component, directly inspired by the multivariate GARCH literature.[1] The extensive out-of-sample forecasting comparison performed by Bauwens et al. [5], although not identifying a clear winner, shows that the MMReDCC model gives remarkably good performances in important financial applications such as Value-at-Risk forecasting and portfolio allocation. However, their results are limited to a relatively low dimensional setting (10 assets) and to a short-term forecasting horizon (1 day).

This paper extends the work by Bauwens et al. [5] along these directions: estimation for high-dimensional systems and multi-step forecasting. We contribute to the first line of research developing of a computationally feasible procedure for the estimation of vast dimensional MMReDCC models. Under this respect, it is important to remark that, although the introduction of a dynamic secular component in the structure of the model adds a major element of flexibility and enables to obtain more accurate forecasts than standard models reverting to constant mean levels (see Bauwens et al. [5]), it also dramatically increases the number of parameters to be estimated. Specifically, the long term component incorporates a scale intercept matrix with number of parameters equal to $n(n+1)/2$, where

[1] We refer to Engle [14] and Ghysels et al. [20] as leading references for detailed discussions of the DCC model and MIDAS regressions.

$n$ denotes the number of assets. In a vast dimensional framework, this quickly translates into the impossibility of estimating the model.

Therefore, we propose to overcome this estimation issue by proposing an iterative procedure inspired by the covariance targeting of Engle [15]. More precisely, based on a Method of Moments estimator, we profile out the parameters of the intercept matrix and iteratively maximize the likelihood in terms of the other parameters of interest. We refer to this as the Iterative Moment-Based Profiling (IMP) estimator, as opposed to the Quasi Maximum Likelihood (QML) estimator which directly maximizes the likelihood over the full parameter vector. It is worth noting that the proposed estimation procedure can be considered a *switching algorithm* in the sense discussed by Boswijk [9] and Cubadda [13] since the maximization of the overall likelihood is obtained by switching between optimizations over different blocks of parameters. This idea has a long standing tradition in the econometric analysis of time series. A simple, well known example of switching algorithm is given by the Cochrane-Orcutt iterative estimation procedure. Compared to conventional switching algorithms, the procedure that is here implemented incorporates an additional targeting step. In particular, it reduces the dimension of the optimization problem to be solved by concentrating out some of the parameters, the elements of the intercept matrix, by means of an iteratively re-computed moment-based estimator. A comprehensive simulation study is performed to assess the finite-sample properties of the proposed estimator which is found to deliver unbiased estimates and to quickly converge, as no more than three iterations are required in general.

The second relevant contribution of the paper is the development of a resampling based procedure for the generation of multi-step ahead forecasts of the realized covariance matrices. The multiplicative component structure of the MMReDCC model makes the derivation of a closed-form expression for the h-step predictor troublesome. Hence, to solve this issue we use a distribution-free procedure based on residual bootstrap. The bootstrap has been a standard tool for generating multi-step forecasts from non-linear and non-Gaussian time series models for more than two decades (see e.g. Clements and Smith [12]). Its use has been later extended to univariate volatility modeling (see e.g. Pascual and Ruiz [29]; Shephard and Sheppard [31]). More recently, Fresoli and Ruiz [19] have proposed a simple resampling algorithm that makes use of residual bootstrap to compute multi-step forecasts from DCC models. The bootstrap procedure which is implemented in this paper builds on the work of Fresoli and Ruiz [19] but the algorithm is adapted to the dynamic modeling of realized covariance matrices.

Finally, the results of two different applications to real data are presented and discussed. In the first one, we focus on a low dimensional setting (10 assets), in which both the IMP and one-step QML estimation procedures are feasible, and compare the estimates obtained by means of both algorithms. In the second application the MMReDCC model estimated by the IMP is used to generate forecasts of the realized covariance matrix, up to 20 days ahead, and compared to existing benchmarks not accounting for short and long term (co)volatility dynamics. For the dataset considered, in correspondence of a forecasting horizon equal to 1 day, the MMReDCC model is outperformed by the benchmarks while it dominates for longer horizons up to 10 days. Over the longest horizon considered (20 days), there is no clear winner and, hence, no practical advantage derives from adopting a component structure for realized covariances.

The remainder of the paper is organized as follows. Section 2 briefly recalls the structure of the MMReDCC model and explains the causes of the curse of dimensionality issue. In Section 3 we introduce the IMP algorithm and in Section 4 we discuss the results of a Monte Carlo experiment aimed at assessing the finite sample statistical properties of the proposed estimation procedure. Section 6 presents the empirical results from the in-sample estimation comparison and the out-of-sample forecasting exercise. Section 7 concludes with some final remarks.

## 2. The MMReDCC model

Let $C_t$ be a $n \times n$ positive definite and symmetric (PDS) realized estimator of the latent integrated covariance (IC) matrix of daily returns. In the following, unless otherwise stated, we will refer to $C_t$ as the realized covariance (RC), although any other consistent PDS estimator could be used. Conditionally on the set consisting of all relevant information up to and including day $t-1$, $C_t$ is assumed to follow a $n$-dimensional central Wishart distribution:

$$C_t | I_{t-1} \sim W_n(\nu, S_t/\nu), \quad \forall t = 1, \ldots, T \tag{1}$$

where $\nu$ ($> n-1$) is the degrees of freedom parameter and $S_t$ is the PDS conditional mean matrix of order $n$. Under the assumption of absence of microstructure noise and other biases (see Barndorff-Nielsen and Shephard [4]), $S_t$ represents the conditional covariance matrix of returns, which is our object of interest.

In the MMReDCC model, $S_t$ is designed to directly capture the long run movements in the levels around which realized (co)variances (and by extension, correlations) fluctuate from day to day. To this extent, the model features a multiplicative decomposition of the conditional covariance matrix $S_t$ into a smoothly varying or *secular* component $M_t = L_t L_t'$ and a short-lived component $S_t^\star$, such that $S_t$ can be rewritten as $S_t = L_t S_t^\star L_t'$, where the matrix square root $L_t$ can be obtained by a Cholesky factorization of $M_t$. These components can then be modeled separately.

First, the secular component is specified parametrically and extracted by means of a MIDAS filter assumed to be a weighted sum of $K$ lagged realized covariance matrices over a long horizon, where the number of lags spanned in the MIDAS specification is usually chosen to minimize the trade-off between the highest in-sample likelihood value and the number of observations lost to initialize the filter. It is expressed as

$$M_t = \Lambda + \theta \sum_{k=1}^{K} \phi_k(\omega) C_{t-k}. \tag{2}$$

In the right hand side of Eq.(2), the first term $\Lambda$ is a $n \times n$ symmetric and semi-positive definite matrix of constant parameters, $\theta$ is a positive scalar and $\phi_k(\cdot)$ is a weight function parametrized according to the restricted Beta polynomial

$$\phi_k(\omega) = \frac{\left(1-\frac{k}{K}\right)^{\omega-1}}{\sum_{j=1}^{K}\left(1-\frac{j}{K}\right)^{\omega-1}}$$

The scalar parameter $\omega$ dictates the shape of the function and in order to achieve a time-decaying pattern of the weights, it is constrained to $\omega > 1$. For identification it also holds $\sum_{k=1}^{K}\phi_k(\omega) = 1$.

Second, the dynamics of the short term component $S_t^\star$ is specified according to a scalar DCC parametrization that enables a separate treatment of conditional volatilities and correlations, thus allowing for a high degree of flexibility. Therefore, assuming that $S_t^\star = D_t^\star R_t^\star D_t^\star$, where $D_t^* = diag\{S_t\}^{*1/2}$, their scalar specifications correspond to the following equations:

$$S_{ii,t}^\star = (1-\gamma_i-\delta_i) + \gamma_i C_{ii,t-1}^\star + \delta_i S_{ii,t-1}^\star, \qquad \forall i = 1,\dots,n \tag{3}$$

$$R_t^\star = (1-\alpha-\beta)I_n + \alpha P_{t-1}^\star + \beta R_{t-1}^\star, \tag{4}$$

where $\boldsymbol{\gamma} = \{\gamma_1, ..., \gamma_n\}$, $\boldsymbol{\delta} = \{\delta_1, ..., \delta_n\}$, $P_t^\star = (diag\{C_t^\star\})^{-1/2} C_t^\star (diag\{C_t^\star\})^{-1/2}$ and $C_t^\star = L_t^{-1} C_t (L_t')^{-1}$. The matrix $C_t^\star$ is the realized covariance matrix purged of its long term component, and the matrix $P_t^\star$ is the corresponding short term realized correlation matrix. Mean reversion to unity in Eq.(3) and to an identity matrix in Eq.(4) is needed for identification of the different components.

The parameters can be estimated by maximizing the following Wishart (quasi) log-likelihood function in one step:

$$\ell_T(\boldsymbol{\psi}) = -\frac{1}{2}\sum_{t=1}^{T}\left\{\log|S_t(\boldsymbol{\psi})| + \mathrm{tr}\left[S_t(\boldsymbol{\psi})^{-1}C_t\right]\right\}. \tag{5}$$

The finite-dimensional parameter vector $\boldsymbol{\psi} = \{vech(\Lambda), \theta, \omega, \boldsymbol{\gamma}, \boldsymbol{\delta}, \alpha, \beta\}$[2], has length $\{n_\Lambda + 2n + 4\}$ where $n_\Lambda = n(n+1)/2 = O(n^2)$ denotes the number of unique parameters included in the intercept matrix $\Lambda$ of Eq.(2). It is not difficult to guess that, as $n$ increases, the curse of dimensionality problem quickly worsens, leading to the number of parameters listed in the first two rows of Table 1. Observe that estimation becomes already cumbersome after $n = 20$ and almost impossible between 50 and 100, as the dimension of $\boldsymbol{\psi}$ is likely to be higher than the number of available observations in the time series.

On the other hand, the last row of Table 1 shows that an intuitive way to keep the model tractable is to avoid estimating the parameters of the matrix $\Lambda$. This would be sufficient to reduce the order to $2n + 4 = O(n)$, thus making the model estimable also for large $n$.

[2]Note that $\boldsymbol{\psi}$ do not include the degree of freedom parameter $\nu$, as the first order conditions for the estimation of the parameter vector $\boldsymbol{\psi}$ do not depend on $\nu$ by linearity in $\nu$ (see [5]).

**Table 1:** Model parameters

| | $n = 5$ | $n = 10$ | $n = 20$ | $n = 50$ | $n = 100$ |
|---|---|---|---|---|---|
| $n_\Lambda$ | 15 | 55 | 210 | 1275 | 5050 |
| $\psi$ | 29 | 79 | 254 | 1379 | 5254 |
| $\tilde{\psi}$ | 14 | 24 | 44 | 104 | 204 |

Note: Entries report number of model parameters to be estimated as the cross-sectional dimension $n$ increases; $n_\Lambda$ denotes the number of unique parameters contained in the $\Lambda$ matrix, $\psi$ denotes the full vector of model parameters and $\tilde{\psi}$ the vector of parameters excluding $n_\Lambda$.

In the following section we put forward a feasible estimation procedure that aims at overcoming the direct estimation of the long term component intercept matrix, thus crucially mitigating the computational complexity of the model.

## 3. An Iterative Moment based Profiling (IMP) algorithm

In this section we discuss an iterative procedure for fitting the MMReDCC model to large dimensional datasets. The basic idea underlying the proposed algorithm is to concentrate out of the likelihood maximization the parameters of the intercept matrix $\Lambda$ using a technique that builds upon the covariance targeting discussed in Pedersen and Rahbek [30] for BEKK and Engle et al. [17] for DCC models. First of all, notice that from Eq.(2) and the following relation

$$\Lambda = E(M_t) - \theta \sum_{k=1}^{K} \phi_k(\omega) E(C_{t-k}),$$

a moment based estimator of the $\Lambda$ intercept matrix is

$$\hat{\Lambda} = \frac{1}{T} \sum_{t=1}^{T} \left[ M_t - \theta \sum_{k=1}^{K} \phi_k(\omega) C_{t-k} \right]. \tag{6}$$

Obviously, given the latent nature of $M_t$, the estimator in Eq.(6) cannot be computed in practice and hence the covariance targeting approach cannot be applied in the usual way. It is worth noting that, if $L_t$ and $S_t^\star$ were assumed to be independent, given $E(S_t^\star) = I_n$, it would hold that $E(C_t) = E(M_t)$, implying that an asymptotically equivalent version of Eq.(6) could be explicitly computed replacing $M_t$ by $C_t$. However, this is not the approach we intend to pursue here, since the assumption of independence of the short and long term sources is difficult to justify and would result in a rather counterintuitive and arbitrary constraint. Hence, we propose to adopt a different method.

By noting from Eq.(6) that no estimate of $\Lambda$ makes sense regardless of the value of $(\theta, \omega)$, we make this dependence explicit and obtain an estimate of $\Lambda$ as a function of

$(\theta, \omega)$, i.e. $\hat{\Lambda}(\theta, \omega)$. In this way, a different estimate of $\Lambda$ is required for each different value of the other two parameters. Therefore, by replacing $\hat{\Lambda}(\theta, \omega)$ for $\Lambda$ in the Wishart QML function stated in Eq.(5), the following moment based QML approximation obtains:

$$\tilde{\ell}_T(\tilde{\boldsymbol{\psi}}) = -\frac{1}{2}\sum_{t=1}^{T}\Big\{\log|\tilde{L}(\theta,\omega)_t S_t^*(\tilde{\boldsymbol{\psi}})\tilde{L}_t'(\theta,\omega)| + tr\big\{\big[\tilde{L}_t(\theta,\omega)S_t^*(\tilde{\boldsymbol{\psi}})\tilde{L}_t'(\theta,\omega)\big]^{-1}C_t\big\}\Big\} \quad (7)$$

with $\tilde{\boldsymbol{\psi}} = (\omega, \theta, \boldsymbol{\psi}_{S^*})'$, $\boldsymbol{\psi}_{S^*}' = (\boldsymbol{\gamma}, \boldsymbol{\delta}, \alpha, \beta)$ and

$$\tilde{M}_t(\theta,\omega) = \tilde{L}_t(\theta,\omega)\tilde{L}_t'(\theta,\omega) = \hat{\Lambda}(\theta,\omega) + \theta\sum_{k=1}^{K}\phi_k(\omega)C_{t-k}. \quad (8)$$

The method we propose consists of estimating the parameters in $\tilde{\boldsymbol{\psi}}$ by a block-wise maximization of the moment-based QML function given in Eq.(7). First, conditional on some reasonable initial guess of $(\theta, \omega)$, $\tilde{\ell}_T(\tilde{\boldsymbol{\psi}})$ is maximized with respect to the short term parameters $\psi_{S^*}$ and then, conditional on $\hat{\boldsymbol{\psi}}_{S^*}$, the same function is maximized with respect to $(\theta, \omega)$. The procedure is iterated for $j = 0, \dots, J$ until some pre-specified convergence criterion is met.

To initialize the algorithm at $j = 0$, one can reasonably use as starting values the parameter estimates obtained by fitting the model to low dimensional subsets of data; also, an initial guess for the long term component $M_{t,0}$ could be either provided in a naive way, *i.e.* using the series of observed realized covariance matrices directly, or in a more sophisticated manner, by fitting to the data a nonparametric kernel smoother with an optimized bandwidth parameter. Note that in order to guarantee the positive definiteness of $\tilde{M}_t(\theta, \omega)$ in Eq.(8), it suffices to initialize $M_{t,0}$ from a PDS matrix and to impose $\theta > 0$. Given that the observed series of $C_t$, for every $t$, is PDS by definition, $\hat{\Lambda}(\theta, \omega)$ is assured to be at least semi-positive definite at each iteration $j > 0$.

Once $\Lambda_j(\theta_j, \omega_j)$ has been computed at the initial iteration $j = 0$, for every $j > 0$ the steps conducted in the algorithm are as follows.

*Step 1* Plug $\Lambda_{j-1}(\theta_{j-1}, \omega_{j-1})$ into Eq.(2), then get $\tilde{M}_{t,j}$ and $\tilde{L}_{t,j} = \text{chol}(\tilde{M}_{t,j})$;

*Step 2* For each asset $i = 1, ...n$, obtain the short term GARCH(1,1) parameters of Eq.(3) as follows

$$\{\gamma_{i,j}, \delta_{i,j}\} = \underset{\{\gamma_i,\delta_i\}}{\arg\max} \quad \tilde{\ell}_T\left(\theta_{j-1}, \omega_{j-1}, \alpha_{j-1}, \beta_{j-1}\right);$$

*Step 3* Conditional on the estimated vectors $\boldsymbol{\gamma}_j = (\gamma_{1,j}, \dots, \gamma_{n,j})'$ and $\boldsymbol{\delta}_j = (\delta_{1,j}, \dots, \delta_{n,j})'$, maximize the same log-likelihood function with respect to the short term DCC correlation parameters:

$$\{\alpha_j, \beta_j\} = \underset{\{\alpha,\beta\}}{\arg\max} \quad \tilde{\ell}_T\left(\theta_{j-1}, \omega_{j-1}, \boldsymbol{\gamma}_j, \boldsymbol{\delta}_j\right);$$

*Step 4* Finally, conditional on the vector of short term parameter estimates $\boldsymbol{\phi}_{S^*} = \{\boldsymbol{\gamma}_j, \boldsymbol{\delta}_j, \alpha_j, \beta_j\}$, maximize $\tilde{\ell}_T$ with respect to $\{\theta_j, \omega_j\}$; these estimates are used to compute an updated version of $\Lambda_j(\theta_j, \omega_j)$;

*Step 5* Check for convergence otherwise update all parameter estimates and go back to Step 1.

It is worth to stress that although $\tilde{\ell}_T(\tilde{\boldsymbol{\psi}})$ could look like a profile likelihood, it is not since $\hat{\Lambda}(\theta,\omega)$ is not a QML estimator but a feasible moment estimator. This motivates our choice to refer to Steps $1-5$ as the Iterative Moment based Profiling algorithm, or IMP for short. This implies that $\tilde{\boldsymbol{\psi}}$ is typically less efficient than the standard QML estimator which maximizes Eq.(5) in one step. We will come back to this issue in Section 6.1.

## 4. Simulation study

A Monte Carlo study is conducted to analyse the finite sample properties of the IMP estimator.

We assume the MMReDCC to be the true DGP and we generate 500 processes of length $T = 1000$ and 2000[3] for $n = 10, 20, 40$ and 50, with true parameter values inspired by the estimates given in Bauwens et al. [5], as summarized in Table 2.

It is important to stress that, in order to initialize the algorithm, parameter values have to be carefully chosen. This is a standard requirement in every optimization based procedure where the initial amount of information on the model parameters can be limited or even null. In our situation we are mainly concerned with the impact that different choices of $M_{t,0}$, more than the remaining set of parameters, may have on the convergence of the IMP algorithm. We evaluate this by performing a robustness check based on the two possible initializations of $M_{t,0}$ mentioned in Section 3.

Namely, in a first set of repetitions $M_{t,0}$ is computed by fitting to the series of simulated realized covariance matrices a Nadaraya-Watson kernel estimator with a single bandwidth parameter for the whole covariance matrix. As in Bauwens et al. [5] and Bauwens et al. [6], the optimal bandwidth is selected by least squares cross-validation criterion, where the six-month rolling covariance is used as the reference for the computation of least squares. In the second (equivalent) simulation study, $M_{t,0}$ is simply obtained by substituting in Eq. (6) the observed $C_t$ for the latent matrix $M_t$ at each $t$. In both cases, the initial scalar model parameters are set equal to the values listed in Panel B of Table 2.

The in-sample bias is evaluated by the Relative Bias (RB), computed as $RB = \frac{1}{500}\sum_{i=1}^{500}\frac{\hat{\psi}_i-\psi}{\psi}$, along with the interquartile range (IQR), mean, minimum and maximum of the obtained parameter estimates. To save on space, we report averaged bias results for the parameters of the MIDAS intercept matrix in a separate table.

Table 3 reports results from the first simulation exercise. The emerging picture looks encouraging. The finite sample bias for the parameters driving the short term volatility and correlation components are very small, being smaller than five per cent in most of the

[3] We simulate series of length $T + 1000$ and discard the first 1000 observations to reduce the impact of initial conditions.

**Table 2:** Simulation setting

| **Panel A: Parameters** | | | | | |
|---|---|---|---|---|---|
| *Long term component* | | | | | |
| $\theta$ | | | 0.5 | | |
| $K$ | | | 264 | | |
| $\omega$ | | | 15 | | |
| $\Lambda$ | | | $\Lambda_{i,i} = 0.02$, $\Lambda_{i,j} = 0.002$ for $i \neq j$ | | |
| *Short term components* | | | | | |
| $\gamma_i$ | | | $\sim U(\gamma_0 - 0.02, \gamma_0 + 0.02)$, $\gamma_0 = 0.2$ | | |
| $\delta_i$ | | | $\sim U(2\delta_0 + \gamma_i - 1 + 0.01, 1 - \gamma_i - 0.01)$, $\delta_0 = 0.7$ | | |
| $\alpha$ | | | 0.2 | | |
| $\beta$ | | | 0.7 | | |
| *General* | | | | | |
| $\nu$ | | | $2n$ | | |
| T | | | 1000, 2000 | | |
| initial discarded observations | | | 1000 | | |
| convergence tolerance | | | 0.0001 | | |
| **Panel B: Initial values** | | | | | |
| $\theta_0$ | $\omega_0$ | $\gamma_{i,0}$ | $\delta_{i,0}$ | $\alpha_0$ | $\beta_0$ |
| 0.8 | 10 | 0.05 | 0.90 | 0.05 | 0.9 |

Note: In Panel A, for every $i = 1, ..., n$ it holds $\{\gamma_i + \delta_i\} < 1$. Entries of Panel B are scalar parameters chosen to initialize the algorithm in both sets of simulation exercises.

cases, with one exception recorded for $\bar{\gamma}$ at $T = 1000$. As for the scalar parameters in the MIDAS specification, the bias for $\theta$ is negative in seven out of eight cases (exception occurs for $n = 10$ at $T = 2000$) and ranging from the maximum of 5.8% (in absolute value) for $n = 10$ and $T = 1000$ to the lowest value of 0.1% for $n = 50$ and $T = 2000$. The bias on the $\omega$ parameter, also generally negative, tends to decrease with $n$ but is usually of higher order (from 1.1 to 12% in absolute value). A similar behavior is observed for the IQR measure, which decreases across $n$ and $T$ but remains on higher values for the parameter $\omega$. However, this does not represent a major concern as the Beta weight function is not very sensitive to small variations of this parameter and therefore we do not expect the likelihood function to be either.

Table 4 gives an idea of the robustness of the results to a different initialization of the long term component. Entries can be directly compared to those in Table 3. As hoped for, the initial choice has a minor impact on the overall accuracy of the estimator, as the parameter biases are in the same range of magnitude and the comments made earlier are still valid under this alternative scenario. The only valuable remark can be made after closer

**Table 3:** Simulation exercise I: summary statistics

| | T= 1000 | | | | | | T= 2000 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\bar{\gamma}$ | $\bar{\delta}$ | $\alpha$ | $\beta$ | $\theta$ | $\omega$ | $\bar{\gamma}$ | $\bar{\delta}$ | $\alpha$ | $\beta$ | $\theta$ | $\omega_2$ |
| | 0.197 | 0.705 | 0.2 | 0.7 | 0.5 | 15 | 0.197 | 0.705 | 0.2 | 0.7 | 0.5 | 15 |
| **n=10** | | | | | | | | | | | | |
| **RB** | 0.098 | -0.036 | 0.020 | 0.003 | -0.058 | -0.120 | -0.039 | -0.002 | 0.033 | 0.003 | 0.053 | -0.095 |
| **IQR** | 0.048 | 0.093 | 0.006 | 0.010 | 0.044 | 1.641 | 0.032 | 0.068 | 0.006 | 0.008 | 0.037 | 1.819 |
| **Mean** | 0.202 | 0.699 | 0.204 | 0.702 | 0.475 | 14.820 | 0.2 | 0.713 | 0.207 | 0.702 | 0.526 | 13.58 |
| **Min** | 0.176 | 0.660 | 0.191 | 0.679 | 0.393 | 7.460 | 0.153 | 0.669 | 0.183 | 0.523 | 0.072 | 1.949 |
| **Max** | 0.214 | 0.735 | 0.220 | 0.728 | 0.709 | 18.943 | 0.22 | 0.803 | 0.37 | 0.817 | 1.000 | 17.74 |
| **n=20** | | | | | | | | | | | | |
| **RB** | 0.049 | -0.009 | 0.019 | 0.001 | -0.056 | -0.110 | 0.036 | 0.011 | 0.024 | 0.002 | -0.014 | -0.080 |
| **IQR** | 0.046 | 0.083 | 0.003 | 0.006 | 0.019 | 0.713 | 0.031 | 0.079 | 0.002 | 0.004 | 0.015 | 0.622 |
| **Mean** | 0.202 | 0.701 | 0.204 | 0.701 | 0.472 | 14.782 | 0.209 | 0.708 | 0.205 | 0.702 | 0.496 | 13.801 |
| **Min** | 0.171 | 0.633 | 0.197 | 0.687 | 0.430 | 12.802 | 0.197 | 0.678 | 0.200 | 0.694 | 0.001 | 2.440 |
| **Max** | 0.219 | 0.744 | 0.211 | 0.711 | 0.532 | 16.759 | 0.221 | 0.739 | 0.220 | 0.748 | 0.598 | 15.270 |
| **n=40** | | | | | | | | | | | | |
| **RB** | 0.028 | 0.023 | 0.015 | 0.002 | -0.049 | -0.080 | 0.033 | 0.029 | 0.022 | 0.002 | -0.014 | -0.072 |
| **IQR** | 0.042 | 0.077 | 0.002 | 0.002 | 0.011 | 0.372 | 0.030 | 0.064 | 0.001 | 0.002 | 0.007 | 0.263 |
| **Mean** | 0.208 | 0.715 | 0.203 | 0.701 | 0.476 | 14.810 | 0.209 | 0.719 | 0.204 | 0.702 | 0.493 | 13.925 |
| **Min** | 0.190 | 0.656 | 0.060 | 0.695 | 0.446 | 3.735 | 0.181 | 0.671 | 0.182 | 0.674 | 0.172 | 1.000 |
| **Max** | 0.217 | 0.762 | 0.222 | 0.799 | 0.705 | 16.500 | 0.221 | 0.761 | 0.223 | 0.744 | 0.837 | 14.760 |
| **n=50** | | | | | | | | | | | | |
| **RB** | 0.027 | 0.011 | 0.016 | 0.001 | -0.045 | 0.012 | 0.029 | 0.027 | 0.017 | 0.001 | -0.011 | -0.037 |
| **IQR** | 0.042 | 0.076 | 0.001 | 0.002 | 0.008 | 0.293 | 0.030 | 0.056 | 0.001 | 0.002 | 0.007 | 0.220 |
| **Mean** | 0.208 | 0.716 | 0.203 | 0.701 | 0.473 | 15.182 | 0.208 | 0.720 | 0.203 | 0.701 | 0.494 | 14.442 |
| **Min** | 0.162 | 0.644 | 0.200 | 0.697 | 0.455 | 13.150 | 0.191 | 0.657 | 0.199 | 0.695 | 0.474 | 12.089 |
| **Max** | 0.220 | 0.814 | 0.207 | 0.705 | 0.525 | 15.830 | 0.222 | 0.763 | 0.207 | 0.707 | 0.529 | 16.238 |

inspection of the results for $n = 50$. Here, the difference between the two initialization choices appears more pronounced, as the bias results reported in the bottom panel of Table 4 are slightly bigger than those of Table 3. Figure C.2 contains plots of the Monte Carlo standard deviations of the estimated $\theta, \omega, \alpha$ and $\beta$ parameters against the cross-section size. In all cases, standard deviations tend to decline as the cross-section dimension grows, with a faster decline when $T = 2000$. The two approaches produce similar parameter standard deviations, with slightly bigger values recorded for $\theta$ and $\omega$ under the second simulation experiment in correspondence to the higher cross-section sizes. These findings mainly justify our choice of the nonparametric smoother as the initialization method in high dimensional framework.

If we move to analyzing the bias results for the scale MIDAS intercept matrix, Table 5 shows that under both set of simulation exercises the estimator $\hat{\Lambda}(\theta, \omega)$ well approximates

**Table 4:** Simulation exercise II: summary statistics

| | T= 1000 | | | | | | | T= 2000 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\bar{\gamma}$ | $\bar{\delta}$ | $\alpha$ | $\beta$ | $\theta$ | $\omega$ | | $\bar{\gamma}$ | $\bar{\delta}$ | $\alpha$ | $\beta$ | $\theta$ | $\omega_2$ |
| | 0.197 | 0.705 | 0.2 | 0.7 | 0.5 | 15 | | 0.197 | 0.705 | 0.2 | 0.7 | 0.5 | 15 |
| | **n=10** | | | | | | | **n=10** | | | | | |
| **RB** | -0.077 | 0.013 | 0.019 | 0.002 | -0.032 | -0.007 | **RB** | 0.043 | 0.006 | 0.025 | 0.002 | -0.012 | -0.069 |
| **IQR** | 0.070 | 0.113 | 0.007 | 0.010 | 0.043 | 1.664 | **IQR** | 0.028 | 0.046 | 0.004 | 0.007 | 0.024 | 0.870 |
| **Mean** | 0.191 | 0.708 | 0.204 | 0.701 | 0.484 | 14.892 | **Mean** | 0.211 | 0.707 | 0.205 | 0.701 | 0.494 | 13.959 |
| **Min** | 0.107 | 0.588 | 0.191 | 0.678 | 0.393 | 6.678 | **Min** | 0.189 | 0.654 | 0.196 | 0.686 | 0.435 | 9.614 |
| **Max** | 0.231 | 0.833 | 0.218 | 0.723 | 0.927 | 19.237 | **Max** | 0.232 | 0.748 | 0.213 | 0.715 | 0.623 | 15.535 |
| | **n=20** | | | | | | | **n=20** | | | | | |
| **RB** | 0.048 | -0.002 | 0.018 | 0.002 | -0.045 | -0.008 | **RB** | 0.006 | 0.039 | 0.023 | 0.002 | -0.011 | -0.064 |
| **IQR** | 0.042 | 0.085 | 0.003 | 0.005 | 0.021 | 0.719 | **IQR** | 0.030 | 0.060 | 0.002 | 0.004 | 0.013 | 0.510 |
| **Mean** | 0.207 | 0.702 | 0.204 | 0.701 | 0.477 | 14.887 | **Mean** | 0.206 | 0.721 | 0.205 | 0.701 | 0.494 | 14.047 |
| **Min** | 0.196 | 0.649 | 0.197 | 0.689 | 0.429 | 6.751 | **Min** | 0.164 | 0.680 | 0.199 | 0.692 | 0.467 | 11.108 |
| **Max** | 0.220 | 0.740 | 0.214 | 0.713 | 0.887 | 16.573 | **Max** | 0.225 | 0.772 | 0.209 | 0.710 | 0.555 | 15.241 |
| | **n=40** | | | | | | | **n=40** | | | | | |
| **RB** | 0.046 | 0.017 | 0.017 | 0.003 | -0.045 | 0.005 | **RB** | 0.050 | 0.016 | 0.021 | 0.002 | -0.012 | -0.057 |
| **IQR** | 0.042 | 0.080 | 0.002 | 0.003 | 0.010 | 0.421 | **IQR** | 0.029 | 0.053 | 0.001 | 0.002 | 0.007 | 0.245 |
| **Mean** | 0.209 | 0.713 | 0.203 | 0.702 | 0.477 | 15.078 | **Mean** | 0.210 | 0.718 | 0.204 | 0.701 | 0.494 | 14.148 |
| **Min** | 0.197 | 0.682 | 0.193 | 0.696 | 0.116 | 6.895 | **Min** | 0.194 | 0.658 | 0.202 | 0.697 | 0.479 | 8.433 |
| **Max** | 0.219 | 0.756 | 0.216 | 0.807 | 0.955 | 49.985 | **Max** | 0.223 | 0.768 | 0.212 | 0.708 | 0.726 | 14.638 |
| | **n=50** | | | | | | | **n=50** | | | | | |
| **RB** | 0.029 | 0.028 | 0.016 | 0.002 | -0.053 | 0.015 | **RB** | 0.028 | 0.018 | 0.020 | 0.002 | -0.017 | -0.048 |
| **IQR** | 0.041 | 0.071 | 0.001 | 0.002 | 0.009 | 0.342 | **IQR** | 0.030 | 0.061 | 0.001 | 0.001 | 0.005 | 0.195 |
| **Mean** | 0.208 | 0.715 | 0.203 | 0.701 | 0.473 | 15.225 | **Mean** | 0.208 | 0.721 | 0.204 | 0.701 | 0.492 | 14.278 |
| **Min** | 0.190 | 0.632 | 0.200 | 0.698 | 0.457 | 14.190 | **Min** | 0.184 | 0.674 | 0.202 | 0.698 | 0.437 | 11.178 |
| **Max** | 0.220 | 0.766 | 0.206 | 0.705 | 0.494 | 15.844 | **Max** | 0.216 | 0.766 | 0.209 | 0.713 | 0.508 | 14.742 |

Note: Summary statistics of the second set of simulations where $M_{t,0}$ is initialized from the series of realized covariance matrices, see Section 3. To save on space, $\bar{\gamma}$ and $\bar{\delta}$ are reported as averaged values across series and replications. RB denotes the Relative Bias computed over 500 replications. True parameter values used to simulate the process at the top of the table.

the true $\Lambda$ matrix at all cross-section dimensions, with the parameter bias (averaged across diagonal and off-diagonal elements) clearly improving with increasing $n$. Again, the direct comparison of Panel A and B confirms that the algorithm initialized from the series of realized covariance matrices overall performs no worse than the one initialized from a nonparametric smoother.

To summarize, the simulation study carried out in this section suggests that the proposed algorithm works quite accurately in finite samples and converges irrespective of the initialization choice made. Overall, the moment-based estimator used for iteratively targeting the constant intercept matrix in the secular component does not create a severe

**Table 5:** Bias results for the scale MIDAS intercept matrix.

| Panel A: Simulation exercise I | | | | Panel B: Simulation exercise II | | | |
|---|---|---|---|---|---|---|---|
| T=1000 | | T=2000 | | T=1000 | | T=2000 | |
| n=10 | | n=10 | | n=10 | | n=10 | |
| $\mathbf{RB}_{\{i,i\}}$ | 0.080 | $\mathbf{RB}_{\{i,i\}}$ | 0.033 | $\mathbf{RB}_{\{i,i\}}$ | 0.075 | $\mathbf{RB}_{\{i,i\}}$ | 0.060 |
| $\mathbf{RB}_{\{i,j\}}$ | 0.079 | $\mathbf{RB}_{\{i,j\}}$ | 0.000 | $\mathbf{RB}_{\{i,j\}}$ | 0.066 | $\mathbf{RB}_{\{i,j\}}$ | 0.040 |
| n=20 | | n=20 | | n=20 | | n=20 | |
| $\mathbf{RB}_{\{i,i\}}$ | 0.073 | $\mathbf{RB}_{\{i,i\}}$ | 0.068 | $\mathbf{RB}_{\{i,i\}}$ | 0.072 | $\mathbf{RB}_{\{i,i\}}$ | 0.058 |
| $\mathbf{RB}_{\{i,j\}}$ | 0.062 | $\mathbf{RB}_{\{i,j\}}$ | 0.048 | $\mathbf{RB}_{\{i,j\}}$ | 0.060 | $\mathbf{RB}_{\{i,j\}}$ | 0.044 |
| n=40 | | n=40 | | n=40 | | n=40 | |
| $\mathbf{RB}_{\{i,i\}}$ | 0.073 | $\mathbf{RB}_{\{i,i\}}$ | 0.062 | $\mathbf{RB}_{\{i,i\}}$ | 0.073 | $\mathbf{RB}_{\{i,i\}}$ | 0.058 |
| $\mathbf{RB}_{\{i,j\}}$ | 0.061 | $\mathbf{RB}_{\{i,j\}}$ | 0.046 | $\mathbf{RB}_{\{i,j\}}$ | 0.159 | $\mathbf{RB}_{\{i,j\}}$ | 0.043 |
| n=50 | | n=50 | | n=50 | | n=50 | |
| $\mathbf{RB}_{\{i,i\}}$ | 0.072 | $\mathbf{RB}_{\{i,i\}}$ | 0.004 | $\mathbf{RB}_{\{i,i\}}$ | 0.072 | $\mathbf{RB}_{\{i,i\}}$ | 0.058 |
| $\mathbf{RB}_{\{i,j\}}$ | 0.057 | $\mathbf{RB}_{\{i,j\}}$ | 0.037 | $\mathbf{RB}_{\{i,j\}}$ | 0.058 | $\mathbf{RB}_{\{i,j\}}$ | 0.043 |

Note: $RB_{\{i,i\}}$ denotes averaged values over diagonal terms, while $RB_{\{i,j\}}$ denotes averages over off diagonal terms. Panel (a) reports summary statistics of the first simulation exercise where $M_{t,0}$ is initialized from a nonparametric smoother while Panel (b) reports results from the second simulation exercise where the series of observed realized covariance matrices are used.

bias problem in the estimation of the remaining dynamic parameters, thus representing a feasible solution to alleviate the curse of dimensionality issue that would otherwise prevent the use of the MMReDCC model in high dimensional applications.

## 5. Multi-step Forecasting

Models featuring short and long-run dynamics are particularly attractive for computing multi-step-ahead predictions, as their component dynamic structure is possibly expected to be beneficial for longer-term forecasts. Unfortunately, the complex nonlinear structure of the MMReDCC model makes the analytical derivation of closed-form solutions troublesome. In order to overcome this problem, we propose to compute multi-step predictions by means of a procedure based on bootstrap resampling.

At the outset, notice that Eq.(1) can be alternatively written as

$$C_t = S_t^{1/2} U_t (S_t^{1/2})' \tag{9}$$

where the $U_t$ are a sequence of iid random variables such that $E(U_t) = I_n$, $S_t^{1/2}$ is any PDS matrix such that $S_t^{1/2}(S_t^{1/2})' = S_t$.
The bootstrap procedure we use for generating multi-step-ahead forecasts of the realized covariance matrix $C_t$ is described in the following.

*Step 1* Estimate the model on $\{C_t, t = 1, \ldots, T\}$, obtain the estimated conditional covariances $\hat{S}_t$.

*Step 2* Compute the estimated standardized residuals

$$\hat{U}_t = \hat{S}_t^{-1/2} C_t (\hat{S}_t^{-1/2})', \qquad t = 1, \ldots, T$$

and rescale them to enforce their sample mean to be equal to $I_n$, namely:

$$\tilde{U}_t = (\hat{E}_u^{-1/2}) \hat{U}_t (\hat{E}_u^{-1/2})',$$

where $\hat{E}_u = (1/T) \sum_{t=1}^{T} \hat{U}_t = I_n$. The rescaled $\tilde{U}_t$ can then be used to generate bootstrap replicates of $C_{T+j}$, for $j = 1, \ldots, h$, where $h$ denotes the chosen forecast horizon.

*Step 3* Draw with replacement a bootstrap sample of length $h$ from the empirical CDF of $\tilde{U}_t$:

$$\{\tilde{U}_{T+1|T}, \ldots, \tilde{U}_{T+h|T}\}$$

*Step 4* For $j = 1 \ldots, h$, recursively generate a sequence of bootstrap replicates of $C_{T+j}$ as follows

$$\begin{aligned}
M_{T+j|T} &= \hat{\Lambda}(\theta, \omega) + \hat{\theta} \sum_{k=1}^{K} \phi_k(\hat{\omega}) \tilde{C}_{T-k+j|T} \\
L_{T+j|T} &= M_{T+j|T}^{1/2} \\
C^*_{T+j|T} &= L_{T+j|T} C_{T+j} (L'_{T+j|T})^{-1} \\
P^*_{T+j|T} &= (diag\{C^*_{T+j|T}\})^{-1/2} C^*_{T+j|T} (diag\{C^*_{T+j|T}\})^{-1/2} \\
S^*_{ii,T+j|T} &= (1 - \hat{\gamma}_i - \hat{\delta}_i) + \hat{\gamma}_i C^*_{ii,T+j-1|T} + \hat{\delta}_i S^*_{ii,T+j-1|T} \\
R^*_{T+j|T} &= (1 - \hat{\alpha} - \hat{\beta}) I_n + \hat{\alpha} P^*_{T+j-1|T} + \hat{\beta} R^*_{T+j-1|T} \\
S^*_{T+j|T} &= (diag\{S^*_{T+j|T}\})^{1/2} R^*_{T+j|T} (diag\{S^*_{T+j|T}\})^{1/2} \\
S_{T+j|T} &= L_{T+j|T} S^*_{T+j|T} L'_{T+j|T} \\
C_{T+j|T} = &= (S^{1/2}_{T+j|T}) \tilde{U}_{T+j} (S^{1/2}_{T+j|T})'
\end{aligned}$$

*Step 5* Repeat steps 3-4 B times, where B is set sufficiently large (e.g. B=10000). As a result the procedure will generate an array of $h \times B$ bootstrap replicates $C^{(b)}_{T+j|T}$ $(b = 1, \ldots, B)$.

*Step 6* Finally, the h-steps-ahead forecast can be computed as

$$\hat{S}_{T,j} = \frac{1}{B} \sum_{b=1}^{B} C^{(b)}_{T+j|T}.$$

Even if our primary interest is in forecasting from MMReDCC models, the proposed forecasting procedure is very general and can be readily adapted to any model that admits

**Table 6:** Application I. Tickers

| Dataset 1: 2242 obs | Dataset 2: 1499 obs | Dataset 3: 1499 obs |
|---|---|---|
| **Period: 02/2001 - 12/2009** | **Period: 01/2007 - 12/2012** | **Period: 01/2007 - 12/2012** |
| AA, AXP, BAC, KO, DD, | ACAS, AET, AFL, AIG, AIZ, | STI, STT, TMK, TROW, UNH, |
| GE, IBM, JPM, MSFT, XOM | ALL, AMP, AXP, BAC, BBT | UNM, USB, WFC, WU, ZION |

the representation in Eq.(9) where where $S_t$ is modeled as a function of past information $\mathcal{I}_{t-1}$. For example, in the empirical application which is being presented in Section 6.2, we will use it to generate multi-step forecasts of $C_t$ from the cRDCC model. To this purpose, the dynamic equations in step 4 must be replaced with those pertaining to the specific model of interest.

## 6. Empirical Applications

This section illustrates two different empirical applications. The first application gives a useful hint of the in-sample accuracy of the IMP estimator as opposed to the standard QML estimator in the ideal case where both can be computed. The second one is performed in a large dimensional system and aims at evaluating both the full-sample fit of the model and its forecasting performance. Specifically, we evaluate the ability of the MMReDCC model to provide accurate multi-step-ahead covariance predictions against existing competitors not accounting for time-varying long term dynamics.

### *6.1. Small sample accuracy comparison*

As regards the in-sample performance, we are mainly interested in the efficiency of the IMP estimator as opposed to the QML one, which maximizes the likelihood over the full parameter vector and can only be used in low dimensional cases where the model can be estimated in one step (recall Table 1). To this extent, we fix the cross-sectional dimension equal to ten assets and use three different datasets to our estimation purpose. An overview of the data being used is given in Table 6 and more in details in Appendix A. The first panel comprises the assets used in Bauwens et al. [5] and includes series of daily realized covariance matrices estimated on five minute intraday returns over the period February 2001 to December 2009; the second and third panels comprise arbitrary selected subsamples of the dataset used in the work of Boudt et al. [10] which includes series of daily realized covariance matrices obtained with the *CholCov* estimator over the period January 2007 to December 2012.[4] As already mentioned before, the choice of the realized estimator is

[4]Our analysis focuses on open-to-close covariance matrices, whereby noisy overnight returns have not been included in the construction of the estimators. We refer to the cited papers for further details.

**Table 7:** Application I. In-sample parameter estimates

| Dataset 1 | | | | Dataset 2 | | | | Dataset 3 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| QML | | IMPL | | QML | | IMP | | QML | | IMP | |
| $\gamma_i$ | $\delta_i$ | $\gamma_i$ | $\delta_i$ | $\gamma_i$ | $\delta_i$ | $\gamma_i$ | $\delta_i$ | $\gamma_i$ | $\delta_i$ | $\gamma_i$ | $\delta_i$ |
| 0.36 (0.04) | 0.55 (0.05) | 0.38 (0.06) | 0.56 (0.04) | 0.15 (0.03) | 0.75 (0.06) | 0.18 (0.05) | 0.77 (0.05) | 0.20 (0.05) | 0.73 (0.12) | 0.18 (0.07) | 0.78 (0.08) |
| 0.35 (0.63) | 0.59 (0.03) | 0.41 (0.07) | 0.51 (0.04) | 0.23 (0.05) | 0.69 (0.07) | 0.23 (0.05) | 0.71 (0.06) | 0.21 (0.06) | 0.70 (0.23) | 0.22 (0.04) | 0.72 (0.05) |
| 0.34 (0.04) | 0.57 (0.06) | 0.37 (0.05) | 0.53 (0.06) | 0.24 (0.06) | 0.61 (0.09) | 0.26 (0.06) | 0.64 (0.07) | 0.25 (0.05) | 0.66 (0.07) | 0.22 (0.06) | 0.69 (0.06) |
| 0.33 (0.04) | 0.60 (0.06) | 0.31 (0.04) | 0.59 (0.05) | 0.20 (0.28) | 0.65 (0.41) | 0.19 (0.08) | 0.73 (0.09) | 0.12 (0.07) | 0.79 (0.20) | 0.05 (0.01) | 0.93 (0.02) |
| 0.34 (0.03) | 0.64 (0.05) | 0.33 (0.04) | 0.60 (0.04) | 0.22 (0.1) | 0.49 (0.06) | 0.19 (0.08) | 0.72 (0.10) | 0.18 (0.14) | 0.76 (0.41) | 0.16 (0.03) | 0.79 (0.04) |
| 0.26 (0.03) | 0.64 (0.05) | 0.25 (0.03) | 0.66 (0.04) | 0.18 (0.09) | 0.67 (0.12) | 0.18 (0.06) | 0.76 (0.08) | 0.29 (0.10) | 0.58 (0.17) | 0.24 (0.07) | 0.67 (0.09) |
| 0.38 (0.03) | 0.51 (0.05) | 0.37 (0.03) | 0.51 (0.03) | 0.18 (0.2) | 0.56 (0.09) | 0.08 (0.04) | 0.89 (0.05) | 0.14 (0.05) | 0.80 (0.50) | 0.20 (0.13) | 0.59 (0.15) |
| 0.38 (0.04) | 0.51 (0.05) | 0.37 (0.03) | 0.52 (0.04) | 0.18 (0.09) | 0.75 (0.09) | 0.18 (0.04) | 0.76 (0.04) | 0.21 (0.10) | 0.71 (0.11) | 0.22 (0.06) | 0.71 (0.07) |
| 0.37 (0.04) | 0.52 (0.05) | 0.39 (0.04) | 0.52 (0.04) | 0.18 (0.06) | 0.73 (0.08) | 0.18 (0.05) | 0.76 (0.05) | 0.22 (0.06) | 0.73 (0.21) | 0.17 (0.10) | 0.76 (0.09) |
| 0.36 (0.04) | 0.50 (0.04) | 0.37 (0.04) | 0.53 (0.05) | 0.14 (0.04) | 0.77 (0.08) | 0.13 (0.04) | 0.83 (0.05) | 0.15 (0.06) | 0.69 (0.10) | 0.02 (0.08) | 0.74 (0.03) |
| $\alpha$ | $\beta$ | $\alpha$ | $\beta$ | $\alpha$ | $\beta$ | $\alpha$ | $\beta$ | $\alpha$ | $\beta$ | $\alpha$ | $\beta$ |
| 0.07 (0.00) | 0.84 (0.01) | 0.07 (0.00) | 0.88 (0.00) | 0.017 (0.00) | 0.80 (0.1) | 0.018 (0.00) | 0.91 (0.05) | 0.02 (0.00) | 0.96 (0.01) | 0.01 (0.00) | 0.90 (0.04) |
| $\theta$ | $\omega_2$ | $\theta$ | $\omega_2$ | $\theta$ | $\omega_2$ | $\theta$ | $\omega_2$ | $\theta$ | $\omega_2$ | $\theta$ | $\omega_2$ |
| 0.91 (0.05) | 4.56 (0.06) | 0.88 (0.05) | 4.52 (0.05) | 0.84 (0.18) | 5.63 (2.4) | 0.81 (0.18) | 3.60 (0.65) | 0.90 (0.14) | 3.46 (0.71) | 0.80 (0.22) | 2.67 (0.51) |
| **Frobenius Norm** | | | | **Frobenius Norm** | | | | **Frobenius Norm** | | | |
| 1.60 | | | | 3.83 | | | | 2.81 | | | |
| **Loglik** | | **Loglik** | | **Loglik** | | **Loglik** | | **Loglik** | | **Loglik** | |
| -18317 | | -18323 | | -28571 | | -28576 | | -29026 | | -29056 | |

Note: Each panel reports parameter estimates and corresponding standard errors in brackets. The Frobenius norm measures the difference between the two long term MIDAS intercepts and is computed as $\sqrt{\sum_{i,j=1}^{n} |\hat{\Lambda}_{i,j}^{QML} - \hat{\Lambda}_{i,j}^{IMP}|^2}$, where the superscripts $QML, IMP$ denote respectively the one-step quasi-maximum likelihood and the iterative profile likelihood estimators. Number of in-sample observations is 2242 for Dataset 1 and 1499 for Dataset 2 and 3.

not an issue here as the model can be fitted to any series of realized variance-covariance matrices as long as they are guaranteed to be PDS. Estimation results for the MMReDCC model by both the IMP and the QML estimators are reported in Table 7.

In the three cases considered, the estimators appear to deliver similar in-sample estimates. Short term GARCH coefficients tend to be quite homogeneous across assets and generally significant; the same applies to the short term correlation estimates. As for the parameters driving the long term component, it can be noticed that the estimated $\theta$ and $\omega$ coefficients are regularly lower for the IMP than for the QML model. This is in line with the prevailing negative bias reported from the simulation study. The QML estimator, as expected, performs slightly better than the IMP but the small difference in the log-likelihood values stresses the evidence that the loss, in terms of goodness of fit, is negligible and that the proposed IMP algorithm represents a reliable option even when the model can be estimated in the standard way.

### *6.2. Forecasting performance*

In this subsection we push the analysis to a higher dimension, with the aim of assessing the usefulness of the MMReDCC model in a classical forecasting framework. As benchmarks we consider the Consistent RDCC (cRDCC) model of Bauwens et al. [7] as the closest competitor and a simple Exponentially Weighted Moving Average (EWMA) approach. The simple EWMA predictor appears a natural candidate due to its widespread diffusion among practitioners and in risk management systems like RiskMetrics; if applied to the realized covariance matrices it obtains as follows:

$$S_t = (1 - \lambda)C_{t-1} + \lambda S_{t-1},$$

where the $\lambda$ parameter is set equal to the value 0.94 (see also Golosnoy et al. [21]).

On the other hand, the choice of the cRDCC as a benchmark is supported by two main reasons. First, it assumes that conditional volatilities and correlations mean revert to constant quantities, thus it can be considered as a simplified version of the MMReDCC model despite not being formally nested. Second, the findings of Boudt et al. [10] show that the cRDCC model favorably compares with some widely used competitors, such as the HEAVY (Noureldin et al. [28]) and the cDCC (Aielli [1] model, in forecasting Value at Risk. In order to estimate the cRDCC in high dimension, we apply a three stage QML estimation procedure as suggested by [7], where the constant long term covariance matrix is consistently targeted by the unconditional covariance. This drastically reduces the number of parameters to be estimated to $2n + 2$.

The investment universe comprises 50 of the most liquid equities of the S&P 500 traded over the period May 1997 – July 2008, for a total of 2524 observations. Tickers are reported in Table 8 while descriptive statistics of the data are given in Appendix B.

Before turning to the out-of-sample analysis, it is worth first looking at the estimates obtained by fitting the MMReDCC and cRDCC models over the full sample period. As emerges from Panel A of Table 9, the MMReDCC outperforms the cRDCC in terms of the AIC and BIC criteria, which are both minimized for the MMReDCC. The univariate GARCH(1,1) parameters $\bar{\gamma}$ and $\bar{\delta}$, reported in averaged values across series, largely agree with each other, while the correlation estimates are markedly different across models.

To closely examine the fit of the models, consider the conditional correlations between randomly selected stocks, APOL and GCI, presented in Figure 1. The parameter estimates from the MMReDCC produce large and more persistent shifts in the conditional correlation of the assets, including a marked increase starting beginning of May, 2007 and lasting until the end of the sample. The cRDCC model, on the contrary, delivers a conditional

**Table 8:** Application II. Selected constituents of the $S\&P$ 500 on May 1997

| Stock | Issue Name | Stock | Issue Name | Stock | Issue Name |
|---|---|---|---|---|---|
| AA | Alcoa | BMY | Bristol-Myers Squibb Company | ETR | Entergy Corporation |
| ABT | Abbott Laboratories | C | Citigroup | F | Ford Motor Co. |
| ADI | Analog Devices | CAG | ConAgra Foods | FDO | Family Dollar Stores |
| AFL | Aflac Incorporated | CAH | Cardinal Health | FISV | Fiserv |
| AIG | American International Group | CL | Colgate-Palmolive Co. | GCI | Gannett Co. |
| ALL | Allstate | CLX | Clorox | GE | General Electric |
| APD | Air Products & Chemicals | CMA | Comerica Incorporated | DTE | DTE Energy Company |
| APOL | Apollo Education Group | CMS | CMS Energy Corp. | EIX | Edison International |
| AVY | Avery Dennison Corporation | COF | Capital One Financial Corporation | EMN | Eastman Chemical Co. |
| AXP | American Express Company | COST | Costco Wholesale Corporation | GIS | General Mills |
| AZO | AutoZone | CPB | Campbell Soup | GPC | Genuine Parts Company |
| BAC | Bank of America | CSC | Computer Sciences Corporation | GPS | The Gap |
| BAX | Baxter Corporation | CTAS | Cintas Corporation | HD | The Home Depot |
| BBBY | Bed Bath & Beyond | CTL | CenturyLink | HNZ | HNZ Grou |
| BDX | Becton, Dickinson and Company | DOV | Dover Corporation | HPQ | Hewlett-Packard Company |
| BHI | Braveheart Resources | DOW | The Dow chemical Company | KO | Coca Cola |
| HON | Honeywell International | T | AT&T | | |

**Table 9:** Full sample estimates and Implemented loss functions.

**Panel A: Full sample estimates**

| | MMReDCC | cRDCC |
|---|---|---|
| $\bar{\gamma}$ | 0.396 | 0.351 |
| $\bar{\delta}$ | 0.601 | 0.592 |
| $\alpha$ | 0.013 | 0.027 |
| $\beta$ | 0.951 | 0.875 |
| $\theta$ | 0.719 | |
| $\omega$ | 2.068 | |
| Lik | 800141 | 677064 |
| AIC | -632 | -599 |
| BIC | -629 | -598 |

**Panel B: Implemented Loss functions**

| | | |
|---|---|---|
| Frob | Frobenius distance | $\mathrm{tr}\left[(C_t - H_t)'(C_t - H_t)\right]$ |
| Sfrob | Squared Frobenius distance | $\sum_{i=1}^{n} \lambda_i$ |
| Euclid | Euclidean distance | $\mathrm{vech}(C_t - H_t)'\mathrm{vech}(C_t - H_t)$ |
| ST | Stein | $\mathrm{tr}(H_t^{-1}C_t) - \log\lvert H_t^{-1}C_t\rvert - n$ |
| vND | von Neumann Divergence | $\mathrm{tr}\left(C_t \log C_t - C_t \log H_t - C_t + H_t\right)$ |
| QLIK | Qlike | $\log\lvert H_t\rvert + \mathrm{tr}\left(H_t^{-1}C_t\right)$ |

Note: Panel A reports full sample estimates from the MMReDCC and cRDCC model, with AIC and BIC criteria rescaled by the number of observations. Panel B contains the loss functions chosen to evaluate the models forecasting ability. $H_t$ denotes the predicted conditional covariance matrix while $C_t$ is the realized measure; $\lambda_i$ are the eigenvalues of $(C_t - H_t)^2$ and $n$ denotes the number of assets.

correlation which is nearly constant and exhibits little variation even near the spread of the financial crisis events in 2008. Given the close similarity between the models, this can be reasonably explained by the fact that the parameters $\theta$ and $\omega$ driving the long term (co)volatilities dynamics have a crucial role in allowing for a major flexibility of the model and thus for a better responsiveness in periods of higher market volatility.

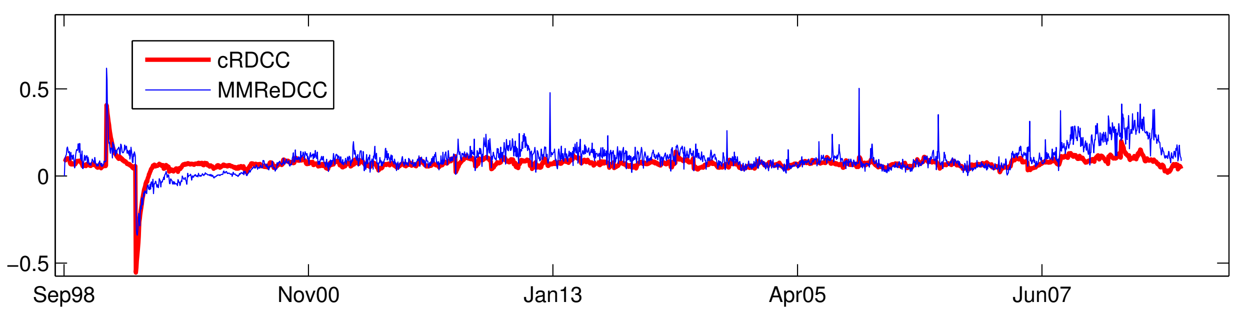
To determine whether the MMReDCC model can lead to considerable forecasting gains we compute forecasts of the conditional covariance matrix of daily returns at different horizons making use of the bootstrap procedure outlined in Section 5. A similar approach is also applied to the cRDCC model, while predictions from the EWMA are obtained analytically reminding that $E(C_{t+h}|\Im_t) = E(C_{t+h-1}|\Im_t)$.

To shorten the computational time, estimation is performed using a fixed-rolling window scheme with window length equal to 2024 observations, that shifts forward every 20 days, over which parameter estimates are kept fixed. Given the full sample size of 2524 observations, this leads to a number of re-estimations of each model equal to 25.

The out-of-sample period starts on July, 2006 and covers roughly the last 500 observations of the sample, ending just before the heat of the financial crisis (July 2008). The horizons considered for predictions are $h = 1, 5, 10$ and 20 days.

The comparison of the models forecasting ability is performed using the six consistent[5] loss functions defined in Panel B of Table 9, for which we report averaged values over the out-of-sample period. In order to evaluate the significance of the loss function differences we employ the Model Confidence Set (MCS) approach of Hansen et al. [23], which identifies the single model or the set of models having the best forecasting performance at a given confidence level.[6]

**Figure 1:** Estimated correlation of APOL-GCI.




Results are summarized by horizon in Table 10. According to Panel A, at the shortest horizon the MMReDCC model is outperformed by its competitors on all the selected criteria, and excluded by the MCS at both the 90 and 75% confidence levels. This suggests that for the data at hand, accurate one-step-ahead predictions can be obtained by employing simpler models that do not necessarily account for a time-varying long run level. At this

[5]The term *consistent* is used according to Laurent et al. [26].

[6]The MCS is computed at the 90 % and 75% confidence levels, with block-length bootstrap parameter and number of bootstrap samples used to obtain the distribution under the null respectively equal to 2 and 10000.

stage the choice between the EWMA and the cRDCC model appears almost indifferent, despite the latter being more often included in the MCS.

As we move further in time the situation is quickly reversed: at the 5-day horizon, the MMReDCC model minimizes five out of six loss functions while at the 10-day horizon it appears to deliver the optimal covariance forecasts according to the whole set of losses. This gain is confirmed by the inclusion of the model in the MCS resulting from all the selected criteria, differently from the competitors which are almost never included (EWMA and cRDCC are included three and two times, respectively, in the 75% MCS at $h = 5$, but never at $h = 10$). The predominance of the MMReDCC remains quite stable even at the longest horizon ($h = 20$), but the difference in the forecast accuracy between the MMReDCC and the benchmarks becomes smaller, with the cRDCC performing almost as good as the MMReDCC in terms of MCS inclusions (cRDCC excluded only in the von Neumann 75% MCS). These results appear to be in line with those of [21], where already at the 10-ahead horizon the differences in the forecast accuracy between their best component CAW model and the selected benchmarks were smaller than at shorter horizons.

Overall, the out-of-sample performance of the MMReDCC model in a moderately volatile time period appears to be good relative to the competing models especially at medium-term horizons, when it yields the most accurate forecasts. In light of these empirical results, it appears that the introduction of an additional component capturing the secular movements in the volatility and covolatility dynamics is well justified and useful to enhance a higher forecasting accuracy.

## 7. Conclusion

The inference procedures derived in the paper allow to extend the range of applicability of the MMReDCC model to large dimensional portfolios such as those routinely encountered in standard risk management practice. In order to reach this objective, we face two crucial well-known topics in multivariate time series modeling, such as high-dimensional estimation and multi-step forecasting. To the first aim, we propose a feasible estimation procedure, the Iterative Moment based Profiling (IMP) algorithm, which profiles out the parameters of the scale MIDAS intercept matrix and iteratively maximizes the likelihood in terms of the other parameters of interest.

Whilst not providing a rigorous asymptotic theory, we discuss the finite sample properties of the estimator via a comprehensive simulation study, which demonstrates that the algorithm is computationally simple and is converging irrespective of the initialization method employed. We also compare the standard one-step QML estimator against the IMP estimator in a small dimensional framework exercise and find that not only the two estimators deliver very similar in-sample estimates, but the efficiency loss of the IMP in terms of likelihood values can be considered negligible. From the computational point of view, we find that our algorithm is reliable and easy to apply despite the large number of parameters involved in the MMReDCC. Our application exemplifies its usefulness when the model is applied to the realized covariances of 50 stocks but, given its flexibility, we fairly believe that it could be feasibly extended to dataset of even larger dimension.

**Table 10:** Multi-step-ahead forecast evaluation

| | **Horizon 1** | | | **Horizon 5** | | | **Horizon 10** | | | **Horizon 20** | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **MMReDCC** | **EWMA** | **cRDCC** | **MMReDCC** | **EWMA** | **cRDCC** | **MMReDCC** | **EWMA** | **cRDCC** | **MMReDCC** | **EWMA** | **cRDCC** |
| *Panel A: Loss functions* | | | | | | | | | | | | |
| **Frob** | 0.133 | **0.125** | 0.126 | 0.138 | **0.135** | 0.139 | **0.144** | 0.148 | 0.148 | **0.158** | 0.167 | 0.160 |
| **Sfrob** | 0.040 | **0.034** | 0.037 | **0.036** | **0.036** | **0.036** | **0.037** | 0.038 | 0.038 | **0.041** | 0.042 | 0.043 |
| **Euclid** | 0.086 | **0.080** | 0.082 | **0.087** | 0.088 | 0.088 | **0.091** | 0.093 | 0.093 | **0.099** | 0.102 | 0.101 |
| **ST** | 65.159 | 59.018 | **55.449** | **59.227** | 61.684 | 60.245 | **61.522** | 63.611 | 63.289 | 67.748 | **66.269** | 66.667 |
| **vND** | 0.018 | 0.017 | **0.017** | **0.018** | 0.019 | 0.019 | **0.019** | 0.022 | 0.020 | **0.022** | 0.025 | **0.022** |
| **QLIK** | -370.64 | -376.78 | **-380.35** | **-376.66** | -374.20 | -375.64 | **-374.43** | -372.34 | -372.67 | -368.07 | **-369.55** | -369.15 |
| *Panel B: 90 % MCS* | | | | | | | | | | | | |
| **Frob** | 0.014 | **0.689** | **1.000** | **0.486** | **1.000** | **0.232** | **1.000** | 0.001 | 0.015 | **0.224** | **0.619** | **1.000** |
| **Sfrob** | 0.044 | **0.259** | **1.000** | **0.614** | **1.000** | **0.614** | **1.000** | 0.052 | 0.052 | **0.547** | **0.547** | **1.000** |
| **Euclid** | 0.023 | **0.464** | **1.000** | **1.000** | **0.311** | **0.420** | **1.000** | 0.001 | 0.016 | **0.506** | **0.506** | **1.000** |
| **ST** | 0.000 | 0.000 | **1.000** | **1.000** | 0.000 | 0.018 | **1.000** | 0.000 | 0.003 | **1.000** | 0.000 | **0.281** |
| **vND** | 0.000 | **0.263** | **1.000** | **1.000** | 0.002 | 0.028 | **1.000** | 0.000 | 0.000 | **1.000** | 0.000 | **0.159** |
| **QLIK** | 0.000 | 0.000 | **1.000** | **1.000** | 0.000 | 0.017 | **1.000** | 0.000 | 0.004 | **1.000** | 0.000 | **0.275** |
| *Panel C: 75 % MCS* | | | | | | | | | | | | |
| **Frob** | 0.013 | **0.695** | **1.000** | **0.497** | **1.000** | 0.227 | **1.000** | 0.003 | 0.016 | **1.000** | 0.218 | **0.615** |
| **Sfrob** | 0.047 | **1.000** | **0.268** | **0.631** | **1.000** | **0.631** | **1.000** | 0.051 | 0.051 | **1.000** | **0.545** | **0.545** |
| **Euclid** | 0.018 | **0.461** | **1.000** | **1.000** | **0.315** | **0.429** | **1.000** | 0.001 | 0.015 | **1.000** | **0.511** | **0.511** |
| **ST** | 0.000 | 0.000 | **1.000** | **1.000** | 0.000 | 0.015 | **1.000** | 0.000 | 0.003 | **1.000** | 0.000 | **0.278** |
| **vND** | 0.000 | **0.265** | **1.000** | **1.000** | 0.002 | 0.029 | **1.000** | 0.000 | 0.000 | **1.000** | 0.000 | 0.160 |
| **QLIK** | 0.000 | 0.000 | **1.000** | **1.000** | 0.000 | 0.014 | **1.000** | 0.000 | 0.003 | **1.000** | 0.000 | **0.274** |

Note: Panel A reports averaged values of the loss functions listed in Table 9 over the out-of-sample period, where the best performing model within each row is in bold. Entries in Panel B and C are p-values of the MCS with 10% and 25% size, respectively. Included models in bold.

As for the second topic, we develop a bootstrap approach to the generation of multi-step-ahead predictions. In an application to a portfolio of 50 US stocks we provide compelling evidence that the MMReDCC model is useful for out-of-sample forecasting purposes even when one has to deal with realistic high dimensional datasets. If compared with existing multivariate competitors not accounting for time-varying long-term dynamics, the MMReDCC is found to deliver the most accurate predictions especially at medium-term horizons, thus indicating the importance of allowing for a long-run component.

In this respect, the model lends itself to several useful applications. For example, an extension being currently explored includes the incorporation of influential macroeconomic or financial variables directly into the specification of the long-term (co)volatility dynamics. This has been addressed by Engle et al. [16] in the univariate framework but no attempt has been made so far in a reasonably large multivariate setting. Over longer horizons ($\geq$ 1 month), the inclusion of low-frequency macro-variables is possibly expected to favorably impact on the overall fit of the model and to further improve its forecasting accuracy.

## Appendix A. Application I: constituents and descriptive statistics of daily realized variances.

| Symbol | Issue name | Mean | Max. | Min. | Std.dev. | Skewness | Kurtosis |
|---|---|---|---|---|---|---|---|
| | **Dataset 1: February, 2001 – December, 2009** | | | | | | |
| AA | Alcoa | 5.458 | 277.308 | 0.074 | 16.811 | 7.178 | 72.570 |
| AXP | American Express | 5.055 | 176.478 | 0.112 | 11.094 | 7.529 | 84.686 |
| BAC | Bank of America | 1.934 | 57.543 | 0.075 | 3.362 | 7.319 | 85.006 |
| KO | Coca Cola | 2.455 | 43.106 | 0.084 | 3.412 | 4.724 | 36.234 |
| DD | Du Pont | 2.073 | 115.378 | 0.126 | 4.155 | 13.296 | 288.066 |
| GE | General Electric | 4.944 | 160.241 | 0.294 | 8.935 | 7.635 | 92.124 |
| IBM | International Business Machines | 4.420 | 201.879 | 0.077 | 9.154 | 8.536 | 133.699 |
| JPM | JP Morgan | 2.529 | 63.874 | 0.163 | 3.728 | 6.442 | 68.505 |
| MSFT | Microsoft | 3.196 | 114.256 | 0.097 | 7.114 | 7.232 | 75.484 |
| XOM | Exxon Mobil | 1.414 | 56.505 | 0.039 | 2.254 | 9.715 | 180.206 |
| | **Dataset 2: January, 2007 – December, 2012** | | | | | | |
| ACAS | American Capital | 8.576 | 331.786 | 0.060 | 20.844 | 7.226 | 78.667 |
| AET | Aetna | 8.163 | 771.525 | 0.109 | 26.593 | 17.882 | 467.969 |
| AFL | Aflac Incorporated | 9.113 | 675.348 | 0.133 | 27.345 | 13.811 | 284.791 |
| AIG | American International Group | 8.799 | 555.098 | 0.103 | 26.382 | 11.459 | 185.778 |
| AIZ | Assurant | 8.613 | 325.167 | 0.101 | 23.230 | 7.712 | 79.082 |
| ALL | The Allstate Coprporation | 8.213 | 543.714 | 0.186 | 24.593 | 11.277 | 189.052 |
| AMP | Ameriprise Financial | 7.679 | 264.761 | 0.129 | 17.790 | 6.098 | 54.262 |
| AXP | American Express Company | 8.076 | 945.750 | 0.095 | 30.571 | 21.795 | 618.891 |
| BAC | Bank of America | 8.450 | 332.586 | 0.130 | 22.830 | 8.458 | 96.824 |
| BBT | BB&T Corporation | 9.093 | 613.826 | 0.087 | 28.801 | 11.267 | 184.837 |
| | **Dataset 3: January, 2007 – December, 2012** | | | | | | |
| STI | SunTrust Banks | 8.510 | 388.707 | 0.086 | 22.765 | 8.088 | 94.838 |
| STT | State Street Corporation | 8.985 | 315.656 | 0.039 | 23.192 | 7.265 | 74.391 |
| TMK | Torchmark Corporation | 8.748 | 537.273 | 0.022 | 24.479 | 10.543 | 176.515 |
| TROW | T.Rowe Price Group | 8.991 | 425.263 | 0.073 | 25.019 | 8.753 | 108.221 |
| UNH | UnitedHealth Group | 8.344 | 378.667 | 0.130 | 21.065 | 8.534 | 112.479 |
| UNM | Unun Group | 8.046 | 309.086 | 0.044 | 19.440 | 8.564 | 107.272 |
| USB | U.S.Bancorp | 10.176 | 2534.073 | 0.079 | 69.496 | 32.392 | 1164.053 |
| WFC | Wells Fargo & Company | 9.481 | 525.034 | 0.106 | 30.162 | 10.717 | 156.791 |
| WU | The Western Union Company | 8.775 | 484.124 | 0.056 | 25.204 | 9.954 | 145.509 |
| ZION | Zions Bancorporation | 15.302 | 6855.823 | 0.136 | 197.794 | 30.674 | 1005.621 |

## Appendix B. Application II: descriptive statistics of daily realized variances.

**Estimation sample: May 12, 1997 to July 17, 2006 (2024 observations)**

| Stock | Mean (e-03) | Max. | Min.(e-03) | Std.dev.(e-03) | Skewness | Kurtosis |
|---|---|---|---|---|---|---|
| ABT | 0.434 | 0.025 | 0.026 | 0.948 | 15.958 | 333.606 |
| AFL | 0.491 | 0.027 | 0.020 | 1.208 | 13.097 | 225.997 |
| APD | 0.490 | 0.075 | 0.020 | 1.896 | 32.234 | 1221.466 |
| AA | 0.540 | 0.012 | 0.040 | 0.759 | 7.886 | 94.971 |
| ALL | 0.497 | 0.098 | 0.012 | 2.319 | 37.311 | 1554.460 |
| AXP | 0.549 | 0.041 | 0.014 | 1.553 | 16.921 | 374.949 |
| AIG | 0.391 | 0.073 | 0.027 | 1.724 | 38.011 | 1597.332 |
| ADI | 1.375 | 0.043 | 0.070 | 2.153 | 7.635 | 103.080 |
| APOL | 1.276 | 0.080 | 0.043 | 2.556 | 16.984 | 470.125 |
| T | 0.620 | 0.046 | 0.017 | 1.659 | 15.676 | 339.927 |
| AZO | 0.497 | 0.046 | 0.021 | 1.250 | 25.934 | 909.714 |
| AVY | 0.416 | 0.064 | 0.019 | 1.643 | 30.923 | 1136.941 |
| BHI | 1.000 | 0.098 | 0.060 | 3.284 | 23.106 | 603.142 |
| BAC | 0.477 | 0.054 | 0.015 | 1.681 | 20.789 | 578.406 |
| BAX | 0.447 | 0.059 | 0.022 | 1.884 | 22.785 | 609.476 |
| BDX | 0.481 | 0.036 | 0.021 | 1.109 | 19.322 | 541.972 |
| BBBY | 1.078 | 0.029 | 0.051 | 1.579 | 7.060 | 92.898 |
| BMY | 0.565 | 0.051 | 0.028 | 2.005 | 19.683 | 450.186 |
| CPB | 0.470 | 0.083 | 0.012 | 1.940 | 38.241 | 1616.776 |
| COF | 0.909 | 0.092 | 0.023 | 3.300 | 20.993 | 547.443 |
| CAH | 0.465 | 0.044 | 0.016 | 1.705 | 18.142 | 393.755 |
| CTL | 0.492 | 0.069 | 0.026 | 2.049 | 23.613 | 687.584 |
| CTAS | 1.104 | 0.113 | 0.018 | 2.839 | 31.100 | 1212.341 |
| C | 0.624 | 0.086 | 0.021 | 2.305 | 27.085 | 952.213 |
| CLX | 0.481 | 0.074 | 0.026 | 2.203 | 27.445 | 840.327 |
| CMS | 0.764 | 0.083 | 0.034 | 2.854 | 20.911 | 544.730 |
| KO | 0.314 | 0.009 | 0.015 | 0.464 | 7.565 | 101.565 |
| CL | 0.401 | 0.058 | 0.026 | 1.569 | 27.867 | 957.179 |
| CMA | 0.345 | 0.025 | 0.011 | 1.011 | 16.800 | 350.599 |
| CSC | 0.764 | 0.105 | 0.031 | 2.884 | 25.713 | 861.526 |
| CAG | 0.530 | 0.072 | 0.014 | 2.129 | 25.460 | 758.407 |
| COST | 0.773 | 0.135 | 0.042 | 3.358 | 33.314 | 1277.539 |
| DOV | 0.441 | 0.050 | 0.031 | 1.334 | 27.171 | 949.990 |
| DOW | 0.474 | 0.033 | 0.018 | 0.994 | 20.169 | 609.262 |
| DTE | 0.296 | 0.068 | 0.017 | 1.552 | 41.597 | 1818.052 |
| EMN | 0.432 | 0.089 | 0.027 | 2.141 | 35.535 | 1433.408 |
| EIX | 1.103 | 0.251 | 0.022 | 6.892 | 26.622 | 885.136 |
| ETR | 0.332 | 0.029 | 0.018 | 0.779 | 26.216 | 937.670 |
| FDO | 0.911 | 0.093 | 0.043 | 2.519 | 26.543 | 911.464 |
| FISV | 0.913 | 0.063 | 0.041 | 1.715 | 24.450 | 850.641 |
| F | 0.635 | 0.024 | 0.050 | 1.053 | 10.141 | 170.597 |
| GCI | 0.272 | 0.010 | 0.015 | 0.370 | 12.616 | 289.030 |
| GPS | 1.007 | 0.049 | 0.032 | 2.730 | 10.649 | 142.751 |
| GE | 0.445 | 0.037 | 0.013 | 1.233 | 19.948 | 515.188 |
| GIS | 0.231 | 0.008 | 0.013 | 0.334 | 9.989 | 181.369 |
| GPC | 0.427 | 0.089 | 0.018 | 2.039 | 40.541 | 1745.937 |
| HNZ | 1.240 | 0.075 | 0.030 | 2.547 | 19.600 | 511.929 |
| HPQ | 0.291 | 0.030 | 0.014 | 0.918 | 24.562 | 727.020 |
| HD | 0.902 | 0.047 | 0.022 | 1.965 | 11.388 | 195.042 |
| HON | 0.643 | 0.078 | 0.028 | 2.418 | 23.919 | 682.976 |

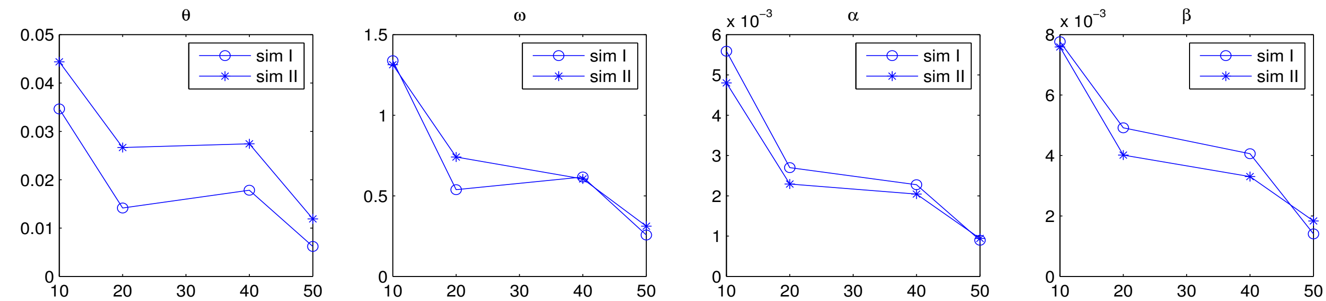
**Forecasting sample: July 18, 2006 to July 18, 2008 (500 observations)**

| Stock | Mean (e-03) | Max. | Min.(e-03) | Std.dev.(e-03) | Skewness | Kurtosis |
|---|---|---|---|---|---|---|
| ABT | 0.191 | 0.004 | 0.020 | 0.263 | 8.887 | 122.525 |
| AFL | 0.268 | 0.004 | 0.015 | 0.406 | 4.815 | 35.550 |
| APD | 0.261 | 0.003 | 0.013 | 0.292 | 4.633 | 38.977 |
| AA | 0.559 | 0.007 | 0.051 | 0.682 | 4.078 | 25.551 |
| ALL | 0.232 | 0.002 | 0.012 | 0.304 | 3.423 | 18.970 |
| AXP | 0.498 | 0.015 | 0.010 | 0.919 | 8.966 | 127.909 |
| AIG | 0.523 | 0.010 | 0.017 | 1.020 | 4.375 | 27.892 |
| ADI | 0.432 | 0.014 | 0.046 | 0.816 | 11.614 | 168.440 |
| APOL | 0.939 | 0.079 | 0.035 | 4.173 | 14.929 | 261.548 |
| T | 0.278 | 0.004 | 0.023 | 0.399 | 6.116 | 50.180 |
| AZO | 0.303 | 0.010 | 0.016 | 0.517 | 13.313 | 242.173 |
| AVY | 0.243 | 0.005 | 0.019 | 0.446 | 7.402 | 70.878 |
| BHI | 0.506 | 0.009 | 0.079 | 0.563 | 8.511 | 108.889 |
| BAC | 0.495 | 0.016 | 0.012 | 1.223 | 7.682 | 79.279 |
| BAX | 0.190 | 0.004 | 0.020 | 0.306 | 7.569 | 80.297 |
| BDX | 0.150 | 0.003 | 0.017 | 0.210 | 8.356 | 104.636 |
| BBBY | 0.464 | 0.010 | 0.029 | 0.627 | 7.907 | 106.865 |
| BMY | 0.303 | 0.007 | 0.017 | 0.521 | 7.153 | 72.678 |
| CPB | 0.152 | 0.002 | 0.011 | 0.206 | 4.746 | 30.567 |
| COF | 0.923 | 0.013 | 0.028 | 1.481 | 3.995 | 25.175 |
| CAH | 0.186 | 0.005 | 0.011 | 0.294 | 9.695 | 135.019 |
| CTL | 0.279 | 0.021 | 0.022 | 1.008 | 17.453 | 350.144 |
| CTAS | 0.293 | 0.006 | 0.034 | 0.395 | 8.196 | 99.256 |
| C | 0.648 | 0.014 | 0.019 | 1.276 | 5.411 | 41.978 |
| CLX | 0.153 | 0.007 | 0.012 | 0.416 | 13.180 | 203.335 |
| CMS | 0.219 | 0.003 | 0.020 | 0.242 | 5.684 | 49.133 |
| KO | 0.112 | 0.002 | 0.005 | 0.181 | 7.972 | 78.506 |
| CL | 0.125 | 0.004 | 0.015 | 0.212 | 11.526 | 172.718 |
| CMA | 0.632 | 0.023 | 0.017 | 1.400 | 9.231 | 131.210 |
| CSC | 0.285 | 0.005 | 0.025 | 0.412 | 6.514 | 62.089 |
| CAG | 0.179 | 0.005 | 0.016 | 0.313 | 9.470 | 120.784 |
| COST | 0.338 | 0.010 | 0.030 | 0.564 | 11.158 | 166.572 |
| DOV | 0.245 | 0.003 | 0.025 | 0.327 | 5.352 | 39.200 |
| DOW | 0.347 | 0.007 | 0.028 | 0.616 | 6.593 | 55.276 |
| DTE | 0.177 | 0.002 | 0.015 | 0.202 | 4.481 | 34.565 |
| EMN | 0.292 | 0.005 | 0.024 | 0.429 | 6.244 | 57.701 |
| EIX | 0.223 | 0.005 | 0.026 | 0.320 | 7.899 | 96.229 |
| ETR | 0.203 | 0.003 | 0.017 | 0.281 | 5.482 | 44.736 |
| FDO | 0.770 | 0.017 | 0.033 | 1.332 | 7.121 | 70.492 |
| FISV | 0.276 | 0.006 | 0.031 | 0.366 | 7.898 | 96.806 |
| F | 0.802 | 0.009 | 0.091 | 1.101 | 4.107 | 23.288 |
| GCI | 0.305 | 0.012 | 0.018 | 0.658 | 12.646 | 218.866 |
| GPS | 0.564 | 0.007 | 0.028 | 0.681 | 4.354 | 30.343 |
| GE | 0.213 | 0.016 | 0.016 | 0.718 | 19.569 | 416.499 |
| GIS | 0.108 | 0.002 | 0.011 | 0.137 | 6.537 | 58.387 |
| GPC | 0.182 | 0.002 | 0.017 | 0.189 | 5.554 | 49.714 |
| HNZ | 0.309 | 0.003 | 0.043 | 0.280 | 4.217 | 28.816 |
| HPQ | 0.121 | 0.002 | 0.016 | 0.138 | 5.344 | 43.491 |
| HD | 0.289 | 0.005 | 0.024 | 0.387 | 6.301 | 64.372 |
| HON | 0.394 | 0.004 | 0.042 | 0.469 | 3.953 | 25.552 |

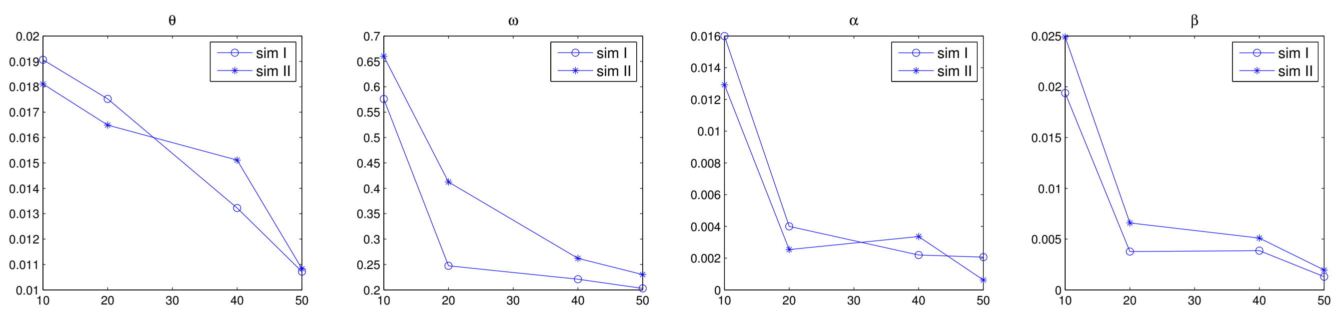
## Appendix C. Figures

**Figure C.2:** Standard deviation of the IMP Monte Carlo estimated scalar parameters $\theta, \omega, \alpha$ and $\beta$ against the cross-section dimension ranging from 10 to 50. Results from the two simulation studies for $T = 1000, 2000$ jointly reported respectively in Panel (a) and (b).

**(a)** $T = 1000$




**(b)** $T = 2000$



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