Macroeconomic stress–testing with nonlinear BVARs*

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Abstract

Motivated by policy-makers' needs to build statistical models to properly probe macroeconomic tail risks in stress-testing exercises, we build on our in-house linear vector-autoregressive (VAR) models to include nonlinearities, motivated by the desire to capture potentially different economic dynamics during stressful times. We estimate two types of regime–switching models: Threshold VAR and Markov Switching VAR. For each of the models, we estimate regimes which carry the interpretation of recessionary/normal and financially stressful/stable periods. Using the recursiveness assumption to identify financial and output shocks and conditional on shocks of one standard deviation, we show that (i) *financial shocks* hitting during times of recessions create disproportionately more severe contractions in output; (ii) *output growth shocks* hitting financially stressful times will result in disproportionately further financial stress. We also demonstrate the power of *feedback loop* between the real and the financial sectors when extremely large shocks hit the economy in normal/financially stable periods. Afterwards, we perform unconditional and conditional forecasting exercises, and find that the threshold VAR model performs relatively better in reduced-form *conditional* macro tail forecasting. Our findings provide useful information for policy makers to investigate and to forecast macroeconomic tail risks, as well as to calibrate their stress scenarios conditional on structural shocks.

Keywords: Nonlinearity, Threshold VAR, Markov Switching VAR, Generalised Impulse Responses, Density Forecasts

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1 Introduction

Stress tests are new and developing tools for policy–makers to assess the resilience of financial systems. They are designed to examine the health of the banking system and individual institutions under some *hypothetical adverse macroeconomic scenarios*. The analysis provides a forward looking assessment of capital adequacy and is used as a basis for devising measures for maintaining financial stability.[1]

An important challenge accompanying stress test is the construction of models for scenario design purposes. Scenarios should be constructed such that they adequately include the risks that the banks might face domestically but also global risks. Moreover they should also be severe but plausible at the same time (see Bank of England (2013)). This paper contributes to the aspect of probing macroeconomic risks by studying the traditional vector-autoregressive models. Our objective is to provide credible tools to enable policy makers to study structural shock transmission during stressful times and to forecast macroeconomic tail risks.

In finance, value–at–risks models are commonly used to measure the loss risk on a specific portfolio of financial exposures. This concept has been recently adopted to macroeconomics. Boucher and Maillet (2015) estimates value–at–risk of US output using quantile regressions, and produce a fan chart for out–of–sample forecasting for industrial production growth. This use of quantile regression highlights the importance of outliers which are otherwise overlooked in the basic regressions, a reflection of the presence of nonlinearities in the data. Outliers are associated with extreme events, which undoubtedly provide valuable information for modelling and forecasting future tail events. This point is shown forcefully in Covas et al. (2014) who use fixed effect quantile autoregressive models to capture the nonlinear dynamic of banks losses and revenues and to project capital shortfalls. Recently, White et al. (2015) provides a theoretical framework to estimate and make inference in multi-variate, multi-quantile models.

While acknowledging the usefulness of quantile regressions, this paper takes a more traditional regime–switching modelling approach to capture the data nonlinearities in the United Kingdom. This is motivated by our desire to assign proper economic meanings to the regimes and to perform more structural analyses. It is widely discussed that the variance of shocks can be time–varying (see Primiceri (2005)). Moreover, there is reason to believe that an economy displays structural breaks in means and variances during stressful times. As discussed by Drehmann et al. (2007), linear approximations might sufficiently work in the middle of the distributions but can behave badly in the tails. Simply, linear models might not be able to catch tail risks and capture the possible impacts of adverse scenarios.

More specifically, we build a quarterly linear Bayesian Vector Autoregression (henceforth BVAR) model as specified in Aikman et al. (2011). We consider a Threshold VAR (TVAR) model and

[1] Please refer to Bank of England (2015).

a Markov switching VAR (MSVAR) for a system summarising (i) the real economy (proxied by *annualised* real GDP growth and inflation rates); (ii) the monetary policy (proxied by the short-term interest rate); (iii) financial sector stress (proxied by corporate bond spreads); (iv) banking sector performance (proxied by the *annualised* aggregate return of UK banks in excess of the market). The system can be easily extended to include other variables which reflect the features of the UK being a small open economy.

As discussed in Kalliovirta et al. (2014), TVARs model the conditional expectation of a time series given its history, where the conditional expectation is specified as a convex combination of two or more linear VAR models. In our set-up, we consider two variants of TVAR. First identifies the regimes as recessionary and non-recessionary and second defines the regimes as financially stressful or not. Recessionary regimes are defined as two consecutive quarters registering the real GDP growth being below the estimated threshold (which is close to 0 percent), whereas financially stressful regimes are defined as the corporate bond spreads rise above an estimated threshold (which is about 290 basis points). As for our MSVAR, we assume a *joint dynamic* for both variance and coefficient regimes to identify structural shocks.[2] These two nonlinear models, TVAR and MSVAR, are going to be compared against the linear BVAR model.

All of the three models are estimated with data between 1965:Q2 and 2014:Q2. The TVAR model manages to capture all the recessionary periods and financially stressful periods. The MSVAR model generates estimated regimes of high volatility during financially volatile times and regimes of high mean during the stagflation periods in the 1970s.

To study structural shock transmission under different regimes, we adopt the common recursiveness assumption in the macroeconomic literature. Our identification strategy is closely in line with Christiano et al. (1998), Gilchrist and Zakrajšek (2012) and broadly similar to Hubrich and Tetlow (2015). We study financial shocks (proxied by exogenous jump in the corporate bond spreads), negative output growth shocks and monetary policy shocks, and compute the corresponding impulse responses using the generalised impulse response functions described in Koop et al. (1996). The nonlinear TVAR models successfully capture the following results: (i) financial shocks hitting during times of recessions create disproportionately more severe recessions. One-standard-deviation rise in corporate bond spreads in recessions (the size of which is 60 percent larger than that in non-recessionary regimes) lead to six times larger the maximum drop in real output growth. Aggregate bank excess returns also drop significantly; (ii) negative output growth shocks hitting during times of financial stress will lead to disproportionately higher financial stress. In particular, negative growth shocks in financially stressful regimes generate a surge in corporate bond spreads seven times as large to growth shocks of a similar size in the financially non-stressful world; (iii) As for interest rate shocks, we find some evidence

[2]The extension of MSVAR model which exploits two independent Markov chains, one for the slope and intercept coefficients, the other one for the covariance matrix, is given in Appendix however will not be followed in the body of the paper.

that the drop in output growth is deeper but much less persistent in recessions. Aggregate bank excess returns drop in recessions as opposed to a rise as predicted by the linear model. These results, failed to be captured by the linear BVAR model, provide useful information to calibrate macro stress scenarios and to probe tail risks conditional on structural shocks.

Another interesting result is the evidence to support the feedback loop between the real and financial sectors. Conditional on extremely large shocks of the magnitude of three standard deviations (akin to tail events), economic contractions and resulting financial stress are disproportionately larger when such shocks hit at the *good* states of the world. For example, a severe drop in output growth rate during non-recessionary regimes create disproportionately sharp and persistent rise in corporate bond spreads. The sharp rise in financial stress then feeds back to the real economy: the subsequent fall in output growth becomes disproportionately more persistent and more negative. A very similar story applies when extremely large financial shocks hit financially non-stressful regimes: huge financial shocks as embodied by substantial spikes in corporate bond spreads lead to disproportionately deeper and more persistent contraction of the real output. These results indirectly lend support to the recent proposal of the counter-cyclical approach in the design of stress scenarios as set out in Bank of England (2015): '[i]mportantly, the severity is likely to be greater in a boom, for example, when growth in credit is rapid and asset prices unsustainably high' (page 14).

After the structural exercises, we turn to forecasting analysis which is more reduced-form in nature. We re-estimate the model with data until 2007:Q2, right before the Great Recession set in. We then produce multi-step ahead predicative densities for the variables in the system and compare them against the outturns during the Great Recession period. We have the following two questions in mind. Firstly, do nonlinear models produce unconditional density forecasts more informative in the tail? Secondly, could we produce more reliable predictive densities when we condition on variable paths consistent with stressful times? Notice that we are not after predicting the timing of crises or any stress events, but assessing the capabilities of different models to generate a broad view on tail risks.

The answers to the two questions are sequentially 'slightly more' and 'potentially substantial'. We find that unconditional density forecasts produced by MSVAR and TVAR do not exhibit huge advantages in generating a broad view of tail risk forecasts. On the other hand, when we follow Waggoner and Zha (1999) to perform conditional forecasting based on variable paths of *GDP growth rates* and *corporate bond spreads* during the 2008–09 Great Recession, TVAR, and to a lesser extent MSVAR, show substantial improvements in the tail density forecasts for the non-conditioned variables. Our findings confirm the importance of incorporating nonlinearities in modelling macro data, and demonstrate the usefulness of such nonlinear models to generate reasonable tail forecasts for policy-making purposes.

On the stress-testing front, this paper is directly related to the literature which uses VAR models for stress-testing. Hoggarth et al. (2005) proposes a parsimonious linear VAR model to account for the dynamics between banks' write-off to loan ratios and key macroeconomic variables. Drehmann et al. (2007) uses a VAR with third-order approximations to study the dynamics between corporate defaults and macroeconomic variables, and the local projections method to study the relevant impulse responses. Covas et al. (2014) uses fixed effect quantile autoregressive models to capture the nonlinear dynamic of banks losses, and shows that such a framework delivers superior out-of-sample forecasting performance relative to the standard linear VAR framework.[3] Our paper is also related to an emerging literature which develops methods to stress test banks using granular balance sheet data, such as Kapinos and Mitnik (2015) and Pritsker (2015).

This paper is also highly associated with an active area of empirical research using nonlinear VAR models. Sims and Zha (2006) employs a multivariate regime switching model for US monetary policy in a structural VAR framework. Their findings indicate no changes in the coefficients across different regimes but changes in the variances of structural variances. The interaction between inflation expectations and nominal and real macroeconomic variables in the UK after World War II is investigated by Barnett et al. (2010) using a Markov switching structural VAR model. The results show that the impact of the shocks to inflation expectations to real inflation changed after 1970s. Similar results are found for oil price shocks and real demand shocks.

Barnett et al. (2012) investigates the performance of a variety of models with time-varying parameters in forecasting UK GDP growth, inflation and interest rate. The paper compares numerous econometric models in terms of their forecasting performance. Alessandri and Mumtaz (2014) constructs a set of linear and nonlinear econometric models to study the predictive densities, especially focus on tails to assess the power of financial indicators for output and inflation in the US. Among other contributions, they found evidence on nonlinear models generating noisier central forecasts compared to their linear peers but they perform better in predicting distributions. Hubrich and Tetlow (2015) investigates the interaction between a financial stress index for the US and real activity, inflation, monetary policy using a Markov switching VAR model. The empirical findings support the inadequacy of single regime models to capture the dynamics of the economy.

The remainder of this paper proceeds as follows. We introduce the econometric models used in this paper in section 2. This section also provides details on which priors we use and how they are incorporated to the Gibbs sampler. Section 3 presents the features of our data set. We illustrate the estimation results of the proposed procedure in section 4. Section 5 provides our analysis on the structural shock transmission in our models. Section 6 follows with the

[3] Schuermann (2014) provides an in-depth discussion on the scenario design and compares the scenarios in various stress-testing exercises. Breuer et al. (2009) discusses scenario design using a quantitative risk management framework.

forecasting results. Section 5 reports the impulse response analysis. We conclude with section 7 and some other details such as the graphs of individual series are given in the appendix.

2 Models

In this section, we present the models under study, namely, the threshold VAR, the Markov switching VAR and the linear VAR models. We will give a brief overview of each of the models, and will refer the readers to the appendix for technical details.

2.1 Threshold VAR

Threshold VAR model comprises an explicit threshold variable which allows regimes to switch. The associated model is

$$Y_t = \left[c_1 + \sum_{j=1}^{P} \beta_{1,j} Y_{t-j} + v_t \right] R_t + \left[c_2 + \sum_{j=1}^{P} \beta_{2,j} Y_{t-j} + v_t \right] (1 - R_t)$$

where

$$R_t = 1 \Longleftrightarrow Z_{t-d} \leq Z^* \text{ and } Z_{t-d+1} \leq Z^*$$

The threshold Z^* is assumed to have a normal prior, $Z^* \sim N(\bar{Z}, \bar{V})$ where $\bar{Z}$ is the sample mean of the threshold variable.

The delay parameter, d, is also reffered as threshold lag. We determine the state of the economy by looking at the threshold variable on the determined delay of the chosen variable, Y_i. However the important feature of the model we are considering is that we define the states such that regime 1 only occurs, $R_t = 1$ if the threshold variable is below the estimated threshold for *two consecutive quarters*. Regime 1 occurs for those two consecutive periods both and the following periods which will occur after that also satisfy this condition. Therefore, by taking into account the definition of recessions, we set the delay parameter $d = 1$ to be in line with the lag restriction which is taken to be 2.

We impose normal inverse Wishart priors following Bańbura et al. (2010). To estimate this model, we follow Alessandri and Mumtaz (2014) in using the Gibbs sampling algorithm which includes a Metropolis Hastings step for sampling the threshold value in each simulation. Technical details are reported in the appendix.

We employ two TVAR exercises. We investigate the features of a TVAR model where we use output growth as the threshold variable (henceforth TVAR–Y). Additionally, we use corporate bond spreads as the threshold variable (henceforth TVAR–S). Both models have different implications. The former identifies recessionary and non–recessionary regimes whereas the latter classifies financially stressful and non–stressful states of the economy.

2.2 Markov-Switching Vector Autoregression Models

The Markov switching VAR Model (MSVAR) is written as

$$Y_t = c_{S_t} + B_{1,S_t}Y_{t-1} + B_{2,S_t}Y_{t-2} + \cdots + B_{l,S_t}Y_{t-L} + v_t, \tag{1}$$

where $e_t \sim N(0, \Omega_{S_t})$. Both coefficients and the covariance matrix are state dependent for $S_t = 1, ..., M$. The data vector Y contains the five macro-financial variables of interest. The lag length is given by L.

The model can also be represented in the following compact way

$$\begin{aligned} Y_t =& c_{S_t} + \sum_{j=1}^{L} B_{j,S_t}Y_{t-j} + v_t \\ Y =& XB + v, \end{aligned}$$

where $Y = (Y_1, \dots, Y_T)'$ is a $T \times N$ matrix, $X = (X_1, \cdots, X_T)'$ is a $T \times k$ matrix where $X_t = (Y'_{t-1}, \dots, Y'_{t-L}, 1)'$, $e = (e_1, \dots, e_T)'$ and $B_S = (B_{1,S}, \dots, B_{L,S}, c_S)'$ is a $k \times N$ matrix where $k = NL + 1$.

The regime switches follow a joint dynamic for coefficients and variance at the same time. The latent regimes S_t are assigned as $S = 1, 2$. The switch between these latent states is governed by the transition matrix, P,

$$\begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}$$

where $p_{ij} = prob(S_t = i|S_{t-1} = j)$ indicates regime i is followed by regime j. There are no restrictions on regime switches, i.e they are left unrestricted to jump back and forth. The columns sum up to 1.

We impose the normal inverse Wishart priors as in TVAR model. We assume that the VAR coefficients B are normally distributed:

$$p(B) \sim N(B_0, H)$$

where B_0 is a $(N \times (N \times L + 1)) \times 1$ vector of prior mean and H is a $(N \times (N \times L + 1)) \times (N \times (N \times L + 1))$ matrix whose diagonal elements refer the variance of the prior.

2.3 Benchmark Bayesian VAR

Bayesian VAR model is used as a benchmark to our nonlinear VAR models. We perform estimation and forecasting exercises for BVAR model to compare the results with MSVAR and TVAR models. This linear model is given by

$$Y_t = c + B_1 Y_{t-1} + B_2 Y_{t-2} + \cdots + B_l Y_{t-L} + v_t, \tag{2}$$

We impose the same priors as in nonlinear models and estimate this model by the usual Gibbs sampler.

3 Data

Our quarterly data run from 1965:Q2 to 2014:Q2. We have a 5 variable system including the output growth rate (proxied by quarterly change in real GDP), inflation rate (proxied by the quarterly change in consumer price index), excess bank returns, corporate bond spreads (proxied by the difference between UK Corporate Bond Yield and UK 10–year Government Bond Yield) and the short term interest rate (proxied by the three-month interest rate). Except for the short term interest rate and corporate bond spreads, all series are annualised. The choice of variables reflects our goal of capturing the overall dynamics in the economy and linking the real economic sector to the banking and financial sectors. In the Appendix section, Tables 1 and 2 describe the basic statistics of the data, and figure 5 plots the five variables of interest.

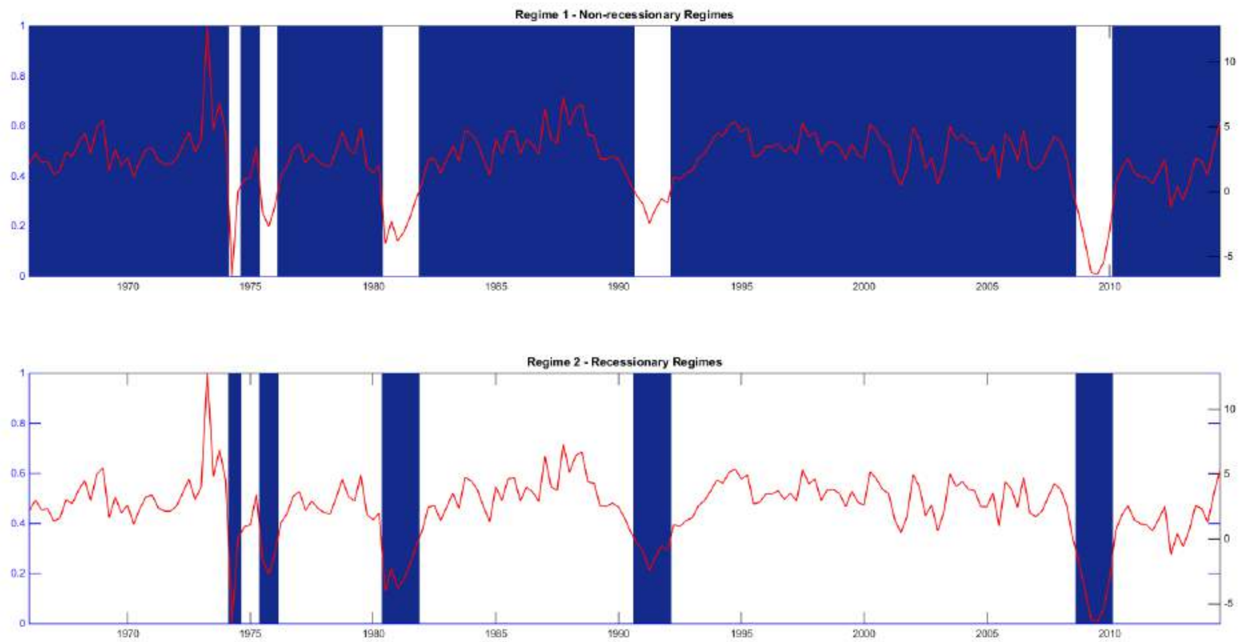
4 Full sample estimation results

This section reports the estimation results for both nonlinear models using the full sample from 1965:Q2 to 2014:Q2. Figure 1 and 2 illustrate the recessionary and non–recessionary regimes and financial stress and financial non-stress regimes modelled by the TVAR–Y and TVAR–S models, respectively. Figure 3 reports the estimated regime probabilities addressing high and low stress states.

Due to the set-up of the threshold VAR model, the regime changes are abrupt and the economy is either in one regime or the other. Therefore the probabilities accompanying the regimes are either 0 or 1. As discussed before, the GDP growth rate is used as one of the threshold variables in the estimation with the threshold values being endogenously estimated. Recessions are defined as the GDP growth being under the estimated threshold for two consecutive quarters. We are particularly interested in capturing the recessionary periods with this model. By definition, the first regime in figure 1 can be labelled as *nonrecessionary* whereas the second regime as *recessionary* periods.

The first two recessions coincide with the mid–1970s recessions. They are associated with the 1973 oil crisis and stagflation that are followed by the decline of traditional British industries and inefficient production caused by excessive union wage demands. The next is the recession in the early 1980s due to deflationary government policies including spending cuts, pursuance of monetarism to reduce inflation, and switches from a manufacturing economy to a services economy. The fourth is the recession in the early 1990s recession which started in the third quarter of 1990 and went on for five quarters. That was mainly driven by US savings and loan crisis. The last recession is the Great Recession when the GDP hit the bottom by dropping almost -7% in the first quarter of 2009.

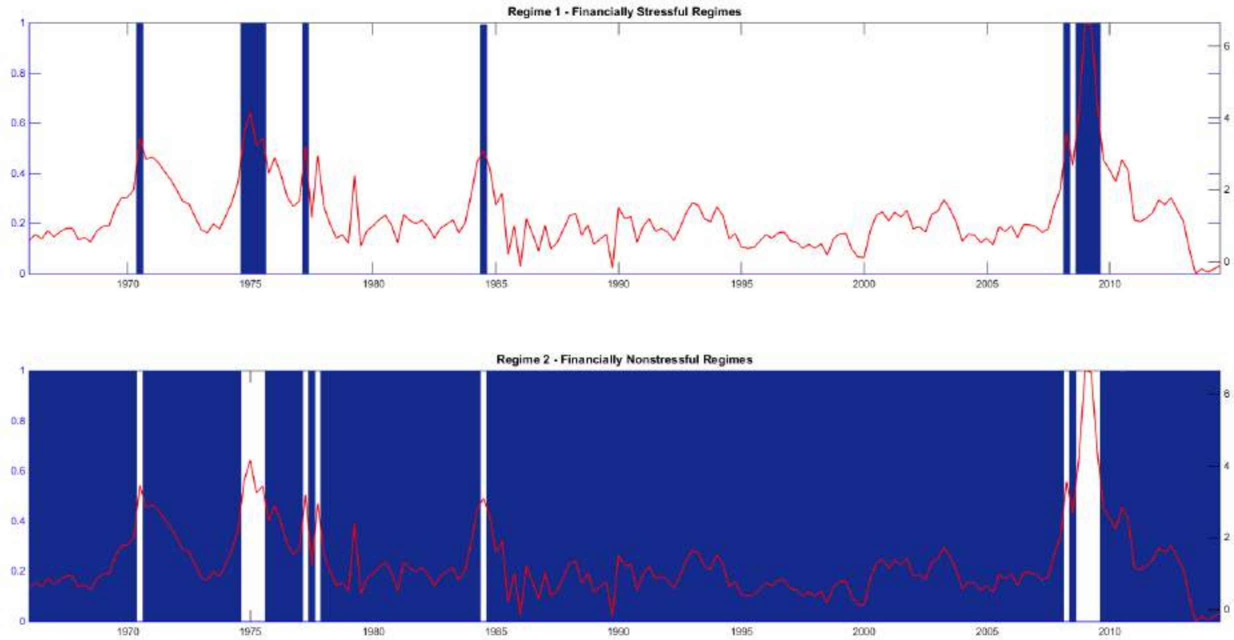
Figure 1: Full sample regimes for TVAR–Y



Notes: The threshold variable is the real GDP growth which is overlapped with the probabilities of each regime.

Similarly, figure 2 presents the regimes where the corporate bond spreads are used as the threshold variable in the estimation of TVAR model. The first financial stress period corresponds to the second quarter of 1970. The second reflects the impact of oil crisis and stagflation on the corporate bond market over the periods 1974:Q3–1975:Q2. The third and fourth single period financial stress regimes are associated with 1977:Q1 and 1984:Q2. The former is the introduction of small number of floating-rate issues, both of gilt-edged and corporate loan stocks to the life assurance companies and pension funds. The latter is the degulation of the Eurobond market and openning up of the international markets. The last financial stress periods show the effects of the Great Recession onto the market and overlap with the recessionary regimes with an exception of the second quarter of 2008.

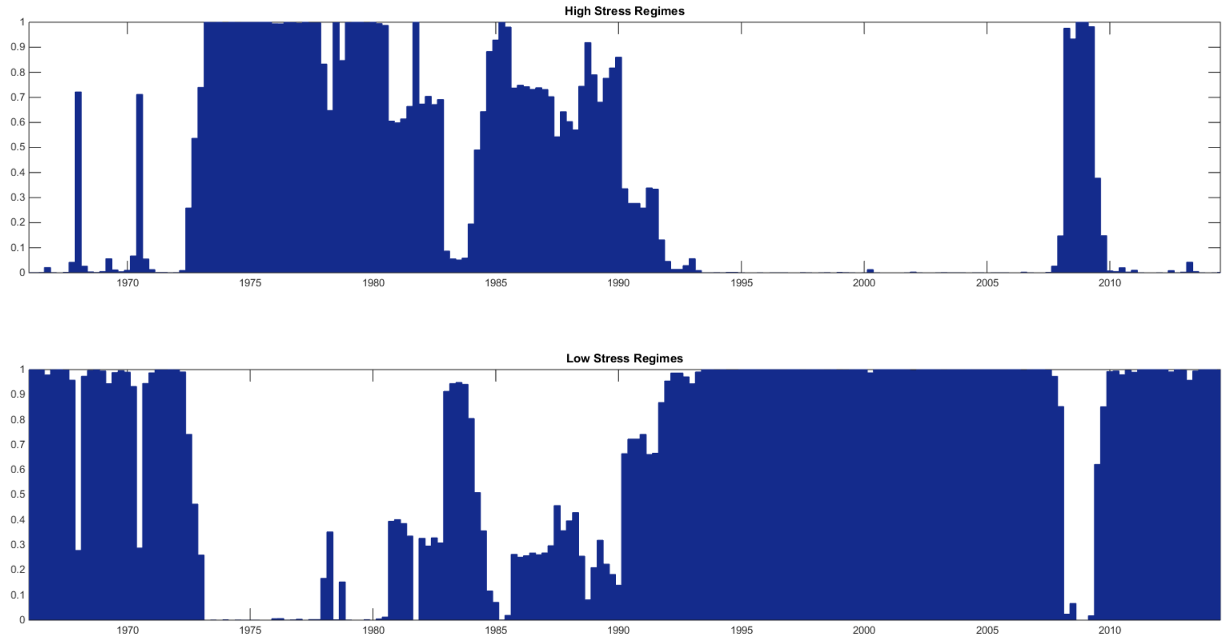
Figure 2: Full sample regimes for TVAR–S



Notes: The threshold variable is the corporate bond spreads series which is overlapped with the probabilities of each regime.

The regimes given by the MSVAR mode in figure 3 suggest similar interpretation. In accordance with the data we use, shown in figure 5, the MSVAR model does not only catch recessionary or financially stressful periods but also time periods where inflation rate and short term interest rates were both high, approximately around the time period 1972–1992. Labelling these regimes as high stress and low stress periods, we observe that high stress periods start around 1973s and mute by early 1990s with a significant low stress periods in between 1982 and 1985. Stable periods lasting around fifteen years after 1992 are followed by the last stress periods, clearly addressing the Great Recession. These regimes are in line with the regimes found by TVAR models by capturing the recessionary and high financial stress periods. On the other hand, high stress regime on the top panel of figure 3 addresses extended periods of stress.

Figure 3: Full sample regimes for MSVAR



5 Impulse Response Analysis of Structural Shocks

The design of stress scenarios usually rests on severe shocks materialising to the economy. For example, in the stress-testing exercise in 2014 in the UK, policy makers design a 'tail risk' scenario where the core part of the scenario involves disappointing output growth leads to gloomy prospects of the UK economy, which leads to significant depreciation of the sterling and a huge (35 percent) drop in house prices (see Bank of England (2014)). In the 2015 scenario, it was designed that shocks originated from disappointing global growth rates and further disinflationary pressures, which impact the UK economy materially through exports (see Bank of England (2015)). Our current setup, although involving only domestic variables, can be easily extended to include more international variables to study global shocks.

Our setup of different models provides us with a convenient platform to study structural shocks based on historical data. We are particularly interested in the potential differences in the shock transmission under different regimes. As discussed in the introduction, this is important because it is commonly believed that the linear model is generally inadequate to explore possible tail risks, and to investigate different shock transmission during stressful times. Our results in this section will help policy makers to calibrate tail risk scenarios, both in terms of the severity and persistence of shocks. Moreover, the set-up also allows policy makers to think about how the aggregate banking sector responds (the excess bank returns), which is a crude form of stress-testing. This will also offer information to policy makers to form proper policy responses if we these severe shocks were indeed to materialise in the future.

5.1 Shock identification

We adopt the Cholesky decomposition (the recursivess assumption) for our purposes, which is very common in the empirical macroeconomic literature to identify macro structural shocks. As is well known, the order of variables matter. In our case, we rank the variables in the following order: (i) real GDP growth; (ii) inflation rate; (iii) aggregate bank excess returns; (iv) corporate bond spreads; (v) short-term interest rate. This order is consistent with the monetary policy literature that real variables respond to shocks in monetary policy shocks with a time lag. But monetary policy can respond to shocks from the real sector and the financial sector contemporaneously. Our identification strategy is closely in line with Christiano et al. (1998), Gilchrist and Zakrajšek (2012) and broadly similar to Hubrich and Tetlow (2015).[4]

In this section we consider the following three structural shocks: (i) output growth shocks; (ii) interest rate shocks; (iii) corporate bond shocks which proxies for shocks in the financial market.

5.2 Generalised impulse response functions

Impulse response functions derived from univariate linear models are shown size, shock and composition *independent* by Koop et al. (1996). In the case of nonlinear models, however, these features no longer hold. Once nonlinearity is introduced to a univariate model, impulse responses become history and shock *dependent*. Furthermore, when the interest lies in modelling multivariate space, the impulse response functions come to be shock composition dependent in addition to becoming shock size and history dependent. Hence, we are presenting the generalised impulse response functions (GIRFs) following Koop et al. (1996). The advantage of the method is that we fully take into account of the possibility of the system switching regimes in the future.

This section presents the estimated impulse response functions of the three models we employ. As BVAR model is linear, we calculate linear impulse responses of BVAR model. However for TVAR and MSVAR models we need a more sophisticated way of estimating the impulse response

[4]For robustness, we checked alternative orderings of the last three variables. Results are mostly robust except when we order corporate spreads after the interest rate. Putting financial stress variables at the end corresponds to the ordering of Hubrich and Tetlow (2015). But we want to note that Hubrich and Tetlow (2015)'s is monthly set-up, whereas ours is a quarterly setup. The implication for a quarterly setup makes putting the policy variable after the financial variables more convincing. This is also in line with Gilchrist and Zakrajšek (2012) who places excess bond premium (a component of credit spreads) at the end.

functions. Following Koop et al. (1996), we calculate these impulse responses as

$$GIRF = E(y^{*,p}_{t+k}|y_t, \Upsilon, \triangle) - E(y^{*}_{t+k}|y_t, \Upsilon)$$

where k is the forecast horizon, Υ denotes the hyperparameters, $\triangle$ indicates the perturbed shocks while superscript p marks the forecasts with the perturbed path of errors. Appendix reports the steps on the nonlinear impulse response functions for both TVAR and MSVAR models.

5.3 Results

This section reports the selected impulse responses of our analysis. Each subsection specifies the types of shocks we consider which are also discussed in section 5.1. All figures are given in Appendix.

5.3.1 Financial shocks as proxied by shocks in the corporate bond spreads

Figures 6, 7 and 9 show the impulses responses corresponding to a one standard-deviation shock of the corporate bond spreads of TVAR–Y, TVAR–S and linear model. Gilchrist and Zakrajšek (2012) explains that credit spreads can reflect the changes in the quality of corporate firms' balance sheet and their external finance as well as the capital position of financial intermediaries who supplies credit. Nevertheless, we interpret any exogenous rise in the corporate bond spreads as shocks to the financial intermediation process which is orthogonal to shocks to the real sector.

Our TVAR–Y and TVAR–S models (figures 6 and 7) demonstrate what the linear BVAR model (figure 9) does not do: the shock transmission mechanisms are very different in different states of the world.

In TVAR–Y (Figure 6), one standard deviation financial shocks in the recessionary world (80 basis point jump versus 50 basis point jump in bond spreads) generate significant and deep decline in output growth (maximum fall of 50 basis points in growth 5 quarters after the shock). If such shocks happen in the non-recessionary world, the recessionary impact is much shallower. The financial shock also leads to a deeper and more persistent drop in the short-term interest rate in the recessionary world. Interestingly, this shock leads to a rise in inflation rate, which can be explained by the stagflationary period in the 1970s in the sample.

The most interesting implication is the response of the excess bank returns (a proxy for bank's profits), which is the most pertinent to our top-down stress-testing analysis. In the recessionary regimes, financial shocks lead to almost 10 pp drop in aggregate bank excess returns two quarters after the shock, whereas in the non-recessionary world there is a slightly rise in excess returns. This reflects that aggregate excess returns of banks are seriously affected when financial shocks hit in the bad state of the world.

The GIRFs in TVAR–S, figure 7 presents a very similar story in the sense that financial shocks in a originally financially stressful regime leads to much greater contraction of output and greater drop in aggregate bank returns (although the significance is marginal) when compared to the financially non-stressful world.

Responses of MSVAR model is in line with those of TVAR–Y and TVAR–S models' apart from size differences. In response to a financial shock, the contraction of output growth more severe in high stress periods than nonstressful periods. Five quarters after the shock hits, real GDP growth troughs almost at -0.5pp in the high stress periods, whereas the trough for the nonstressful regime is -0.2pp. Corporate bond spreads jump almost twice as much on the stressful times compared to stable periods. Banks' profitability is seriously affected in the bad state of the world. On the other hand, our observation related to excess bank returns rising in the good state holds for the MSVAR framework.

These results are consistent with the predictions of the theoretical literature. During times where the real economy is in recession or the financial market is under stress (where the balance sheets of the financial and non-financial sectors are considered weak, any shocks in the financial market will further amplify the recessionary effects through the well-known financial accelerator mechanisms as described in Kiyotaki and Moore (1997) and Bernanke et al. (1999). This undoubtedly leads to the erosion of aggregate banks' profits. This set of results are particularly important to policy makers when probing tail risks caused by shocks in the financial market. Our results shows that financial shocks deepen recessions when the economy is already in recession or financially stressed.

5.3.2 Negative output shocks

Another shock of interest is negative shocks to the output growth in the economy. Since our identification scheme does not allow us to distinguish aggregate supply shocks from aggregate demand shocks [5], we consider that this shock as proxying exogenous changes in the real ecoonomy that reduces output growth, which could originate domestically (productivity or demand shocks) or internationally through the export-import channel.

Figures 10 and 11 report the GIRFs of TVAR–Y and TVAR–S with respect to one-standard-deviation shocks in the output growth in different regimes. Since inflation jumps on impact,

[5] Aggregate supply shocks are characterised by a fall in output growth and a rise in inflation rate, whereas aggregate demand shocks are characterised by both decrease in output growth and inflation. These two shocks can be identified by sign restrictions, which is left for future research.

the models seem to be picking up aggregate supply shocks in this context. Relative to the non-recessionary/financially non-stressful regimes, these shocks lead to a larger and more persistent rise in the inflation rate, which is again heavily driven by the stagflation experience in the UK and the interest rate. The rise in corporate bond spreads is also significant, especially in the financially-stressful regimes in Figure 11: on impact the spread can rise up to 22 basis points in the financially stressful regimes as opposed to 3 basis points otherwise. This shows that when the economy is under financial stress, further bad shocks in the real economy will lead to heightened stress in the financial system. This point is not picked up by the linear BVAR response as shown in figure 13.

GIRFs of the MSVAR model tell a very similar story. As in TVAR–S model, bond spreads respond to an output shock with a big jump in the bad state of the world whereas the response in the good state of the world is very insignificant. Inflationary effects of the shock are not very significatly different for both states. Short term interest rate, however, persistently stays low in both states of the world. The effect in the stressful periods is almost three times more severe in the stressful periods when the interest rate troughs six quarters after the shock. However, there are no significant changes to the aggregate bank excess returns in any of the models.

5.3.3 Interest rate shocks

We also consider the GIRFs of short term interest rate shocks, defined as any interest rate movement unexplained by the systematic responses of policy makers to variations in the state of the economy (see Christiano et al. (1998)).

We first start with the linear model's impulse responses which are shown in figure 14. We see that the identified interest rate is persistent and leads to hump-shaped response of real output growth where the trough occurs at a 20 pp (annualised) fall in growth rate three years after the shock. Corporate spreads also rise significantly after two years of the shock. These results are consistent with the tradtional monetary policy literature. Again, the inflation rate rises, which is attributable to the influence of the stagflation sample. Aggregate bank excess return goes by an annualised of 3 percent one quarter after the shock.

Figure 15 shows the GIRFs of the interest rate shock under recessionary and non-recessionary regimes. There are several major differences. First, the drop in output growth is deeper but much less persistent in recessions. Second, aggregate bank returns drop by about 2 pp (annualised) in recessions as opposed to a rise otherwise. This may be evidence that banks are particularly vulnerable to monetary policy shocks during recession. Slightly puzzling is the initial significant drop in corporate bond spreads in recessions. [6] [7]

[6] Our other TVAR-S model produces a boom conditional on an unexpected rise in short-term interest rate. The results seem counter-intuitive so we do not report the results here.

[7] As a robustness check, we also investigate the GIRFs of MSVAR model, subject to the caveats stated in

In theory, contractionary monetary policy should lead prices to fall. Regardless of the models we use, we find positive relationship between monetary tightening and inflation rate which is a well acknowlegded phenomenon in the literature, known as *price puzzle*. First identified by Sims (1992) and labelled byEichenbaum (1992), the reason of and solutions for the price puzzle have widely discussed. Sims (1992) suggested that puzzle arises when central bank raise the interest rates to offset the expectations of higher inflation but this response is not enough to prevent inflation actually from rising. This explanation is investigated by Castelnuovo and Surico (2010) leading to an explanation where price puzzle is historically limited to subsamples which are associated with a weak interest rate response to inflation.

The literature offers some measures to overcome the price puzzle. Hürtgen and Cloyne (2014) proposes a new identification strategy used by Romer and Romer (2004) which builds on Romer and Romer (1989) to build a proxy by looking at the historical records of the data to pick up the dates when the contractionary monetary policy actions were taken. Rather than using interest rate as a policy variable, they propose to use this proxy to overcome price puzzle. Christiano et al. (1996) argued that when the commodity prices are included in VARs, the issue is resolved. Balke and Emery (1994) inherits a similar approach by using different variables to solve the price puzzle, such as the spread between long and short term interest rates, oil prices, stock prices, unit labor costs. Among all, only the spread between long and short term interest rates appear to resolve the puzzle. Particularly for our analysis, we might inherit one of the solutions suggested in the literature.

5.3.4 Extremely large shocks in 'good' states of the world and the feedback loop

The generalised impulse response functions also allow us to investigate the differences in shock transmissions when extremely large shocks hit in different states of the world. As stated in Bank of England (2013), one way to explore the severity of the scenarios is 'to recognise the variation in the probability and impact of systemic stresses over time'. For example, as credit conditions ease and leverage builds up, the banking system may be susceptible to more severe shocks. Conversely, in a downturn, with tightening credit conditions and lower leverage, a less severe scenario might be more appropriate' (page 21). It implies that the probability of a tail shock hitting a booming economy is larger than when the economy is in bust. Our set-up enables us to carry out an experiment in this regard.

We repeat our GIRFs simulations with shocks of *three standard deviations*, which happen with a 0.1 percent of probability in a symmetric normal distribution (arguably this is a tail event). We find two points worthwhile to mention. First, for the TVAR–Y model, when compared

an earlier footnote. We find that aggregate bank excess returns rise in periods indicated by high volatility but decrease in other periods. So the responses of aggregate bank excess returns do not seem to be robust.

to the standard one-standard-deviation shock in the output growth (Figure 10), we find that a three-standard deviation shock in the non-recessionary regime (amounting to a 4 pp (annualised) drop in output growth in non-recessionary regimes versus a 8 pp decrease in the recessionary ones; see Figure 16) leads to spikes in corporate spreads which are four times (20 bp) and six times (30 bp) as high respectively on impact and five quarters after the shock. The persistence in financial stress then feeds back to the economy, leading to more protracted decline in real output and the fall of short-term interest rates. These responses are not only disproportionately larger when compared to the one-standard-deviation shock scenario, but also disproportionately larger relative to the responses in the recessionary regimes.

Second, in the TVAR–S model, when compared to the standard one-standard-deviation shock in credit spreads (Figure 7), we find that a three-standard deviation financial shock in the non-financially stressful regime (amounting to a 120bp spike in spreads in financially non-stressful regimes versus a 200bp jump in financially stressful ones; see Figure 17) leads to a very deep and persistent recession, with the trough occurring three years after the shock and the contraction reaching 1.2 pp, as opposed to the much shallower recession of a contraction of 0.1 pp at the trough of the fourth quarter otherwise. Again, due to feedback effects, the persistence and rise in corporate spreads are more protracted. These responses are not only disproportionately larger when compared to the one-standard-deviation shock scenario, but also disproportionately larger relative to the responses in the financially stressful regimes.

Our results not only provide evidence of feedback loops, and indirectly lends support to the Bank of England's counter-cyclical approach in the design of stress scenarios as set out in Bank of England (20015, pg. 14): '[i]mportantly, the severity is likely to be greater in a boom, for example, when growth in credit is rapid and asset prices unsustainably high'.

6 Forecasting analysis

In this section, we investigate how well these models are able to forecast, with special attention paid to the *recessionary* tails. To that end, we perform two types of forecasting exercises: unconditional and conditional forecasting. Specifically, we have two questions in mind. Firstly, do nonlinear models produce unconditional density forecasts more informative in the tail? Secondly, could we produce more reliable predictive densities when we condition on variable paths consistent with stressful times? It worths emphasizing that we are not after predicting the timing of stresses. We are rather interested in checking whether the models are able to provide a broad view of macro tail risks. Both exercises involves estimating the three models using data until 2007:Q2 and produce multi-step ahead (pseudo-out-of-sample) forecasts for 12 quarters. We will then compare the predictive densities against the Great Recession in 2008–09. All fan charts are given in the Appendix.

6.1 Unconditional forecasting

The rationale for unconditional exercise is to investigate whether the nonlinear models will be able to generate forecast densities which properly entail tail risks. Figure 18 reports the fan charts for the five variables (across rows, in the order of output growth, inflation rate, excess bank returns, short-term interest rate and corporate bond spreads) for our three models (across columns, in the order of BVAR, TVAR–Y and MSVAR).[8] The black lines are the median forecasts. The shaded areas are the error bands from 20^{th} to 80^{th} percentiles with 5% increments. The charts are overlapped with the realisations of the series which are given by the red lines. Visually speaking, whereas there may be some slight improvement in the coverage of tails for MSVAR and TVAR–Y models, none of these models are capable of capturing the tails risks, especially the drop in output growth and the spike in financial volatility during the 2008–09 Great Recession, a truly extreme event in the post-war sample.

6.2 Conditional forecasting

Our next exercise is conditional forecasting. We would like to investigate whether our models will manage to better capture tail risks if we *a priori* feed into specific paths of variables under stress.[9] This will enable the models to exploit the correlational dynamics between the conditioning variable and the other variables being modelled.

We employ the conditional forecasting techniques proposed by Waggoner and Zha (1999) and used by Bańbura et al. (2015). The algorithm first involves sampling the residuals of the VAR system which are consistent with the exogenously imposed scenario paths, and then constructs forecast densities of other variables *conditional on the imposed paths.* In this exercise, we separately impose the actual outturns of (i) the path of GDP growth rates between 2007:Q3 and 2010:Q2; and (ii) the path of corporate bond spreads in the same period. Figures 19 and 20 report the corresponding fan charts. We have special interest in assessing the tail forecasting power of the non-conditioned variables, including the output growth rates, corporate bond spreads and bank returns.

The following observations are in order judging by the fan charts. First, the linear BVAR model consistently falls short of producing reasonable probe of macro tail risks. Second, the TVAR–Y model shows even bigger improvement in the forecast densities as shown in the second columns of Figures 19 and 20. Especially in the case of conditioning of GDP growth rate, the

[8] To keep the charts concise, we only report TVAR–Y results for the forecasting exercises.

[9] Conditional forecasting is a common exercise among policy makers. For example, in the Inflation Reports produced by the Bank of England, the fan chart projections for GDP growth and CPI inflation are generated conditional on 'market interest rate expectations' and 'the stock of purchased assets financed by the issuance of central bank reserves'.

TVAR–Y model is able to generate very good densities to forecast the tails of inflation rate and aggregate bank excess returns. For the conditioning of corporate bond spreads, the TVAR–Y model is able to generate very good density to forecast the tails of output growths and bank excess returns. Third, the MSVAR models show some improvement in the forecast densities of the non-conditioning variables, compared with the linear BVAR model. This can be seen by the second columns of Figure 19: when conditioning on the path of GDP growth rate, the predictive density for bank returns has improved. A similar story can be seen from Figure 20 for the predictive density of excess bank returns when the financial volatility is the conditioning variable.

Interestingly, all models fail to generate forecast densities covering the possibility of zero interest rates. This can be explained by the data sample that zero interest rates have never occurred before, and that the nonlinear model is heavily influenced by the stagflation periods in the 1970s where recessions are associated with high inflation and high nominal interest rates.

These results highlight the importance of incorporating nonlinearities in modelling macro-financial variables. While the evidence is still preliminary and crude, these models have the potential to generate reasonable predictive tail densities in the conditional forecasting exercise. This also appears to support Clements and Smith (2000) that nonlinear models can potentially perform better than the linear counterparts in terms of the density forecast precision, as long as the data contain nonlinear features.

7 Concluding remarks

In this paper, we build a VAR framework, estimated by Bayesian techniques, to properly account for macroeconomic tail risks in stress-testing exercises. We utilise regime switching models to estimate the regimes governed by different time periods. Our methodology is motivated by the notion of capturing different economic dynamics during recessionary/stressful states of the world. These regimes are identified by TVAR–Y, TVAR–S and MSVAR models. Although these model leads to different interpretations of regimes, we are able to associate them with recessionary/nonrecessionary and financially stressful/stable periods in the TVAR models. MSVAR model produces regimes that are attributed to high stress and stable time periods.

Estimating the regimes aids us to identify the impacts of various shocks to the economy in particular regimes. We obtain substantial evidence on the fact that financial shocks cause more severe contractions in output growth during recessionary periods as well as more severe effects of output and interest rate shocks in the bad states of the world. Furthermore, we demostrate the feedback loop between the real and financial sectors as a result of severely adverse shocks hitting the economy in nonrecessionary/stable time periods.

To catch a glimpse of the forecasting performances of the proposed models, we perfom unconditional and conditional forecasting exercises. Forecast comparison broadly shows that unconditional forecasting exercise does not provide satisfactory forward looking perspective to the future paths of the variables. Generally TVAR–Y model outperforms other models although the evidence is crude. The results practically hightlight the importance of incorporating nonlinearities.

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Appendix

A Priors

We impose the normal inverse Wishart priors following Bańbura et al. (2010). For the prior means, we assume that the variables included in the VAR system follow an AR(1) process. The priors of the variance H, on the other hand, has a sophisticated structure and defined as

$$\begin{aligned} \left(\frac{\lambda_1}{\ell\lambda_3}\right)^2 &\quad \text{if} \quad i = j \\ \left(\frac{\sigma_i \lambda_1 \lambda_2}{\sigma_j \ell^{\lambda_3}}\right)^2 &\quad \text{if} \quad i \neq j \\ (\sigma_i \lambda_4)^2 &\quad \text{for the constant} \end{aligned}$$

where i is the dependent variable in i^{th} equation and j is the independent variables in that equation. Therefore when $i = j$, it refers to the coefficients on the own lags of variable i. the variances σ_i and σ_j are the OLS estimations of the variances from AR regressions by using the VAR variables. The lag length in that particular step is shown by ℓ. The parameters λs are to control the tightness of the prior.

- λ_1 controls the standard deviation of the prior on own lags.
- λ_2 is weight of own lag of dependent variable versus other lags. When it is 1, there is no difference on the lags of the dependent variable and other variables. It basically is to control the standard deviation of the prior on lags of variables other than the dependent variable.
- λ_3 represents the lag decay. When it increases the coefficients on higher lags are shrunk to zero more tightly.
- λ_4 controls the prior variance on the constant term.

There is no convention on the choice of the hyperparameters which control the priors. Nevertheless, Canova (2007) and Blake and Mumtaz (2012) report their own hyperparameters. In the light of these, we choose our hyperparameters as $\lambda_1 = 0.1, \lambda_2 = 1, \lambda_3 = 1$ and $\lambda_4 = 10^5$ which are quite similar to what they use with slight differences to incorporate them with our models and data.

B The Gibbs Sampling algorithm for TVAR

The Gibbs sampler for TVAR works the following way:

1. Given the initial values of the threshold values, we separate the observations into two regimes.

2. We already know which regime coefficients and covariances we should evaluate, after the first step. In this step, we draw the coefficients and covariances.

3. Given the starting values for coefficients and covariances, we draw the threshold value.

$$Z^*_{new} = Z^*_{old} + \Psi^{1/2}\varepsilon$$

   where $\Psi^{1/2}$ is scaling factor to keep the acceptance rate in an interval and $\varepsilon \sim N(0,1)$. Since the posterior of the threshold value is not known, we need to perform Metropolis Hastings algorithm, along with Gibbs sampler. The acceptance rate is $\frac{f(Y_t|Z^*_{new})}{f(Y_t|Z^*_{old})}$ where $f(.)$ is the posterior density of the old and new threshold. The scaling factor is chosen to ensure that the acceptance rate is in 20–40% interval for providing variability to the model and not just capturing the results locally.

4. As for threshold, the posterior density of the delay parameter is not identified. After drawing the threshold, we should sample the delay parameter. Chen and Lee (1995) showed that the conditional posterior probability function of this parameter is multinomial distribution with probability $\frac{L(Y_t)}{\sum^d L(Y_t)}$ where $L(.)$ is the likelihood function. Hence, we draw d from discrete sample.

5. We run 100,000 draws and burn in the first 60,000 to ensure convergence.

C The Gibbs Sampling algorithm for MSVAR

Following Barnett et al. (2012), we adopt the Gibbs sampling data augmentation algorithm to estimate the model:

1. Sampling covariance states, S_t:
   We start with sampling the variance states. Given the starting values of VAR parameters and the covariances, we use multi-move Gibbs sampling that was proposed by Kim and Nelson (1999). This method, in a nutshell, basically predicts the unobserved state and then updates it by running a simulation smoother to obtain a draw from the joint posterior densities conditional on data and the parameters, namely $f(S_t|Y_t, c_S, B_{1,S}, \ldots, B_{L,S}, P, Q)$.

- We first calculate $f(S_T|Y_T)$. Hamilton filter (Hamilton (1989)) which is elaborated below provides $f(S_t|Y_t)$ for $t = 1, 2, \ldots, T$.
- The next step is calculating $f(S_t|S_{t+1}, Y_t)$. Kim and Nelson (1999) show that

$$f(S_t|S_{t+1}, Y_t) \propto f(S_t|S_{t+1})f(S_t|Y_t) \tag{3}$$

  Hamilton filter provides $f(S_t|Y_t)$ and $f(S_t|S_{t+1})$. Therefore we sample S_t from (3) separately which it also holds for the coefficient regimes.

2. Sampling the covariances, Ω_S:
Given the states, we sample the covariance matrices from inverse Wishart distribution,

$$\Omega_S \sim iW(\bar{H}_S^0, \varphi_S)$$

where $H_3 = I(S_t = 3)(\bar{E}_t'\bar{E}_t)$ and $H_4 = I(s_t = 4)(\bar{E}_t'\bar{E}_t)$ and $E_t = Y_t - \mu - B_jY_{t-j}$. $I(.)$ denotes an indicator function for selecting the variance regimes 3 and 4 given coefficient regimes. Bars indicate the average across $S_t = 1, 2$. The parameter φ_S is the number of the observations in each regime.

3. Sampling the VAR coefficients, $c_{S_t}, B_{1,S_t}, B_{2,S_t}, \ldots, B_{L,S_t}$:
Given the states and the covariances, we sample $c_{S_t}, B_{1,S_t}, B_{2,S_t}, \ldots, B_{L,S_t}$.
For that, we collect the VAR coefficients for regime $S = j$ into the $(N \times (N \times L + 1))$ vector, $vec(B)$. The conditional posterior of the VAR coefficients is

$$vec(B) \mid \Omega_s, Y \sim N(vec(\tilde{B}), \Omega_S \otimes (X^{*'}X^*)^{-1}).$$

4. Sampling the transition probabilities, P:
As the last step we sample the transition probabilities for both of the independent Markov chains. Prior for the non-zero elements of the transition matrices, p_{ij}, is of the following form, as in Barnett et al. (2010),

$$p_{ij}^0 = D(u_{ij})$$

where $D(.)$ represents the Dirichlet distribution. The posterior distributions of the transition probabilities are

$$p_{ij} = D(u_{ij} + \eta_{ij})$$

where η_{ij} denotes the number of times regime i is followed by regime j for mean regimes and ζ_{ij} denotes the number of times regime i is followed by regime j for variance regimes.

5. We employ 50,000 iterations for the Gibbs sampling. We discard the first 10,000 draws as burn in and keep every tenth draw to ensure convergence.

C.1 MSVAR with two independent Markov chains

We can expand the MSVAR system by exploiting two independent Markov Chains to drive variance and coefficient regimes separately by followingBarnett et al. (2010). Considering two independent Markov chains, one for the slope and intercept coefficients, the other one for the covariance matrix, we assume two regimes for each of the chain. As precisely explained by Kalliovirta et al. (2014), MSVAR models describe time series that switch between regimes with each regime has the dynamics of a linear VAR model. The regime switches are determined by a latent indicator that follows a time-homogenous Markov chain with the transition probabilities depending on the most recent regime but not on past observations.

In this specific case we can also make use of time varying transition probabilities which are indicated by the subscript t. Naturally, the representation of the model and the Gibbs sampling algorithm have to be modified accordingly. Here, the MSVAR model is written as

$$Y_t = c_{S_t} + B_{1,S_t} Y_{t-1} + B_{2,S_t} Y_{t-2} + \cdots + B_{l,S_t} Y_{t-L} + v_t, \tag{4}$$

where $e_t \sim N(0, \Omega_{s_t})$. This VAR model incorporates regime changes both in its coefficients, denoted by $S_t = 1, ..., M$, and the variance of the error terms, denoted by $s_t = 1, ..., m$. The changes of coefficient and variance regimes are independent of each other. The data vector Y contains the five macro-financial variables of interest. The lag length is given by L.

The first set of latent regimes S_t is assigned as $S = 1, 2$ which we refer as variance regimes. The regimes associated with the coefficients are denoted as $s = 3, 4$. They are assumed to follow two independent first order Markov chains. Therefore we have two transition matrices, one for each regime, P and Q,

$$\begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{pmatrix}$$

where $p_{ij} = prob(S_t = i | S_{t-1} = j)$ indicates regime i is followed by regime j in variance regimes and $q_{ij} = prob(s_t = i | s_{t-1} = j)$ indicates regime i is followed by regime j in coefficient regimes. The probability of high variance regime is followed by low variance regime is p_{21} while q_{21} corresponds to high coefficient regime being followed by low coefficient regime. The columns sum up to 1. There are no restrictions on regime switches, i.e they are left unrestricted to jump back and forth.

For Gibbs sampling, we impose the normal inverse Wishart priors following Bańbura et al. (2010) as in the previous MSVAR case. We assume that the VAR coefficients B are normally distributed:

$$p(B) \sim N(B_0, H)$$

where B_0 is a $(N \times (N \times L+1)) \times 1$ vector of prior mean and H is a $(N \times (N \times L+1)) \times (N \times (N \times L+1))$ matrix whose diagonal elements refer the variance of the prior. It can be shown that the conditional posterior for the coefficients is given by $H(b|\Omega_{s_t}, Y_t) \sim N(M^*, V^*)$ where

$$\begin{aligned} M^* =& (H^{-1} + \Omega_{s_t}^{-1} \otimes X_t' X_t)^{-1}(H^{-1}b + \Omega_{s_t}^{-1} \otimes X_t' X_t \hat{b}) \\ V^* =& (H^{-1} + \Omega_{s_t}^{-1} \otimes X_t' X_t)^{-1} \end{aligned}$$

where $\hat{b}$ is a $(N \times (N \times L+1)) \times 1$ vector that denotes the OLS estimates of the VAR coefficients in a vector format $\hat{b} = vec((X_t' X_t)^{-1} X_t' Y_t)$. the conjugate prior for the VAR covariance matric is an inverse Wishart distribution with prior scale matrix S and prior degrees of freedom α, $p(\Omega_{s_t}) \sim (S, \alpha)$.[10]

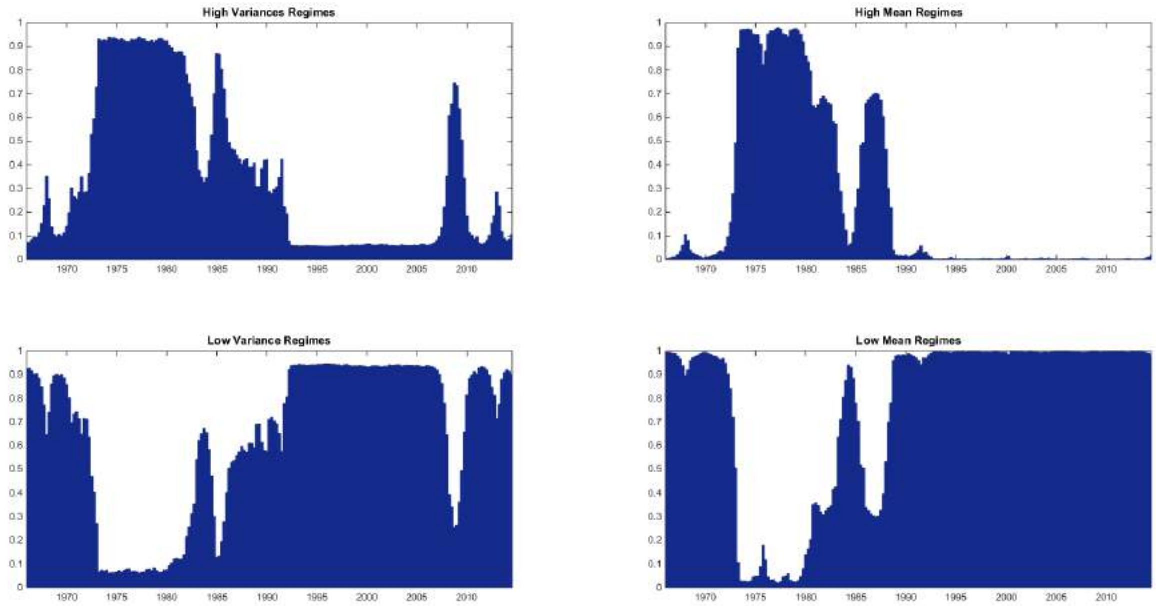
The MSVAR model is estimated with Bayesian methods following Barnett et al. (2012). All the steps given in C hold for this specific case with an additional step after the second step to sample the coefficient states, s_t. After sampling the variance regimes and the covariances in the previous two steps, we rewrite the model as in Barnett et al. (2010),

$$Y^* = A_{j,s} X^* + V_t \tag{5}$$

where $Y^* = I(S_t = 1)[(\Omega_1^{-1} \otimes I_t^*)^{-1/2} \times vec(Y_t)] + I(S_t = 2)[(\Omega_2^{-1} \otimes I_t^*)^{-1/2} \times vec(Y_t)]$, $X^* = I(S_t = 1)[(\Omega_1^{-1} \otimes I_t^*)^{-1/2} \times (X_t \otimes I_N)] + I(S_t = 2)[(\Omega_2^{-1} \otimes I_t^*)^{-1/2} \times (X_t \otimes I_N)]$ where Y_t includes the observations corresponding to the relevant coefficients states, $s_t = 3, 4$ which are determined by the starting values of the coefficient regimes in the first iteration and t^* shows the number of observations in that particular regime. This new representation is an MSVAR model with homoscedastic covariance matrix. We can make use of multi move Gibbs sampling to draw the coefficient states, $f(s_t|Y_t, c_S, B_{1,S}, \ldots, B_{L,S}, P, Q)$. The estimation results of this extension is given by table 4. The first set of regimes are identified as high variance regimes given that they successfully capture high volatiliy periods before Great Moderation and around the Great Recession. The second set of the regimes are attributed to the high and low mean states. High mean states appear to capture high inflation interest rate periods before 90s.

[10]For technical details, we refer the readers to Blake and Mumtaz (2012).

Figure 4: Full sample regimes for MSVAR with 2 independent Markov chains



Note that our way of identifying structural shocks in this MSVAR model is not in line with Sims et al. (2008) and Hubrich and Tetlow (2015), who estimate a *structural* MSVAR from rather than a reduced-form model from the outset. This leads to a major difference: in this current setup set-up, the structural identification matrix is dependent on variance regimes, whereas in the case of Sims et al. (2008) and Hubrich and Tetlow (2015), the structural identification matrix is dependent on the coefficient regimes. Hence, we do *not* pursue this extension to the stage where we would exploit structural shocks and compare forecasting performances of the proposed models.

D Generalized Impulse Response Functions

We compute the nonlinear impulse response functions of MSVAR and TVAR models by following Koop et al. (1996), Baum and Koester (2011) and Afonso et al. (2011). The steps of the algorithm are given below for both models.

D.1 GIRFs for MSVAR

1. Run the estimation with the whole sample. This step is already fulfilled in section 4.

2. Given the regimes, regime specific coefficients and variance and given a Gibbs draw and time t, run the forecasting algorithm by projecting ergodic probabilities. The errors in this step are drawn from the normal distribution. Refer the resulting forecasts y^{M}_{t+k} where k is the forecast horizon and superscript M marks the MSVAR model. The output is a $(horizon \times N)$ matrix of forecasts for all N variables and these forecasts serve as a baseline.

3. Run the forecasting algorithm with perturbed shocks such that shock is a vector of zeros except for the variable we are interested in shocking which is 1. Refer these forecasts as $y^{M,p}_{t+k}$ where the additional superscript addresses the perturbed shocks. The output is a $(horizon \times N \times 1)$ matrix *for a given shock*. If one is interested in shocking all the variables, the resulting matrix is size of $(horizon \times N \times N)$.

4. Repeat both steps 2 and 3 for $Simm = 500$.

5. Take the mean of the resulting forecasts over Simm and the difference between the means such that $\frac{1}{Simm}\sum_{Simm} y^{M,p}_{t+k} - \frac{1}{Simm}\sum_{Simm} y^{M}_{t+k}$. This difference is for a given Gibbs draw and given time period.

6. Repeat steps 2 to 5 for all Gibbs draws and for all T.

7. Take the mean of the time varying impulse response functions from the previous step over T. The output of this step gives the GIRFs of MSVAR model.

D.2 GIRFs for TVAR

The following steps are employed for both TVAR–Y and TVAR–S models.

1. Similarly, run the estimation for the whole sample as in section 4.

2. Given the regimes, regime specific coefficients and variances separate the observations according to the regimes. In our exercise we can call them recessionary and expansionary observations. Without loss of generality, we explain the steps concerning the recessionary observations. Same steps are applicable to the expansionary observations.

3. Given a Gibbs draw, pick a random history from recessionary observations.

4. Run the forecasting algortihm such that there is no restriction on the error terms. Refer these set of forecasts are y^{Th}_{t+k} where th indicates the TVAR model and k is forecast horizon.

5. Run the forecasting algorithm with the perturbed shocks. Refer these forecasts as $y^{Th,p}_{t+k}$.

6. Repeat steps 3 to 5 for $Simm = 500$.

7. Take the means of the forecasts over $Simm$ and calculate the difference between the means such that $\frac{1}{Simm}\sum_{Simm} y_{t+k}^{Th,p} - \frac{1}{Simm}\sum_{Simm} y_{t+k}^{Th}$.

8. Repeat steps 3 to 7 for all Gibbs draws and all histories. The result of this step is the time varying impulse response functions.

9. Take the mean of the resulting impulse response functions from the previous step over all possible histories. The output is the ultimate GIRFs of recessionary regime in TVAR model.

10. Repeat steps 3 to 9 for the expansionary regime.

E Variables

In our system we use 5 macroeconomic variables, Real GDP growth, Inflation Rate, Aggregate Bank Excess Returns, Corporate Bond Spreads. The tables in this section give the desciptive statistics and the correlation matirx of these variables. The charts for these variables for the whole data span of Q2:1965 to Q2:2014 are also given below. The dashed vertical lines indicate the observation in Q2:2007 which is the quarter when we separate the data for forecasting purposes. The forecasting results are given for the quarters after that particular observation.

Table 1: Summary statistics

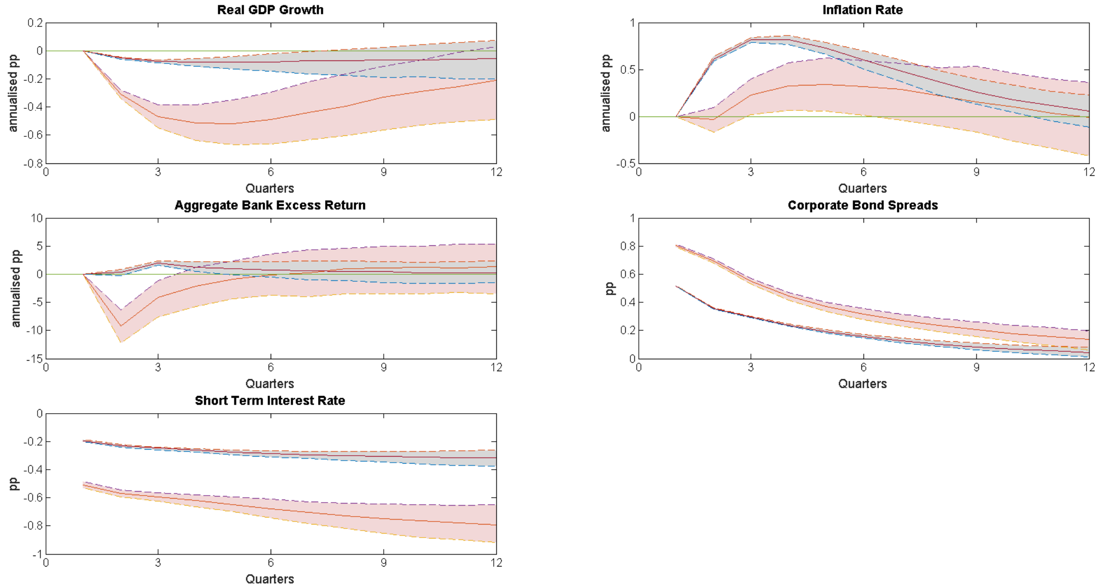
	Real GDP Growth	Inflation Rate	Agg. Bank Excess Returns	Corporate Bond Spreads	Short Term Interest Rate
Mean	2.39	5.55	1.26	1.30	7.19
Median	2.60	3.88	0.43	1.03	6.56
Maximum	12.78	41.90	120.85	6.64	16.27
Minimum	-6.52	-5.96	-112.36	-0.33	0.32
Std Deviation	2.49	6.11	33.34	1.02	3.88
Skewness	-0.82	2.06	0.11	2.07	0.15
Kurtosis	6.23	9.78	4.53	9.67	2.54
Observations	197	197	197	197	197

Table 2: Correlation coefficients of variables

	Real GDP Growth	Inflation Rate	Agg. Bank Excess Returns	Corporate Bond Spreads	Short Term Interest Rate
Real GDP Growth	1				
Inflation Rate	-0.2070	1			
Agg. Bank Excess Returns	0.0053	0.0223	1		
Corporate Bond Spreads	-0.4356	0.2302	-0.0151	1	
Short Term Interest Rate	0.0002	0.4850	0.0001	-0.1141	1

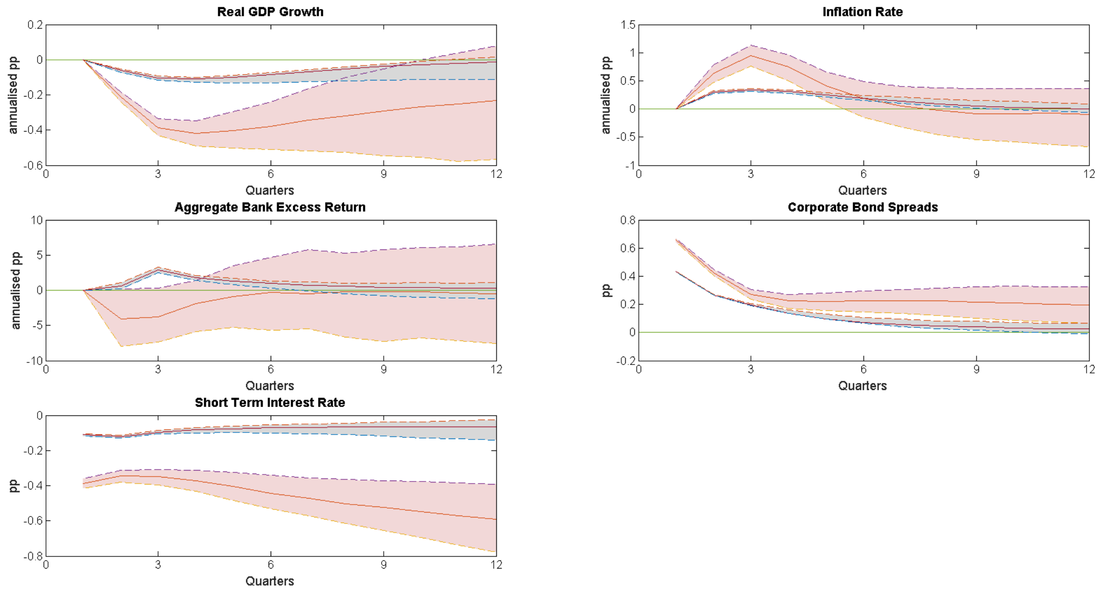
Figure 5: Variables

Figure 6: Generalised Impulse Responses to a 1 SD adverse shock to corporate bond spreads in TVAR–Y model



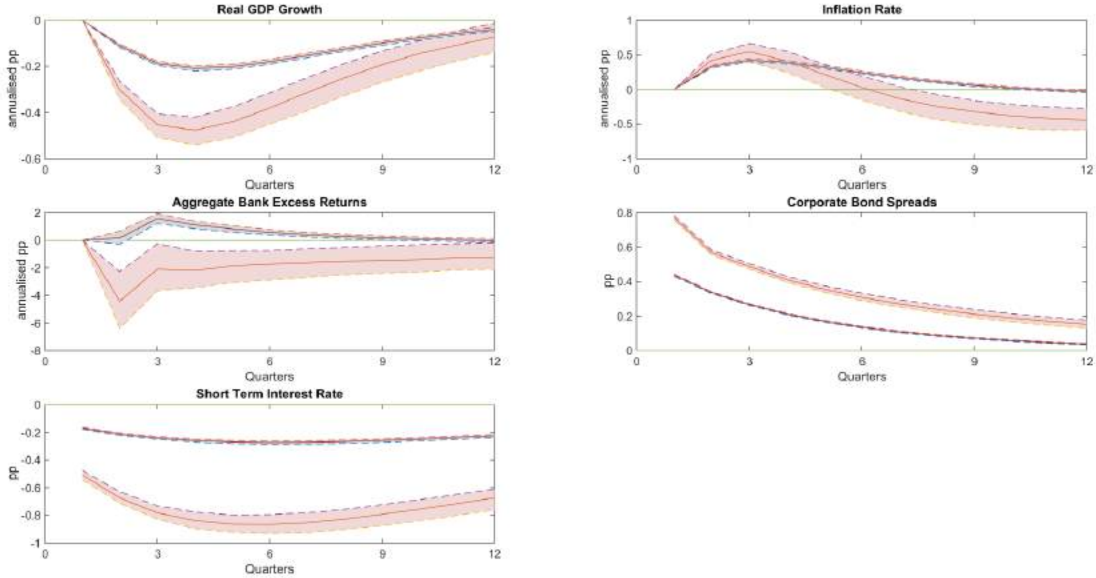
Notes: The error bands correspond to the 68% confidence intervals. The pink shaded area corresponds to the error bands of recessionary regimes. The grey shades correspond to those of nonrecessionary regimes.

Figure 7: Generalised Impulse Responses to a 1 SD adverse shock to corporate bond spreads in TVAR–S model



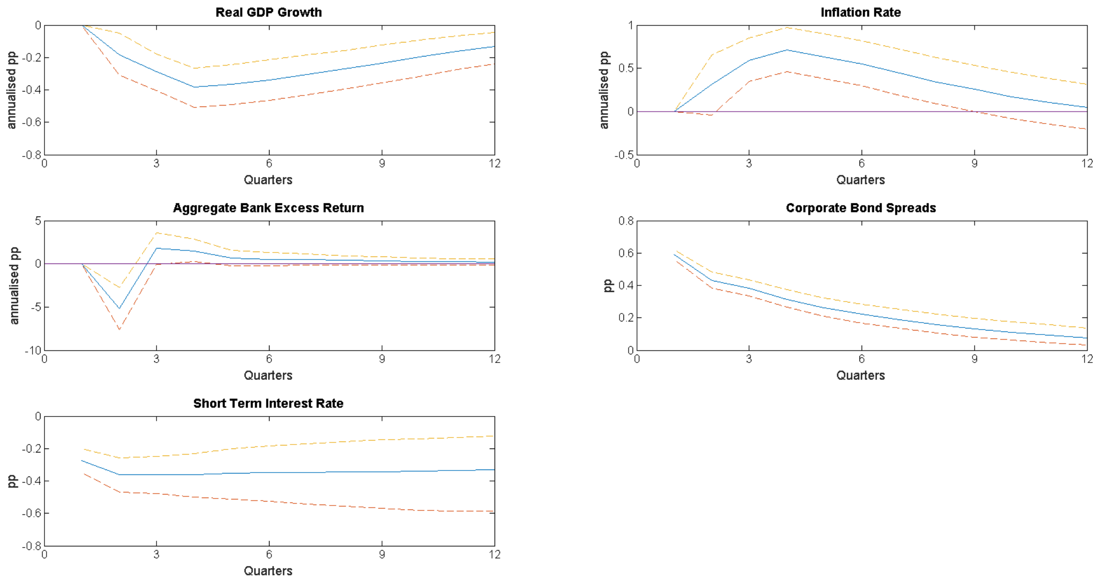
Notes: The error bands correspond to the 68% confidence intervals. The pink shaded area corresponds to the error bands of financial stress regimes. The grey shades correspond to those of financial nonstress regimes.

Figure 8: Generalised Impulse Responses to a 1 SD adverse shock to corporate bond spreads in MSVAR model



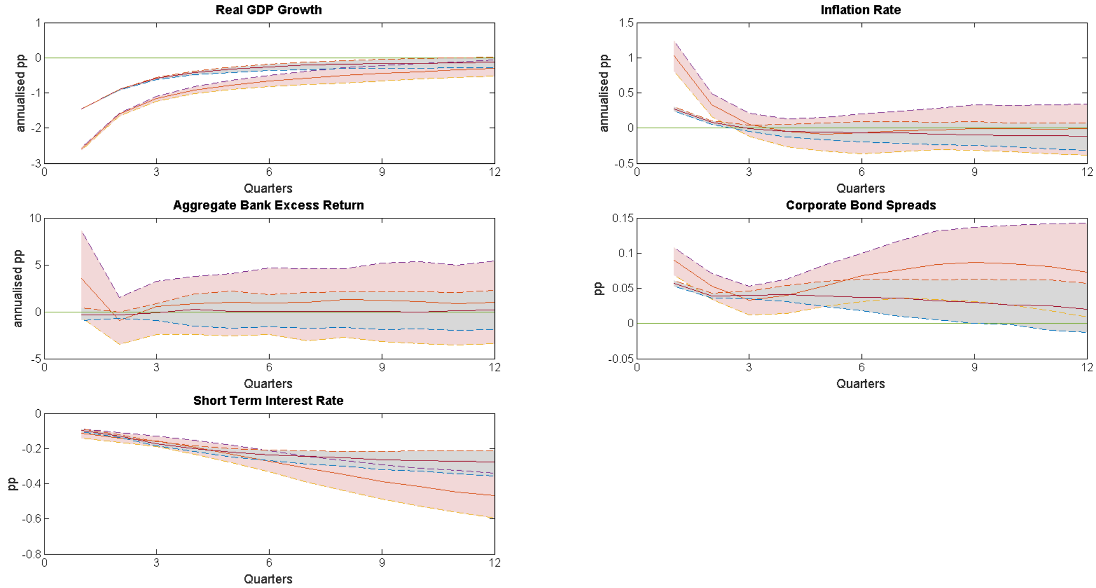
Notes: The error bands correspond to the 68% confidence intervals. The pink shaded area corresponds to the error bands of high stress regimes. The grey shades correspond to those of low stress regimes.

Figure 9: Impulse Responses to a 1 SD adverse shock to corporate bond spreads in BVAR model



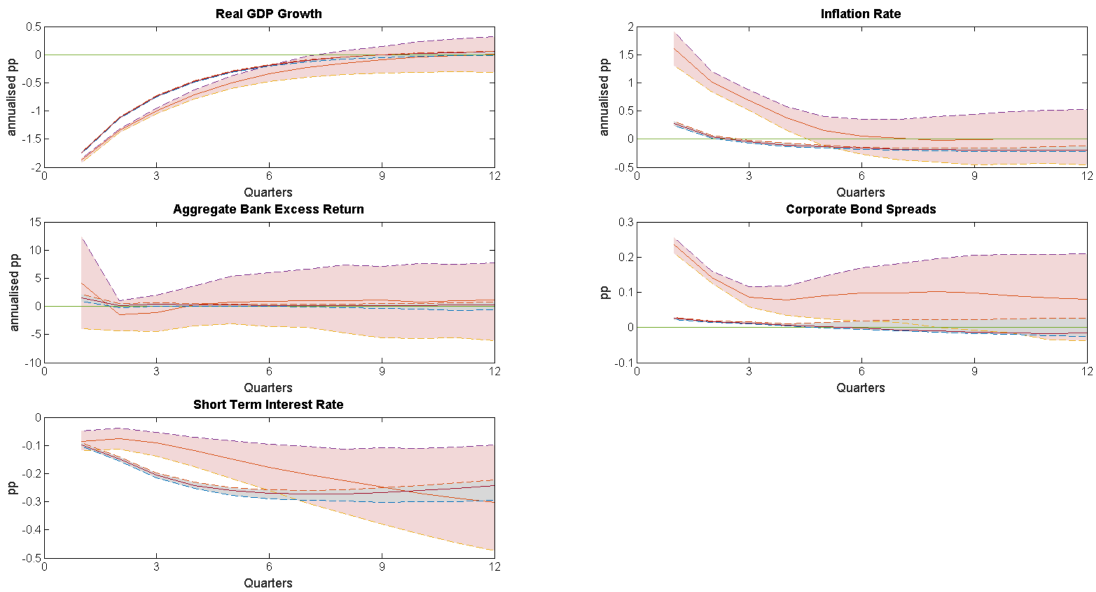
Notes: The error bands correspond to the 68% confidence intervals.

Figure 10: Generalised Impulse Responses to a 1 SD adverse shock to real GDP growth in TVAR–Y model



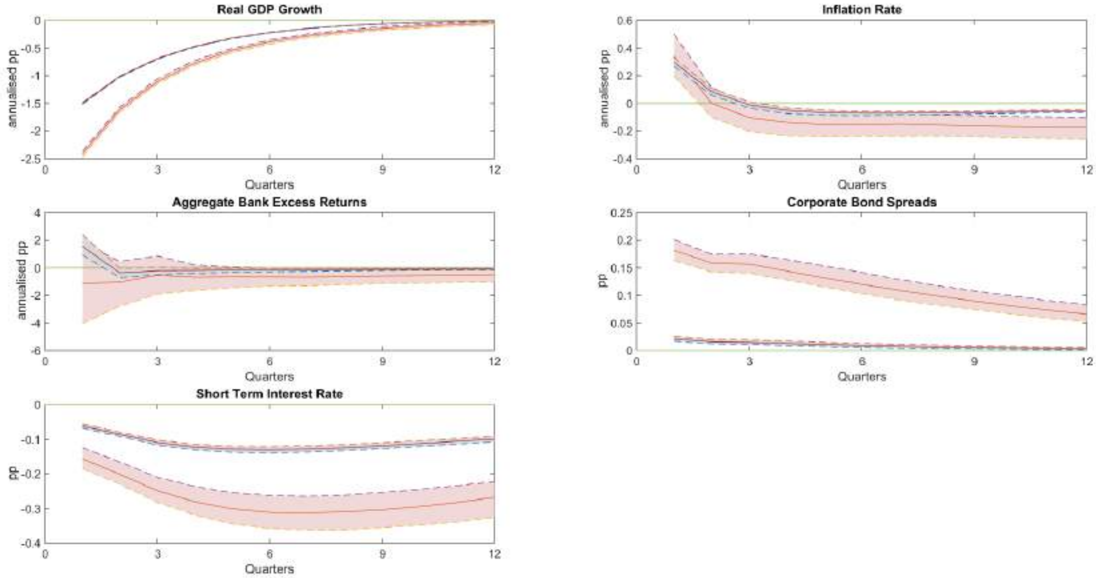
Notes: The error bands correspond to the 68% confidence intervals. The pink shaded area corresponds to the error bands of recessionary regimes. The grey shades correspond to those of nonrecessionary regimes.

Figure 11: Generalised Impulse Responses to a 1 SD adverse shock to real GDP growth in TVAR–S model



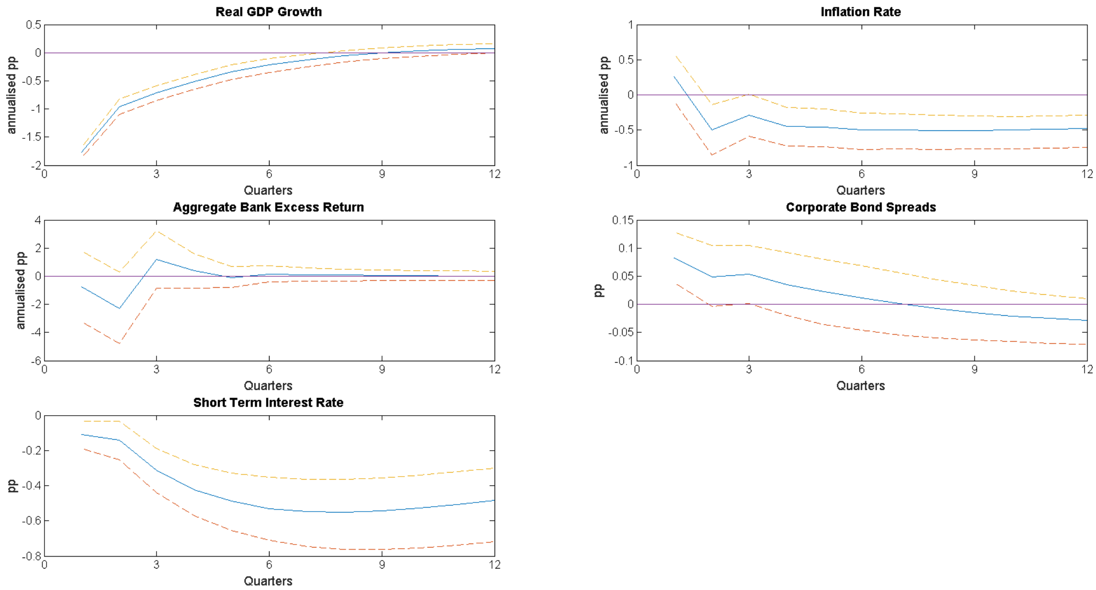
Notes: The error bands correspond to the 68% confidence intervals. The pink shaded area corresponds to the error bands of financial stress regimes. The grey shades correspond to those of financial nonstress regimes.

Figure 12: Generalised Impulse Responses to a 1 SD adverse shock to real GDP growth in MSVAR model



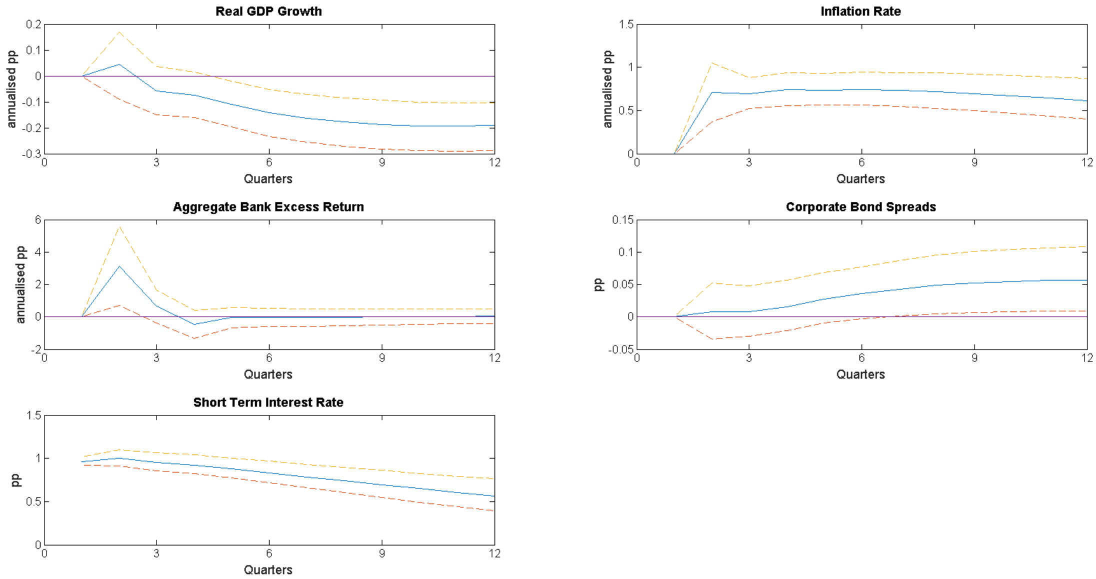
Notes: The error bands correspond to the 68% confidence intervals. The pink shaded area corresponds to the error bands of high stress regimes. The grey shades correspond to those of low stress regimes.

Figure 13: Impulse Responses to a 1 SD adverse shock to real GDP growth in BVAR model



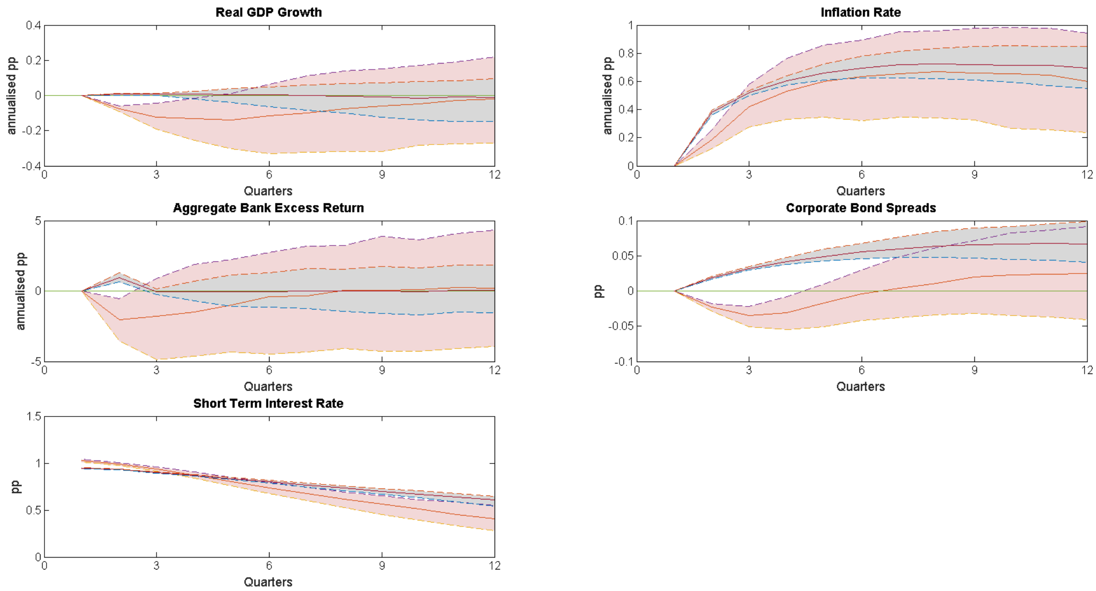
Notes: The error bands correspond to the 68% confidence intervals.

Figure 14: Impulse Responses to a 1 SD adverse shock to short term interest rate in BVAR model



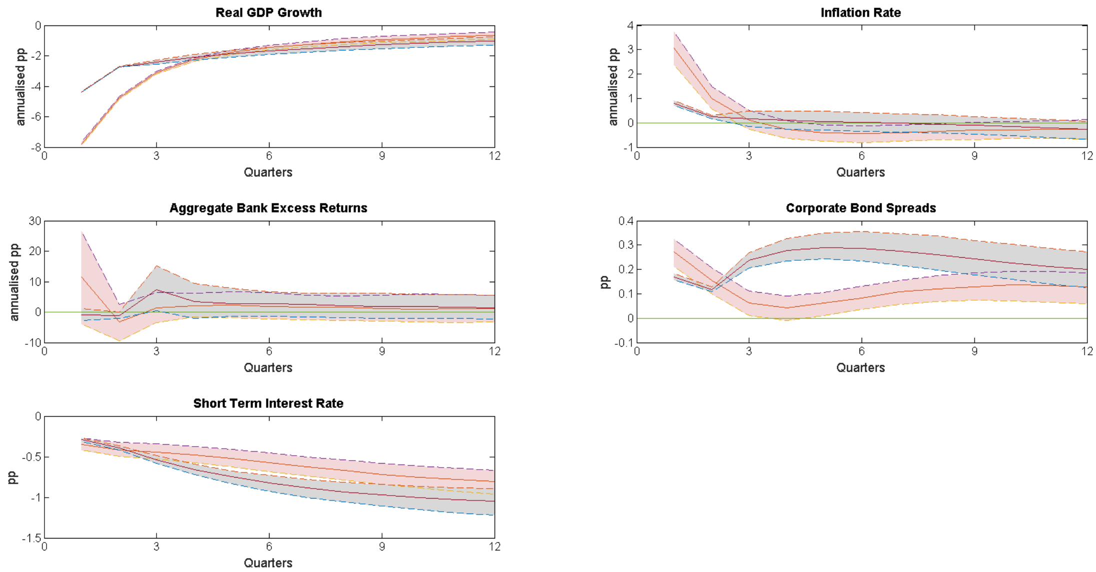
Notes: The error bands correspond to the 68% confidence intervals.

Figure 15: Generalised Impulse Responses to a 1 SD adverse shock to short term interest rate in TVAR–Y model



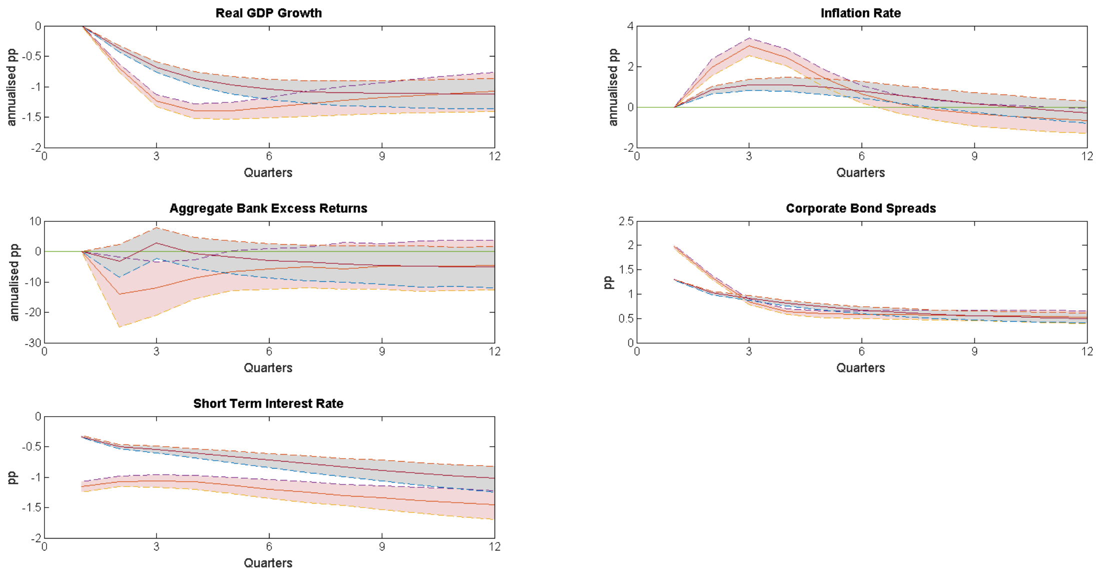
Notes: The error bands correspond to the 68% confidence intervals. The pink shaded area corresponds to the error bands of recessionary regimes. The grey shades correspond to those of nonrecessionary regimes.

Figure 16: Generalised Impulse Responses to a **3 SD adverse shock to real GDP growth** in TVAR–Y model (to be compared with Figure 10)



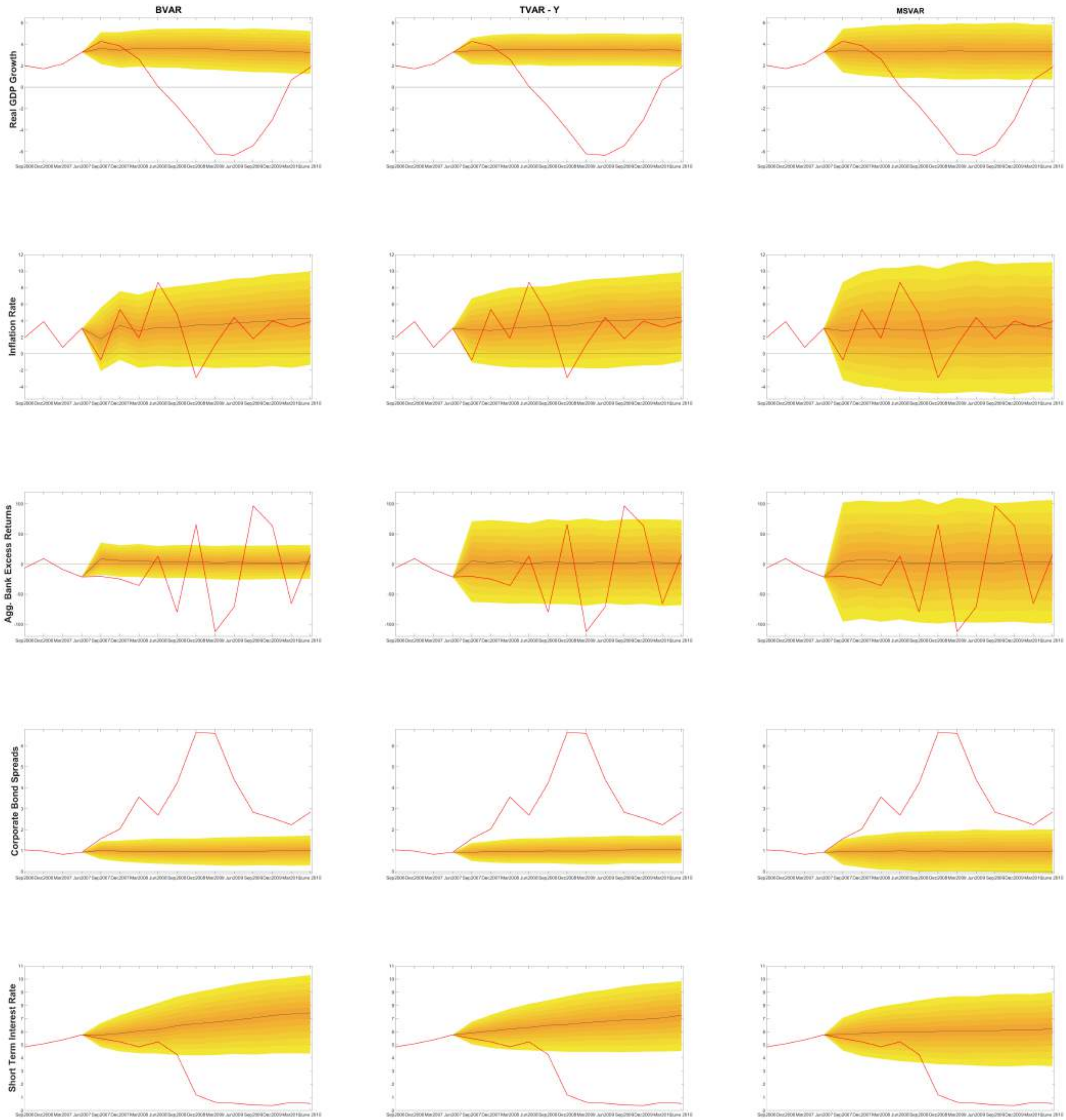
Notes: The error bands correspond to the 68% confidence intervals. The pink shaded area corresponds to the error bands of recessionary regimes. The grey shades correspond to those of nonrecessionary regimes.

Figure 17: Generalised Impulse Responses to a **3 SD adverse shock to corporate bond spreads** in TVAR–S model (to be compared with Figure 7)



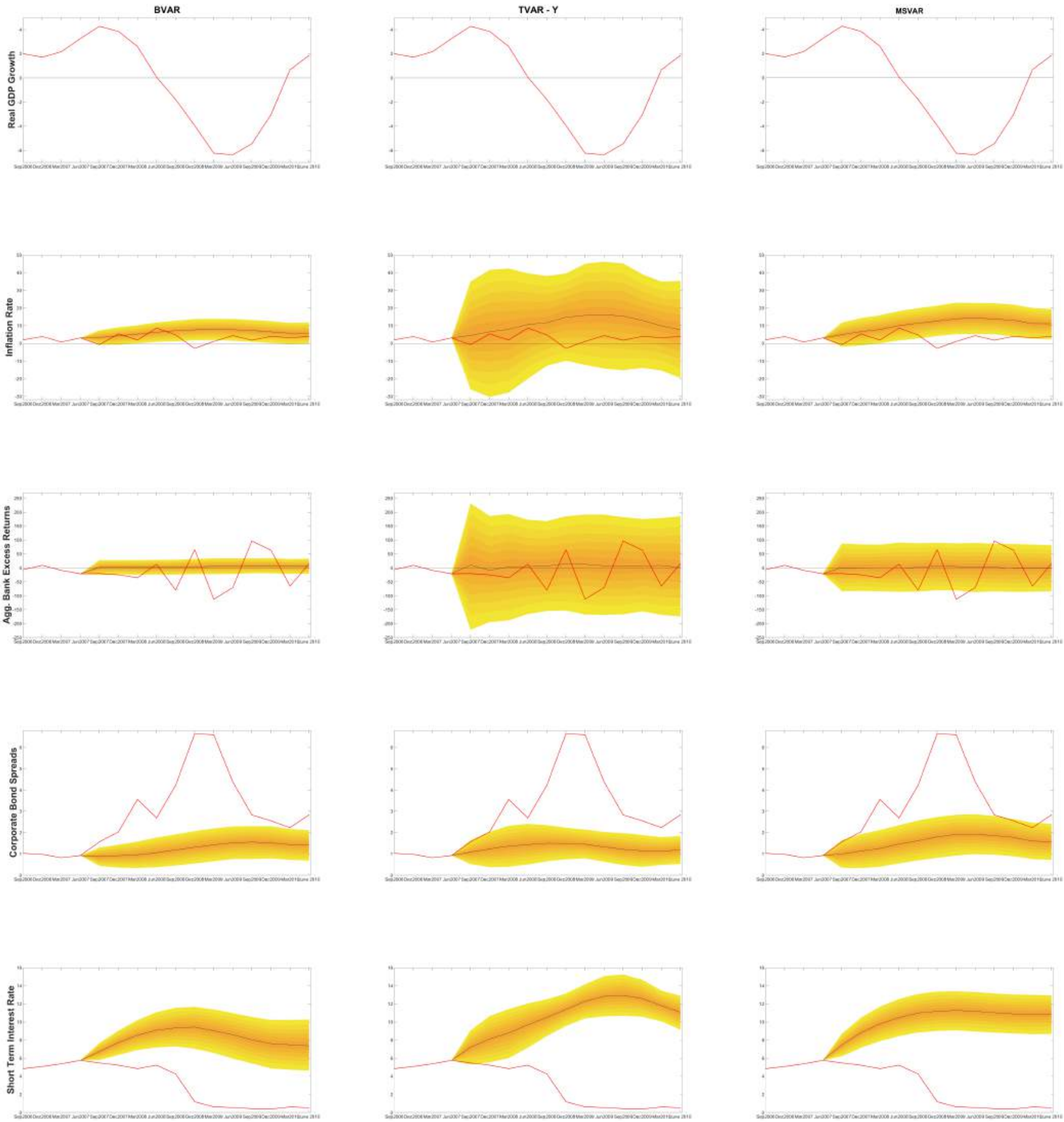
Notes: The error bands correspond to the 68% confidence intervals. The pink shaded area corresponds to the error bands of financial stress regimes. The grey shades correspond to those of financial nonstress regimes.

Figure 18: Unconditional predictive densities for the three models



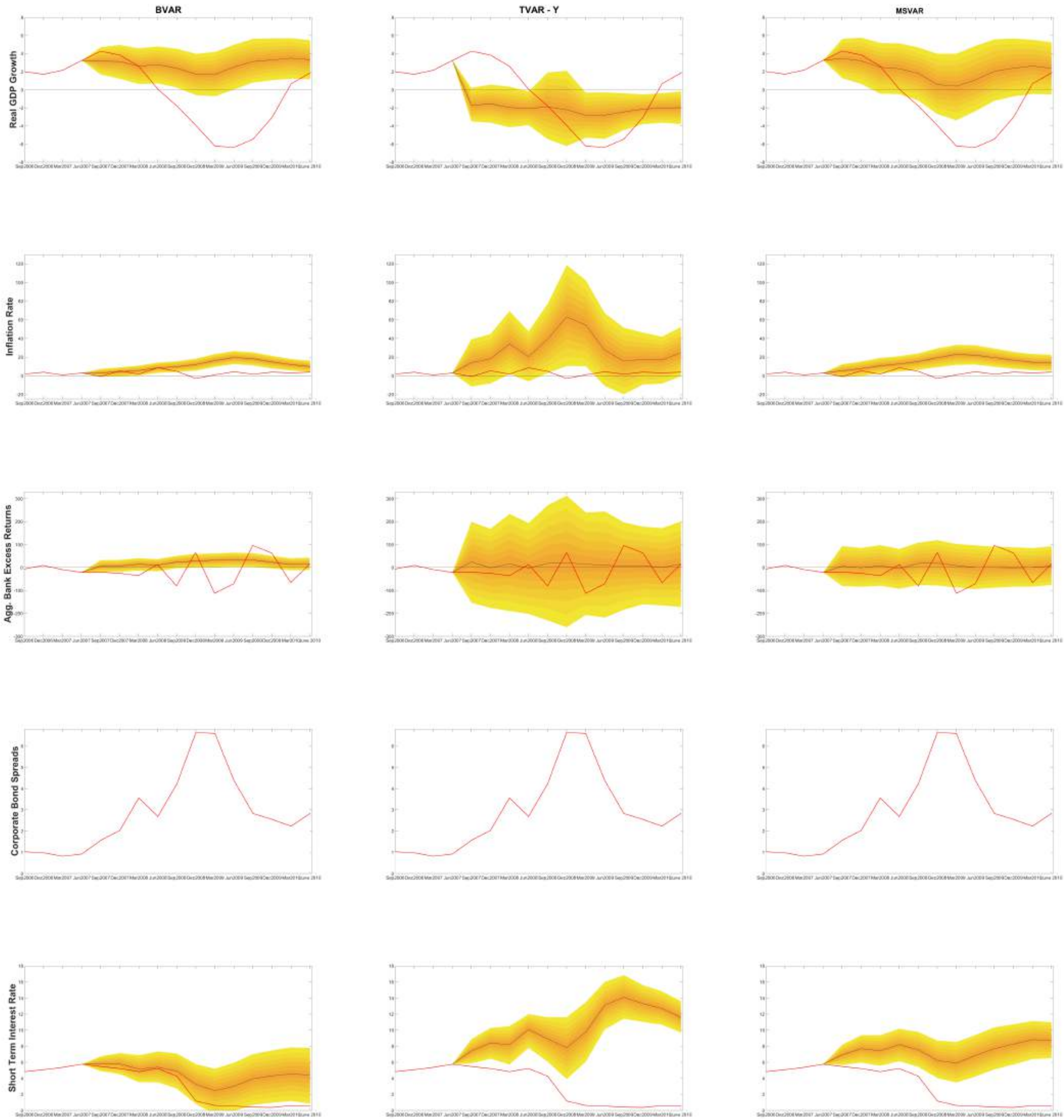
Notes: Unconditional forecast densities of BVAR, TVAR–Y, and MSVAR models are respectively shown in the the first, second and third columns. The variables in order: Real GDP Growth, Inflation Rate, Aggregate Bank Excess Returns, Corporate Bond Spreads and Short Term Interest Rate. The first three variables are annualised. Actual outturns are indicated by the red lines. Fan charts indicate forecast bands between 20^{th} and 80^{th} percentile with 5% increments. The densities correspond to the 12-horizon pseudo-out-of-sample forecasts between 2007:Q3 and 2010:Q2, generated based on data between 1965:Q2 and 2007:Q2.

Figure 19: Conditional predictive densities based on the path of GDP growth rates



Notes: Conditional forecast densities of BVAR, TVAR–Y, and MSVAR models are respectively shown in the the first, second and third columns. The variables in order: Real GDP Growth, Inflation Rate, Aggregate Bank Excess Returns, Corporate Bond Spreads and Short Term Interest Rate. The first three variables are annualised. Actual outturns are indicated by the red lines. Fan charts indicate forecast bands between 20^{th} and 80^{th} percentile with 5% increments. The densities correspond to the 12-horizon pseudo-out-of-sample forecasts between 2007:Q3 and 2010:Q2, generated based on data between 1965:Q2 and 2007:Q2.

Figure 20: Conditional predictive densities based on the path of Corporate Bond Spreads



Notes: Conditional forecast densities of BVAR, TVAR–Y, and MSVAR models are respectively shown in the the first, second and third columns. The variables in order: Real GDP Growth, Inflation Rate, Aggregate Bank Excess Returns, Corporate Bond Spreads and Short Term Interest Rate. The first three variables are annualised. Actual outturns are indicated by the red lines. Fan charts indicate forecast bands between 20^{th} and 80^{th} percentile with 5% increments. The densities correspond to the 12-horizon pseudo-out-of-sample forecasts between 2007:Q3 and 2010:Q2, generated based on data between 1965:Q2 and 2007:Q2.