

# Risk Aversion, Macro Factors and non-fundamental Components in Euro Area Yield Spreads: A Macro-Financial Analysis

Fabian Herrmann<sup>1</sup>

*TU Dortmund University*

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## Abstract

This paper investigates the effects of economic fundamentals and a common risk factor that is not accounted for by Euro area fundamentals on Euro area yield spreads. In particular, it seeks to disentangle the effects of changes in risk aversion and the common risk factor. For this purpose, I use a multi-country macro-finance model of the term structure, where changes in risk-aversion are captured by one single variable. This risk aversion variable is identified from restrictions on the pricing kernel to be the single source of time variation in the prices of risk. The model is applied to yield data of French, German, Italian and Spanish government bonds and the estimation is conducted using Bayesian estimation techniques. The results show that although economic fundamentals were the most dominant driver of Euro area yield spreads, the common risk factor accounts for a non-negligible part in Euro area yield spreads. Notably, the contribution of common risk factor shocks to the yield spreads increased from 2012 onwards. Among the economic factors, changes in risk aversion were the most important source for variations in yield spreads.

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<sup>1</sup>TU Dortmund University, Vogelpothsweg 87, 44227 Dortmund, Germany. Phone: +49 231 755 3441. Email: fabian.herrmann@tu-dortmund.de. Financial support from German Science Foundation (DFG) through SFB 823 is gratefully acknowledged.

# 1 Introduction

From the start of the European Economic and Monetary Union (EMU), a convergence process between the sovereign bond yields of Euro area countries has been observed, despite large differences in fiscal position among those countries. Though interest rate differentials did not vanish completely, they stabilized around a remarkably low level, indicating that country-specific factors did only play a minor role in this period. This convergence is widely referred to the elimination of exchange rate risk and the gradual reduction of inflation risk in Euro area sovereign yields by the introduction of common currency. However, since the onset of the European debt crisis in late 2009, a dramatic surge in the yield spreads of bonds of Euro area sovereigns vis-à-vis German government bonds did occur.

The rise in yield spreads was accompanied by an increase in sovereign debt of several Euro area countries. However, not did only the spreads of sovereign yields of highly indebted countries vis-à-vis Germany rose, but also the spreads of countries with solid fiscal fundamentals (cf. ECB, 2014, p. 75), indicating that also other factors than only credit risk accounted for the rise in yield spreads.

In particular at the beginning of the European debt crisis, in addition to credit risk, the effects of changes in risk aversion are found to be an important component in yield spreads (cf. Haugh et al. 2009, Barrios et al. 2009, Oliviera et al. 2010, Caceres et al., 2010, or Favero et al. 2010). However, recent evidence by e.g. Di Cesare et al. (2012), Hördahl and Tristani (2013), De Santis (2015) or Dewachter et al. (2015) suggests that the surge in sovereign spreads of Euro-area countries cannot be fully explained by changes in fundamentals and country-specific fiscal factors. These authors conclude that also contagion or redenomination risk have played a non-negligible role for the dynamics of yield spreads during the European debt crisis.

This paper investigates the effects of economic fundamentals and a common factor that is unrelated to economic fundamentals on Euro area yield spreads using a macro-finance model of the term structure. Specifically, this paper seeks to disentangle the effects of changes in risk aversion and this common risk factor to quantify their respective contribution to yield spreads of Euro

area sovereigns vis-à-vis Germany while taking account for country-specific fiscal variables, the European business cycle, and monetary policy and their dynamics and interactions. In contrast to the existing literature, the risk aversion measure used in this work is directly derived from Euro area bond market within the macro-finance model.

The results show that the common risk factor played a non-negligible role for yield spreads, accounting for a substantial increase in yield spreads during for the financial crisis and the European debt crisis. Notably, the contribution of common risk factor shocks to the yield spreads increased from 2012 onwards. However, the most dominant drivers of yield spreads have been economic shocks. Among the economic shocks, changes in the risk aversion variable were the most important source for variation in sovereign yield spreads, revealing the importance of measuring risk aversion in Euro area bond markets adequately.

Studying the driver of yields and yield spreads is of interest to practitioners and researchers alike. Not do only play sovereign bonds an important role for asset pricing, sovereign yields are also used as reference rates for key interest rates. Moreover, understanding the determinants of yields is important for understanding the transmission of monetary policy. Likewise, spreads between Euro area sovereign yields may indicate impairments of the transmission process of monetary policy (cf. ECB, 2014). In addition, higher sovereign yields lead to higher marginal (re-) funding costs of governments and thus have the potential to increase the debt burden of a country.

Since the beginning of the EMU, a large empirical literature analyzes Euro area sovereign yields. Traditionally, the literature focuses on a set of variables describing credit risk, investors' risk aversion, and liquidity risk (cf. ECB, 2014). Most of those studies use reduced-form regressions of yield spreads of Euro area countries vis-à-vis Germany at a specific maturity on different determinants. In contrast, a small but growing literature relies on affine term structure models to explore the determinants of Euro area sovereign yields (e.g. Geyer et al., 2004, Borgy et al. 2011, Hördahl and Tristani, 2013, Monfort and Renne, 2011, or Dewachter et al., 2015). By cross-section restrictions derived from no-arbitrage assumptions, these models tie the movements of yields across maturities closely together. They allow to employ information

from the cross-section and are suitable to capture the interaction and dynamics of macro variables and the prices of risk. In the empirical literature, investors' risk aversion is usually proxied by US corporate bond spreads or a US stock market volatility index (e.g. Codogno et al. 2003, Attinasi et al., 2010, Favero et al., 2010, Favero and Missale, 2011, or Bernoth et al., 2012). Although the correlation between these both variables seems to be high, this measure is unable to infer the underlying determinants that drive risk aversion as noted by Manganelli and Wolswijk (2009).<sup>2</sup> Section (2) is dedicated to a more detailed literature review of the determinants of yields.

In order to assess the effect of different determinants on the evolution of sovereign yield spreads I use a multi-country macro-finance model. The model features a unique pricing kernel reflecting the integration of financial markets in a currency union while it still allows for country-specific variables to affect one country's yield curve. Specifically, the yield curve of a country is driven by common variables capturing the European business cycle, a unified monetary policy, the common risk variable, time-varying risk aversion, and also a country-specific fiscal variable capturing default risk.

The common risk factor is meant to capture dynamics in Euro area yield spreads that are unrelated to dynamics in the common economic fundamentals, i.e. a part in Euro area yield spreads that cannot be accounted for by macroeconomic variables. This factor is identified from information contained in cross-country yield curves. Gathering these information requires estimating the term structure of sovereign yields of different European sovereigns jointly. As in Hördahl and Tristani (2013), the common risk factor is modeled as an unobservable variable and is, by construction, unrelated to the economic fundamentals. Therefore, it might be a proxy for contagion effects or redenomination risk.

Changes in risk aversion are measured by a risk aversion variable. Specifically, following Dewachter et al. (2014) and Ireland (2015), by imposing restric-

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<sup>2</sup>Manganelli and Wolswijk (2009) instead suggest to use the risk-free short-term interest rate as a proxy for risk aversion. While indeed evidence by Manganelli and Wolswijk (2009) or Bekaert et al. (2013) indicate an inverse relationship between the short-term interest rate and risk aversion, risk aversion should potentially also respond to other macroeconomic developments (see e.g. Dewachter et al. 2014) and this response does not have to coincide with those of the monetary policy authority to macroeconomic developments. Therefore, in this work, the short-term interest rate is, together with other Euro area-wide factors, one potential driver of changes in risk aversion.

tions on the stochastic discount factor, this risk aversion variable is identified from the term structure of default-free government bonds as the only driver of time variation in the prices of risk. But while Dewachter et al. (2014) and Ireland (2015) use this variable to analyze term premia movements, in this paper the risk aversion variable is used to explore the effects of changes in risk aversion on yield spreads. The risk aversion variable does not only respond to distortions in economic fundamentals and the common risk factor but also exhibits an exogenous dynamic. Thus, it also allows risk aversion to alter exogenously and enables the analysis of changes in risk aversion that are not related to economic developments.

The affine term structure model can be cast into a state-space representation. The effects of fundamental shocks on different state variables are identified, similar to VAR models, by timing restrictions. Monetary policy is described by a monetary policy rule in the spirit of Taylor (1993). In addition to the standard macro variables, monetary policy potentially also responds to movements in the risk aversion variable as in Ireland (2015). To the extent that monetary policy responds on movements in the risk aversion variable, including this channel is required to capture expected monetary policy properly and thus for separating changes of risk aversion from changes in expected future short rates. Indeed, as shown by Ireland (2015) for the US and Herrmann (2015) for the EU, the respective central bank does respond to movements of this risk aversion variable.

In addition to the Euro area business cycle variables, the country-specific fiscal variable, the risk aversion variable and the common risk factor, a time-varying long-run trend component in inflation, interpreted as the central bank's inflation target around which inflation is stabilized, is employed. The long-run trend component helps to shape the expectation of long-term bond yields.

The model is estimated by Bayesian estimation techniques. The posterior function is evaluated using an Adaptive Metropolis (AM) algorithm in the lines of Haario, Saksman and Tamminen (2001).

## 2 Literature review

A broad literature investigates the determinants of Euro area sovereign yield spreads. Traditionally, these determinants are referred to credit risk, liquidity risk, and global risk aversion. More recently, also redenomination risk or systemic risk as drivers of Euro area yield spreads are considered. The credit risk or default risk, measuring a countries creditworthiness, is typically proxied by variables describing the fiscal position of a country (debt-to-GDP ratio, deficit-to-GDP ratio, the debt maturity, or interest expenditure-to-GDP, etc.). Instead of using historical values of the fiscal fundamentals, often the expected fiscal variables are used in order to capture the forward-lookingness of financial markets (cf. Laubach, 2009, Borgy et al. 2012, or Hördahl and Tristani, 2013). The literature finds that the importance of credit risk in sovereign bond spreads increased since the start of the financial crisis and even more since the European sovereign debt crisis (see e.g. Barrios et al., 2009, or Attinasi et al., 2010). Liquidity risk measures the liquidity of sovereign bonds of a specific country. Typically, liquidity risk is proxied by the bid-ask spreads, the amount of outstanding public debt of a country, trading volumes or turn-over ratios. Global risk aversion is typically proxied by the spread of U.S.-Corporate Bonds over U.S. treasury bonds or a volatility index of US stock markets (e.g. Codogno et al. 2003, Attinasi et al., 2010, Favero et al., 2010, Favero and Missale, 2011, or Bernoth et al., 2012).

Although a broad spectrum of different modeling approaches is used, the literature on the determinants of yield spreads can be roughly categorized into two strands. The first strand regresses sovereign bond yields or sovereign bond yield spreads at different maturities on different sets of explanatory variables, representing macro fundamentals, credit risk, liquidity risk and global risk aversion (e.g. Barrios et al., 2009 Beber et al, 2009, Attinasi et al., 2010, Favero et al. 2010, Manganelli and Wolswijk, 2009, Schuknecht et al., 2011, Bernoth et al., 2012, or Di Cesare et al. 2013). The second strand of the literature uses no-arbitrage term structure models in order to examine the determinants of euro area sovereign yield spreads. Among these authors are Geyer et al. (2004), Ang and Longstaff (2013), Battestini et al. (2013), Borgy et al. (2012), Hördahl and Tristani (2013), and Dewachter et al. (2015). While

Geyer et al. (2004) employ a purely latent factor model, Borgy et al. (2012) investigate the determinants of yield spreads using only macro variables as factors. Focusing on the effects of fiscal variables on spreads, they find that the importance of fiscal variables for euro area yield spreads increased since the beginning of the financial crisis. Notably, among those papers Hördahl and Tristani (2013) are the only one whose model accounts for non-linear effects of fiscal fundamentals on sovereign yield spreads. Instead of relying on an affine term structure model, they employ a quadratic, no-arbitrage term structure model. They argue that the non-linear model helps to explain the surge in Euro area yield spreads during the European debt crisis.

While the importance of credit risk and global risk aversion, not only during but also before the onset of the European debt crisis, seems to be unambiguous (e.g. Codogno et al, 2003, Geyer et al., 2004, Manganelli and Wolswijk, 2009, Favero et al., 2010, Attinasi et al., 2010, Laubach, 2011, Schuknecht et al., 2011, or Bernoth et al., 2012), the evidence for the relevance of liquidity risks for sovereign bond yields seems to be less striking. Beber et al (2009), using intra-day European bond quotes from the beginning of 2003 until the end of 2004, or Haugh (2009), using a panel regression including an interaction term between their proxy for risk aversion and their liquidity proxy, stress the importance of liquidity, in particular for smaller European economies, and in particular in times of high market distress. Meanwhile, other authors find no or only less explanatory power of liquidity for sovereign yield spreads (e.g. Codogno et al., 2003, Geyer et al., 2004, Pagano and von Thadden, 2004, Favero et al., 2010, or Bernoth et al., 2012). Bernoth et al. (2012) find that liquidity played only a role in European sovereign bond yields before the start of the EMU, but not after the start of the EMU. Moreover, while Beber et al. (2009) and Manganelli and Wolswijk (2009) find a positive relationship between their proxy for risk aversion and between their proxy for liquidity, meaning that in times of market stress investors value liquidity more than in “normal” times, Favero et al. (2010) found exactly the opposite relation. The different results may arise due to their different measures for risk. Favero et al. (2010) use a risk measure derived from the U.S. bond markets to proxy aggregate risk. Manganelli and Wolswijk (2009), instead, employ a Euro-area risk-free short-term interest rate, as a proxy for international (Euro area) risk

aversion. In contrast, Beber et al. (2009) controls only for country-specific risk factors and not for an aggregate risk factor.

Evidence by Geyer et al. (2004), Caceres et al. (2010), Amisano and Tristani (2013), Ang and Longstaff (2013), Hördahl and Tristani (2013), De Santis (2014,2015), Di Cesare et al. (2013), Giordano et al. (2013), or Dewachter et al. (2015) suggests that also systemic risk or redenomination risk and financial contagion effects may be drivers of Euro area sovereign yield spreads. Geyer et al. (2004) find evidence for a common factor in yield spreads which they interpret as “systematic risk”. They conclude that systematic risk arises from a “small but positive probability of a general failure of the EMU”. Caceres et al. (2010) use a GARCH model to investigate the effects of changes in global risk aversion and country-specific risk, via fundamentals and or contagion, on euro area sovereign yield spreads. They find evidence for contagion in euro area yield spreads and that the source of contagion changed over time. Amisano and Tristani (2013) use a Markov switching VAR to examine contagion in euro area sovereign bond spreads. Considering a normal and a crisis regime, they find that the risk of falling into the crisis regime depends on macroeconomic fundamentals, on risk aversion, and on the other countries regime dynamics. Ang and Longstaff (2013) investigate the effects of country-specific shocks and systemic shock on the CDS spreads of states of the U.S. and euro area countries. Using CDS spreads in a multifactor affine framework, they find a stronger impact of systemic risk in the euro area than in the U.S. states. Di Cesare et al. (2013) show that the surge in euro area sovereign yield spreads during the debt crisis cannot be fully explained by country-specific fiscal variables and macroeconomic fundamentals, but by a common non-fundamental factor. They argue that this common factor is the perceived risk of a break-up in the euro area. De Santis (2014) analyzes spill-over effects and contagion in the euro area sovereign bond market using a VAR model. Employing a sample from the beginning of 2006 until the end of 2012, he finds that country-specific fiscal variables and spill-over effects from Greece contribute to the developments in sovereign yield spreads. Giordano et al. (2013) categorize contagion into different types of contagion. Using a dynamic panel approach, in contrast to other authors, they do not find evidence for pure contagion, that is, a contagion that is completely unrelated fundamentals in euro area bond markets during the



debt crisis. Hördahl and Tristani (2013) construct a quadratic, no-arbitrage term structure model for defaultable sovereign bonds. Using yield spreads of five different Euro area countries vis-à-vis German yields at corresponding maturities, they find that economic fundamentals, but also an unobservable non-fundamental factor contribute significantly to the surge in spreads of most of the considered Euro area countries. De Santis (2015) proposes a measure for redenomination risk in the euro area using CDS spread data. He finds that redenomination risk shocks adversely affect euro area yield spreads. He also finds evidence for spill-over effects of redenomination shocks, concluding that these effects are a source of systemic risk. Finally, Dewachter et al. (2015) use a multi-issuer, no-arbitrage affine term-structure model with unspanned macro factors. Their findings show that economic fundamentals are the dominant drivers of euro area sovereign bond spreads. However, also shocks unrelated to economic fundamentals have played an important role during the European debt crisis.

### **3 The model**

This section develops a multi-country no-arbitrage affine term structure model for the Eurozone. The model is a multi-country extension of the affine term structure model proposed by Ireland (2015).

The model section is structured as follows. The first part describes the structural macroeconomic dynamics and casts the macro part into its state representation. The state variables are then used as pricing factors in the term structure model in the second subsection. Cross-equation restrictions, based on the assumption of no-arbitrage, are employed to tie the movements of yields closely together. The risk aversion variable is identified from restrictions on the prices of risk. Finally, the last subsection discusses the properties of the risk aversion variable. In particular, this subsection demonstrates that the risk aversion variable is the only driver of the term premium of the default-free government bonds.

### 3.1 The State Equation

The model contains nine variables, six of them are observable, and three are unobservable. The observables are the short-term interest rate  $r_t$ , the output gap  $g_t^y$ , the inflation rate  $\pi_t$ , and the fiscal variables of the three sovereigns whose sovereign bonds might face credit risk or markets consider them to be subject to credit risk. The latent variables are the common time-varying central bank's inflation target  $\pi_t^*$ , the risk aversion variable  $v_t$  (which captures all movements in the prices of risk, as in Dewachter and Iania, 2012, Dewachter et al., 2014, and Ireland, 2015), and a common risk factor  $C_t$ . This common risk factor is meant to capture non-fundamental risks, i.e. the part of the spread between yields of a potentially defaultable government bond and of a non-defaultable reference bond of the same maturity that cannot be justified by country-specific economic factors and euro area economic fundamentals. The analysis focuses only on countries belonging to the Euro area. Therefore, monetary policy for all countries is conducted by a single central bank. I follow Ireland (2015) closely in the specification of the dynamics of the Euro-area variables while the specification of the country-specific variables is based on Borgy et al. (2012).

The central bank's policy is depicted as choosing an inflation rate target and adjusting the short-term interest rate accordingly. The incorporation of an unobservable time-varying inflation target is a common approach in the recent macro-finance term structure literature (as in e.g. Dewachter and Lyrio, 2006, Hördahl et al., 2006, Rudebusch and Wu, 2008, or Hördahl and Tristani, 2012, Ireland, 2015). It allows, on the one hand, for some variation in the conduction of monetary policy, and it helps, on the other hand, to capture movements in long-term nominal government bond yields which arise due to changes in central bank's inflation target. In fact, Barr and Campbell (1997) for the UK and Gürkaynak et al. (2005) for the US find that movements in long-term interest rates occur mainly due to changes in expected inflation. In practice, the central bank's inflation target is supposed to follow at stationary AR(1) process. Specifically,

$$\pi_t^* = (1 - \rho_{\pi^*}) \pi^* + \rho_{\pi^*} \pi_{t-1}^* + \sigma_{\pi^*} \varepsilon_{\pi^* t}, \quad (1)$$

where  $\pi^*$  is the steady state level of the inflation target,  $\rho_{\pi^*} \in [0, 1)$ ,  $\sigma_{\pi^*} > 0$

and the shock  $\varepsilon_{\pi^*t}$  is standard normally distributed. As in Hördahl et al. (2006), Rudebusch and Wu (2008), Hördahl and Tristani (2012), or Ireland (2015), this restriction is imposed to ensure stationarity of the inflation target process. A non-stationary inflation target leads to non-stationary inflation and non-stationary nominal short-term interest rate. As shown by Campbell, Lo, and MacKinley (1997 p. 433) or Spencer (2008) for models with homoscedastic shocks, a unit root in the nominal short-term interest rate translates in undefined asymptotic long-term bond yields. Thus, imposing stationarity of the inflation target process ensures that the term structure part of the model is well-behaved.

By defining the inflation gap, and the interest rate gap, as in Ireland (2015), the notation is simplified. Specifically, the inflation rate gap is defined as the deviation of the inflation rate from central bank's inflation target,

$$g_t^\pi \equiv \pi_t - \pi_t^*,$$

and the interest rate gap is defined as the deviation of the interest rate from the inflation target,

$$g_t^r \equiv r_t - \pi_t^*.$$

The central bank's policy rule for the short term nominal interest rate can then be specified in terms of the interest rate gap, the inflation gap and the output gap. Specifically, the central bank sets the interest rate according to the following interest rate rule in the spirit of Taylor (1993),

$$g_t^r - g^r = \rho_r (g_{t-1}^r - g^r) + (1 - \rho_r) [\rho_\pi g_t^\pi + \rho_y (g_t^y - g^y) + \rho_v v_t] + \sigma_r \varepsilon_{rt}, \quad (2)$$

where  $\rho_r, \rho_r \in (0, 1)$ , is the interest rate smoothing parameter,  $\rho_\pi, \rho_\pi > 0$ ,  $\rho_y, \rho_y > 0$ , and  $\rho_v$  are the central bank's response parameters on inflation, the output gap and the variation in the term premium variable, respectively,  $\sigma_r, \sigma_r > 0$ , is a volatility parameter, and  $g^r$  and  $g^y$  are the steady state values of  $g_t^r$  and  $g_t^y$ , respectively. The shock  $\varepsilon_{rt}$  is supposed to be standard normally distributed and represents the interest rate policy shock. The notation of the policy rule incorporates the assumption that the central bank is on average able to implement its inflation target. Thus, in the steady state, the actual inflation rate equals the central bank's target rate. While the response parameter  $\rho_\pi$  and  $\rho_y$  are restricted to be non-negative, the response parameter  $\rho_v$ ,

is unconstrained. As demonstrated in Section (3.3), the risk aversion variable is identified as the only source for fluctuations in term premia. Thus, a positive value of  $\rho_v$  implies that the central bank tends to tighten monetary policy in response to a rise in term premia. Goodfriend (1993) and McCallum (2005) argue that this should be the case if the central bank regards an increase in premia as an indicator of “inflation scares” or as an indicator of policy laxity.<sup>3</sup> In contrast, Bernanke (2006) argues that, to the extent that aggregate demand also depends on long-term interest rates, a rise in the term premium requires the central bank to lower the short-term interest rate in order to offset the effects of the decline in premia and to retain the economic condition, all else being equal. Thus, the coefficient  $\rho_v$  should be negative. This so called practitioner view, as labeled and discussed by Rudebusch, Sack, and Swanson (2007), states that optimal monetary policy should account for movements in premia by adjusting the interest rate contrary to the directions of the premia movements. Apparently, if  $\rho_v$  is zero, the central bank does not react on changes in term premia at all.

To the extent that the central bank does respond on term premia movements, including this response parameter is important for modeling the expectation of future short-term interest rates. Therefore, it is crucial for separating the yield of risk-free bonds into expectation part and term premium part, and thus for identifying movements in risk aversion. In fact, Herrmann (2015) for the Euro Area and Ireland (2015) for the US find that there is a systematic response of the respective central bank on term premia movements.

The dynamics of the output gap and the inflation gap are modeled as in more conventional structural VAR models. While this specification allows for a fairly high degree of flexibility, restrictions on the contemporaneous relationship of these variables are imposed to ensure identification of the structural model.

Specifically, the output gap depends on its own lags, on lags of the interest

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<sup>3</sup>To be precise, McCallum (2005) suggests that the central bank should tighten monetary policy if the interest rate spread between long-term bond yields and the short-term rate increases, given that the expectation hypothesis holds and that the premium follows an AR(1) process. A rise in the long-short rate spread might be due to two reasons: an increase in future expected short rates or an increase in the term structure premium. In McCallum’s specification of the interest rate rule, the central bank reacts on the long-short spread, and with it, in general, on fluctuations in the term premium. However, the cause for the rise in the spread is not identified.

rate gap, on lags of the inflation gap and on lags of the term premium variable, on the innovations of the inflation target  $\varepsilon_{\pi^*t}$ , on the innovations of inflation  $\varepsilon_{\pi t}$ , and on its own innovations  $\varepsilon_{yt}$ ,

$$g_t^y - g^y = \rho_{yr} (g_{t-1}^r - g^r) + \sum_{i=1}^3 \rho_{y\pi}^i g_{t-i}^\pi + \sum_{i=1}^3 \rho_{yy}^i (g_{t-i}^y - g^y) \quad (3)$$

$$+ \rho_{yv} v_{t-1} + \sigma_{y\pi} \sigma_\pi \varepsilon_{\pi t} + \sigma_{y\pi^*} \sigma_{\pi^*} \varepsilon_{\pi^*t} + \sigma_y \varepsilon_{yt},$$

where the volatility parameter  $\sigma_y$  is non-negative, and  $\varepsilon_{yt}$  is supposed to be standard normally distributed. The evolution of the inflation gap depends on own lags, on lags of the interest rate gap, on lags of the output gap, on lags of the term premium variable, on innovations of the inflation target  $\varepsilon_{\pi^*t}$ , and on its own innovations  $\varepsilon_{\pi t}$ ,

$$g_t^\pi = \rho_{\pi r} (g_{t-1}^r - g^r) + \sum_{i=1}^3 \rho_{\pi\pi}^i g_{t-i}^\pi + \sum_{i=1}^3 \rho_{\pi y}^i (g_{t-i}^y - g^y) \quad (4)$$

$$+ \rho_{\pi v} v_{t-1} + \sigma_{\pi\pi^*} \sigma_{\pi^*} \varepsilon_{\pi^*t} + \sigma_\pi \varepsilon_{\pi t},$$

where the volatility parameter  $\sigma_\pi$  is non-negative and  $\varepsilon_{\pi t}$  is standard normally distributed.

The fiscal variable of a country is given by the change in the change in the debt-to-GDP ratio of the respective country. In the choice of the change of the debt-to-GDP ratio as the measure of fiscal sustainability, I follow Borgy et al. (2012) and Dewachter et al. (2015). Specifically, similar to Borgy et al. (2012), the dynamics of the fiscal variables are modeled by an AR(1) process,

$$d_t^i = \rho_d^i d_{t-1}^i + \sigma_d^i \varepsilon_{dt}^i \quad \forall i \in \text{fr, it, es} \quad (5)$$

where  $d_t^i$  denotes the fiscal variable of a country  $i$ ,  $\rho_d^i \in [0, 1)$  is the persistence parameter and  $\sigma_d^i > 0$  is the volatility parameter. The shock  $\varepsilon_{dt}^i$  is standard normally distributed. For parsimonious reasons, the specification supposes that the debt-to-GDP growth rate is exogenous from the other state variables. Omitting feedback effects from the European business cycle to the national fiscal variables helps to reduce the number of parameters in an already highly parameterized model.

The model features a latent common risk aversion variable which can affect the yield spreads of the Euro area sovereigns. . This factor potentially captures the effects of redenomination risk or contagion on yield spreads. Similar

to Hördahl and Tristani (2013), the common risk variable is supposed to be unrelated to economic fundamentals, but is allowed to exhibit an endogenous dynamic through a feedback effect  $\rho_C$  and a structural shock  $\varepsilon_{Ct}$  developments. Specifically, the dynamic of the common risk variable is given by the following AR(1) process

$$C_t = \rho_C C_{t-1} + \sigma_C \varepsilon_{Ct}, \quad (6)$$

where  $\rho_C < |1|$ ,  $\sigma_C > 0$  and the shock  $\varepsilon_{Ct}$  is standard normally distributed.

The risk aversion variable is supposed to be the most endogenous variable in the economy. It is identified from the time variation in the prices of risk in the stochastic discount factor. By construction, all movements in the prices of risk are attributed to the risk aversion variable (see Section (3.2)). Movements in the prices of risk are in turn identified from the default-free reference term structure. Specifically, the evolution of the risk aversion variable is given by

$$\begin{aligned} v_t = & \rho_{vv} v_{t-1} + \sigma_{vr} \sigma_r \varepsilon_{rt} + \sigma_{v\pi} \sigma_\pi \varepsilon_{\pi t} + \sigma_{vy} \sigma_y \varepsilon_{yt} + \sigma_{v\pi^*} \sigma_{\pi^*} \varepsilon_{\pi^* t} \\ & + \sigma_{vC} \sigma_C \varepsilon_{Ct} + \sigma_{vd}^{fr} \sigma_d^{fr} \varepsilon_{dt}^{fr} + \sigma_{vd}^{es} \sigma_d^{es} \varepsilon_{dt}^{es} + \sigma_{vd}^{it} \sigma_d^{it} \varepsilon_{dt}^{it} + \sigma_v \varepsilon_{vt}, \end{aligned} \quad (7)$$

where the volatility parameter  $\sigma_v$  is non-negative, and  $\varepsilon_{vt}$  is standard normally distributed.

Though the specification of the risk aversion variable follows Ireland (2015), the interpretation of this variable is more closely related to the return forecasting factor of Dewachter and Iania (2012) and Dewachter et al. (2014a): It allows for endogenous dynamics, through a feedback effect ( $\rho_{vv}$ ) and a risk aversion shock ( $\varepsilon_{vt}$ ). This shock is meant to account for not macro related shifts in risk aversion. In addition, the common macro variables are allowed to affect the risk aversion variable directly by the contemporaneous effect of their structural shocks ( $\varepsilon_{rt}$ ,  $\varepsilon_{\pi t}$ ,  $\varepsilon_{yt}$ ,  $\varepsilon_{\pi^* t}$ ,  $\varepsilon_{Ct}$ ). Moreover, the country-specific fiscal variables are also allowed to affect the risk aversion variable. This specification potentially allows the model to account for flight-to-safety motives.

The chosen structure imposes restrictions in order to identify the structural model. As in Ireland (2015), shocks to the inflation target  $\varepsilon_{\pi^* t}$  affect the interest rate gap, the inflation gap, the output gap and the risk aversion variable only contemporaneously. All further effects of fluctuations in the central bank's inflation target affect the economy only if the change in the inflation gap and interest rate gap are not fully offset by a proportional adjustment

of the interest rate and the inflation rate. This specification imposes a form of long-run monetary neutrality (see Ireland, 2015). To disentangle the effects of different fundamental disturbances on the economy's variables, the following restrictions on the contemporaneous relationship of these variables are imposed.

The central bank responds immediately to changes in the risk aversion variable while the risk aversion variable only responds to interest rate shocks. While the interest rate responds immediately on fluctuations of the output gap and the inflation gap, changes in the short term interest rate do not affect the output gap and the inflation gap immediately, but with one period lag. The output gap shock  $\varepsilon_{yt}$  does only affect the inflation gap with a lag of one period, while a shock to the inflation gap affects the output gap contemporaneously. Moreover, the fiscal variables are modeled by an autoregressive process, as already discussed above. In addition, as in Borgy et al. (2012) and Hördahl and Tristani (2013), direct feedbacks from the national fiscal variables back to the Euro area business cycle are omitted. However, through their effects on  $v_t$ , they can affect the economy.

Define the vectors  $X_t$  and  $\varepsilon_t$  containing the state variables and the structural disturbances, respectively, by

$$X_t = \left[ r_t \quad g_t^y \quad g_{t-1}^y \quad g_{t-2}^y \quad g_t^\pi \quad g_{t-1}^\pi \quad g_{t-2}^\pi \quad \pi_t^* \quad C_t \quad v_t \quad d_t^{fr} \quad d_t^{it} \quad d_t^{es} \right]'$$

and

$$\varepsilon_t = \left[ \varepsilon_{rt} \quad \varepsilon_{yt} \quad 0 \quad 0 \quad \varepsilon_{\pi t} \quad 0 \quad 0 \quad \varepsilon_{\pi^* t} \quad \varepsilon_{Ct} \quad \varepsilon_{vt} \quad \varepsilon_d^{fr} \quad \varepsilon_d^{it} \quad \varepsilon_d^{es} \right]'$$

then eq., (1) - (6) can be expressed as

$$P_0 X_t = \mu_0 + P_1 X_{t-1} + \Sigma_0 \varepsilon_t. \quad (8)$$

For the specific form of the matrices  $P_0$ ,  $P_1$ ,  $\mu_0$ , and  $\Sigma_0$  see Appendix (A.1). Eq. (8) displays the structural form of the model. Multiplying by  $P_0^{-1}$  yields the reduced form representation of the state equation,

$$X_t = \mu + P X_{t-1} + \Sigma \varepsilon_t, \quad (9)$$

where

$$\begin{aligned} \mu &= P_0^{-1} \mu_0, \\ P &= P_0^{-1} P_1 \end{aligned}$$

and

$$\Sigma = P_0^{-1}\Sigma_0.$$

### 3.2 The Term Structure Model

Affine term structure models, as developed by Duffie and Kan (1996) and Dai and Singleton (2000), are a particular class of term structure models where the time  $t$  yield  $y_t^{(\tau)}$  of  $\tau$ -period zero coupon bond is modeled as an affine function of the state vector  $X_t$ ,

$$y_t^{(\tau)} = A_\tau + B_\tau' X_t,$$

where both coefficients  $A_\tau$  and  $B_\tau$  depend on the maturity  $\tau$ . Though yields are linear affine in the state vector  $X_t$ ,  $A_\tau$  and  $B_\tau'$  are highly non-linear functions of the parameters of the state vector and of the prices of risk. The particular functional form of these coefficients is derived from cross-equation restrictions, which in turn stem from the assumption of the absence of arbitrage opportunities.

The outlined model follows the discrete-time version by Ang and Piazzesi (2003). Restrictions on the prices of risk similar to those in Dewachter and Iania (2012), Dewachter et al. (2014) and Ireland (2015) are imposed to permit the risk aversion variable  $v_t$  to be the only source of fluctuations in the prices of risk and with it in the term premium. In order to study the role of default risk in this affine set-up, I employ the extension of affine term structure models to defaultable bond as proposed by Duffie and Singleton (1999). This subsection is structured as follows: the first part derives the default-risk-free bond prices and discusses the restrictions on the prices of risk. The second part derives the prices of defaultable bonds.

#### 3.2.1 Default risk-free bond pricing and the prices of risk

The short-end of the yield curve, the nominal short-term risk-free interest rate, is modeled as an affine function of the state vector  $X_t$ . The short-term interest rate equation is given by

$$r_t = \delta_0 + \delta_1' X_t, \tag{10}$$

where  $\delta_0$  is a scalar, and  $\delta_1'$  is a 1x13 selection vector indicating the position of  $g_t^r$  and  $\tau_t$  in  $X_t$ . The short-term rate, and thus the short-end of the yield



curve, is from eq. (2) under the control of the central bank. The coefficients  $\delta_0$  and  $\delta_1$  are restricted to ensure consistency between the macro part and the term structure part of the model. This requires  $\delta_0 = 0$ , and

$$\delta'_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix},$$

so that eq. (10) corresponds to the definition of the interest rate gap.

The prices of government bonds are supposed to be arbitrage free. As shown in Harrison and Kreps (1979) or Duffie (2001, pp. 108) the assumption of the absence of arbitrage opportunities guarantees for the existence of a positive stochastic discount factor. Following, among many others, Ang and Piazzesi (2003), the stochastic discount factor which is used to price all bonds in the economy is given by the following log-normal process

$$m_{t+1} = \exp\left(-r_t - \frac{1}{2}\lambda'_t\lambda_t - \lambda'_t\varepsilon_{t+1}\right), \quad (11)$$

where  $\lambda_t$  are the time-varying prices of risk. If all elements in  $\lambda_t$  are equal to zero, investors are risk neutral. The prices of risk are supposed to be affine functions of the state variables, taking the functional form

$$\lambda_t = \lambda_0 + \Lambda_1 X_t, \quad (12)$$

where  $\lambda_0$  is a  $13 \times 1$  vector and  $\Lambda_1$  is a  $13 \times 13$  matrix.

In the following, restrictions on the matrix  $\lambda_1$  are imposed. First, in order to identify the risk aversion variable  $v_t$  as the only source for time-variation in the prices of risk, all elements in  $\Lambda_1$ , except the 10th column, are restricted to be equal to zero. This restriction is in the spirit of Cochrane and Piazzesi (2005,2008) who found that one single factor - a linear combination of the spot rate and four forward rates - accounts for most of the variation in term premia. But instead of using an observable combination of interest rates, the risk aversion variable is modeled as a latent variable (as in Dewachter and Iania, 2012, Dewachter et al. 2014, and Ireland, 2015). Second, I assume that only contemporaneous state variables can be priced. Finally, as in Ireland (2015), the risk aversion variable itself is not a source for priced risk. After

applying the restrictions on the matrix  $\Lambda_1$ ,  $\Lambda_1$  reads

$$\Lambda_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda^r & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda^y & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda^\pi & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda^{\pi^*} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda^C & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda^v & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda^{dfr} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda^{des} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda^{dit} & 0 & 0 & 0 \end{bmatrix},$$

and the corresponding vector  $\lambda_0$  reads

$$\lambda'_0 = \begin{bmatrix} \lambda_0^r & \lambda_0^y & 0 & 0 & \lambda_0^\pi & 0 & 0 & \lambda_0^{\pi^*} & \lambda_0^C & \lambda_0^v & \lambda_0^{dfr} & \lambda_0^{des} & \lambda_0^{dit} \end{bmatrix}.$$

From eq. (12) these restrictions work to attribute all movements in the prices of risk  $\lambda_t$  to the variable that is ordered at the 10th position in  $X_t$ , that is, the risk aversion variable  $v_t$ . As demonstrated in section (3.3), from the restricted form of  $\Lambda_1$  also all time-variation in term premia are attributed to the risk aversion variable.

Let  $P_t^{\tau+1}$  denote the price of a risk-free zero-coupon bond maturing at time  $t + \tau$ , then, given the no-arbitrage assumption, the pricing kernel  $m_{t+1}$ , and the affine prices of risk  $\lambda_t$ , from the no-arbitrage condition

$$P_t^{\tau+1} = E_t(m_{t+1}P_{t+1}^\tau),$$

it can be shown that the bond price  $P_t^{\tau+1}$  can be written as an exponentially affine function of the state vector  $X_t$ . Specifically, the price of a  $t + \tau$ -period risk-free zero-coupon bond  $P_t^{\tau+1}$  at period  $t$  is given by

$$P_t^{\tau+1} = \exp(\bar{A}_{\tau+1} + \bar{B}'_{\tau+1}X_t). \quad (13)$$

The coefficients  $\bar{A}_{\tau+1}$  and  $\bar{B}_{\tau+1}$  can be computed by the standard recursive formulas as provided by Ang and Piazzesi (2003).

### 3.2.2 Pricing of defaultable bonds

Following Duffie and Singleton (1999), the no-arbitrage affine term structure model can be extended to price also defaultable bonds. Duffie and Singleton (1999) show that under the assumption that the recovery value of a defaulting bond is a fraction of the bond’s value conditional on no default would occur (the so-called “recovery of market value” assumption), there exists some recovery-adjusted default intensity process  $s_{j,t}$  (see Appendix (A.2)). Defaultable bonds can then be priced using the same formulas, simply by replacing the risk-free short-term interest rate  $r_t$  by the default-adjusted short-term interest rate  $r_{j,t+1}^* = r_t + s_{j,t+1}$ . Then, bond prices can be expressed by

$$\tilde{P}_{j,t}^{\tau+1} = E_t \left( \exp \left( -r_{j,t+1}^* - \frac{1}{2} \lambda_t' \lambda_t - \lambda_t' \varepsilon_{t+1} \right) \tilde{P}_{j,t+1}^\tau \right),$$

where  $\tilde{P}_{j,t}^{\tau+1}$  denotes the time- $t$  price of a  $\tau + 1$ -period defaultable bond of country  $j$ . If the “recovery-adjusted default intensities” (see e.g. Monfort and Renne, 2011)  $s_{j,t}$  of a country  $j$  is also an affine function of the state vector,

$$s_{j,t} = \psi_{j,0} + \psi_{j,1} X_t,$$

then one can proceed as in standard valuation models for default-risk free bonds and bond prices can be computed by applying the standard recursive formulas. Hence, the price of a zero-coupon defaultable bond can be expressed by

$$\tilde{P}_{j,t}^{\tau+1} = \exp \left( \bar{A}_{j,\tau+1} + \bar{B}'_{j,\tau+1} X_t \right) \quad (14)$$

where the specific solution of the pricing matrices  $\bar{A}_{j,\tau+1}$  and  $\bar{B}'_{j,\tau+1}$  can be computed by the standard recursive formulas. However, these formulas come along with an intense computational cost since the pricing matrices have to be calculated for each period  $\tau = 1, \dots, 60$ , each country  $j$  and each evaluation of the log-posterior function. Therefore, in practice, I apply an algorithm based by Borgy et al. (2012). Instead of computing the pricing matrices  $\bar{A}_{j,\tau+1}$  and  $\bar{B}'_{j,\tau+1}$  recursively, this algorithm computes only selected nested bond maturities and concatenate country-specific pricing matrices, so as to compute parts of the pricing matrices for all countries simultaneously. As demonstrated by Borgy et al. (2012) this algorithm reduces computation time considerably. The solution for the pricing matrices  $\bar{A}_{j,\tau+1}$  and  $\bar{B}'_{j,\tau+1}$  and the algorithm are discussed in Appendix (A.3).

Finally, the dependence of the adjusted default intensities of a country  $j$  on the state variables, that is, the elements in the vector  $\psi_{j,1}$  need to be specified. Instead of estimating all elements in  $\psi_{j,1}$ , I follow, among others, Borgey et al. (2012) and impose restrictions on  $\psi_{j,1}$ . This helps to conserve the number of parameters that need to be estimated. First, the German term structure is supposed to be free of default risk, thus  $\psi_{ger,1} = 0_{13 \times 1}$ . Noteworthy, in this case, the solution of  $\bar{A}_{ger,\tau+1}$  and  $\bar{B}'_{ger,\tau+1}$  reduces to the solution for pricing matrices of the risk-free bonds  $\bar{A}_{\tau+1}$  and  $\bar{B}_{\tau+1}$ , respectively. Thus, the German term structure is the reference structure. It is used to identify the time variation in the prices of risk. Second, as in Borgey et al. (2012) and Dewachter et al. (2015), the spread between risk-free and defaultable bonds depends on common and country-specific factors. In particular, the spread between the yield on a defaultable bond of country  $j$  and the yield of a risk-free bond with the same maturity is assumed to depend on the common, euro area economic fundamental, the common risk factor, and the country-specific fiscal variable of country  $j$ . However, it does not depend on the fiscal variables of the other countries. Finally, only contemporaneous variables are allowed to affect spreads.

### 3.2.3 Bond yields

The continuously compounded bond yields  $y_{j,t}^{(\tau)}$  are defined by

$$y_{ger,t}^{(\tau)} = -\frac{\log(P_t^{\tau+1})}{\tau},$$

and

$$y_{j,t}^{(\tau)} = -\frac{\log(\tilde{P}_{j,t}^{\tau+1})}{\tau} \quad \forall j \neq ger.$$

Given the bond prices  $P_t^{\tau+1}$  and  $\tilde{P}_{j,t}^{\tau+1}$  from eq. (13) and eq. (14), the yields are given by

$$y_{ger,t}^{(\tau)} = A_{\tau+1} + B'_{\tau+1} X_t, \tag{15}$$

and

$$y_{j,t}^{(\tau)} = A_{j,\tau+1} + B'_{j,\tau+1} X_t \quad \forall j \neq ger, \tag{16}$$

respectively, where  $A_{\tau+1} = -\bar{A}_{\tau+1}/\tau$ ,  $B'_{\tau+1} = -\bar{B}'_{\tau+1}/\tau$ ,  $A_{j,\tau+1} = -\bar{A}_{j,\tau+1}/\tau$  and  $B'_{j,\tau+1} = -\bar{B}'_{j,\tau+1}/\tau$ .

### 3.3 Term Premia

It remains to show how the restrictions applied on the prices of risk in section (3.2) work. It can be shown that all movements in term premia can be attributed to the risk aversion variable  $v_t$ . Term structure premia can be captured in different forms (see e.g. Cochrane and Piazzesi, 2008, or Joslin et al., 2014). Similar to Dewachter and Iania (2012) I focus in this analysis on the return premium (as classified by Cochrane and Piazzesi, 2008).

The *return* premium is defined as the expected excess holding period return (or short expected excess return). It is the expected return from buying a long term bond in period  $t + i$  and selling it in the subsequent period  $t + i + 1$  in excess of the expected return from buying a one-period bond. Formally, the  $i + 1$ -period *return* premium is defined as

$$E_t \left( hpr_{t+i+1}^{(\tau)} \right) = E_t \left( hpr_{t+i+1}^{(\tau)} - y_{t+i}^{(1)} \right)$$

where  $hpr_{t+i+1}^{(\tau)}$  is the holding period return defined by

$$hpr_{t+i+1}^{(\tau)} \equiv p_{t+i+1}^{(\tau-1)} - p_{t+i}^{(\tau)},$$

where  $p_t^{(\tau)}$  is the log price of a zero-coupon bond maturing in  $t + \tau$  periods,  $p_t^{(\tau)} \equiv \log \left( P_t^{(\tau)} \right)$  and  $y_{t+i}^{(1)}$  is the yield of a one-period bond. The holding period return  $hpr_{t+i+1}^{(\tau)}$  is the return from buying a bond at time  $t$  that matures in  $t + \tau$  periods and selling this bond the period after.

To bring the return premium in a computationally more tractable form the expected holding period return and the expected short rate have to be calculated. The expected future short rates are given from eq. (10) by  $E_t(r_{t+j}) = \delta_1' E_t(X_{t+j})$ . To calculate the expected future short-term interest, it proves to be helpful to demean the state equation, eq. (9). Let  $\bar{\mu}$  be the unconditional mean of the state vector, then from eq. (9)  $\bar{\mu}$  is given by  $\bar{\mu} = (I - P)^{-1} \mu$  and the demeaned state equation reads  $X_{t+1} - \bar{\mu} = P(X_t - \bar{\mu}) + \Sigma \varepsilon_{t+1}$ . Then, the time- $t$  conditional expected future short rate for period  $t + j$ ,  $\forall j > 0$ , can be computed by

$$E_t(r_{t+j}) = \delta_1' (I - \delta' P^j) \bar{\mu} + \delta_1' P^j X_t.$$

The expected holding period return can be calculated by plugging the model implied log prices,  $p_t^{(\tau)} = \bar{A}_\tau + \bar{B}_\tau' X_t$ , into the definition of the  $i + 1$ -period

holding period return: Thus plugging the expected short-term interest rate and the expected holding period return into the definition of the  $i + 1$ -period return premium and rearranging terms (see Appendix (A.4)) yields,

$$E_t \left( hprx_{t+i+1}^{(\tau)} \right) = \bar{B}'_{\tau-1} \Sigma \left[ \lambda_0 + \Lambda_1 (I - P^i) \bar{\mu} + \Lambda_1 P^i X_t \right] - \frac{1}{2} \bar{B}'_{\tau-1} \Sigma \Sigma' \bar{B}_{\tau-1} \quad (17)$$

If  $i = 0$ , then eq. (17) is the one-period return premium. Precisely, the one-period return premium of a bond with maturity  $\tau$  is given by

$$E_t \left( hprx_{t+1}^{(\tau)} \right) = \bar{B}'_{\tau-1} \Sigma (\lambda_0 + \Lambda_1 X_t) - \frac{1}{2} \bar{B}'_{\tau-1} \Sigma \Sigma' \bar{B}_{\tau-1}. \quad (18)$$

Due to the restricted form of  $\Lambda_1$  the only source of variation in  $E_t \left( hprx_{t+1}^{(\tau)} \right)$  is the variable that is ordered at the 10<sup>th</sup> position in  $X_t$ , that is, the risk aversion variable  $v_t$ . Thus, eq. (18) reveals that all variation over time in one-period return premia arises solely from fluctuations in  $v_t$  for all bond maturities. If all elements in the matrix  $\Lambda_1$  are equal to zero, then the one-period return premium is constant. Likewise, if  $v_t$  is constant over time, the return premium is constant.

## 4 Estimation

### 4.1 Data

My sample contains monthly data on the Euro area from the Beginning of 2000 until the End of 2014. I consider government bonds from the four biggest economies in the Euro area: Germany, France, Italy and Spain. The German term structure is taken as the reference term structure and considered to be free of default risk. As noted by De Santis (2015), the expected probability of a credit event in Germany is considered to be negligible. Not only is the country relatively large and plays a central role in the Euro area, but also, as shown by De Santis (2014) does the German Bund yield comove with the OIS rate. The sample contains data for the country-specific fiscal variables, the Euro area business cycle, and the risk-free short-term interest rate.

The model requires zero-coupon yield data. However, government bonds with maturities of more than one year usually do pay coupons. The zero-coupon yield data need to be constructed from these data. All zero-coupon

yields are constructed using the same method to ensure the comparability of yields across countries. Specifically, the zero-coupon bond yields are estimated from the prices of government bonds of each of the four countries using the Nelson-Siegel (1987) model. The data for the end-of-month government bond prices of each country is taken from Datastream. Appendix (A.5) describes the data selection and the estimation of the zero-coupon yields in more detail. After constructing the zero-coupon yield data, for the subsequent estimation, yields with maturities of 3 months, and 1, 2, 3, 4, and 5 years for the German term structure are selected and yield with maturities of 1 and 5 years for the French, Italian and Spanish term structure are selected. Figure (1) and (2) depict the estimated one-year and five-year yields of the government bonds of France, Germany, Italy and Spain.

The Euro-area variables are the inflation rate, the output gap, and the short-term interest rate. While the first two variables capture the European business cycle, the latter captures monetary policy. The inflation rate is measured by the annual rate of change of the seasonally adjusted HICP of the euro area. The output gap is defined as the percentage (logarithmic) deviation of actual output from trend output. Since GDP data is only available at a quarterly frequency, I use the seasonally adjusted industrial production index of the Euro area as a proxy for output (as e.g. Clarida, Galí and Gertler, 1998, or Favero, 2006). Trend output is constructed using the one-sided HP filter with a smoothing parameter equal to 14,400. The euro-area-wide, risk-free monetary policy rate is proxied by the 3-month rate of zero-coupon German government bonds. In choosing the 3-month rate as the rate with the shortest maturity, I follow the practice of the Bundesbank (cf. Schich, 1997).<sup>4</sup>

The fiscal variable of a country is measured by the change in the debt-to-GDP ratio of the respective country. The data for the debt-to-GDP ratio is taken from Datastream. Since the debt-to-GDP ratio is only available on a quarterly basis, the missing observations need to be constructed. Instead of simply interpolating the data, I follow Hördahl and Tristani (2013) and suppose

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<sup>4</sup>The trading volume of government bonds decreases considerably for short residual maturities so that their prices seem to be significantly influenced by low liquidity (see BIS, 2005, p.9). Therefore, prices of bonds with residual maturities shorter than three months are excluded. Thus, the estimation of zero-coupon yields with residual maturities shorter than three months is basically an out-of-sample forecast.

an autoregressive law of motion for the debt-to-GDP ratio. Specifically, in a preceding step, by presuming the autoregressive law of motion the time-path of the missing observations is constructed by the Kalman filter. Finally, I suppose that the long-run mean of the change in the debt to GDP ratio is zero (as in Borge et al., 2012).

## 4.2 The state space system

The macro part and the affine term structure model form a state-space system. The state equation, given by eq. (9), describes the dynamic of the state vector, while the observables - output gap, inflation, the short-term interest, the fiscal variables and the long-term government bond yields - are linked to the state vector by measurement equations.

For the estimation, a version of the state-state space model without constant terms is employed. By dropping the constant terms appearing in eq. (9), (15) and (16), and using demeaned data the estimation is simplified. In particular, under the assumption that the central bank is able - on average - to implement its target inflation rate, so that the average of the actual inflation rate equals the average target inflation rate, the steady state values of  $g^r$ ,  $\tau$ , and  $g^y$  can be calibrated to match the data averages of the short-term interest rate, the output gap, the fiscal variables and inflation. More precisely, by setting the steady-state inflation target equal to the steady-state value of the inflation target, the steady-state value of the interest rate gap  $g^r$  can be calculated from the average of the short-term interest rate net the average of the inflation rate. The steady state value of the output gap is set equal to the sample mean of the output gap. The fiscal factors are assumed to have a mean of zero. As in Borge et al. (2012), this implies that the debt-to-GDP ratio of each country is stationary. Moreover, as demonstrated in Ireland (2015), the values of the elements in  $\lambda_0$  can be calibrated so that the steady state values of yields match the average yields. The state equation then reads

$$X_t = PX_{t-1} + \Sigma\varepsilon_t,$$

and the measurement equation can then be written by

$$Z_t = UX_t + V\eta_t,$$



where  $Z_t$  is a vector of observables,  $U$  is a matrix connecting the observables to the state vector,  $\eta_t$  is a vector of *i.i.d.* distributed errors and  $V$  is a matrix capturing the volatility parameters of these errors. The vector of observables  $Z_t$  consists of the government bond yields of the four countries, the country-specific fiscal variables and the three observables capturing the European business cycle and monetary policy. To simplify notation, define the vector containing all yields of country  $i$  by  $\mathbf{y}_t^i = \left[ y_{i,t}^{12} \ y_{i,t}^{24} \ y_{i,t}^{36} \ y_{i,t}^{48} \ y_{i,t}^{60} \right]'$  for country  $i = ger$  and  $\mathbf{y}_t^i = \left[ y_{i,t}^{12} \ y_{i,t}^{60} \right]'$  for country  $i \neq ger$ , then the vector of observables reads

$$Z_t \equiv \begin{bmatrix} \mathbf{y}_t^{ger} \\ \mathbf{y}_t^{fr} \\ \mathbf{y}_t^{it} \\ \mathbf{y}_t^{es} \\ d_t^{fr} \\ d_t^{fr} \\ d_t^{fr} \\ r_t \\ g_t^y \\ \pi_t \end{bmatrix}.$$

The matrix  $U$  is given by

$$U \equiv \left[ \mathbf{B}' \ \mathbf{B}'_{fr} \ \mathbf{B}'_{es} \ \mathbf{B}'_{it} \ U_{fr}^f \ U_{it}^f \ U_{es}^f \ U^r \ U^y \ U^\pi \right]',$$

where the  $B$ -matrices are

$$\begin{aligned} \mathbf{B} &= \begin{bmatrix} B'_{12} & B'_{24} & B'_{36} & B'_{48} & B'_{60} \end{bmatrix}, \\ \mathbf{B}_{fr} &= \begin{bmatrix} B'_{fr,12} & B'_{fr,60} \end{bmatrix}, \\ \mathbf{B}_{es} &= \begin{bmatrix} B'_{es,12} & B'_{es,60} \end{bmatrix}, \\ \mathbf{B}_{it} &= \begin{bmatrix} B'_{it,12} & B'_{it,60} \end{bmatrix}. \end{aligned}$$

The remaining elements in the  $U$ -matrix are given by

$$\begin{aligned}
U_{fr}^f &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \\
U_{fr}^f &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \\
U_{fr}^f &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \\
U^r &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
U^y &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
U^\pi &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}.
\end{aligned}$$

The matrix  $V$  contains the volatility parameters of the yield errors. The errors are attached to avoid stochastic singularity. The problem of stochastic singularity arises in macro-finance term structure models because a high dimensional vector of observables (the yield data and the observable macro variables) is fitted to a lower dimensional state vector. Instead of attaching errors to some selected yields, I assume that all yields are affected by error terms, as in Chib and Ergashev (2009). The last columns of  $V$  are equal to zero, reflecting that the short-term interest rate, the output gap, the inflation rate and the changes of the debt-to-GDP ratios of the three countries are not measured with errors.

### 4.3 Estimation method

The model captures the effect of national fiscal variables, investors' risk aversion, the European business cycle, a time-varying inflation target and a common non-fundamental risk factor in sovereign yields. Changes in risk aversion are identified from the default-free term structure. The non-fundamental risk aversion variable is given by the part of sovereign yields that cannot be accounted for by European fundamentals and country-specific fiscal factors. Due to the interaction of risk aversion and the non-fundamental risk factor it is not possible to split the estimation into separate steps (e.g. estimating first the risk-free term structure and the macro dynamics together and then the term structure of each of the other counties separately). Instead, the term structures of the four countries under consideration need to be jointly estimated. This complicates the estimation considerably.

To estimate the state space model, I apply Bayesian estimation techniques.

As often noted in the literature, even the estimation of pure affine term structure model is computationally challenging and time-consuming (see e.g. Christensen et al., 2011, or Chib and Ergashev, 2009). Adding the macro-dynamics enhances these difficulties due to the complexity of the macroeconomic interactions with the term structure and vice versa (see also Rudebusch and Wu, 2008). The parameters in the  $B_{j,\tau}$  matrices of the observation equations are highly non-linear functions of the underlying parameters of the state equations and the prices of risk. This non-linearity, as demonstrated by Chib and Ergashev (2009), can produce multimodal likelihood functions. Applying Bayesian estimation techniques allow employing a priori information which helps to down-weight regions of the parameter space which are not economically reasonable and help to rule out economically implausible parameter values. As a result, the posterior distribution can be smoother than the likelihood function (see Chib and Ergashev, 2009). Moreover, the usage of prior information is helpful when dealing with short data sets.

#### 4.3.1 Posterior and Likelihood function

Formally, let  $Z$  denotes the data set,  $Z = (Z_1, \dots, Z_T)'$ , where  $T$  is the number of total observations, and let  $\theta$  denotes the vector of all parameters contained in the matrices  $P$ ,  $\Sigma$ ,  $\Lambda$  and  $V$ , then from Bayes rule, the joint posterior distribution of  $\theta$ ,  $\pi(\theta|X)$ , is obtained by combining the likelihood function of the observables, the prior distribution of the parameter vector and a norming constant, thus,

$$\pi(\theta|Z) \propto L(Z|\theta)p(\theta),$$

where  $L(Z|\theta)$  is the likelihood function, and  $p(\theta)$  is the prior distribution. Denote by  $\mathcal{Z}_{t-1}$  all available information of the observable variables at time  $t-1$ ,  $\mathcal{Z}_{t-1} \equiv (Z_1, \dots, Z_{t-1})'$ . If the initial state  $X_0$  and the innovations  $\{\varepsilon_t, \eta_t\}_{t=1}^T$  are multivariate Gaussians, then the conditional distribution of the observables  $Z_t$  on  $\mathcal{Z}_{t-1}$  is also Gaussian (see Hamilton, 1994, p. 385)

$$Z_t|\mathcal{Z}_{t-1} \sim N(UX_{t|t-1}, R_{t|t-1}),$$

where  $X_{t|t-1}$  denotes the one step ahead forecast,  $X_{t|t-1} \equiv E[X_t|\mathcal{Z}_{t-1}, \theta]$ , and  $R_{t|t-1}$  denotes the conditional variance,  $R_{t|t-1} \equiv Var(Z_t|\mathcal{Z}_{t-1}, \theta)$ .<sup>5</sup> Since two

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<sup>5</sup>See Appendix (A.6) for the explicit expressions of the prediction and updating equations of the mean and the variance.

of the state variables are latent, the likelihood  $L(Z|\theta)$  is constructed using the standard Kalman filter recursions (see Harvey, 1991). Hence, the joint density of the date set  $Z$  given  $\theta$  can be written as

$$L(Z|\theta) = \prod_{t=1}^T (2\pi)^{-\frac{T}{2}} [\det(R_{t|t-1})]^{-\frac{1}{2}} \\ \times \exp\left(-\frac{1}{2} (Z_t - UX_{t|t-1})' (R_{t|t-1})^{-1} (Z_t - UX_{t|t-1})\right).$$

At the start of the recursions, the initial matrix of the variance of the forecast errors is set equal to the unconditional variance of the state variables.

Since the posterior density is, in general, not known in closed form, I apply Markov Chain Monte Carlo (MCMC) methods (the Adaptive-Metropolis algorithm) to simulate draws from the joint posterior distribution.

### 4.3.2 MCMC Method

The choice of the proposal density of the Metropolis-Hastings algorithm is crucial for the speed of the convergence of the chain (cf. Rosenthal, 2010). The scaling of the posterior distribution is often done by trial and error. But not only is the scaling of the proposal density “by hand” in general time-consuming, improving the proposal distribution manually also becomes very hard, if not infeasible, in high-dimensional problems. Therefore, I employ the Adaptive Metropolis (AM) algorithm as introduced by Haario et al. (2001) to evaluate the posterior. The main idea of the AM algorithm is to run a chain that alters its proposal distribution by using all information about the posterior cumulated so far. Thus, the algorithm improves on the fly. Precisely, the covariance of the proposal distribution is updated each step using all available information. Apart from the updating scheme, the algorithm is identical to the standard random walk Metropolis-Hastings algorithm. Due to the adaptive nature of the algorithm, it is non-Markovian, but Haario et al. (2001) show that it still has the correct ergodic properties.

Let  $\theta_0, \dots, \theta_{j-1}$ , denote the sampled parameters until  $j - 1$  iterations, where  $\theta_0$  is the initial set of parameters. I follow Haario et al. (2001) and let the proposal distribution, denoted by  $q(\cdot|\theta_0, \dots, \theta_{j-1})$ , be a multivariate Gaussian distribution with the mean at the current value of the parameter vector  $\theta_{j-1}$  and a covariance matrix  $C_t$ . The algorithm starts with a pre-

specified strictly positive proposal distribution covariance  $C_0$ . After an initial period  $n_0$  the adaption takes place by updating the covariance of the proposal distribution according to  $C_j = s_d \text{Cov}(\theta_0, \dots, \theta_j) + s_d \varepsilon I_d$ , where  $s_d$  is a parameter that depends only on the dimension  $d$  of the parameter vector  $\theta$  and  $\varepsilon > 0$  is a (very small) constant employed to prevent  $C_j$  from becoming singular. In practice, the calculation of the covariance  $C_j$  is simplified using the following recursion formula (see Haario et al., 2001):

$$C_{j+1} = \frac{j-1}{j} C_j + \frac{s_d}{j} \left( \bar{\theta}_{j-1} \bar{\theta}'_{j-1} - (j+1) \bar{\theta}_j \bar{\theta}'_j + \theta_j \theta'_j + \varepsilon I_d \right).$$

where  $\bar{\theta}_j = \frac{1}{j+1} \sum_{i=0}^j \theta_i$ .

The AM algorithm is given by the following steps:

1. Set the number of total iterations  $n$  and specify the initial period  $n_0$  ( $n_0 < n$ ) after which the adaption starts. Chose an (arbitrary) positive definite initial covariance matrix  $C_0$  and specify the initial parameter vector  $\theta_0$ . Set  $C_j = C_0$  and  $\theta_{j-1} = \theta_0$ .
2. Draw a candidate  $\theta_j^*$  from  $q(\cdot | \theta_{j-1}, C_j)$
3. Compute  $\alpha(\theta_j^*, \theta_{j-1}) = \min \left[ 1, \frac{\pi(\theta_j^* | \cdot)}{\pi(\theta_{j-1} | \cdot)} \right]$ .
4. Set  $\theta_j = \theta_j^*$  with probability  $\alpha(\theta_j^*, \theta_{j-1})$   
and set  $\theta_j = \theta_{j-1}$  with probability  $1 - \alpha(\theta_j^*, \theta_{j-1})$ .
5. Update  $C_{j+1} = \begin{cases} C_0, & j \leq n_0 \\ s_d \text{Cov}(\theta_0, \dots, \theta_j) + s_d \varepsilon I, & j > n_0 \end{cases}$ .
6. Repeat step 2-5 until  $j = n$ .

Haario et al. (2001) note that the choice of an appropriate initial covariance  $C_0$  helps to speed up the algorithm and thus to increase efficiency. Therefore, I use a scaled down version of the inverse of the Hessian matrix computed at the posterior mode for the initial covariance matrix. The initial parameter vector is set to the parameter values at the mode. For the choice of the scaling parameter  $s_d$  I follow Haario et al. (2001) (whose choice, in turn, is based on Gelman et al. (1996)) and set  $s_d = (2.4)^2 / d$ . The initial period is set to  $n_0 = 20,000$  and the number of draws is set to  $n = 1,500,000$ .

As noted by Chib and Ergashev (2009), the mode of the posterior can in general not be found using Newton-like optimization methods. Therefore, I employ the Covariance Matrix Adaption Evolution Strategy (CMA-ES) algorithm. The CMA-ES is a stochastic method for numerical parameter optimization of non-linear, non-convex functions with many local optima. It belongs to the class of evolutionary optimization algorithms (Hansen and Ostermeier, 2001). The computation of the mode is conducted by the software package Dynare (Adjemian et al., 2011).

## 4.4 Parameter Restrictions and Prior Distributions

### 4.4.1 Parameter Restrictions

For the estimation, restrictions are imposed to ensure either the stationarity of the macro dynamics, the stability of the arbitrage recursions, or the identification of the model. Stationarity of the state dynamics requires the eigenvalues of the matrix  $P$  to be less than unity in absolute value,  $|eig(P)| < 1$ . A similar restriction has to be imposed to guarantee the stability of the no-arbitrage recursions (see e.g. Dai and Singleton, 2001). Specifically, the eigenvalues of  $P - \Sigma\Lambda$  have to be less than unity in absolute value,  $|eig(P - \Sigma\Lambda)| < 1$ . For identification purposes, the scaling of the latent variable  $v_t$  and  $C_t$  have to be pinned down, since multiplicative transformations of the latent factor lead to observationally equivalent systems. To pin down the scale of the latent variables, the scaling parameters of these variables are set equal to  $\sigma_v = 0.01$  and  $\sigma_C = 0.01$ . In the same spirit, the direction in which an increase in the risk aversion variable  $v_t$  moves the prices of risk, needs to be specified. Following Ireland (2015), without loss of generality, the constraint  $\Lambda^\pi \leq 0$  is imposed. Finally, similar to Dewachter et al. (2014a) and Ireland (2015), to guarantee that  $v_t$  only moves the prices of risk associated with the other four state variable, the restriction  $\Lambda^v = 0$  is imposed. This imposes that the risk aversion variable is not itself a sourced for priced risk.

### 4.4.2 Prior Distribution

Using prior information from previous studies and restricting parameters to lie in an economically reasonable region helps to reduce the complexity of the maximization problem by down-weighting economically non-meaningful regions of

the parameter space (see Chib and Ergashev, 2009, for a more detailed discussion). The first part of the table (1) displays the prior distributions of the parameters of the monetary policy rule and the parameters associated with the endogenous dynamic of the other state variables. I follow closely Smets and Wouters (2003) for the choice of the priors for the Taylor rule coefficients. Since the parameter capturing the degree of interest rate smoothing  $\rho_r$  is supposed to be in the interval between 0 and 1, it is assumed that  $\rho_r$  is Beta distributed. I set the prior mean equal to 0.8 and the standard deviation equal to 0.05, assuming a high degree of interest rate inertia. The parameter governing central bank's reaction to the deviation of the actual inflation rate from its target rate is assumed to be Gamma distributed with a mean of 1.5 and a standard deviation of 0.25. I employ the Gamma distribution to ensure that the parameter  $\rho_\pi$  cannot be negative. The prior mean satisfies the Taylor principle. Likewise, I also suppose that the prior for the parameter of central bank's reaction to deviation from the output gap is Gamma distributed. The prior mean is chosen to correspond to the Taylor rule coefficient of 0.5. Finally, the coefficient of central bank's response to movements in term premia  $\rho_v$  is assumed to be Normal distributed with a mean of 0 and a standard deviation of 0.25. The choice of the prior means implies that monetary policy is, a priori, characterized by a standard Taylor rule.

The choice of the priors of the parameters describing the dynamics of the macroeconomy is also displayed in the first part of table (1). As described in Section (3.1), these dynamics are modeled as in a structural VAR model. The priors for the VAR part (eq. 2 - 7) are chosen in the spirit of Minnesota (see Litterman, 1986) by assuming that almost all coefficients are normal distributed and by setting the prior means of most of the coefficients equal to zero except for these coefficients corresponding to the first own lags of the dependent variables. These coefficients are set equal to 0.9 as suggested by Koop and Korobilis (2010). The choice of the prior means reflects the assumption that these variables exhibit a high degree of persistence, but do not follow a unit root process. The standard deviation of the prior distribution of the parameters is weighted by the lag length, implying that with increasing lag length the coefficients are shrunk towards zero. As in Dewachter et al. (2014a), I set the standard deviations for the coefficients on the first lags

equal to 0.15. Departing from Minnesota and following Dewachter and Iania (2011) and Dewachter et al. (2014a), I choose a negative prior mean for the parameters  $\rho_{yr}^1$  and  $\rho_{\pi r}^1$ . These choices capture beliefs that an increase in the interest rate dampens economic activity. For the parameters  $\rho_{yv}$  and  $\rho_{\pi v}$  I choose a relatively uninformative prior. Precisely, I set the prior mean equal to zero and the standard deviation equal to 0.25, assuming that movements in the term premium do not affect output and inflation a priori. The coefficient of the inflation target process is Beta distributed with a mean of 0.9 and a standard deviation of 0.1. Employing the Beta distribution guarantees that the process of the inflation target is stationary while avoiding that the central bank's inflation target jumps erratically. Finally, the persistence parameters of the common factor  $\rho_C$  and the persistence parameters of the change in the debt-to-GDP ratios  $\rho_d^i$  of country  $i$ ,  $\forall i \in \{fr, es, it\}$ , are also assumed to be Beta distributed with a mean of 0.9 and a standard deviation of 0.1. The overall choice of these priors satisfies the stationarity of the macro dynamics.

The second part of table (1) presents the prior distributions of the volatility parameters associated with the structural shocks, the yield errors, and the prior distributions of the co-movement parameters. The prior distributions of the volatility parameters corresponding to the structural shocks and the yield errors follow, similar to Dewachter (2008), the Inverse Gamma distribution with a mean of 0.01 and 0.0001, respectively, and a standard deviation of 0.2 and 0.001, respectively, corresponding to a mean of 1 percentage point of the structural shocks and a mean of 0.01 percentage points of the yield errors. This specification captures the beliefs that these errors should be rather small. I employ the Inverse Gamma distribution to prevent the volatility parameter from being negative or equal to 0. It is worth pointing out that the table (1) displays a reparameterized version of the volatility parameters of the yield errors. A reparameterization is performed since the Inverse-Gamma distribution (the traditional distribution for variances) is not very flexible in dealing with very small numbers, as discussed by Chib and Ergashev (2010). Therefore, the transformation  $\sigma_j^* \equiv s * \sigma_j$ ,  $\forall j \in \{12, 24, 36, 48, 60\}$ , and  $\sigma_k^{i*} \equiv s * \sigma_k^i$ ,  $\forall k \in \{12, 60\}$  and  $\forall i \in \{fr, es, it\}$ , is performed, where  $s$  is given by  $s = 1000$ . The prior distributions for the co-movement parameters follow a Normal distribution with a mean of 0 and a standard deviation of



2. Noteworthy, the choice of the priors satisfies the stationarity condition and the stability condition of the no-arbitrage recursions. Hence, under the chosen prior specification  $|eig(P)| < 1$  and  $|eig(P - \Sigma\lambda_1)| < 1$  hold.

For the elements in the vectors  $\psi^{es}$ ,  $\psi^{fr}$ , and  $\psi^{it}$  I use relatively uninformed priors. In particular, the elements in the vectors  $\psi^{es}$ ,  $\psi^{fr}$ , and  $\psi^{it}$  are supposed to be Normal distributed with the mean equal to 0 and the standard deviation equal to 2. Finally, for the choice of the prior distributions of the parameters in  $\Lambda_1$  (the elements in the matrix of the prices of risk), I follow Dewachter and Iania (2011) and Dewachter et al. (2014). The last part of table (1) presents the priors for the prices of risk. I use relatively uninformative priors, reflected by the choice of large standard deviations. More precisely, each element in the matrix of the prices of risk is assumed to be Normal distributed with a mean of 0 and a standard deviation of 25.

## 5 Results

This section presents the results of the estimation. Table (2) - (4) list the estimated parameters. The tables report the posterior modes, the posterior means, and the 90% highest posterior density (HPD) interval of the estimated parameters. While the posterior mode is obtained by maximizing the (log-) posterior distribution, the latter results are obtained by using the Adaptive Metropolis algorithm outlined in Section (4.3.3). In this model, a part of the spreads is explained by a common risk factor. Figure (3) displays the time path of the common risk factor. The common risk factor has been above its steady state level during the financial crisis and the European debt crisis.

In the literature of macro-finance term structure models, the standard deviations of the yield errors are used to evaluate the fit of the model. The bottom part of table (4) presents the standard deviations of these errors. With standard deviations of these errors around 8 and 11 basis points for French bond yields, around 26 and 33 basis points for Spanish bonds yields and around 4 and 28 basis points for Italian bond yields, the fit of the yield curves is reasonably good (cf. Borgy et al., 2012, or Hördahl and Tristani, 2013). The model's fit of the German term structure is remarkably good.

The estimates of the interest rate rule parameters are given in the first

four rows in table (2). Notably, all four parameter estimates are significantly different from zero, including the ECB’s response parameter to movements in the risk aversion variable  $\rho_v$ . The posterior mean of  $\rho_v$  is significantly different from zero and negative,  $\rho_v = -0.2565$ . Using a macro-finance model and an index of Euro-area government bonds, Herrmann (2015) also finds a negative coefficient. Since from eq. (18) term premia are a proportional to the risk aversion variable, this implies that the ECB lowered the interest rate in response to a rise in term premia. In line with the practitioner view (see Rudebusch et al., 2007), this indicates that the central bank counteracted changes in term premia.<sup>6</sup>

The estimated values of the other three parameters of the interest rate rule are similar to those from studies using a more standard interest rate rules specification for the Euro Area (e.g. Andrés et al., 2006, or Smets and Wouters, 2003). The estimate of the interest rate inertia  $\rho_r = 0.8188$  reflects a high degree of interest rate smoothing. The estimate of the coefficient measuring central bank’s response to changes in the output gap is  $\rho_y = 0.1198$ . The estimated coefficient of the central bank’s response to a change in inflation is larger than one,  $\rho_\pi = 1.3197$ , satisfying the Taylor principle.

In the following, rather than interpreting each of the remaining estimates separately, I describe the results of the parameter estimation jointly by computing impulse response functions (IRFs) of the yield spreads to selected shocks of the economy, by decomposing the forecast error variance of the yield spreads and by performing a historical shock decomposition of yield spreads. These methods help to examine the dynamic of yield spreads, to describe the propagation of different shocks and to reveal the relevance of different shocks for variation in the yield spreads. All yield spreads are calculated with respect to Germany.

## 5.1 Impulse Response Functions

Each of the following figures shows the impulse response of the yield spreads to a particular shock. Each shock is of a size of one-standard-deviation. The first row of each figure gives the graphs of the impulse responses of the one-year

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<sup>6</sup>In contrast, Ireland (2015), who estimated the same parameter, but for the Fed with US data, finds a significantly positive coefficient.

spreads of France ( $y_{t,fr}^{12} - y_{t,ger}^{12}$ ), Spain ( $y_{t,es}^{12} - y_{t,ger}^{12}$ ), and Italy ( $y_{t,it}^{12} - y_{t,ger}^{12}$ ). The second row contains the graphs of the impulse responses of the five-year yield spreads of France ( $y_{t,fr}^{60} - y_{t,ger}^{60}$ ), Spain ( $y_{t,es}^{60} - y_{t,ger}^{60}$ ), and Italy ( $y_{t,it}^{60} - y_{t,ger}^{60}$ ). The last row contains the graph of the impulse response of the risk aversion variable  $v_t$ . The gray shaded areas cover the 90 percentage HPD interval. The IRF (displayed by the blue line) is computed as the mean impulse response. The yield spreads are shown in annualized percentage points. One period corresponds to one month.

First, I display the impulse responses of the yield spreads to some selected economic shocks. The impulse responses to the risk aversion shock  $\varepsilon_{vt}$  are presented in figure (4). The yields spreads of both maturities of all countries rise significantly on impact. Over a horizon of five years, the impulse responses of the yield spreads converge slowly back to their steady state. The magnitude of the impact responses to the risk aversion shock is significantly larger for the spreads of Italy (around 30 basis points) and Spain (around 25 basis points) than the magnitude of the impact response of French yield spreads (around five basis points). Trivially, the risk aversion variable rises on impact and converges than back to its steady state.

Figure (5) displays the impulse responses to a rise in the French debt-to-GDP growth rate. The figure highlights that only the one-year yield spread and the five-year yield spread of France are affected by an increase the debt-to-GDP growth rate of France. The respond of all other spreads is not significantly different from zero. The same applies for a shock to the change in the debt-to-GDP ratio of Italy and Spain. Figure (6) and (7) shows that each shock does only affect the yield spread of the respective country vis-à-vis Germany. All other spreads do not respond significantly. Thus, the results provide no evidence for flight-to-safety effects running from the country-specific fiscal variable to the other countries of the EMU.

Finally, figure (8) shows the impulse responses of the yield spreads and the risk aversion variable to the common risk factor shock. The yield spreads of all countries rise significantly and persistently. The increase in the yield spreads is of stronger magnitude for Spain and even stronger for Italy, than for France. While all spreads rise, the risk aversion variable drops, implying that investors require a negative term premium.

## 5.2 Variance decomposition

To identify the main drivers of movements in bond yield spreads and to assess the relative importance of different shocks for the variability of the yield spreads, I compute the forecast error variance decomposition (FEVD). The FEVD helps to quantify the contribution of each of the structural shocks to the forecast error variance of the different yield spreads. Formally, the fraction of the forecast error variance of variable  $i$  due shock  $j$  for horizon  $h$ , denoted by  $\phi_{i,j}(h)$ , is defined by

$$\phi_{i,j}(h) = \frac{\omega_{i,j}(h)}{\Omega_i(h)},$$

where  $\omega_{i,j}(h)$  is the forecast error variance of variable  $i$  due to shock  $j$  at horizon  $h$  and  $\Omega_i(h)$  is the total error forecast variance of variable  $i$  at horizon  $h$ .

Similar to Dewachter et al. (2015), I divide the contribution of the different shocks into three groups: *economic factors*, the *idiosyncratic shocks*, and the *common risk factor*. *Economic factors* are those variables that concern the economic environment of the countries in the Euro area and the country-specific fiscal variable. These variables are the Euro area-wide inflation rate, the euro area output gap, the global risk aversion variable, the monetary policy rate, the central bank's inflation target rate and the change in debt-to-GDP ratio. For the illustration purposes, the economic factors in table (5) exclude the risk aversion variable. This variable is reported separably in table (5). The *idiosyncratic shocks* cover all country-specific variation in the yield spread that cannot be explained by the models' variables. They are given by the errors of the yields of the sovereign under consideration.<sup>7</sup> The *common risk factor* is given by the common factor  $C_t$ . This factor captures common dynamics in yield spreads that are not related to the other global economic factors. The FEVD is performed for the yield spreads of one- and five-year maturity for different horizons. Table (5) display the FEVD of the yield spreads.

Both, economic factors and the common risk factor are important drivers of Euro area sovereign yield spreads. Within the group of economic factors,

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<sup>7</sup>Notably, the errors attached to the German government bond yields play only a neglectable role in the forecast error variance of all yield spreads. Thus, the idiosyncratic shocks reflect the yield-specific shocks of the respective country whose yield spread with respect to German bond yields is under consideration.

the risk aversion variable takes a pronounced role. For intermediate forecast horizons (from one year up to three years), it accounts for around 39 and 43 percent of the forecast error variance in the one-year yield spreads of France and for between 46 and 62 percent of the forecast error variance in the five-year yield spreads of France. Risk aversion shocks are also important for the yield spreads of Spain and Italy. They account for between 55 and 68 percent, and 51 and 68 percent in the variability of the Spanish one-year yield spread and the Spanish five-year yield, respectively, on an intermediate forecast horizon. For the Italian yield spreads, the risk aversion variable accounts for between 45 and 64 percent and between 43 and 70 percent in the one-year yield spread and the five-year yield spread, respectively, both on an intermediate forecast horizon. Notably, risk aversion shocks are more pronounced for shorter forecast horizons, while their importance in yields spreads of all maturities decreases with the horizon.

Also common risk factor shocks contribute substantially to the variability of sovereign yield spreads. The effects of variation in the common risk factor are more pronounced for longer forecast horizons. In fact, for longer forecast horizons, common risk factor shocks are the main source for variations in the yield spreads, accounting for between 65 and 77 percent of the variations in yield spreads on a 10-year forecast horizon. But also for intermediate horizons, shocks to the common risk factor play a non-negligible role. For the one-year yield spread of French government bonds, shocks to the common risk factor account for between 23 and 41 percent in the forecast error variance, and for the five-year yield spread, they account for between 9 and 28 percent in the forecast error variance, both for an intermediate forecast horizon. The same holds true for Spanish and Italian yield spreads. Between 7 and 23 percent of all variation in the one-year yield spread of Spanish government bonds and between 19 and 40 percent of all variations in the one-year yield spread of Italian government bonds are attributable to shocks to the common risk factor. The common risk factor shocks also account for sizeable movements in the five-year yield spread of both countries. It accounts for between 42 and 65 percent in the Spanish five-year yield spread and between 32 and 59 percent in the Italian five-year yield spread. Idiosyncratic shocks, however, play only a role for short horizons. They do not contribute substantially for the forecast error variance

of yield spreads for longer forecast horizons.

### 5.3 Historical Shock Decomposition

The historical shock decomposition of the yield spreads is performed to identify the contribution over time of each group of factors to bond yield spreads. Figure (9) - (11) presents the historical decomposition of the five-year yield spreads for government bonds of France, Italy, and Spain with respect to the German bond yield of the same maturity. Each of the figures contains four panels. Each panel shows the historical values of the yield spread and the contribution of a variable or a group of factors to the respective yield spread. The first panel in each figure displays the contribution over time of economic shocks (including shocks of the risk aversion variable) to the respective yield spread, the second panel in each figure shows the contribution over time of idiosyncratic shocks, and the third panel depicts the contribution over time of the common risk factor to the yield spread. The last panel displays the contribution of risk aversion shocks separated from the contribution of the other economic factors to the yield spreads. This helps to visualize the importance of risk aversion shocks for the yields spreads.

In all of the three yield spreads, economic shocks have played the most important role for their evolution. Within the group of economic factors, shocks to the risk aversion variable are the most important drivers, accounting for a substantial part in this group. For the Spanish and Italian five-year yield spreads, shocks to the risk aversion variable explain most of the spread between 2010 and the Beginning of 2012. From 2012 onwards until 2014, the importance of shocks to the risk aversion variable for the yield spread decreases slowly. Shocks to the risk aversion variable also explain a large part in the French yield spread though their contribution for the spread is not as pronounced as for the Spanish and the Italian yield spread. Notably, within the group of economic factors, shocks to the short-term interest rate had a negative contribution to the yield spreads, indicating that monetary policy worked to reduce spreads.

Shocks to the common risk factor also had a substantial impact on yield spreads. In particular, during the financial crisis and the intensification of the European debt crisis in late 2011, common risk factor shocks had a positive

contribution to the yield spreads of all three countries. The absolute contribution of common risk factor shocks to the yield spreads is larger for the Spanish and the Italian yield spread. For example, in mid-2013, the common risk factor shock explains 117 basis points in the Spanish yield spread and 177 basis points in the Italian yield spread, highlighting that spreads of Euro-area countries cannot be fully justified by economic and country-specific factors only. These results are in line with the findings of previous studies (see Di Cesare et al., 2012, Hördahl and Tristani, 2013, De Santis, 2015, and Dewachter et al., 2015).<sup>8</sup> Moreover, for the yield spreads of all three countries, the positive contribution of the common risk factor shocks increases from 2011 onwards, partially offsetting the effects of the decrease of the contribution of shocks to the risk aversion variable to the yield spreads.

Over the whole sample period, idiosyncratic shocks had only a small contribution to yield spreads. Only during the debt crisis, their contribution to the yield spreads increases. Specifically, at the Beginning of 2012, idiosyncratic shocks contributed significantly to the French five-year yield spread, and between 2012 and 2013 to the Spanish five-year yield spread. For the Italian five-year yield spread, however, the contribution of idiosyncratic shocks is negligible.

From this findings follows that though the common risk factor played a non-negligible role for yield spreads, accounting for a substantial increase in yield spreads during for the financial crisis and the European debt crisis, the most important drivers of yield spreads have been economic shocks. In particular shocks to the risk-aversion variable had a huge impact on yield spreads, revealing the importance of measuring risk aversion in Euro area bond markets adequately.

## 6 Conclusion

In this work, I evaluated the effects of economic fundamentals and a common risk factor on Euro area yield spreads. Specifically, using a multi-country

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<sup>8</sup>However, while Dewachter et al. (2015) find that common non-fundamental shocks (i.e. the part of the spread that cannot be explained by country-specific and euro area economic fundamentals, and international influences), are in particular important for yield spreads around the End of 2011, my findings show that the yield spreads of Italy and Spain before 2012 are largely explained by exogenous changes in risk aversion.

macro-finance model of the term structure, where changes in risk-aversion are captured by one single variable, I am interested in disentangling the effects of changes in risk aversion and the common risk factor in Euro area yield spreads. In contrast, to the existing literature on Euro area yield spreads, the risk aversion measure used in this work is directly derived from the pricing kernel. By restricting the prices of risk in the pricing kernel, one single variable is identified to account for all time-variation in the prices of risk. This risk aversion variable responds contemporaneously to distortions of the economy, but also exhibits an autonomous dynamic. The common risk factor is identified as a common factor in Euro area yield spreads that is not related to Euro area economic fundamentals, i.e. the part of the spread that cannot be accounted for by common Euro area economic fundamentals. This common risk factor potentially captures contagion effects or redenomination risk. Furthermore, exclusion restrictions on the state process, similar to those from more conventional VARs, are entailed to identify structural shocks.

In line with the results of Di Cesare et al. (2013), De Santis (2015), or Dewachter et al. (2015), a non-negligible part of the Euro area yield spreads cannot be explained by economic fundamentals, but is accounted for by the common risk factor. Although the contribution of the common risk factor has been important for yield spreads, the substantial part of the yield spreads is explained by economic fundamentals. Within in the group of economic factors, shocks to the risk aversion variable are the most important driver of yield spreads. This finding underlines the importance of measuring risk aversion in Euro area bond markets adequately.

I like to emphasize two aspects of my findings. First, until the end of 2011, shocks to the risk aversion variable are able to explain spreads very well. In fact, for the Spanish and Italian five-year yield spreads with respect to the yield of German government bonds of the same maturity, shocks to the risk aversion variable explain large parts of the spreads between 2010 and the Beginning of 2012. Shocks to the risk aversion variable also explain a large part of the French yield spread though their contribution for the spread is not as pronounced as for the Spanish and the Italian yield spread. However, from 2012 onwards until 2014 the importance of shocks to the risk aversion variable for the yield spread decreases, although they remain a dominant driver of yield spreads. Second,



common risk factor shocks had, in particular, during the financial crisis and the intensification of the European debt crisis in 2012, a positive contribution to the yield spreads of France, Italy, and Spain. Moreover, the importance of the common risk factor shocks increased from 2011 onwards until the end of my sample in December 2014.

# A Appendix

## A.1 Parameter Matrices

The matrix  $P_0$  is given by

$$P_0 = \begin{bmatrix} 1 & \bar{\rho}_y & 0 & 0 & \bar{\rho}_\pi & 0 & 0 & 0 & 0 & \bar{\rho}_v & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

where  $\bar{\rho}_y = -(1 - \rho_r) \rho_y$ ,  $\bar{\rho}_\pi = -(1 - \rho_r) \rho_\pi$  and  $\bar{\rho}_v = -(1 - \rho_r) \rho_v$ . The matrix  $P_1$  is given by

$$P_1 = \begin{bmatrix} \rho_r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{yr} & \rho_{yy}^1 & \rho_{yy}^2 & \rho_{yy}^3 & \rho_{y\pi}^1 & \rho_{y\pi}^2 & \rho_{y\pi}^3 & 0 & 0 & \rho_{yv} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{\pi r} & \rho_{\pi y}^1 & \rho_{\pi y}^2 & \rho_{\pi y}^3 & \rho_{\pi\pi}^1 & \rho_{\pi\pi}^2 & \rho_{\pi\pi}^3 & 0 & 0 & \rho_{\pi v} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{\pi^*} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_C & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{vv} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_d^{fr} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_d^{es} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_d^{it} \end{bmatrix},$$

and the matrix  $\Sigma_0$  is given by

$$\Sigma_0 = \begin{bmatrix} \Sigma_0^1 & \Sigma_0^2 \end{bmatrix},$$

where the sub-matrices  $\Sigma_0^1$  and  $\Sigma_0^2$  are given by

$$\Sigma_0^1 = \begin{bmatrix} \sigma_r & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_y & 0 & 0 & \sigma_{y\pi}\sigma_\pi & 0 & 0 & \sigma_{y\pi^*}\sigma_{\pi^*} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_\pi & 0 & 0 & \sigma_{\pi\pi^*}\sigma_{\pi^*} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{\pi^*} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma_{vr}\sigma_r & \sigma_{vy}\sigma_y & 0 & 0 & \sigma_{v\pi}\sigma_\pi & 0 & 0 & \sigma_{v\pi^*}\sigma_{\pi^*} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$\Sigma_0^2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \sigma_C & 0 & 0 & 0 & 0 \\ \sigma_{vC}\sigma_C & \sigma_v & \sigma_{vd}^{fr}\sigma_d^{fr} & \sigma_{vd}^{es}\sigma_d^{es} & \sigma_{vd}^{it}\sigma_d^{it} \\ 0 & 0 & \sigma_d^{fr} & 0 & 0 \\ 0 & 0 & 0 & \sigma_d^{es} & 0 \\ 0 & 0 & 0 & 0 & \sigma_d^{it} \end{bmatrix}.$$

Finally, the vector  $\mu_0$  is given by

$$\mu_0 = \begin{bmatrix} (1 - \rho_r) (g^r - \rho_y g^y) \\ -\rho^{\pi r} g^r - (\rho_{\pi y}^1 + \rho_{\pi y}^2 + \rho_{\pi y}^3) g^y \\ 0 \\ 0 \\ (1 - (\rho_{yy}^1 + \rho_{yy}^2 + \rho_{yy}^3)) g^y - \rho_{yr} g^r \\ 0 \\ 0 \\ (1 - \rho_{\pi^*}) \pi^* \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

## A.2 Pricing of Defaultable Bonds

Consider the time  $t$  price of a defaultable zero-coupon bond  $\tilde{P}_{j,t}^\tau$  issued by the sovereign of country  $j$  maturing in  $\tau$ -periods that promises to pay a certain amount at maturity. If no default has occurred until time  $t$ , the value of this bond is given by the present value of the recovery payment in the case of default between period  $t$  and  $t + 1$  plus the present value of the bond if no default occurred,

$$\tilde{P}_{j,t}^\tau = E_t \left( m_{t+1} \tilde{P}_{j,t+1}^{\tau-1} \mid D_{j,t+1} = 0 \right) + E_t \left( m_{t+1} \tilde{P}_{j,t+1}^{\tau-1} \mid D_{j,t+1} = 1 \right) \quad (19)$$

where  $D_{j,t}$  is a default indicator variable taking the values 0 in the event of no-default prior to time  $t$  and 1 in the event of default at/or prior to time  $t$ . Duffie and Singleton (1999) assume that the recovery value of the bond is equal to a fraction  $\omega$  of what the bond would have been worth in the event of no-default (the so-called “recovery to market value assumption”).

Simplify the notation by defining the expected value of the bond in  $t + 1$  in the case of no default by

$$E_t \left( \exp(-\tilde{s}_{j,t+1}) m_{t+1} \tilde{P}_{j,t+1}^{\tau-1} \right) \equiv E_t \left( m_{t+1} \tilde{P}_{j,t+1}^{\tau-1} \mid D_{j,t+1} = 1 \right),$$

where  $\tilde{s}_{j,t+1}$  is the time  $t$  conditional default probability of issuer  $j$  that it

survives until  $t + 1$ .<sup>9</sup> Then, the present market value of the bond in the case of default in period  $t + 1$  can be written by

$$E_t \left( m_{t+1} \tilde{P}_{j,t+1}^{\tau-1} \mid D_{j,t+1} = 0 \right) = E_t \left( (1 - \exp(-\tilde{s}_{j,t+1})) m_{t+1} \omega \tilde{P}_{j,t+1}^{\tau-1} \right),$$

and the present value of the bond is given by

$$\begin{aligned} \tilde{P}_{j,t}^{\tau} &= E_t \left( (1 - \exp(-\tilde{s}_{j,t+1})) m_{t+1} \omega \tilde{P}_{j,t+1}^{\tau-1} + \exp(-\tilde{s}_{j,t+1}) m_{t+1} \tilde{P}_{j,t+1}^{\tau-1} \right) \\ &= E_t \left( (1 - \exp(-\tilde{s}_{j,t+1})) \omega + \exp(-\tilde{s}_{j,t+1}) m_{t+1} \tilde{P}_{j,t+1}^{\tau-1} \right). \end{aligned}$$

Finally, define the “recovery-adjusted default intensities”  $s_{j,t}$  (see e.g. Monfort and Renne, 2011) by

$$\exp(-s_{j,t+1}) \equiv (1 - \exp(-\tilde{s}_{j,t+1})) \omega + \exp(-\tilde{s}_{j,t+1}),$$

then the market value of the bond is given by

$$\tilde{P}_{j,t}^{\tau} = E_t \left( \exp(-s_{j,t+1}) m_{t+1} \tilde{P}_{j,t+1}^{\tau-1} \right).$$

Note, that if the recovery rate is equal to zero ( $\omega = 0$ ), then the recovery-adjusted default intensity  $s_{j,t}$  would be equal to the default probability  $\tilde{s}_{j,t+1}$ . However, since the recovery rate is, in general, larger than zero,  $s_{j,t}$  reflects the adjusted default intensity of country  $j$ , rather than actual default intensities.

### A.3 Pricing Matrices

Borgy et al. (2012) depart from the standard formulas to compute the matrices  $\bar{A}_{i,\tau}$  and  $\bar{B}_{i,\tau}$  in eq. (14), as provided by Ang and Piazzesi (2003) and suggest an improved algorithm to compute the pricing matrices different countries with a high data frequency. Instead of computing each of the pricing matrices  $\bar{A}_{i,\tau}$  and  $\bar{B}_{i,\tau} \forall \tau = 1, \dots, 60$  recursively, the idea behind their algorithm is to compute only selected nested bond maturities and to concatenate. As demonstrated by Borgy et al. (2012), this algorithm reduces computation time significantly, in particular for increasing numbers of yield curves and a high frequency of data. Starting from the no-arbitrage condition, pricing of defaultable bonds of a country  $i$  under the risk-neutral measure is given by

$$\bar{P}_{i,t}^{\tau+1} = E_t^Q \left( \exp(-r_t - s_{t+1}^i) \bar{P}_{i,t+1}^{\tau} \right).$$

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<sup>9</sup>Thus, the time  $t$  survival probability of an issuer  $j$  until time  $t + 1$  is given by  $E_t(\exp(-\tilde{s}_{j,t+1}))$ .

By iterating, we get

$$\bar{P}_{i,t}^{\tau+1} = E_t^Q \left( \exp \left( -r_t - s_{t+1}^i \dots - r_{t+\tau} - s_{t+\tau+1}^i \right) \right).$$

Now remember that the short term interest rate  $r_t$  and the default intensities  $s_{t+1}^i$  both are affine in  $X_t$ ,

$$r_t = \delta_1 X_t$$

and

$$s_{t+1}^i = \psi_0 + \psi_1^i X_{t+1}.$$

Moreover, it can be shown (see e.g. [Gourieroux., 2003](#)) that the pricing factors

$X_t$  (under the risk-neutral measure) follow the autoregressive process

$$X_t = \mu^* + P^* X_{t-1} + \Sigma \varepsilon_t^*,$$

where  $\varepsilon_t^* \sim N(0, I)$  and

$$\begin{aligned} \mu^* &= \mu - \Sigma \lambda_0, \\ P^* &= (P - \Sigma \Lambda_1). \end{aligned}$$

Thus,

$$\begin{aligned} \bar{P}_{i,t}^{\tau+1} &= E_t^Q \left( \exp \left( \begin{array}{c} -\delta_1 X_t - (\psi_0^i + \psi_1^i X_{t+1}) - \delta_1 X_{t+1} \\ -(\psi_0^i + \psi_1^i X_{t+2}) \dots - \delta_1 X_{t+\tau} - (\psi_0^i + \psi_1^i X_{t+\tau+1}) \end{array} \right) \right) \\ &= \exp(-\tau \psi_0^i) E_t^Q \left( \exp \left( -\delta_1 X_t - \tilde{\psi}_1^i (X_{t+1} + \dots + X_{t+\tau}) - \psi_1^i X_{t+\tau+1} \right) \right) \end{aligned}$$

where  $\tilde{\psi}_1^i$  is defined by

$$\tilde{\psi}_1^i = \psi_1^i + \delta_1.$$

Now, define

$$F(i)_{t,t+\tau} \equiv -\delta_1 X_t - \tilde{\psi}_1^i (X_{t+1} + \dots + X_{t+\tau}) - \psi_1^i X_{t+\tau+1}$$

and note that if  $X_{t+1}, \dots, X_{t+\tau}$  are Gaussian under the risk-neutral measure, then also  $F(i)_{t,t+\tau}$  is Gaussian under the risk neutral measure. More precisely, let  $F(i)_{t,t+\tau}$  be Gaussian distributed

$$F(i)_{t,t+\tau} \sim N^Q \left( \chi_{0,\tau+1}^i + \chi_{1,\tau+1}^i X_t, \Omega_{i,t} \right),$$

then, one can express the price of an defaultable government bond of country  $i$  with maturity  $\tau$  by  $\bar{P}_{i,t}^\tau = \exp(\bar{A}_{i,\tau+1} + \bar{B}_{i,\tau+1}X_t)$ , where from

$$\begin{aligned}\bar{P}_{i,t}^{\tau+1} &= \exp(-\tau\psi_0^i) E_t^Q \left( \exp \left( F(i)_{t,t+\tau+1} \right) \right) \\ &= \exp \left( -\tau\psi_0^i + \chi_{0,\tau+1}^i + \frac{1}{2}\Omega_{i,t} + \chi_{1,\tau+1}^i X_t \right)\end{aligned}$$

the coefficients  $\bar{A}_{i,\tau+1}$  and  $\bar{B}_{i,\tau+1}$  are given by

$$\bar{A}_{i,\tau+1} = \chi_{0,\tau+1}^i + \frac{1}{2}\Omega_{i,t}, \quad (20)$$

$$\bar{B}_{i,\tau+1} = \chi_{1,\tau+1}^i. \quad (21)$$

Finally, in order to calculate the coefficients  $\bar{A}_{i,\tau+1}$  and  $\bar{B}_{i,\tau+1}$  it remains to compute  $\chi_{0,\tau}^i$ ,  $\chi_{1,\tau}$  and  $\Omega_{i,t}$ . However, since I employ a version of the model without constant terms, it is only necessary to calculate  $\chi_{1,\tau+1}^i$ . Computation of the conditional expectation of  $F(i)_{t,t+\tau}$  is done by

$$\begin{aligned}E_t^Q \left( F(i)_{t,t+\tau} \right) &= E_t^Q \left( -\delta_1 X_t - \tilde{\psi}_1^i [X_{t+1} + \dots + X_{t+\tau}] - \psi_1^i X_{t+\tau+1} \right) \\ &= E_t^Q \left( \begin{array}{c} -\tilde{\psi}_1^i \left[ \mu^* + P^* X_t + \dots + \sum_{k=0}^{j-1} \mu^* (P^*)^k + (P^*)^\tau X_t \right] \\ -\delta_1 X_t - \psi_1^i (P^*)^{\tau+1} X_t \end{array} \right) \\ &\quad + E_t^Q \left( \begin{array}{c} -\tilde{\psi}_1^i \left[ \Sigma \varepsilon_t + \dots + \sum_{j=0}^{\tau-1} (P_1^*)^j \Sigma \varepsilon_t \right] \\ -\psi_1^i \sum_{j=0}^{\tau} (P_1^*)^j \Sigma \varepsilon_t \end{array} \right) \\ &= -\delta_1 X_t - \tilde{\psi}_1^i [\tau I + (\tau-1)P + (\tau-2)P^2 + \dots + P^{\tau-1}] \mu^* \\ &\quad - \tilde{\psi}_1^i [P^* + \dots + (P^*)^\tau] X_t - \psi_1^i (P^*)^{\tau+1} X_t \\ &= -\tilde{\psi}_1^i [P^* [(P^*)^\tau - I] [I - (P^*)]^{-1} - \tau I] [I - (P^*)]^{-1} \mu^* \\ &\quad - \left[ \delta_1 + \tilde{\psi}_1^i P^* [(P^*)^\tau - I] [I - (P^*)]^{-1} + \psi_1^i (P^*)^{\tau+1} \right] X_t\end{aligned}$$

where I used in the second equality that  $E_t^Q X_{t+j} = E_t^Q \left[ \sum_{k=0}^{j-1} \mu^* (P^*)^k \right] + E_t^Q \left[ (P_1^*)^j X_t + \sum_{k=0}^{j-1} (P_1^*)^k \Sigma \varepsilon_t \right]$  and in the fourth equality that

$$\begin{aligned}&[\tau I + (\tau-1)P + (\tau-2)P^2 + \dots + P^{\tau-1}] \mu^* \\ &= [P^* [(P^*)^\tau - I] [I - (P^*)]^{-1} - \tau I] [I - (P^*)]^{-1} \mu^*\end{aligned}$$

and

$$P^* + \dots + (P^*)^\tau = P^* [(P^*)^\tau - I] [I - (P^*)]^{-1}.$$

Thus,

$$E_t^Q \left( F(i)_{t,t+\tau} \right) = \chi_{0,\tau+1}^i + \chi_{1,\tau+1}^i X_t$$

where

$$\chi_{1,\tau+1}^i = - \left[ \delta_1 + \tilde{\psi}_1^i P^* [(P^*)^\tau - I] [(P^*) - I]^{-1} + \psi_1^i (P^*)^{\tau+1} \right], \quad (22)$$

$$\chi_{0,\tau+1}^i = -\tilde{\psi}_1^i \left[ P^* [(P^*)^\tau - I] [I - (P^*)]^{-1} - \tau I \right] [I - (P^*)]^{-1} \mu^*. \quad (23)$$

Note that the terms  $P^* [(P^*)^\tau - I] [(P^*) - I]^{-1}$  and  $(P^*)^{\tau+1}$  in eq. (22) do not depend on the debtor, thus, these terms do not need to be calculated for each debtor separately.

#### A.4 Computation of the $i + 1$ -period return premium

The return premium is given by (for  $\tau > i$ )

$$\begin{aligned} E_t \left( hpr_{t+i+1}^{(\tau)} \right) &= E_t \left( hpr_{t+i+1}^{(\tau)} \right) - E_t \left( y_{t+i}^{(1)} \right). \\ &= E_t \left( p_{t+i+1}^{(\tau-1)} - p_{t+i}^{(\tau)} \right) - E_t \left( y_{t+i}^{(1)} \right) \end{aligned}$$

Plugging the log prices and the expected short rate into the equation above yields

$$E_t \left( hpr_{t+i+1}^{(\tau)} \right) = \bar{A}_{(\tau-1)} + \bar{B}'_{(\tau-1)} E_t X_{t+i+1} - \bar{A}_{(\tau)} - \bar{B}'_{(\tau)} E_t X_{t+i} - \delta' \bar{\mu} - \delta' P^i (X_t - \bar{\mu})$$

Next, it is well known that the pricing matrices  $\bar{A}_t$  and  $\bar{B}_\tau$  can be expressed recursively by

$$\bar{A}_{\tau+1} = \bar{A}_\tau + \bar{B}'_\tau (\mu - \Sigma \lambda_0) + \frac{1}{2} \bar{B}'_\tau \Sigma \Sigma' \bar{B}_\tau - \delta_0, \quad (24)$$

$$\bar{B}'_{\tau+1} = \bar{B}'_\tau (P - \Sigma \lambda_1) - \delta'_1, \quad (25)$$

with initial conditions for  $\bar{A}_\tau$  and  $\bar{B}_\tau$  are given by  $\bar{A}_1 = \delta_0 = 0$ , and  $\bar{B}'_1 = -\delta'_1$  (see, amongst many others, Ang and Piazzesi, 2003).

Using  $E_t X_{t+j} = \bar{\mu} + P^j (X_t - \bar{\mu})$ ,  $\mu = (I - P) \bar{\mu}$ , eq. (24), rearranging, and



collecting terms yields

$$\begin{aligned}
E_t \left( hpr x_{t+i+1}^{(\tau)} \right) &= -\bar{B}'_{(\tau-1)} (\mu - \Sigma \lambda_0) - \frac{1}{2} \bar{B}'_{(\tau-1)} \Sigma \Sigma' \bar{B}_{\tau-1} + \bar{B}'_{(\tau-1)} E_t X_{t+i+1} \\
&\quad - \bar{B}'_{(\tau)} E_t X_{t+i} - \delta' \bar{\mu} - \delta' P^i (X_t - \bar{\mu}) \\
&= -\bar{B}'_{(\tau-1)} (\mu - \Sigma \lambda_0) - \frac{1}{2} \bar{B}'_{(\tau-1)} \Sigma \Sigma' \bar{B}_{\tau-1} + \bar{B}'_{(\tau-1)} \bar{\mu} \\
&\quad - \bar{B}'_{(\tau-1)} P^{i+1} \bar{\mu} - \bar{B}'_{(\tau)} \bar{\mu} + \bar{B}'_{(\tau)} P^i \bar{\mu} - \delta' \bar{\mu} + \delta' P^i \bar{\mu} \\
&\quad + \bar{B}'_{(\tau-1)} P^{i+1} X_t - \bar{B}'_{(\tau)} P^i X_t - \delta' P^i X_t \\
&= c + [\bar{B}'_{(\tau-1)} P^{i+1} - \bar{B}'_{(\tau)} P^i - \delta'_1 P^i] X_t
\end{aligned}$$

where  $c$  is defined by

$$\begin{aligned}
c &\equiv -\bar{B}'_{(\tau-1)} (\mu - \Sigma \lambda_0) - \frac{1}{2} \bar{B}'_{(\tau-1)} \Sigma \Sigma' \bar{B}_{\tau-1} - \bar{B}'_{(\tau-1)} P^{i+1} \bar{\mu} \\
&\quad + \bar{B}'_{(\tau-1)} \bar{\mu} - \bar{B}'_{(\tau)} \bar{\mu} - \delta' \bar{\mu} + \delta' P^i \bar{\mu} + \bar{B}'_{(\tau)} P^i \bar{\mu} \\
&= \bar{B}'_{(\tau-1)} \Sigma \lambda_0 - \frac{1}{2} \bar{B}'_{(\tau-1)} \Sigma \Sigma' \bar{B}_{(\tau-1)} \\
&\quad + [\bar{B}'_{(\tau-1)} (P - P^{i+1}) - \delta'_1 + \delta'_1 P^i - \bar{B}'_{(\tau)} + \bar{B}'_{(\tau)} P^i] \bar{\mu}
\end{aligned}$$

Now use eq. (25) to see that

$$\begin{aligned}
c &= \bar{B}'_{(\tau-1)} \Sigma \lambda_0 - \frac{1}{2} \bar{B}'_{(\tau-1)} \Sigma \Sigma' \bar{B}_{(\tau-1)} + [\bar{B}'_{(\tau-1)} (P - P^{i+1}) - \delta'_1 + \delta'_1 P^i] \bar{\mu} \\
&\quad - [[\bar{B}'_{(\tau-1)} (P - \Sigma \lambda_1) - \delta'_1] + [\bar{B}'_{(\tau-1)} (P - \Sigma \lambda_1) - \delta'_1] P]^i \bar{\mu} \\
&= \bar{B}'_{(\tau-1)} \Sigma [\lambda_0 + \lambda_1 (I - P^i) \bar{\mu}] - \frac{1}{2} \bar{B}'_{(\tau-1)} \Sigma \Sigma' \bar{B}_{(\tau-1)}.
\end{aligned}$$

and

$$E_t \left( hpr x_{t+i+1}^{(\tau)} \right) = c + \bar{B}_{\tau-1} \Sigma \lambda_1 P^i X_t.$$

Hence,

$$E_t \left( hpr x_{t+i+1}^{(\tau)} \right) = \bar{B}'_{\tau-1} \Sigma [\lambda_0 + \lambda_1 [(I - P^i) \bar{\mu} + P^i X_t]] - \frac{1}{2} \bar{B}'_{\tau-1} \Sigma \Sigma' \bar{B}_{\tau-1}$$

Note that the  $i+1$ -period return premium depends on the state of the economy only due to the term  $\lambda_1 P^i X_t$ . As long as not only the elements in the last columns of  $P^i$  but also other elements in the columns in  $P^i$  are different from zero and  $P^i \neq I$ , all variation in the variables in  $X_t$  affect  $E_t \left( hpr x_{t+i+1}^{(\tau)} \right)$ . For  $i = 0$  follows  $P^i = I$  so that the 1-period return premium reads

$$\begin{aligned}
E_t \left( hpr x_{t+1}^{(\tau)} \right) &= \bar{B}'_{\tau-1} \Sigma \lambda_0 - \frac{1}{2} \bar{B}'_{\tau-1} \Sigma \Sigma' \bar{B}_{\tau-1} + \bar{B}_{\tau-1} \Sigma \lambda_1 X_t \\
&= \bar{B}'_{\tau-1} \Sigma [\lambda_0 + \lambda_1 X_t] - \frac{1}{2} \bar{B}'_{\tau-1} \Sigma \Sigma' \bar{B}_{\tau-1}.
\end{aligned}$$

Due to the restricted form of  $\lambda_1$  the only source of variation in  $E_t \left( hprx_{t+1}^{(\tau)} \right)$  is the variable that is ordered at the last position in  $X_t$ .

## A.5 Zero-Coupon Yield Data

The model uses yields data of zero-coupon government bonds from four European countries (France, Germany, Italy, and Spain). However, usually most bond bear indeed coupon payments, in particular, those issued with a maturity of more than one year. Thus, a method to extract zero coupon rates from the prices of coupon-bearing bonds is needed. In order to construct zero-coupon bond data different methods are in used in practice (BIS, 2005), which can be broadly categorized into parametric and spline-based approaches.

Following Gürkaynak, Sack, and Wright (2005), I will use a parametric model. The basic idea of parametric models is to specify a single function defined over the entire maturity domain. In particular, following Borgy et al. (2012), I choose the Nelson-Siegel model as proposed by Nelson and Siegel (1987). In the following, I will briefly discuss the Nelson-Siegel model and the estimation approach.

The Nelson-Siegel function for the instantaneous forward rates  $f$  at a given point in time  $t$  is defined by

$$f_t^\tau(\theta) = \beta_0 + \beta_1 \exp\left(-\frac{\tau}{\tau_1}\right) + \beta_2 \frac{\tau}{\tau_1} \exp\left(-\frac{\tau}{\tau_1}\right),$$

where  $\tau$  denotes the time to maturity, and  $\theta = (\beta_0, \beta_1, \beta_2, \tau_1)'$  denotes the parameters of the Nelson-Siegel Function. It can be shown that to the corresponding spot rate function for a given point in time  $t$  is given by

$$y_t^\tau(\theta) = \beta_0 + (\beta_1 + \beta_2) \left(1 - \exp\left(-\frac{\tau}{\tau_1}\right)\right) - \beta_2 \left(-\frac{\tau}{\tau_1}\right) n$$

where  $\beta_0$  can be interpreted as the instantaneous asymptotic rate and the term  $(\beta_0 + \beta_1)$  as the asymptotic spot rate.

Consider one particular coupon bearing bond at time  $t$  that matures in  $\tau$  periods. The present value of a coupon-bearing bond is calculated as the discounted sum of coupon payments and the bond's repayment on maturity. Thus, the price of a coupon-bearing bond will be equal to

$$P_{t,\tau} = \sum_{i=1}^{\tau} d_{t,i} C + d_{t,\tau} V, \quad (26)$$

where  $C$  denotes the coupon payment,  $V$  is the bond's repayment on maturity, and the discount function which gives the price of a zero-coupon bond paying one Euro at maturity is defined by

$$d_{t,i} = \exp(-y_t^i(\theta) i).$$

For given parameters from the discount function together with eq. (26) model based bond prices can be computed. Hence, in the estimation process, the parameters of the Nelson-Siegel spot rate function are chosen as to minimize the distance between the observed bond prices at time  $t$  and the calculated bond prices. Specifically, the minimization problem is given by

$$\hat{\theta}_t = \arg \min_{\theta} \sum_{j=1}^N w_j \left( P_t^j - \hat{P}_t^j \right)^2$$

where  $N$  is the total number of observed dirty bond prices at date  $t$ ,  $P_t^j$  denotes the observed dirty prices of coupon bonds with different maturity at time  $t$ ,  $\hat{P}_t^j$  denotes the model-implied prices of coupon bonds, and  $w_j$  is a weighting factor. Another approach would seek to minimize the sum of squared yield errors (as opposed to minimizing the sum of squared pricing errors). However, minimizing the sum of yield errors is computationally more time consuming since it requires to solve additionally for the yields after calculating bond prices.

As noted by Svensson (1995) minimizing the squared sum of pricing errors (instead of minimizing the sum of squared yield errors) leads to an unsatisfactory fit of yields of bonds relatively short residual maturity.<sup>10</sup> In order to correct for this shortcoming, different weights are chosen for different residual maturities. In particular, I set the optimization weight, following the practice of e.g. the Belgian central bank or the Spanish central bank (BIS, 2005) equal to the inverse of the modified duration times the observed dirty price.

The data for the prices of coupon government bonds is taken from Datastream. In order to calculate the bonds' cash flows accrued interest and the respective day-count conventions are taken into account. In the spirit Gürkaynak et al. (2005) and following the practice of the ECB (ECB, 2008) different

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<sup>10</sup>Intuitively, the smaller (modified) duration (which is the elasticity of bond prices to changes in yield to maturity changes) of bonds with shorter/longer residual maturities makes their prices more/less sensitive to yield changes. Choosing equal weights would lead to an overfitting of the long-end of the yield curve at the expense of the fit of the short-end of the yield curve.

filters on the bond date are applied in order to detect and remove outliers that would bias the estimation results. In particular, I exclude all bonds from the estimation that are issued before 1990, and prices of bonds with a residual maturity less than 1 month. In order to prevent noise from the yield estimation outliers are traced separately for a number of residual maturity brackets. Specifically, bond yields that deviate more than two standard deviations from the average yield in this bracket are considered as outliers and excluded. The procedure is iterated in order to account for potentially large outliers in the first round that would distort the average yield and the standard deviation. For the size of each maturity bracket, I follow the specification of the ECB.

Finally, due to the absence of information on the trading volume of bonds, for each point in time at which the estimation has been conducted the yields are checked manually. Since the trading volume of bonds usually decreases considerably for shorter maturities, this may lead to large outliers at the short end of the yield curve. Moreover, in some maturity brackets may not be enough bond yields to apply the outliers removal algorithm. This approach helps to eliminate outliers that would otherwise result into unrealistic high or low short-term rates (e.g. short term rates above 50 percentage points).

## A.6 The Likelihood Function

The likelihood function reads

$$L(Z|\theta) = \prod_{t=1}^T (2\pi)^{-\frac{T}{2}} [\det(R_{t|t-1})]^{-\frac{1}{2}} \times \exp\left(-\frac{1}{2} (Z_t - UX_{t|t-1})' (R_{t|t-1})^{-1} (Z_t - UX_{t|t-1})\right).$$

where  $R_{t|t-1}$  denotes the conditional variance,

$$R_{t|t-1} \equiv \text{Var}(Z_t | \mathcal{Z}_{t-1}, \theta) = U \Xi_{t|t-1} U' + VV'$$

$X_{t|t-1}$  denotes the one step ahead forecast,

$$X_{t|t-1} \equiv E[X_t | \mathcal{Z}_{t-1}, \theta] = PX_{t-1|t-1}$$

with

$$X_{t|t} \equiv X_{t|t-1} + \Xi_{t|t-1} U (U' \Xi_{t|t-1} U + VV')^{-1} (Z_t - UX_{t|t-1}),$$

and  $\Xi_{t+1|t}$  denotes the mean squared error of the forecasts

$$\begin{aligned}\Xi_{t+1|t} &\equiv E \left[ (X_{t+1} - X_{t|t}) (X_{t+1} - X_{t+1|t})' \right] \\ &= P \left( \Xi_{t|t-1} - \Xi_{t|t-1} U (U' \Xi_{t|t-1} U + VV')^{-1} U' \Xi_{t|t-1} \right) P' + \Sigma \Sigma' .\end{aligned}$$

The Kalman filter is implemented by iterating on  $X_{t|t-1}$  and  $\Xi_{t|t-1}$  for given initial values  $\Xi_{1|0}$  and  $X_{1|0}$ .

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Table 1: Summary of the prior distribution

<b>Taylor Rule and Persistence Parameter</b>							
Param.	type	mean	std. dev.	Parameter	type	mean	std. dev.
$\rho_r$	$B$	0.80	0.050	$\rho_{y\pi}^1$	$N$	0.00	0.150
$\rho_\pi$	$G$	1.50	0.250	$\rho_{y\pi}^2$	$N$	0.00	0.075
$\rho_y$	$G$	0.5	0.150	$\rho_{y\pi}^3$	$N$	0.00	0.050
$\rho_v$	$B$	0.00	0.250	$\rho_{yy}^1$	$N$	0.90	0.150
$\rho_{\pi r}$	$N$	-0.20	0.150	$\rho_{yy}^2$	$N$	0.00	0.075
$\rho_{\pi\pi}^1$	$N$	0.90	0.150	$\rho_{yy}^3$	$N$	0.00	0.050
$\rho_{\pi\pi}^2$	$N$	0.00	0.075	$\rho_d^i$	$B$	0.90	0.100
$\rho_{\pi\pi}^3$	$N$	0.00	0.050	$\rho_C$	$B$	0.90	0.100
$\rho_{\pi y}^1$	$N$	0.00	0.150	$\rho_{\pi^*}$	$B$	0.90	0.100
$\rho_{\pi y}^2$	$N$	0.00	0.075	$\rho_{yv}$	$N$	0.00	0.250
$\rho_{\pi y}^3$	$N$	0.00	0.050	$\rho_{\pi v}$	$N$	0.00	0.250
$\rho_{yr}$	$N$	-0.20	0.150				
<b>Volatility and co-movement Parameters</b>							
Param.	type	mean	std. dev.	Parameter	type	mean	std. dev.
$\sigma_{vr}$	$N$	0.00	2.00	$\sigma_y$	$IG$	0.01	0.200
$\sigma_{v\pi}$	$N$	0.00	2.00	$\sigma_{\pi^*}$	$IG$	0.01	0.200
$\sigma_{vy}$	$N$	0.00	2.00	$\sigma_d^{fr}$	$IG$	0.01	0.200
$\sigma_{v\pi^*}$	$N$	0.00	2.00	$\sigma_d^{es}$	$IG$	0.01	0.200
$\sigma_{vC}$	$N$	0.00	2.00	$\sigma_d^{it}$	$IG$	0.01	0.200
$\sigma_{vd}^{fr}$	$N$	0.00	2.00	$\sigma_{12}^*$	$IG$	0.1	1.000
$\sigma_{vd}^{es}$	$N$	0.00	2.00	$\sigma_{24}^*$	$IG$	0.1	1.000
$\sigma_{vd}^{it}$	$N$	0.00	2.00	$\sigma_{36}^*$	$IG$	0.1	1.000
$\sigma_{y\pi}$	$N$	0.00	2.00	$\sigma_{48}^*$	$IG$	0.1	1.000
$\sigma_{y\tau}$	$N$	0.00	2.00	$\sigma_{60}^*$	$IG$	0.1	1.000
$\sigma_{\pi\tau}$	$N$	0.00	2.00	$\sigma_{12}^{i*}$	$IG$	0.1	1.000
$\sigma_r$	$IG$	0.01	0.20	$\sigma_{60}^{i*}$	$IG$	0.1	1.000
$\sigma_\pi$	$IG$	0.01	0.20				
<b>Prices of Risk and Spread Parameters</b>							
Param.	type	mean	std. dev.	Parameter	type	mean	std. dev.
$\psi_d^i$	$N$	0.00	2.00	$\Lambda^r$	$N$	0.00	25.00
$\psi_{gy}^i$	$N$	0.00	2.00	$\Lambda^{\pi^*}$	$N$	0.00	25.00
$\psi_\pi^i$	$N$	0.00	2.00	$\Lambda^C$	$N$	0.00	25.00
$\psi_r^i$	$N$	0.00	2.00	$\Lambda^\pi$	$N$	0.00	25.00
$\psi_v^i$	$N$	0.00	2.00	$\Lambda^y$	$N$	0.00	25.00
$\psi_C^i$	$N$	0.00	2.00	$\Lambda^{d,i}$	$N$	0.00	25.00
$\psi_{\pi^*}^i$	$N$	0.00	2.00				

Summary of the prior distributions of the Parameters. Type of the distribution is either  $N$ ,  $B$ ,  $G$ , or  $IG$  where  $N$  denotes the Normal distribution,  $B$  the Beta distribution,  $G$  the Gamma distribution, and  $IG$  the Inverse-Gamma distribution. The prior distribution holds for all countries  $i$ ,  $\forall i = fr, es, it$ .

Table 2: Results: Posterior Distribution (Part I)

Param.	Prior Mean	Post. Mode	Post. Mean	90% HPD Interval		Prior
$\rho_r$	0.800	0.8055	0.8188	0.7838	0.8525	$\mathcal{B}$
$\rho_\pi$	1.500	1.2295	1.3197	0.9305	1.7182	$\mathcal{G}$
$\rho_y$	0.500	0.1026	0.1198	0.0977	0.1416	$\mathcal{G}$
$\rho_v$	0.000	-0.1871	-0.2565	-0.3361	-0.1671	$\mathcal{N}$
$\rho_{vv}$	0.900	0.9876	0.9767	0.9572	0.9981	$\mathcal{B}$
$\rho_{\pi^*}$	0.900	0.9847	0.9849	0.9823	0.9872	$\mathcal{B}$
$\rho_C$	0.900	0.9990	0.9987	0.9977	0.9997	$\mathcal{B}$
$\rho_d^{fr}$	0.900	0.9951	0.9899	0.9805	0.9999	$\mathcal{B}$
$\rho_d^{es}$	0.900	0.9938	0.9907	0.9828	0.9997	$\mathcal{B}$
$\rho_d^{it}$	0.900	0.9967	0.9916	0.9839	0.9999	$\mathcal{B}$
$\rho_{\pi v}$	0.000	0.0094	0.0141	0.0086	0.0202	$\mathcal{N}$
$\rho_{yv}$	0.000	-0.0280	-0.0354	-0.0688	-0.0060	$\mathcal{N}$
$\rho_{\pi r}$	-0.200	-0.0222	-0.0081	-0.0277	0.0109	$\mathcal{N}$
$\rho_{\pi\pi}^1$	0.900	0.8738	0.8588	0.7818	0.9340	$\mathcal{N}$
$\rho_{\pi\pi}^2$	0.000	-0.0302	-0.0354	-0.1142	0.0462	$\mathcal{N}$
$\rho_{\pi\pi}^3$	0.000	-0.0013	0.0106	-0.0274	0.0480	$\mathcal{N}$
$\rho_{\pi y}^1$	0.000	0.0041	0.0078	-0.0105	0.0271	$\mathcal{N}$
$\rho_{\pi y}^2$	0.000	-0.0159	-0.0209	-0.0438	0.0003	$\mathcal{N}$
$\rho_{\pi y}^3$	0.000	0.0160	0.0142	0.0052	0.0248	$\mathcal{N}$
$\rho_{yr}$	-0.200	-0.5834	-0.6308	-0.7576	-0.4989	$\mathcal{N}$
$\rho_{y\pi}^1$	0.000	-0.1792	-0.1786	-0.3785	0.0310	$\mathcal{N}$
$\rho_{y\pi}^2$	0.000	-0.0504	-0.0446	-0.1597	0.0658	$\mathcal{N}$
$\rho_{y\pi}^3$	0.000	-0.0169	-0.0191	-0.1009	0.0632	$\mathcal{N}$
$\rho_{yy}^1$	0.900	1.0921	1.0561	0.9628	1.1373	$\mathcal{N}$
$\rho_{yy}^2$	0.000	0.0760	0.0686	-0.0147	0.1520	$\mathcal{N}$
$\rho_{yy}^3$	0.000	-0.1232	-0.0831	-0.1537	-0.0121	$\mathcal{N}$

Summary of the posterior distributions of the Parameters. Type of the distribution is either  $N$ ,  $B$ ,  $G$ , or  $IG$  where  $N$  denotes the Normal distribution,  $B$  the Beta distribution,  $G$  the Gamma distribution, and  $IG$  the Inverse-Gamma distribution.

Table 3: Results: Posterior Distribution (Part II)

Param.	Prior Mean	Post. Mode	Post. Mean	90% HPD Interval		Prior
$\psi_d^{fr}$	0.000	0.0063	0.0046	0.0024	0.0067	$\mathcal{N}$
$\psi_d^{es}$	0.000	0.0118	0.0110	0.0074	0.0145	$\mathcal{N}$
$\psi_d^{it}$	0.000	0.0312	0.0282	0.0145	0.0418	$\mathcal{N}$
$\psi_C^{fr}$	0.000	0.1300	0.1050	0.0789	0.1284	$\mathcal{N}$
$\psi_C^{es}$	0.000	0.6110	0.5188	0.3963	0.6299	$\mathcal{N}$
$\psi_C^{it}$	0.000	0.7499	0.6337	0.4945	0.7637	$\mathcal{N}$
$\psi_v^{fr}$	0.000	0.0373	0.0297	0.0158	0.0423	$\mathcal{N}$
$\psi_v^{es}$	0.000	0.1986	0.1779	0.1345	0.2220	$\mathcal{N}$
$\psi_v^{it}$	0.000	0.2590	0.2386	0.1882	0.2871	$\mathcal{N}$
$\psi_r^{fr}$	0.000	-0.1345	-0.1354	-0.1918	-0.0731	$\mathcal{N}$
$\psi_r^{es}$	0.000	-0.7454	-0.7367	-0.9402	-0.4979	$\mathcal{N}$
$\psi_r^{it}$	0.000	-0.8329	-0.8058	-1.0896	-0.5077	$\mathcal{N}$
$\psi_{gy}^{fr}$	0.000	0.0461	0.0390	0.0247	0.0526	$\mathcal{N}$
$\psi_{gy}^{es}$	0.000	0.2257	0.2082	0.1478	0.2729	$\mathcal{N}$
$\psi_{gy}^{it}$	0.000	0.2630	0.2370	0.1553	0.3152	$\mathcal{N}$
$\psi_\pi^{fr}$	0.000	-0.2272	-0.2062	-0.3041	-0.0984	$\mathcal{N}$
$\psi_\pi^{es}$	0.000	-0.9888	-0.8829	-1.2403	-0.4902	$\mathcal{N}$
$\psi_\pi^{it}$	0.000	-1.6730	-1.5219	-2.0060	-1.0702	$\mathcal{N}$
$\psi_{\pi^*}^{fr}$	0.000	0.1314	0.1123	0.0631	0.1611	$\mathcal{N}$
$\psi_{\pi^*}^{es}$	0.000	0.5193	0.4525	0.2188	0.6706	$\mathcal{N}$
$\psi_{\pi^*}^{it}$	0.000	0.6256	0.5334	0.2421	0.8267	$\mathcal{N}$
$\Lambda^r$	0.000	-1.2197	-0.5040	-1.8484	0.9038	$\mathcal{N}$
$\Lambda^\pi$	0.000	-2.3014	-1.7231	-3.3438	-0.0038	$\mathcal{N}$
$\Lambda^y$	0.000	-4.4859	-3.3326	-6.9010	-0.0715	$\mathcal{N}$
$\Lambda^{\pi^*}$	0.000	2.7985	1.6422	-0.4218	3.7850	$\mathcal{N}$
$\Lambda^C$	0.000	0.1624	-1.1453	-2.3036	0.0408	$\mathcal{N}$
$\Lambda^{d,fr}$	0.000	-5.2790	-16.2097	-25.1001	-6.7510	$\mathcal{N}$
$\Lambda^{d,es}$	0.000	3.1854	1.6828	-3.3680	6.7816	$\mathcal{N}$
$\Lambda^{d,it}$	0.000	-0.7113	5.7761	-0.2233	11.7661	$\mathcal{N}$

Summary of the posterior distributions of the Parameters. Type of the distribution is either  $N$ ,  $B$ ,  $G$ , or  $IG$  where  $N$  denotes the Normal distribution,  $B$  the Beta distribution,  $G$  the Gamma distribution, and  $IG$  the Inverse-Gamma distribution.



Table 4: Results: Posterior Distribution (Part III)

Param.	Prior Mean	Post. Mode	Post. Mean	90% HPD Interval	Prior
$\sigma_r$	0.010	0.0027	0.0027	0.0025 0.0030	$IG$
$\sigma_\pi$	0.010	0.0021	0.0022	0.0020 0.0024	$IG$
$\sigma_y$	0.010	0.0104	0.0115	0.0100 0.0129	$IG$
$\sigma_{\pi^*}$	0.010	0.0022	0.0021	0.0019 0.0023	$IG$
$\sigma_d^{fr}$	0.010	0.0111	0.0110	0.0100 0.0121	$IG$
$\sigma_d^{es}$	0.010	0.0058	0.0059	0.0054 0.0064	$IG$
$\sigma_d^{it}$	0.010	0.0198	0.0199	0.0181 0.0215	$IG$
$\sigma_{\pi\pi^*}$	0.100	-0.3849	-0.4003	-0.5581 -0.2461	$\mathcal{N}$
$\sigma_{\pi y}$	0.000	2.7227	2.4750	1.5677 3.2642	$\mathcal{N}$
$\sigma_{y\pi^*}$	0.000	-2.6588	-2.0921	-3.0013 -1.0745	$\mathcal{N}$
$\sigma_{vr}$	0.000	1.3138	1.6533	0.6604 2.6566	$\mathcal{N}$
$\sigma_{v\pi}$	0.000	4.2305	4.4241	3.1736 5.7097	$\mathcal{N}$
$\sigma_{vy}$	0.000	-0.7412	-0.4990	-0.8098 -0.1899	$\mathcal{N}$
$\sigma_{v\pi^*}$	0.000	-2.7101	-2.9299	-4.1029 -1.7058	$\mathcal{N}$
$\sigma_{vC}$	0.000	-2.1047	-1.6241	-2.1521 -1.0935	$\mathcal{N}$
$\sigma_{vd}^{fr}$	0.000	-0.0895	-0.0895	-0.5162 0.4048	$\mathcal{N}$
$\sigma_{vd}^{es}$	0.000	-0.1842	-0.2508	-0.6153 0.1196	$\mathcal{N}$
$\sigma_{vd}^{it}$	0.000	-1.0895	-0.2804	-2.3436 2.1357	$\mathcal{N}$
$\sigma_{12}^*$	0.100	0.0784	0.0793	0.0715 0.0876	$IG$
$\sigma_{24}^*$	0.100	0.0192	0.0199	0.0157 0.0241	$IG$
$\sigma_{36}^*$	0.100	0.0349	0.0353	0.0318 0.0386	$IG$
$\sigma_{48}^*$	0.100	0.0170	0.0174	0.0140 0.0204	$IG$
$\sigma_{60}^*$	0.100	0.0935	0.0954	0.0864 0.1048	$IG$
$\sigma_{12}^{fr*}$	0.100	1.1125	1.1305	1.0213 1.2519	$IG$
$\sigma_{60}^{fr*}$	0.100	0.8722	0.8996	0.8075 0.9933	$IG$
$\sigma_{12}^{es*}$	0.100	2.6575	2.7167	2.4883 2.9544	$IG$
$\sigma_{60}^{es*}$	0.100	3.3813	3.4152	3.1168 3.7130	$IG$
$\sigma_{12}^{it*}$	0.100	2.8446	2.8613	2.6307 3.0933	$IG$
$\sigma_{60}^{it*}$	0.100	0.0459	0.0881	0.0229 0.1591	$IG$

Summary of the posterior distributions of the Parameters. Type of the distribution is either  $N$ ,  $B$ ,  $G$ , or  $IG$  where  $N$  denotes the Normal distribution,  $B$  the Beta distribution,  $G$  the Gamma distribution, and  $IG$  the Inverse-Gamma distribution.

Table 5: Variance Decomposition

	1-year yield spread				5-year yield spread			
France								
h	Eco	RA	Idio	C	Eco	RA	Idio	C
3 months	12.90	34.57	34.22	18.31	11.89	56.39	25.69	6.03
1 year	12.26	43.11	21.53	23.10	13.86	62.04	14.87	9.23
3 years	11.85	39.19	7.97	40.99	21.14	46.17	04.86	27.83
5 years	13.19	25.46	04.05	57.30	21.03	30.43	02.52	46.02
10 years	9.63	11.40	01.53	77.44	14.10	14.13	00.99	70.78
Spain								
h	Eco	RA	Idio	C	Eco	RA	Idio	C
3 months	20.82	60.48	11.55	7.15	16.13	64.90	17.38	1.59
1 year	17.97	67.56	6.49	7.98	18.50	68.60	09.68	3.22
3 years	19.25	55.97	02.22	22.56	26.38	51.38	03.21	19.03
5 years	20.34	36.52	01.13	42.01	25.56	34.92	01.72	37.80
10 years	13.90	16.37	00.43	69.30	16.70	16.63	00.69	65.98
Italy								
h	Eco	RA	Idio	C	Eco	RA	Idio	C
3 months	13.25	61.64	10.08	15.03	13.00	75.68	00.00	11.32
1 year	11.40	63.92	05.47	19.21	14.25	70.31	00.00	15.44
3 years	13.10	45.85	01.70	39.35	20.23	43.68	00.00	36.09
5 years	14.38	28.30	00.83	56.49	19.20	27.67	00.00	53.13
10 years	10.10	12.39	00.31	77.20	12.42	12.64	00.00	74.94

*Eco*, *RA*, *Idio*, and *C* denote the contribution to the FEVD of the economic shocks (excluding risk aversion shocks), risk aversion shocks, yield errors and the common risk factor, respectively

Figure 1: One-year bond yields of Euro area sovereigns

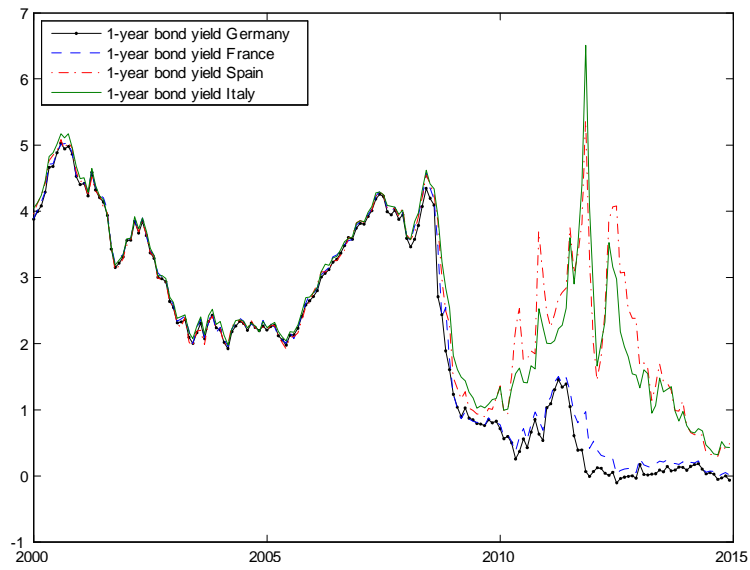


Figure 2: Five-year bond yields of Euro area sovereigns

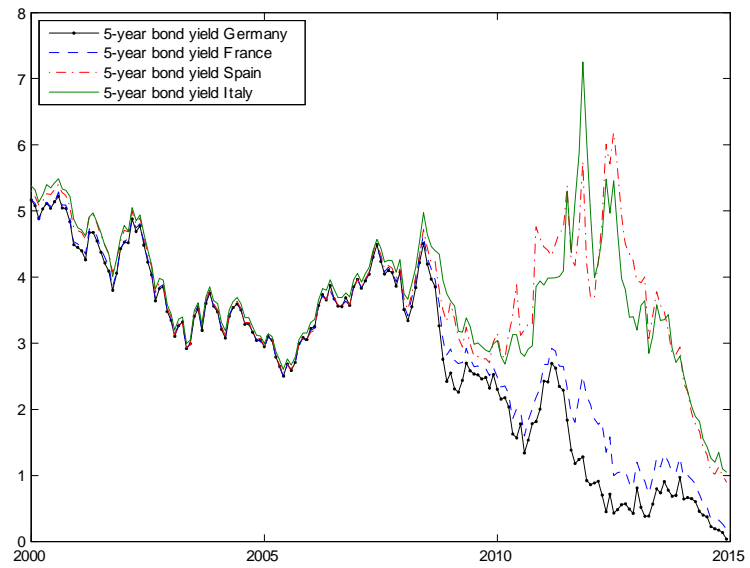


Figure 3: Estimated time path of the common risk factor

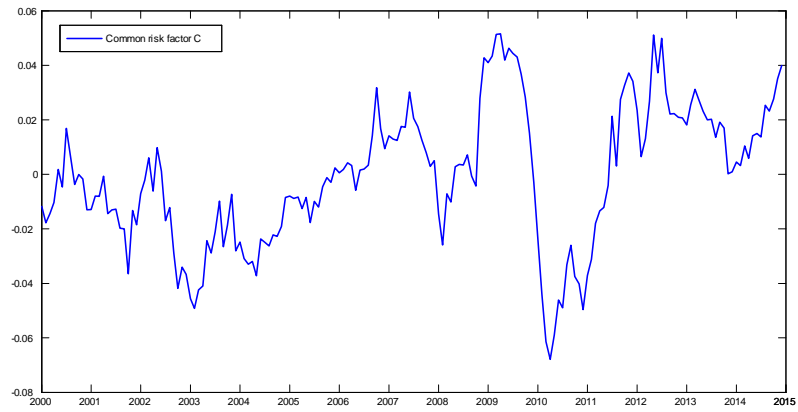
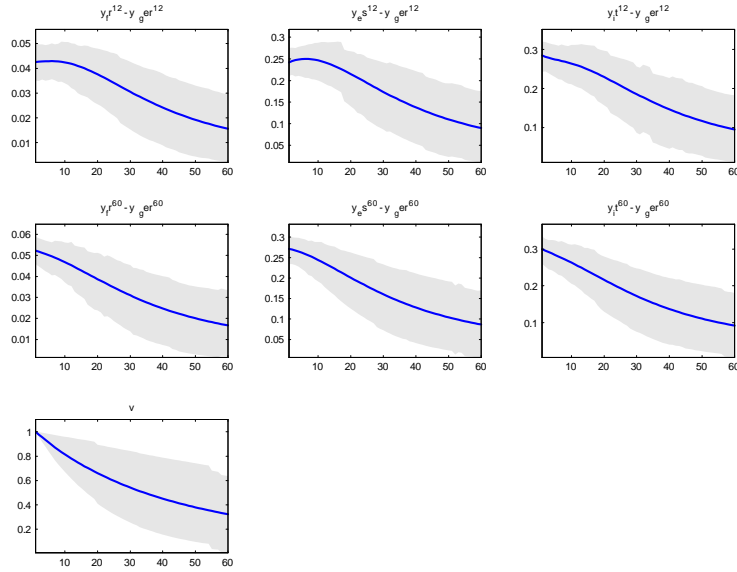
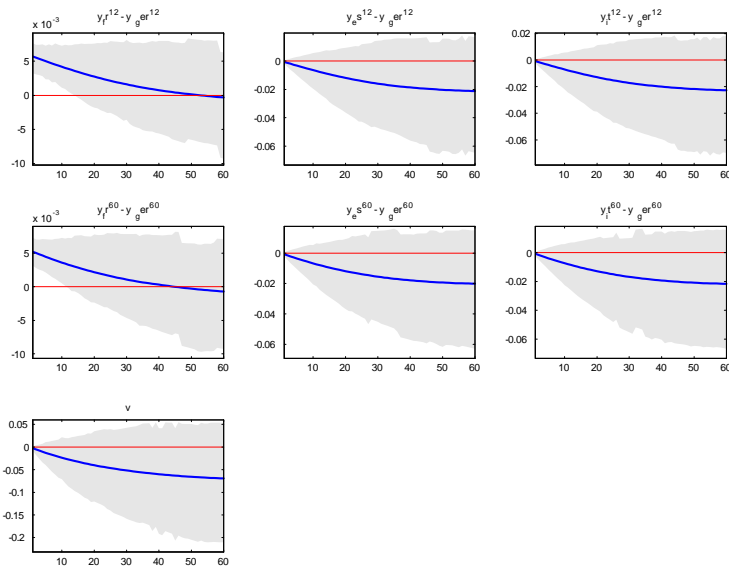


Figure 4: IRFs to a one-standard-deviation risk aversion shock



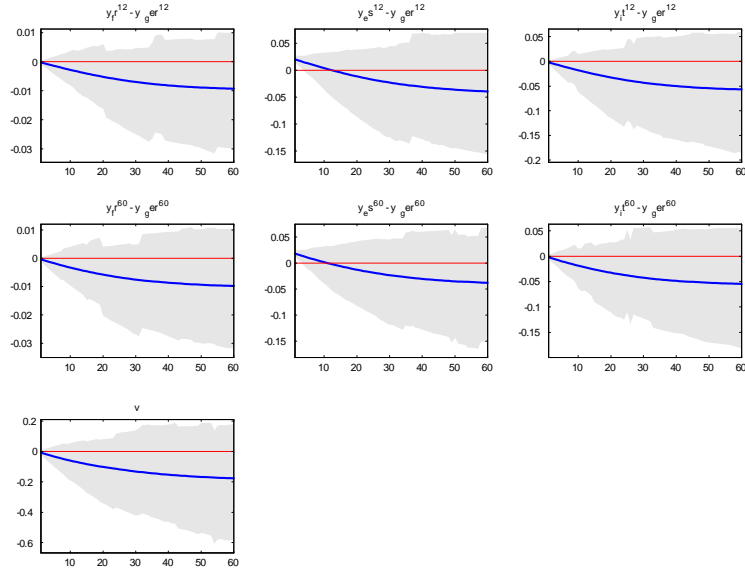
All yield spreads are calculated with respect to the yield of German government bonds of the same maturity. The yield spreads are shown in annualized percentage points. The grey shaded areas cover the 90 percent HPD interval.

Figure 5: IRFs to a one-standard-deviation shock to  $d_t^f r$



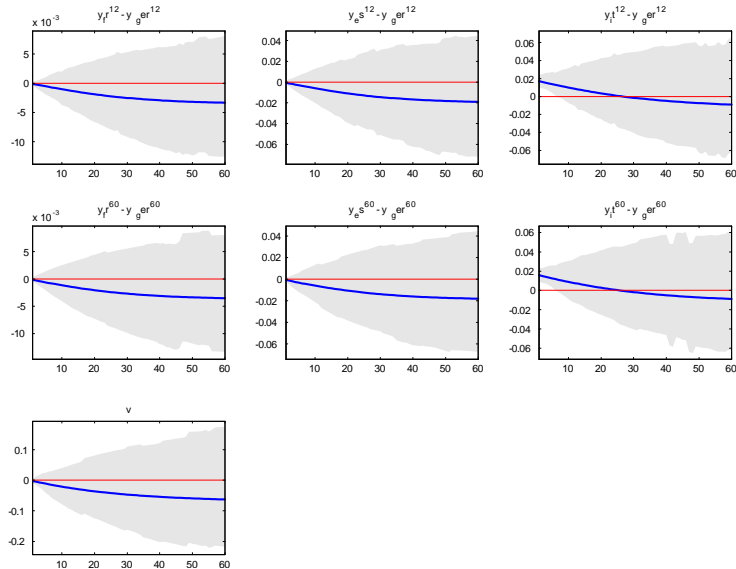
All yield spreads are calculated with respect to the yield of German government bonds of the same maturity. The yield spreads are shown in annualized percentage points. The grey shaded areas cover the 90 percent HPD interval.

Figure 6: IRFs to a one-standard-deviation shock to  $d_t^{es}$



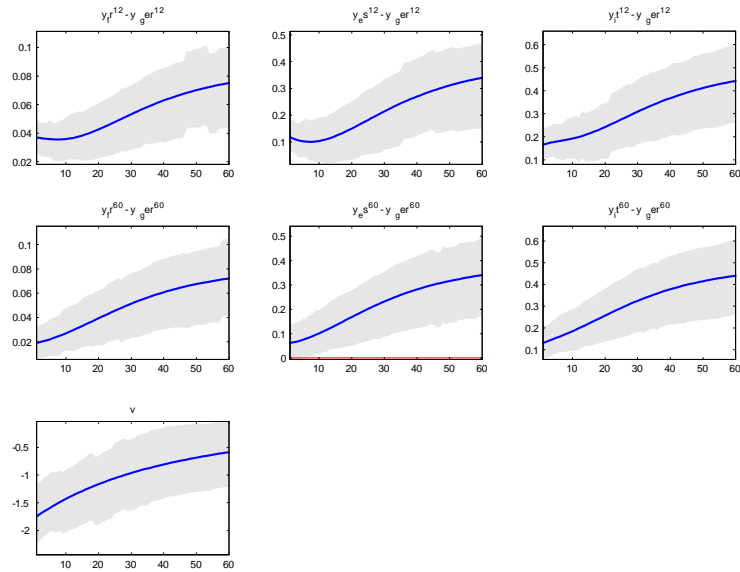
All yield spreads are calculated with respect to the yield of German government bonds of the same maturity. The yield spreads are shown in annualized percentage points. The grey shaded areas cover the 90 percent HPD interval.

Figure 7: IRFs to a one-standard-deviation shock to  $d_t^{it}$



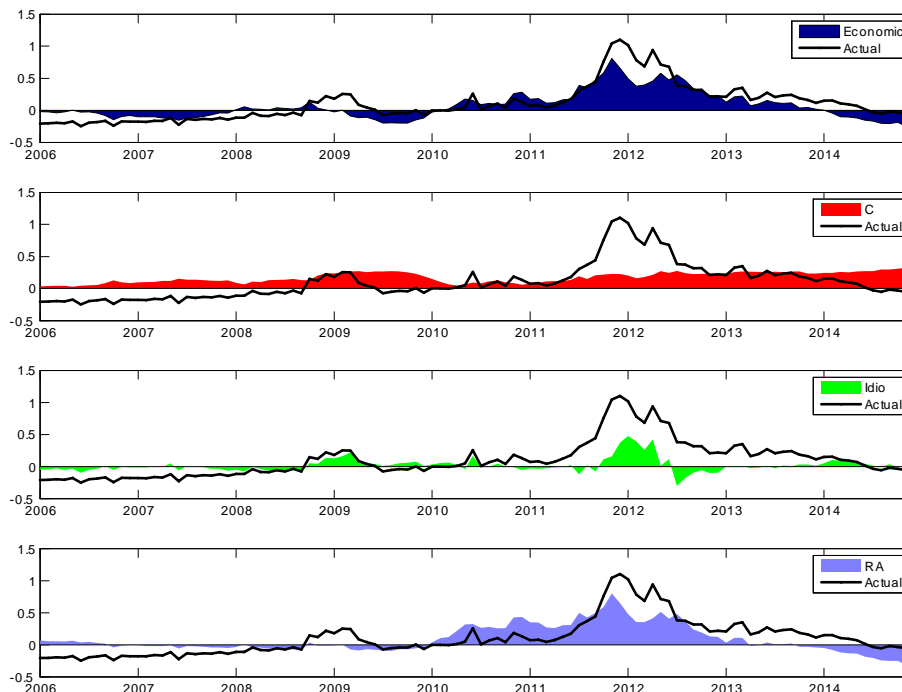
All yield spreads are calculated with respect to the yield of German government bonds of the same maturity. The yield spreads are shown in annualized percentage points. The grey shaded areas cover the 90 percent HPD interval.

Figure 8: IRFs to a one-standard-deviation common risk factor shock



All yield spreads are calculated with respect to the yield of German government bonds of the same maturity. The yield spreads are shown in annualized percentage points. The grey shaded areas cover the 90 percent HPD interval.

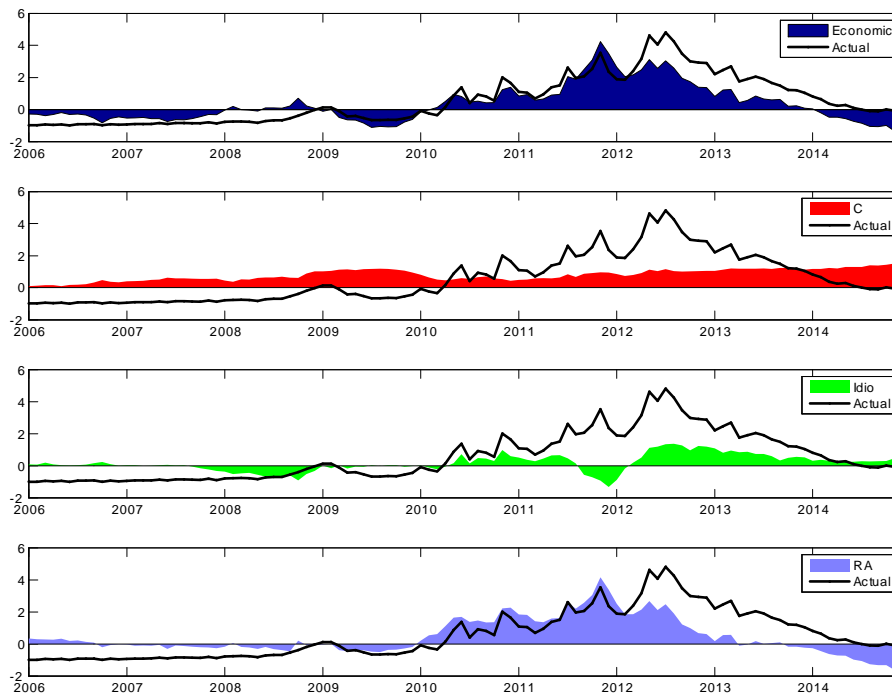
Figure 9: Historical Shock Decomposition of the five-year yield spread of France



The spread is shown in annualized percentage points. The figure presents the historical decomposition of the five-year French yield spread with respect to Germany. *Economic* factors contain country-specific and Euro area wide economic fundamentals (including risk aversion shocks); *C* denotes the common risk factor shock; idiosyncratic shocks are given by the yield errors; the last row depicts the risk aversion shocks separately from the other economic factors. The initial values are not displayed.

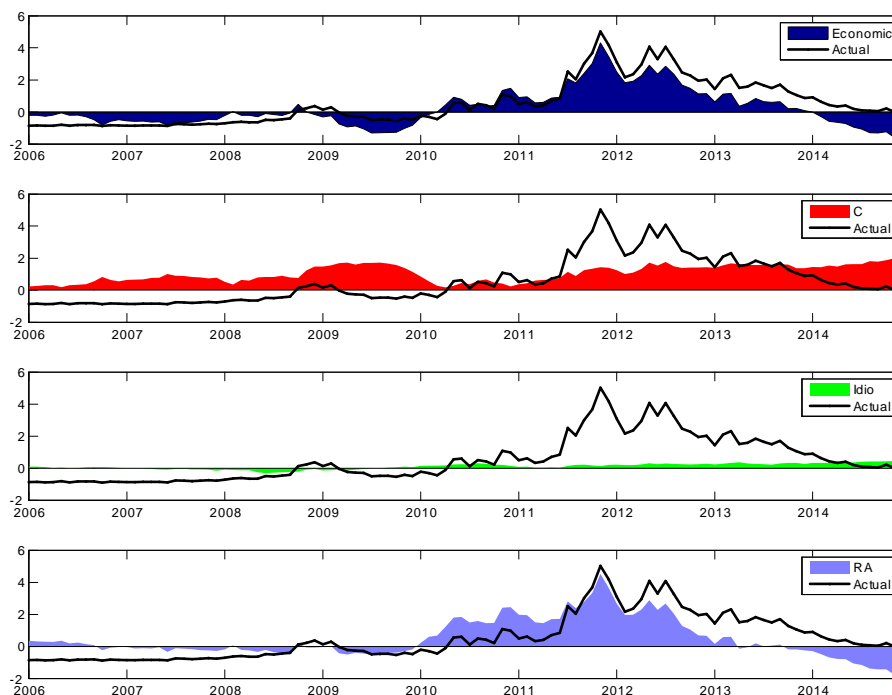


Figure 10: Historical Shock Decomposition of the five-year yield spread of Spain



The spread is shown in annualized percentage points. The figure presents the historical decomposition of the five-year Spanish yield spread with respect to Germany. *Economic* factors contain country-specific and Euro area wide economic fundamentals (including risk aversion shocks); *C* denotes the common risk factor shock; idiosyncratic shocks are given by the yield errors; the last row depicts the risk aversion shocks separately from the other economic factors. The initial values are not displayed.

Figure 11: Historical Shock Decomposition of the five-year yield spread of Italy



The spread is shown in annualized percentage points. The figure presents the historical decomposition of the five-year Italian yield spread with respect to Germany. *Economic* factors contain country-specific and Euro area wide economic fundamentals (including risk aversion shocks); *C* denotes the common risk factor shock; idiosyncratic shocks are given by the yield errors; the last row depicts the risk aversion shocks separately from the other economic factors. The initial values are not displayed.