

# Do Estimated Taylor Rules Suffer from Weak Identification?

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## Abstract

Over the last decade, applied researchers have estimated forward looking Taylor rules with interest rate smoothing via Nonlinear Least Squares. A common empirical finding for post Volcker samples, based on asymptotic theory, is that the Federal Reserve adheres to the Taylor Principle. We explore the possibility of weak identification and spurious inference in estimated Taylor rule regressions with interest rate smoothing. We argue that the presence of smoothing subjects the parameters of interest to the Zero Information Limit Condition analyzed by Nelson and Startz (2007, *Journal of Econometrics*). We demonstrate that confidence intervals based on standard methods such as the delta-method can have severe coverage problems when interest rate smoothing is persistent. We then demonstrate that alternative methodologies have better finite sample coverage. We reconsider the results of three recent empirical studies and show that the evidence supporting the Taylor Principle can be reversed over half of the time.

KEYWORDS: Interest Rate Smoothing; Nonlinear Least Squares; Spurious Inference; Zero-Information-Limit-Condition.

JEL CLASIFICACIONES: C12, C22, E52.

## 1. Introduction

In a recent paper, Nelson and Startz (2007) document that a class of models, used widely in empirical work, suffer from weak identification. This occurs when a particular parameter, not necessarily the parameter of interest, approaches a critical point. They refer to this as the Zero Information Limit Condition (ZILC). These models include Nonlinear regression, IV with weak instruments, ARMA models, etc. They document that for models in which the ZILC holds, asymptotic theory is a poor approximation to the finite sample distribution of the parameter of interest. Standard errors of this parameter are underestimated, and the size of the  $t$ -test depends on the properties of the underlying DGP.

Over the last decade, applied researchers have estimated forward looking Taylor rules with interest rate smoothing via Nonlinear Least Squares (NLS) using real time data. This was a break from tradition when ex-post data were used, necessitating instrumental variables and estimation by GMM; *e.g.* Clarida, Galí and Gertler (2000). Using the delta method, which is an asymptotic approximation, a common finding from NLS estimation is that the confidence interval for the response of the Federal Reserve to changes in expected inflation lies entirely above unity, supporting the empirical conclusion that the central bank follows the Taylor Principle. We show in this paper that this nonlinear monetary policy regression model falls into the same framework as that discussed in Nelson and Startz (2007), suggesting that the parameter estimates in Taylor rules may suffer from weak identification. In this case, confidence intervals based on the delta-method will not have correct finite sample coverage.

In this paper, we demonstrate that the parameters of interest in estimated Taylor rules are in fact subject to the ZILC. That is, the NLS estimators of the response to changes in expected inflation and the business cycle contain no information about the true parameter values as the degree of persistence of interest rate smoothing increases. Asymptotic theory is a poor approximation to the actual finite sample distribution of the parameter estimates, and the coverage probability of confidence intervals is too small for the empirically relevant range of interest rate smoothing.

We reconsider three recent empirical studies using methods which generate more accurate confidence intervals, and demonstrate that over one half of the results

supporting the finding that the Taylor Principle holds in various post-Volcker subsamples is reversed.

## 2. A Nonlinear Taylor Rule Regression Model which is subject to the Zero Information Limit Condition

### 2.1 Nelson and Startz (2007 JoE)

Nelson and Startz (2007) consider the following nonlinear regression model:

$$y_t = \gamma(x_t + \beta z_t) + \varepsilon_t, \quad (2.1)$$

where  $\beta$  is the parameter of interest. This model is identified if  $\gamma \neq 0$  and Nelson and Startz focus on the behavior of  $\hat{\beta}_{NLS}$  as  $\gamma$  approaches the Zero Information Limit Condition (ZILC), in this case zero. They show that when the regression errors are Normal,  $\gamma$  controls the amount of information about  $\beta$  that is contained in the data for a given sample size. In particular, the asymptotic variance of  $\hat{\beta}_{NLS}$  is proportional to  $\gamma^{-2}$ . They demonstrate that as  $\gamma$  approaches zero, information contained in  $\hat{\beta}_{NLS}$  goes to zero, and its variance diverges. In the special case where  $x_t$  and  $z_t$  are standardized to have unit variance, and correlation  $\rho_{xz}$ , the asymptotic variance of  $\hat{\beta}_{NLS}$  is given by the following expression:

$$\text{var}(\hat{\beta}_{NLS}) = \left(\frac{\sigma^2}{T}\right) \left(\frac{1}{\gamma^2}\right) \frac{1 + 2\beta\rho_{xz} + \beta^2}{(1 - \rho_{xz}^2)} \quad (2.2)$$

In the limiting case that  $\gamma = 0$  the variance of  $\hat{\beta}$  becomes infinite. Nelson and Startz also show through a series of Monte Carlo simulations that the standard error of  $\hat{\beta}_{NLS}$  is understated relative to the asymptotic formula. However,  $t$ -tests for  $\beta$  may be undersized or oversized, depending on the DGP, which for the model above boils down to the value of  $\rho_{xz}$ .

## 2.2A Simple Taylor Rule Model with Interest Rate Smoothing

In order to determine whether estimated Taylor rules suffer from weak identification, we extend the nonlinear model of Nelson and Startz to resemble a Taylor rule regression with interest rate smoothing. For simplicity, we consider the case in which the Fed focuses only on expected inflation while ignoring business cycle considerations. We can think of such a model as follows:

$$r_t = (1 - \gamma)r_{t-1} + \gamma\beta z_t + \varepsilon_t. \quad (2.3)$$

In this case, the dependent variable is the Federal Funds Rate (FFR),  $z_t$  is expected inflation,  $\beta$  is the response to changes in expected inflation, and  $(1 - \gamma)$  is the degree of interest rate smoothing, which corresponds to  $\rho$  in our empirical parameterizations of Taylor rules. It is easily seen that as the degree of smoothing increases,  $\gamma$  approaches zero, and the Taylor rule coefficient of interest becomes unidentified.

Some differences between the models are worth noting.  $\gamma$  appears twice on the right hand side of the regression and we have lagged  $r_t$  as a regressor, both due to smoothing. Unlike the model of Nelson and Startz, we do not need  $x_t$  to identify  $\gamma$ , so it has been dropped to keep the model tractable. Dropping  $x_t$  leaves only one right hand side correlation to consider; that between  $z_t$  and  $r_{t-1}$ .

With Normal errors, it can be shown that the Information matrix for  $\hat{\beta}$  and  $\hat{\gamma}$  is equal to:

$$I(\hat{\beta}, \hat{\gamma}) = \begin{bmatrix} \frac{\gamma^2}{\sigma^2} Tm_{zz} & \left( -\frac{\gamma}{\sigma^2} Tm_{rz} + \frac{\gamma\beta}{\sigma^2} Tm_{zz} \right) \\ \left( -\frac{\gamma}{\sigma^2} Tm_{rz} + \frac{\gamma\beta}{\sigma^2} Tm_{zz} \right) & \left( \frac{1}{\sigma^2} Tm_{rr} + \frac{\beta^2}{\sigma^2} Tm_{zz} - \frac{2\beta}{\sigma^2} Tm_{rz} \right) \end{bmatrix}, \quad (2.4)$$

with determinant  $\Delta = \frac{\gamma^2}{\sigma^4} T^2 (m_{zz} m_{rr} - m_{rz}^2)$ .  $m_{zz}$  is the 2<sup>nd</sup> sample moment of  $z_t$ , etc. This

implies the following asymptotic variance for  $\hat{\beta}_{NLS}$ :

$$\text{var}(\hat{\beta}_{NLS}) = \left( \frac{\sigma^2}{T} \right) \left( \frac{1}{\gamma^2} \right) \frac{m_{rr} - 2\beta m_{rz} + \beta^2 m_{zz}}{(m_{rr}m_{zz} - m_{rz}^2)}. \quad (2.5)$$

This clearly demonstrates that as  $\gamma$  approaches zero,  $\hat{\beta}_{NLS}$  contains no information about the true value of  $\beta$ , suggesting that estimated Taylor rules may be subject to spurious inference.

### 2.3A More General Taylor Rule Model with Interest Rate Smoothing

Most estimated Taylor rules consider some sort of business cycle activity in addition to expected inflation, typically the output gap. We extend equation (2.3) to allow for this as follows:

$$r_t = (1 - \gamma)r_{t-1} + \gamma[\beta z_t + \omega \tilde{y}_t] + \varepsilon_t, \quad (2.6)$$

where  $\tilde{y}_t$  is the deviation of output from its trend or potential, and  $\omega$  is the change in the FFR when output deviates from its target. In this framework, it is possibly more transparent that as interest rate smoothing increases, the terms we care about in the squared bracket are being multiplied by a smaller number, suggesting that the ZILC may cause problems for inference on  $\beta$  and  $\omega$  that is based on asymptotic theory.

The algebra demonstrating this is as follows...

[To be added]

### 3. A Review of the Literature on the Delta Method for the ratio of Coefficients

The problem of computing confidence intervals for ratios of parameters has a long tradition in economics, being particularly important within the literature estimating elasticities and/or long-run multipliers. In these setups, the well-known delta Wald-type method constitutes a general procedure to approximate the standard error of a nonlinear combination of estimates based on a first-order Taylor series expansion. However, this methodology is only valid asymptotically, provided that the transformation function is differentiable, with nonzero and bounded derivatives.

To fix ideas, consider the following first-order dynamic regression model:

$$y_t = \rho y_{t-1} + \theta_0 + \theta_1 x_t + \theta_2 z_t + \varepsilon_t \quad (3.1)$$

where the long-run elasticity, or long-run multiplier depending on the context at hand, is defined as the ratio of the coefficient on a regressor to one minus the coefficient on the lagged dependent variable.

Within this framework, a  $100 \cdot (1 - \alpha)\%$  confidence interval for the ratio is given by the following expression:

$$DCI(\alpha) = \left[ \frac{\hat{\theta}_j}{(1 - \hat{\rho})} \pm z_{\alpha/2} \left( \hat{G}' \hat{\Sigma} \hat{G} \right)^{1/2} \right]$$

$$\hat{G} = \left[ \frac{1}{(1 - \hat{\rho})}, \frac{\hat{\theta}_j}{(1 - \hat{\rho})^2} \right]'$$

where  $z_{\alpha/2}$  is the normal two-tailed  $\alpha$ -level cutoff point and  $\hat{\Sigma}$  is the estimated variance-covariance matrix associated with  $(\hat{\theta}_j, \hat{\rho})$ .

Notice that this transformation of the parameter vector becomes problematic (i.e., unbounded) as  $\rho$  approaches unity. In other words, this ratio is weakly identified over a subset of the parameter space, and as shown by Dufour (1997), standard procedures that are bounded by construction can have zero coverage probability. Therefore, this

provides a clear connection between the ZILC of Nelson and Startz (2007) and inappropriateness of the delta-method that motivates our investigation.

A number of recent studies have compared the relative performance of alternative methods, such as the bootstrap procedure of Krinsky and Robb (1986) and the modification to Fieller’s (1940,1954) original approach, and have concluded that the latter performs remarkably well in a variety of settings – see for example Hirschberg et al. (2008), Bolduc et al (2010), and Bernard et al. (2007). Like the delta-method, Fieller’s method also relies on asymptotic theory but confidence intervals are computed by inverting a test that does not require identifying the ratio, and are neither symmetric nor bounded.

In the next section we describe four alternative methodologies in detail, and in section 5 we conduct a series of Monte Carlo experiments to assess their relative performance within the context of a dynamic regression model that resembles a Taylor rule with interest rate smoothing.

#### **4. Methods of Computing Confidence Intervals for the Ratio of Parameter Estimates**

In this section, we discuss four methods of computing confidence intervals for the ratio of coefficients.

##### *4.1 The Delta-Method*

The basic Taylor rule equation to be estimated is:

$$r_t = \rho r_{t-1} + (1 - \rho)[\mu + \beta E_t \pi_{t+h} + \omega \hat{y}_t] + \varepsilon_t \quad (4.1)$$

where the Taylor Principle is satisfied if  $\beta > 1$ . There are two equivalent ways of testing a hypothesis about the value of  $\beta$ . One is to estimate Equation (4.1) by NLS, and compute:

$$\hat{\beta}_{NLS} \pm 1.96 asyse(\hat{\beta}_{NLS}).$$

This can be done quite easily using canned software packages. Asymptotically, this confidence interval has 95% coverage, and a  $t$ -test based on the same output has asymptotic size of 5%. Equivalently, one can linearize Equation (4.1) as follows:

$$r_t = \rho r_{t-1} + \theta_0 + \theta_1 E_t \pi_{t+h} + \theta_2 \hat{y}_t + \varepsilon_t \quad (4.2)$$

where  $\hat{\beta}_{OLS}$  can be computed as  $\frac{\hat{\theta}_{1,OLS}}{1 - \hat{\rho}_{OLS}}$ . Since this is the ratio of OLS estimates, the delta-method can be used to construct the asymptotic variance of  $\hat{\beta}_{OLS}$ . Both this method and NLS of Equation (4.1) give numerically equivalent results. Since  $\beta = \frac{\theta_1}{1 - \rho}$ , the delta-method is only valid when  $\rho < 1$ , which ensures that the derivatives of the transformation are bounded and continuously differentiable. We also note here that when  $\rho = 1$ , the ZILC holds.

#### 4.2 Fieller

It is well known that Wald-type tests are not invariant to the formulation of the null hypothesis. An alternative formulation could be written as follows:

$$\theta_1 - \lambda_0(1 - \rho) = 0$$

where  $\lambda_0$  is the value of the ratio of parameters under the null hypothesis. In order to compute a confidence interval, one could invert the corresponding  $t$ -statistic as follows:

$$\left| \frac{\hat{\theta}_1 - \lambda_0(1 - \hat{\rho})}{\text{asyv}(\hat{\theta}_1) + 2(\lambda_0)\text{asyc}(\hat{\theta}_1, \hat{\rho}) + (\lambda_0)^2 \text{asyv}(\hat{\rho})} \right| \leq z_{\alpha/2}$$

which requires solving a quadratic inequality such that:

$$A(\lambda_0)^2 + 2B(\lambda_0) + C \leq 0$$

where:



$$\begin{cases} A = (1 - \hat{\rho}) - (z_{\alpha/2})^2 \text{asyv}(\hat{\rho}) \\ B = -\hat{\theta}_1(1 - \hat{\rho}) - (z_{\alpha/2})^2 \text{asyv}(\hat{\theta}_1, \hat{\rho}) \\ C = \hat{\theta}_1^2 - (z_{\alpha/2})^2 \text{asyv}(\hat{\theta}_1) \end{cases}$$

Following Bernard et al. (2007), it can be shown that if  $A > 0$  the bounded solution is given by:

$$\left[ \frac{-B - \sqrt{\Delta}}{A}, \frac{-B + \sqrt{\Delta}}{A} \right]$$

if and only if  $\Delta \equiv B^2 - AC > 0$ .

Otherwise, the solution could either be an unbounded interval or the entire real line.

#### 4.3 *Krinsky and Robb*

They propose a bootstrap procedure to compute a confidence interval for a ratio of parameters by sampling from their asymptotic distribution, then computing the ratio for each draw, and finally trimming the lower/upper 2.5% tails. The result is an approximation to an asymptotic 95% confidence interval, and this methodology has been widely used by empirical researchers in situations where the delta-method is expected to fail.

### 5. Finite Sample Size of the Delta-Method, Fieller, and Krinsky and Robb

In this section, we compare the finite sample size of the  $t$ -statistic for the Taylor rule coefficient for the delta-method, Fieller's method, and Krinsky and Robb, when estimation is performed by NLS in the presence of interest rate smoothing. We consider various parameterizations of the Taylor rule, parameter values, departures from Normality in the errors and HAC corrections.

We consider  $T = 25, 50, 75, 100, 150, 250,$  and  $1000$ . The first five sample sizes are empirically relevant, and we know of no empirical work with 250 observations, but use it and the much more unrealistic 1000 observations to see how quickly asymptotic

theory takes hold. We consider the entire range for  $\rho$ , but will limit our discussion to values of  $\rho$  greater than 0.7 for two reasons. First, values of  $\rho$  less than 0.7 are far from the ZILC of unity, and as expected, all 4 methods work quite well in small samples. Second, values of  $\rho$  greater than 0.7 are empirically relevant aside from being close to the ZILC: the three empirical studies that we analyze have estimated values of  $\rho$  no less than 0.74.

We begin by considering a Taylor rule parameterization where data are generated according to:

$$r_t = \rho r_{t-1} + (1 - \rho)[\mu + \beta E_t \pi_{t+h} + \omega \hat{y}_t] + \varepsilon_t$$

with uncorrelated standard Normal regressors and errors. We use the estimated values of  $\mu$  and  $\omega$  from one of Orphanides' (2004) specifications, and consider six values of  $\beta$ : 0.75, 1.0, 1.5, 2.0, 2.5 and 5. We use all three methods to test the null hypothesis that  $\beta$  is equal to its true value, and plot the empirical rejection frequencies. The delta-method rejection line is in blue, Fieller in red, and Krinsky and Robb in green. We plot the rejection frequencies only up to 0.25, so that their differences are more visually apparent, although we do note that for very small sample sizes, the delta-method often goes above 25%, while this is never true for Fieller and Krinsky and Robb.

Figure 1 presents the results for  $\beta = 0.75$ , which corresponds to the case where the Fed raises the nominal interest rate less than one for one with an increase in expected inflation, so that the real interest rate falls.

Figure 2 presents the results for  $\beta = 1$ , which corresponds to the case where the Fed raises the nominal interest rate one for one with an increase in expected inflation so that the real interest rate remains unchanged. For  $T = 25$  and 50, very small sample sizes, the delta-method is dominated by both other methods. Around a smoothing parameter of 0.8, the delta-method becomes sized at 10%, and this size distortion worsens approaching the ZILC point of  $\rho = 1$ , reaching as high at 35% for  $T = 25$ . The situation is much better for Fieller and Krinsky and Robb. Even for these small samples, these tests do not become doubly-sized until around  $\rho = .95$ , which is higher than most empirical estimates, although we know that there is a downward mean and median-bias

in the NLS estimate of this coefficient. It isn't until  $\rho$  approached unity that the size distortions worsen, with Krinsky and Robb never going over 15%. For  $T = 75$  and 100, moderate but empirically relevant sample sizes, a similar pattern emerges. The size distortion of the delta-method is mitigated, as predicted by asymptotic theory, but it still reaches twice its nominal size around  $\rho = .80$  and peaking around 20% at the ZILC point. Fieller and Krinsky and Robb are much better behaved, only becoming oversized when  $\rho$  exceeds 0.95. We finally consider  $T = 150$  and 250. These are interesting sample sizes, since one of our studies has 150 observations, and no empirical studies have 250. For the former, although the delta-method continues to improve with the larger sample size, Krinsky and Robb is the clear winner, with a maximum size of around 7% when interest rate smoothing is high. Fieller is in the middle. When we increase the sample size to 250, Krinsky and Robb is again the clear winner, being almost correctly sized for the entire range of  $\rho$ . It may be worth noting that as  $\rho$  approaches unity for this sample size, the delta-method has better size than the Fieller.

Figure 3 presents the results for  $\beta = 1.5$ , which corresponds to Taylor's (1993) original specification, where the Fed raises the nominal interest 1.5 to 1 when expected inflation increases, so that the real interest rate increases to slow down real economic activity. For  $T = 25$  and 50, the delta-method gets worse in terms of being oversized, approaching 10% actual size around  $\rho = .70$ , and getting more oversized as interest rate smoothing increases, peaking around 27-40%. For these sample sizes, Fieller doesn't get oversized until  $\rho = .95$ , and the Krinsky and Robb procedure only eclipses 10% actual size very close to the ZILC point. For  $T = 75$  and 100, the delta-method slightly improves, is dominated by Fieller, and Krinsky and Robb are slightly oversized in the limit. For  $T = 150$  and 250, the delta-method is still the poorest performer, save for slightly better size than Fieller close to the ZILC, while Krinsky and Robb is almost properly sized for the entire range of interest rate smoothing.

Figure 4 – 6 report actual size for  $\beta = 2.0, 2.5$  and 5, the latter two corresponding to very high increases in the Fed Funds Rate, when expected inflation increases. The main qualitative differences remain, while the main quantitative difference is that both Fieller and Krinsky and Robb become more oversized as  $\rho$  approaches unity for the

higher Taylor Rule coefficients, which are probably unrealistic values of the response of the Fed to increases in expected inflation.

Finally, while we do not plot the results for  $T = 1000$ , it is the case the Fieller and Krinsky and Robb are properly sized, while the delta method is slightly oversized, around 8-9% close the Zero Information Limit Condition. Given these simulation results, we now turn to three recent empirical studies on the Taylor Rule estimated by NLS, taking Krinsky and Robb as the most reliable procedure to produce a confidence interval with correct size.

## 6. Confidence Intervals of Taylor Rule Coefficients with Better Finite Sample Coverage Properties

In this section, we re-examine three empirical studies which estimate Taylor rules with interest rate smoothing and forward looking data, using nonlinear least squares, which again is equivalent to using the delta-method on the linearized version of the model. We compare the confidence intervals computed from the delta-method, Fieller, and Krinsky and Robb, where the latter two methods were demonstrated in the previous section to have better coverage in finite samples, especially when we approach the ZILC.

### 6.1 Orphanides (2004 JMCB)

We start by re-examining Orphanides (2004 ), the first paper to estimate Taylor rules with forward looking data by NLS. He estimates equations of the form<sup>1</sup>:

$$r_t = \rho r_{t-1} + (1 - \rho)[\mu + \beta E_t \pi_{t+h} + \omega \hat{y}_t] + \varepsilon_t \quad (6.1)$$

where  $r_t$  is the nominal Federal Funds rate,  $E_t \pi_{t+h}$  is the forecast of inflation  $h$  horizons into the future,  $\hat{y}_t$  is the estimated output gap, and  $\rho$  is the degree of interest rate smoothing.

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<sup>1</sup> In the published version of his paper, Orphanides uses AR(2) smoothing. We cannot replicate his results using his working paper data, but we will consider AR(2) smoothing in this paper with the same data.

Orphanides considers two samples, 1966:1-1979:2 and 1979:3-1995:4, corresponding the pre and post Volcker regimes. He also considers inflation forecasts from 1 to 4 quarters ahead. For these 8 regressions, the Taylor Principle is estimated to hold twice at the nominal 5% level, during the post Volcker regime for the 3 and 4 quarter ahead inflation forecasts. Of course, these findings of significance are based on the delta-method, which is only valid asymptotically, and the sample size in this case is quite small;  $T = 66$ . We consider whether this finding is robust to using methods which generate confidence intervals with more accurate coverage properties.

Table 1 reports the three sets of confidence intervals for each inflation forecast horizon. The first column is the delta-method, which can be directly inferred from Orphanides' Table 1, by computing:

$$\hat{\beta} \pm 1.96 \text{asyse}(\hat{\beta}).$$

The second column is Fieller, and column 3 is Krinsky and Robb (KR).

The published version of his paper uses AR(2) smoothing. Using the data from his working paper, we also consider AR(2) smoothing. The results are reported in Table 1. For the 3 quarter ahead inflation forecast, all 3 confidence intervals have lower bounds less than unity. This reverses 1 of his 2 findings that the Taylor Principle held post-Volker. For the 4 quarter ahead inflation forecast, all first 3 confidence intervals have lower bounds greater than unity, thereby not reversing his finding, but the lower bounds of the confidence intervals are very close to unity.

## 6.2 *Nikolsko-Rzhevskyy (2011 JMCB)*

Nikolsko-Rzhevskyy (2011) estimates Taylor Rules with AR(1) interest rate smoothing from 1982:1-2007:1, 101 quarterly observations, for the “nowcast” of inflation, as well as for forecast horizons 1-6. Based on the delta-method, for the seven regressions, he finds that the Taylor Principle holds for  $h = 2 - 6$ . In Table 2, we report his asymptotic confidence intervals for these horizons, as well as those based on Fieller and KR. For  $h = 2 - 4$ , both Fieller and KR reverse the conclusion that  $\beta > 1$ . In contrast, for  $h = 5 - 6$  his conclusion is not changed.

Although he does not report it, we redo Nikolsko-Rzhevskyy's regressions with AR(2) smoothing, reported in Table 3. Using the delta-method, we find that the Taylor Principle holds for  $h = 2 - 6$ . However, just as with AR(1) smoothing, the results for  $h = 2 - 4$  are overturned when considering Fieller and KR. Overall, this reverses 3 of the 5 findings that the Taylor principle held from 1982:1 - 2007:1

### *6.3 Coibion and Gorodnichenko (2011 AER)*

Coibion and Gorodnichenko (2011) estimate 6 Taylor rule regressions, using data up through 1979, and data after 1982. Two of these specifications have significant estimates of  $\beta$  being larger than unity. The first is a forward looking Taylor rule with a 2 quarter ahead inflation forecast, forecasted output gap, and forecasted output growth as regressors. The second has a 2 quarter ahead inflation forecast, contemporaneous gap and growth as regressors, called a mixed Taylor rule. Both specifications use AR(2) smoothing.

Table 4 reports the 3 sets of confidence intervals. For the forward looking specification, all 3 confidence intervals are consistent with the Taylor Principle holding. For the mixed Taylor rule as well, all 3 confidence intervals support Coibion and Gorodnichenko's finding that the Taylor Principle was followed post 1982. We also redo their regressions by omitting output growth, to better compare their results with Orphanides and Nikolsko-Rzhevskyy. The results are starkly different. While Fieller breaks down, the delta-method and KR consistent with virtually every realistic value of  $\beta$ , as well as a large range of unrealistic positive and negative values.

## **7. Conclusions**

In this paper we demonstrate that forward looking Taylor rules with interest rate smoothing estimated by NLS are subject to a variety of problems related to the difference in the predictions based on asymptotic theory and the actual finite sample distributions of the parameters of interest. We use methods to construct confidence intervals for the Fed's change in its nominal interest rate target to changes in expected inflation that have better properties than the delta-method.

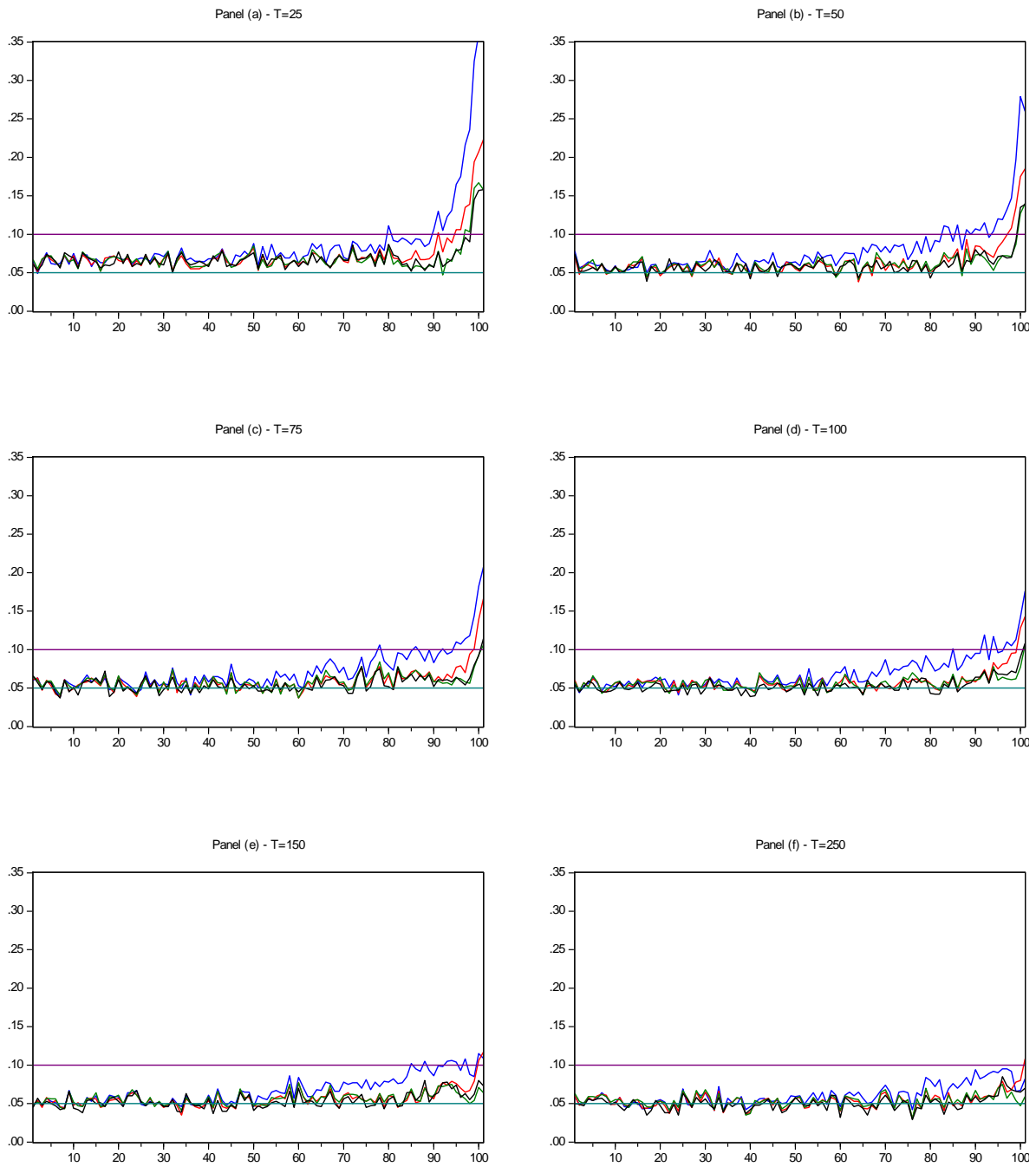
We reconsider 3 empirical studies which estimate Taylor Rule regressions via NLS, and find that over half of the failures to reject are overturned. Specifically, many examples of apparently informative confidence intervals based on the delta-method widen when better procedures are used to become much less informative, in that they do not rule out that the Taylor Principle did not hold for a variety of samples and sample sizes.

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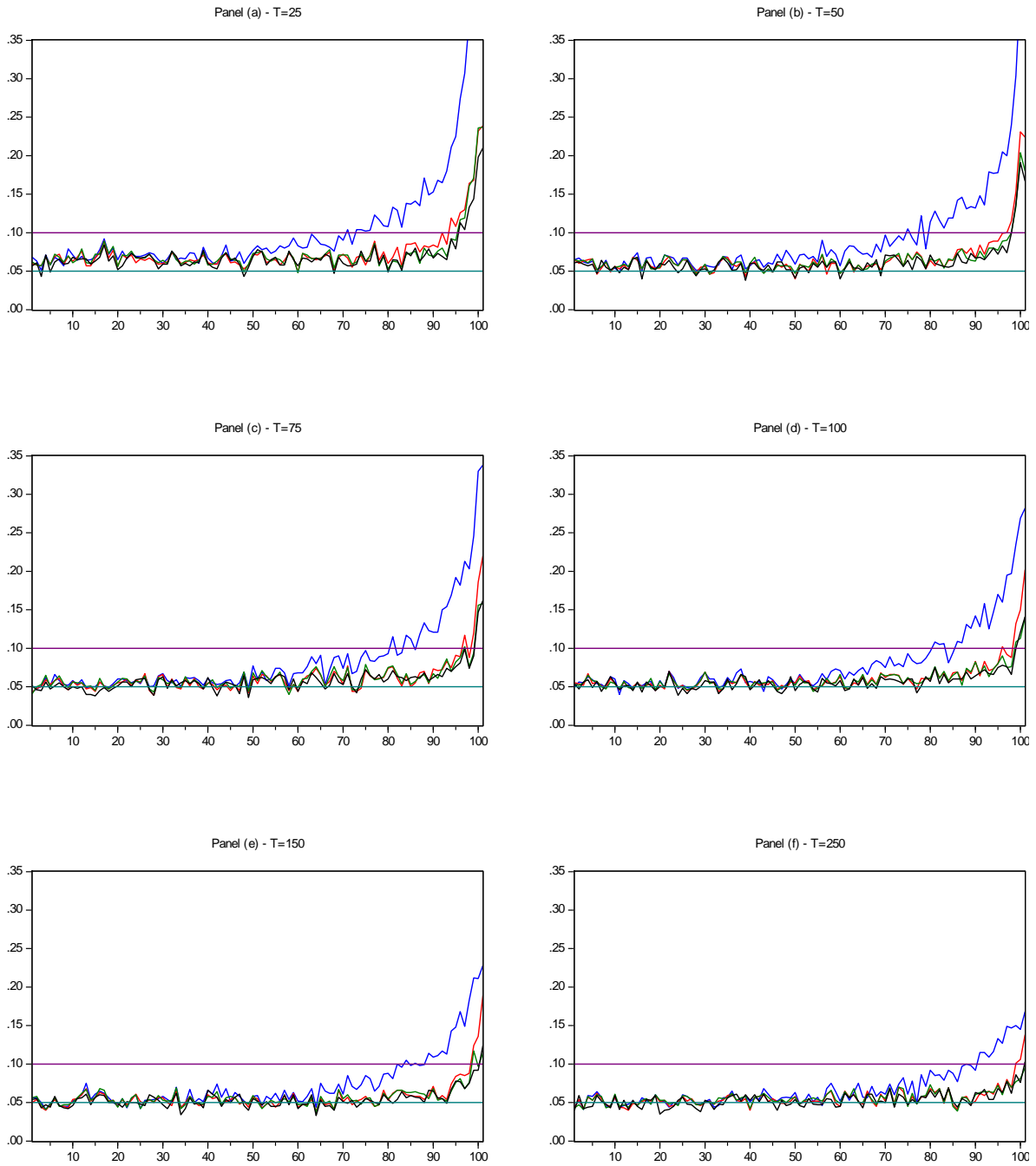
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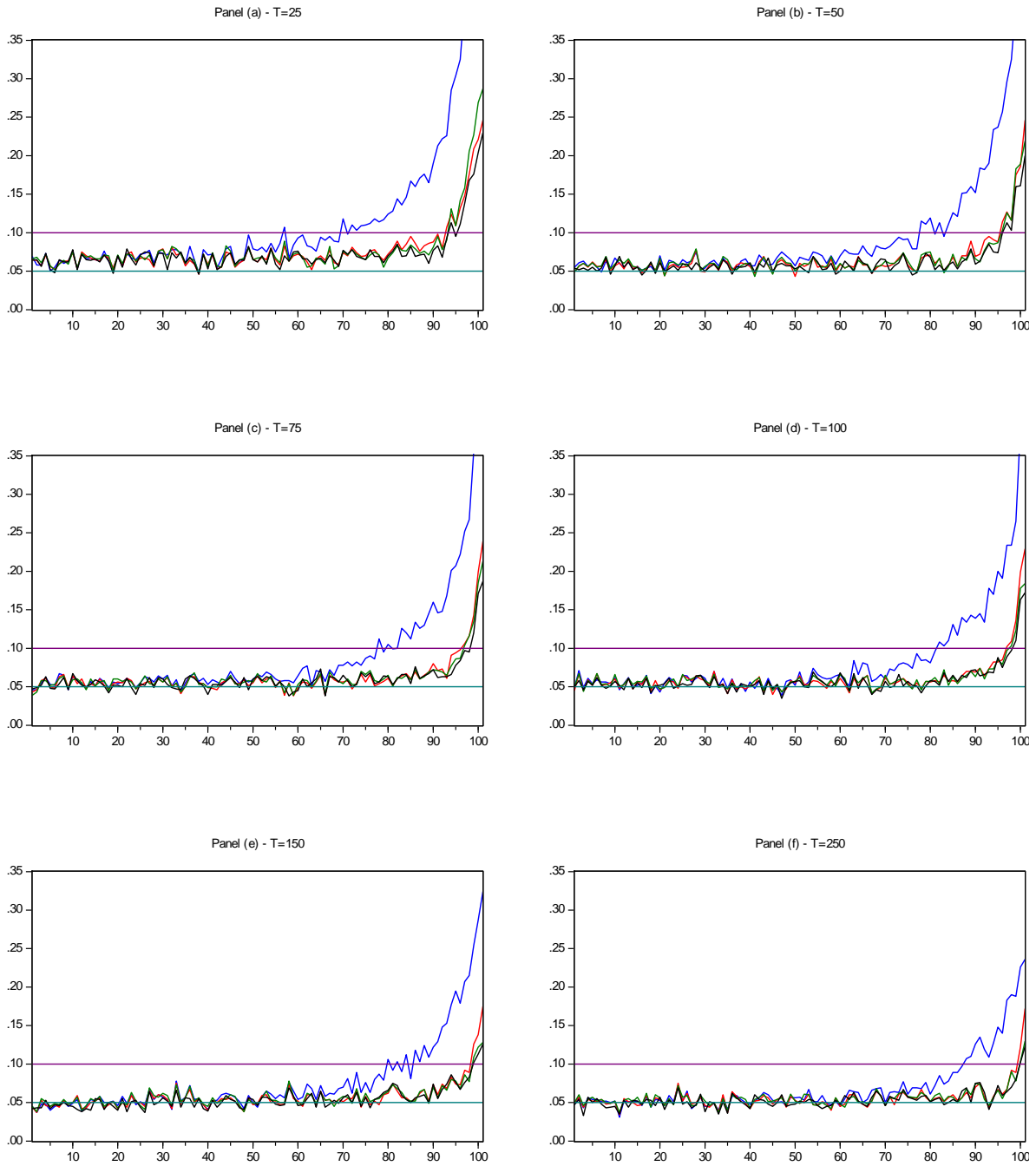
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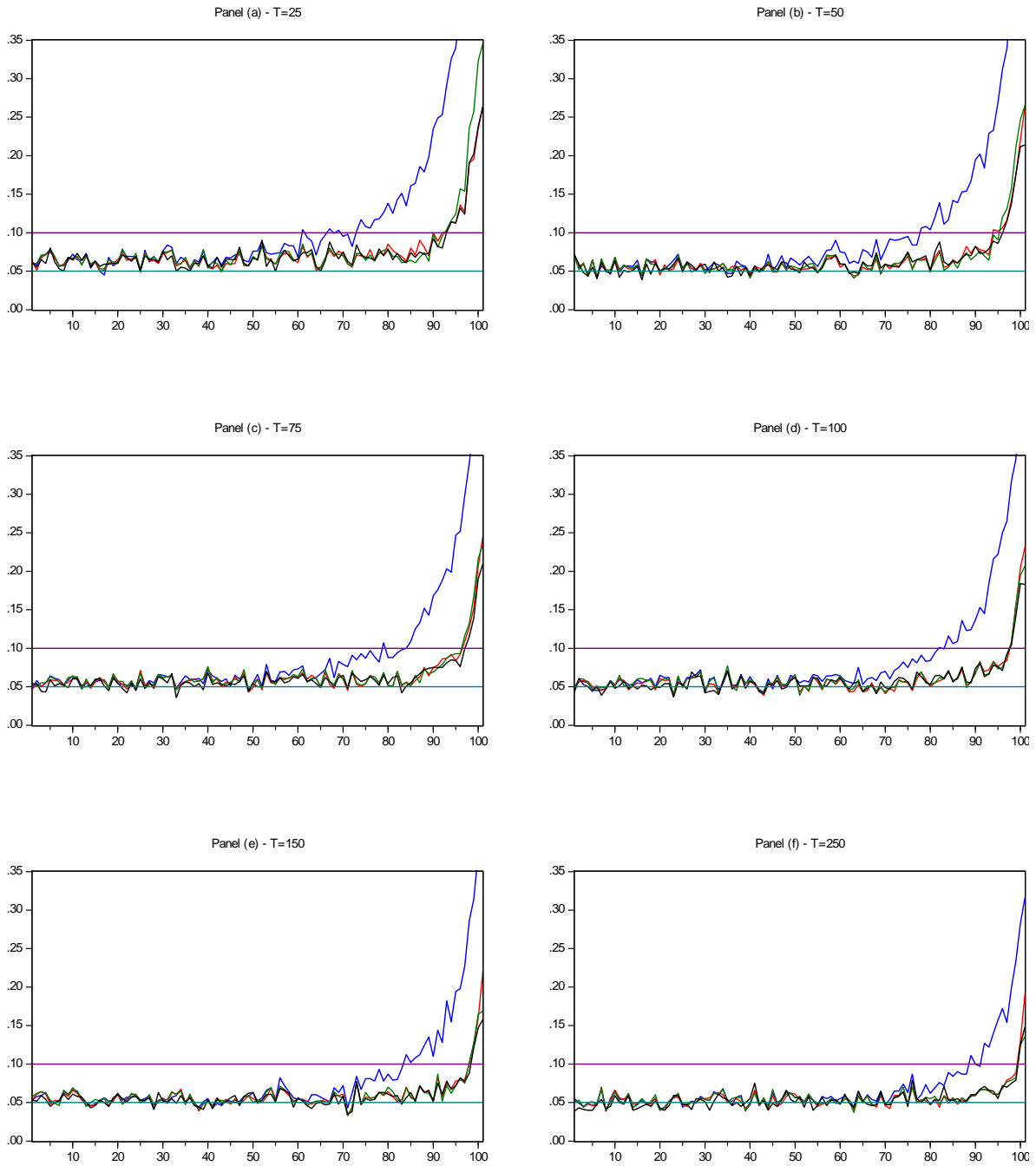
**Figure 1 - Actual Size of a Nominal 5% Test, Beta=1**



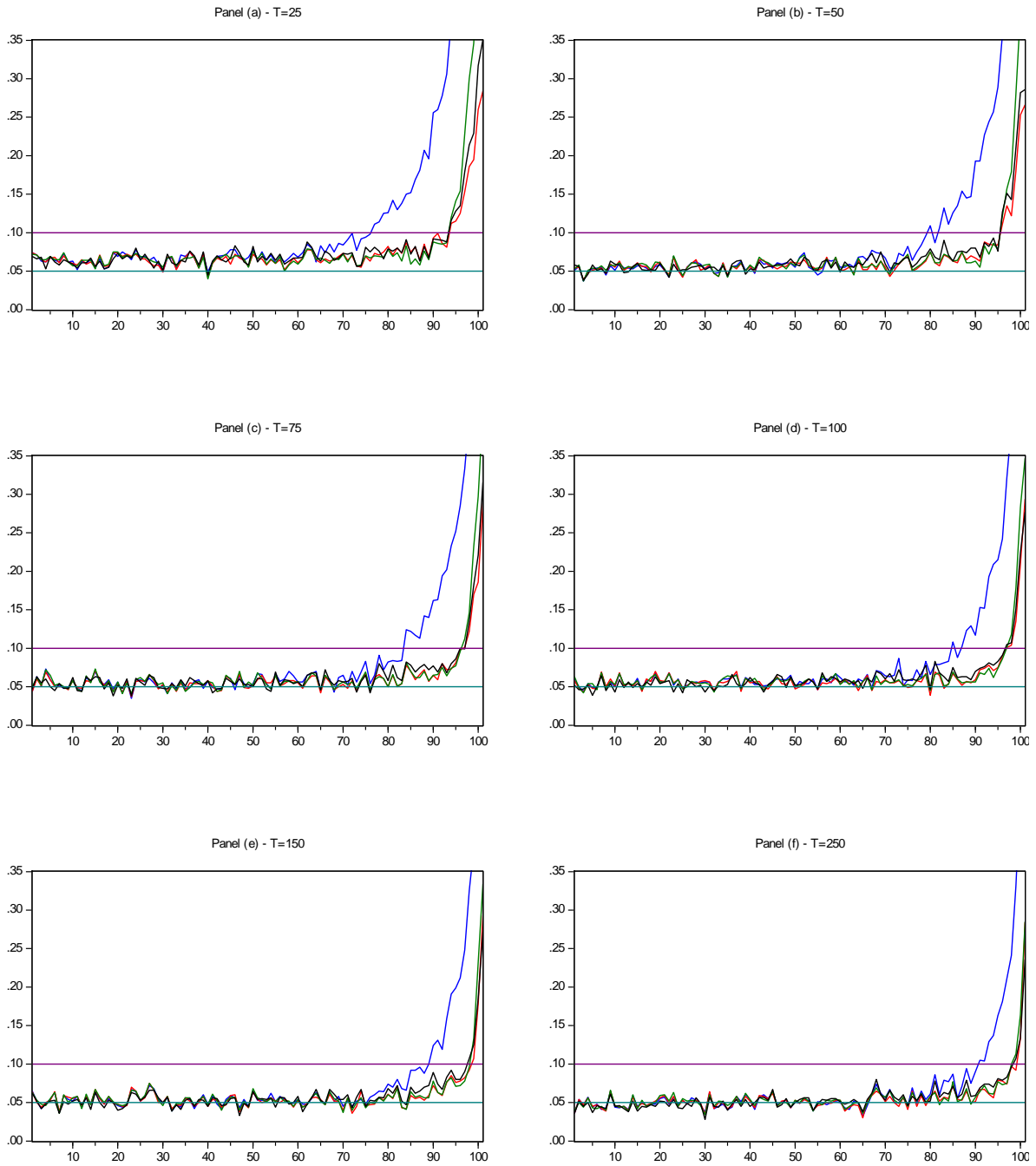
**Figure 2 - Actual Size of a Nominal 5% Test, Beta=1.5**



**Figure 3 - Actual Size of a Nominal 5% Test, Beta=2**



**Figure 4 - Actual Size of a Nominal 5% Test, Beta=2.5**



**Figure 5 - Actual Size of a Nominal 5% Test, Beta=5**

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Table 1 - Orphanides (2004) / 95% Confidence Intervals

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Sample: 1979:3-1995:4 ( $T = 66$ ) - AR(1) smoothing

Horizon	$\hat{\rho}$	<i>Delta Method</i>	<i>Fieller</i>	<i>Krinsky-Robb</i>
$h = 3$	0.76	[1.093, 2.686]	[1.062, 2.845]	[1.053, 2.829]
$h = 4$	0.74	[1.178, 2.729]	[1.164, 2.864]	[1.186, 2.853]

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Note: Newey-West HAC standard errors.

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Table 2 - Orphanides (2004) / 95% Confidence Intervals

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Sample: 1979:3-1995:4 ( $T = 66$ ) - AR(2) smoothing

Horizon	$\hat{\rho}$	<i>Delta Method</i>	<i>Fieller</i>	<i>Krinsky-Robb</i>
$h = 3$	0.76	<b>[0.935, 2.758]</b>	<b>[0.918, 2.921]</b>	<b>[0.935, 2.949]</b>
$h = 4$	0.74	[1.034, 2.795]	[1.032, 2.940]	[1.019, 2.926]

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Note: Newey-West HAC standard errors.

Table 3 - Nikolsko-Rzhevskyy (2011) / 95% Confidence Intervals

Sample: 1982:1 – 2007:1 ( $T = 101$ ) - AR(1) smoothing				
Horizon	$\hat{\rho}$	<i>Delta Method</i>	<i>Fieller</i>	<i>Krinsky–Robb</i>
$h = 2$	0.85	[1.195, 3.690]	<b>[0.717, 4.402]</b>	<b>[0.697, 4.363]</b>
$h = 3$	0.85	[1.193, 3.969]	<b>[0.984, 4.425]</b>	<b>[0.959, 4.479]</b>
$h = 4$	0.84	[1.339, 3.963]	<b>[0.961, 4.350]</b>	<b>[0.957, 4.388]</b>
$h = 5$	0.82	[1.686, 3.995]	[1.395, 4.339]	[1.412, 4.357]
$h = 6$	0.83	[1.503, 3.938]	[1.431, 4.243]	[1.423, 4.176]

Note: Newey-West HAC standard errors.

Table 4 - Nikolsko-Rzhevskyy (2011) / 95% Confidence Intervals

Sample: 1982:1 – 2007:1 ( $T = 101$ ) - AR(2) smoothing				
Horizon	$\hat{\rho}$	<i>Delta Method</i>	<i>Fieller</i>	<i>Krinsky–Robb</i>
$h = 2$	0.85	[1.183, 3.659]	<b>[0.652, 4.228]</b>	<b>[0.691, 4.180]</b>
$h = 3$	0.85	[1.178, 3.927]	<b>[0.932, 4.352]</b>	<b>[0.956, 4.368]</b>
$h = 4$	0.84	[1.259, 3.976]	<b>[0.768, 4.316]</b>	<b>[0.804, 4.357]</b>
$h = 5$	0.82	[1.571, 4.049]	[1.127, 4.337]	[1.096, 4.372]
$h = 6$	0.83	[1.463, 3.912]	[1.416, 4.273]	[1.436, 4.309]

Note: Newey-West HAC standard errors.



Table 5 – Coibion and Gorodnichenko (2011) / 95% Confidence Intervals

Sample: post 1982 ( $T = 158$ )				
Specification	$\hat{\rho}$	<i>Delta Method</i>	<i>Fieller</i>	<i>Krinsky–Robb</i>
<i>Forward</i>	0.94	[1.351, 3.715]	[1.290, 4.987]	[1.260, 4.758]
<i>Mixed</i>	0.92	[1.409, 2.994]	[1.472, 3.376]	[1.464, 3.372]

Note: Newey-West HAC standard errors.

Table 6 - Coibion and Gorodnichenko (2011) / 95% Confidence Intervals

Sample: post 1982 ( $T = 158$ )– No output growth				
Specification	$\hat{\rho}$	<i>Delta Method</i>	<i>Fieller</i>	<i>Krinsky–Robb</i>
<i>Forward</i>	0.96	<b>[0.356, 4.626]</b>	–	<b>[-2.198, 8.856]</b>
<i>Mixed</i>	0.96	<b>[0.187, 4.314]</b>	–	<b>[-2.778, 7.823]</b>

Note: Newey-West HAC standard errors.