Do Seasonal Adjustments Induce Noncausal Forecastability?

An Application to Inflation Rates

Alain Hecq* **Lenard Lieb** **Sean Telg**

Maastricht University

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Abstract

Recent papers find that mixed causal-noncausal (MAR) models outperform purely causal autoregressive processes when modelling and forecasting the U.S. inflation rate. It is argued that this can be seen as evidence that inflation is a forward-looking variable. This intuitive claim is indeed supported by structural economic models allowing for both forward- and backward-looking components such as the hybrid new Keynesian Phillips curve. Seasonally adjusted prices are used in those applications before constructing inflation series.

In this paper, we investigate the effect of seasonal adjustment filters on the identification of MAR models through information criteria. By means of Monte Carlo simulations, we find that standard seasonal filters might induce spurious autoregressive dynamics, a well-known phenomenon already documented in the literature. Symmetrically, we show that those filters also generate a spurious noncausal component in the seasonally adjusted series. An empirical application on European inflation data illustrates these results. In particular, whereas inflation rates are forecastable on seasonally adjusted series, they appear to be white noise using raw data.

Keywords: seasonality, inflation, Census X-11, seasonal adjustment filters, mixed causal-noncausal models. **JEL:** C22, E37.

*Corresponding author: Alain Hecq, Maastricht University, School of Business and Economics, P.O. Box 616, 6200 MD Maastricht, The Netherlands. E-mail: a.hecq@maastrichtuniversity.nl; URL: http://researchers-sbe.unimaas.nl/alainhecq/

1 Introduction

Hundreds of theoretical and empirical papers have been devoted to accurately model and forecast the inflation rate, the main target variable meticulously watched by the European Central Bank as well as by many national and international institutions. To that end, univariate and multivariate approaches have been proposed, among which (vector) autoregressive moving average models, mixed frequency regressions, experts' judgements, many variants of the factor model, different forms of linear and nonlinear Phillips' curves (see e.g., Stock and Watson, 1999), etc.

Several recent papers also advocate the use of mixed causal-noncausal autoregressive processes (see e.g., Lanne and Saikkonen, 2011) when forecasting a series with a dependence on both past and future values. This is certainly the case for inflation, since several models (e.g., the hybrid new Keynesian Phillips Curve, hybrid NKPC hereafter), explicitly include both past inflation and future expectations of the rate of inflation. The univariate mixed causal-noncausal model is often denoted by MAR(r, s), where r represents the number of lags and s the number of leads of the dependent variable. Lanne, Luoto and Saikkonen (2012) find that the MAR(1,4), namely an equation with one lag and four leads of the inflation rate, is the best model for that variable and that there are substantial forecasting gains over its pseudo-causal AR(5) counterpart.[1] As a direct response to a paper by Stock and Watson (2007), Lanne and Luoto (2012) further argue that the U.S. inflation rate has not become harder to forecast as long as the noncausal autoregression is taken into account. In yet another paper, Lanne and Luoto (2013) deduce that the hybrid NKPC has an MAR(r, s) representation and find that the inflation rate based on CPI and on GDP deflator follows an MAR(1,3) and MAR(1,4) process respectively.

The importance of forward-looking behavior in inflation is still an ongoing debate, though. For instance Rudd and Whelan (2007), Nason and Smith (2008) find only little evidence for a forward-looking component based on estimated NKPC's for the U.S. There is however also much

[1] Gouriéroux and Jasiak (2014) define the concept of a corresponding pseudo-causal model, which is a purely causal AR(p) where the number p equals the amount of lags and leads used in the MAR(r, s) specification, i.e., $p = r + s$.

evidence in favor of adding such a lead term: see e.g., Galí and Gertler (1999) and Lanne and Luoto (2013). From an economic point of view, future expectations of agents are often assumed to be rational in the inflation framework, an opportunity that Lanne and Saikkonen (2008) take to show that MAR(r, s) models can be interpreted as solutions to rational expectations models. The reason for this is that such models typically have a non-fundamental interpretation (dependence on future values) preventing the agents from making structural errors based on solely past information. They argue that mixed models take the agent's true information set into account without explicitly specifying it.

This being said, virtually all studies use seasonally adjusted prices for building inflation rates. The literature on the side effects and consequences of using seasonally adjusted series instead of the raw ones is enormous in econometrics.[2] In this paper, we show by Monte Carlo simulations that the model selection procedure for MAR(r, s) models is heavily affected by this filtering operation. More specifically, we apply the linear X-11 seasonal adjustment filter to three different data generating processes with deterministic seasonality (strong white noise, purely causal AR(1) and purely noncausal AR(1)) and find that spurious dynamics are created, of which in many cases also noncausal ones. This results does not come as a complete surprise given the existing literature. It is well known that seasonal adjustment filters might create spurious autocorrelation. Since the (partial) autocorrelation function is completely symmetric, the effect is also observed in reverse time. In fact, we want to emphasize this issue for mixed causal-noncausal models, as they are more difficult to identify and to estimate than purely causal autoregressive models (that is, OLS and Gaussian MLE are not applicable). Our simulation results are illustrated in comparing MAR(r, s) model selection on both raw and seasonally adjusted quarterly inflation rates for 32 countries and one overall Europe measure.

The remainder of this paper is organized as follows. Section 2 formalizes the notion of mixed causal-noncausal models and comments on the identifiability and estimation of such models.

[2]See inter alia, Ghysels and Perron (1993), Maravall (1997) and Hecq (1998).

The results of the simulation study are collected in Section 3. Section 4 details the empirical application. Section 5 summarizes and concludes.

2 Mixed Causal-Noncausal Models

Brockwell and Davis (1991, 2002) originally advocated the use of noncausal models as they offer the possibility to rewrite a process with explosive roots in calendar time into a process in reverse time with roots outside the unit circle. Additional important empirical features of the noncausal approach have been put forward in the recent literature. Beyond the improvement in terms of forecasting accuracy already mentioned in the introduction (see also Lanne, Nyberg and Saarinen, 2012; Lof and Nyberg, 2015) as well as their closeness to the concept of non-fundamental shocks (see Lippi and Reichlin, 1993a, 1993b; Alessi, Barigozzi and Capasso, 2011; Beaudry, Fève, Guay and Portier, 2015) simple linear noncausal models are able to mimic nonlinear processes such as bubbles or asymmetric cycles (Gouriéroux and Zakoïan, 2013; Gouriéroux and Jasiak, 2015; Hecq, Lieb and Telg, 2015).

2.1 Model Representation

The univariate mixed causal-noncausal autoregressive model MAR(r, s), for a stationary time series y_t, $t = 1...T$ is usually written as

$$(1-\phi_1 L-...-\phi_r L^r)(1-\varphi_1 L^{-1}-...-\varphi_s L^{-s})y_t = \varepsilon_t, \tag{1}$$

$$\phi(L)\varphi(L^{-1})y_t = \varepsilon_t, \tag{2}$$

with L the backshift operator, i.e., $Ly_t = y_{t-1}$ gives lags and $L^{-1}y_t = y_{t+1}$ produces leads. When $\varphi_1 = ... = \varphi_s = 0$, the process y_t is a purely causal autoregressive process, denoted AR(r,0) or simply AR(r):

$$\phi(L)y_t = \varepsilon_t. \tag{3}$$

Model specification (3) can be seen as the standard backward-looking AR process, with y_t being regressed on y_{t-1} up to y_{t-r}. The process in (2) becomes a purely noncausal AR$(0, s)$ model

$$\varphi(L^{-1})y_t = \varepsilon_t, \tag{4}$$

when $\phi_1 = ... = \phi_r = 0$. Model specification (4) is the counterpart of (3), since it is a purely forward-looking AR process. That is, y_t does not depend on its past values, but rather on its future values y_{t+1} up to y_{t+s}. Models of the form (2) that contains both lags and leads of the dependent variable, are called mixed causal-noncausal models.

The roots of both the causal and noncausal polynomials are assumed to lie outside the unit circle, that is $\phi(z) = 0$ and $\varphi(z) = 0$ for $|z| > 1$ respectively. These conditions imply that the series y_t admits a two-sided moving average (MA) representation $y_t = \sum_{j=-\infty}^{\infty} \psi_j \varepsilon_{t-j}$, such that $\psi_j = 0$ for all $j < 0$ implies a purely causal process y_t (w.r.t. ε_t) and a purely noncausal model when $\psi_j = 0$ for all $j > 0$ (Lanne and Saikkonen, 2011). In order to identify the causal from the noncausal component, the error term ε_t is assumed *iid* (and not only weak white noise) but non-Gaussian with $E(|\varepsilon_t|^{\delta}) < \infty$, for some $\delta \in (0, 1)$.[3] This ensures that the stochastic process ε_t (and accordingly y_t) can have infinite first and second moment.

Following Gouriéroux and Jasiak (2015), we define the unobserved causal and noncausal components of the process y_t as follows:

$$u_t \equiv \phi(L)y_t \leftrightarrow \varphi(L^{-1})u_t = \varepsilon_t,$$

and

$$v_t \equiv \varphi(L^{-1})y_t \leftrightarrow \phi(L)v_t = \varepsilon_t.$$

[3]Proposition 2.1 in Gouriéroux and Zakoïan (2013) shows that assuming the summability condition $\sum_{j=-\infty}^{\infty} |\psi_j|^{\delta} < \infty$ for some $\delta \in (0, 1)$ ensures absolute summability of the MA coefficients, i.e., the convergence of the series y_t with probability 1. Then the process y_t is well defined for any *iid* sequence ε_t such that $E(|\varepsilon_t|^{\delta}) < \infty$ for some $\delta \in (0, 1)$. This extension is extremely useful in the context where $E(|\varepsilon_t|)$ is not defined, like in the Cauchy case.

The specification of these filtered values is very useful in simulating, estimating and forecasting mixed causal-noncausal processes.

2.2 Hybrid New Keynesian Phillips Curve

Lanne and Luoto (2013) directly link mixed causal-noncausal models to the analysis of inflation. They show that the hybrid NKPC in its regression form,

$$\pi_t = \alpha \mathbb{E}_t(\pi_{t+1}) + \beta \pi_{t-1} + \gamma x_t + \epsilon_t, \tag{5}$$

where π_t denotes inflation, $\mathbb{E}_t(\cdot)$ the conditional expectation at time t, x_t a measure for marginal costs and ϵ_t an error term, can be represented as an MAR(r, s) model as in (2). By means of replacing $\mathbb{E}_t(\pi_{t+1})$ by the future realized value of inflation π_{t+1} and putting the parts $\alpha\mathbb{E}_t(\pi_{t+1})$, γx_t and $-\alpha\pi_{t+1}$ into the error term, they show that one obtains an MAR(1,1) with a newly defined disturbance term, say ξ_t, that is likely to be autocorrelated. ξ_t thus consists of three parts: (i) the expectation error $\alpha(\mathbb{E}_t(\pi_{t+1}) - \pi_{t+1})$, (ii) marginal costs γx_t and (iii) an *iid* error ϵ_t. Following the literature on rational expectations models, the expectation error is assumed *iid*. The variable x_t is however likely to create autocorrelation, as economic theory indicates that marginal costs are an important explanatory variable for inflation. By assuming an MAR$(r-1, s-1)$ structure on ξ_t, they show that the four polynomials [two from MAR(1,1), two from MAR$(r-1, s-1)$], can be rewritten as two, i.e., a causal polynomial $\phi(L)$ of order r and a noncausal polynomial $\varphi(L^{-1})$ of order s. The resulting model is mixed causal-noncausal with an *iid* error term, as introduced in (2). Figure 1 shows a simulated path from an MAR(2,2) model. It can be seen that such models are able to capture the well-known fluctuating behavior of inflation as well as sudden peaks or throughs that could, for example, be caused by a macroeconomic policy measure.

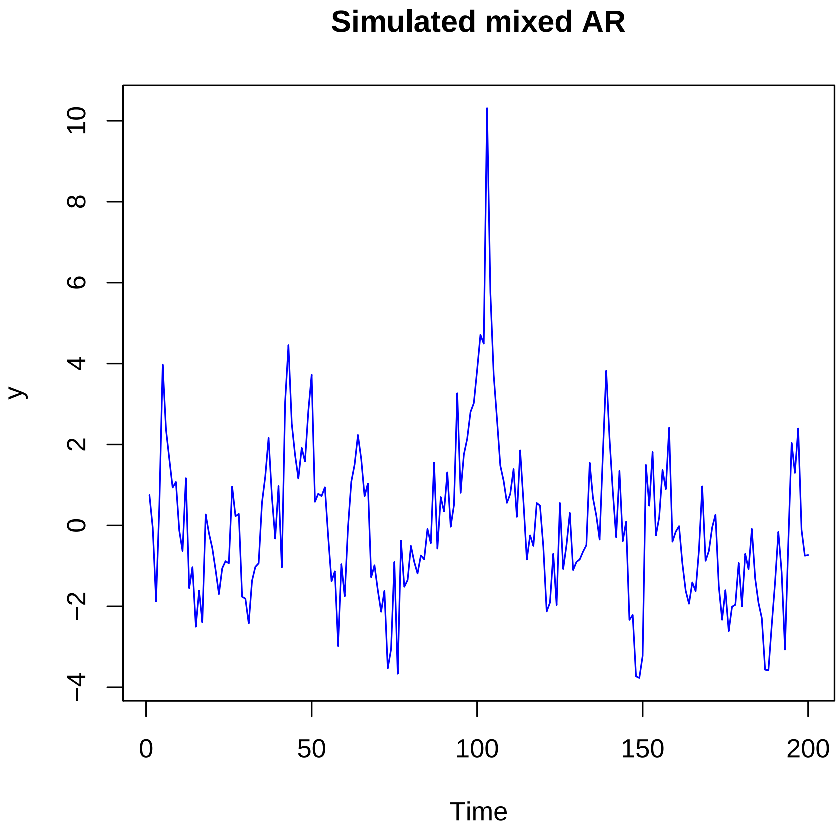


Figure 1: Simulated MAR(2,2) process with $\phi = (0.2, 0.3)$ and $\varphi = (0.3, 0.1), \varepsilon_t \sim t_5$

2.3 Estimation

The non-Gaussianity assumption ensures the identifiability of the causal (similarly, backward-looking) and the noncausal (forward-looking) part (Breidt et al., 1991). Most papers by Lanne, Saikkonen and coauthors use Student's t_ν-distributions with a degree of freedom $\nu \geq 2$ as an alternative to the Gaussian distribution. Gouriéroux and coauthors rely on the Cauchy or a mixture of Cauchy and Normal distributions. In this paper, we also consider a non-standardized t-distribution for the error process. Lanne and Saikkonen (2011) show that the parameters of mixed causal-noncausal autoregressive models of the form (2) can be consistently estimated by approximate maximum likelihood (AML).[4] Letting ε_t be a sequence of *iid* zero mean random variables with probability density function

$$f_\varepsilon(\varepsilon_t|\sigma,\nu) = \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})\sqrt{\pi\nu}\sigma}\left(1+\frac{1}{\nu}\left(\frac{\varepsilon_t}{\sigma}\right)^2\right)^{-\frac{\nu+1}{2}},$$

the corresponding (approximate) log-likelihood function, conditional on the observed data $(y_1,...,y_T)$ can be formulated as

$$\begin{aligned} l_y(\phi,\varphi,\alpha,\sigma,\nu|y_1,...,y_T) &= (T-(r+s))\left[\ln(\Gamma((\nu+1)/2)) - \ln(\sqrt{\nu\pi}) - \ln(\Gamma(\nu/2)) - \ln(\sigma)\right] \\ &\quad -(\nu+1)/2\sum_{t=r+1}^{T-s}\ln(1+((\phi(L)\varphi(L^{-1})y_t-\alpha)/\sigma)^2/\nu). \end{aligned} \qquad (6)$$

The scale parameter is denoted by σ, ν are degrees of freedom, and $\Gamma(\cdot)$ is the gamma function. Thus, the AMLE corresponds to the solution of the problem:

$$\hat{\theta}_{ML} = \arg\max_{\theta\in\Theta} l_y(\theta|y_1,..,y_T),$$

with $\theta = [\phi,\varphi,\alpha,\sigma,\nu]'$ and Θ is a permissible parameter space containing the true value of θ, say θ_0, as an interior point. Since an analytical solution of the score function is not directly available, gradient based (numerical) procedures like the Berndt-Hall-Hall-Hausman (BHHH) and Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithms can be used to find $\hat{\theta}_{ML}$. If $\nu > 2$, and hence $E(|\varepsilon_t|^2) < \infty$, the MLE is $\sqrt{T}$-consistent and asymptotically normal. Lanne and Saikkonen (2011) also show that the local ML estimator is asymptotically normally distributed, and a consistent estimator of the limiting covariance matrix is obtained from the standardized Hessian of the log-likelihood.

Hecq, Lieb and Telg (2015) propose an alternative method to compute standard errors,

[4]The term 'approximate' stems from the fact that the sample used in the likelihood contains only $T-(r+s)$ terms. As shown in Breidt et al. (1991), this quantity is only an approximation of the true joint density of the data vector $y=(y_1,...,y_T)$.

which does not require the numerical optimization of the Hessian matrix and hence is more numerically stable. Based on the unobserved causal and noncausal components discussed above, it is straightforward to show (see e.g., Fonseca, Ferreira, and Mignon, 2008) that in case of the mixed causal-noncausal model

$$\sqrt{T}(\hat{\phi}_{ML} - \phi_0) \sim \mathcal{N}(0, (\nu+3)/(\nu+1)\sigma^2 \Upsilon_{\varphi}^{-1}), \tag{7}$$

$$\sqrt{T}(\hat{\varphi}_{ML} - \varphi_0) \sim \mathcal{N}(0, (\nu+3)/(\nu+1)\sigma^2 \Upsilon_{\phi}^{-1}), \tag{8}$$

holds. We use the notations $v_t(\varphi) = \varphi(L^{-1})y_t$, $u_t(\phi) = \phi(L)y_t$ and then $\Upsilon_\varphi = E[v_t^2(\varphi)]$ and similarly $\Upsilon_\phi = E[u_t^2(\phi)]$. The variance-covariance matrices Υ_ϕ and Υ_φ can be consistently estimated by $\sum_{t=r+1}^{T-s} u_t^2(\hat{\phi}_{ML})/(T-(r+s))$ and $\sum_{t=r+1}^{T-s} v_t^2(\hat{\varphi}_{ML})/(T-(r+s))$, respectively. For large ν, i.e., $\nu \to \infty$, l_y approaches the Gaussian (log)-likelihood, and the model parameters cannot be consistently estimated anymore.

2.4 Implementation of MAR(r, s) Models

In order to estimate mixed causal-noncausal models we take the following strategy:

1. We determine a purely autoregressive process using information criteria, AIC, BIC or HQ. Even in the presence of a noncausal component we obtain an estimate of the pseudo-causal order $p = r + s$. Simulation results in Hecq et al. (2015) favor the use of BIC for univariate time series.

2. We test for the null of normality on the residuals of the AR(p) using e.g., the Jarque-Bera and one-sample Kolmogorov-Smirnov (KS) test. It should be noted, that in case the null hypothesis of normality is not rejected, there is no need to consider noncausal or mixed causal-noncausal models, as the causal and noncausal polynomial cannot be distinguished.

3. We test for *iid*-ness. If the true process is *iid* causal, then the errors from the AR(p) must

be *iid*. One should reject the *iid* null hypothesis if we estimate a pseudo-causal model using data generated by an *iid* noncausal process. Several tests can be used (e.g., BDS test) but we look at the residuals from the pseudo-causal AR(p), denoted $\hat{\varepsilon}_t$, and we run a multivariate regression from $\hat{\varepsilon}_t$ on $\hat{\varepsilon}^2_{t-1}$ up to $\hat{\varepsilon}^2_{t-m}$:

$$\hat{\varepsilon}_t = \mu + \delta_1 \hat{\varepsilon}^2_{t-1} + ...\delta_m \hat{\varepsilon}^2_{t-m} + u_t,$$

and test for *iid*-ness using the null hypothesis $H_0 : \delta_1 = ... = \delta_m = 0$. This test is asymptotically $\chi^2_{(m)}$ distributed under normality. Because this is not the case here, its distribution can be tabulated. This is the natural extension to the procedure proposed by Gouriéroux and Zakoïan (2013) for $m = 1$ lag.

4. After having established the order p that captures the autocorrelation in the series and found that non-normality is detected and the *iid*-ness of the pseudo-causal residuals can be rejected, the next step is to select a model among all MAR(r, s) specifications with $r + s = p$. To do so, we apply the Student's t AMLE for each potential model (within $p = r + s$); we take the one that gives the highest value for the likelihood.

3 Simulation Study

We consider three data generating processes for the stationary time series $y_{1,t}, y_{2,t}$ and $y_{3,t}$

$$y_{1,t} = -6D_{1t} + 1.5D_{2t} - 0.5D_{3t} + 5D_{4t} + \varepsilon_t,$$

$$y_{2,t} = 0.7y_{2,t-1} - 6D_{1t} + 1.5D_{2t} - 0.5D_{3t} + 5D_{4t} + \varepsilon_t,$$

$$y_{3,t} = 0.7y_{3,t+1} + -6D_{1t} + 1.5D_{2t} - 0.5D_{3t} + 5D_{4t} + \varepsilon_t,$$

where D_{it}, $i = 1, 2, 3, 4$ are quarterly seasonal dummies with values 1 for the corresponding quarter and zero otherwise; $y_{1,t}$ is a strong white noise, $y_{2,t}$ is a causal AR(1) and $y_{3,t}$ is a noncausal AR(1). For the three processes the error term ε_t is *iid* t-distributed with 3 degrees of freedom.

On each series, we apply the X-11 linear seasonal filter $\Psi^{SA}(L)$ (see Ghysels and Perron, 1993) and perform a model selection on the adjusted series

$$\begin{aligned} y_{1,t}^{SA} &= \Psi^{SA}(L) y_{1,t}, \\ y_{2,t}^{SA} &= \Psi^{SA}(L) y_{2,t}, \\ y_{3,t}^{SA} &= \Psi^{SA}(L) y_{3,t}. \end{aligned}$$

The X-11 program consists of a set of moving average filters which are applied to the data sequentially (main steps are summarized in Appendix A). The linear approximation to the quarterly X-11 filter, $\Psi^{SA}(L)$, is a moving average of order 57 of which the weights sum up to one. The filter uses both leads and lags, as it takes 28 quarters before and 28 quarters after every data point into account. The final weights, rounded to 3 decimals, are given in Table 1.

lags/leads		lags/leads		lags/leads	
0	0.856	10	0.025	20	-0.003
1	0.051	11	0.012	21	0.000
2	0.041	12	-0.053	22	0.002
3	0.050	13	0.021	23	0.000
4	-0.140	14	0.016	24	0.000
5	0.055	15	-0.005	25	0.000
6	0.034	16	-0.010	26	0.000
7	0.029	17	0.000	27	0.000
8	-0.097	18	0.008	28	0.000
9	0.038	19	-0.002		

Table 1: Filter weights of the linear quarterly X-11 filter

The Census X-11 program was the most widely applied adjustment procedure by statistical agencies. More recent versions, such as the so-called X13-ARIMA, consist in first identifying and estimating an ARMA model on the series (with outliers, breaks, calendar effects, etc.) with the aim to extrapolate the variable in the past and in the future before taking a set of moving average filters similar to $\Psi^{SA}(L)$. This is done to preserve the number of observations that would be lost in the X-11 method without applying back- and forecasting operations. We consider a simple linear filter $\Psi^{SA}(L)$ in this study because we want to isolate the effects coming from the moving average adjustment.

Fifteen MAR(r, s) models are estimated on $y_{1,t}^{SA}$, $y_{2,t}^{SA}$ and $y_{3,t}^{SA}$ by AMLE, assuming a Student's $t-$distribution, for $r + s = p$ where $p = 1, ..., 4$. We then rely on BIC for selecting the specification that minimizes that criterion. The results are collected in Table 2. We also consider model selection where, in the first step, the original variables $y_{1,t}$, $y_{2,t}$ and $y_{3,t}$ are regressed on four quarterly deterministic seasonal dummies. Model selection is performed afterwards on the residuals from this regression. The results are collected in Table 3. We display the results for three different sample sizes ($T = 100, 400$ and 700); 1000 replications are used and we add a burn-in period of 50 observations in both sides to delete the possible effect of initial values on the simulated series.

3.1 Case 1: X-11 Seasonal Adjustment

Table 2 reports the frequencies with which BIC selects the different MAR(r, s) specifications on $y_{1,t}^{SA}$, $y_{2,t}^{SA}$ and $y_{3,t}^{SA}$. At $T = 100$, we see that the percentages with which the correct model is selected lie around 80% for all series. These results are not extremely bad, but relatively low when compared to usual results by BIC. Furthermore, it can be seen that the remaining percentages mostly go to either a MAR(r, s) of one order higher than the true data generating process or to models with $r+s = 4$, where especially the purely causal an noncausal specifications are selected. When T increases, the frequency with which a model of order 4 is selected increases by quite a margin for $y_{2,t}^{SA}$ and $y_{3,t}^{SA}$ and dramatically for $y_{1,t}^{SA}$. The correct model selection for the causal and noncausal AR(1) specifications stays around 72% to 85%, even though it becomes less accurate as T increases (despite $T = 400$ performing better than $T = 100$). For $T = 700$, we see that in 97.3% of the cases BIC either selects a MAR(4,0) or MAR(0,4) for $y_{1,t}^{SA}$ instead of the correct white noise specification. These results are in line with Ghysels et al. (1993), who indeed find that the X-11 adjustment affects the time series properties of the data and not only at the seasonal frequencies. (Partial) autocorrelation functions are heavily affected and in our case, it seems plausible to argue that the X-11 filter creates artificial autocorrelation up to order four due to the large weight at that order in the $\Psi^{SA}(L)$ filter. In an almost equal amount of cases, the purely causal MAR(4,0) and purely noncausal MAR(0,4) maximize the log-likelihood (or similarly minimize BIC).

	$T = 100$			$T = 400$			$T = 700$		
	$y_{1,t}^{SA}$	$y_{2,t}^{SA}$	$y_{3,t}^{SA}$	$y_{1,t}^{SA}$	$y_{2,t}^{SA}$	$y_{3,t}^{SA}$	$y_{1,t}^{SA}$	$y_{2,t}^{SA}$	$y_{3,t}^{SA}$
MAR(0,0)	75.5	0.0	0.0	19.5	0.0	0.0	1.9	0.0	0.0
MAR(1,0)	6.4	82.1	4.4	4.3	83.1	0.0	0.3	73.8	0.0
MAR(0,1)	5.1	5.4	82.7	3.8	0.0	84.5	0.5	0.0	72.2
MAR(2,0)	0.6	4.1	0.2	0.2	1.8	0.0	0.0	1.9	0.0
MAR(1,1)	0.5	3.2	3.6	0.0	4.4	4.3	0.0	5.2	5.4
MAR(0,2)	0.2	0.4	3.6	0.4	0.0	1.9	0.0	0.0	1.9
MAR(3,0)	0.6	0.8	0.0	0.0	0.6	0.0	0.0	0.2	0.0
MAR(2,1)	0.1	0.3	0.3	0.0	0.2	1.0	0.0	0.0	1.2
MAR(1,2)	0.0	0.0	0.3	0.0	0.4	0.1	0.0	0.6	0.1
MAR(0,3)	0.1	0.0	0.7	0.0	0.0	0.8	0.0	0.0	0.7
MAR(4,0)	4.8	2.6	0.2	38.3	8.3	0.0	50.6	16.9	0.0
MAR(3,1)	0.0	0.2	0.8	0.2	0.0	0.9	0.0	0.0	1.8
MAR(2,2)	0.7	0.3	0.3	0.0	0.0	0.1	0.0	0.1	0.0
MAR(1,3)	0.3	0.3	0.1	0.0	1.2	0.0	0.0	1.3	0.0
MAR(0,4)	5.1	0.3	2.8	33.3	0.0	6.4	46.7	0.0	16.7

Table 2: Frequency (in percentages) with which model is selected (X-11 seasonal adjustment)

3.2 Case 2: No Seasonal X-11 Adjustment

If we now use seasonal dummy variables instead of the seasonal filter, we do not see the patterns of case 1. It can be seen in Table 3 that the amount of times the correct model is selected is relatively high. Frequencies increase with the number of time observations, making the model selection consistent. In the few cases the right model is not selected, the chosen model has at most a single order more than the correct specification. For $y_{1,t}$ it selects almost equally the MAR(1,0) and MAR(0,1), while for $y_{2,t}$ it picks either the causal MAR(2,0) or the MAR(1,1). For $y_{3,t}$, the noncausal MAR(0,2) and MAR(1,1) are often the second best choice.

	$T = 100$			$T = 400$			$T = 700$		
	$y_{1,t}$	$y_{2,t}$	$y_{3,t}$	$y_{1,t}$	$y_{2,t}$	$y_{3,t}$	$y_{1,t}$	$y_{2,t}$	$y_{3,t}$
MAR(0,0)	92.9	0.0	0.0	93.9	0.0	0.0	96.6	0.0	0.0
MAR(1,0)	2.5	89.2	2.9	2.7	95.8	0.0	1.3	96.4	0.0
MAR(0,1)	3.2	3.2	90.3	2.9	0.0	96.4	1.9	0.0	97.3
MAR(2,0)	0.3	3.6	0.1	0.2	2.1	0.0	0.1	1.5	0.0
MAR(1,1)	0.4	2.4	2.4	0.2	1.8	1.8	0.0	1.5	0.7
MAR(0,2)	0.2	0.3	2.7	0.1	0.0	1.6	0.1	0.0	1.8
MAR(3,0)	0.1	0.5	0.0	0.0	0.0	0.0	0.0	0.1	0.0
MAR(2,1)	0.1	0.2	0.5	0.0	0.3	0.1	0.0	0.2	0.0
MAR(1,2)	0.1	0.2	0.3	0.0	0.0	0.1	0.0	0.2	0.0
MAR(0,3)	0.0	0.0	0.4	0.0	0.0	0.0	0.0	0.0	0.2
MAR(4,0)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
MAR(3,1)	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0
MAR(2,2)	0.0	0.2	0.1	0.0	0.0	0.0	0.0	0.1	0.0
MAR(1,3)	0.0	0.2	0.1	0.0	0.0	0.0	0.0	0.0	0.0
MAR(0,4)	0.2	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0

Table 3: Frequency (in percentages) with which model is selected (no X11 adjustment)

In the light of cases 1 and 2, we conclude that using raw series and exploiting the correct deterministic seasonal features of the series, does not induce spurious dynamics. X-11 type of filters typically do create both causal and noncausal autoregressive parts (due to their two-sidedness) even if they are applied to white noise series. We do, however, not claim that these filters should not be used. It has to be mentioned that the removal of seasonality remains a challenging task. The data generating processes considered here only take into account deterministic seasonality. In case of data containing stochastic seasonality, deterministic terms (like e.g., quarterly dummies) will not capture the true seasonal dynamics. Moreover, the power of most seasonal unit root tests is relatively low (see e.g., Del Barrio Castro, Rodrigues, Taylor, 2015), which heavily complicates the exercise of detecting their presence in the data.

4 Empirical Application

We consider Harmonized Consumer Price Index (HCPI) series for 32 European countries and one overall Europe measure (see table 4 for countries investigated). Raw data are obtained from the Eurostat database and range from 1996Q1 until 2014Q4, which accounts for 76 quarterly observations (available for most countries). While it is obvious that prices are available monthly, we sample the monthly series (point-in-time sampling) to obtain quarterly data. Although we indeed lose time observations, we intentionally apply this transformation to compare our findings with results found in the papers quoted in the introductory section on quarterly inflation series.

We first apply a simple seasonal unit root test (HEGY test, see Hylleberg et al., 1990) on the natural logarithm of the raw prices. The HEGY test is based on the following auxiliary regression

$$\Delta_4 y_t = \alpha + \sum_{s=1}^{3} \beta_s D_{st} + \gamma T_t + \pi_1 z_{1,t-1} + \pi_2 z_{2,t-1} + \pi_3 z_{3,t-2} + \pi_4 z_{3,t-1} + \sum_{i=1}^{p} \zeta_i \Delta_4 y_{t-i} + \varepsilon_t,$$

where D_{st} are seasonal dummies, T_t is a time trend and $\Delta_4 = (1 - L^4)$, $z_{1,t} = (1 + L + L^2 + L^3)y_t, z_{2,t} = -(1 - L + L^2 - L^3)y_t$ and $z_{3,t} = -(1 - L^2)y_t$. This test checks for the presence of both seasonal and nonseasonal unit roots in the quarterly time series data.[5] In particular, three test-statistics are computed (two t-statistics and one F-statistic), where the null hypotheses are given as *(i)* $H_0 : \pi_1 = 0$: unit root at the zero frequency (nonseasonal stochastic trend), *(ii)* $H_0 : \pi_2 = 0$, this implies two cycles per year, *(iii)* $H_0 : \pi_3 = \pi_4 = 0$, the series contains roots

[5] Alternatively, modified ($\mathcal{M}$) seasonal unit root tests, like introduced by Del Barrio Castro et al. (2015), could be used. It has been shown that these tests have good finite sample size as well as power properties. However, as we only apply the seasonal unit root test for illustrative purposes, we decide to restrict ourselves to the original HEGY test.

i and $-i$ (seasonal unit roots at annual frequencies). Hylleberg et al. (1990) indicate that the following transformations have to be made in order to remove the seasonal and nonseasonal unit roots in y_t: (*i*) if $\pi_1 = 0$, $(1 - L)$, (*ii*) if $\pi_2 = 0$, $(1 + L)$ and (*iii*) if $\pi_3 = \pi_4 = 0$, $(1 + L^2)$. The resulting transformed series is checked for an additional unit root at the zero frequency by the Augmented Dickey Fuller (ADF) test, where the standard regression equation is augmented with quarterly dummies. This additional step allows to determine the degree of integration of the inflation rate.

In the second case, the log transformed data is immediately adapted by a seasonal adjustment filter. We use the seasonal adjustment procedures in Eviews 9 (X-13 and TRAMO/SEATS) on the monthly prices and then we also obtain quarterly point-in-time sampled series. In that case the ADF test is first employed to see whether the price series are non-stationary and still a second time to assess whether its first difference (inflation) is integrated of order zero.

After having transformed prices, we apply model selection on both raw and seasonally adjusted inflation series. Since mixed causal-noncausal AR processes are not identified by Gaussian likelihood, the first step in modelling a time series with a potential forward-looking component is to check for signs of noncausality. We first estimate pseudo-causal AR(p) models by OLS and choose the model order $p = r + s$ that adequately captures the autocorrelation in the series (using BIC). Then, diagnostic tests for autocorrelation are performed to see whether additional lags are needed. We restrict ourselves to the case where $p_{\max} = 8$. The null hypothesis of normality is tested on the resulting residuals by means of the Jarque-Bera test. In case this null cannot be rejected, there is no need to consider mixed causal-noncausal models, as the backward- and forward-looking components cannot be distinguished from each other. In case the null is rejected, all MAR(r, s) specification for the selected pseudo-causal order p are considered. The model that maximizes the log-likelihood is chosen to be the final model.

4.1 Results

Table 4 shows the results of the HEGY test on the natural logarithm of raw quarterly HCPI series. Critical values are from Franses and Hobijn (1997). The reported ADF statistic is on the first difference of log prices (i.e., the rate of inflation). Rejections of the null hypotheses at a 5% significance level are indicated by asterisks. Test results indicate that the presence of a zero frequency unit root is rejected for a few countries (Czech Republic, Romania and Turkey), while the presence of seasonal unit roots at annual frequencies is rejected in all cases except for Switzerland (for which there are only 35 observations available). The possibility of prices containing two cycles per year is rejected for almost all countries, except for Lithuania, Slovenia, Spain and again Switzerland. For these countries, we report results of two situations: (*i*) we ignore the seasonal unit roots and force the first difference and (*ii*) we apply the appropriate transformation according to the HEGY test. This means that the transformation $(1 + L)$ is applied to the logarithm of the HCPI for Lithuania, Slovenia and Spain. For Switzerland, $(1 + L)(1 + L^2)$ is the correct transformation.

From the ADF test, we deduce that inflation in the countries Bulgaria, Hungary, Ireland, Latvia, The Netherlands, Poland and Slovenia is not stationary. For the HEGY transformed series, we find similar results for inflation in Lithuania and Slovenia. Hence, model selection is not performed for these series. We want to stress that one should not take the results of the ADF test for granted. It is known that the ADF test has relatively low power in the presence of noncausality (Saikkonen and Sandberg, 2016).

From there onwards, we determine the pseudo-causal model order p by estimating various AR models up to order $p = 8$ including an intercept and quarterly dummies. We find that BIC selects white noise, i.e., AR(0), for 14 of the remaining stationary series. For four series, an AR(p) with $p > 0$ is found, but the normality of the residuals cannot be rejected. This means that we can only perform model selection for mixed causal-noncausal models on two remaining series: inflation in Greece and Iceland. BIC selects the purely noncausal MAR(0,4)

	Tests on log levels			Tests on inflation			
Country	$H_0:$ $\pi_1 = 0$	$H_0:$ $\pi_2 = 0$	$H_0:$ $\pi_3 = \pi_4 = 0$	ADF statistic $H_0:$ unit root	Pseudo model	Jarque-Bera $H_0:$ normality	MAR(r, s)
Austria	-2.82	-4.51*	34.81*	-7.13*	AR(0)	reject	-
Belgium	-3.46	-4.60*	44.48*	-6.97*	AR(0)	not reject	-
Bulgaria	-0.51	-4.69*	12.31*	-2.39	-	-	-
Croatia	-1.40	-5.34*	24.96*	-6.72*	AR(0)	not reject	-
Cyprus	0.01	-4.48*	14.26*	-8.44*	AR(0)	not reject	-
Czech Republic	-3.66*	-3.94*	52.64*	-	-	-	-
Denmark	-1.89	-4.06*	50.24*	-8.06*	AR(0)	not reject	-
Estonia	-3.09	-5.44*	43.98*	-5.41*	AR(7)	not reject	-
Europe (overall)	-2.30	-5.24*	38.89*	-6.58*	AR(0)	not reject	-
Finland	-2.18	-3.39*	6.90*	-7.52*	AR(0)	reject	-
France	-2.38	-5.13*	36.08*	-7.46*	AR(0)	not reject	-
Germany	-2.48	-4.80*	33.67*	-8.11*	AR(0)	not reject	-
Greece	-2.59	-4.24*	27.77*	-3.96*	AR(4)	reject	MAR(0,4)
Hungary	-2.64	-6.60*	18.10*	-2.56	-	-	-
Iceland	-2.20	-5.05*	40.33*	-4.76*	AR(1)	reject	MAR(0,1)
Ireland	-0.44	-3.73*	65.62*	-2.66	-	-	-
Italy	-1.90	-3.48*	70.62*	-3.20*	AR(2)	not reject	-
Latvia	-1.58	-4.15*	52.58*	-2.50	-	-	-
Lithuania	-2.59	-2.70	13.57*	-2.93*	AR(6)	not reject	-
Lithuania (HEGY)	-	-	-	-2.55	-	-	-
Luxembourg	-2.27	-4.29*	34.15*	-8.11*	AR(0)	reject	-
Malta	-2.48	-4.23*	27.26*	-8.87*	AR(0)	not reject	-
Netherlands	-2.14	-3.43*	11.07*	-3.10*	AR(4)	reject	MAR(0,4)
Norway	-2.45	-6.51*	19.85*	-8.45*	AR(2)	not reject	-
Poland	-2.88	-6.19*	25.54*	-2.78	-	-	-
Portugal	-0.43	-3.32*	58.84*	-4.10*	AR(2)	not reject	-
Romania	-3.65*	-7.63*	122.90*	-	-	-	-
Slovakia	-0.73	-2.92*	64.09*	-3.17*	AR(2)	reject	MAR(0,2)
Slovenia	-1.07	-2.52	13.35*	-1.48	-	-	-
Slovenia (HEGY)	-	-	-	-2.56	-	-	-
Spain	1.00	-1.66	14.83*	-3.69*	AR(6)	not reject	-
Spain (HEGY)	-	-	-	-3.70*	AR(4)	reject	MAR(3,1)
Sweden	-1.34	-4.91*	32.04*	-8.29*	AR(0)	reject	-
Switzerland	-1.25	-0.69	4.95	-6.84*	AR(0)	not reject	-
Switzerland (HEGY)	-	-	-	-3.78*	AR(3)	not reject	-
Turkey	-4.07*	-4.66*	34.92*	-	-	-	-
United Kingdom	-1.48	-5.37*	24.87*	-7.13*	AR(0)	not reject	-
c.v. (5%)	-3.39	-2.82	6.55	-2.86		5.99	

Table 4: HEGY test on prices, ADF test and MAR(r, s) identification on quarterly inflation rates (not s.a.)

and MAR(0,1) to be the best respectively. For the HEGY transformed series, we cannot reject the null of normality for the residuals of Switzerland, while an MAR(3,1) is found for Spain.

Table 5 shows the results for seasonally adjusted inflation data. The methods used to seasonally adjust the data are both X-13 and TRAMO/SEATS. This latter adjustment method (see Maravall, 1997), merely used by Eurostat, is based on an unobserved components decomposition but is not free from filters.[6] We find similar results for the two procedures. Note that we do not apply the HEGY test to the seasonally adjusted series, as seasonal effects (and thus also seasonal roots) are assumed to be removed by applying the filters. Hence, it suffices to perform the ADF-test for both the price series and inflation. Since all price series where found to contain a unit root, we only report the ADF-statistics for the rate of inflation. We find that for both adjustment procedures, the null of a unit root cannot be rejected for approximately five series. It is interesting to see that these are not always the same series; i.e., the seasonal adjustment procedure apparently directly affects the stationarity at the zero frequency. Similar to Ghysels et al. (1993), we see that the time series properties of the data are affected and not only at the seasonal frequencies.

When the pseudo-causal model is selected, the white noise specification, i.e., AR(0) with intercept, is less often selected than for the raw data. For TRAMO/SEATS only four series are found to have an AR(0) structure; for X-13 this number equals five. For the remaining pseudo-causal AR(p) models with $p > 0$, we find that the null of normality can be rejected in 14 cases for TRAMO/SEATS and 13 times for X-13 seasonal adjustment. These numbers are in great contrast with the two cases that are found for the raw data application. For TRAMO/SEATS nine series are found to contain at least one noncausal component (six are purely noncausal) and for X-13 this is the case for six series (five are purely noncausal).

[6] In particular, the two-sided, centered, symmetric Wiener-Kolmogorov filter is used to estimate the signal in an observed process y_t. Theoretically, the filter uses an infinite amount of lags and leads, but in practice this is truncated to a large number m, typically representing 3-5 years of data (for more details, see e.g., Maravall, 2006).

	TRAMO/SEATS				X-13			
Country	ADF-statistic H_0 : unit root	Pseudo model	Jarque-Bera H_0 : normality	MAR(r,s)	ADF-statistic H_0 : unit root	Pseudo model	Jarque-Bera H_0 : normality	MAR(r,s)
Austria	-5.97*	AR(2)	reject	MAR(1,1)	-6.74*	AR(1)	reject	MAR(1,0)
Belgium	-5.90*	AR(1)	reject	MAR(1,0)	-6.09*	AR(1)	reject	MAR(1,0)
Bulgaria	-5.57*	AR(4)	not reject	-	-5.83*	AR(4)	not reject	-
Croatia	-7.02*	AR(1)	not reject	-	-6.48*	AR(3)	not reject	-
Cyprus	-9.61*	AR(0)	not reject	-	-9.46*	AR(0)	not reject	-
Czech Republic	-5.22*	AR(6)	not reject	-	-5.24*	AR(6)	not reject	-
Denmark	-6.87*	AR(1)	reject	MAR(0,1)	-6.84*	AR(1)	reject	MAR(0,1)
Estonia	-3.32*	AR(6)	not reject	-	-3.88*	AR(5)	not reject	-
Europe (overall)	-6.03*	AR(2)	reject	MAR(0,2)	-5.90*	AR(1)	reject	MAR(1,0)
Finland	-6.43*	AR(1)	reject	MAR(0,1)	-6.63*	AR(1)	reject	MAR(0,1)
France	-7.02*	AR(1)	not reject	-	-6.69*	AR(1)	not reject	-
Germany	-7.75*	AR(1)	not reject	-	-7.44*	AR(0)	reject	-
Greece	-4.22*	AR(2)	not reject	-	-2.88*	AR(1)	reject	MAR(0,1)
Hungary	-3.81*	AR(3)	not reject	-	-3.30*	AR(1)	not reject	-
Iceland	-5.07*	AR(1)	reject	MAR(1,0)	-2.92*	AR(2)	reject	MAR(0,2)
Ireland	-5.93*	AR(2)	not reject	-	-5.39*	AR(2)	not reject	-
Italy	-4.79*	AR(2)	reject	MAR(0,2)	-2.94*	AR(2)	reject	MAR(2,0)
Latvia	-2.87*	AR(1)	reject	MAR(0,1)	-3.14*	AR(1)	reject	MAR(1,0)
Lithuania	-4.18*	AR(2)	not reject	-	-2.83	-	-	-
Luxembourg	-7.69*	AR(0)	reject	-	-7.24*	AR(0)	reject	-
Malta	-7.69*	AR(0)	reject	-	-7.90*	AR(0)	not reject	-
Netherlands	-3.81*	AR(2)	reject	MAR(1,1)	-2.27	-	-	-
Norway	-9.04*	AR(2)	not reject	-	-9.49*	AR(2)	not reject	-
Poland	-3.29*	AR(2)	not reject	-	-3.03*	AR(2)	not reject	-
Portugal	-3.88*	AR(2)	reject	MAR(1,1)	-3.75*	AR(2)	reject	MAR(0,2)
Romania	-2.43	-	-	-	-1.88	-	-	-
Slovakia	-3.41*	AR(4)	reject	MAR(4,0)	-2.93*	AR(2)	reject	MAR(2,0)
Slovenia	-7.39*	AR(2)	reject	MAR(0,2)	-7.97*	AR(3)	reject	MAR(3,0)
Spain	-5.95*	AR(3)	reject	MAR(0,3)	-5.93*	AR(1)	not reject	-
Sweden	-7.81*	AR(0)	reject	-	-8.34*	AR(0)	not reject	-
Switzerland	-3.67*	AR(1)	reject	MAR(0,1)	-4.71*	AR(1)	reject	MAR(0,1)
Turkey	-1.82	-	-	-	-3.85*	AR(4)	reject	MAR(1,3)
United Kingdom	-6.56*	AR(1)	reject	MAR(1,0)	-6.71*	AR(4)	reject	MAR(4,0)
c.v. (5 %)	-2.86		5.99		-2.86		5.99	

Table 5: Quarterly inflation rates (s.a.) and MAR(r,s) identification

Hence, we find that, in general, larger order pseudo-causal models are selected by BIC when seasonally adjusted data is considered. The different methods affect the time series in such a way that different autoregressive dynamics are detected by BIC, both in the amount of AR parts and whether these are causal or noncausal. An important point to take into account is however that the number of observations for every time series is relatively low (76 at maximum).

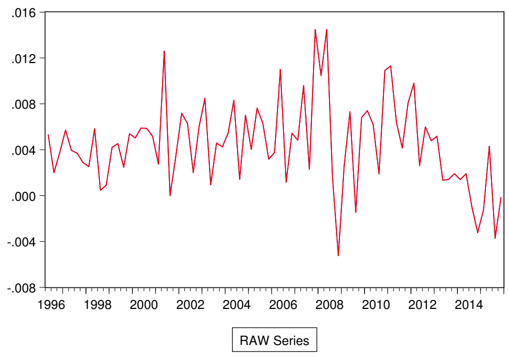
It has to be mentioned that these findings are likely to extend to cases beyond the exercise of seasonal adjustment. As pointed out in Kaiser and Maravall (1999), there are typically two different uses of trends in applied work. In short-term monitoring and seasonal adjustment, trends are defined as the difference between the observed series (y_t) and the sum of the seasonal (S_t) and irregular component (I_t). Hence, these trends often only contain variation of the series within the range of cyclical frequencies. For this reason, they are referred to as short term trend of trend-cycle. In the X-11 procedure, this trend is produced by a Henderson moving average filter (see appendix A).

The second use of trends is in business cycle analysis where long-term trends are of interest. The standard method to estimate cycles is to apply ad-hoc filters like the Hodrick-Prescott (HP) filter to series *that have already been seasonally adjusted*. In a recent paper, De Jong and Sakarya (2015) derive a new representation of the HP filter which highlights that it is a symmetric weighted average similar to the filters considered in this paper. They further state that, in case of a unit root process, the weak dependence of the cyclical component suggests that the unit root is absorbed into the trend component. As the filter (and autocorrelation function) is symmetric, this introduces spurious autocorrelation identically in calendar and reverse time.

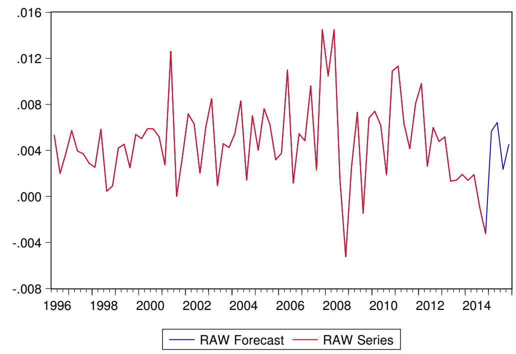
4.2 Forecasting Inflation in Europe

In this subsection, we extend the analysis by examining whether seasonal adjustment filters have a clear impact on forecasting. As an example, we focus on the inflation rate of Europe. In Table 4, an MAR(0,0) with quarterly dummies is found to be the best model for the raw data based on BIC. For the seasonally adjusted data, Table 5 indicates an MAR(0,2) and an MAR(1,0) for inflation adjusted with the TRAMO/SEATS and X-13 procedure respectively.

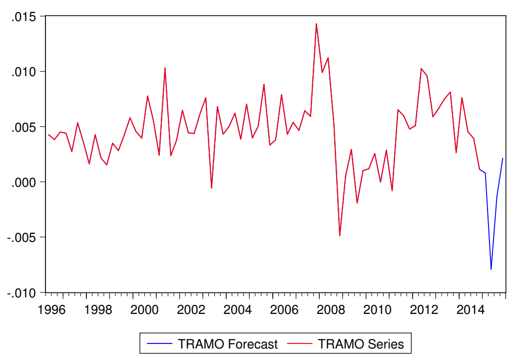
Figure 2 shows the raw realized rate of inflation and forecasts based on the three aforementioned models. Since the best model for the raw data does not possess an autoregressive structure, its forecasts are computed using solely deterministics. Hence, one can forecast the pattern of the quarterly dummies based on a simple OLS regression. For the other models, we find that the null hypothesis of normality can be rejected according to the results in the previous section. For this reason, we use the forecasting method proposed by Lanne et al. (2012),



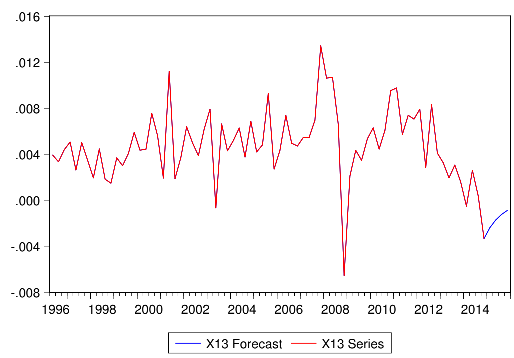
(a) Raw realized inflation (based on HCPI) from 1996Q1 until 2015Q4



(b) Raw inflation (red), forecasts MAR(0,0) with deterministics (blue)



(c) TRAMO/SEATS seasonally adjusted inflation (red), forecasts (blue)



(d) X-13 seasonally adjusted inflation series (red), forecasts (blue)

Figure 2: Realized and forecasted rate of inflation for raw and seasonally adjusted data

which can be applied directly to purely causal, purely noncausal and mixed causal-noncausal autoregressive processes. As discussed in Section 2.3, we assume that the error term follows a t_ν-distribution. From these specifications, we produce four one-step-ahead point forecasts for all seasonally adjusted series. As we are only interested in a forecasted trajectory of values, we restrict ourselves to point forecasts. Alternatively, one could consider density forecast procedures as introduced in papers by Lanne et al.(2012) and Gouriéroux and Jasiak (2015).[7]

[7] In particular, Gouriéroux and Jasiak (2015) introduce a risk measure to assess the probability with which a bubble may burst. This method may be interesting for anticipating a large peak or trough in the rate of inflation.

It can be seen that no forecast is completely accurate in predicting the expected sign of the rate of inflation. Whereas the realized values indicate the presence of inflation in the second quarter and deflation in the remaining quarters, the raw forecast shows inflation in all four time periods considered. It does however mimic the movement of the realized values considerably well, as can be seen in Figures 2a and 2b. The forecasts of TRAMO/SEATS and X-13 seasonally adjusted data cannot directly be compared to the predictions of the raw data, as they are not based on the same time series process. For this reason, we cannot compute forecast performance measures such as the RMSFE. Graphically, it can be observed that the X-13 forecasts have opposite signs from the raw forecasts. The rate of deflation however rapidly becomes smaller over the four quarters, which results in the final forecasted value to come very close to the final realized value. TRAMO/SEATS overestimates both the drop in and the subsequent recovery of the inflation rate. Interestingly, we see that the forecasts differ between raw and seasonally adjusted data (in terms of sign and magnitude), as well as between data based on different seasonal adjustment procedures.

5 Conclusion

In this paper, we investigate the effect of seasonal adjustment on model selection for the inflation rate of 32 European countries and one overall Europe measure. In particular, we are interested whether seasonal adjustment may spuriously affect the noncausality found in different time series. Since raw data are directly available, we can compare model selection by BIC where one (*i*) deterministically removes seasonality or (*ii*) applies a predefined seasonal adjustment filter. We find that almost half of the series is found to be white noise in the first case, while this number is much lower in the second case. Besides, pseudo-causal models of larger order are selected in the second instance, which makes it more worthwhile to investigate the presence of noncausality (which is confirmed in approximately half of the cases). This is exactly in line with simulation results presented earlier in this paper.

As such, it seems valid to argue that model selection for mixed causal-noncausal models is heavily affected by the seasonal adjustment method performed. We do not claim that one method is better than the other, as only removing deterministic seasonality might be inappropriate when stochastic seasonality is present. We do however show, by simulations, that performing the famous X-11 filter on time series can create spurious autoregressive dynamics (even when the series is simply a white noise). We find that these dynamics can be both causal and noncausal.

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Appendix A - X-11 filter

The derivation of the X-11 filter, denoted $v_{X11}(L)$, typically starts with the assumption that the observed data y_t is a function of different components:

$$y_t = f(TC_t, S_t, TD_t, H_t, I_t),$$

where TC_t is the trend-cycle, S_t the seasonal, TD_t the trading day, H_t the holiday and I_t the irregular component. The function f can take an additive or multiplicative form. The TD_t and H_t parts are often removed deterministically (e.g., by use of dummies). The X-11 filter now originates from the combination of sequentially applying 4 different filters to the data aiming to isolate the remaining components. The four main steps can be summarized as follows. Firstly, a centered five-term MA is subtracted from the original series (i.e., the trend-cycle is subtracted from the original series) using the filter

$$SQ(L) = -0.125L^2 - 0.250L + 0.750 - 0.250L^{-1} - 0.125L^{-2}.$$

The series obtained is a first estimate of the seasonal plus noise ($S_t + I_t$) part of the series. Secondly, in order to obtain an estimate of the purely seasonal (S_t) part, the following filter is applied:

$$M_1(L) = \frac{1}{9}(L^4 + 1 + L^{-4}).$$

For the seasonal components to sum to unity, the $SQ(L)$ filter is applied again. The resulting series accounts for a first estimate of the seasonal component. Thirdly, this estimate is subtracted from the original series (i.e, we obtain a first estimate for y_t^{SA}) after which a Henderson moving average is applied to obtain a second estimate of the trend cycle component:

$$HQ(L) = -0.073L^2 + 0.294L + 0.558 + 0.294L^{-1} - 0.073L^{-2}.$$

The result is subtracted from the original series to obtain a second estimate of the seasonal plus noise part. Fourthly, a final estimate of the seasonal component is obtained by using the filter

$$M_2(L) = \frac{1}{15}(L^4 + 1 + L^{-4})(L^8 + L^4 + 1 + L^{-4} + L^{-8})$$

Again we want the seasonal components to sum up to unity, so the $SQ(L)$ filter is applied once more. After taking these steps, the $v_{X11}(L)$ filter emerges. A more detailed explanation can be found in Ghysels and Perron (1993) or Laroque (1997).

Appendix B - Graphs

This appendix contains the graphs of the inflation rate of the 32 countries and one overall Europe measure used in this paper. Figures 3 and 4 are the rates of inflation computed according to the raw prices series, while Figures 5 and 6 are computed using TRAMO/SEATS seasonally adjusted price series. Figure 7 shows the raw and the two seasonally adjusted (TRAMO/SEATS and X-13) inflation rates for Europe.

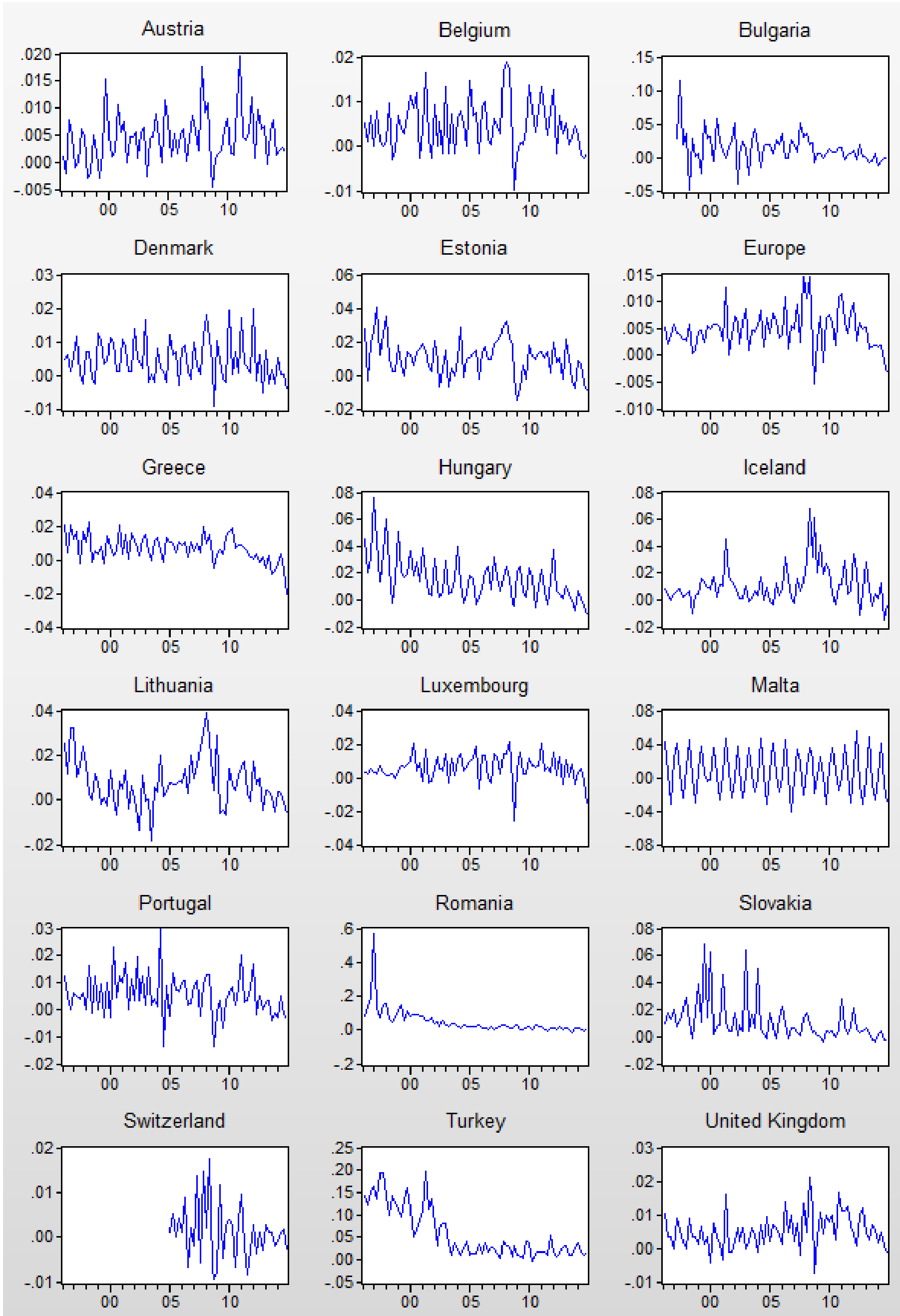

Figure 3: Raw inflation rates for first set of countries

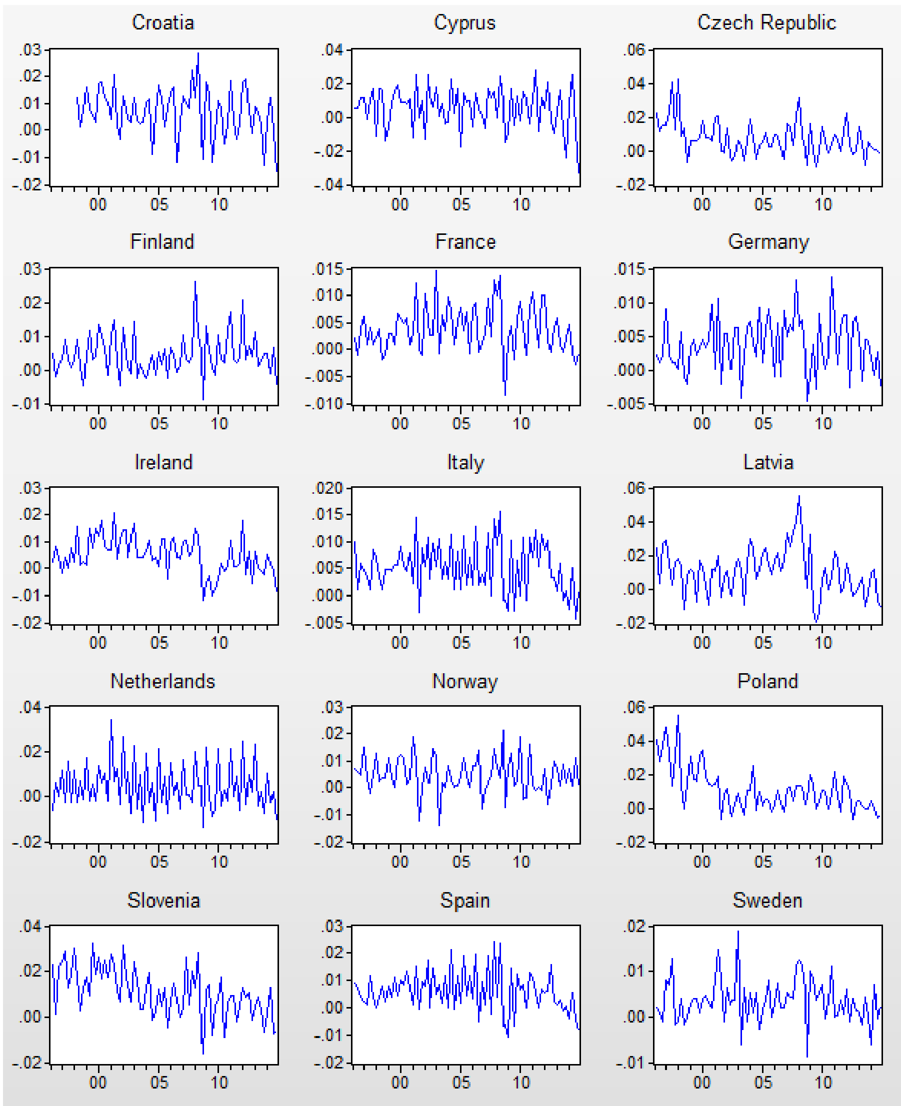

Figure 4: Raw inflation rates for second set of countries

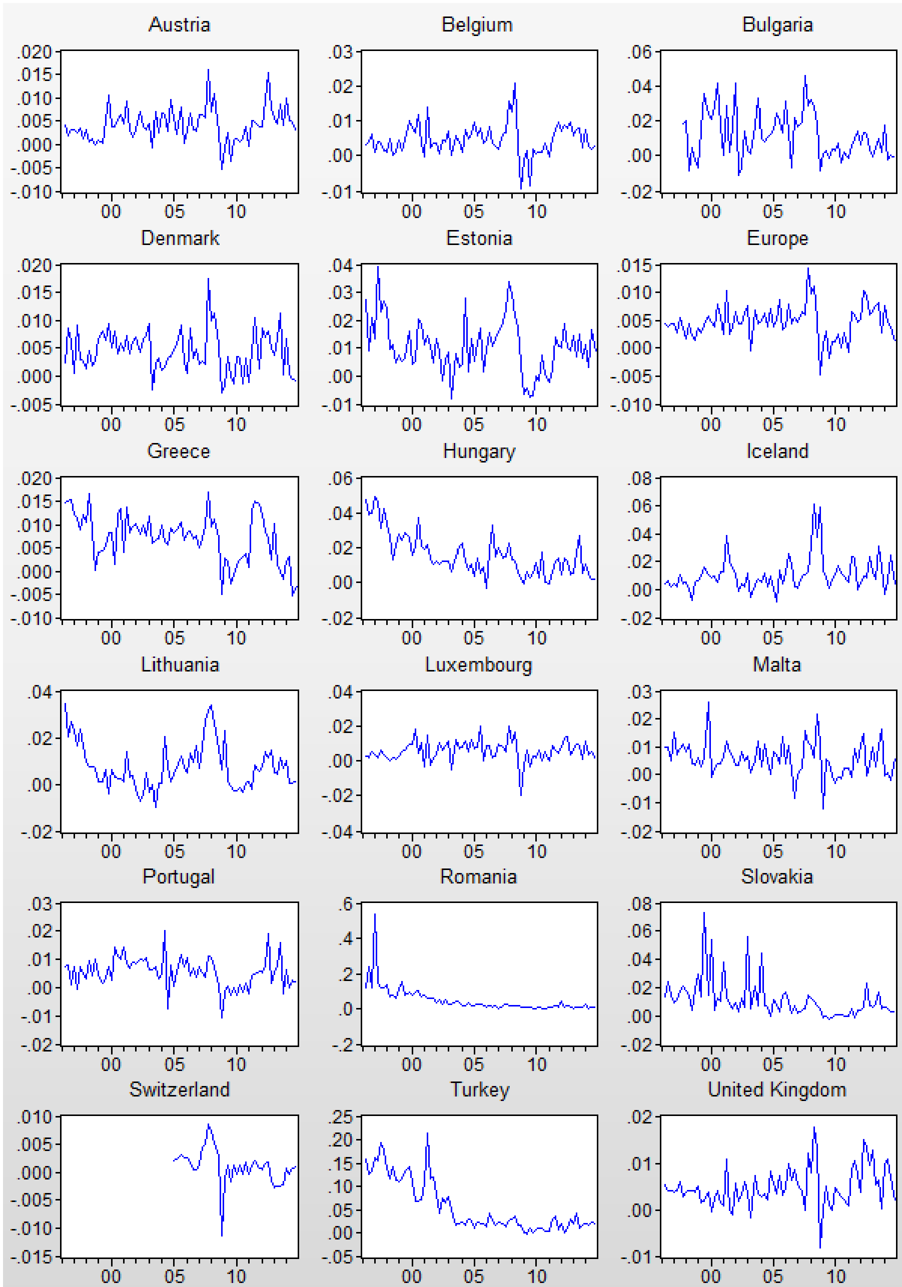

Figure 5: Seasonally adjusted inflation rates for first set of countries

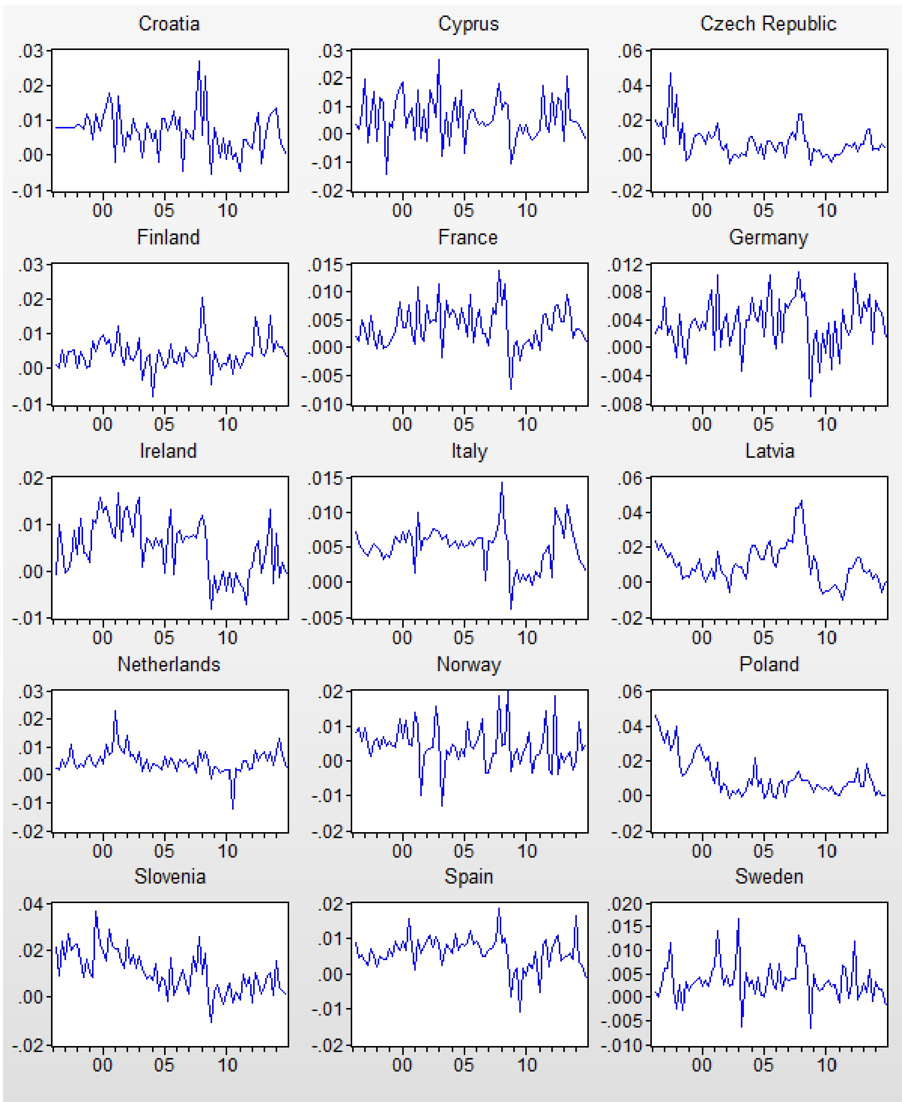

Figure 6: Seasonally adjusted inflation rates for second set of countries

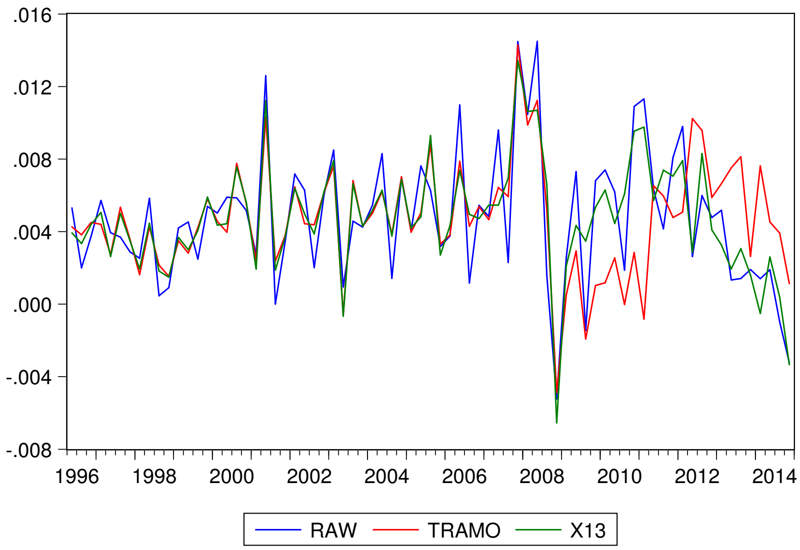

Figure 7: Raw and seasonally adjusted (TRAMO/SEATS and X-13) rate of inflation for Europe