Monitoring Parameter Constancy with Endogenous Regressors

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Abstract

This paper proposes monitoring tests for parameter change in linear regression models with endogenous regressors. We consider a CUSUM-type test based on the instrumental variable (IV) estimation, as the IV method is standard for models with endogenous regressors. In addition, we propose a test based on the residuals from the least squares (LS) estimation. We show that for a given boundary function, both tests have the same limiting distribution under the null hypothesis, whereas their powers are different. In particular, when a structural change occurs early in a monitoring period, the test based on the LS method tends to detect it more rapidly than that based on the IV method. We apply our methods to investigate the Japanese Phillips curve and show that the LS based test performs well to detect a change in 2007, while neither test finds evidence of a change after 2013.

JEL classification: C12, C22, C26

Key words: structural change, CUSUM test, instrumental variable, Phillips curve

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1. Introduction

This paper investigates monitoring tests for structural change in models with endogenous regressors. Although tests for structural change have been an important issue in the time series literature, with many tests for structural change proposed, there is interest in monitoring parameter stability every time we update data with the stable in-sample or training period. For example, against the background of the Bank of Japan introducing its 2% inflation target in January 2013, we would like to check whether the inflation rate is really starting to increase. In this case, as pointed out by Chu, Stinchcombe, and While (1996), it is well known that the repeated use of in-sample structural change tests, which are sometimes called retrospective tests, is invalid because the size of tests cannot be controlled.

In the econometric literature, Chu, Stinchcombe, and White (1996) considered monitoring tests in linear regression models using the CUSUM and fluctuation tests, and Leisch, Hornik, and Kuan (2000) further developed tests using recursive and moving estimates. In general, the performance of these tests depends not only on the test statistics themselves but also on the boundary functions, and several versions of the tests and boundary functions have been investigated by Zeileis (2005) and Zeileis, Leisch, Kleiber, and Hornik (2005), among others. Horváth, Hušková, Kokoszka, and Steinebach (2004, hereafter HHKS) considered CUSUM monitoring tests with a boundary function depending on some parameter $\gamma \in [0, 1/2)$. They found by simulations that the test with smaller γ is suitable for the break that occurs relatively long after the monitoring period, while the test with larger γ is good at detecting early change. This property is confirmed theoretically by deriving the limiting distribution of the stopping time in Aue and Horváth (2004) and Aue, Horváth, and Reimherr (2009). Tests with $\gamma = 1/2$ were also considered by Horváth, Kokoszka, and Steinebach (2007), Aue, Horváth, Kokoszka, and Steinebach (2008), and Aue and Kühn (2008). Monitoring tests have further been developed for models other than linear regressions, such as the autoregressive models of Carsoule and Franses (2003) and Lee, Lee, and Na (2009), GARCH models or GARCH errors of Berkes, Gombay, Horváth, and Kokoszka (2004) and Aue, Horváth, Hušková, and Kokoszka (2006), generalized linear models of Xia, Guo, and Zhao (2009), linear models with endogenous regressors of Xia, Guo, and Zhao (2011), change in autocorrelation functions of Na, Lee, and Lee (2011), multivariate models of Groen, Kapetanios, and Price (2013), dependent functional linear models of Aue, Hörmann, Horváth, and Hušková (2014), and causal time series of Bardet and Kengne (2014).

All the research above, except for Xia, Guo, and Zhao (2011), is based on the assumption that regressors are exogenous. However, econometric models include endogenous regressors in many empirical analyses, and therefore we need to develop monitoring tests for such models. Although Xia, Guo, and Zhao (2009) considered simultaneous equation models and proposed the weighted CUSUM-type monitoring test based on the GMM estimation, it is not necessarily easy to find the appropriate instrumental variables (IVs), which prevents the monitoring scheme from working in practice.

In this paper, we consider linear regression models with endogenous regressors and investigate CUSUM-based monitoring tests. As a benchmark, we first consider the test with the IV estimation. In addition, we propose a monitoring test with the least squares (LS) method regardless of endogenous regressors, as considered by Perron and Yamamoto (2015). The key feature of the LS estimation is that although the LS estimator of the coefficient is biased, it consistently estimates some biased parameters; thus, the forecast errors using the LS estimator behave like a zero-mean process before the break. In addition, because the level of the forecast errors changes after the break, the structural break can be detected even using the LS estimator. The advantage of using the LS method is that we do not have to find appropriate IVs. Moreover, the estimation residuals based on the LS method have smaller variance than those based on the IV method and thus we can expect the monitoring test with the LS method to be more powerful and able to detect the break earlier. In fact, this property is theoretically confirmed in Section 3. By using monitoring tests, we investigate the Japanese Phillips curve and find that the CUSUM-based test with the LS method is good at detecting a structural change. However, we find no evidence of a change after the introduction of the 2% inflation target by the Bank of Japan.

The rest of the paper is organized as follows. The model and assumptions are introduced in Section 2. The CUSUM-based monitoring tests with the IV and LS methods are investigated in Section 3. We show that they have the same limiting distribution as derived in HHKS.

In addition, we derive the limiting distributions of the stopping times and show that the delay time based on the LS estimation is shorter than that based on the IV method. The finite sample property is investigated via Monte Carlo simulations in Section 4. The tests developed in this paper are implemented to the Japanese Phillips curve in Section 5. Section 6 provides concluding remarks.

2. Model and Assumptions

Let us consider the following linear model:

$$y_t = x_t'\beta_t + u_t \qquad \text{for } t = 1, 2, \cdots, m, m+1, \cdots \tag{1}$$

where y_t and u_t are scalar and $x_t := [x_{1t}, x_{2t}, \cdots, x_{pt}]'$ is a p-dimensional regressor with $x_{1t} = 1$, meaning that the first element of the regressors is constant. We suppose that the parameter $\beta_t := [\beta_{1t}, \beta_{2t}, \cdots, \beta_{pt}]'$ is constant for $t = 1, \cdots, m$ meaning that $\beta_t = \beta_0 := [\beta_{10}, \beta_{20}, \cdots, \beta_{p0}]'$ for $t = 1, \cdots, m$. We want to test for stability in β_t for $t = m+1, m+2, \cdots$ every time we update data. Then, the testing problem is

$$H_0: \ \beta_t = \beta_0 \quad \forall t \geq m+1 \quad \text{vs.} \quad H_1: \begin{cases} \beta_t = \beta_0 & : \ t = m+1, \cdots, m+k^*-1 \\ \beta_t = \beta_* & : \ t = m+k^*, m+k^*+1, \cdots \end{cases} \tag{2}$$

for some $k^* \geq 1$, where $\beta_* \neq \beta_0$. Note that the sample period for $t = 1, \cdots, m$ is sometimes called a training period or in-sample period.

Since x_t is assumed to be stationary in the following assumption, model (1) can be expressed by using the demeaned regressor such that

$$y_t = \tilde{x}_t'\tilde{\beta}_t + u_t, \tag{3}$$

where $\tilde{x}_t := [1, \tilde{x}_{2t}, \cdots, \tilde{x}_{pt}]'$ with $\tilde{x}_{jt} := x_{jt} - \mu_{xj}$ and $\mu_{xj} := E[x_{jt}]$ for $j = 2, \cdots, p$ and $\tilde{\beta}_t := [\tilde{\beta}_{1t}, \beta_{2t}, \cdots, \beta_{pt}]'$ with $\tilde{\beta}_{1t} := \beta_{1t} + \mu_{x2}\beta_{2t} + \cdots + \mu_{xp}\beta_{pt}$. By using model (3), $\tilde{\beta}_t = \tilde{\beta}_0$ under the null hypothesis, whereas it changes to $\tilde{\beta}_*$ at some $k^* \geq 1$ under the alternative, where $\tilde{\beta}_0$ and $\tilde{\beta}_*$ are defined following the definition of $\tilde{\beta}_t$ with $\tilde{\beta}_{10} := \beta_{10} + \mu_{x2}\beta_{20} + \cdots + \mu_{xp}\beta_{p0}$ and $\tilde{\beta}_{1*} := \beta_{1*} + \mu_{x2}\beta_{2*} + \cdots + \mu_{xp}\beta_{p*}$, respectively. In model (3), we can see that $\tilde{\beta}_0 - \tilde{\beta}_* = [\tilde{\beta}_{10} - \tilde{\beta}_{1*}, 0, \cdots, 0]'$ and as shown in a later section, the power of the test increases as $\delta_\beta := \tilde{\beta}_{10} - \tilde{\beta}_{1*}$ rises in absolute values.

For model (1) or (3), we make the following common assumption:

Assumption CA *(a) $E[u_t] = 0$ and the following relations hold for some $\nu > 2$:*

$$\sup_{r \geq 1/m} \frac{1}{(mr)^{1/\nu}} \left| \sum_{t=m+1}^{m+mr} u_t - \sigma_u W_{1,m}(mr) \right| = O_p(1), \tag{4}$$

$$\sum_{t=1}^{m} u_t - \sigma_u W_{2,m}(m) = o_p(m^{1/\nu}), \tag{5}$$

where $\{W_{1,m}(r)\}$ and $\{W_{2,m}(r)\}$ are sequences of Brownian motions and are independent.
(b) $x_{2t}, \cdots, x_{pt}$ are stationary with $\Sigma_{xx} := E[\tilde{x}_t \tilde{x}_t'] > 0$, $E[|\tilde{x}_{jt}|^{\mu}] < \infty$ for some $\mu > 2$ $(j = 2, \cdots, p)$ and $\left| \frac{1}{m} \sum_{t=1}^{m} \tilde{x}_t \tilde{x}_t' - \Sigma_{xx} \right| = O(m^{-\tau})$ a.s.
(c) $\sigma_{xu} := E[\tilde{x}_t u_t]$ is not necessarily equal to zero.

The conditions in which (4) and (5) hold are discussed in Section 2 of Aue and Horváth (2004). According to their examples, (4) and (5) hold for not only an i.i.d. sequence but also a dependent sequence with some regularity conditions. Assumption CA(b) excludes nonstationary regressors such as a linear trend and a unit root process. The condition on the second moment is the same as supposed in HHKS. Assumption CA(c) implies that x_t may be endogenous, meaning that the LS estimator of β would be biased.

3. Monitoring Tests

3.1. CUSUM test with the IV method

We first consider a CUSUM-type test with residuals obtained by the IV estimation, because the regressor x_t may be correlated with the error term u_t. Although Xia, Guo, and Zhao (2011) proposed monitoring tests with endogenous regressors using the weighted CUSUM from the GMM estimation with the monitoring period proportional to m, we focus on the simple unweighted CUSUM from the IV estimation.

Let $z_t := [z_{1t}, z_{2t}, \cdots, z_{qt}]'$ be a q-dimensional vector $(q \geq p)$ with $z_{1t} = 1$. Similar to x_t, we also define the demeaned version of z_t as $\tilde{z}_t := [1, \tilde{z}_{2t}, \cdots, \tilde{z}_{qt}]'$ with $\tilde{z}_{jt} := z_{jt} - \mu_{zj}$ and $\mu_{zj} := E[z_{jt}]$ for $j = 2, \cdots, q$. The conditions that the IVs must satisfy are standard and given in the following assumption:

Assumption IV *(a)* $z_{2t}, \cdots, z_{qt}$ *are stationary with* $\Sigma_{zz} := E[\tilde{z}_t \tilde{z}_t'] > 0$, $E[|\tilde{z}_{jt}|^\mu] < \infty$ *for some* $\mu > 2$ $(j = 2, \cdots, q)$ *and* $\frac{1}{\sqrt{m}} \sum_{t=1}^m (\tilde{z}_t \tilde{z}_t' - \Sigma_{zz}) = O_p(1)$.
(b) $E[\tilde{z}_t \tilde{x}_t'] \xrightarrow{p} \Sigma_{zx}$ *with* $rank(\Sigma_{zx}) = p$.
(c) $\frac{1}{\sqrt{m}} \sum_{t=1}^m \tilde{z}_t u_t = O_p(1)$.

Let $\hat{\beta}_{IV}$ be the IV estimator in the training period defined by

$$\hat{\beta}_{IV} := \left[\left(\sum_{t=1}^m x_t z_t' \right) \left(\sum_{t=1}^m z_t z_t' \right)^{-1} \left(\sum_{t=1}^m z_t x_t \right) \right]^{-1} \left(\sum_{t=1}^m x_t z_t' \right) \left(\sum_{t=1}^m z_t z_t' \right)^{-1} \left(\sum_{t=1}^m z_t y_t \right)$$

and the detection statistic (detector) be defined by

$$\Gamma_{m+k}^{IV} := \left| \frac{1}{\hat{\sigma}_{IV}} \sum_{t=m+1}^{m+k} \left(y_t - x_t' \hat{\beta}_{IV} \right) \right|$$

for $k = 1, 2, \cdots$, where $\hat{\sigma}_{IV}^2$ is the consistent estimator of σ_u^2 based on the in-sample estimation residuals. We reject the null hypothesis if Γ_{m+k}^{IV} crosses a boundary function for some $k \geq 1$; otherwise, H_0 is not rejected. Following HHKS, we consider the boundary function given by

$$g(m, k) := d\sqrt{m} \left(\frac{m+k}{m} \right) \left(\frac{k}{m+k} \right)^\gamma, \tag{6}$$

where $0 \leq \gamma < \min(\tau, 1/2)$. The constant d is determined by the given significance level α meaning that

$$\lim_{m \to \infty} P\left(\tau^{IV}(m) < \infty \right) = \alpha \quad \text{under } H_0,$$

where $\tau^{IV}(m)$ is the stopping time defined by

$$\tau^{IV}(m) := \inf \left\{ k \geq 1 : \Gamma_{m+k}^{IV} \geq g(m, k) \right\}$$

and $\tau(m)^{IV} := \infty$ if $\Gamma_{m+k}^{IV} < g(m, k)$ for all $k \geq 1$. As a result, we reject the hypothesis of stability if $\Gamma_{m+k}^{IV} \geq g(m, k)$ for some $k \geq 1$.

The following theorem describes the asymptotic behavior of the IV-based detector.

Theorem 1 *Suppose Assumptions CA and IV hold.*
(a) Under H_0,

$$\lim_{m \to \infty} P\left(\sup_{1 \leq k < \infty} \Gamma_{m+k}^{IV} \leq g(m, k) \right) = P\left(\sup_{0 \leq r \leq 1} |W(r)|/r^\gamma \leq d \right),$$

where $\{W(r)\}$ *is a Brownian motion.*

(b) Under H_1 *with the first element of* $\tilde{\beta}_0 - \tilde{\beta}_*$ *not equal to* 0*,*

$$\sup_{1 \le k < \infty} \Gamma^{IV}_{m+k}/g(m,k) \to \infty.$$

The null limiting distribution is the same as that derived in HHKS and the values of d are given by Table 1 in HHKS for the given values of α and γ. The condition for the consistency of the test is also the same as the standard CUSUM-type test.

We next derive the limiting distribution of the stopping time. Typically, while we cannot necessarily reject the null hypothesis as soon as a structural change occurs, we can detect it with lags after the break date. Since it would be more desirable for this delay time to be short, it is important to investigate the asymptotic property of the stopping time. As in the existing literature, we focus on the case where a structural change occurs shortly after the training period in order to derive the limiting distribution.

Assumption ST *(a) The change point* k^* *satisfies*

$$k^* = O(m^\theta) \quad \textit{for some} \quad 0 \le \theta \le \frac{1-2\gamma}{4(1-\gamma)}.$$

(b) $c_1 \le |\delta_\beta| \le c_2$ *for some* $0 < c_1 < c_2 < \infty$*.*

Assumption ST satisfies the conditions given by Aue, Horváth, and Reimherr (2009). Assumption ST(a) implies that we need to focus on the early structural change to derive the limiting distribution of the stopping time. Assumption ST(b) may be slightly relaxed such that $\beta_0 - \beta_* = O(\log(m))$. Under this assumption, we can obtain the same result as in the case with exogenous regressors given in Aue, Horváth, and Reimherr (2009).

Theorem 2 *Suppose that Assumptions CA, IV, and ST hold. If* $\mu > 8(1-\gamma)/(1-2\gamma)$*, then,*

$$\lim_{m \to \infty} P\left(\tau^{IV}(m) \le a^{IV}_m + b^{IV}_m z\right) = \Phi(z)$$

for all real z*, where* $\Phi(\cdot)$ *is the cumulative distribution function of a standard normal distribution and*

$$a^{IV}_m = \left(c^{IV}_m - \frac{1}{c^{IV}_m|\delta_\beta|}\sum_{t=m+k^*}^{m+c_m}(x_t-\mu_x)'(\beta_*-\beta_0)\right)^{1/(1-\gamma)},$$

$$b_m^{IV} = \frac{\sqrt{c_m^{IV}}\sigma_u}{(1-\gamma)|\delta_\beta|} \quad and \quad c_m^{IV} = \left(\frac{\sigma_u d\, m^{1/2-\gamma}}{|\delta_\beta|}\right)^{1/(1-\gamma)}.$$

As proven in Aue, Horváth, and Reimherr (2009), c_m^{IV} is a dominating term and thus $a_m^{IV}/c_m^{IV} \xrightarrow{p} 1$, which implies $\tau^{IV}(m)/c_m^{IV} \xrightarrow{p} 1$. As a result, for a given value of γ, the delay time would be shorter for a smaller variance of the error term, σ_u^2, and/or a larger magnitude of the break, $|\delta_\beta|$.

3.2. CUSUM test with the LS method

Although it is common practice to rely on the IV method for models with endogenous regressors, Perron and Yamamoto (2015) investigated retrospective tests for structural change and the break point estimator based on the LS method. They found that the LS method leads to a more desirable result than the IV method except in special cases. Although the LS estimator of the coefficient is biased, it consistently estimates some biased parameters, which remain stable before the break, while it changes to another biased parameter after the break, meaning that the retrospective tests based on the LS estimator still work well even though the estimator is biased. In the case of the monitoring test, we estimate the coefficient only once in the training period; however, as we see below, the forecast errors using the LS estimator behave like a zero-mean process before the break, while the level of the forecast errors changes after the break. As a result, the future structural break is detectable even using the forecast errors based on the LS estimator.

Let us express demeaned model (3) as

$$\begin{aligned} y_t &= \tilde{x}_t'\tilde{\beta}_t + u_t \\ &= \tilde{x}_t'\tilde{\beta}_t^+ + u_t^+, \end{aligned} \tag{7}$$

where $\tilde{\beta}_t^+ := \tilde{\beta}_t + \Sigma_{xx}^{-1}\sigma_{xu}$ and $u_t^+ := u_t - \tilde{x}_t'\Sigma_{xx}^{-1}\sigma_{xu}$. Note that the regressor $\tilde{x}_t$ is uncorrelated with the error term u_t^+. For model (7), the coefficient before and after the break becomes

$$\tilde{\beta}_0^+ = \tilde{\beta}_0 + \Sigma_{xx}^{-1}\sigma_{xu} \quad \text{and} \quad \tilde{\beta}_*^+ = \tilde{\beta}_* + \Sigma_{xx}^{-1}\sigma_{xu}. \tag{8}$$

Assumption LS *(a) The following relations hold for some* $\nu > 2$*:*

$$\sup_{r \geq 1/m} \frac{1}{(mr)^{1/\nu}} \left| \sum_{t=m+1}^{m+mr} u_t^+ - \sigma_u^+ W_{1,m}^+(mr) \right| = O_p(1),$$

$$\sum_{t=1}^{m} u_t^+ - \sigma_u^+ W_{2,m}^+(m) = o_p(m^{1/\nu}),$$

where $\{W_{2,m}^+(r)\}$ *are sequences of Brownian motions and they are independent and* $\sigma_u^{+2} = \sigma_u^2 - \sigma_{xu}' \Sigma_{xx}^{-1} \sigma_{xu}$.

(b) $\frac{1}{\sqrt{m}} \sum_{t=1}^{m} \tilde{x}_t u_t^+ = O_p(1)$.

Note that Assumptions LS(a) and (b) correspond to Assumptions CA(a) and IV(c).

Let $\hat{\beta}_{LS}^+$ be the LS estimator in the training period and the CUSUM detection statistic be defined by

$$\Gamma_{m+k}^{LS} := \left| \frac{1}{\hat{\sigma}_{LS}} \sum_{t=m+1}^{m+k} \left(y_t - x_t' \hat{\beta}_{LS}^+ \right) \right|$$

for $k = 1, 2, \cdots$, where $\hat{\sigma}_{LS}^2$ is the consistent estimator of σ_u^{+2} in the training period. We consider the same boundary function (6) again. In this case, the stopping time is defined by

$$\tau^{LS}(m) := \inf \left\{ k \geq 1 : \Gamma_{m+k}^{LS} \geq g(m,k) \right\}$$

and $\tau(m)^{LS} := \infty$ if $\Gamma_{m+k}^{LS} < g(m,k)$ for all $k \geq 1$ as in the IV case.

Theorem 3 *Suppose Assumptions CA and LS hold. Then, we obtain the same result as Theorem 1 with* Γ_{m+k}^{IV} *replaced by* Γ_{m+K}^{LS}.

We cannot find any theoretical advantage of the LS detector over the IV detector from Theorems 1 and 3. However, if we focus on the stopping time, we can find an important difference.

Theorem 4 *Suppose that Assumptions CA, LS, and ST hold. If* $\mu > 8(1-\gamma)/(1-2\gamma)$*, then,*

$$\lim_{m \to \infty} P\left(\tau^{LS}(m) \leq a_m^{LS} + b_m^{LS} z \right) = \Phi(z)$$

for all real z, where

$$a_m^{LS} = \left(c_m^{LS} - \frac{1}{c_m^{LS}|\delta_\beta|} \sum_{t=m+k^*}^{m+c_m} (x_t - \mu_x)'(\tilde{\beta}_*^+ - \tilde{\beta}_0^+) \right)^{1/(1-\gamma)},$$

$$b_m^{LS} = \frac{\sqrt{c_m^{IV}}\sigma_u^+}{(1-\gamma)|\delta_\beta|} \quad \text{and} \quad c_m^{LS} = \left(\frac{\sigma_u^+ d\, m^{1/2-\gamma}}{|\delta_\beta|} \right)^{1/(1-\gamma)}.$$

Note that there seems to be two differences between Theorems 2 and 4: the second terms in a_m^{IV} and a_m^{LS} and variances σ_u^2 and σ_u^{2+}. However, we can easily see from definition (8) that $\beta_*^+ - \beta_0^+ = \beta_* - \beta_0$ and then the difference is only in the variances. Since $\sigma_u^2 - \sigma_u^{+2} = \sigma_{xu}' \Sigma_{xx}^{-1} \sigma_{xu} \geq 0$, we can see that the delay time of the LS detector would be shorter than that of the IV detector. We formally state this result in the following corollary:

Corollary 1 *Suppose that Assumptions CA, IV, LS, and ST hold. If* $\mu > 8(1-\gamma)/(1-2\gamma)$, *then,*

$$\left(\frac{\tau^{LS}(m)}{\tau^{IV}(m)} \right)^{2(1-\gamma)} \xrightarrow{p} \frac{\sigma_u^{+2}}{\sigma_u^2} \leq 1.$$

Note that the equality holds only if $\sigma_{xu} = 0$, the case where x_t is exogenous, because Σ_{xx} is positive definite. Therefore, this corollary implies that the LS detector is always more desirable irrespective of whether x_t is endogenous or exogenous.

3.3. Change in the correlation

Although the monitoring test based on the LS method is asymptotically superior to the test based on the IV method if only the coefficient β sustains structural change, another issue is related to the LS estimation in models with endogenous regressors; the LS method is affected by a change in the correlation between x_t and u_t. In fact, Perron and Yamamoto (2015) showed that the distribution of the break point estimator is affected by this change. In our case, the LS estimator $\hat{\beta}_{LS}$ is obtained in the training period and we do not estimate β in the out-of-sample period, meaning that the change in σ_{xu} does not affect the test through the estimation of β. However, it does affect the test through the change in $\sigma_u^{+2} = \sigma_u^2 - \sigma_{xu}' \Sigma_{xx}^{-1} \sigma_{xu}$.

Let us first consider the effect of the change in σ_{xu} on the LS monitoring test under the null hypothesis. If σ_{xu} changes to σ_{xu}^* during the monitoring period, while β_0 is still stable, then the appropriate scale parameter in Γ^{LS}_{m+k} becomes $\sigma^{*2}_u := \sigma^2_u - \sigma^{*\prime}_{xu}\Sigma^{-1}_{xx}\sigma^*_{xu}$ (see (7)), and the LS detector must be divided by σ^*_u after the break in σ_{xu}. This implies that the LS monitoring test would become more conservative if $\sigma^{*2}_u/\sigma^{+2}_u < 1$, while it would be more liberal for $\sigma^{*2}_u/\sigma^{+2}_u > 1$. For the same reason, the power of the LS monitoring test decreases if $\sigma^{*2}_u/\sigma^{+2}_u < 1$ compared with the case of no change in σ_{xu} but increases in the case of $\sigma^{*2}_u/\sigma^{+2}_u > 1$.

We next consider the effect on the stopping time under the alternative. Suppose that β_0 and σ_{xu} change such that

$$\beta_0 \to \beta_* \qquad \text{and} \qquad \sigma_{xu} \to \sigma^*_{xu} \tag{9}$$

at $k = k^*$. In this case, the coefficient in transformed model (7) changes from $\tilde{\beta}^+_0$ to $\tilde{\beta}^\dagger_* := \tilde{\beta}^+_* + \Sigma^{-1}_{xx}\sigma^*_{xu}$. Then, following Aue, Horváth, and Reimherr (2009), we have the same result for the stopping time $\tau^{*LS}(m)$ as in Theorem 4 with a^{LS}_m, b^{LS}_m and c^{LS}_m replaced by

$$a^{*LS}_m = \left(c^{*LS}_m - \frac{1}{c^{*LS}_m|\delta^*_\beta|} \sum_{t=m+k^*}^{m+c_m} (x_t - \mu_x)'(\tilde{\beta}^\dagger_* - \tilde{\beta}^+_0) \right)^{1/(1-\gamma)},$$

$$b^{*LS}_m = \frac{\sqrt{c^{*IV}_m}\sigma^*_u}{(1-\gamma)|\delta^*_\beta|} \quad \text{and} \quad c^{*LS}_m = \left(\frac{\sigma^+_u d\, m^{1/2-\gamma}}{|\delta^*_\beta|} \right)^{1/(1-\gamma)}.$$

Note that not σ^*_u but σ^+_u, the probability limit of $\hat{\sigma}^+_u$, appears in c^{*LS}_m. In this case, we have the following corollary:

Corollary 2 *Suppose that Assumptions CA, LS, and ST hold. If both β_0 and σ_{xu} sustain a structural change given by* (9)*, we have*

$$\frac{\tau^{*LS}(m)}{\tau^{LS}(m)} \xrightarrow{p} 1. \tag{10}$$

This corollary implies that even if the correlation between the endogenous regressors and errors changes, the stopping time is asymptotically equivalent to the case of the change only in β_0.

As a whole, the effect of the change in the correlation affects the size and power of the LS detector only though the change in the variance of the (modified) error term. Although this seems to be a disadvantage of the LS method, we should note that the IV detector is also affected if we allow for the change in σ_u^2. If we allow for both changes in σ_u^2 and σ_{xu}, the effect on the detectors depends on the directions of the changes in σ_u^2 and σ_u^{+2} and we cannot say that one of the detectors dominates the other.

The effect of the change in the correlation between x_t and u_t in finite samples is investigated in the next section.

3.4. Monitoring tests with a bounded monitoring period

In practice, the monitoring period is often restricted as opposed to being unbounded. For example, the Bank of Japan's 2% inflation target of 2013 was expected to be achieved within a few years. In this case, we would like to monitor the inflation rate; however, the monitoring period would be at most three years. In such a case, the monitoring tests considered in the previous section would be conservative under the null hypothesis, while they would lose power under the alternative because the boundary function (the adjustment parameter d) is chosen in the case of $1 \leq k < \infty$.

To control the empirical sizes of the tests, we suppose that the monitoring period is restricted such that $1 \leq k \leq \kappa m$, where κ is a positive integer value. That is, the monitoring period is restricted to be proportional to the training period. In this case, it is straightforward to see that

$$\lim_{m\to\infty} P\left(\sup_{1\leq k<\kappa m} \Gamma_{m+k}^{IV} \leq g(m,k)\right) = P\left(\sup_{0\leq r\leq \kappa/(1+\kappa)} |W(r)|/r^{\gamma} \leq d\right) = \alpha$$

under the null hypothesis with the significance level α. Thus, the parameter d of the boundary function should be determined depending on κ.

Table 1 presents the values of d corresponding to $\kappa = 1, 2, \cdots, 8$, which are obtained by simulations with 100,000 replications, in which a Brownian motion is approximated by using 10,000 independent normal random variables. As expected, the value of d rises as the monitoring period lengthens. We use these values in the next section.

4. Finite Sample Property

In this section, we investigate the finite sample property of the monitoring tests in the previous section. The data-generating process we consider is similar to that in HHKS and is given by

$$y_t = \beta_{1t} + \beta_{2t} x_t + u_t, \quad x_t = (1-\phi) + \phi x_{t-1} + v_t,$$

where $\beta_{1t} = 1$ and $\beta_{2t} = 1$ for $t = 1, \cdots, m$, and the IV variable z_t is generated by

$$z_t = 0.5(1-\phi) + \phi z_{t-1} + w_t,$$

$$\text{where} \quad \begin{bmatrix} u_t \\ v_t \\ w_t \end{bmatrix} \sim i.i.d.N\left(0, \begin{bmatrix} 1 & \sigma_{uv}\sqrt{1-\phi^2} & 0 \\ \sigma_{uv}\sqrt{1-\phi^2} & 1-\phi^2 & \sigma_{vw}(1-\phi^2) \\ 0 & \sigma_{vw}(1-\phi^2) & 1-\phi^2 \end{bmatrix}\right),$$

meaning that $Var(x_t) = 1$, $Var(z_t) = 1$, and σ_{uv} and σ_{vw} denote the correlation coefficients between u_t and v_t and between v_t and w_t, respectively. We fix $\sigma_{uv} = 0.4$, while σ_{vw} is set to 0.2, 0.5, and 0.8. The initial values of x_t and z_t at $t = 0$ are set to 0. The serial correlations in x_t and z_t are controlled by ϕ and we set $\phi = 0$, 0.4, and 0.8. To investigate the finite sample property of the tests, we need to stop the monitoring period at some point $m + \bar{m}$ and we choose $\bar{m} = \kappa m$ with $\kappa = 1$, 4, and 8, while the training period m is 50, 100, and 250. We use the critical values adjusted for the given monitoring periods. The parameter γ in boundary function (6) is set to 0.25 and 0.45. Note that, as pointed out by HHKS and Aue and Horváth (2004), the CUSUM detector with larger values of γ tends to detect an earlier break than that with smaller values of γ. The significance level is set to 0.05, the number of replications is 3,000, and all the computations are conducted by using the GAUSS matrix language.

Table 2 reports the empirical sizes of the tests. We can see that when $\phi = 0$, the empirical sizes of both the IV and the LS detectors are close to the nominal size for all cases. However, as the serial correlations in the regressor and IV strengthen, they tend to over-reject the null hypothesis. In particular, when the correlation between the regressor and IV is not strong, the IV detector suffers from severe size distortion. As a whole, the IV must be strongly correlated with the regressor in order for the IV detector to perform as well as the LS detector under the null hypothesis. By comparing the difference in γ, we see that the sizes of both tests with $\gamma = 0.45$ are slightly closer to the nominal size than those with $\gamma = 0.25$.

To investigate the performance of the tests under the alternative, we set $\beta_{1t} = 1 + h/\sqrt{m}$ and $\beta_{2t} = 1 + h/\sqrt{m}$ for $h = 1$ and 2. The empirical powers of the tests are summarized in

Tables 3 and 4. Table 3 corresponds to the case where a structural change occurs just after the training period ($k^* = 1$). In general, both tests are more powerful for longer monitoring periods. The LS detector is more powerful than the IV detector for $\phi = 0$ and $\phi = 0.4$, while it is less powerful in some cases for $\phi = 0.8$, but this is because the test with the IV method suffers from severe size distortion for $\phi = 0.8$. By comparing the difference in γ, we see that both tests with $\gamma = 0.25$ seem slightly more powerful than those with $\gamma = 0.45$; however, this may be because of the size distortions.

The empirical powers of the tests with $k^* = m/2$ are reported in Table 4. The relative performance in this case is preserved, although, as expected, both tests are less powerful than in the case with $k^* = 1$.

For the monitoring tests, not only the empirical power but also the delay time, the time period required to actually detect the change after the true break point, is an important measure of the finite sample performance of the tests. We conducted simulations with a middle to large magnitude of change ($h = 2$ and 4) to ensure a sufficient number of break detections and summarized the distributional property, minimum value, quartiles, and maximum value. Tables 5 and 6 report the cases where $k^* = 1$, $\sigma_{vw} = 0.5$, and $\bar{m} = 8m$.[2] Note that the delay times are always nonnegative in Table 5 because the structural change occurs as soon as the monitoring period starts ($k^* = 1$), while the minimum values in Table 6 are negative because both tests may detect the break before the true break point, which is related with the type I error. As expected, both tests tend to detect the larger magnitude of the break earlier after the break. When the serial correlation in the error term is not strong ($\phi = 0$ and 0.4), the CUSUM test based on the LS method is good at detecting the structural change as soon as it occurs compared with that based on the IV method. In particular, when $h = 4$, the maximum delay time by the IV detector is very large. On the contrary, when $\phi = 0.8$ and $h = 2$, the distribution of the break date detected by the IV detector is located to the left compared with that by the LS detector; however, the maximum delay time is still very large for $h = 4$. As pointed out in the literature such as HHKS, tests with $\gamma = 0.45$ can detect the early break ($k^* = 1$) compared with those with $\gamma = 0.25$ as in Table 5, whereas tests with

[2] We do not report the other cases to save space. Their relative performance was basically the same as in Tables 5 and 6. Details are available upon request.

$\gamma = 0.25$ perform slightly better for $k^* = [m/2]$.

We next investigate the effect of a change in σ_{uv} on the finite sample performance of the LS detector. In Table 7, we report the empirical size of the LS monitoring test with σ_{uv} changing at $k^* = 1$. We set $\sigma_{uv} = 0$ or 0.4 before the break, while it changes to $\sigma_{uv} \pm 0.4$ after the break. In the table, the entries of the column of LS(+) are the rejection frequencies of the test with the positive change in the correlation, while the column of $LS(-)$ corresponds to the negative change. For the purpose of the comparison, we also report the rejection frequencies of the LS test with no change in the column of $LS(0)$. As expected from the theoretical investigation presented in the previous section, the rejection frequencies decline for a positive change in the correlation, while a negative change results in higher rejection frequencies. We note that the differences between rejection frequencies are relatively minor when σ_{uv} before the break is 0, while the size of the test is affected when it is 0.4. As a whole, the rejection frequencies are more affected by a change in the correlation when ϕ is closer to 1. A similar tendency is observed when $k^* = [m/2]$; nevertheless, the differences are smaller compared with the case of $k^* = 1$.

We also conducted simulations under the alternative; both β_0 and σ_{uv} sustain a structural change. As a whole, the same tendency is observed for the power of the tests, while the differences are relatively minor for all cases. On the contrary, we cannot find a significant difference in the stopping time for positive and negative changes in σ_{uv} up to the third quartile. However, the maximum delay time of the test with a positive change in σ_{uv}^2 ($\sigma_u^{*2}/\sigma_u^{+2} < 1$) tends to be slightly greater than that with a negative change ($\sigma_u^{*2}/\sigma_u^{+2} > 1$) for $m = 50$ and 100 (although the reverse relation is observed for $m = 250$). Finally, we cannot find any other systematic property of the delay time as far as our simulations are concerned (we omit the details to save space).

5. Empirical Application

In this section, we implement the monitoring tests proposed in the previous section to the Phillips curve by using Japanese monthly data and investigate a) how the two monitoring tests perform in an empirical analysis and b) whether the policy change by the Bank of Japan affected the Phillips curve. Following Galí and Gertler (1999), the hybrid Phillips curve in its reduced form is given by

$$\pi_t = \beta_1 + \beta_1 \pi_{t-1} + \beta_2 E_t \pi_{t+1} + \beta_3 x_t + u_t, \tag{11}$$

where π_t is the inflation rate, x_t is an economic variable, and E_t denotes the conditional expectation using the information up to time t. We use the growth rate of the consumer price index (all items, excluding fresh food) and the first difference in unemployment for π_t and x_t, respectively, while $E_t p_{t+1}$ is replaced by p_{t+1} in view of rational expectations.[3]

We first test for structural changes in the sample period ranging from 1981.01 to 2012.12 because we need to choose the training period without a structural change. We implement the UDmax and WDmax tests for the null hypothesis of no break against the alternative that the number of breaks is at most five, which were proposed by Bai and Perron (1998), and we observe possible evidence of multiple breaks at the 1% significance level in panel (a) of Table 5. We also test for the null of ℓ against the alternative of $\ell+1$ breaks for $\ell = 1$, 2, 3, and 4 by the supF($\ell + 1|\ell$) test by Bai and Perron (1998). We can see that the SupF(2|1) test rejects the null at the 1% significance level, whereas the other tests do not reject the null hypothesis. Following the procedure proposed by Bai and Perron (2006), the number of breaks based on these tests is estimated as two. We confirm this result by implementing the modified BIC (MBIC1 and MBIC2) proposed by Kurozumi and Tuvaandorj (2011); the number of breaks is estimated as two again, as in panel (b) of Table 2. In panel (c), we report the estimated break dates and 95% confidence intervals (CIs). The first break is estimated at May 1994 and CI(BP), the CI by Bai and Perron (1998), is from November 1993 to November 1994, while the second break is at September 2007 and the CI is from December 2006 to November 2008. However, it is known that the CI based on the limiting distribution of the break point estimator tends to suffer from under-coverage for a small size of a break. Hence, we also calculate the CIs by the method based on the sup-type test proposed by Kurozumi and Yamamoto (2015), which are denoted CI(KY) in Table 8. We can see that the CIs based on the two methods are relatively close for the first break, whereas CI(KY) covers earlier dates

[3]Galí and Gertler (1999) proposed using the marginal cost as x_t, while Yamamoto (2014) investigated the Japanese New Keynesian Phillips curve. However, it is difficult to find a proxy variable of the marginal cost for monthly data and thus we consider unemployment as an economic indicator, as used by Giacomini and Rossi (2009) and Yamamoto (2014).

than CI(BP). Overall, we find two structural changes and thus three regimes in this sample period.

Considering the above results, the first training period we choose is from January 1995 to September 2004. For the CUSUM monitoring test based on the IV method, we estimate model (11) using two sets of IVs: π_{t-1} to π_{t-4} and x_{t-1} to x_{t-4} (IV1) and π_{t-1} to π_{t-4} and x_{t-1} to x_{t-3} (IV2). Figure 1(a) shows the behavior of the monitoring tests with the two boundary functions with $\kappa = 1$ at the 95% significance level ($\gamma = 0.25$ and $\gamma = 0.45$). We can see that the LS-based test rejects the null of stability in June 2008, four months earlier than October 2008 at which IV1 rejects the null, while IV2 does not cross the boundary function for the whole monitoring period. Since the difference between IV1 and IV2 is only whether x_{t-4} is included or not, it seems that the result of the IV monitoring test is heavily affected by the IVs chosen by the researcher.

The second training period we choose is from January 2009 to December 2012. Note that since the Bank of Japan introduced its 2% inflation target in January 2013, we can test whether the change in policy affected the Phillips curve. Figure 1(b) shows that none of the tests crosses the boundary functions in the monitoring period. Thus, although the monitoring tests entail the delay time to detect the change, it seems the Japanese Phillips curve is relatively stable even after the introduction of the 2% inflation target.

6. Concluding Remarks

In this paper, we investigated monitoring tests in models with endogenous regressors. We proposed constructing CUSUM-based monitoring statistics by using the IV and LS methods. We showed theoretically that the monitoring test based on the LS method works better than that based the IV method. We also confirmed by Monte Carlo simulations that this theoretical result holds in finite samples. Although the monitoring test with the LS estimation may be affected by a change in the correlation between the regressors and error term, this test is useful in practice, particularly when it is difficult to find the appropriate IVs. Even if we can find such IVs, the monitoring test based on the IV method may be heavily affected by the choice of the IVs used for the estimation of the model, as observed in the empirical analysis and thus the LS-based test complements the monitoring test based on the IV method.

Appendix

Proof of Theorem 1: Since $\tilde{x}_{jt}$ for $j = 2, \cdots, p$ and $\tilde{z}_{jt}$ for $j = 2, \cdots, q$ are a stationary process with mean zeros, we can see from Assumptions CA and IV that the first element of $\beta_{IV} - \beta_0$ is $(1/m)\sum_{t=1}^{m} u_t(1 + o_p(1))$, while the rest are $O_p(1/\sqrt{m})$. Then, in exactly the same manner as Lemma 5.3 in HHKS, we can see that

$$\sup_{1\le k<\infty} \left| \sum_{t=m+1}^{m+k} (y_t - x_t'\hat{\beta}_{IV}) - \left(\sum_{t=m+1}^{m+k} u_t - \frac{k}{m}\sum_{t=1}^{m} u_t \right) \right| / g(m,k) = o_p(1)$$

for a given value of $0 < d < \infty$. Then, following the proof of Theorems 2.1 and 2.2 of HHKS, we obtain the result.■

Proof of Theorem 2: Let C denote a generic constant that may differ from place to place. According to the Rosenthal inequality, we can see that

$$\begin{aligned} E\left[\left|\sum_{t=1}^{k} \tilde{x}_{jt}\right|^{\mu}\right] &\le C\left[\sum_{t=1}^{k} E\left[|\tilde{x}_{jt}|^{\mu}\right] + \left(\sum_{t=1}^{k} E\left[\left|\sum_{t=1}^{k} \tilde{x}_{jt}\right|^{2}\right]\right)^{\mu/2}\right] \\ &\le C\, k^{\mu/2} \qquad (12) \end{aligned}$$

because $\mu > 2$. Then, in exactly the same manner as the proof of Theorem 3.1 of Aue, Horváth, and Reimherr (2009), we obtain the results.■

Proof of Theorems 3 and 4: Since (12) holds, all the assumptions made in HHKS and Aue, Horváth, and Reimherr (2009) hold and then we obtain the results. From Theorem 1 in Aue, Horváth, and Reimherr (2009), a_m^{LS} is expressed as

$$a_m^{LS} = \left(c_m^{LS} - \frac{1}{c_m^{LS}|\delta_\beta|} \sum_{t=m+k^*}^{m+c_m} (\tilde{x}_t - E[\tilde{x}_t])'(\tilde{\beta}_*^{+} - \tilde{\beta}_0^{+}) \right)^{1/(1-\gamma)}.$$

Since $E[\tilde{x}_t] = [1, 0, \cdots, 0]$ and $\mu_x = [1, \mu_{x2}, \cdots, \mu_{xp}]'$, we have $\tilde{x}_t - E[\tilde{x}_t] = x_t - \mu_x$.■

Proof of Corollary 2: We first note that $\tilde{\beta}_*^{\dagger} - \tilde{\beta}_0^{+}$ in the second term of a_m^{*LS} no longer equals $\tilde{\beta}_*^{+} - \tilde{\beta}_0^{+}$ in a_m^{LS}. However, as shown by Aue, Horváth, and Reimherr (2009), these terms are dominated by c_m^{*LS} and c_m^{LS}, respectively. These authors showed that $a_m^{*LS}/c_m^{*LS} \xrightarrow{p} 1$ and $a_m^{LS}/c_m^{LS} \xrightarrow{p} 1$. In fact, their result implies that

$$\frac{\tau^{*LS}(m)}{c_m^{*LS}} \xrightarrow{p} 1 \quad \text{and} \quad \frac{\tau^{LS}(m)}{c_m^{LS}} \xrightarrow{p} 1. \qquad (13)$$

Therefore, the effect of the change in correlation through $\tilde{\beta}_*^\dagger - \tilde{\beta}_0^+$ disappears asymptotically.

We next investigate the effect of a change in σ_{xu} through the value of δ_β^*. However, since the first element of $\tilde{x}_t$ is constant, while the others are mean-zero stationary variables, Σ_{xx} must be a block-diagonal matrix such as

$$\Sigma_{xx} = \begin{bmatrix} 1 & 0 \\ 0 & \Sigma_{xx,2} \end{bmatrix} \quad \text{and} \quad \sigma_{xu} = \begin{bmatrix} 0 \\ \sigma_{xu,2} \end{bmatrix}, \quad \text{say.}$$

Thus, although a change in σ_{xu} affects the second to the last elements of $\tilde{\beta}_t^+$, we can see that the first element of $\tilde{\beta}_t^+$ is free of the correlation change. Therefore, we have $\delta_\beta = \delta_\beta^*$, which implies $c_m^{*LS} = c_m^{LS}$. Then, by using (13), we conclude that

$$\frac{\tau^{*LS}(m)}{\tau^{LS}(m)} = \frac{\tau^{*LS}(m)}{c_m^{*LS}} \frac{c_m^{LS}}{\tau^{LS}(m)} \xrightarrow{p} 1. \blacksquare$$

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Table 1: Critical Values

γ	$\alpha = 0.01$	0.025	0.05	0.1	$\alpha = 0.01$	0.025	0.05	0.1
	$\kappa = 1$				$\kappa = 5$			
0	1.9803	1.7577	1.5785	1.3833	2.5602	2.2763	2.0428	1.7842
0.15	2.2369	2.0019	1.8076	1.5951	2.6898	2.3974	2.1614	1.9039
0.25	2.4521	2.2036	1.9997	1.7716	2.7983	2.5042	2.2708	2.0142
0.35	2.7221	2.4675	2.2523	2.0192	2.9543	2.6640	2.4289	2.1824
0.45	3.1664	2.9094	2.6910	2.4541	3.2524	2.9897	2.7651	2.5199
0.49	3.5304	3.2736	3.0397	2.7906	3.5625	3.3052	3.0738	2.8268
	$\kappa = 2$				$\kappa = 6$			
0	2.2868	2.0335	1.8282	1.5933	2.6032	2.3055	2.0707	1.8110
0.15	2.4856	2.2143	1.9993	1.7598	2.7143	2.4242	2.1839	1.9231
0.25	2.6453	2.3671	2.1478	1.9047	2.8165	2.5223	2.2860	2.0285
0.35	2.8481	2.5776	2.3525	2.1094	2.9660	2.6750	2.4391	2.1917
0.45	3.2114	2.9517	2.7328	2.4919	3.2598	2.9952	2.7687	2.5241
0.49	3.5494	3.2892	3.0583	2.8115	3.5644	3.3076	3.0757	2.8282
	$\kappa = 3$				$\kappa = 7$			
0	2.4224	2.1553	1.9381	1.6907	2.6298	2.3305	2.0932	1.8295
0.15	2.5854	2.3078	2.0838	1.8340	2.7371	2.4417	2.2008	1.9376
0.25	2.7276	2.4374	2.2113	1.9633	2.8338	2.5362	2.2993	2.0397
0.35	2.9073	2.6231	2.3893	2.1456	2.9752	2.6850	2.4474	2.1986
0.45	3.2307	2.9715	2.7489	2.5067	3.2650	2.9976	2.7727	2.5269
0.49	3.5530	3.2973	3.0669	2.8199	3.5667	3.3084	3.0768	2.8300
	$\kappa = 4$				$\kappa = 8$			
0	2.5046	2.2264	1.9993	1.7472	2.6537	2.3510	2.1101	1.8441
0.15	2.6501	2.3629	2.1286	1.8762	2.7550	2.4563	2.2118	1.9485
0.25	2.7741	2.4748	2.2482	1.9931	2.8482	2.5466	2.3073	2.0494
0.35	2.9363	2.6474	2.4126	2.1667	2.9852	2.6904	2.4548	2.2036
0.45	3.2418	2.9840	2.7584	2.5149	3.2680	3.0012	2.7755	2.5291
0.49	3.5582	3.3025	3.0713	2.8244	3.5668	3.3099	3.0775	2.8312

Table 2: Size of the Tests

$\bar{m}$	$\gamma = 0.25$ IV σ^2_{vw}			LS	$\gamma = 0.45$ IV σ^2_{vw}			LS
	0.2	0.5	0.8		0.2	0.5	0.8	
	$m = 50$, $\phi = 0$							
$1m$	0.061	0.060	0.061	0.062	0.047	0.046	0.048	0.048
$4m$	0.065	0.063	0.060	0.067	0.056	0.054	0.056	0.054
$8m$	0.066	0.063	0.063	0.064	0.050	0.054	0.057	0.053
	$m = 50$, $\phi = 0.4$							
$1m$	0.127	0.076	0.068	0.071	0.104	0.064	0.053	0.052
$4m$	0.139	0.084	0.073	0.072	0.128	0.072	0.062	0.062
$8m$	0.151	0.085	0.072	0.072	0.135	0.073	0.061	0.061
	$m = 50$, $\phi = 0.8$							
$1m$	0.285	0.165	0.107	0.100	0.258	0.144	0.090	0.078
$4m$	0.350	0.215	0.129	0.119	0.323	0.197	0.112	0.106
$8m$	0.365	0.210	0.123	0.117	0.340	0.189	0.108	0.098
	$m = 100$, $\phi = 0$							
$1m$	0.052	0.054	0.053	0.054	0.046	0.045	0.044	0.048
$4m$	0.052	0.049	0.052	0.057	0.049	0.046	0.049	0.048
$8m$	0.057	0.057	0.058	0.058	0.045	0.047	0.047	0.050
	$m = 100$, $\phi = 0.4$							
$1m$	0.097	0.059	0.055	0.056	0.091	0.053	0.047	0.049
$4m$	0.106	0.062	0.056	0.061	0.096	0.056	0.050	0.053
$8m$	0.107	0.064	0.065	0.060	0.097	0.056	0.051	0.051
	$m = 100$, $\phi = 0.8$							
$1m$	0.260	0.119	0.068	0.073	0.241	0.104	0.063	0.062
$4m$	0.307	0.135	0.083	0.084	0.296	0.122	0.070	0.075
$8m$	0.309	0.137	0.089	0.074	0.303	0.126	0.078	0.064
	$m = 250$, $\phi = 0$							
$1m$	0.048	0.042	0.044	0.048	0.042	0.044	0.044	0.045
$4m$	0.048	0.049	0.052	0.046	0.043	0.045	0.046	0.041
$8m$	0.053	0.051	0.053	0.053	0.049	0.049	0.050	0.047
	$m = 250$, $\phi = 0.4$							
$1m$	0.066	0.046	0.044	0.050	0.065	0.046	0.043	0.043
$4m$	0.069	0.051	0.050	0.048	0.072	0.048	0.047	0.036
$8m$	0.079	0.054	0.054	0.058	0.075	0.053	0.050	0.044
	$m = 250$, $\phi = 0.8$							
$1m$	0.206	0.076	0.053	0.062	0.204	0.071	0.049	0.050
$4m$	0.222	0.080	0.058	0.057	0.221	0.081	0.060	0.048
$8m$	0.238	0.089	0.067	0.063	0.235	0.080	0.061	0.053

Table 3: Power of the Tests ($k^* = 1$)

h	$\bar{m}$	$\gamma = 0.25$ IV σ^2_{vw} 0.2	0.5	0.8	LS	$\gamma = 0.45$ IV σ^2_{vw} 0.2	0.5	0.8	LS
		$m = 50$, $\phi = 0$							
	$1m$	0.225	0.287	0.298	0.320	0.163	0.215	0.220	0.233
1	$4m$	0.332	0.410	0.420	0.472	0.268	0.329	0.342	0.370
	$8m$	0.364	0.444	0.460	0.511	0.271	0.348	0.363	0.411
	$1m$	0.599	0.729	0.747	0.821	0.514	0.642	0.666	0.727
2	$4m$	0.757	0.900	0.915	0.955	0.696	0.847	0.876	0.922
	$8m$	0.777	0.929	0.947	0.976	0.719	0.890	0.908	0.950
		$m = 50$, $\phi = 0.4$							
	$1m$	0.287	0.297	0.304	0.299	0.223	0.234	0.237	0.217
1	$4m$	0.392	0.430	0.440	0.447	0.333	0.347	0.353	0.349
	$8m$	0.425	0.457	0.471	0.487	0.350	0.375	0.380	0.382
	$1m$	0.638	0.736	0.752	0.802	0.563	0.650	0.671	0.706
2	$4m$	0.788	0.897	0.915	0.949	0.746	0.853	0.874	0.914
	$8m$	0.809	0.928	0.945	0.970	0.765	0.889	0.913	0.936
		$m = 50$, $\phi = 0.8$							
	$1m$	0.422	0.358	0.332	0.263	0.370	0.308	0.270	0.195
1	$4m$	0.532	0.492	0.469	0.392	0.475	0.429	0.398	0.309
	$8m$	0.569	0.528	0.506	0.430	0.518	0.460	0.426	0.338
	$1m$	0.692	0.728	0.742	0.730	0.631	0.655	0.670	0.638
2	$4m$	0.862	0.895	0.913	0.905	0.821	0.858	0.875	0.847
	$8m$	0.872	0.916	0.943	0.934	0.835	0.884	0.913	0.885
		$m = 100$, $\phi = 0$							
	$1m$	0.235	0.260	0.261	0.299	0.167	0.184	0.191	0.216
1	$4m$	0.351	0.398	0.407	0.450	0.262	0.307	0.311	0.344
	$8m$	0.390	0.445	0.443	0.498	0.296	0.340	0.347	0.385
	$1m$	0.657	0.746	0.755	0.822	0.563	0.646	0.657	0.718
2	$4m$	0.820	0.915	0.921	0.963	0.754	0.862	0.868	0.924
	$8m$	0.846	0.939	0.948	0.975	0.795	0.897	0.904	0.948
		$m = 100$, $\phi = 0.4$							
	$1m$	0.274	0.274	0.274	0.279	0.211	0.200	0.197	0.203
1	$4m$	0.386	0.407	0.413	0.421	0.313	0.321	0.320	0.317
	$8m$	0.430	0.451	0.457	0.471	0.342	0.355	0.358	0.359
	$1m$	0.673	0.748	0.755	0.804	0.592	0.659	0.664	0.700
2	$4m$	0.824	0.914	0.920	0.952	0.775	0.865	0.873	0.901
	$8m$	0.857	0.942	0.951	0.968	0.815	0.900	0.910	0.939
		$m = 100$, $\phi = 0.8$							
	$1m$	0.405	0.330	0.302	0.242	0.350	0.261	0.233	0.177
1	$4m$	0.521	0.446	0.428	0.378	0.465	0.371	0.350	0.280
	$8m$	0.565	0.492	0.479	0.410	0.503	0.406	0.384	0.316
	$1m$	0.726	0.754	0.756	0.748	0.665	0.680	0.675	0.641
2	$4m$	0.860	0.903	0.920	0.911	0.823	0.858	0.873	0.847
	$8m$	0.899	0.937	0.948	0.943	0.870	0.900	0.912	0.894

Table 3: (continued)

h	$\bar{m}$	$\gamma = 0.25$ IV σ^2_{vw} 0.2	0.5	0.8	LS	$\gamma = 0.45$ IV σ^2_{vw} 0.2	0.5	0.8	LS
		$m = 250$, $\phi = 0$							
	$1m$	0.260	0.271	0.272	0.304	0.193	0.203	0.202	0.224
1	$4m$	0.368	0.386	0.392	0.439	0.280	0.293	0.293	0.328
	$8m$	0.398	0.421	0.425	0.479	0.306	0.325	0.327	0.373
	$1m$	0.724	0.767	0.769	0.831	0.622	0.670	0.676	0.741
2	$4m$	0.888	0.921	0.923	0.965	0.826	0.867	0.868	0.923
	$8m$	0.902	0.942	0.946	0.975	0.842	0.894	0.898	0.943
		$m = 250$, $\phi = 0.4$							
	$1m$	0.277	0.276	0.276	0.287	0.220	0.211	0.210	0.208
1	$4m$	0.391	0.388	0.390	0.419	0.303	0.300	0.302	0.310
	$8m$	0.414	0.425	0.427	0.451	0.341	0.336	0.334	0.356
	$1m$	0.723	0.771	0.768	0.811	0.641	0.678	0.679	0.715
2	$4m$	0.888	0.922	0.927	0.956	0.837	0.870	0.871	0.913
	$8m$	0.903	0.944	0.947	0.971	0.855	0.893	0.902	0.931
		$m = 250$, $\phi = 0.8$							
	$1m$	0.382	0.297	0.287	0.250	0.345	0.247	0.231	0.179
1	$4m$	0.477	0.412	0.403	0.372	0.423	0.327	0.317	0.279
	$8m$	0.512	0.444	0.436	0.415	0.456	0.359	0.348	0.309
	$1m$	0.757	0.769	0.767	0.764	0.697	0.689	0.686	0.663
2	$4m$	0.891	0.922	0.927	0.932	0.850	0.872	0.873	0.866
	$8m$	0.906	0.940	0.947	0.951	0.874	0.897	0.904	0.900

Table 4: Power of the Tests ($k^* = [m/2]$)

h	$\bar{m}$	$\gamma = 0.25$ IV σ^2_{vw} 0.2	0.5	0.8	LS	$\gamma = 0.45$ IV σ^2_{vw} 0.2	0.5	0.8	LS
		$m = 50$, $\phi = 0$							
	$1m$	0.089	0.105	0.107	0.116	0.059	0.064	0.068	0.074
1	$4m$	0.250	0.297	0.308	0.342	0.176	0.213	0.218	0.242
	$8m$	0.297	0.366	0.379	0.430	0.202	0.259	0.268	0.314
	$1m$	0.207	0.261	0.270	0.286	0.122	0.163	0.167	0.188
2	$4m$	0.657	0.794	0.821	0.885	0.556	0.691	0.711	0.792
	$8m$	0.726	0.886	0.908	0.949	0.646	0.801	0.831	0.902
		$m = 50$, $\phi = 0.4$							
	$1m$	0.164	0.124	0.116	0.116	0.121	0.084	0.073	0.074
1	$4m$	0.313	0.320	0.318	0.330	0.251	0.234	0.230	0.237
	$8m$	0.366	0.373	0.393	0.403	0.282	0.280	0.280	0.296
	$1m$	0.271	0.275	0.280	0.277	0.192	0.184	0.182	0.181
2	$4m$	0.697	0.802	0.823	0.868	0.614	0.700	0.722	0.768
	$8m$	0.759	0.885	0.907	0.936	0.688	0.806	0.830	0.881
		$m = 50$, $\phi = 0.8$							
	$1m$	0.311	0.214	0.165	0.129	0.273	0.171	0.120	0.093
1	$4m$	0.476	0.409	0.372	0.308	0.429	0.331	0.285	0.219
	$8m$	0.536	0.453	0.426	0.368	0.471	0.368	0.330	0.270
	$1m$	0.414	0.348	0.308	0.261	0.342	0.270	0.231	0.181
2	$4m$	0.785	0.803	0.827	0.798	0.713	0.711	0.729	0.690
	$8m$	0.848	0.879	0.902	0.891	0.795	0.816	0.832	0.813
		$m = 100$, $\phi = 0$							
	$1m$	0.087	0.090	0.088	0.093	0.059	0.060	0.060	0.066
1	$4m$	0.245	0.287	0.287	0.322	0.158	0.186	0.196	0.210
	$8m$	0.316	0.362	0.365	0.406	0.216	0.251	0.252	0.283
	$1m$	0.200	0.221	0.226	0.259	0.125	0.135	0.136	0.165
2	$4m$	0.715	0.816	0.821	0.880	0.600	0.699	0.709	0.786
	$8m$	0.795	0.890	0.903	0.946	0.708	0.809	0.825	0.896
		$m = 100$, $\phi = 0.4$							
	$1m$	0.128	0.096	0.090	0.093	0.103	0.069	0.065	0.065
1	$4m$	0.293	0.293	0.292	0.307	0.215	0.202	0.201	0.196
	$8m$	0.361	0.366	0.372	0.387	0.272	0.257	0.257	0.268
	$1m$	0.244	0.234	0.235	0.249	0.169	0.146	0.143	0.157
2	$4m$	0.724	0.811	0.820	0.865	0.628	0.699	0.709	0.762
	$8m$	0.800	0.894	0.904	0.939	0.728	0.810	0.829	0.876
		$m = 100$, $\phi = 0.8$							
	$1m$	0.289	0.158	0.115	0.098	0.258	0.130	0.088	0.076
1	$4m$	0.456	0.335	0.315	0.268	0.395	0.266	0.227	0.184
	$8m$	0.505	0.418	0.394	0.344	0.433	0.316	0.289	0.234
	$1m$	0.392	0.297	0.258	0.227	0.322	0.216	0.181	0.149
2	$4m$	0.792	0.803	0.816	0.804	0.716	0.713	0.713	0.692
	$8m$	0.862	0.895	0.907	0.897	0.810	0.825	0.835	0.821

Table 4: (continued)

d	$\bar{m}$	$\gamma = 0.25$ IV σ^2_{vw} 0.2	0.5	0.8	LS	$\gamma = 0.45$ IV σ^2_{vw} 0.2	0.5	0.8	LS
		$m = 250$, $\phi = 0$							
	$1m$	0.087	0.093	0.089	0.095	0.055	0.058	0.059	0.062
1	$4m$	0.251	0.264	0.266	0.310	0.165	0.174	0.176	0.204
	$8m$	0.314	0.334	0.335	0.393	0.219	0.229	0.230	0.270
	$1m$	0.209	0.219	0.220	0.251	0.133	0.142	0.143	0.165
2	$4m$	0.775	0.815	0.815	0.885	0.659	0.694	0.697	0.776
	$8m$	0.844	0.890	0.894	0.943	0.752	0.803	0.809	0.879
		$m = 250$, $\phi = 0.4$							
	$1m$	0.107	0.091	0.090	0.086	0.080	0.062	0.058	0.063
1	$4m$	0.275	0.271	0.267	0.297	0.193	0.179	0.180	0.188
	$8m$	0.338	0.339	0.339	0.374	0.244	0.237	0.233	0.256
	$1m$	0.229	0.226	0.224	0.238	0.159	0.152	0.148	0.153
2	$4m$	0.776	0.819	0.818	0.868	0.678	0.697	0.702	0.754
	$8m$	0.844	0.891	0.892	0.933	0.763	0.804	0.808	0.867
		$m = 250$, $\phi = 0.8$							
	$1m$	0.248	0.118	0.102	0.084	0.226	0.090	0.069	0.065
1	$4m$	0.394	0.298	0.290	0.266	0.336	0.217	0.190	0.177
	$8m$	0.447	0.356	0.345	0.333	0.384	0.256	0.244	0.228
	$1m$	0.348	0.247	0.238	0.211	0.296	0.178	0.157	0.139
2	$4m$	0.799	0.818	0.818	0.817	0.715	0.711	0.710	0.694
	$8m$	0.857	0.885	0.890	0.901	0.802	0.812	0.818	0.817

Table 5: Delay Time ($k^* = 1$, $\sigma_{vw} = 0.5$, $\bar{m} = 8m$)

		$\gamma = 0.25$					$\gamma = 0.45$				
h		min	1Q	2Q	3Q	max	min	1Q	2Q	3Q	max
		$m = 50$, $\phi = 0$									
2	IV	2	18	33	65	395	0	11	26	62	396
	LS	2	17	28	51	396	0	11	23	49	399
4	IV	0	6	10	16	230	0	3	6	12	256
	LS	1	6	9	13	62	0	3	5	9	86
		$m = 50$, $\phi = 0.4$									
2	IV	1	18	32	64	396	0	10	25	58	393
	LS	2	18	30	54	393	0	11	24	52	399
4	IV	0	6	10	17	254	0	3	6	12	234
	LS	1	6	9	13	84	0	3	6	10	87
		$m = 50$, $\phi = 0.8$									
2	IV	2	16	31	63	392	0	9	23	57	399
	LS	2	19	33	64	397	0	12	27	63	394
4	IV	0	6	11	19	398	0	3	7	15	321
	LS	1	6	10	16	312	0	3	6	12	167
		$m = 100$, $\phi = 0$									
2	IV	4	40	69	126	790	0	25	55	122	789
	LS	4	36	60	105	788	0	24	51	105	798
4	IV	2	14	21	32	188	0	6	13	24	394
	LS	3	13	19	26	99	0	6	12	19	106
		$m = 100$, $\phi = 0.4$									
2	IV	4	38	67	126	796	0	23	54	121	780
	LS	4	38	63	110	799	0	25	53	111	795
4	IV	2	14	21	32	772	0	6	13	25	663
	LS	3	13	19	27	132	0	7	12	20	119
		$m = 100$, $\phi = 0.8$									
2	IV	3	35	63	125	781	0	19	48	117	786
	LS	4	41	70	129	790	0	27	58	127	799
4	IV	2	13	21	35	780	0	6	13	26	697
	LS	3	14	20	31	187	0	7	13	24	371
		$m = 250$, $\phi = 0$									
2	IV	13	102	178	327	1994	0	64	141	311	1996
	LS	15	94	155	263	1995	0	62	129	257	1965
4	IV	6	37	54	77	440	0	18	33	58	474
	LS	9	34	48	65	395	0	17	29	46	421
		$m = 250$, $\phi = 0.4$									
2	IV	13	99	173	322	1971	0	60	135	294	1992
	LS	15	97	162	268	1970	0	66	134	262	1995
4	IV	5	36	54	77	547	0	17	33	57	620
	LS	10	35	49	66	330	0	17	30	48	396
		$m = 250$, $\phi = 0.8$									
2	IV	10	93	165	304	1979	0	53	125	279	1984
	LS	14	106	178	308	1976	0	70	148	300	1999
4	IV	5	34	53	80	765	0	15	32	59	1855
	LS	6	37	53	72	312	0	18	33	54	397

Table 6: Delay Time ($k^* = [m/2]$, $\sigma_{vw} = 0.5$, $\bar{m} = 8m$)

		$\gamma = 0.25$					$\gamma = 0.45$				
h		min	1Q	2Q	3Q	max	min	1Q	2Q	3Q	max
		$m = 50$, $\phi = 0$									
2	IV	−21	33	62	115	368	−24	34	66	126	373
	LS	−17	32	54	95	374	−24	34	61	113	375
4	IV	−21	14	22	34	313	−24	14	23	36	346
	LS	−17	13	19	27	146	−24	13	19	29	165
		$m = 50$, $\phi = 0.4$									
2	IV	−20	31	60	112	372	−24	31	64	125	375
	LS	−18	32	57	100	375	−24	34	63	118	375
4	IV	−20	14	22	34	370	−24	13	23	36	332
	LS	−18	13	20	29	122	−24	13	20	30	161
		$m = 50$, $\phi = 0.8$									
2	IV	−22	23	51	102	375	−24	20	52	114	374
	LS	−20	33	62	113	373	−24	32	66	125	375
4	IV	−22	11	21	36	373	−24	10	22	39	369
	LS	−20	13	22	33	176	−24	13	23	36	214
		$m = 100$, $\phi = 0$									
2	IV	−32	69	126	225	749	−48	72	138	261	749
	LS	−43	66	115	194	750	−49	71	131	229	744
4	IV	−32	30	46	66	532	−48	30	48	71	737
	LS	−43	27	40	56	199	−49	27	42	60	221
		$m = 100$, $\phi = 0.4$									
2	IV	−33	68	124	222	749	−49	70	137	253	749
	LS	−45	70	120	200	747	−49	73	137	234	746
4	IV	−33	29	45	66	613	−49	30	47	72	557
	LS	−45	28	41	58	216	−49	28	43	61	255
		$m = 100$, $\phi = 0.8$									
2	IV	−46	58	118	216	750	−49	55	125	250	750
	LS	−46	73	131	221	746	−49	77	148	264	746
4	IV	−46	25	43	67	501	−49	24	45	71	731
	LS	−46	29	45	64	245	−49	30	47	69	304
		$m = 250$, $\phi = 0$									
2	IV	−103	185	318	574	1867	−124	195	354	664	1865
	LS	−85	170	289	498	1875	−124	180	325	567	1871
4	IV	−103	78	115	166	1005	−124	79	121	177	1222
	LS	−85	71	103	141	452	−124	71	106	148	486
		$m = 250$, $\phi = 0.4$									
2	IV	−104	181	320	583	1864	−124	189	344	661	1875
	LS	−85	174	298	515	1871	−124	185	340	591	1868
4	IV	−104	77	115	166	1201	−124	77	120	177	1412
	LS	−85	73	105	143	504	−124	73	108	151	509
		$m = 250$, $\phi = 0.8$									
2	IV	−105	169	309	558	1864	−124	173	340	633	1873
	LS	−103	185	326	561	1851	−124	192	363	653	1870
4	IV	−105	73	113	168	1489	−124	72	117	177	1719
	LS	−103	76	112	157	702	−124	77	116	169	891

Table 7: Size of the Tests with a Break in Correlation ($k^* = 1$)

		$\gamma = 0.25$			$\gamma = 0.45$		
σ^2_{uv}	$\bar{m}$	LS(+)	LS(−)	LS(0)	LS(+)	LS(−)	LS(0)
		$m = 50$, $\phi = 0$					
	1m	0.064	0.062	0.063	0.050	0.047	0.048
0	4m	0.062	0.063	0.061	0.056	0.056	0.056
	8m	0.064	0.063	0.064	0.059	0.056	0.057
	1m	0.029	0.111	0.062	0.015	0.104	0.048
0.4	4m	0.040	0.106	0.067	0.022	0.110	0.054
	8m	0.037	0.101	0.064	0.022	0.106	0.053
		$m = 50$, $\phi = 0.4$					
	1m	0.072	0.066	0.069	0.053	0.055	0.050
0	4m	0.071	0.070	0.066	0.059	0.065	0.063
	8m	0.071	0.068	0.069	0.065	0.062	0.060
	1m	0.023	0.131	0.071	0.013	0.122	0.052
0.4	4m	0.038	0.128	0.072	0.022	0.133	0.062
	8m	0.038	0.124	0.072	0.019	0.127	0.061
		$m = 50$, $\phi = 0.8$					
	1m	0.090	0.090	0.090	0.071	0.073	0.070
0	4m	0.104	0.105	0.102	0.083	0.088	0.084
	8m	0.103	0.098	0.103	0.085	0.082	0.085
	1m	0.047	0.159	0.100	0.034	0.133	0.078
0.4	4m	0.067	0.174	0.119	0.048	0.166	0.106
	8m	0.071	0.173	0.117	0.046	0.160	0.098
		$m = 100$, $\phi = 0$					
	1m	0.053	0.053	0.055	0.044	0.043	0.042
0	4m	0.053	0.053	0.052	0.047	0.050	0.049
	8m	0.062	0.061	0.060	0.049	0.048	0.047
	1m	0.023	0.099	0.054	0.012	0.109	0.048
0.4	4m	0.030	0.090	0.057	0.018	0.102	0.048
	8m	0.035	0.089	0.058	0.019	0.102	0.050
		$m = 100$, $\phi = 0.4$					
	1m	0.053	0.054	0.053	0.044	0.047	0.045
0	4m	0.056	0.055	0.054	0.052	0.052	0.051
	8m	0.065	0.062	0.062	0.049	0.052	0.049
	1m	0.015	0.119	0.056	0.009	0.122	0.049
0.4	4m	0.027	0.113	0.061	0.012	0.129	0.053
	8m	0.030	0.104	0.060	0.015	0.122	0.051
		$m = 100$, $\phi = 0.8$					
	1m	0.063	0.064	0.063	0.052	0.055	0.050
0	4m	0.072	0.071	0.071	0.062	0.061	0.058
	8m	0.080	0.074	0.075	0.066	0.063	0.063
	1m	0.021	0.144	0.073	0.014	0.135	0.062
0.4	4m	0.039	0.148	0.084	0.025	0.144	0.075
	8m	0.039	0.124	0.074	0.023	0.137	0.064

Table 7: (continued)

		$\gamma = 0.25$			$\gamma = 0.45$		
σ^2_{uv}	$\bar{m}$	LS(+)	LS(−)	LS(0)	LS(+)	LS(−)	LS(0)
		$m = 250$, $\phi = 0$					
	1m	0.043	0.044	0.044	0.044	0.047	0.043
0	4m	0.050	0.053	0.052	0.048	0.047	0.047
	8m	0.053	0.052	0.053	0.049	0.052	0.052
	1m	0.018	0.093	0.048	0.006	0.110	0.045
0.4	4m	0.021	0.083	0.046	0.008	0.101	0.041
	8m	0.031	0.091	0.053	0.010	0.104	0.047
		$m = 250$, $\phi = 0.4$					
	1m	0.043	0.045	0.045	0.044	0.046	0.045
0	4m	0.051	0.055	0.053	0.048	0.049	0.048
	8m	0.053	0.052	0.052	0.051	0.053	0.054
	1m	0.012	0.118	0.050	0.004	0.132	0.043
0.4	4m	0.016	0.099	0.048	0.005	0.122	0.036
	8m	0.022	0.105	0.058	0.006	0.124	0.044
		$m = 250$, $\phi = 0.8$					
	1m	0.047	0.049	0.047	0.045	0.049	0.043
0	4m	0.055	0.058	0.057	0.052	0.052	0.052
	8m	0.061	0.060	0.060	0.060	0.058	0.059
	1m	0.014	0.131	0.062	0.006	0.139	0.050
0.4	4m	0.019	0.120	0.057	0.007	0.142	0.048
	8m	0.022	0.119	0.063	0.009	0.131	0.053

Table 8: Empirical Results

(a) Tests for Structural Breaks

UDmax= 53.10***	WDmax= 64.91***		
SupF(2|1) = 43.83***	supF(3|2) = 13.34	supF(4|3) = 13.34	supF(5|4) = 8.51

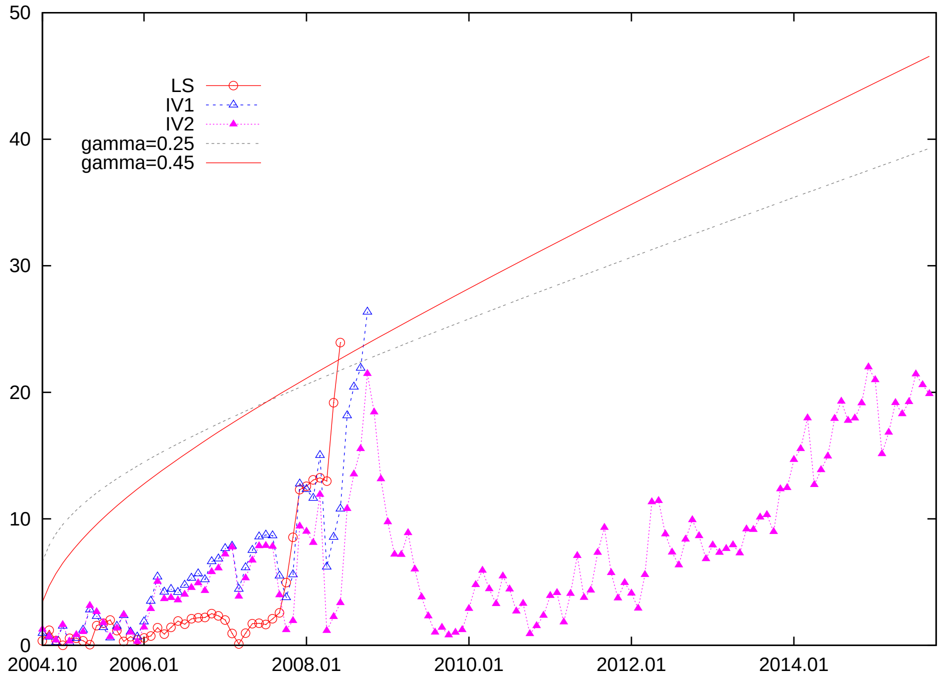
(b) Estimation of the Number of Breaks

MBIC1: 2	MBIC2: 2

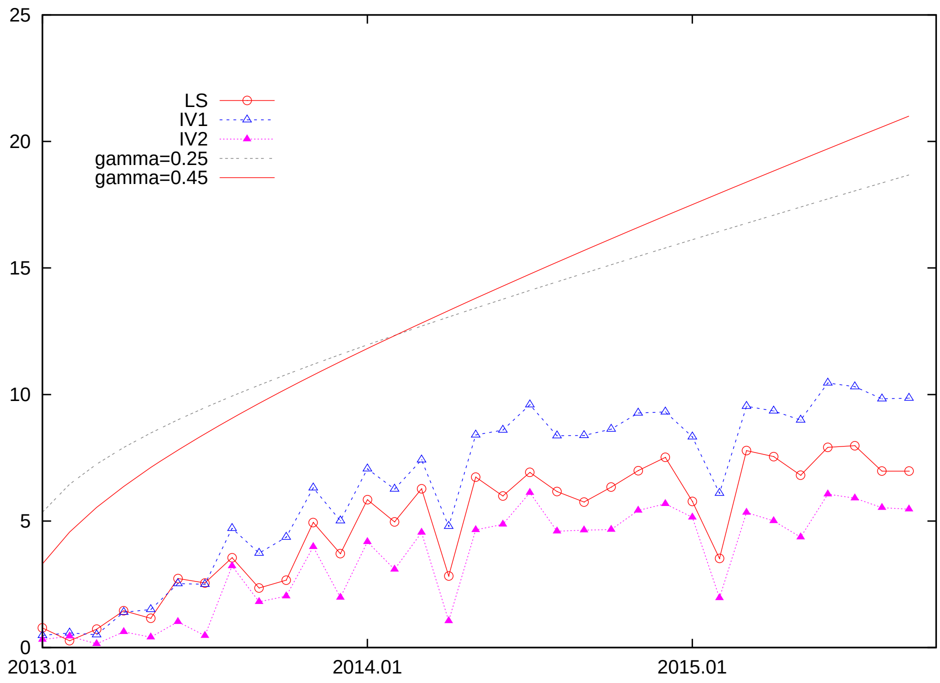
(c) Estimated Break Point and 95% Confidence Intervals

1st break: 1994.05	2nd break: 2007.09
CI (BP): [1993.11-1994.11]	CI (BP): [2006.12-2008.11]
CI (KY): [1993.07-1994.08]	CI (KY): [2004.09-2008.07]

Note: $***$, $**$ and $*$ denote 1%, 5% and 10% significance, respectively. The WDmax test statistic depends on the significance level and the result in the table is when it is 0.01. The same test rejects the null hypothesis with the 5% and 10% levels, respectively. Following Bai and Perron (1998), each break date is the right end of the sub-sample and the new regime starts from 1 month after the break date. The confidence intervals by Kurozumi and Yamamoto (2015) are obtained using the sub-sample 1981.01-2006.12 for the first break and 1994.12-2012.12 for the second break, respectively, where these sub-samples are chosen following the confidence intervals obtained by Bai and Perron (1998).



(a) training period: 1995.01–2004.09



(b) training period: 2009.01–2012.12

Figure 1: CUSUM monitoring