Household choices in fragile families and their effects on children's cognitive and non-cognitive skills

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Abstract

I present a structural model of labor and child care options to analyze the impact of single mothers' decisions on their children's cognitive and non-cognitive development. Mothers decide their child care arrangements and labor supply according to their preferences and constraints. I estimate the parameters of a production function of skills applying simulation methods with data from the Fragile Families Study. Results indicate that maternal time is the most significant factor in the production of skills of young children. Counterfactual exercises show the importance of institutional child care on both cognitive and non-cognitive measures.

JEL: D13, J22, J24

1 Introduction

According to Becker and Tomes (1986), parents are utility-maximizing agents who are concerned about the welfare of their children. They make labor supply decisions to provide consumption to the household and invest in child development. They also decide the amount of time to dedicate to child care, which contributes to the development process of their children. The time they take care of their children and the investments in child development are inputs in

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the production of a child's ability (Todd and Wolpin, 2003). Among these factors, parental care is a critical input in the child's development (Cunha et al., 2006). As reported by Almond and Currie (2011), parents' absence due to their participation in the labor market raises concerns about the effects on children's skills. In single-mother households, the economic resources and available time to spend with their children can be limited. Therefore, they are more likely to choose external sources of child care (either paid or unpaid), which might affect their children's cognitive and non-cognitive skills with a consequent impact on the total household's utility.

I focus on single mothers and their labor and child care choices to assess how these decisions affect their children's cognitive and non-cognitive outputs. Single mothers make sequential decisions of labor supply and time allocation, as well as non-parental child care options (either formal and informal) given their preferences, and time and budget constraints. I apply a structural model proposed by Del Boca et al. (2014) to analyze the contribution of each child care choice on the production function of children's outcomes nested within an explicit model of household behavior.

Many articles describe the importance of cognitive skills in producing social and economic success[1]. Heckman et al. (2006) emphasize the parallel importance of non-cognitive ability in future educational and labor market outcomes. I analyze children's cognitive ability with the Peabody Picture Vocabulary Test (PPVT; Dunn et al., 1997), a test that assesses a child's verbal ability. Because children's non-cognitive skills cover different dimensions of physical health, and social and emotional behaviors, I focus on children's behavioral problems measured by the Child Behavioral Checklist (Achenbach and Rescorla, 2000).

The effects of labor supply and child care decisions on child development are affected by endogeneity due to the correlation of mothers' choices with mothers' and children's unobserved characteristics, and also by the simultaneity of the choices. This structural model accounts for different sources of endogeneity by modeling the mother's decision-making process simultaneously when presented with different child care choices. The single mother optimally chooses inputs in the production of her child's skills to maximize the total household's utility depending on her leisure time, household consumption, and the child's ability.

The model is estimated with data from the Fragile Families and Child Wellbeing study. It consists of a sample of unwed new-parents that followed their children from birth to age five in the U.S. The data set contains information about parents' time allocation and child care choices but at a few points in time. Then, I simulate paths of exogenous and endogenous variables over the child's development process. I retrieve the estimated parameters using the Method of Simulated Moments.

This study contributes to the literature by proposing a structural model that includes a wide set of child care options to simultaneously estimate the effects of household choices on the child's cognitive and non-cognitive skills given the

[1]For instance, Cawley et al. (2001) provide an analytic summary of evidence. Moreover, Herrnstein and Murray (1994) describe how cognitive ability can predict a range of social behaviors.

household's preferences, time and budget constraints. In particular, I examine the following questions: What are the consequences of labor supply and child care decisions in single-mother households on children's outcomes? What are the main determinants in the production of skills for children under five? Can government support improve the child's cognitive and non-cognitive outcomes?

Even though I consider a series of simplifying assumptions that allow us to obtain closed-form solutions of the endogenous variables and to decrease computational burden, the model fits well and yields close estimates of the simulated variables. I found that maternal time strengthens the production of child's ability during the first five years of the development process but at a decreasing rate. Mother's education is another important factor in the production of cognitive and non-cognitive skills because it enhances the maternal care. Formal care provides stronger positive effects than informal care on child development, specially in non-cognitive outcomes. The child's own ability boosts the current level of ability specially at the age of four and five.

The estimation of the parameters of the production function of cognitive development allows me to perform counterfactual exercises and evaluate different policy recommendations. I analyze the effect of cash transfers on child development. The results demonstrate that *unconditional* transfers do not offer significant impact on the average cognitive skill of children. However, a subsidy on child development that can be offered to mothers who send their children 40 hours per week to an institutional child care starting at the age of three, has a considerable positive impact on child development. The impacts of this policy on non-cognitive measures are modest, for which *unconditional* transfers provide stronger results on children.

Because formal child care includes options that vary in quality, I extend the original model to evaluate simultaneously the contribution of the different options of institutional child care. I observe that Head-Start enhances the child's cognitive ability, but it is less effective than daycare at the age of four, and weaker than kindergarten at the age of five. However, Head-Start produces a considerable positive impact in non-cognitive measures of ability, for both Externalizing and Internalizing scores at the ages of four and five.

The rest of this paper is organized as follows. Section 2 describes the most recent literature that analyzes structural models of child development. Section 3 presents the main details of the theoretical model. Section 4 examines the main characteristics of the data set. Section 5 provides the empirical strategy. Section 6 offers the estimation results of the single-mother model. Section 7 exhibits counterfactuals exercises to study possible policy recommendations. Section 8 performs a robustness check. Section 9 concludes.

2 Literature review

The childhood development literature analyzes children's cognitive and non-cognitive development, and their physical well-being. Numerous studies examine the effects of nutritional conditions, socioeconomic status, and parenting on child development[2]. The time that parents spend with their children constitutes a critical determinant in the production of child development (Cunha et al., 2006).

One potential factor that influences the time that parents spend with their children is household structure. During the last 40 years, we observe an increasing trend of births in single-mother households, with high percentages in the black and Hispanic populations (see Figure 1).

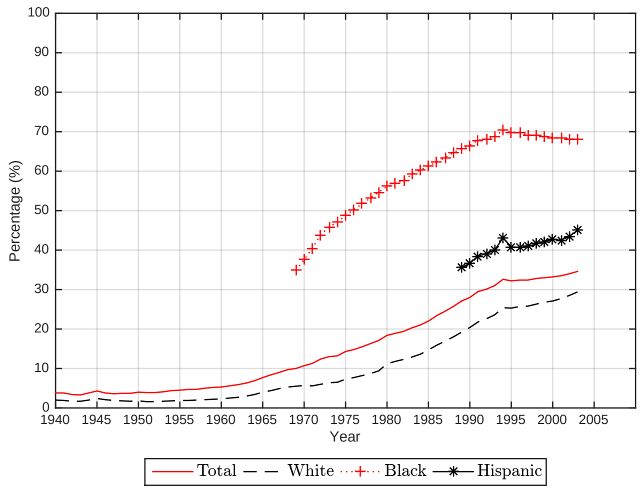

Figure 1: Evolution of unmarried births as a percent of all births, by ethnicity.

Source: own calculation with data from National Center for Health Statistics. Vital Statistics of the United States, 2003, Volume I, Natality.

Household structure affects parents' child care choices because it affects the amount of available economic and caregiving support (Markowitz et al., 2014). A mother living with the child's biological father will most easily arrange work and caregiving to fill in necessary gaps. Both parents will have more available resources and time to dedicate to their children. When the mother is living with her extended family, she may be more likely to ask her relatives to take care of the child. The time that relatives spend with the child will be convenient, flexible, and inexpensive for the mother. By contrast, if a mother lives alone, she may be more likely to choose paid child care because parental care may be inconvenient or unavailable. Single mothers will have fewer economic resources[3], and less access to quality child care. Therefore, they may rely on care

[2]For further references, see Blau (1999), and Berger et al. (2009).

[3]According to the U.S. Census Bureau, in 2012 the poverty rates for children living in single-mother and nuclear-family households were 47.2% and 11.1%, respectively (Haskins, 2015). In Table 2 from the on-line appendix we observe that in 1997, single mothers had a median income of $16,236; compared to a median family income of $52,553 in nuclear households. We also observe single mothers are more likely to be above the poverty line, to have less education, and to rent their homes.

from relatives or friends who might be less educated, and less likely to provide educationally rich settings to young children (Goelman and Pence, 1987).

Moreover, mothers' participation in the labor market raises concerns about their child care arrangements and the effects on their children's cognitive development (Almond and Currie, 2011). For instance, Bianchi (2000) shows that between 1978 and 1998, the total average of working hours of married women with children under 6 increased from 583 to 1,094[4]. The author also reports that, according to the Current Population Surveys of the years 1965 and 1994, the use of center care as a primary child care arrangement increased from 7.9 to 51.7 % for mothers in the labor force and from 4.8 to 44% for mothers not in the labor force.

The effects of maternal employment and child care on child development has been extensively studied in the literature, especially in disadvantaged households (see Currie, 2001 and Blau and Currie, 2006). Because of the endogeneity of child care and child outcomes, numerous studies employ regression with covariates, fixed effects models, or instrumental variables approaches to avoid biased estimates. Most of the studies find a negative association between maternal employment during the first years of life and children's outcomes, and also by the substitution of maternal care for low quality child care. For instance, Bernal and Keane (2011) exploit the exogenous variation in welfare policy rules facing single mothers. They use welfare policy variables as instruments to estimate a child cognitive ability production function. They estimate that a year of child care reduces child test scores by 2.1%.

Economic factors affect parents's available time to dedicate to their children and, as a consequence, their children's skills (Blau, 1999). Previous studies focused attention on estimating a child's technology of skill formation without modeling parent's preferences or budget constraints. This literature is based on Todd and Wolpin (2003, 2007). They describe how to specify and estimate a production function of cognitive development, recognizing child development as a cumulative process. Similarly, Cunha and Heckman (2008) and Cunha et al. (2010) estimate models of the evolution of cognitive and non-cognitive skills to explore the impact of the household environment and investments on the child's ability during the life cycle. They find that, in disadvantage households, it is optimal to invest more in the early stages of childhood than in later stages.

Building on these previous studies, formal structural models of parental investment incorporate the production function of skill formation into a household maximization problem. They include investment in goods and time as factors that stimulate the production function of the child's ability.

Bernal (2008) is one of the first studies that evaluates the impact of both maternal employment and non-parental child care on child outcomes applying a structural approach. Using data from the National Longitudinal Survey of Youth, the author finds that a mother who works full-time and uses child care during one year reduces her child's test scores by 1.8%.

[4]Table 1 in the on-line appendix includes the evolution in the employment trends for women between 1978 and 1998. Also, Table 2 presents the main characteristics of the working women.

Similarly, Del Boca et al. (2014) use the Panel Study of Income Dynamics and Child Development Supplement to estimate a Cobb-Douglas production function of cognitive skills. Parents make labor supply decisions and provide time and money inputs into their children's quality production function. Results show that parents' time inputs enhance the cognitive development of young children, but at a decreasing rate.

The model that I propose follows the strategy developed by Del Boca et al. (2014). This original article includes parental time and physical goods as inputs in the production function of skills. I posit a technology of child's ability depending on different child care options, mother's education, and the previous level of skill. I study the effects of single mothers' decisions on their children's cognitive and non-cognitive skills. As pointed out by Kautz et al. (2014), because both cognitive and non-cognitive skills can be shaped and changed over the life cycle, they are called skills.

To measure cognitive ability, I use the Peabody Picture Vocabulary Test (PPVT; Dunn et al., 1997), which was administered by the Fragile Families study during rounds two and three. Children's non-cognitive skills are analyzed through behavioral problems measured with subscales derived from the Child Behavioral Checklist (Achenbach and Rescorla, 2000). In order to focus on the contribution of each child care option on the child's ability, I do not include material inputs in the production of skills. As Del Boca et al. (2014) found in their article, physical goods do not represent a critical factor on the cognitive ability of young children. Also, the Fragile Families Study does not include data about investment in material resources[5]. Some assumptions are considered in order to obtain closed-form solutions of the expressions that will allow me to simulate paths of endogenous and exogenous variables. The assumptions let me reduce computational burden, and ease interpretation of the results. I also explore the benefits of including several institutional child care options, such as daycare, Head-Start, and kindergarten. These specific options were not included in the original article.

3 Model

The model follows the framework proposed by Becker and Tomes (1986) and Ribar (1992), with the empirical application developed by Del Boca et al. (2014). Household preferences are described by a single mother's utility function, with the ability of the child as a factor, and subject to a child's technology of skill formation, plus budget and time constraints.

3.1 Timing and household's preferences

The model begins at the birth of the child. A single mother makes sequential decisions during the child's development, being t the child's age. The mother

[5] Del Boca et al. (2014) do not have information about physical investments in their dataset, however, they can still identify the model.

invests in child quality from the first period, $t = 1$, through the last development period, T. At the terminal point, the child starts the next development stage (primary school)[6]. I will not include additional model decisions beyond that point. The focus is on the child's experience prior to the terminal period.

Each period, the single mother decides her labor supply (h_t), the amount of time to dedicate to her child (τ_t), the informal child care (π_t), and the institutional child care (ν_t, also called formal child care) to maximize her expected lifetime utility. The model focuses on a single-mother household with only one child who is under five years old. During this period, the mother does not get married to the child's biological father, nor engage or live with a new partner. The total household's utility depends on three components. The first component is the utility that the mother receives from leisure, represented by l_t. The second component is the utility derived from consumption, which consists on expenditures on market goods consumed by the whole household (denoted by c_t). The last component is the utility that the mother obtains from the child's development outcome, k_t. The single-mother values the current level of her child's ability because of paternalism. The child's outcome can be a cognitive or non-cognitive measure of ability. Hence, the total Cobb-Douglas utility function of the household can be represented as

$$u(l_t, c_t, k_t) \quad = \quad \alpha_1 \ln l_t + \alpha_2 \ln c_t + \alpha_3 \ln k_t. \tag{1}$$

The time that the mother spends with her child represents one type of investment in the child; no direct utility is derived from this decision. The parameters $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$ define the elasticities of the factors in the utility function.

The mother decides how to allocate the household income into consumption and investment in the child's development. Investment represents the amount of money that a mother spends on the different child care arrangements. Then, the total budget constraint is represented by

$$c_t + p_\nu \nu_t + p_\pi \pi_t \quad = \quad w_t h_t + I_t, \tag{2}$$

where w_t denotes the mother's hourly wage. The variable I_t represents non-labor income such as government transfers[7]. It also includes financial assistance from the father that is the result of mutual agreements (child support). The wage and non-labor income are exogenous in the model. The variable ν_t represents the amount of time that the child spends in a formal child care, for instance, daycare, Head-Start, or a kindergarten institution[8]. This child care option has a cost of p_ν per hour, which remains constant during the period of analysis, and does not depend on the quality of care[9]. The variable π_t stands for the

[6] The terminal period is set to $T = 5$; Bernal (2008) follows a similar strategy. Del Boca et al. (2014) consider their model with $T = 16$. The child's quality k_{T+1} is an initial condition for the next stage of the development process.

[7] This variable is included to consider that in disadvantaged households, government support is a important factor in their budget constraint.

[8] The main characteristics of these child care options are described in Section 4.

[9] Section 8 presents a modified version of the model where I consider all the paid child care options separately with their corresponding price.

amount of time that the child spends in informal child care, such as with a non-resident relative. This type of child care has an hourly cost of p_π that regards transportation cost or compensation to the relative for the service[10]. The total income is the aggregation of the labor income ($w_t h_t$), and the external transfer. Because the model focuses on single-mother households with low levels of income there are no borrowing and savings that can affect the household's decisions[11].

The mother has a total time endowment (T^m) of 112 hours[12]. Each period, she can allocate her time endowment across three choices. The first option represents leisure time. The mother can also devote some time to her child, or work in the labor market. Then, her time constraint can be represented by

$$T^m = l_t + h_t + \tau_t. \tag{3}$$

3.2 Technology of Skill Formation

The production function of child's skill is a simplified version of Cunha and Heckman (2008). It depends on parents' investment in the child (which includes the formal and informal options of child care), mother's education, and the previous level of the child's skill. Parents' cognitive and non-cognitive skills are not a factor in the child's production of ability[13,14]. At age t, the child's output k_t is produced according to the following Cobb-Douglas production function

$$\ln k_{t+1} = \delta_t^\tau \ln \tau_t + \delta_t^\nu \ln \nu_t + \delta_t^\pi \ln \pi_t + \delta_t^{\mathcal{E}} \ln \mathcal{E}^m + \delta_t^k \ln k_t. \tag{4}$$

The concavity feature of this Cobb-Douglas representation is the result of limiting factors. The productivity parameters, δ_t^j, for $j = \tau, \nu, \pi, \mathcal{E}, k$, vary through time according to the age of the child, but are the same for all the households[15].

[10] I follow the strategy proposed by Ribar (1992) to include hours of formal and informal child care in both the production function of skills and in the budget constraint. This approach helps to avoid degenerate solutions, that is, single mothers might use an infinite amount of child care (see also Lokshin, 2004). In our model, we estimate positive values for both prices. In particular, p_ν represents a shadow price that captures the effects of the indirect costs of non-market care. Prices of formal and informal child care are estimated because of data limitations, that is, we cannot consider a particular data generation process to simulate the prices in the model.

[11] Caucutt and Lochner (2012), using data from NLSY, CNLSY, and 2006 March CPS, investigate the importance of family borrowing constraints in determining human capital investments in children at early and late ages. They obtain evidence that suggests that borrowing constraints bind for at least some families with young children.

[12] I consider that the mother has 16 hours hours available per day during one week.

[13] The FFCWS data set does not include information about parent's non-cognitive skills. Besides, only a few observations of parents' cognitive measures are included in round two.

[14] No material inputs are considered in the production function because, as Del Boca et al. (2014) found in their article, physical goods do not represent a critical factor in the production of skills for young children. Morover, the FFCWS study does not include this information. Therefore, I decided to use mother's education in the production function.

[15] This assumption reflects that the marginal productivity of factors are not constant across time. A factor can be more productive in the child's ability at a particular stage of the development process. This result is based on the work of Cunha et al. (2010); Heckman and Masterov (2007); Heckman (2007).

3.3 Dynamic programing problem

According to the current level of child quality, wage offers, and the exogenous transfer, the household solves the following Bellman equation

$$\begin{aligned} V_t(k_t, w_t, I_t) &= \max \alpha_1 \ln l_t + \alpha_2 \ln c_t + \alpha_3 \ln k_t + \beta \mathbb{E}_t V_{t+1}(k_{t+1}, w_{t+1}, I_{t+1}) \\ \text{s.t.} & \qquad (5) \\ T^m &= l_t + h_t + \tau_t \\ w_t h_t + I_t &= c_t + p_\nu \nu_t + p_\pi \pi_t. \end{aligned}$$

Single mothers optimally decide their labor supply and child care choices that maximize the expected discounted household utility over the development stage. The conditional expectation operator ($\mathbb{E}_t$) at time t is taken with respect to the random variables that appear at time $t+1$. These variables are the mother's accepted wages (w_{t+1}), and the non-labor income (I_{t+1}). The state variables at $t = 1$ are k_1, w_1, and I_1.

3.4 Terminal condition and optimal solutions

The solution of the model will be represented by the sequence $\mathbf{\Upsilon}_t^* = \left\{h_t^*, l_t^*, \tau_t^*, \nu_t^*, \pi_t^*, c_t^*\right\}_{t=1}^{T}$. As this model involves a dynamic programming problem with a terminal condition, the optimal solutions can be obtained by the application of backward induction. According to the backward induction process, the first step consists in finding the optimal amount of labor supply, maternal time, and child care options at time T. Then, we can compute the optimal solutions of the leisure time, and consumption for the same period. A further description of this methodology is provided in Appendix A.1. Having obtained the solutions for the period T, we can apply the same strategy for the remaining periods $t = T-1, ..., 1$. For any period t, the optimal solutions, conditional on h_t, are:

$$\pi_t^* = \frac{(w_t h_t + I_t)\lambda_t^\pi}{(\alpha_2 + \lambda_t^\nu + \lambda_t^\pi)p_\pi}, \qquad (6)$$

$$\nu_t^* = \frac{(w_t h_t + I_t)\lambda_t^\nu}{(\alpha_2 + \lambda_t^\nu + \lambda_t^\pi)p_\nu}, \qquad (7)$$

$$\tau_t^* = \frac{(T^m - h_t)\lambda_t^\tau}{\alpha_1 + \lambda_t^\tau}. \qquad (8)$$

Because of the adopted functional form of child's ability, τ_t^*, ν_t^* and π_t^* are restricted to be positive numbers[16]. The optimal solution of the labor supply is determined as follows:

$$h_t = \frac{T^m w_t(\alpha_2 + \lambda_t^\nu + \lambda_t^\pi) - I_t(\alpha_1 + \lambda_t^\tau)}{w_t(\alpha_1 + \alpha_2 + \lambda_t^\tau + \lambda_t^\nu + \lambda_t^\pi)}. \qquad (9)$$

[16]Different specifications of utility and production functions were considered but they did not allow to obtain closed-forms solutions of the endogenous variables. With this specification, we can derive analytic solutions for the endogenous variables that enables us to reduce computational burden.

I allow for corner solution of the labor supply to consider single mothers who might decide not to work and spend more time with their children or in leisure activities. Then, the actual labor supply is given by:

$$h_t^* = \begin{cases} h_t, & \text{if } h_t > 0, \\ 0, & \text{if } h_t \leq 0. \end{cases} \tag{10}$$

The optimal choices will allow us to determine the level of the child's ability during each period, denoted by $\{k_t^*\}_{t=1}^T$. Recall from equation (4),

$$\ln k_{t+1}^* = \delta_t^\tau \ln \tau_t^* + \delta_t^\nu \ln \nu_t^* + \delta_t^\pi \ln \pi_t^* + \delta_t^{\mathcal{E}} \ln \mathcal{E}^m + \delta_t^k \ln k_t^*. \tag{11}$$

Each period, the vector of endogenous variables $\mathbf{\Upsilon}_t^*$ depends on the parameters $\mathbf{\Lambda}_t = (\alpha_1, \alpha_2, \alpha_3, \beta, \psi, \delta_t^\tau, \delta_t^\nu, \delta_t^\pi, \delta_t^{\mathcal{E}}, \delta_t^k)$, prices $\mathbf{P} = (p_\nu, p_\pi)$, and exogenous variables $\mathbf{\Phi}_t = (w_t, I_t, \mathcal{E}^m)$.

4 Data

I estimate the model with data from the Fragile Families and Child Wellbeing Study (FFCWS). It follows a cohort of unwed new parents and their children to provide information about their conditions, capabilities, and the well-being of their children. The baseline was conducted between 1998 and 2000 in all US cities with a population of 200,000 people or more[17]. They interviewed mothers and fathers at the hospital after the birth of their children. Parents were re-interviewed when their babies were one (round one), three (round two), and five years old (round three)[18,19]. Given the characteristics of the dataset, households in the sample are likely to be low income, to have nonresident fathers, and to have mothers with low levels of education[20]. According to the full sample, 39.69% of the mothers have less than high school education. Moreover, 25.28% of the mothers only completed high school, 24.30% obtained a college degree, and 10.71% received graduate education. In this dataset, half of the mothers initially sampled are African American, and more than one-fourth are Hispanic[21].

This dataset focuses on couples who were not married at the time of giving birth to their children. After the children were born, or during the different rounds of the study, parents might have decided to get married. The data set includes individuals with a number of transitions in their marital status: a single

[17]The included cities are: Austin, TX; Baltimore, MD; Boston, MA; Chicago, IL; Corpus Christi, TX; Indianapolis, IN; Jacksonville, FL; Nashville, TN; New York, NY; Norfolk, VA; Philadelphia, PA; Pittsburgh, PA; Richmond, VA; San Antonio, TX; San Jose, CA; Toledo, OH; Detroit, MI; Milwaukee, WI; Newark, NJ; Oakland, CA; and Jacksonville, FL.

[18]Round one collected the data between June 1999 and March 2002, round two between April 2001 and December 2003, and round three between July 2003 and February 2006.

[19]Section D of the online appendix contains more information about the different rounds of the study, construction of the variables, and relations between the main variables.

[20]The online appendix includes an analysis of the relationship between the household's poverty level and education of the mother at the baseline.

[21]Section D of the online appendix presents more information about parents' ethnicity.

mother might have decided to marry the child's biological father, to marry a new partner, to remain single and take care of her child by herself, or to choose different combinations of these alternatives[22]. As I described in Section 3.1, the model focuses on single-mother households during the first five years of their children. Therefore, I restrict the sample to mothers who did not get married during the three rounds of the study either with their children's biological fathers nor new partners and did not have another child. Although the FFCWS might follow families with more than one child, it only collects information of one child in the household. With these restrictions, I can track 176 single-mother households with one child during the three rounds of the data[23].

Table 1 contains the summary statistics of the final sample, with the main characteristics of the mothers, children, and the different time allocations by round. The sample reflects the basic features of the FFCWS: 60% of the mothers are black, and more than 60% have at most high school education. This low level of education leads to a low level of annual income: during the entire period, the average household income is equal to $28,012.92.

4.1 Outcome variables

As pointed by Heckman et al. (2006) and Cunha and Heckman (2008), both cognitive and non-coginitve skills are relevant measures of child outcomes. There are different measures of skills that provide sufficient information about child's quality. I will focus on measures of children's skills during the critical period of the first five years of their lives (Kautz et al., 2014). I use the children's PPVT score, a test of receptive vocabulary (Dunn et al., 1997)[24], as a measure of the child's cognitive skill. This test was administered to children at the ages of three and five by the FFCWS. It assesses the size and range of words that children understand. The average PPVT score for children in rounds two and three were equal to 26.63 and 65.10, respectively[25].

Non-cognitive skills are measured through behavioral problems using subscales derived from the Child Behavioral Checklist (CBCL, Achenbach and Rescorla 2000). This measure evaluates maladaptive behavioral and emotional

[22]For instance, at round three 53% of the unmarried mothers had one transition, 23% experienced 2 transitions, 16% observed 3 transitions, and 8% recorded 4 or more transitions. A "transition" is the change from a marital or cohabitating status to another.

[23]There are 4,898 observations at the baseline in the FFCWS. There are only 922 families with one child during the three rounds of the data. I dropped 52 extra families because the mother got married during the five years of analysis. After we eliminate families that present missing observations of the relevant variables, we obtain the final sample of 176 families.

[24]This variable identifies the child's achievement during the learning process. Dunn et al. (1997) describes the two possible forms of this test, Form III-A and Form III-B, each of which contains 204 items grouped into 17 sets of 12 items each. The FFCWS used the Form III-A. More information is provided in Section D of the online appendix.

[25]In the online appendix I present an analysis of the distribution of the cognitive and non-cognitive measures by ethnicity and mother's education. In general, children whose mothers have more than high school education perform better than children whose mothers only have a primary education or high school. Also, white mothers' children have higher scores than children whose mothers are Hispanic, black, or from other ethnicities.

Table 1: Summary statistics

	$t=0$		$t=1$		$t=3$		$t=5$	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Mother's education level:								
Primary	0.30	0.46						
High School	0.35	0.48						
College	0.32	0.47						
Graduate	0.03	0..18						
Mother's ethnicity:								
White	0.16	0.37						
Black	0.60	0.49						
Hispanic	0.23	0.42						
Other	0.01	0.11						
Mother's age			23.24	4.99	24.87	5.01	27.02	5.00
Annual HH income/$1,000			25.71	25.02	26.15	22.87	29.46	26.28
Weekly non-labour income			57.14	82.90	75.54	103.81	83.03	121.91
Proportion woking			0.99	0.07	0.99	0.08	0.99	0.08
Child is male	0.52	0.50						
Outcome variables:								
PPVT raw score					26.63	14.34	65.10	17.82
Externalizing score					30.06	7.69	46.45	7.86
Internalizing score					38.83	4.82	40.26	4.05
Endogenous variables:								
Leisure, l_t					35.63	26.10	32.37	23.63
Working, h_t			36.38	10.81	36.08	10.03	36.48	9.56
Maternal care, τ_t					42.87	30.18	44.55	22.46
Formal care, ν_t					15.78	24.84	27.76	18.19
Informal care, π_t					29.14	27.31	15.84	20.99
Formal care options:								
Daycare, ν_t^D					15.78	24.84	4.35	11.82
Head-Start, ν_t^H							13.88	18.89
Kindergarten, ν_t^K							9.51	15.21
N	176							

Notes:
Own calculations using data from the FFCWS. The table includes the average and standard deviation of the main variables included in the model.

problems. It consists of three-point Likert items in which mothers report whether their child's behavior is not true (0), sometimes or somewhat true (1), or often or very true (2). The CBCL covers two dimensions of analysis: Externalizing and Internalizing problems. Externalizing problems refer to the child's aggressive and rule-breaking behavior[26]. Internalizing problems correspond to children's scores on the anxious/depressive and withdrawn behavior. Items are summed to form the Externalizing and Internalizing indexes. The scales are normalized so that higher numbers correspond to more socially desirable behavior[27].

4.2 Control variables

The dataset contains information about child care choices since the child turned three years old. Institutional child care is a variable that reflects the time the child spends at different paid child care options. The FFCWS includes daycare at round two. At round three it also consists of the time spent in preschool institutions such as Head-Start and kindergarten.

Daycare are paid care institutions that provide supervision, protection, and safety to children. State legislation specifies the starting age, minimum, and maximum number of hours. This paid car option offers developmental learning opportunities that contribute to the cognitive, physical, emotional, and social aspects of child development. Head-Start is a preschool program, aimed to help three and four year old disadvantaged children to improve their cognitive skills. This program begun in 1965 as part of the "War on Poverty" program (Blau and Currie, 2006). Kindergarten is a care option for five-year-old children, which can be part of a public or private school system.

The variable π_t covers the time that the child is being taken care by the non-resident father, siblings, maternal grandparents, other maternal relatives, paternal grandparents, other paternal relatives, or the mother's friends.

The variable transfer includes government assistance programs, such as income assistance (e.g., unemployment insurance, workers' compensation, SSI). It also consists of Temporary Assistance for Needy Families program (TANF, which mandates that mothers meet work requirements to receive benefits and limits lifetime welfare benefits to a maximum of five years). Finally, it includes the Supplemental Nutritional Assistance Program (SNAP, commonly referred as Food Stamps), and the Earned Income Tax Credit (EITC).

4.3 OLS Estimations

Table B1 reports estimates of the impacts of child care choices on the child outcomes at age five. Columns (1), (2), and (3) refer to the PPVT, Externalizing, and Internalizing scores, respectively. They include maternal time, formal and informal child care, mother's education, and the child's previous level of ability (k_{t-1}, score at age 3). Both the dependent and independent variables are expressed in logs, as in equation (4). In column (1) we observe that when the child is five years old, the PPVT increases with all the different child care options, however the estimates are not significant. The most relevant factors are the mother's education and the child's previous level of ability.

[26] The online appendix describes the items included to make up these scales.

[27] A similar strategy is applied in Heckman et al. (2013).

According to column (2), different child care options have distinct impacts on the child's Externalizing score. The different child care options and mother's education enhance the child's non-cognitive measure, but without statistical significance. For the Internalizing score (column 3), maternal time, informal care, and mother's education reduce the child's ability, but as in the previous outcome, there are no significance in any of the explanatory variables, except for the previous level of ability.

I also split the variable ν_t to consider more in detail the effects of different formal care options on child development. We observe that at the age of five, daycare, Head-Start, and kindergarten have a positive effect on the child's PPVT score (column 4), with significant results at the 10% level for daycare and kindergarten. In this new specification, the Externalizing score increases with all the different child care arrangements, but without significant estimates (column 5). Finally, the Internalizing score decreases with the maternal time, informal care, and mother's education and increases with the different options of formal child care (column 6).

These results should be interpreted with caution. The estimated parameters of a production function of skills are generally biased and inconsistent when we do not take into account the endogeneity of the inputs. The estimation strategy (presented in the next section) produces unbiased results by solving the simultaneity problem of the endogenous decisions.

5 Econometric strategy

5.1 Description of the DGP

In this section, I describe the Data Generation Process (DGP) of the parameters and exogenous variables included in solutions of the endogenous variables (see Section 3.4) following the approach proposed by Del Boca et al. (2014). The primitive parameters that form the vectors $\mathbf{\Lambda}_t$ and $\mathbf{\Phi}_t$ are generated according to the following assumptions.

Household preference parameters ($\boldsymbol{\alpha}$) are assumed to be fixed over time but different across households, such that $\alpha_j \in (0, 1)$, and $\sum_{j=1}^{3} \alpha_j = 1$[28]. Preference parameters depend on a vector $\boldsymbol{\zeta}$, which is a draw from a bivariate normal distribution with mean μ_ζ and covariance matrix Σ_ζ, such that

$$\boldsymbol{\zeta} \sim \mathcal{N}(\mu_\zeta, \Sigma_\zeta), \quad \text{where } \boldsymbol{\zeta} = \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix}, \mu_\zeta = \begin{bmatrix} \mu_{\zeta_1} \\ \mu_{\zeta_2} \end{bmatrix}, \text{ and } \Sigma_\zeta = \begin{bmatrix} \sigma^2_{\zeta_1} & \sigma_{\zeta_1,\zeta_2} \\ \sigma_{\zeta_1,\zeta_2} & \sigma^2_{\zeta_2} \end{bmatrix}.$$

I define $\Gamma = 1 + \exp(\zeta_1) + \exp(\zeta_2)$, then, the preference parameters adopt the

[28]This condition implies that the utility function adopts a Cobb-Douglas representation with constant returns to scale. Therefore, utility increases in each argument, and also, only the relative value of the utility matters.

following functional forms

$$\alpha_1 = \exp(\zeta_1)/\Gamma, \quad (12)$$
$$\alpha_2 = \exp(\zeta_2)/\Gamma, \quad (13)$$
$$\alpha_3 = 1/\Gamma. \quad (14)$$

Therefore, μ_ζ and Σ_ζ are the primitive parameters to estimate.

The productivity parameters ($\boldsymbol{\delta}$) vary over time but not across households; families possess the same child's ability production function. Based on Heckman and Masterov (2007), the adopted functional forms allow us to obtain monotonicity in t. The input-specific productivity parameters are generated according to the following expressions,

$$\delta_t^j = \exp(\gamma_0^j + \gamma_1^j \times t), \text{ for } j = \tau, \nu, \pi, \mathcal{E}, k.$$

The structural parameters to estimate are γ_0^j and γ_1^j.

The wage offer presents the following structure,

$$\ln w_t = \mu_{w,t} + \epsilon_{w,t}, \quad \text{with } \epsilon_{w,t} \sim \mathcal{N}(0, \sigma_w^2), \forall t.$$

The term $\mu_{w,t}$ is the mean of the log wage draws of the mother at time t. The empirical specification adopted for the estimation is the following

$$\mu_{w,t} = \mu_w^0 + \mu_w^1 \mathcal{E}^m + \mu_w^2 Age_t + \mu_w^3 Age_t^2,$$

where $\mathcal{E}^m$ represents the mother's years of education (which is time invariant), and Age_t and Age_t^2 are variables associated with the age of the mother.

The non-labor income process reflects that households might not receive transfers during some periods. This can be represented by a truncated version of a latent variable process, such that,

$$I_t^* = \mu_I + \epsilon_{I,t}, \text{ with } \epsilon_{I,t} \sim \mathcal{N}(0, \sigma_I^2). \quad (15)$$

Hence, the actual non-labor income is given by,

$$I_t = \max(0, I_t^*), \forall t. \quad (16)$$

Also, we need to estimate the mother's valuation of the child's cognitive ability at time T (ψ), and the prices p_ν and p_π. Finally, the starting level of the child's ability in the first period is specified as follows,

$$k_1 = \exp\big(\vartheta_0 + \vartheta_1 \mathcal{E}^m + \vartheta_2 \mathcal{E}^f + \vartheta_3 LBW\big), \quad (17)$$

where $\mathcal{E}^m$ and $\mathcal{E}^f$ represent parents' education. These two components try to capture a genetic component in the initial condition of child's ability. Finally, LBW is an indicator variable if the child had a low birth weight. This factor seizes the risk in the initial condition of the child's ability of suffering from poor health, reduced cognitive ability, and eventually low education levels.

With the description of the preference and productivity parameters, and exogenous variables, the vector of structural parameters to estimate is given by, $\mathbf{\Theta} = \big[\mu_{\zeta_1}, \mu_{\zeta_2}, \sigma_{\zeta_1}^2, \sigma_{\zeta_1,\zeta_2}, \sigma_{\zeta_2}^2, \gamma_0^\tau, \gamma_1^\tau, \gamma_0^\nu, \gamma_1^\nu, \gamma_0^\pi, \gamma_1^\pi, \gamma_0^{\mathcal{E}}, \gamma_1^{\mathcal{E}}, \gamma_0^k, \gamma_1^k, \mu_w^0, \mu_w^1, \mu_w^2, \mu_w^3, \sigma_w^2, \mu_I, \sigma_I^2, \psi, p_\nu, p_\pi, \vartheta_0, \vartheta_1, \vartheta_2, \vartheta_3\big]$.

5.2 Method of Simulated Moments (MSM)

The dynamic model presented in Section 3 does not have an analytical expression of theoretical moment functions that can be evaluated directly[29]. For such situations, McFadden (1989) and Pakes and Pollard (1989) proposed to use simulation instead of solving the moment conditions analytically. Let $\{\widetilde{\mathbf{X}}_t^{(s)}(\mathbf{\Theta})\}_{i=1}^{\mathfrak{N}(n)}$ be the data simulated from the model with parameter $\mathbf{\Theta}$ and random seed s for individuals $i = 1, ..., \mathfrak{N}(n)$. Therefore, $\widetilde{\mathbf{X}}_t^{(s)}(\mathbf{\Theta}_0)$ is drawn from the same distribution as the original $\mathbf{X}_t$ and share the same moment characteristics. The parameter $\mathbf{\Theta}$ minimizes the distance between the sample moments of the data and those of the simulated data[30,31], such that,

$$\widehat{\mathbf{\Theta}}^{MSM} = \arg\min_{\mathbf{\Theta}} \big(M_N - M_S(\mathbf{\Theta})\big)' W_N \big(M_N - M_S(\mathbf{\Theta})\big). \tag{18}$$

The matrix W_n is the inverse of the variance-covariance matrix of the data moments, such that,

$$W_N = \left(\frac{1}{R}\sum_{r=1}^{R}(\mathcal{M}_N^r - M_N)(\mathcal{M}_N^r - M_N)'\right)^{-1},$$

where $\mathcal{M}_N^r$ is computed for each r resample from the original data[32].

5.3 Identification

If we could observe the child's output score in two successive periods (for instance at age three and four, and the production factors at age three), the estimation of the production parameters would be straightforward. According to the specification expressed in equation (4), we would have,

$$\begin{aligned}
\ln k_{t+1} &= \delta_\tau^1 \ln \tau_t + \delta_t^\nu \ln \nu_t + \delta_t^\pi \ln \pi_t + \delta_t^{\mathcal{E}} \mathcal{E}^m + \delta_t^k \ln k_t \\
&= \exp(\gamma_0^\tau + \gamma_1^\tau \times t) \ln \tau_t + \exp(\gamma_0^\nu + \gamma_1^\nu \times t) \ln \nu_t + \\
&\quad + \exp(\gamma_0^\pi + \gamma_1^\pi \times t) \ln \pi_t + \exp(\gamma_0^{\mathcal{E}} + \gamma_1^{\mathcal{E}} \times t) \ln \mathcal{E}^m + \\
&\quad + \exp(\gamma_0^k + \gamma_1^k \times t) \ln k_t \\
&\equiv \Gamma(t, \tau_t, \nu_t, \pi_t, \mathcal{E}^m, k_t; \gamma).
\end{aligned}$$

[29]Traditional GMM requires a set of moment conditions to be known. That is, given g a set of moment conditions such that $E[g(\mathbf{X};\Theta_0)] = 0$, the GMM minimizes a distance measure

$$\widehat{\Theta}^{GMM} = \arg\min_{\Theta} \left(\frac{1}{n}\sum_{i=1}^{n} g(X_i;\Theta)\right)' W_n \left(\frac{1}{n}\sum_{i=1}^{n} g(X_i;\Theta)\right),$$

where W_n is a certain positive definite weighting matrix of $q \times q$-dimension, which may depend on the sample but not on Θ. This method requires that the moments are known analytically or easy to calculate.

[30]A good reference to analyze this estimator is Gourieroux and Monfort (1997).

[31]Given $M_N = \frac{1}{n}\sum_{i=1}^{n} g(\mathbf{X}_i)$, and $M_S(\mathbf{\Theta}) = \frac{1}{\mathfrak{N}(n)}\sum_{i=1}^{\mathfrak{N}(n)} g(\widetilde{\mathbf{X}}_t^{(s)};\mathbf{\Theta})$.

[32]Where $r = 1, ..., R$. In the estimation I set $R = 200$.

Then, we could estimate the model with a non-linear least squares estimator,

$$\hat{\gamma}_{NNLS} \quad = \quad \arg\min \left(\ln k_{t+1} - \Gamma(t, \tau_t, \nu_t, \pi_t, \mathcal{E}, k_t; \gamma) \right)^2.$$

The main problem in the estimation is the missing data in the sample. This model considers periods of one year, but the FFCWS collects data at age three and five. Therefore, this gap in the data makes it impossible to use successive observations of child quality along with inputs to estimate the parameters directly. Also, this strategy does not allow us to recover the preference parameters.

The only tractable way to fill in this gap is to simulate paths of the endogenous variables over the entire period using the DGP described in the previous subsection. The FFCWS provides enough information to identify the model, with the help of the distributional and functional form assumptions of technology, preference, and wage processes (described in Sections 3 and 5.1.).

In order to obtain the basic trend in the data for the variables mother's labor supply and care, formal and informal child care, and child's cognitive and non-cognitive scores, I use the average and standard deviation of these variables at different ages of the child, and the probability of the mother working.

Mothers' preference parameters are identified with the correlation of the endogenous choice variables and the exogenous variables (such as wages and non-labor income). The identification of the productivity parameters is obtained by computing a number of contemporaneous and lagged correlations between the observed labor supply, maternal time, and child quality. Moreover, I include as moments the mother's average working hours by level of education and age.

To identify the parameters of the wage equation, I count as moments the mother's average salary by her age and education. In particular, I study the average salary of the mother with 12 and more than 16 years of education, to include high school and at least college degree education, respectively. Moreover, the parameters in the non-labor income are identified with it average and standard deviation, and with the average of the non-labor income for those households who receive less that \$100 per week.

5.4 Estimation

The data available from the FFCWS consist of a sample of households with observed characteristics at a few points in time[33]. At the baseline we know most of the time-invariant characteristics[34]. We also observe the age of the mother, and other household's characteristics that are necessary for defining the sample[35]. In the following three rounds of the data, we have access to the household decisions and child's characteristics at various ages. For instance, at round one (when the child is on average one year old) we have data of household

[33]The online appendix includes more information about the details of the variables observed at different points in time.

[34]The time-invariant variables in the model collected at the baseline are: mother and father's education ($\mathcal{E}^m$ and $\mathcal{E}^f$, respectively), and the indicator of low birth weight (LBW).

[35]For instance, the number of household members ($members_t$) and the presence of just one child in the household who is less than five years old ($I_t\{children < 5\}$).

income, mother's age, child's age, working hours, and (accepted) hourly salary. In rounds two and three, we have data of leisure time, working hours, accepted wages, child care arrangements, and the measures of child quality.

Following the strategy described by Del Boca et al. (2014), I use simulation methods to estimate the model. First, I define a set of sample characteristics, which summarizes the relationships in the sample at each survey date and across survey dates. I represent the vector of sample characteristics with M_N. For each household i, I generate a set of S sample paths over the development period as follows: the empirical process begins with the data collected in round one ($t = 1$). Given the mother's characteristics at the sample date (education and age), I draw from the distribution of shocks to wages and non-labor incomes to determine the initial wage and non-labor income draws. Moreover, I draw from the distribution of household preferences, and this draw stays with the household over the entire sample path. I compute the labor supply and investment decisions with expressions (6)-(9), and the child's initial condition according to equation (17). Then, I can calculate the child's ability at the end of the first period (k_{t+1}).

I repeat this process at $t = 2$ using the DGP described previously. I use the child's computed level of skill at the end of $t = 1$ as the current level of ability. Since the wage and non-labor income processes are assumed to be conditionally (on observable characteristics) independently distributed over time, I draw new wage offers and non-labor income.

At $t = 3$, even though we have data of the child's current level of ability, I use the simulated score of this period. The reason is that the simulated child's score computed at $t = 2$ might not be equal to the real score obtained from the data. The same consideration applies for the following two periods. At $t = 4$ there are no observations on child quality or investments, but using the DGP I can simulate these variables. I determine the child's quality at $t = 4$ with the previous level of child's ability and the inputs obtained from the simulation. Finally, at $t = 5$, I repeat the mechanism described for $t = 3$, since at this period I also have the complete set of information available. This strategy lets me "fill in" the portion of the empirical sample path between data points.

For each household i, I repeat this strategy S times, so that in the end I have $S \times N$ sample paths. Using the simulated data set, I then compute the analogous simulated sample characteristics to those determined from the actual data sample. The characteristics of the simulated sample depend on $\mathbf{\Theta}$, the vector of all primitive parameters that characterize the model. Also, the simulated data depends on a vector of pseudorandom number draws made to generate the sample paths. With the actual and simulated statistics, I construct the objective function to minimize (equation 18), using the Nelder Mead Simplex Optimization. This estimation strategy is applied for each outcome k_t (PPVT, Externalizing, and Internalizing scores).

6 Results

6.1 Preference Parameters

Table 2 presents the estimated preference parameters for the three outcomes, with β set to 0.95. The table includes the averages and standard deviations of the parameters $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$. Single mothers have strong preferences on child's quality (according to the parameter α_3) for both cognitive and non-cognitive measures. Moreover, the averages and standard deviations of the preference parameters present similar values across the three models. These estimated preferences of our cognitive measure are in line with the results obtained by Del Boca et al. (2014). They found that nuclear families possess a strong preference for child cognitive ability, with an average coefficient equal to 0.353. In their model, preferences for leisure are equal to 0.196 and 0.194 for the mother and father, respectively. Finally, the authors obtain that parent's preference for consumption is equal to 0.257.

Table 2: Estimation results: preference parameters$^{(*)(**)}$

	Cognitive	Non-cognitive	
	PPVT	Externalizing	Internalizing
Mean of α_1	0.240	0.241	0.259
	(0.027)	(0.026)	(0.024)
Mean of α_2	0.126	0.135	0.054
	(0.018)	(0.071)	(0.007)
Mean of α_3	0.634	0.624	0.687
	(0.038)	(0.065)	(0.024)
SD of α_1	0.244	0.222	0.225
	(0.016)	(0.008)	(0.019)
SD of α_2	0.138	0.127	0.140
	(0.012)	(0.062)	(0.004)
SD of α_3	0.368	0.332	0.302
	(0.016)	(0.041)	(0.020)
Terminal payoff to child quality (ψ)	5.177	1.468	9.527
	(4.124)	(28.722)	(23.177)

Notes:

($*$) Parameter estimates obtained by the application of the MSM using data from FFCWS, with the objective function represented by equation (18). The endogenous variables ($\boldsymbol{\Upsilon}_t^* = \{h_t^*, l_t^*, \tau_t^*, \nu_t^*, \pi_t^*, c_t^*\}$, for $t = 1, ..., T$) are simulated with the DGP described in Section 5.1.

($**$) Standard errors in parenthesis obtained by bootstrap with 200 replications.

The valuation of the child's ability at the terminal period (ψ) differs with respect to the outcome. To interpret this number, we have to analyze the meaning of this parameter in the terminal period. If we assume the household lives infinitely with a parameter $\beta = 0.95$, the terminal value of the child quality would be $(1-\beta)^{-1} = 20$. The obtained parameter estimates indicate that single mothers pose a modest value of child quality in the following periods. Del Boca et al. (2014) found a coefficient ψ equal to 28.89 in nuclear families. Their result might be due to a stronger preference that those households place on child development. However our estimates are not significant.

6.2 Productivity Parameters

The structural parameter estimates of the child's production function of skills are included in Table 3. The interpretation of the parameters γ_0^j and γ_1^j is the following: γ_0^j represents the intercept of the function δ_t^j, and γ_1^j is the time slope; where δ_t^j is the productivity of the input j at time t for a particular outcome k_t. By the monotonicity condition, the function δ_t^j should be either increasing or decreasing. Table 4 presents the evolution of the parameters δ_t^j at the age of three, four, and five, respectively.

Table 3: Estimation results: primitive technology parameters$^{(*)(**)}$

	Cognitive		Non-cognitive			
	PPVT		Externalizing		Internalizing	
	Intercept, γ_0^j	Slope, γ_1^j	Intercept, γ_0^j	Slope, γ_1^j	Intercept, γ_0^j	Slope, γ_1^j
Maternal time, τ_t	-0.996	-0.222	-0.967	-0.085	-0.889	-0.336
	(0.094)	(0.025)	(0.037)	(0.007)	(0.042)	(0.011)
Formal care, ν_t	-2.521	-0.034	-2.655	0.081	-2.553	-0.083
	(0.186)	(0.042)	(0.050)	(0.009)	(0.067)	(0.025)
Informal care, π_t	-2.062	-0.374	-1.715	-0.228	-1.7182	-0.425
	(0.252)	(0.059)	(0.108)	(0.044)	(0.079)	(0.045)
Mother's education, $\mathcal{E}$	-0.417	-0.368	-0.365	-0.384	0.236	-0.238
	(0.23)	(0.060)	(0.113)	(0.033)	(0.032)	(0.012)
Current skill, k_t	-1.625	0.313	-1.593	0.223	-1.5395	0.207
	(0.026)	(0.006)	(0.026)	(0.007)	(0.028)	(0.006)

Notes:

(*) Parameter estimates obtained by the application of the MSM using data from FFCWS, with objective function represented by equation (18). The endogenous variables ($\boldsymbol{\Upsilon}_t^* = \{h_t^*, l_t^*, \tau_t^*, \nu_t^*, \pi_t^*, c_t^*\}$, for $t = 1, ..., T$) are simulated with the DGP described in Section 5.1.

(**) Standard errors in parenthesis obtained by bootstrap with 200 replications.

According to the estimation results of Table 4, the maternal time is the most productive child care factor of cognitive and non-cognitive measures of skills, but its productivity decreases with the age of the child. For the PPVT score, the informal care presents the lowest productivity among the different options. The productivity of formal care does not present much variation during the period of analysis. Furthermore, for the Externalizing and Internalizing scores, maternal time is the most productive factor but at a decreasing rate, and the productivity of formal care exceeds the contribution of informal care.

Mother's education presents a strong productivity on the different measures of child's ability. This results might reflect that better parenting style is associated with a high level of education, which enhances both, cognitive and non-cognitive child's skills.

Table 4: Estimation results: productivity parameters$^{(*)(**)}$

	Cognitive			Non-cognitive					
	PPVT			Externalizing			Internalizing		
	3	4	5	3	4	5	3	4	5
Maternal time, δ_t^τ	0.190	0.152	0.122	0.294	0.270	0.248	0.150	0.107	0.076
	(0.018)	(0.016)	(0.014)	(0.011)	(0.011)	(0.010)	(0.008)	(0.007)	(0.006)
Formal care, δ_t^ν	0.073	0.070	0.068	0.090	0.097	0.105	0.061	0.056	0.051
	(0.010)	(0.009)	(0.009)	(0.006)	(0.008)	(0.009)	(0.006)	(0.007)	(0.008)
Informal care, δ_t^π	0.041	0.028	0.020	0.091	0.072	0.058	0.050	0.033	0.021
	(0.009)	(0.006)	(0.005)	(0.007)	(0.007)	(0.007)	(0.008)	(0.007)	(0.006)
Mother's education, $\delta_t^\mathcal{E}$	0.219	0.151	0.105	0.219	0.149	0.102	0.620	0.489	0.385
	(0.056)	(0.041)	(0.032)	(0.022)	(0.016)	(0.013)	(0.022)	(0.024)	(0.025)
Current skill, δ_t^k	0.504	0.690	0.943	0.397	0.496	0.620	0.399	0.491	0.604
	(0.015)	(0.023)	(0.034)	(0.010)	(0.014)	(0.020)	(0.010)	(0.013)	(0.016)

Notes:
(∗) Given $\delta_t^j = \exp(\gamma_0^j + \gamma_1^j \times t)$, for $j = \tau, \nu, \pi, \mathcal{E}$, and k. Parameter estimates obtained by the application of the MSM using data from FFCWS, with objective function represented by equation (18). The endogenous variables ($\boldsymbol{\Upsilon}_t^* = \{h_t^*, l_t^*, \tau_t^*, \nu_t^*, \pi_t^*, c_t^*\}$, for $t = 1, ..., T$) are simulated with the DGP described in Section 5.1.
(∗∗) Standard errors in parenthesis obtained by bootstrap with 200 replications.

The productivities of the formal child care and previous level of skill grow with the age of the child for both, the Externalizing and Internalizing scores. These results indicate that as the child starts to interact with other children and teachers in daycare, Head-Start, or kindergarten institutions, his or her behavioral problems decreases. These results reflect the importance of investment in child development. Even though this model does not include investment in physical goods, it demonstrates that household's investment in formal care is beneficial for the development of their children. Besides, the increasing productivity of the current level of skill indicates the cumulative process of child development. By the time the child is above five years old, the current level of child's ability is the most productive factor.

The estimated results of our cognitive measure of ability present similar conclusions to Del Boca et al. (2014). The authors found a decreasing productivity of maternal time, and an increasing productivity of the child's ability. In their model, investments in physical good present a positive effect on child development, which represents the increasing importance of child goods investments. In our case, investments in child development is represented by paid child care options. I include informal care as an additional alternative of outsourcing child care, but we obtain a modest and decreasing productivity of these factor during the five years of analysis.

Bernal (2008) discusses the importance of maternal time in the production of child's cognitive ability. The author finds that when a mother works full-time and uses child care during one year, the child's test scores reduces approximately

1.8%. The model I propose extends the analysis to non-cognitive measures of child's ability. My estimates also reveal the importance of maternal time for child development over other child care options, and the single mother's trade-off in deciding the child care arrangements. I perform a further analysis by observing the effects on children's outcomes of exchanging child care options. In Table 5 I analyze the effects on an outcome k_t at age t of three different experiments: in the first row I increase the formal care option one hour per week and reduce maternal time in the same amount. Results indicate that the three outcomes (at the ages of three, four, and five) are negatively affected by this decision, but at a decreasing rate. In row (2) of Table 5, we increase informal care and reduce maternal time one hour per week. Similarly, the results show a negative impact on the child's scores. Finally, in the last row we increase informal care and reduce formal care one hour per week. The results present the same pattern as in the previous exercises but the magnitudes differ.

Table 5: Effects on children's outcomes of exchanging care options

	Cognitive			Non-cognitive					
	PPVT			Externalizing			Internalizing		
	3	4	5	3	4	5	3	4	5
$\nu_t - \tau_t$	-0.12	-0.08	-0.05	-0.20	-0.17	-0.14	-0.09	-0.05	-0.03
$\pi_t - \tau_t$	-0.15	-0.12	-0.10	-0.20	-0.20	-0.19	-0.10	-0.07	-0.06
$\pi_t - \nu_t$	-0.03	-0.04	-0.05	0.00	-0.03	-0.05	-0.01	-0.02	-0.03

Notes:

The table includes the effects on the child's outcome of combining the child care options. For instance, row (1) shows the effects on the child's outcome by increasing formal care (ν_t) one our per week and reducing maternal time (τ_t) one hour, holding the other option constant.

6.3 Parameters of Wages, Non-labor Income and Prices

Table 6 presents the parameter estimates of mother's wage, non-labor income, and prices. The parameter μ_w^1 represents the increment in the logarithm of the wage offer according to her years of education. The parameters μ_w^2 and μ_w^3 reflect the standard concavity feature of the income process with respect to the age of the mother.

According to the DGP, the non-labor income has a truncated distribution. I found results that are in line with the estimated parameters of Del Boca et al. (2014). The estimated results are the same for the three outputs estimated. The average of the latent non-labor income is equal to 2.999. Also, the estimated variance is equal to 179.870. These two parameters give rise to the variable I_t^* according to expression (16). Then, we can recover the non-labor income by computing the maximum between 0 and I_t^*.

Finally, the estimated prices differ among the three outcomes. The prices of informal care are equal to 3.364, 4.492, and 5.845 for the PPVT, Externalizing, and Internalizing models, respectively. The price of informal care corresponds

Table 6: Estimation results: estimated parameters of the accepted wages, non-labor income, and initial level of ability$^{(*)(**)}$

	Cognitive	Non-cognitive	
	PPVT	Externalizing	Internalizing
Intercept, μ_w^0	0.135	0.303	0.856
	(0.172)	(0.088)	(0.078)
Mother's education, μ_w^1	0.031	0.034	0.026
	(0.012)	(0.006)	(0.006)
Mother's age, μ_w^2	0.085	0.084	0.076
	(0.007)	(0.002)	(0.003)
Mother's age squared, μ_w^3 ($\times$ 1000)	-0.764	-0.970	-1.389
	(0.206)	(0.084)	(0.090)
Variance of innovation (mother's wage), σ_w^2	0.405	0.473	0.514
	(0.057)	(0.037)	(0.035)
Intercept, μ_I	2.999	2.993	2.993
	(13.361)	(14.335)	(13.915)
Variance of innovation (Non-labor income), σ_I^2	179.870	179.870	179.870
	(14.455)	(15.015)	(15.617)
Price of informal care, p_π	3.364	4.492	5.845
	(0.911)	(0.560)	(1.284)
Price of formal care, p_ν	9.746	7.683	10.440
	(1.440)	(1.063)	(1.468)
Intercept, ϑ_0	0.289	0.288	0.286
	(0.021)	(0.005)	(0.006)
Mother's education, ϑ_1	0.122	0.123	0.124
	(0.006)	(0.002)	(0.002)
Father's education, ϑ_2	0.097	0.097	0.098
	(0.004)	(0.002)	(0.002)
Low birth weight, ϑ_3	-0.353	-0.348	-0.350
	(0.021)	(0.007)	(0.007)

Notes:

($*$) Parameter estimates obtained by the application of the MSM using data from FFCWS, with objective function represented by equation (18). The endogenous variables ($\boldsymbol{\Upsilon}_t^* = \{h_t^*, l_t^*, \tau_t^*, \nu_t^*, \pi_t^*, c_t^*\}$, for $t = 1, ..., T$) are simulated with the DGP described in Section 5.1.

($**$) Standard errors in parenthesis obtained by bootstrap with 200 replications.

to unpaid care provider's time in alternative activities. This can also be a measure of unpaid care. If this type of care is provided for some non-monetary compensation (such as room, food, etc.), the shadow cost would represent this agreement. The estimated prices in the PPVT, Externalizing, and Internalizing models of formal child care are equal to 9.746, 7.683, and 10.440, respectively. These estimates represent the hourly cost of the institutional child care option. The estimated prices are in line with the trends of child care costs discussed by Child Care Aware of America (2014).

6.4 With-in Sample Fit

Table 7 displays the sample fit of the simulated model for the wages and non-labor income variables. The model fits well the data in these variables.

Table 7: Sample fit of accepted wages and non-labor income

	Actual†	Simulated‡		
		Cognitive	Non-cognitive	
		PPVT	Externalizing	Internalizing
Mother's wage, w_t				
Average	9.735	10.169	10.706	10.397
St. dev.	7.169	5.475	6.091	5.862
Average (age < 30)	9.460	9.288	10.095	10.447
Non-labor income, I_t				
Average	71.903	71.903	71.903	71.903
St. dev.	104.370	104.371	104.370	104.370
Proportion with $I_t \geq 0$	0.716	0.713	0.713	0.713

(†) Actual data refers to the information obtained from the FFCWS.

(‡) Simulated data corresponds to the data obtained by simulation using the estimated parameters $\hat{\boldsymbol{\Theta}}$.

Table 8 shows the sample fit of the endogenous variables of the model. In general, the model fits the data at $t = 3$ and $t = 5$, which is when we have access to the real data from the FFCWS. The first panel of Table 8 includes the average time of the different child care arrangements, working hours, and probability of working. In general, the simulated average time in formal and informal care fits well the data for the three outcomes[36]. The average working hours marginally differ across the three models, but remain close to the actual data. This model involves single mothers with almost 100% of participation in the labor market, and the results reflect this feature in the simulated data.

The second panel of Table 8 includes the actual and simulated scores of the cognitive and non-cognitive children's outcomes. The results show the expected characteristics of these variables: the averages level of cognitive and non-cognitive skills are increasing with the age of the child with a good sample fit. The online appendix presents the list of moments considered in the estimation process for the three models analyzed. This information allows the reader to analyze more in detail how well the model fits the data.

7 Counterfactual exercises

This section discusses whether policies can be effective to adjust the development process of children under five years old in disadvantaged households. Using the

[36] According to the American Health and Retirement Study, grandparents spent spent 1250 hours in the previous 12 months caring for their grandchildren (Zanella and Rupert, 2011). Our estimates indicate that single mothers use informal care 1,440 hours at the age of three, and 1,056 hours at the age of five.

Table 8: Sample fit of endogenous choice variables (averages)

	Actual[†]	Simulated[‡]		
		Cognitive	Non-cognitive	
		PPVT	Externalizing	Internalizing
Maternal time, τ_t:				
$t = 3$	42.872	50.517	45.516	48.049
$t = 5$	44.545	45.187	40.604	45.386
Formal care, ν_t:				
$t = 3$	15.784	18.901	18.304	19.059
$t = 5$	27.756	26.493	23.894	29.057
Informal care, π_t:				
$t = 3$	29.142	31.228	31.739	28.136
$t = 5$	15.841	22.167	22.343	21.656
Working hours, h_t:				
$t = 3$	36.080	34.706	35.715	32.701
$t = 5$	36.483	38.222	36.976	39.472
Probability of $h_t > 0$:				
$t = 3$	0.994	0.981	0.982	0.958
$t = 5$	0.994	0.986	0.983	0.981
Children's outcomes[⋆], k_t:				
$t = 3$		22.230 [26.625]	28.966 [30.062]	39.221 [38.829]
$t = 5$		66.200 [65.102]	47.383 [46.449]	39.965 [40.261]

Notes:

(†) Actual data refers to the information obtained from the FFCWS.

(‡) Simulated data corresponds to the data obtained by simulation using the estimated parameters $\hat{\boldsymbol{\Theta}}$.

(⋆) Actual data in square brackets.

estimated parameters of our structural model, we can analyze counterfactual exercises that help the policy maker to decide optimal strategies to improve children's outcomes. I present two different policies: in Section 7.1 I analyze an exogenous increase in the accepted wages, while in Section 7.2 I study the effects on children's outcomes of different transfer schemes that provide financial assistance to single mothers.

7.1 Wage increases

In this section we analyze the effects on children outcomes of rising the accepted wages in 10 and 20%. The first column of Table 9 includes the baseline results from the simulated data (the numbers coincide with the simulated averages of Tables 7 and 8). In column (2) we observe the effects on children outcomes and the child care arrangements by increasing the accepted wage in 10%. In general, the three child's outcomes increase by 1%. Single mothers decide to work more hours, to reduce maternal time, and to buy more external sources of child care. Column (3) presents the experiment of a 20% increase in the accepted wages. The results reveal that children's outcomes slightly increase with respect to the baseline, maternal care reduces, and single mothers spend more time in the labor market. These decisions lead single mothers to spend a higher proportion of their income in external sources of care.

Table 9: Effects of exogenous wage increases on the endogenous variables

	t	Baseline (1)	$w_t\triangle 10\%$ (2)	$w_t\triangle 20\%$ (3)
Child's PPVT score				
k_t	3	22.23	22.43	22.62
	5	66.20	67.57	68.86
Endogenous variables, (averages at $t = 5$**)**				
Mother's working hours, h_t		38.23	38.68	39.06
Maternal time, τ_t		45.18	44.89	44.64
Mother's leisure time, l_t		28.59	28.56	28.43
Consumption/1000, c_t		0.16	0.17	0.19
Household's utility/1000, u_t		0.06	0.06	0.06
Formal child care, ν_t		26.52	28.95	31.42
Informal care, π_t		22.18	24.23	26.29
Child's Externalizing score				
k_t	3	28.94	29.38	29.77
	5	47.35	48.63	49.80
Endogenous variables, (averages at $t = 5$**)**				
Mother's working hours, h_t		36.96	37.43	37.81
Maternal time, τ_t		40.63	40.35	40.14
Mother's leisure time, l_t		34.40	34.26	34.10
Consumption/1000, c_t		0.21	0.23	0.25
Household's utility/1000, u_t		0.06	0.06	0.07
Formal child care, ν_t		23.90	26.12	28.34
Informal care, π_t		22.40	24.42	26.50
Child's Internalizing score				
k_t	3	39.21	39.57	39.89
	5	39.96	40.49	40.98
Endogenous variables, (averages at $t = 5$**)**				
Mother's working hours, h_t		39.48	39.93	40.33
Maternal time, τ_t		45.37	45.08	44.82
Mother's leisure time, l_t		27.15	27.03	26.90
Consumption/1000, c_t		0.06	0.07	0.08
Household's utility/1000, u_t		0.05	0.06	0.06
Formal child care, ν_t		29.04	31.75	34.45
Informal care, π_t		21.67	23.67	25.68

7.2 Transfers

In this section I explore two different schemes of government support on single-mother households, called *unconditional* and *conditional* transfers. In the *unconditional* setting, the government provides a certain amount of money to a single-mother household which can be used without any restriction. The mother can spend this extra amount of money either on household consumption or on child development. The *conditional* transfer restricts the use of the financial assistance on child development. In our case, the restriction implies that the mother uses the transfer to buy more formal child care. The purpose of this section is to analyze the effects of both types of transfers on the children's cognitive and non-cognitive skills.

Table 10: Transfers

	t	Baseline	Unconditional transfers			Subsidy
			$50	$100	$200	
		(1)	(2)	(3)	(4)	(5)
Child's PPVT score						
k_t	3	22.62	22.55	22.87	23.45	23.67
	5	68.86	68.21	70.17	73.96	75.15
Endogenous variables, (averages at $t = 5$)						
Mother's working hours, h_t		39.06	34.71	31.24	24.63	19.18
Maternal time, τ_t		44.64	47.34	49.47	53.57	63.11
Mother's leisure time, l_t		28.43	30.08	31.42	33.92	31.04
Consumption/1000, c_t		0.19	0.17	0.17	0.19	0.22
Household's utility/1000, u_t		0.06	0.06	0.06	0.07	0.06
Formal child care, ν_t		31.42	27.59	28.68	30.95	40.00
Informal care, π_t		26.29	23.08	24.00	25.90	19.77
Child's Externalizing score						
k_t	3	29.77	29.61	30.23	31.41	29.33
	5	49.80	49.23	51.05	54.60	50.54
Endogenous variables, (averages at $t = 5$)						
Mother's working hours, h_t		37.81	33.43	29.96	23.46	24.64
Maternal time, τ_t		40.14	42.53	44.42	48.02	51.12
Mother's leisure time, l_t		34.10	36.08	37.66	40.56	36.86
Consumption/1000, c_t		0.25	0.22	0.23	0.25	0.28
Household's utility/1000, u_t		0.07	0.06	0.07	0.07	0.07
Formal child care, ν_t		28.34	24.85	25.82	27.85	40.00
Informal care, π_t		26.50	23.24	24.14	26.04	18.50
Child's Internalizing score						
k_t	3	39.89	39.72	40.20	41.11	39.78
	5	40.98	40.65	41.32	42.60	41.77
Endogenous variables, (averages at $t = 5$)						
Mother's working hours, h_t		40.33	35.86	32.34	25.90	16.78
Maternal time, τ_t		44.82	47.67	49.91	54.18	64.61
Mother's leisure time, l_t		26.90	28.51	29.79	31.95	31.45
Consumption/1000, c_t		0.08	0.07	0.07	0.08	0.15
Household's utility/1000, u_t		0.06	0.06	0.06	0.06	0.06
Formal child care, ν_t		34.45	30.30	31.57	34.27	40.00
Informal care, π_t		25.68	22.58	23.53	25.54	18.70

Notes:
Column (1) presents the averages of the endogenous and outcome variables presented in Section 4.1. Columns (2), (3), and (4) use the parameter estimates from the Section 6 and present the averages of the unconditional transfers. In each setting, the program consists in offering $50, $100 or $200 per week during a period of three years. In column (5), the policy consists in subsidizing the institutional child care. Also, it is compulsory that the mother sends the child to an institutional child care 40 hours per week for a period of three years.

The *unconditional* transfer sets the amounts of $50, $100, and $200 per week to households with a child between three and five years old. There are no restrictions in the use of this benefit. In the *conditional* transfer, the government requires the mother to send her child to an institutional child care 40 hours per week starting at the age of three. The government returns the amount of investment on child development; therefore, this mechanism works as a subsidy.

The results of these exercises are included in Table 10. As in the previous exercise, column (1) presents the baseline results for comparative purposes. Columns (2), (3), and (4) offer us the results of the *unconditional* transfer schemes. A transfer of $50 dollars per week reduces the child's PPVT score by 0.30% and 0.94% at the ages of three and five, respectively. This result is the consequence that single mothers work fewer hours, spend more time in leisure activities, and buy fewer hours of formal and informal care. Similarly, the Externalizing and Internalizing scores present the same results. A transfer of $100 per week shows positive effects on children's outcomes. The time they spend with their children increases in 30 more minutes per week, but there is a considerable reduction of working hours. Single mothers "buy" more leisure, less external sources of care, but spend more time with their children. Cognitive and non-cognitive outcomes increase between 1% and 2% depending on the age of analysis. Finally, an *unconditional* transfer of or $200 per week present stronger results on children's outcomes, in particular with considerable increments on the child's PPVT and Externalizing scores at the age of five.

The results of the subsidy experiment are included in Table 10, column (5). This linear child care subsidy changes the budget constraint by increasing the net wage. We observe a reduction in maternal labor supply, given that households whose pre-transfer level of expenditures on the child was substantially less than the amount of the transfer, there is essentially no income effect from receiving it. The effects are stronger for the cognitive child's measure. The PPVT score increases to 23.67 and 75.15 at the ages of three and five, respectively. The significant impact on the PPVT score is derived by sending a child 40 hours per week to institutional child care starting at the age of three. Also, this result is a consequence of an increment of 20 extra hours of maternal care, and a decrease of 25% of informal care. Moreover, single mothers reduce considerably their working hours, which allows them to spend this extra available time in leisure activities. They also can allocate their extra money, derived from the saving in child development, on consumption.

Non-cognitive measures present weaker results than in the previous case. The Externalizing score reduces 1.48 % at the age of three, but increases 1.5% at the age of five. Single mothers increase the maternal time (τ_t) in 10 hours, and reduce informal care in 8 hours. Given that the productivity parameters of the different child care arrangements are slightly similar, the results show modest effects on children's Externalizing scores. The Internalizing score reduces marginally at the age of three, but increases 2.15% in the last year of analysis. Similar conclusions can be derived for this measure of non-cognitive ability.

This linear child care subsidy alters the relative price of the child care options. Single mothers will rely less on non-resident relative care, and consume the 40 hours per week of institutional child care. Therefore, we observe a crowding-out effect due to the implementation of this program (Blau, 2003). Even though not discussed in this section, most of the subsidy programs for child care are structured to have nonlinear settings (Blau and Currie, 2006). These results are similar to the conclusions obtained by Del Boca et al. (forthcoming), who propose a structural model to study the cost effectiveness of unrestricted, restricted, and conditional cash transfers on child cognitive ability. The authors find that a CCT is the most cost effective policy that improves the average child's cognitive ability since they have more limited scope for household consumption relative to an unrestricted transfer. Our results also indicate that a CCT transfer might have a stronger impact on cognitive than on non-cognitive measures of ability for children under five years.

7.3 Discussion

In this section we analyzed the effectiveness of remediation programs for children under five in disadvantaged households with specific policies that tried to benefit their development process. Almond and Currie (2011) offer a summary of the three main policies that seek to boost child development in disadvantaged households. Income enhancement is a policy usually implemented to support financially poor households, either through cash transfers or cash credits. The evidence shows that this policy might alter the consumption pattern of the household and improve the child's nutrition level, but not rise significantly the child's cognitive ability. Our unconditional transfers in Section 7.2, for both cognitive and non-cognitive measures, provide similar conclusions.

A near-cash program is another policy that provides different benefits to the household in order to improve their members' quality of life. We can mention the US Food Stamp Program, or public housing benefits (such as the Moving to Opportunity program, MTO). There is evidence that these policies might provide positive effects on the children's nutrition, but not on their cognitive ability. The closest policy in our model is the unconditional transfer.

Finally, early intervention programs have been more effective for the child's development process. The reason is that they have a direct impact on the child's production function of skill. Almond and Currie (2011) recognize that nurse home visiting programs, and quality early childhood education programs have been the most effective interventions for children under five. In our case, we observe a positive impact of the subsidy on child care. This program provides high-quality care to children in disadvantaged households, and financially relief single mothers, which allow them to dedicate more time to their children.

8 Robustness

8.1 Introduction

According to Cunha and Heckman (2008) and Cunha et al. (2010), it is optimal to invest more in the early stages of childhood than in later stages. In the model presented in Section 3, single mothers only have the option of institutional child care as a way to invest in child development. The FFCWS records information about the time the child spends in daycare, Head-Start, and kindergarten institutions. Therefore, we can use this information to split the variable ν_t from our original specification into more specific child care arrangements to observe the productivity of each factor at different ages. For instance, two recent papers (Kline and Walters, 2014, and Feller et al., 2014) analyze the productivity of Head-Start on child development compared to other alternatives. These studies try to reconcile the results of the randomized Head-Start experiment with prior research. The Head-Start randomized experiment found small effects of the program on test scores[37]. In contrast, prior research found larger short-term effects[38]. With a our model and data from the FFCWS, we can study the contribution of the different formal care options on both the child's cognitive and non-cognitive skills. This analysis can complement the conclusions of cash transfers and subsidies proposed in the previous section.

According to Table 1, single-mother households use daycare on average 15.78 hours per week when their children are three-year old. This child care option complements the 30 and 42 hours of informal and maternal care, respectively. Single mothers decide different options when the child is five years old. For instance, they use 13.88 hours of Head-Start, while daycare use reduces to 4.35 hours per week. Finally, children spend on average almost 10 hours per week in kindergarten institutions.

In this Section I present a new specification to consider that a child can attend a daycare (ν_t^D) starting at $t = 1$. During the first three years of analysis, daycare and informal care (π_t) are the only options to outsource child care. At the age of four, the child can also attend a Head-Start institution (ν_t^H). Finally, at the age of five, the child can attend a Head-Start institution or a kindergarten (ν_t^H), in addition to the informal care option. This conditions are represented by the indicators χ_t^D, χ_t^H, and χ_t^K as follows:

$$\chi_t^D = \begin{cases} 0, \text{if } t > 4, \\ \nu_t^D, \text{otherwise;} \end{cases} \quad \chi_t^H = \begin{cases} 0, \text{if } t < 4, \\ \nu_t^H, \text{otherwise;} \end{cases} \quad \chi_t^K = \begin{cases} 0, \text{if } t < 5, \\ \nu_t^K, \text{otherwise.} \end{cases}$$

Therefore, given this new set of variables, the child's skill production function can be represented by the following functional form,

$$k_{t+1} = \begin{cases} (\tau_t)^{\delta_t^\tau} (\nu_t^D)^{\delta_t^D} (\pi_t)^{\delta_t^\pi} (\mathcal{E}^m)^{\delta_t^{\mathcal{E}}} (k_t)^{\delta_t^k} & \text{if } t \leq 3. \\ (\tau_t)^{\delta_t^\tau} (\nu_t^D)^{\delta_t^D} (\nu_t^H)^{\delta_t^H} (\pi_t)^{\delta_t^\pi} (\mathcal{E}^m)^{\delta_t^{\mathcal{E}}} (k_t)^{\delta_t^k} & \text{if } t = 4. \\ (\tau_t)^{\delta_t^\tau} (\nu_t^H)^{\delta_t^H} (\nu_t^K)^{\delta_t^K} (\pi_t)^{\delta_t^\pi} (\mathcal{E}^m)^{\delta_t^{\mathcal{E}}} (k_t)^{\delta_t^k} & \text{if } t = 5. \end{cases}$$

We can include this information into the original household maximization problem. For instance, at time T, the household's problem can be expressed

[37] Further discussions of the methods and results can be found in Puma et al. (2012).

[38] The following is an incomplete list of good references that found important effects of the Head-Start program: Currie and Thomas (1995); Garces et al. (2002); Ludwig and Miller (2007); Deming (2009); Heckman et al. (2010a,b), and Heckman et al. (2013).

according to the following expression

$$
\begin{aligned}
V_T(k_T, w_T, I_T) &= \max \alpha_1 \ln l_T + \alpha_2 \ln c_T + \alpha_3 \ln k_T + \\
&\quad + \beta \mathbb{E}_T V_{T+1}(k_{T+1}, w_{T+1}, I_{T+1}) \\
\text{s.t.} & \\
T^m &= l_T + h_T + \tau_T \\
w_T h_T + I_T &= c_T + p_H \nu_T^H + p_K \nu_t^K + p_\pi \pi_t .
\end{aligned} \tag{19}
$$

The online appendix includes the analytic derivations of the endogenous variables of this new specification during the different periods of analysis. For example, at time T, we obtain the following closed-form representations of the endogenous variables:

$$\pi_T^* = \frac{(w_T h_T + I_T)\lambda_T^\pi}{(\alpha_2 + \lambda_T^\pi + \lambda_T^H + \lambda_T^K)p_\pi}, \tag{20}$$

$$\nu_T^{H*} = \frac{(w_T h_T + I_T)\lambda_T^H}{(\alpha_2 + \lambda_T^\pi + \lambda_T^H + \lambda_T^K)p_H}, \tag{21}$$

$$\nu_T^{K*} = \frac{(w_T h_T + I_T)\lambda_T^K}{(\alpha_2 + \lambda_T^\pi + \lambda_T^H + \lambda_T^K)p_K}, \tag{22}$$

$$\tau_T^* = \frac{(T^m - h_T)\lambda_T^\tau}{\alpha_1 + \lambda_T^\tau}, \tag{23}$$

$$h_T = \frac{T^m w_T(\alpha_2 + \lambda_T^H + \lambda_T^K + \lambda_T^\pi) - I_T(\alpha_1 + \lambda_T^\tau)}{w_T(\alpha_1 + \alpha_2 + \lambda_T^\tau + \lambda_T^H + \lambda_T^K + \lambda_T^\pi)}, \tag{24}$$

where $\lambda_T^j = \beta \delta_T^j \omega_{T+1}$, for $j = \tau, \nu^H, \nu^K, \pi$; and $\omega_{T+1} = \frac{\partial V_{T+1}(\cdot)}{\partial \ln k_{T+1}}$. As in the original model, we allow for corner solutions of h_t, such that

$$h_t^* = \begin{cases} h_t, & \text{if } h_t > 0, \\ 0, & \text{if } h_t \leq 0. \end{cases}$$

With the modifications of our original model, we obtain a vector of endogenous variables $\mathbf{\Upsilon}_t^* = [h_t^*,\ l_t^*,\ \tau_t^*,\ \pi_t^*,\ c_t^*]$. During the first three years of the child, $\mathbf{\Upsilon}_t^*$ also includes daycare (ν_t^D). At time $t = 4$, in addition to daycare, we add the Head-Start option (ν_t^H). Finally, at $t = 5$, we include Head-Start and Kindergarten (ν_t^K) to $\mathbf{\Upsilon}_t^*$. These endogenous variables depend on the parameters $\mathbf{\Lambda}_t = (\alpha_1,\ \alpha_2,\ \alpha_3,\ \beta,\ \psi,\ \delta_t^\tau,\ \delta_t^D,\ \delta_t^H,\ \delta_t^K,\ \delta_t^\pi,\ \delta_t^{\mathcal{E}},\ \delta_t^k)$, prices $\mathbf{P} = (p_D,\ p_H,\ p_K,\ p_\pi)$ and exogenous variables $\mathbf{\Phi}_t = (w_t,\ I_t, \mathcal{E}^m)$.

The estimation strategy will rely on the MSM as in the original version of the model. The DGP of the preference and technology parameters, labor income, and transfers will adopt the same functional forms considered previously. This expanded model gives us a new vector of structural parameters to estimate, $\mathbf{\Theta} = \Big[\mu_{\zeta_1},\ \mu_{\zeta_2},\ \sigma_{\zeta_1}^2,\ \sigma_{\zeta_1,\zeta_2},\ \sigma_{\zeta_2}^2,\ \gamma_0^\tau,\ \gamma_1^\tau,\ \gamma_0^D,\ \gamma_1^D,\ \gamma_0^H,\ \gamma_1^H,\ \gamma_0^K,\ \gamma_1^K,\ \gamma_0^\pi,\ \gamma_1^\pi,\ \gamma_0^{\mathcal{E}},\ \gamma_1^{\mathcal{E}},\ \gamma_0^k,\ \gamma_1^k,\ \mu_w^0,\ \mu_w^1,\ \mu_w^2,\ \mu_w^3,\ \sigma_w^2,\ \mu_I,\ \sigma_I^2,\ \psi,\ p_\pi,\ p_D,\ p_H,\ p_K,\ \vartheta_0,\ \vartheta_1,\ \vartheta_2,\ \vartheta_3\Big]$.

8.2 Structural Estimation Results

Table B2 the estimation results of the productivity parameters during the last three years of our analysis[39]. In general we observe the main features of the original version of the model: maternal time is the most relevant child care option in the production of both children's cognitive and non-cognitive skills. Mother's education also presents a strong positive contribution on the cognitive and non-cognitive measures. The child's own level of ability has a positive impact on the child's measures of ability, which increases with the age of the child. Informal child care has a small and decreasing impact on the child's PPVT score and non-cognitive measures.

By splitting the original variable institutional child care into different options we can analyze the contribution of each arrangement on the child development. At the age of three, mothers can send their children to a daycare and informal care. Among these two options, daycare presents the more beneficial impact on the child's ability. Results indicate that the productivity of daycare is almost three times bigger than that of informal care for the PPVT, and doubles the productivity of this type of care in non-cognitive measures. When the child is four years old, single mothers can send their children to daycare and Head-Start institutions. Daycare is more productive than Head-Start in the PPVT score but not for the Externalizing and Internalizing scores. Finally, at the age of five, the two available options of formal care are Head-Start and kindergarten. Results in our three measures of ability indicate that kindergarten presents a higher positive impact on the child's ability, in particular for the PPVT score.

These conclusions provide valuable information for policy analysis. Policies that try to foster child development in disadvantaged families should try to improve those factors of production that are more productive in the development process. For instance, our results show a moderate effect on children's outcomes of Head-Start compared to other formal child care options. However, this type of care results more productive than informal care in disadvantaged households.

9 Conclusion

In this paper, I focus on the labor supply and child care decisions of single mothers to evaluate the effects of mothers' decisions on their children's cognitive and non-cognitive development. Due to unobserved characteristics of both mothers and children, and the simultaneity of their decisions, there is a potential endogeneity of the child care inputs on the child's ability. Single mothers are heterogeneous with respect to their constraints and tastes, which affect their labor supply and child care decisions. At the same time, children are heterogeneous with respect to their cognitive and non-cognitive endowments. Moreover, children of working women or children of women who use child care will differ

[39] The online appendix includes the estimation results of the preference, primitive structural, wage, and non-labor income parameters. It also includes the within sample fit obtained from the simulated data.

from those whose mothers stay at home or do not use paid child care.

The structural model that I present in this paper estimates the productivity parameters of a Cobb-Douglas production function of cognitive and non-cognitive measures of ability for children under five years old. The production function of skills is nested within a single-mother household maximization problem. The model allows us to analyze the effects of labor supply and child care decisions on the child's development, given the mother's preferences and constraints. A number of simplifying assumptions are considered to obtain closed-form solutions of the endogenous variables, to reduce computational burden, and to consider diverse productivity parameters over time on child development.

As in Bernal (2008), our results reveal the importance of the maternal time on child development over other child care options, specially for young children. The productivity of institutional child care remains constant during the first five years of analysis for the cognitive ability, but increases with the age of the child for the non-cognitive measures. This result reflects the importance of investment in child development. Single mothers also have the alternative of using informal child care. However, among all the factors of production, this alternative offers the lowest productivity on children's cognitive and non-cognitive ability.

Counterfactual exercises show that *unconditional* transfers do not offer significant impact on the average cognitive skill of children. As explained by Del Boca et al. (forthcoming), conditional transfers are the most cost effective policy that improves the average child's cognitive ability since they have more limited scope for household consumption relative to an unrestricted transfer. However, a subsidy on child development, that can be offered to mothers who send their children 40 hours per week to an institutional child care starting at the age of three, has a considerable positive impact on child development. The impacts of this policy on non-cognitive measures are modest, where *unconditional* transfer provide stronger results on children.

I consider an expanded version of the model by including different options of institutional child care. We observe that Head-Start enhances the child's cognitive ability, but it is less effective than daycare a the age of four, and weaker than kindergarten at the age of five. However, Head-Start results more beneficial for non-cognitive measures of ability, for both Externalizing and Internalizing scores at the ages of four and five.

This paper presents evidence of the availability of policies that children's skill deficits can be improved during early ages in disadvantaged households. Moreover, it supports previous research which finds that maternal time is an important factor for the cognitive development of children. I provide further evidence to demonstrate that it also contributes positively on the child's non-cognitive ability. The proposed framework allows to design policies to improve children development by analyzing alternatives of *conditional* and *unconditional* transfers at different ages of child's development process in disadvantaged households or deciding exogenous wage increases. When designing financial assistance and policies of child development, it should be considered the fragile families's budget and time constraints, and also the preferences of child care care. As we observed in our analysis, those are components that cannot be ignored when defining the child care arrangements. Our analysis provides strong evidence of the importance of mother's education during the child development process. It should be noted that this factor contributes to improve the child rearing habits which enhance the productivity of the maternal time.

This study can be extended by estimating simultaneously both cognitive and non-cognitive child's outputs. A specification of that format would allow to understand the mechanism through which an outcome might foster another type of outcome, when taking into account household's preferences and constraints. In particular when designing policies targeted at an specific outcome.

References

Achenbach, Thomas and Leslie Rescorla. 2000. *Child Behavior Checklist*, Burlington.

Almond, Douglas and Janet Currie. 2011. "Human Capital Development before Age Five." in Orley Ashenfelter and David Card, eds., *Handbook of Labor Economics*, Vol. 4, Elsevier, chapter 15, pp. 1315–1486.

Becker, Gary and Nigel Tomes. 1986. "Human Capital and the Rise and Fall of Families." *Journal of Labor Economics*, *4* (3): S1–39.

Berger, Lawrence M., Christina Paxson, and Jane Waldfogel. 2009. "Income and Child Development." *Children and Youth Services Review*, *31* (9): 978–989.

Bernal, Raquel. 2008. "The Effect of Maternal Employment and Child Care on Children's Cognitive Development." *International Economic Review*, *49* (4): 1173–1209.

Bernal, Raquel and Michael Keane. 2011. "Child Care Choices and Children's Cognitive Achievement: The Case of Single Mothers." *Journal of Labor Economics*, *29* (3): 459–512.

Bianchi, Suzanne. 2000. "Maternal Employment and Time with Children: Dramatic Change or Surprising Continuity?" *Demography*, *37* (4): 401–414.

Blau, David. 1999. "The Effect of Income on Child Development." *The Review of Economics and Statistics*, *81* (2): 261–276.

Blau, David. 2003. in Robert Moffitt, ed., *Means-Tested Transfer Programs in the United States*, NBER Chapters, National Bureau of Economic Research, chapter 20, pp. 443–516.

Blau, David and Janet Currie. 2006. "Pre-School, Day Care, and After-School Care: Who's Minding the Kids?" in E. Hanushek and F. Welch, eds., *Handbook of the Economics of Education*, Vol. 2, Elsevier, pp. 1163 – 1278.

Caucutt, Elizabeth and Lance Lochner. "Early and Late Human Capital Investments, Borrowing Constraints, and the Family." NBER Working Papers 18493, National Bureau of Economic Research 2012.

Cawley, John, James Heckman, and Edward Vytlacil. 2001. "Three Observations on Wages and Measured Cognitive Ability." *Labour Economics*, *8* (4): 419–442.

Child Care Aware of America. 2014. *Parents and the High Cost of Child Care: 2014 Report.*

Cunha, Flavio and James Heckman. 2008. "Formulating, Identifying and Estimating the Technology of Cognitive and Noncognitive Skill Formation." *Journal of Human Resources*, *43* (4).

Cunha, Flavio, James Heckman, and Lance Lochner. 2006. "Interpreting the Evidence on Life Cycle Skill Formation." in In Erik Hanushek and F. Welch, eds., *Handbook of the Economics of Education*, Vol. 1, Elsevier, chapter 12, pp. 697–812.

Cunha, Flavio, James Heckman, and Susanne Schennach. 2010. "Estimating the Technology of Cognitive and Noncognitive Skill Formation." *Econometrica*, *78* (3): 883–931.

Currie, Janet. 2001. "Early Childhood Education Programs." *The Journal of Economic Perspectives*, *15* (2): 213–238.

Currie, Janet and Duncan Thomas. 1995. "Does Head Start Make a Difference?" *American Economic Review*, *85* (3): 341–64.

Del Boca, Daniela, Christopher Flinn, and Matthew Wiswall. 2014. "Household Choices and Child Development." *The Review of Economic Studies*, *81* (1): 137–185.

Del Boca, Daniela, Christopher Flinn, and Matthew Wiswall. forthcoming. "Transfers to Households with Children and Child Development." *Economic Journal.*

Deming, David. 2009. "Early Childhood Intervention and Life-Cycle Skill Development: Evidence from Head Start." *American Economic Journal: Applied Economics*, *1* (3): 111–34.

Dunn, Lloyd, Leota Dunn, and American Guidance Service. 1997. *Examiner's Manual for the PPVT-III, Peabody Picture Vocabulary Test, third edition*, AGS.

Feller, Avi, Todd Grindal, Luke Miratrix, and Lindsay Page. 2014. "Compared to What? Variation in the Impacts of Early Childhood Education by Alternative Care-Type Settings." *SSRN.*

Garces, Eliana, Duncan Thomas, and Janet Currie. 2002. "Longer-Term Effects of Head Start." *American Economic Review*, *92* (4): 999–1012.

Goelman, Hillel and Alan Pence. 1987. "Some aspects of the relationships between family structure and child language development in three types of day care." *Advances in Applied Developmental Psychology*, *2*, 129 – 146.

Gourieroux, Christian and Alain Monfort. 1997. *Simulation-based Econometric Methods* number 9780198774754. In 'OUP Catalogue.', Oxford University Press.

Haskins, Ron. 2015. "No Way Out: Dealing with the Consequences of Changes in Family Composition." in "Families in an Era of Increasing Inequality," Springer, pp. 167–199.

Heckman, James. "The Economics, Technology and Neuroscience of Human Capability Formation." NBER Working Papers 13195, National Bureau of Economic Research 2007.

Heckman, James and Dimitriy Masterov. 2007. "The Productivity Argument for Investing in Young Children ." *Review of Agricultural Economics*, *29* (3): 446–493.

Heckman, James, Jora Stixrud, and Sergio Urzua. 2006. "The Effects of Cognitive and Noncognitive Abilities on Labor Market Outcomes and Social Behavior." *Journal of Labor Economics*, *24* (3): 411–482.

Heckman, James, Rodrigo Pinto, and Peter Savelyev. 2013. "Understanding the Mechanisms through which an Influential Early Childhood Program Boosted Adult Outcomes." *American Economic Review*, *103* (6): 2052–86.

Heckman, James, Seong Hyeok Moon, Rodrigo Pinto, Peter Savelyev, and Adam Yavitz. 2010. "Analyzing Social Experiments as Implemented: A Reexamination of the Evidence from the HighScope Perry Preschool Program." *Quantitative Economics*, *1* (1): 1–46.

Heckman, James, Seong Hyeok Moon, Rodrigo Pinto, Peter Savelyev, and Adam Yavitz. 2010. "The Rate of Return to the HighScope Perry Preschool Program." *Journal of Public Economics*, *94* (1-2): 114–128.

Herrnstein, Richard and Charles Murray. 1994. "The Bell Curve: Intelligence and Class Structure in American life." *New York, Free Press.*

Kautz, Tim, James Heckman, Ron Diris, Bas Ter Weel, and Lex Borghans. "Fostering and Measuring Skills: Improving Cognitive and Noncognitive Skills to Promote Lifetime Success." Technical Report, National Bureau of Economic Research 2014.

Kline, Patrick and Christopher Walters. 2014. "Evaluating Public Programs with Close Substitutes: The Case of Head Start." *IRLE Working Paper*, *1* (123-14).

Lokshin, Michael. 2004. "Household Childcare Choices and Women's Work Behavior in Russia." *Journal of Human Resources*, *39* (4).

Ludwig, Jens and Douglas Miller. 2007. "Does Head Start Improve Children's Life Chances? Evidence from a Regression Discontinuity Design." *The Quarterly Journal of Economics*, *122* (1): 159–208.

Markowitz, Anna, Rebecca Ryan, and Anna Johnson. 2014. "Child Care Subsidies and Child Care Choices: The Moderating Role of Household Structure." *Children and Youth Services Review*, *36*, 230 – 240.

McFadden, Daniel. 1989. "A Method of Simulated Moments for Estimation of Discrete Response Models Without Numerical Integration." *Econometrica*, *57* (5): pp. 995–1026.

Pakes, Ariel and David Pollard. 1989. "Simulation and the Asymptotics of Optimization Estimators." *Econometrica*, *57* (5): 1027–57.

Puma, Michael, Stephen Bell, Ronna Cook, Camilla Heid, Pam Broene, Frank Jenkins, Andrew Mashburn, and Jason Downer. 2012. "Third Grade Follow-up to the Head Start Impact Study Final Report." *OPRE Report. Washington, DC: Office of Planning, Research and Evaluation, Administration for Children and Families, U.S. Department of Health and Human Services.*

Ribar, David. 1992. "Child Care and the Labor Supply of Married Women: Reduced Form Evidence." *The Journal of Human Resources*, *27* (1): pp. 134–165.

Todd, Petra and Kenneth Wolpin. 2003. "On The Specification and Estimation of The Production Function for Cognitive Achievement." *Economic Journal*, *113* (485): F3–F33.

Todd, Petra and Kenneth Wolpin. 2007. "The Production of Cognitive Achievement in Children: Home, School, and Racial Test Score Gaps." *Journal of Human Capital*, *1* (1): 91–136.

Zanella, Giulio and Peter Rupert. "Grandparenting and Labor Supply." in "2011 Meeting Papers" number 404 Society for Economic Dynamics 2011.

A Model solution

A.1 Solution of the endogenous variables

The backward induction process requires the solution of the optimization problem starting from the last one, T. The child's skill at the terminal period is

k_{T+1}. This level of ability represents the starting point for the child's ability at school age. At T, we can express equation (5) as follows[40],

$$V_T(k_T, w_T, I_T) = \max_{\tau_T,\pi_T,\nu_T} \alpha_1 \ln[T^m - h_T - \tau_T] + \alpha_2 \ln[w_T h_T + I_T -$$
$$-p_\pi \pi_T - p_\nu \nu_T] + \alpha_3 \ln k_T + \beta \mathbb{E}_T\big[\alpha_1 \ln l_{T+1} + \alpha_2 \ln c_{T+1} + \psi\alpha_3\big(\delta_T^\tau \ln \tau_T +$$
$$+\delta_T^\nu \ln \nu_T + \delta_T^\pi \ln \pi_T + \delta_T^{\mathcal{E}} \ln \mathcal{E}^m + \delta_T^k \ln k_T\big) + \beta \mathbb{E}_{T+1} \widetilde{V}_{T+2}\big] \quad \text{(A.1)}$$

where ψ represents the value that parents give to child development in the last period[41]. The first order conditions of equation (A.1) are the following:

$$\frac{\partial V_T}{\partial h_T} : \frac{-\alpha_1}{T^m - h_T - \tau_T} + \frac{\alpha_2 w_T}{w_T h_T + I_T - p_\pi \pi_T - p_\nu \nu_T} = 0, \quad \text{(A.2)}$$
$$\frac{\partial V_T}{\partial \pi_T} : \frac{-\alpha_2 p_\pi}{w_T h_T + I_T - p_\pi \pi_T - p_\nu \nu_T} + \beta\psi\alpha_3 \frac{\delta_T^\pi}{\pi_T} = 0, \quad \text{(A.3)}$$
$$\frac{\partial V_T}{\partial \nu_T} : \frac{-\alpha_2 p_\nu}{w_T h_T + I_T - p_\pi \pi_T - p_\nu \nu_T} + \beta\psi\alpha_3 \frac{\delta_T^\nu}{\nu_T} = 0, \quad \text{(A.4)}$$
$$\frac{\partial V_T}{\partial \tau_T} : \frac{-\alpha_1}{T^m - h_T - \tau_T} + \beta\psi\alpha_3 \frac{\delta_T^\tau}{\tau_T} = 0. \quad \text{(A.5)}$$

We can obtain the solutions for τ_T, π_T, and ν_T, conditional on h_T, by solving the system of equations (A.3) to (A.5). Hence,

$$\pi_T^* = \frac{(w_T h_T + I_T)\lambda_T^\pi}{(\alpha_2 + \lambda_T^\nu + \lambda_T^\pi) p_\pi} \quad \text{(A.6)}$$
$$\nu_T^* = \frac{(w_T h_T + I_T)\lambda_T^\nu}{(\alpha_2 + \lambda_T^\nu + \lambda_T^\pi) p_\nu} \quad \text{(A.7)}$$
$$\tau_T^* = \frac{\lambda_T^\tau (T^m - h_T)}{\alpha_1 + \lambda_T^\tau}, \quad \text{(A.8)}$$

where $\lambda_T^j = \beta \delta_T^j \omega_{T+1}$; and $\omega_{T+1} = \frac{\partial V_{T+1}(\cdot)}{\partial \ln k_{T+1}}$. The sequence ω_t represents the single mother's marginal utility from her child's future development. This expression is obtained as follows,

$$\frac{\partial V_{T+1}}{\partial \ln k_{T+1}} = \psi\alpha_3 = \omega_{T+1}$$
$$\frac{\partial V_T}{\partial \ln k_T} = \alpha_3 + \beta \frac{\partial V_{T+1}}{\partial \ln k_{T+1}} \frac{\partial \ln k_{T+1}}{\partial \ln k_T} = \alpha_3 + \beta \delta_T^k \omega_{T+1} = \omega_T$$
$$\cdots$$
$$\frac{\partial V_t}{\partial \ln k_t} = \alpha_3 + \beta \delta_t^\tau \frac{\partial V_{t+1}}{\partial \ln k_{t+1}} = \alpha_3 + \beta \delta_t^k \omega_{t+1} = \omega_t$$
$$\cdots$$
$$\frac{\partial V_1}{\partial \ln k_1} = \alpha_3 + \beta \delta_1^\tau \frac{\partial V_2}{\partial \ln k_2} = \alpha_3 + \beta \delta_1^k \omega_2 = \omega_1$$

[40] Where $V_T(k_T, w_T, I_T) = \max_{h_T,\pi_T,\nu_T} \alpha_1 \ln l_T + \alpha_2 \ln c_T + \alpha_3 \ln k_T + \widetilde{V}_{T+1}$, given that $\widetilde{V}_{T+1} = \max_{l_{T+1}} \alpha_1 \ln l_{T+1} + \alpha_2 \ln c_{T+1} + \psi\alpha_3 \ln k_{T+1} + \beta \mathbb{E}_{T+1} V_{T+2}(w_{T+2}, I_{T+2})$.

[41] The expression $\widetilde{V}_{T+2}$ is equal to $\alpha_1 \ln l_{T+2} + \alpha_2 \ln c_{T+2} + \alpha_3 \ln k_{T+2}$.

By plugging equations (A.6), (A.7), and (A.8) into equation (A.2), we obtain

$$h_T^* = \frac{T^m w_T(\alpha_2 + \lambda_T^\nu + \lambda_T^\pi) - I_T(\alpha_1 + \lambda_T^\tau)}{w_T(\alpha_1 + \alpha_2 + \lambda_T^\tau + \lambda_T^\nu + \lambda_T^\pi)}.$$

With the optimal results of h_T^*, π_T^*, τ_T^*, and ν_T^* we can solve for l_T^*, and c_T^*,

$$\begin{aligned} l_T^* &= T^m - h_T^* - \tau_T^*, \\ c_T^* &= w_T h_T^* + I_T - p_\pi \pi_T^* - p_\nu \nu_T^*. \end{aligned}$$

The remaining periods periods are obtained with the same methodology.

B Additional tables

Table B1: OLS Estimations

VARIABLES	(1) $PPVT_t$	(2) Ext_t	(3) Int_t	(4) $PPVT_t$	(5) Ext_t	(6) Int_t
τ_t	0.407	0.0949	-0.00439	0.430	0.104	-0.00528
	(0.333)	(0.131)	(0.0751)	(0.335)	(0.132)	(0.0758)
ν_t	0.481	0.115	0.0148			
	(0.311)	(0.123)	(0.0704)			
π_t	0.398	0.102	-0.00713	0.417	0.111	-0.00733
	(0.313)	(0.123)	(0.0706)	(0.316)	(0.125)	(0.0714)
ν_t^D				0.566*	0.151	0.0120
				(0.336)	(0.132)	(0.0758)
ν_t^H				0.465	0.114	0.0175
				(0.313)	(0.124)	(0.0710)
ν_t^K				0.552*	0.130	0.00646
				(0.319)	(0.126)	(0.0723)
$\mathcal{E}$	0.538	0.0543	-0.0925	0.494	0.0427	-0.0900
	(0.335)	(0.126)	(0.0730)	(0.338)	(0.127)	(0.0738)
$PPVT_{t-1}$	0.497***			0.503***		
	(0.0890)			(0.0893)		
Ext_{t-1}		0.610***			0.612***	
		(0.0626)			(0.0628)	
Int_{t-1}			0.395***			0.397***
			(0.0579)			(0.0584)
Constant	7.550	18.42	25.95***	5.804	17.67	25.93***
	(29.20)	(11.42)	(6.636)	(29.38)	(11.51)	(6.700)
Observations	176	176	176	176	176	176
R-squared	0.206	0.374	0.231	0.212	0.377	0.232

Table B2: Estimation results: productivity parameters, extended model$^{(*)(**)}$

	Cognitive			Non-cognitive					
	PPVT			Externalizing			Internalizing		
	3	4	5	3	4	5	3	4	5
Maternal time, δ_t^{τ}	0.274	0.228	0.189	0.206	0.188	0.172	0.301	0.285	0.270
	(0.011)	(0.010)	(0.009)	(0.017)	(0.016)	(0.016)	(0.020)	(0.019)	(0.018)
Informal care, δ_t^{π}	0.043	0.028	0.018	0.034	0.020	0.012	0.053	0.036	0.025
	(0.006)	(0.006)	(0.005)	(0.010)	(0.010)	(0.009)	(0.014)	(0.016)	(0.018)
Mother's education, $\delta_t^{\mathcal{E}}$	0.106	0.065	0.040	0.391	0.371	0.352	0.311	0.184	0.109
	(0.024)	(0.018)	(0.013)	(0.029)	(0.031)	(0.033)	(0.034)	(0.028)	(0.023)
Current skill, δ_t^{k}	0.471	0.642	0.874	0.441	0.514	0.599	0.330	0.403	0.493
	(0.008)	(0.012)	(0.019)	(0.012)	(0.016)	(0.021)	(0.010)	(0.013)	(0.018)
Daycare, ν_t^{D}	0.142	0.066		0.089	0.024		0.174	0.096	
	(0.011)	(0.006)		(0.011)	(0.003)		(0.005)	(0.003)	
Head-Start, ν_t^{H}		0.019	0.010		0.035	0.021		0.099	0.080
		(0.006)	(0.004)		(0.011)	(0.008)		(0.009)	(0.006)
Kindergarten, ν_t^{K}			0.110			0.066			0.127
			(0.010)			(0.010)			(0.009)

Notes:

($*$) Given $\delta_t^j = \exp(\gamma_0^j + \gamma_1^j \times t)$, for $j = \tau, \nu^D, \nu^H, \nu^K, \pi, \mathcal{E}$, and k. Parameter estimates obtained by the application of the MSM using data from FFCWS, with objective function represented by equation (18). The endogenous variables ($\boldsymbol{\Upsilon}_t^* = \{h_t^*, l_t^*, \tau_t^*, \nu_t^{D*}, \nu_t^{H*}, \nu_t^{K*}, \pi_t^*, c_t^*\}$, for $t = 1, ..., T$) are simulated with the DGP described in Section 5.1.

($**$) Standard errors in parenthesis obtained by bootstrap with 200 replications.