Detection, Identification and Estimation of Loss Aversion: Evidence from an Auction Experiment

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We provide a novel experimental auction design, in which (i) an exogenous decrease in the probability of winning, conditional on the bid, reduces the optimal bid of a loss averse agent whose reference point is expectations-based; (ii) observed bid distributions uniquely identify the participants’ latent value distribution and loss aversion parameter. Experimental evidence affirms the presence of such reference points. We show that at the estimated magnitudes of loss aversion, (a) conventional Becker-DeGroot-Marschak experiments may lead to large biases in estimated willingness to pay (which our design can correct for); and (b) first-price auctions may fetch moderately higher revenue, compared with second-price auctions.

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1 Introduction

In this paper, we provide and use a novel experimental design to accomplish three objectives. First, we use bid data from an auction environment based on this design, to test for the presence of expectations-based reference dependent preferences; the data support the notion of expectations as reference points. Second, we prove an identification result, that can be used to uniquely infer latent loss aversion parameters and value distributions of the consumer model from observed bid distributions. This is intended as a bridge between the reference dependence literature and the recent empirical auctions methodology of identification and structural estimation. Third, the estimates of loss aversion that this enables helps to put magnitudes to applications: we find that (a) the magnitude of bias in estimates of willingness to pay that are derived using a conventional Becker-DeGroot-Marschak auction\(^1\) can be large, when agents are loss averse; our design can correct for this bias; (b) first-price auctions can fetch moderately higher revenue than second-price auctions, when bidders are loss averse.

The experiment design exploits an exogenous variation in the probability of winning an auction, conditional on submitting a bid, to test for the presence of expectations-based reference dependent preferences. We prove that in this design, a lower probability of winning will cause a loss averse

\(^1\)This is a popular elicitation mechanism; see Becker, DeGroot and Marschak, 1964.
agent to bid lower, for a large interval of intrinsic values for the object. This effect would be absent if preferences were ‘standard’, or if the status quo was the reference point. We provide evidence in favor of expectations-based reference dependence, by replicating an experiment based on this design at 4 educational institutions. Thus we contribute to the nascent literature that empirically documents the importance of expectations as a source of reference points (Abeler et. al., 2011, Crawford and Meng, 2011, Ericson and Fuster, 2010).

The auction underlying the experiment is the Becker-DeGroot-Marschak (BDM) mechanism. In a BDM auction, a single bidder competes against a random draw or bid from a known distribution; if her bid beats the random draw, she wins the object and pays a price equal to the random draw; if her bid is lower, on the other hand, she loses. It is easy to see that if preferences are standard, this is an incentive-compatible mechanism (i.e., it is optimal to bid one’s value for the object). Due to this property, the BDM auction is popular as a mechanism to ‘elicit’ willingness to pay and accept (WTP/WTA); we show that if agents are loss averse, this method results in biased WTP estimates.

If a bidder has reference dependent preferences, it is in general not optimal to bid one’s value in a BDM auction. In a recent paper on reference dependent preferences, Koszegi and Rabin, 2006, provide a descriptive model in which an agent’s utility is the sum of consumption utility (the utility from actual consumption: this is conventionally all there is to utility in many applications), and gain-loss utility with respect to a reference point. With standard preferences, there is only consumption utility; in this case, in a BDM auction, bidding one’s value for the object is a dominant strategy. However, if the outcome of a bid also gives rise to gain or loss sensations, when the outcome is compared to some reference point, this must be traded off as well in computing the optimal bid, and the ‘bid your value’ result does not in general hold.

While earlier work modeled the reference point as exogenous, (often equating it to some ‘status quo’), Koszegi and Rabin, 2006, (henceforth, K-R) marks an important departure: it introduces and models the notion of the reference point as endogenous expectations, and further, as a useful

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2 By standard preferences, we mean that the bidder has an intrinsic value \( v \) for the object; and winning it at a price \( p \) gives a payoff equal to \( v - p \), whereas not winning gives a payoff equal to 0.

3 In early formulations of reference dependence gain or loss with respect to a reference point was alone the carrier of utility (Kahneman and Tversky, 1979).
benchmark, assumes these expectations are rational. In this framework, gain-loss utility is forward looking: realized consumption is compared, and gain or loss assessed, with the immediately prior expectations about consumption. Lange and Ratan, 2010, adapt and develop this framework to analyze standard auctions, and use their framework to compare bidding behavior in induced values vs. homegrown values auctions, in first- and second-price auctions. Our experiment has an auction environment and so the K-R theory for it follows Lange and Ratan’s framework. The reference point in this formulation is a key factor in our experiment.

In the context of an auction, an agent’s bid induces a distribution over outcomes (given the distribution of values of other agents and their bidding strategies). Since this is the distribution of outcomes that is anticipated, gain or loss sensations arise when comparing the actual realized outcome with the possible outcomes in this distribution; thus this anticipated distribution of outcomes is the reference point. (See Section 2 for elaboration of this point).

We employ the Lange-Ratan modeling to analyze optimal bidding by a loss averse agent, in a BDM auction. Our experiment design manipulates the rational expectations-based reference point by assigning individuals to one of two treatments: individuals bid against uniform distributions with supports \([0, K_1]\) and \([0, K_2]\) respectively in the two treatments, with \(K_2 > K_1\). Thus a typical bid \(b\) induces a higher probability of winning in Treatment 1 (i.e. against the distribution on \([0, K_1]\)) than in Treatment 2; more generally, the distribution of outcomes induced differs across the two treatments. Suppose \(b\) is the optimal bid of a loss averse agent who has an intrinsic value \(v\) for the object, when she bids against a uniform distribution on \([0, K]\). We prove (Section 2, Proposition 3) that then for a wide range of possible values of \(v\), expanding the support of the competing uniform distribution from some interval \([0, K]\) by increasing \(K\), reduces the marginal benefit from bidding \(b\); so, it is optimal to reduce the bid if the competing uniform distribution has an expanded support. So if agents have reference points à la Koszegi-Rabin, we expect bids to be lower on average in Treatment 2, than in Treatment 1 (see Proposition 3 for a precise statement).

The treatment effect embedded in the experiment design is absent if the reference point is the status quo. It is easy to see that a status quo reference point does not change when \(K\) is changed. The treatment effect is also absent.

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\(^4\)An outcome in this context is a pair of numbers, the first element of which signifies whether the agent won or lost, and the second element is the price she pays conditional on winning.
if preferences are standard. In that case, if an agent’s value for the object is $v$, it is optimal to bid $v$ in both treatments. Our statistical comparisons show that the average bid in Treatment 2 is appreciably lower than that in Treatment 1 across all 4 replications/institutions (between 10 and 30 percent); despite relatively small sample sizes and large variances at individual institutions, the treatment effect is significant in 3 of the institutions. In the pooled sample as well, the null of zero treatment effect is rejected, providing evidence in favor of expectations-based, reference dependent preferences (Sections 3-5).

Recent research on analysis of auctions follows a methodology of trying to identify nonparametrically the latent value distributions of players from observed bid distributions (Athey and Haile, 2002, Guerre, Perrigne and Vuong, 2000, are pioneering contributions$^5$); this facilitates structural estimation, whether parametric or nonparametric. Guerre, Perrigne and Vuong, 2009, extend this literature by providing nonparametric identification results for risk aversion along with value distributions. Our experiment is designed such that from observed population bid distributions for the two treatments, we can uniquely identify the loss aversion parameter of the model in Section 2, as well as the latent distribution of values (Proposition 4). This result is the first attempt that we know of, to provide a bridge between the literature on reference dependence and the empirical methodology of the auctions literature.

The identification result enables us to estimate loss aversion parameters and latent value distributions. Our parametric likelihood estimates of loss aversion suggest that women are more loss averse than men$^6$. We use the estimation exercise to accomplish two further objectives. First, we use the estimated loss aversion parameters to estimate the interval on which Treatment 2 bids are lower than Treatment 1 bids, if the reference dependence model is the correct specification. We use this interval, and the estimated

$^5$This literature is influential but somewhat sparse, partly owing to negative results; the identification problem itself is hard, involving unobserved bids, bidder heterogeneity, equilibrium play in auction games. See also Donald and Paarsch, 1996, Krasnokutskaya, 2011, Komarova, 2010. The literature on structural estimation of auctions is too numerous to mention, see for example Li, Perrigne and Vuong, 2002, and Haile and Tamer, 2003.

$^6$Ratan, 2010 is the only other paper that undertakes estimation of loss aversion in an auction; but there, the setting is one of induced values, obviating the need to either identify or estimate the distribution of values; and also making the estimation of loss aversion more straightforward.
value distribution, to re-estimate the treatment effect (Section 5.1). Second, as a contribution to experimental methodology, we evaluate the bias in the BDM auction, if it is used for eliciting preferences when agents are actually loss averse. We show that the BDM mechanism will underestimate loss averse participants’ values, we quantify the underestimation, and we suggest methods to bound this bias.

Recent research on reference dependence has focused on expectations as a source of reference points, both because of its intuitive appeal, and its ability to explain phenomena that an exogenous reference point like the status quo cannot. Intuitively, think for example of an accused awaiting a prison sentence. If she expects to go scotfree, a five year jail sentence would cause great distress, relative to a scenario in which she expects a life term in prison for her crime and gets the five-year sentence. Koszegi and Rabin use this idea to explain decisionmaking in several contexts. For instance, a consumer who expects to buy a good, and ends up not buying, may experience lower utility relative to a situation in which she did not expect to buy in the first place.

Koszegi and Rabin, 2006, introduce applications to consumer behavior and labor supply. Heidhues and Koszegi, 2008, study optimal pricing strategies for firms facing loss averse consumers. Lange and Ratan, 2010, develop the Koszegi-Rabin framework in the context of first and second price auctions, and show that if agents are loss averse, one can explain the aggressive bidding in induced-value first-price auction experiments, but this does not translate to similar bidding behavior in commodity auctions. Herweg, Muller and Weinschenk, 2009, and Macera, 2010 use the Koszegi-Rabin framework to develop theory in the context of optimal incentive contracts.

While the above papers develop and apply the theory, there is at this point a dearth of empirical evidence to demonstrate that reference points are indeed based on expectations; earlier theory and empirical work used exogenously given reference points, often the status quo\(^7\).

An interesting application of reference dependence to labor supply began with Camerer et. al. 1997, who find that the supply of labor by New York City cab drivers displays negative wage elasticity, even though income effects cannot be large. This phenomenon was formalized and sought to be

\(^7\)This earlier literature includes applications in several areas, e.g. Genesove and Mayer, 2001 (housing), Barberis, Huang and Santos, 2001 (finance), Camerer et al., 1997, Sydnor, 2006 (insurance), Benartzi and Thaler, 1995 (trading). DellaVigna, 2009 provides a very useful survey.
explained by Koszegi and Rabin, 2006; by making the labor supplied responsive to income and hours targets, which in turn are given by rational expectations. In particular, the theory predicts that work effort can decrease if there is an unexpected increase in wage, but not if the wage increase is expected. Crawford and Meng model this further and test and find support for hypotheses based on their model in the NYC cab driver data (collected and analyzed by Farber (2005, 2008)).

Crawford and Meng use a proxy for expectations. While field data such as the cab driver data above have been important in highlighting phenomena that the K-R theory can explain, and Crawford and Meng work explicitly in the K-R setting, a lab experiment provides a more direct opportunity to define and manipulate an expectations-based reference point and study the effect of changing it on the behavior of agents. At this point, the only papers other than the present one that test the K-R theory directly are Abeler et. al. 2011, Gill and Prowse, 2012 (both real effort experiments), and Ericson and Fuster, 2010 (exchange experiments).

Abeler et. al.’s basic experiment has to do with work effort. Subjects are required to do a repetitive task that has a given piece rate; they can drop out voluntarily at any point. When they drop out, they are awarded either their earnings from the task, or a fixed payment, with equal probability. There are two treatments that differ in the amount of the fixed payment. If preferences are classical or status quo reference dependent, the labor supply should not vary with the fixed payment. Instead, the experimental data show that on average, labor supply varies positively with the fixed payment, in line with the K-R theory.

Ericson and Fuster, 2010 have experiments that vary the probability of prospective ownership of a mug and study the impact on subjects’ (i) willingness to exchange it for a pen (experiment 1), and (ii) willingness to accept money in exchange for the mug (experiment 2). The hypotheses based on the K-R model are that a high probability of prospective ownership makes a subject (i) less willing to exchange the mug, and (ii) increases the minimum amount they are willing to accept in exchange for it. The data bear out the hypotheses.

Abeler et. al. vary the quantum of (fixed) reward, whereas Ericson and

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8 See Spears, 2011, for another early field application.
9 We thank Dean Spears for pointing this paper out to us after reading our first draft.
10 While Abeler et. al. use a decisionmaking environment, Gill and Prowse, 2012, use a 2-player real effort game, in which the players’ efforts are strategic substitutes if they are loss averse, but not otherwise.
Fuster vary the probability of winning an object, across treatments; these provide the variation in expectations and hence reference points. In this aspect, our experiment is closer to the latter; however, in our setting the probability of winning varies conditional on a bid, and the bid itself is the endogenous object of interest. Having a classical auction setting (of a simple kind) enables us to integrate the detection of reference dependence preferences with recent empirical auction identification and estimation methodology, to get loss aversion estimates from the basic experiment. Furthermore, the BDM is similar to a second-price auction, and so our design can suggest potential sources of variation in such auctions to identify loss aversion from field data (see the discussion on second-price auctions in Section 6).

In the context of designing experiments, we feel moreover that in a K-R setting, an auction is a simpler environment to work with. In the Koszegi-Rabin paper, the agent’s decision-making environment is as follows. Prior to the decision period, she knows that one of several choice sets can occur (with known probabilities). At this point, she makes a plan of what to choose, in each of these choice sets. This plan results in expectations, or a distribution (because the actual choice set is not known yet), over how she will consume, which becomes her reference point. Then uncertainty resolves and she faces one of the choice sets. In the K-R equilibrium notion (‘personal equilibrium’), the agent’s optimal choice from this set, subject to her reference point, must equal the choice she had planned from this choice set. In contrast to this setting, an auction has a simpler decision-making environment, since the only action performed is to choose a bid; thus there is no temporal separation between a plan and its execution (see also Section 2). We feel this property makes for particularly clean manipulations of reference points.

In what follows, Section 2 contains a discussion of the reference dependent preferences model and the treatment effect. Section 3 describes the experiment, the data, and the first analysis of the treatment effect and the rejection of the hypothesis of standard preferences. Section 4 contains the identification result and estimation strategy. Section 5 presents the estimation results; it also uses these to estimate a more sophisticated, model based treatment effect. We then evaluate the significance of the estimates of loss aversion in two small applications: we show that (i) the conventional BDM elicitation method can lead to a large bias in estimating willingness to pay, when agents are loss averse at the levels suggested by our estimates; (ii) first-price auctions can fetch modestly to moderately higher revenue than second-price auctions, at these loss aversion magnitudes. We also discuss very briefly the fact that females in our sample are estimated to be more loss averse than males. Section 6 discusses extensions (e.g., Propositions 5 and 6), including a similar treatment effect for second-price auctions, and concludes.
2 The Models

Since optimal bidding in a BDM auction when preferences are standard is well known, we discuss it briefly at a later point in this section; we begin with optimal bidding when preferences are reference dependent. The model is an application of the K-R framework to auctions, as developed by Lange and Ratan, 2010; we try to keep the notation similar. We now give a brief preface to the general model (see K-R and Lange-Ratan for details).

In our application, there are 2 commodities, money (commodity 0) and chocolate (commodity 1). The utility from commodity $t$, $u_t(c_t|r_t)$, depends on the consumption level $c_t$ and reference point $r_t$. Consumption utility is $v_t(c_t) ≡ u_t(c_t|c_t)$ (K-R). Utility is a sum of consumption utility and gain-loss utility; following the simplification in Lange and Ratan, we normalize gains to zero. Then utility from commodity $t$ is assumed to be given by

$$u_t(c_t|r_t) = v_t(c_t) - \theta \max\{0, v_t(r_t) - v_t(c_t)\}$$

with $\theta > 0$ implying that the agent is loss-averse. Total utility $u(c|r)$ (where $c$, $r$ are vectors) is the sum of these utilities over the two commodities. Lange and Ratan show that with this specification, the WTA/WTP ratio (willingness to accept vs. pay) equals $(1 + \theta)^2$ at the margin. If (vector) consumption and reference levels are stochastic with distributions $F_c$ and $F_r$ respectively, then the agent gets an expected utility $U(F_c|F_r) = \int \int u(c|c) dF_c(c) dF_r(r)$. That is, the expectation involves a point by point comparison of all possible consumption and reference vector configurations.

As explained in Lange and Ratan, the auction structure is simpler than the general K-R framework, when one formulates the rational expectations reference point. The reason is that the choice of a bid induces a distribution of possible consumption outcomes (in terms of obtaining the object and quantum of payment); with rational expectations, this is also the reference distribution. So, $F_c = F_r$. We now apply this framework to the experiment at hand.

In the experiment, which is more fully described in Section 3, each subject participates in a Becker-de Groot-Marschak (BDM) auction. The subject must submit a sealed bid for the object, knowing that there will be a competing bid that is randomly generated by a computer from a uniform distribution on $[0, K]$, where $K > 0$ is a given real number. If the subject’s bid is at least as large as the competing bid, the subject wins the object and pays the competing bid; if her bid is lower, she loses.

To fix notation, suppose the random competing bid is from a distribution $F$ with density $f$ on $[0, K]$. In the reference dependent preferences model, if
an agent has intrinsic value $v$ for the object and bids $b$, her payoff is defined by

$$U(v, b, \theta) = \int_0^b (v - p) f(p) dp - \theta (1 - F(b)) \int_0^b p f(p) dp$$

$$- \theta \int_0^b \int_0^p (p - s) f(s) ds f(p) dp - \theta v F(b) (1 - F(b))$$

(1)

Here, $\theta$ is a measure of the degree of loss aversion. (i) The first term corresponds to 'consumption utility', a weighted average of utilities of the type $(v - p)$ of winning at a price $p$. The other terms arise out of reference dependence and loss aversion. (ii) The agent expects to lose the auction, and keep her money, with probability $(1 - F(b))$. In each case of winning with price $p \leq b$, a money loss is experienced relative to this event. The second term gives the expectation of these money losses, multiplied by $\theta$. (iii) The third term captures money losses when the agent wins the auction, pays a price $p$, but expects to pay $s < p$; the term is $\theta$ times the expectation of $(p - s)$ given that she wins the auction, over all $(p - s) > 0$. (iv) The fourth term comes from losses incurred from not winning the object (which happens with probability $(1 - F(b))$), relative to reference outcomes in which the object is won (which happens with probability $F(b)$).

Substituting the uniform distribution ($f(s) = 1/K, F(s) = s/K$, for all $0 \leq s \leq K$) in the payoff function, and assuming that the bid $b \in [0, K]$ gives

$$U(v, b, \theta, K) = \frac{v}{K} (1 - \theta) b + \frac{1}{K} \left( \theta \left( \frac{v}{K} - \frac{1}{2} \right) - \frac{1}{2} \right) b^2 + \frac{\theta}{3K^2} b^3$$

(2)

### 2.1 Optimal Bid Function

The first two Propositions and Corollary 1 describe various aspects of the optimal bid function. A brief preview: for a loss aversion parameter that is not very large (i.e., corresponding to marginal WTA/WTP ratio of less than 2), the optimal bid function is continuous. It is strictly increasing on an interval, until the optimal bid hits $K$; for larger values than this critical one, the optimal bid stays equal to $K$. The function is strictly convex on the stretch on which it is increasing. For values less than $(2/3)K$, the optimal bid is shaded below value; for an interval of values higher than $(2/3)K$, bids exceed values. See Figure 1.

Proposition 1 below characterizes the optimal bid function.

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11 This also corresponds to the utility specification for standard preferences.
Proposition 1 Let \( \theta \in (0, 1) \). Then there exists \( \bar{v} > 0 \) such that the optimal bid function is given by

\[
b(v) = \begin{cases} 
\frac{(1+\theta-(2v/K))\sqrt{(2v/K-(1+\theta))^2-4\theta(1-\theta)v/K}}{2\theta/K} & \text{if } v < \bar{v} \\
K & \text{otherwise}
\end{cases}
\]  

(3)

Proof. See Appendix.

A part of the proof sets up the problem of maximizing the utility (Eq.(2)) subject to the bid being in the interval \([0, K]\). (From Eq.(1), we can see that a bid greater than \(K\) fetches the same utility as bidding \(K\) (the smallest bid at which the agent wins the object with probability 1); so we can restrict bids to this interval). The first order condition for an interior optimum is a quadratic in bid \(b\). The optimal bid function in Proposition 1 has 2 segments. On the first interval \([0, \bar{v}]\), the optimal bid is given by the lower root \(lr(v, \theta, K)\) (or \(lr(v)\) for short) of the first order quadratic equation. If \(0 < \theta < 1\), \(lr(v)\) is increasing and convex. For \(v \geq \bar{v}\), the optimal bid equals \(K\).

Proposition 1 leaves open the possibility of a jump in the optimal bid function, at \(v = \bar{v}\). Whether the optimal bid function is continuous, or has a jump, depends on the magnitude of the loss aversion parameter \(\theta\); a jump discontinuity is introduced if \(\theta\) is greater than a critical level, as the following Proposition shows.

Proposition 2 Let \(\theta_c = \sqrt{2} - 1\). If \(\theta \leq \theta_c\), the optimal bid function is continuous, and \(\bar{v}\) in Proposition 1 solves \(lr(\bar{v}, \theta, K) = K\). If \(\theta > \theta_c\), the optimal bid function has a jump at some value \(\bar{v}\) at which the utility from bidding \(K\) equals the utility from bidding \(lr(\bar{v}, \theta, K)\).

Proof. Appendix.

The critical value \(\theta_c\) of the loss aversion parameter corresponds to a marginal WTA/WTP ratio equal to 2. A value of \(\theta\) significantly larger than this should show up in bid data as a significant gap between bids equal to \(K\) and lower bids. The bid data that we have does not exhibit such a gap, so for simplicity we will restrict the discussion of the treatment effect that follows to the case where the optimal bid function is continuous.\(^{12}\)

\(^{12}\)Strictly speaking, of course, this statement is true provided the bid data correspond to a single loss aversion parameter value, common to all bidders.
We close this discussion with a corollary that is used later. It shows that for a large stretch of values \( v \in (0, \frac{2}{3}K) \), the optimal bid involves shading the bid below one’s value; whereas, for values beyond \( \frac{2}{3}K \), up to a certain point, it is optimal to submit a bid greater than one’s value.

**Corollary 1** Let \( \theta \in (0, \theta_c] \). In the interval of values \([0, \bar{v}]\), the optimal bid function \( b(v) \) is strictly convex; moreover, \( b(v) \) satisfies

\[
\begin{align*}
b(v) &< v \quad \forall \ v \in (0, \frac{2}{3}K) \\
b(v) &= v \quad \text{for} \ v = \frac{2}{3}K \\
b(v) &> v \quad \forall \ v \in (\frac{2}{3}K, \bar{v})
\end{align*}
\]

It is straightforward to verify convexity by differentiating the bid function in Proposition 1 twice: in fact, convexity is satisfied provided \( 0 < \theta < 1 \). When \( v \in [0, \bar{v}] \), Proposition 1 establishes that \( b(v) \) equals the lower root \( lr(v, \theta, K) \) of the quadratic first order condition. It is easy to show that the equation \( lr(v, \theta, K) = v \) has a solution at \( v = \frac{2}{3}K \). The Corollary thus follows, since \( b(v) \) is convex, continuous and strictly increasing in the relevant interval. Figure 1 illustrates.

### 2.2 Theoretical Treatment Effect

We test whether agents’ bidding behavior exhibits reference dependent preferences, as opposed to standard preferences, using a comparative static property of the reference dependence model. The experiment is designed to compare optimal bid functions in 2 alternative scenarios: bidding against a draw from uniform \( U[0, K_1] \), and bidding against a draw from \( U[0, K_2] \), with \( K_2 > K_1 \). These scenarios correspond to Treatments 1 and 2 respectively, of the experimental design (Section 3).

Consider first an agent with standard preferences, and intrinsic value \( v \) for the object. Winning the object at a price \( p \) (when the draw \( p \) is less than her bid) gives the agent a payoff equal to \( v - p \) (so her expected payoff when bidding against a draw from the uniform distribution on \([0, K]\) is given by simply the first term in Eq.(1)). It is well known that in this case, it is a weakly dominant strategy for the agent to bid \( v \). This agent’s bid should therefore be identical across the 2 treatments.

There is a caveat. Bidding against a random draw from \( U[0, K] \), in the current setup an agent with value \( v \geq K \) is equally well off bidding \( K \) instead
of $v$, and winning for sure. Thus the optimal bid function comparable to that for Proposition (1) for reference dependent preferences, is the function $\beta(v)$ defined by: $\beta(v) = v$ for all $v < K$, $\beta(v) = K$ for $v \geq K$. For an agent using this bid function, her bids would be identical across the 2 treatments if $v \leq K_1$. If $v > K_1$, her bid in treatment 2 (being equal to $v$) is higher than her bid of $K_1$ in treatment 1. We would therefore expect the average bid for Treatment 2 to be at least as large as the average bid for Treatment 1, if agents had standard preferences. See Figure 3.

We would not expect a Treatment Effect in this experiment if the reference point is the status quo, either. For example, suppose the status quo is $(0,0)$ (absence of the good, and zero payment for it). Then the utility from bidding $b$, for an agent with value $v$ for the good, is given by $U(v,b,\theta) = \int_{0}^{b} (v-p)1_{K}dp - \theta \int_{0}^{b} \frac{p}{K}dp$. The second term corresponds to loss sensations from paying $p$ when the reference point is 0. It is easy to see that when the optimal bid is an interior optimum (i.e. less than $K$), it equals $v/(1+\theta)$. While this is shaded below the value $v$, it does not depend on the value of $K$; and so the treatment effect should be zero over a relevant range of values.

Now consider the case of an agent with reference dependent preferences and a rational expectations-based reference point. Proposition (3) below establishes that for a large range of values, agents will bid lower in Treatment 2 than in Treatment 1. The intuition is that given a bid $b$, increasing the length of the interval $[0,K]$ from which the competing random draw is generated decreases the agent’s probability of winning. At the margin, there is a utility decrease owing to increased loss sensations, that leads the agent to optimally bid lower.

For simplicity, we state the Proposition for loss aversion parameters that give rise to continuous optimal bid functions.

**Proposition 3** Treatment Effect:

Let $\theta \leq \theta_c \equiv \sqrt{2} - 1$. Suppose an agent has reference dependent preferences. Let her optimal bid functions in the 2 treatments (bidding against draws from $U[0,K_1]$ and $U[0,K_2]$, $K_2 > K_1$) be denoted $b_1(v)$ and $b_2(v)$ respectively. Let $\bar{v}_1$ be the value at which her optimal bid function in Treatment 1 satisfies $lr(\bar{v}_1,\theta,K_1) = K_1$. There exists $v_r > \bar{v}_1$ s.t. for all $v \in (0,v_r)$, $b_2(v) < b_1(v)$.

**Proof.** Appendix.
Figure 2 illustrates Proposition (3). The two bid functions are increasing and convex on the stretches in which the bids are equal to the lower root of the quadratic first order condition. Since $b_2(v)$ lies below $b_1(v)$ initially, and is continuous, it cuts it at $v_\tau$ to the right of $\bar{v}_1$. For $v$ greater than this, it is optimal to bid $K_1$ in Treatment 1, whereas the optimal bid in Treatment 2 is higher. Note that the Treatment effect says that for any $v$ for which it is optimal to bid less than $K_1$ in Treatment 1, it is optimal to bid even lower in Treatment 2. Figure 2 is drawn for a value of $\theta = 0.2$.

3 Experiment, Data and a First Test

The data used in this paper consist of participants' bids in BDM auctions; they are extracted from an experiment that was replicated at 4 educational institutions in Delhi, India in 2010-11 (we use the letters A-D to refer to these institutions). The subjects/participants were students of various undergraduate and Master's courses. The commodity for which BDM auctions were conducted was chocolate; the particular chocolate bar used here was an 80 gram bar of premium dark chocolate. The brand was unknown to the students as this chocolate is a fairly recent boutique brand that is not marketed from stores; it is available at a single delicatessen.

The experiment was replicated with one institution being covered in a day. The short time span and considerable distance between the institutions was designed to prevent any information about the experiment flowing from an institution that was recently visited and another that was not yet covered\(^\text{13}\).

At each institution, students who signed up for the experiment were randomly assigned to one of the two treatments. The two treatments were done one immediately after the other, so no information about the experiment was shared between participants. The sequencing of the two treatments was randomly determined. Given the population chosen for the experiment, the subjects were homogeneous in terms of age (19-23) and their student background; gender was a principal source of possible heterogeneity. So the list of students that signed up was divided into the male and female subgroups, and randomized allocation to the treatments was done separately by gender. Overall, the number of male students was significantly larger. Also, because not everyone that signed up actually showed up for the experiment, the actual proportion of each gender across the treatments is not equal. However,

\(^{13}\)The large size of the city and the heavy institution density also act as barriers to quick flow of information for experiments of this size.
the absentee rate was small.

The subjects were instructed on the auctions in a classroom kind of setting, with an attempt, however, to provide adequate private space to each participant. For their participation in the experiment, each subject was given Rupees 200 at the beginning; this money was one of the items in the folder of materials given to each of them. It was emphasized that for each participant, this was an exercise in individual decision-making. Following a display and description of chocolates (without revealing the brand), about a sixth of a bar of 80 gram chocolate was given to each subject for tasting. This was followed by a description of the auction. The BDM auction was explained in detail, contrasted with the more familiar first-price auction, and illustrated with examples. The consequences of incrementing a bid by some amount, on the probability of winning and on the increase in expected payment were brought out. Questions were asked to test understanding; questions were taken and answered along the way and at the end of this training part.

Training for elicitation methods such as BDM are becoming standardized, which helped the conduct of the experiment (e.g. Lusk and Shogren, 2007, Plott and Zeiler, 2007). One part of the standardized training is to explain the following incentive of an agent in the canonical model: to illustrate that if the willingness to pay is \( v \), then it is not optimal to choose a bid \( b \) that is strictly less than or greater than \( v \). But this is not true for a loss averse agent in the model in Section 2; so we did not include this illustration. On the other hand, we included one institution (Institution A, a strong Master’s Program in Economics) in which students had seen a standard treatment of first-price and second-price auctions with independent private values, as part of their curriculum; for this institution, the subjects were therefore aware of the incentives in the canonical model. The training at the experiment may have benefited from the relatively quantitative backgrounds of students from all the institutions; students at institutions B to D respectively were pursuing a Bachelor’s course in Physics, a Bachelor’s course in Information Technology, and a Master’s course in Business Economics (this course is a combination of economics and the curriculum of an MBA program). None of these students was acquainted with the BDM auction prior to the experiment, while all had heard of first-price auctions.

Subjects were asked to write down bids on the sheet of paper provided to each of them for this purpose, and put it in the provided envelope; they also filled in a short questionnaire on basic demographics and responses to the chocolate. The envelope and sheet of paper of each subject had a unique serial number. Random draws from the competing distribution for the treatment had been recorded on sheets of paper and put in envelopes by enu-
merators; these envelopes used the same set of serial numbers as the ones for the envelopes given to subjects to seal their bids. The envelopes were then matched using common serial number, and the result of each auction was implemented; winners got an 80 gram bar of the chocolate and paid the competing random draw as the price.

The reference dependence theory of Section 2 shows that if an individual is loss averse and has an intrinsic value $v$ in an appropriate interval, then her optimal bid is lower competing against a uniform distribution on $[0, K_2], K_2 > K_1$, than against that on $[0, K_1]$. The experiment design is however between-subject. We felt that having the same individual bid in separate auctions with two competing distributions ran the risk of confusion, unanticipated framing, and priming. If individuals are randomly assigned to the two treatments, with a large enough sample size, then we would expect intrinsic values of the object to be similar on average across the two treatments. So, if preferences were standard, or if the reference point is the status quo, we would expect mean bids to be similar across the treatments; whereas in the case of expectations-based reference dependence, we would expect the mean bid to be lower in Treatment 2.

In fact, we do see this treatment effect, that is implied by the reference dependence model, for each institution. However, owing to the large variances in bids, the small sample sizes at individual institutions (30-40 per treatment) imply that we are powered to detect significance at 3 out of the 4 institutions (even though the effect is of appreciable magnitude at all of them). So we begin our analysis by pooling the data from the 4 institutions. An institution-wise analysis is reported later, in Section 5 (and Table 5).

For this pooled sample, summary statistics for the two treatments are presented in Table 1. There were 299 subjects in all, quite evenly divided across the two treatments. The intervals for the uniform distributions in Treatments 1 and 2 had right supports at Rupees 150 and 200 respectively. While the mean bid is higher in Treatment 1 than in Treatment 2 (by about Rupees 5.52, with a standard error of about 3.58), a simple comparison of these means is not correct as we now argue.

It was shown in Section 2 that for values beyond a right support $K$,
choosing to bid $K$ is optimal (even with standard preferences: of course, in the standard preferences case a bid equal to one’s value is optimal as well). Thus the data show that no one bid above 150 and 200 respectively, in the treatments. (The proportions of bids equal to 150 and 200 for the respective treatments are 6.5 percent and 1.4 percent). Under the null hypothesis that people have standard preferences, a subject with value $v$ between 150 and 200 could then optimally bid 150 if she is in Treatment 1, and $v$ if she is in Treatment 2. So the range of values for which we expect the same bid in the two treatments (under the null hypothesis) should be restricted to $[0, 150]$. Since we observe bids (not values), we achieve this now by excluding from both treatments all bids that are greater than or equal to 150.

Table 2 summarizes the data for this truncated sample. The means (weighted by gender proportions) for the two treatments are 64.15 (treatment 1) and 56.21 (treatment 2); the t-statistic equals 3.12. So the difference in means of about Rupees 8 is highly significant. Since the treatment effect ranks the bids (with bids from treatment 2 being ranked lower), we also performed the nonparametric Wilcoxon test. The p-value, at 0.0358, is again highly significant.

Table 3 reports an OLS regression of bids on the treatment dummy for the truncated sample. The coefficient of the treatment dummy is estimated to be $-7.94$ (standard error 4.3). For a one-tailed test (in the direction that bids are lower in treatment 2 as predicted by the theory), the implied p-value for the treatment dummy coefficient is 0.033.

4 Estimation

Since the data set has only one bid per individual, we cannot estimate loss aversion parameters for each individual. In fact, it is standard in other empirical auction applications (e.g. estimating the degree of risk aversion of bidders) to assume and estimate a common parameter for the entire set of agents (e.g. Guerre, Perrigne and Vuong, 2009). The main source of heterogeneity in our data set that is possible to account for is gender. So, we estimate gender-specific loss aversion parameters.

4.1 Identification

Our estimation is enabled by the following identification result: loosely put, given a pair of observed population bid distributions, arising out of a population of agents submitting bids in 2 BDM treatments (using uniform distribu-
tions on $[0, K_1]$ and $[0, K_2], K_2 > K_1$), we can obtain a unique loss aversion coefficient $\theta$ and value distribution $G$ for the population, that can generate the pair of observed bid distributions. We state this result formally now, for the case of continuous bid functions; so, we restrict the class of reference dependent models to come out of loss aversion parameters $\theta \in (0, \theta_c]$. The identification employs a certain $\alpha$-quantile of the bid distributions. Limiting the space of the loss aversion parameter as above necessitates that the $\alpha$-quantile bid for Treatment 2 be bounded below as in Proposition 4 below. It can be shown that for the more general case of $0 < \theta < 1$, the implied bound is close to 0; so this bid restriction is quite weak for the more general case.

To state Proposition 4, we define $\alpha$ to be the quantile at which the bid in Treatment 1 equals $\frac{2}{3} K_1$. We use the notation $b_{1\alpha}, b_{2\alpha}$ to represent the bids for this $\alpha$-quantile, for the two Treatments; so we have by construction $b_{1\alpha} = \frac{2}{3} K_1$.

**Proposition 4** Suppose $G_1, G_2$ are the observed bid distributions for the two Treatments, and have continuous densities up to their atoms at the bids $K_1$ and $K_2$ respectively. If $b_{2\alpha} \geq lr(\frac{2}{3} K_1, \theta_c, K_2)$, then there exists a unique loss aversion parameter $\tilde{\theta}$, and a value distribution $G$ unique on $[0, \frac{K_2}{1+\tilde{\theta}})$, such that if the population of agents has the distribution $G$ of values and loss aversion parameter $\tilde{\theta}$, their bids in the two Treatments will generate the bid distributions $G_1$ and $G_2$.

**Proof.** Appendix.

The idea behind the proof is partly illustrated in Figure 4. It shows that the bid $b_{1\alpha} = (2/3) K_1$ is optimal for the value $2/3 K_1$, regardless of the loss aversion parameter (this follows from Corollary 1). The proof goes on to show that then the corresponding observed $\alpha$-quantile bid for Treatment 2, $b_{2\alpha}$, is the optimal bid for value $2/3 K_1$, for a unique loss aversion parameter $\tilde{\theta}$. Having recovered $\tilde{\theta}$, we simply take the bid distribution over the longer interval (Treatment 2), and since values are a monotone transform of bids via the optimal bid function $b_2(v)$ (upto $K_2$), we can compute the value distribution $G$ for $v$ in the usual way.

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15 $lr(\frac{2}{3} K_1, \theta_c, K_2)$ is the optimal bid of an agent with value $\frac{2}{3} K_1$ and loss aversion parameter $\theta_c \equiv \sqrt{2} - 1$, playing against the uniform distribution $U[0, K_2]$. 

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4.2 Estimation

We use a parametric Maximum Likelihood Estimation strategy to estimate the value distribution and degree of loss aversion of the subjects from their bids in the experiment/BDM auction. We shall assume that the value $v_i$ (or consumption utility from a unit of the commodity) for individual $i$ is independent of other individuals’ values and is distributed as a normal with mean $\mu$ and variance $\sigma^2$. After working with several specifications, the precise model we estimated assumes that males and females draw their values from normal distributions with different means, and that they have different loss aversion parameters. However, for simplicity of exposition, the immediate discussion uses notation for a single distribution and a single loss aversion parameter. We represent this distribution and density by $\Phi$ and $\phi$ respectively; these are functions of $\mu$ and $\sigma$, which we suppress for simplicity.

Since the optimal bid function $b(v, \theta, K)$ for a treatment with BDM competing distribution being uniform on $[0, K]$ is strictly increasing for values in $[0, K/(1 + \theta)]$ and equal to $K$ thereafter, the contribution of a bid to the likelihood function is as follows.

For $b < K$, the likelihood contribution is given by $\phi(b^{-1}(b, \theta, K))$. For a bid equal to $K$, all we know is that the value exceeds $K/(1 + \theta)$. Thus the likelihood contribution equals $1 - \Phi(K/(1 + \theta))$. The aggregate log-likelihood function adds up the logarithms of these terms.

5 Results

5.1 Loss Aversion Estimates and Treatment Effect

The estimated normal value distribution has a mean of 66.05 for males and a significantly higher 73.27 for females, with standard deviation 31.97 (Table 4). The degree of loss aversion for male subjects is estimated to be 0.20 (corresponding to a marginal WTA/WTP ratio of 1.44). For females it is 40% higher at 0.28 (WTA/WTP equals 1.64). By way of comparison, endowment effect experiments have given WTA/WTP ratios ranging from 2 (Kahneman, Knetsch and Thaler, 1991) to close to 1 (Plott and Zeiler, 2007).

Using these estimates of the value distributions and loss aversion parameters, we are also able to get more informed estimates of weighted means and mean differences for the two treatments. In the test reported in section 3, all bids that are greater than or equal to 150 were dropped. But in fact, in the loss aversion model, (which is the statistically chosen model), Treatment
2 bids are lower than Treatment 1 bids up to the value $v_\tau$ in Figure 2. Separately for males and females, our estimates of the loss aversion parameter give us an estimate of $v_\tau$. Of the bids equal to 150 in Treatment 1, we now retain some; we drop the proportion of bids equal to 150 that equals the probability that $v > v_\tau$, using our estimates of the distribution of values.

We then recompute the difference between the average bids in the two treatments. We do this exercise separately for males and females and then recompute the weighted means. The weighted mean bid for Treatment 1 is estimated at 68.21, while for Treatment 2 it is 57.17, the t-value is 3.69, thus reconfirming a highly significant treatment effect, and one of fairly large magnitude (16 percent).

We present the estimated bid functions for males and females for each of the two treatments in Figure 5. The bid functions for Treatment 2 lie below those for Treatment 1, up to $v = b_2^{-1}(150)$. Beyond this, $b_2()$ lies above $b_1()$, as it does not pay to bid higher than 150 in Treatment 1.

Corollary 1 established that the subjects whose values belong to $(0, \frac{2}{3}K)$ are expected to be bidding conservatively relative to their values, while those whose values are greater than $\frac{2}{3}K$ (for a stretch) bid aggressively. Note that such departure from bidding their true values is therefore more pronounced for females (who have a higher loss aversion $\theta$) than for males.

Table 5 presents estimates of the treatment effect by institution, corresponding therefore to the 4 replications of the experiment. The treatment effect is in the correct direction for all 4 institutions. Its magnitude varies approximately from 10 to 30 percent. The t-statistic is highly significant for 3 of the 4 institutions, and the p-values for the Wilcoxon test indicates significance for these institutions (between 2 percent and 10 percent); for the 4th institution, the sample size was too small to detect significance.

### 5.2 Discussion and Applications

We briefly examine some of the implications of our loss aversion estimates. First, they suggest that women are more loss averse than men. In the context of the BDM experiment, and more generally in auctions, the implication is that conditional on value, women will bid less aggressively for a wide range of values, but that at high values, they will bid more aggressively (Figure 5). As far as we know, gender differences in loss aversion have not been studied systematically. However, there is for instance a large and growing literature on gender differences in competition that suggests that in some environments, women are more reluctant than men to compete, and that
there may also be a gender gap in performance in competitions, relative to performance in individual tasks (see the survey by Niederle and Vesterlund, 2011). Confidence levels and beliefs about performance appear to be robust explanatory variables; differences in risk aversion far less so. However, we are not aware of studies that examine the role that differences in loss aversion may play.

Are the magnitudes of our loss aversion estimates large enough to prompt some rethinking of conventional wisdom in auction or auction-like environments? We examine this briefly using simulations in two contexts: the magnitude of bias in using BDM for elicitation when agents are loss averse, and the magnitude of differences in expected revenue from using first-price versus using second-price auctions, under loss aversion.

1. Preference Elicitation

The BDM auction is a popular way to elicit preferences or willingness to pay. In such applications, it is assumed that the observed bids are the true values of subjects (i.e. the BDM is incentive compatible). But if agents are loss averse, this is not an accurate assumption; we quantify the consequences of this inaccuracy for our data set as follows. For this application, we use the data and estimates for males.

First, using the loss aversion model, we estimated a latent distribution of values for males. Suppose loss aversion is the true model. The mean value is Rupees 66.05. The cumulative distribution can also be regarded as a demand curve for chocolate, assuming unit demand for each individual; we simply have to put values on the vertical axis, and the proportion of population that has values greater than or equal to each given value, on the horizontal axis. (See Figure 6).

If a researcher assumes that preferences are standard, however, she would run the BDM experiment with a single distribution. Suppose this is uniform [0, 200]. Assuming that the bids are equal to values, we estimate a parametric normal value distribution by the method of Maximum Likelihood, with the Treatment 2 data for males. The mean value is now estimated to be about 54. This is considerably lower, by about 18%, than when we assume loss aversion, because in that case values are greater than the optimal bids for much of the value distribution. We plot the corresponding demand curve in Figure 6; it is considerably lower than the estimate under loss aversion. We then repeat this entire exercise for Treatment 1 data for males; this corresponds to assuming that the researcher uses this narrower distribution. The estimated mean value is now about 64; this is actually quite close to the one we estimated in the loss aversion model. The corresponding demand
curve (Figure 6) is also much closer to the estimated demand under loss aversion. The reason is the direction of the Treatment Effect: with loss aversion, the bids are more aggressive with the narrower range (up to 150; Treatment 1), than in Treatment 2; and hence are closer to the underlying values.

Thus using standard BDM elicitation may lead to a large bias, if agents have even the degree of loss aversion estimated for males in our experiment. If nonetheless one uses the standard BDM method, when preferences are actually loss averse, the estimated value distribution will be closer to the truth if the BDM distribution is over a smaller interval. Of course, this runs the risk of a lot of bid censoring.

2. Revenue Differences in Auctions

Revenue equivalence of first- and second-price auctions does not hold under loss aversion. Lange and Ratan, 2010, show that for symmetric independent private values, the first-price auction revenue dominates the second-price auction. We examine how much more revenue a first-price auction would fetch, compared to a second-price auction, at the estimated loss aversion magnitudes, varying the number \( n \) of bidders from small \( n = 2 \) to large \( n = 10 \).

Consider a simple setting with \( n \) bidders, with bidders’ values independently drawn from distribution \( F \) (with density \( f \)) with support \([0, w]\). Suppose the bidders have degree of loss aversion \( \theta \). Let \( G \) and \( g \) be the distribution and density of the highest value out of \((n-1)\) values. Let the unique symmetric equilibria in the first-price and second-price auctions be \( \beta_1(v) \) and \( \beta_2(v) \) respectively. These equilibria are derived in Lange and Ratan, 2010; we provide the expressions, specialized to our model, in the Appendix.

The ex ante expected payment of a bidder in a first-price auction is then \( \int_0^w G(v)\beta_1(v)f(v)dv \). The ex ante expected payment of a bidder in a second-price auction is \( \int_0^w (\int_0^v \beta_2(z)g(z)dz) f(v)dv \). We evaluate these expressions for the estimated value distribution for females.\(^{16}\) Table 6 contains the percentage differences in the expected payments, at different loss aversion parameter values and numbers of bidders.

At the estimated loss aversion parameter for females (\( \theta = 0.28 \)), a first-price auction fetches a moderate 6.05% more revenue than a second-price auction, if the number of bidders is low \( (n = 2) \); at \( \theta = 0.2 \), the estimated loss aversion for males, the revenue difference is 4.03%. These revenue differences drop to below 3% as the number of bidders rises to beyond 5 (the equilibrium

\[^{16}\text{The results in Table 6 are robust to changing the parameters of the value distribution.}\]
bids in the second-price auction rise faster with the number of bidders). A more general model than in this paper may allow for the same agent to have different degrees of loss aversion for different items; for example, she may be much more loss averse if the auctioned object is a rare painting than a bar of chocolate. It may be instructive therefore to assess revenue differences at higher degrees of loss aversion. Table 6 shows that at \( \theta = 0.4 \) (marginal WTA/WTP=1.96), revenue difference is moderate (about 5%) even with a large number of bidders; at even greater degrees of loss aversion, revenue differences are large. These results are robust to variations in the underlying value distribution.

6 Extensions and Conclusion

The experiment in this paper can be extended in several directions, some of which we examine briefly. In our design, the BDM auction is modified by perturbing the interval of the competing bid distribution on the right; given a bid, extending to the right the interval for the competing bid distribution decreases the probability of winning the auction. For an interval of values, this reduces the optimal bid. A similar result obtains if we were to instead perturb the competing bid distribution interval on the left. Given a bid, the probability of winning against a uniform distribution on \([a,K]\) is lower than that against a uniform on \([0,K], 0 < a < K\). We find that for an interval of values, the optimal bid is higher in the latter case. We summarize this in Proposition 5 below.

(i) Changing the support of the competing distribution from below.

**Proposition 5** Consider a BDM auction experiment with two treatments. The competing bids in these are distributed uniformly on intervals \([a,K]\) (Treatment 1) and \([0,K]\), (Treatment 2), with \(0 < a < (2/3)K\). Let the loss aversion parameter \(\theta \leq \theta_c \equiv \sqrt{2} - 1\). Let the agent’s optimal bid function in Treatment \(i\) be denoted \(b_i(v), i = 1,2\). Then there is an interval \([0,\tilde{v}]\) such that for all \(v\) in this interval, \(b_1(v) < b_2(v)\).

**Proof.** See Appendix.

Our design, arising from Proposition 3, is easier to work with, compared to a design arising out of Proposition 5 that varies the lower support of the
BDM distribution. Competing against the distribution $U[a, K]$, it is optimal for bidders with values $v \leq a$ to bid zero, whether or not they are loss averse; (moreover, if bidders are loss averse, it is optimal to bid zero for an interval of values that goes beyond $[0, a]$). So in a comparison of average bids in the two treatments, zero bids need to be excluded. Thus in the $U[a, K]$ case (Treatment 1 in Proposition 5), we would end up excluding, at least, all bidders with values less than $a$. Since the treatment effect holds only up to a value $\hat{v}$, bids above a certain level must be excluded as well. This truncation on both sides can lead to a large reduction in sample size relative to our design; this is particularly the case as $a$ needs to be much larger than 0 to get a treatment effect of reasonable magnitude.

The nature of the treatment effect we use carries over to second-price sealed-bid auctions. In a second-price auction, given a player’s bid, the probability of winning is decreasing in the number of bidders. We can show, in keeping with Propositions 3 and 5 above, that for an interval of values, the equilibrium bid decreases in the number of bidders. We work with the unique symmetric equilibrium $\beta_2(v)$ used in Section 5 above. However, we change notation to call it $b(v, n)$, for an auction with $n$ bidders. This result is a part of separate, ongoing research on other auction formats\textsuperscript{17}. We state the relevant proposition here; a proof sketch is available in the Appendix.

**(ii) A Treatment Effect for Second Price Auctions.**

**Proposition 6** Suppose we have a second-price sealed bid auction with number of bidders $n$, $n \geq 2$. Suppose bidders are symmetric, have reference dependent preferences (as modeled in Section 3), and have values independently distributed on $[0, \hat{v}]$ according to distribution $F$ and density $f$. Let the unique symmetric Bayesian equilibrium bid function with $n$ bidders be $b(v, n)$. Then for all $v > 0$ satisfying $\log(F(v)) < -\frac{1}{(n-1)}$, $b(v, n+1) < b(v, n)$.

The sufficient condition above implies, for example, that if two auctions in this symmetric setting have $n = 4$ and $n+1 = 5$ bidders respectively, then for all players with values in the lowest 71% of the distribution, the equilibrium bid is lower when the number of bidders is larger. Thus varying the number of bidders can be used to control and manipulate expectations-based reference

points in second-price auctions with loss averse bidders; detailed design is left for future research.

Compared to other kinds of auctions, however, the BDM auction perhaps has the virtue of simplicity in an experimental setting. Thus our subjects were really taking part in individual decision making problems. For formats such as the second-price auction with several players, we must work with Bayesian equilibria; for untrained subjects, it would take a lot more time and training to learn how to play well. On the other hand, taking forward the direction of work suggested by Proposition 6 can lead to a framework of detecting and identifying loss aversion from auction data generated in the field; thereby complementing and testing for the robustness of results from lab experiments.

In auction experiment designs such as ours, there are very few observations per bidder, and empirical auctions methodology is then employed to examine identification and estimation of parameters common to all, or large groups of bidders. For example in our application, we estimate loss aversion parameters for males and females as groups, to take account of gender heterogeneity. To estimate loss aversion parameters for individuals, we would need to augment an auction experiment of this kind with, say, an experiment with lotteries conducted separately. Directly within an auction environment, identification of separate loss aversion parameters for money and object is another possible research direction.

In conclusion, our auction experiment design enables simultaneous (a) testing for the presence of reference dependent preferences using observed bids from BDM auctions, and (b) identification of loss aversion and value distribution parameters from observed bid distributions. The bid data from an experiment with this design rejects the hypothesis that bidders have ‘standard’ preferences, or status quo reference points, in favor of the alternative of expectations-based reference points. Facilitated by our identification strategy, we also estimate the extent of loss aversion in this auction context and show that women are more loss averse. Furthermore, the loss aversion estimates are used to throw fresh light on the bias of the BDM auction as a preference elicitation mechanism when subjects are loss averse, and on the magnitude by which first-price auctions revenue dominate second-price auctions, again when bidders are loss averse.

7 Appendix

Proof of Proposition (1). Note from Equation (1) in the text that the
payoff from a bid \( b > K \) equals the payoff from bidding exactly \( K \). Thus we can take the range of optimal bids to be a subset of \([0, K]\). So the optimal bid for intrinsic value \( v \) is derived by choosing \( b \in [0, K] \) to maximize the utility function (Eq.(2)). Differentiating the utility function w.r.t bid \( b \), we get a first order condition for an interior local maximum:

\[
v(1 - \theta)/K + \frac{2}{K}[\theta\left(\frac{v}{K} - \frac{1}{2}\right) - \frac{1}{2}]b + \frac{\theta b^2}{K^2} = 0
\]

Let the lower root of this quadratic equation be denoted \( lr(v, \theta, K) \); this is the expression on the RHS of Equation (3) in the text. When there is no confusion, we will use the shorthand \( lr(v) \) for the lower root. We first note some properties of \( lr(v) \).

**Step 1.** There is a threshold \( \hat{v} > 0 \) such that for all \( 0 \leq v \leq \hat{v} \), \( lr(v) \) is real.

Proof. The discriminant of \( lr(v) \) can be written as

\[
g\left(\frac{v}{K}\right) = 4\theta^2\left(\frac{v}{K}\right)^2 - 8\theta \frac{v}{K} + (1 + \theta)^2
\]

This is a convex quadratic in \( \frac{v}{K} \), with left root (say \( \hat{v}(\theta)/K \)) given by

\[
\hat{v}(\theta)/K = \theta^{-1}\left[1 - (1/2)\sqrt{4 - (1 + \theta)^2}\right]
\]

If \( 0 < \theta < 1 \), then this is positive; and for all \( 0 \leq v < \hat{v}(\theta) \), \( g\left(\frac{v}{K}\right) > 0 \); so, \( lr(v) \) is real.

**Step 2.** \( lr(0) = 0 \). \( lr(v) \) is increasing on \( v \in [0, \hat{v}(\theta)] \).

Proof. \( lr(0) = 0 \) follows by substituting 0 for \( v \) in the expression for \( lr(v) \). Differentiating \( lr(v) \), we have

\[
 lr'(v) = -1 + 2\left(1 - \frac{\theta v}{K}\right) (g(v/K))^{-1/2}
\]

where the discriminant of \( lr(v) \), \( g(v/K) \), is positive for \( v \in [0, \hat{v}(\theta)] \). Rearranging and squaring gives us that \( lr'(v) > 0 \) if and only if \( 4 > (1 + \theta)^2 \), which holds since \( 0 < \theta < 1 \).

**Step 3.** For \( v \in [0, \hat{v}(\theta)] \), the optimal bid is either \( lr(v) \) or \( K \).

Proof. Since the first order condition is a convex quadratic, the lower root \( lr(v) \) satisfies the necessary first and second order conditions for a local max. So, the utility function \( u(v, b, \theta, K) \) is locally concave at \( b = lr(v) \). Since utility is a cubic in \( b \) (with positive coefficient on the cubic term), after the local max at \( lr(v) \) and the subsequent local min at the larger root, utility
increases in $b$. So if $0 \leq lr(v) \leq K$, the utility max will occur either at $lr(v)$ or at the boundary $K$, unless utility at both these bids is negative. But this last is not possible if $0 < \theta < 1$. For in this case, for $v > 0$, a small positive bid always gives positive utility (the first term of Equation (2) in the text is then positive, and dominates the other terms which are an order or two magnitude lower for small $b$). The utility at the local max is thus at least this positive amount.

If $lr(v) > K$ for some $v$, we can take $K$ to be the optimal bid by the argument in the first line of this proof.

**Step 4.** There exists $v' > 0$ such that for all $v \in [0, v')$ the optimal bid $b(v) = lr(v)$.

Proof. For an agent with value $v$, bidding $b = K$ gives a utility equal to $u(v, b = K, \theta, K) = v - K^2 (1 + \theta)$. This is negative for an agent with $v = 0$; whereas bidding $lr(v) = lr(0) = 0$ gives a payoff of 0, which is greater; i.e. $u(0, lr(0), \theta, K) > u(0, K, \theta, K)$. From Step 3, this implies that it is optimal for type $v = 0$ to bid $lr(0)$.

From Step 1, $lr(v)$ is real for $v \in [0, \hat{v}(\theta)]$. Since $lr(v)$ is continuous in this interval, and $u(v, b, \theta, K)$ is continuous (see Eq. 2), $u(v, lr(v), \theta, K)$ is continuous in $v$. So by continuity, $u(v, lr(v), \theta, K) > u(v, K, \theta, K)$ for some interval of $v \in [0, v')$.

**Step 5.** The optimal bid for an agent with value $\hat{v}(\theta)$ equals $K$.

Proof. When $v = \hat{v}(\theta)$, the cubic utility function (Eq. 2) is increasing in $b$ with a point of inflection at $b = lr(\hat{v}(\theta))$. It follows that it is maximized at a bid equal to $K$.

**Step 6.** The optimal bid function $b(v)$ is non decreasing in $v$.

Proof. From Equation (2), we get the cross-partial

$$\frac{\partial^2 u}{\partial v \partial b} = \frac{1 - \theta}{K} + \frac{2 \theta b}{K^2}$$

If $0 < \theta < 1$ (and bid $b \geq 0$), this is strictly positive. So, $u$ is supermodular. Hence, $b(v)$ is non decreasing. (See Topkis (1978)).

From Step 5, $u(\hat{v}(\theta), lr(\hat{v}(\theta)), \theta, K) \leq u(\hat{v}(\theta), K, \theta, K)$. Let $\bar{v}$ be the lowest $v \in [0, \hat{v}(\theta)]$ such that $u(v, lr(v), \theta, K) = u(v, K, \theta, K)$. By Step 4 and the monotonicity of the bid function (Step 6), it follows that the bid function $b(v) = lr(v)$ for $v \in [0, \bar{v})$ and $b(v) = K$ for $v \geq \bar{v}$.

**Proof of Proposition (2).** As in the proof of Proposition 1 (Step 1), let $\hat{v}(\theta)$ be the threshold $v$ at which the lower and higher real roots of the quadratic FOC equal each other. That is,

$$\frac{\hat{v}(\theta)}{K} = \theta^{-1} \left[ 1 - (1/2)\sqrt{4 - (1 + \theta)^2} \right]$$
So the lower root (strictly speaking the single real repeated root)

\[ lr(\hat{v}(\theta), \theta, K) = 1 - \frac{\theta(2\hat{v} K - 1)}{2\theta K} \]

Notice that \( lr(\hat{v}(\theta), \theta, K) > (=, <) K \) according as \( \theta < (=, >) \sqrt{2} - 1 \equiv \theta_c \).

Three cases are then possible.

**Case 1.** \( \theta > \theta_c \). So \( lr(\hat{v}(\theta), \theta, K) < K \). Parametrized at \( \hat{v}(\theta) \), the utility function is increasing in the bid, and the single repeated root is at the point of inflection \( lr(\hat{v}(\theta), \theta, K) \), which is lower than \( K \). So, \( K \) is the optimal bid:

\[ u(\hat{v}, lr(\hat{v}), \theta, K) < u(\hat{v}, K, \theta, K). \]

On the other hand, when \( v = 0 \), we have seen in the earlier proof that \( u(0, lr(0), \theta, K) > u(0, K, \theta, K) \). By continuity (the Intermediate Value Theorem), there exists a smallest \( \bar{v} \) s.t.

\[ u(\bar{v}, lr(\bar{v}), \theta, K) = u(\bar{v}, K, \theta, K). \]

Also, since \( lr(v) \) is increasing, \( lr(\bar{v}) < lr(\hat{v}) < K \). From the proof of Step 6 of Proposition 1, it follows that the optimal bid function \( b(v) = lr(v) \) for \( v \in [0, \bar{v}) \), and \( b(v) = K \) for higher \( v \). Thus there is a jump from \( lr(\bar{v}) \) to \( K \) at the value \( \bar{v} \).

**Case 2.** \( \theta < \theta_c \). So \( lr(\hat{v}(\theta), \theta, K) > K \). On the other hand, we know that \( lr(0, \theta, K) = 0 < K \). By continuity and monotonicity of \( lr(v) \), therefore, there is a unique \( \bar{v} \) s.t.

\[ lr(\bar{v}, \theta, K) = K. \]

So trivially,

\[ u(\bar{v}, lr(\bar{v}), \theta, K) = u(\bar{v}, K, \theta, K) \quad A1 \]

We claim that then, for every \( v < \bar{v} \), \( u(v, lr(v), \theta, K) > u(v, K, \theta, K) \).

For suppose not. Then for some \( v' < \bar{v} \), \( u(v', lr(v'), \theta, K) \leq u(v', K, \theta, K) \).

Differentiate both sides of this equation w.r.t. \( v \).

**Derivative of the LHS:** Since there is a local optimum at \( lr(v') \), the envelope theorem implies that the derivative is equal to \( u_1 \), the partial derivative w.r.t. the first argument (evaluated at the bid \( lr(v') \)). Partially differentiating Eq. (2) gives

\[ u_1 = (1 - \theta) \frac{lr(v')}{K} + \theta \left( \frac{lr(v')}{K} \right)^2 < 1 \]

because \( v' < \bar{v} \) implies \( lr(v') < lr(\bar{v}) = K \).

On the other hand, the RHS \( u(v', K, \theta, K) = v' - K \left( 1 + \frac{\theta}{2} \right) \), so the partial derivative w.r.t. \( v \) equals 1.

So, \( v > v' \) implies \( u(v, lr(v), \theta, K) < u(v, K, \theta, K) \). This contradicts Equation \( A1 \).
Therefore, for \( v \in [0, \bar{v}] \), \( u(v, lr(v), \theta, K) \geq u(v, K, \theta, K) \). Thus the optimal bid function \( b(v) \) equals the lower root \( lr(v) \) until this root equals \( K \); and then stays at \( K \). So the bid function does not have any jump.

**Case 3.** \( \theta = \theta_c \). So \( lr(\hat{v}(\theta), \theta, K) = K \), and therefore \( u(\hat{v}, lr(\hat{v}), \theta, K) = u(\hat{v}, K, \theta, K) \). We can then show that for every \( v < \hat{v} \), \( u(v, lr(v), \theta, K) > u(v, K, \theta, K) \). The proof is identical to that for the similar claim in Case 2. And rules out any jump in the optimal bid function.

**Proof of Proposition (3).** Suppose an agent with intrinsic value \( v \) bids against a random draw from \( U[0, K] \). In the proof, we keep the value \( v \) and the loss aversion parameter \( \theta \) fixed. We treat \( K \) as a parameter, and study the response of the optimal bid \( b \) to a change in \( K \). For the duration of this proof, rename the utility function \( u(v, b, \theta, K) \) as \( u(b, K) \) for short, keeping \( v \) and \( \theta \) fixed and suppressing them. Use the notations \( u_i \) and \( u_{ij} \) for the first and second partial derivatives of \( u(b, K) \).

Suppose \( v \in (0, \bar{v}) \), so that the optimal bid equals the lower root of the quadratic FOC (Equation 4). Call this optimal bid \( b(K) \), as it is parametrized by \( K \). We show that \( b'(K) < 0 \).

Using the implicit function theorem on the first order condition \( u_1(b, K) = 0 \), we get \( b'(K) = -u_{12}/u_{11} \). By local concavity at the optimal \( b \), \( u_{11} < 0 \). So the signs of \( b'(K) \) and \( u_{12} \) (evaluated at the optimal \( b \)) are the same. From the utility function (Eq. (2)), we get the cross partial derivative

\[
u_{12} = \frac{-[v(1 - \theta) - b(1 + \theta)]}{K^2} - \frac{4\theta bv + 2\theta b^2}{K^3}\]

\( u_{12} < 0 \), therefore, if and only if

\[
\frac{2\theta}{K} b^2 + \left[ \frac{4\theta v}{K} - (1 + \theta) \right] b + v(1 - \theta) > 0 \quad A2
\]

Now, at the optimal bid \( b \), notice that the FOC (Eq.(4)) is

\[
\frac{\theta}{K} b^2 + \left[ \frac{2\theta v}{K} - (1 + \theta) \right] b + v(1 - \theta) = 0
\]

Comparing the LHS of this with the LHS of Eq.(A2) above, we see that the inequality in A2 holds at the optimal bid \( b \). So, \( u_{12} < 0 \), and therefore \( b'(K) < 0 \).

So, since \( K_2 > K_1 \), we have that for every \( v \in (0, \bar{v}_1) \), the optimal bid is lower when \( K = K_2 \) (Treatment 2), than under Treatment 1. Due to the form of the continuous bid function, under Treatment 2, the optimal bid is lower than that under Treatment 1 for all \( v > 0 \) until the optimal bid
function under Treatment 2 cuts the optimal bid function under Treatment 1 from below (at say $v_r$).

**Proof of Proposition (4).** To prove the result, we first state and prove the following lemma, that we then use. \( \bar{v} \) in the statement of the lemma is the minimum value at which the optimal bid equals \( K \): it equals \( K/(1 + \theta) \).

**Lemma 1** Let the competing distribution be uniform on \([0, K]\). Fix a value \( v \in (0, \bar{v}) \). Let \( b(\theta) \) denote the optimal bid when the value is \( v \), when the loss aversion parameter equals \( \theta \). Then \( b'(\theta) < (=, >) 0 \) according as \( v < (=, >) \frac{2}{3} K \).

**Proof of Lemma.** Holding \( v \) and \( K \) fixed, write the utility function as

\[
u(b, \theta) = \frac{v}{K} (1 - \theta) b + \frac{b^2}{K} \left( \theta \left( \frac{v}{K} - \frac{1}{2} \right) - \frac{1}{2} \right) + \frac{\theta b^3}{3 K^2}\]

\( b(\theta) \) is the solution to \( \max \{ u(b, \theta) \mid b \in [0, K] \} \). By the Implicit Function Theorem, \( b'(\theta) = -\frac{u_{b\theta}}{u_{bb}} \), where on the right hand is a ratio of second partial derivatives.

Since \( u_{b\theta} < 0 \) at the optimum (established earlier), the sign of \( b'(\theta) \) is the same as that of \( u_{b\theta} \). We now sign \( u_{b\theta} \); this depends on the value of \( v/K \).

Note that

\[
u_{b\theta} = (b/K)^2 + (2(v/K) - 1)(b/K) - (v/K)\]

Suppose \( 0 < (v/K) < (1/2) \). So \( (b/K) < (v/K) \) (by Corollary 1, as \( (v/K) < (2/3) \)); so \( (b/K)^2 < (v/K) \) as well. It follows that \( u_{b\theta} < 0 \).

Next, we sign \( u_{b\theta} \) for the range \( (4v/K - 1) > 0 \), i.e. \( (v/K) > (1/4) \).

\( u_{b\theta} \) is a convex quadratic in \( (b/K) \) with real roots; the lower root is negative and so irrelevant; the positive root is

\[
\frac{b}{K} = \frac{1 - (2v/K) + \sqrt{4(v/K)^2 + 1}}{2}
\]

Call this positive root \( R(v/K) \) for short. Because \( u_{b\theta} \) is a convex quadratic, it follows that for positive values of \( (b/K) \), \( u_{b\theta} < (=, >) 0 \) according as \( (b/K) < (=, >) R(v/K) \).

Note now that \( (v/K) = (2/3) \) solves the equation \( (v/K) = R(v/K) \); this is established by a simple rearrangement of the equation \( (v/K) = R(v/K) \). At the same time, Corollary 1 implies that for \( (v/K) = (2/3) \), the optimal
bid $b$ satisfies $b = v$. So, for $(v/K) = (2/3)$, $(b/K) = R(b/K)$; so $u_{b} = 0$.

Next, if $(1/4) < (v/K) < R(v/K)$, a rearrangement of the second inequality establishes that $(v/K) < (2/3)$; so by Corollary 1, the optimal bid satisfies $(b/K) < (v/K)$, which is less than $R(v/K)$. So, $u_{b} < 0$. The reverse inequality, for $(v/K) > (2/3)$, follows similarly.

Proof of the Proposition. The argument is recursive: from the observed population bid distributions for the two treatments, we can find a unique $\theta = \hat{\theta}$ that will generate the bids from values. We then use $\theta$ to find the unique value distribution (unique on the interval $[0, K_2]$) that generates the two bid distributions.

Let $\alpha$ be the quantile of the bid distribution for treatment 1 at which the bid equals $(2/3)K_1$. Call this bid $b_{\alpha}$. From Corollary 1, this bid is equal to the value, for Treatment 1 (i.e. for $K = K_1$). So $(2/3)K_1$ is the $\alpha$-quantile of the distribution of values; call this value $v_\alpha$. Since $K_2 > K_1$, $v_\alpha < (2/3)K_2$. Lemma 1 shows that for values less than $(2/3)K_2$ in Treatment 2, $b'(\theta) < 0$. So the lowest possible bid at the value $v_\alpha$, for Treatment 2, over all admissible $\theta$, occurs at the highest such $\theta = \theta_e$: the bid equalling $lr(v_\alpha, \theta_e, K_2)$. $b(\theta)$ is continuous by the Maximum Theorem; so for any observed $\alpha$-quantile bid $b_{2\alpha} \geq lr(v_\alpha, \theta_e, K_2)$ for Treatment 2, there exists $\theta$ such that $b_{2\alpha} = lr(v_\alpha, \theta, K_2)$.

Moreover, because $v_\alpha < (2/3)K_2$, Lemma 1 implies that $b(\theta)$ is strictly decreasing in $\theta$; so the $\theta$ at which it is optimal to bid $b_{2\alpha}$ at the value $v_\alpha$ is unique.

So, from our specifically chosen $\alpha$-quantile, we have established that there is a unique $\theta = \hat{\theta}$ for which the observed $\alpha$-quantile bids for both treatments $b_{1\alpha}, b_{2\alpha}$, are optimal.

Having obtained $\hat{\theta}$, consider Treatment 2, which has the larger interval $[0, K_2]$ for the competing distribution. The optimal bid function $b(v, \hat{\theta}, K_2)$ is strictly increasing for values in $[0, K_2]$; so the bid distribution for Treatment 2 can be inverted in the standard way to get the value distribution in this interval.

Proof of Proposition 5. (Sketch).

Step 1. By standard optimization methods (as in the proofs of Propositions 1 and 2), it can be shown that there is a partition of the space of values, $\{[0, v_1), [v_1, v_2), [v_2, v_3), [v_3, \infty]\}$ such that $b_1(v) = 0$ for $v \in [0, v_1)$; $b_1()$ is strictly increasing and strictly convex on $[v_1, v_3)$, with $v_1 > b_1(v_1) = a$ (i.e. with shading of bid below value for low values); $b_1(v) = K$ for $v \geq v_3$; and the graph of $b_1()$ cuts the 45° line at $v_2$ (i.e., for some interval of high values, bids exceed values). Furthermore, $v_2 = (K + \sqrt{K^2 + 4a^2})/2$ and $v_3 = (K - a\theta)/(1 + \theta)$. Denote the utility function for the Treatment 1 case
as $u(b, v, a, K)$, where $b$ refers to bid, and $a$ and $K$ are the extreme points of the competing distribution. For $v \in (v_1, v_3)$, the optimal bid $b_1(v)$ is an interior optimum (i.e. $a < b_1(v) < K$), solving $(\partial u/\partial b) = 0$.

Step 2. Fix the value $v$ of an agent, and fix $K$. Change notation to suppress $v$ and $K$, and write the utility function as $u(b, a)$. As a function of the parameter $a$, let the optimal bid be $b(a)$. We show that for $v \in (v_1, \hat{v})$, where $\hat{v} > v_2$, $b'(a) < 0$. This suffices to prove Proposition 5.

By the Implicit Function Theorem, at an interior optimum, $b'(a) = -u_{ba}/u_{bb}$. Since $u_{bb} < 0$, $b'(a)$ and $u_{ba}$ share the same sign.

Evaluating the cross partial at any value of $b$ (not necessarily the optimal $b_1(v)$), we have

$$u_{ba} = \frac{(b - v)(1 + \theta) - 2\theta a}{(K - a)^2}$$

So, $b'(a) < 0$ at all values of $v$ at which the interior optimum $b_1(v) < v$, in Treatment 1. So at all such $v$, we have $b_1(v) < b_2(v)$.

Step 3. We now show that $b_1(v)$ cuts the 45° line only at the value $v_2$ (and not also at some lower value). Since $b_1(v)$ is strictly increasing and strictly convex on $(v_1, v_3)$, and $v_1 < v_2 < v_3$, it can cut the 45° degree line at most twice; since it cuts it at $v_2$, it can possibly cut it only once before, at some lower value $v' \in [v_1, v_2)$. Now, from Step 1, we know that $a = b_1(v_1) < v_1$, so such a $v'$ must be lower than $v_1$. Contradiction.

Step 4. It follows from Step 3, that $b_2(v) > b_1(v)$ on the interval $[v_1, v_2)$ and by continuity, on $[v_1, \hat{v})$ for some $\hat{v} > v_2$. Moreover, since $b_2(v) > 0$ for $v \in (0, v_1)$, whereas $b_1(v) = 0$ on this interval, we have $b_2(v) > b_1(v)$ for all $v \in (0, \hat{v})$.

\textbf{Proof of Proposition 6.} (Sketch).

Suppose values are distributed on $[\bar{v}, \check{v}]$ according to $F$. Let $m = n - 1$, where $n$ is the number of bidders in the second-price auction. Then from Lange and Ratan, 2010, the equilibrium (bid function) can be written as

$$b(v, m) = \frac{v(1 - \theta + 2\theta F^m(v))}{1+\theta} + \frac{2\theta}{(1+\theta)^2} \int_{\bar{v}}^v m z(1 - \theta + 2\theta F^m(z))F^{m-1}(z)\exp\left(\frac{2\theta}{1+\theta}(F^m(v) - F^m(z))\right)dF(z)$$

We wish to show that for a range of values $v$, $b(v, m + 1) < b(v, m)$. Although $m = n - 1$ takes integer values, we will allow $m$ to take all real values in an interval, show that $\partial b(\cdot)/\partial m < 0$, and appeal to the Fundamental Theorem of Calculus to establish the result.

Clearly, the first term of $b(v, m)$ is decreasing in $m$, for all $v$. Now differentiate the term under the integral w.r.t. $m$.

This equals

$$m z(1 - \theta + 2\theta F^m(z))F^{m-1}(z)f(z)\frac{\partial}{\partial m}\exp\left(\frac{2\theta}{1+\theta}(F^m(v) - F^m(z))\right)$$
\[ + \exp\left(\frac{2\theta}{1+\theta}(F^m(v) - F^m(z))\right)f(z)\frac{\partial}{\partial m}mz(1 - \theta + 2\theta F^m(z))F^{-1}_m(z) \]

For this to be negative, with \(0 < \theta < 1\), it suffices for the two derivatives above to be negative. Now

\[ \frac{\partial}{\partial m} \exp\left(\frac{2\theta}{1+\theta}(F^m(v) - F^m(z))\right) = \exp\left(\frac{2\theta}{1+\theta}(F^m(v) - F^m(z))\right) \frac{2\theta}{1+\theta} \frac{F^m(v) \log(F(v)) - F^m(z) \log(F(z))}{F^m(v) - F^m(z)} \]

This is negative if \(F^m(x) \log(F(x))\) is decreasing in \(x\). Now,

\[ \frac{d}{dx} F^m(x) \log(F(x)) = F^{m-1}(x)f(x)(1 + m \log(F(x))) \]

This is negative if \(1 + m \log(F(x)) < 0\).

Finally,

\[ \frac{\partial}{\partial m} mz(1 - \theta + 2\theta F^m(z))F^{-1}_m(z) = z(1 - \theta)F^{m-1}(z) + 2\theta F^{2m-1}(z)(1 + m \log(F(z))) \]

Again, this is negative if \((1 + m \log(F(z))) < 0\).

Symmetric Equilibria in First- and Second-Price Auctions with Loss Aversion (used in Section 5.2).

First-Price Auction.

\[ \beta_1(v) = \int_0^v \frac{z(1 - \theta)(1 - 2G(z))dG(z)}{G(v)(1 + \theta(1 - G(v)))} \]

Second-Price Auction.

\[ \beta_2(v) = \int_0^v \left[ \frac{(1 - \theta) + 2\theta(G(s) + sG'(s))}{1 + \theta} \exp\left(\frac{2\theta}{1+\theta}(G(v) - G(s))\right) \right] ds \]

References


### Table 1 Summary statistics of the data

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Treatment 1 (K=150)</th>
<th>Treatment 2 (K=200)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>68.03</td>
<td>62.51</td>
</tr>
<tr>
<td>Standard error</td>
<td>3.41</td>
<td>3.75</td>
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<tr>
<td>Number of bids</td>
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<td>144</td>
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<tr>
<td>Male-female ratio</td>
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<td>1.82</td>
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<tr>
<td>Mean (Males)</td>
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<td>59.60</td>
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<tr>
<td>Standard error (Males)</td>
<td>4.31</td>
<td>4.27</td>
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<tr>
<td>Mean (Females)</td>
<td>70.93</td>
<td>67.82</td>
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<tr>
<td>Standard error (Females)</td>
<td>5.43</td>
<td>7.19</td>
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<tr>
<td>Proportion of bids equal to K</td>
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<td>0.014</td>
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</table>

### Table 2 Summary statistics of truncated* data

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Treatment 1 (K=150)</th>
<th>Treatment 2 (K=200)</th>
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<tbody>
<tr>
<td>Mean</td>
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<td>56.21</td>
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<tr>
<td>Standard error</td>
<td>3.10</td>
<td>2.97</td>
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<tr>
<td>Number of bids</td>
<td>141</td>
<td>134</td>
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<td>1.98</td>
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<tr>
<td>Mean (Males)</td>
<td>64.04</td>
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<tr>
<td>Standard error (Males)</td>
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<tr>
<td>Mean (Females)</td>
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<td>Standard error (Females)</td>
<td>4.88</td>
<td>5.49</td>
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*Retaining bids that are strictly between zero and 150
### Table 3 Regression of Bids on Treatment Dummy

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>Std Error</th>
<th>p value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>64.15</td>
<td>3.00</td>
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<tr>
<td>Treatment</td>
<td>-7.940</td>
<td>4.30</td>
<td>0.065†</td>
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† This implies a p-value of 0.033 for the one-tailed alternative. Note: Truncated sample (as in Table 2): Sample Size=275.

### Table 4 Maximum Likelihood Estimation - Results

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>Std error</th>
<th>p value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (males)</td>
<td>66.05</td>
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<td>&lt; 10^{-16} .</td>
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<tr>
<td>Std dev</td>
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<td>&lt; 10^{-16} .</td>
</tr>
<tr>
<td>Theta (males)</td>
<td>0.20</td>
<td>0.001</td>
<td>&lt; 10^{-16} .</td>
</tr>
<tr>
<td>Difference in theta</td>
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<td>&lt; 10^{-16} .</td>
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<tr>
<td>Difference in means</td>
<td>7.22</td>
<td>3.15</td>
<td>0.022</td>
</tr>
</tbody>
</table>

Sample Size: 299. Mean (females) = Mean (males) + Difference in means

Theta=Loss Aversion Parameter

Theta (females) = Theta (males) + Difference in theta

### Table 5 Institution-wise Treatment Effect Estimates

<table>
<thead>
<tr>
<th>Institution</th>
<th>Treatment 1</th>
<th>Treatment 2</th>
<th>t-stat</th>
<th>Wilcoxon test</th>
<th>p-value</th>
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</thead>
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<td></td>
<td>Weighted Mean</td>
<td>Weighted Mean</td>
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</tr>
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<td>A</td>
<td>85.95</td>
<td>73.02</td>
<td>9.18</td>
<td></td>
<td>0.059</td>
</tr>
<tr>
<td>B</td>
<td>65.00</td>
<td>51.49</td>
<td>3.57</td>
<td></td>
<td>0.093</td>
</tr>
<tr>
<td>C</td>
<td>58.27</td>
<td>52.46</td>
<td>0.49</td>
<td></td>
<td>0.348</td>
</tr>
<tr>
<td>D</td>
<td>60.48</td>
<td>39.08</td>
<td>3.21</td>
<td></td>
<td>0.016</td>
</tr>
</tbody>
</table>

Note: The sample is truncated using the estimates in section 5. Sample sizes for treatments 1 and 2 respectively are: (41,42) for A, (39,35) for B, (37,35) for C, (31,25) for D.
Table 6 Percentage Difference in Revenue for First- vs. Second-Price Auction

<table>
<thead>
<tr>
<th>Theta</th>
<th>n=2</th>
<th>n=6</th>
<th>n=10</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>4.03</td>
<td>1.81</td>
<td>1.55</td>
</tr>
<tr>
<td>0.28</td>
<td>6.05</td>
<td>2.98</td>
<td>2.61</td>
</tr>
<tr>
<td>0.4</td>
<td>9.45</td>
<td>5.17</td>
<td>4.64</td>
</tr>
<tr>
<td>0.99</td>
<td>31.99</td>
<td>24.98</td>
<td>24.0</td>
</tr>
</tbody>
</table>

Note. Theta is the loss aversion parameter; 0.2 and 0.28 are the estimates for males and females.)
Figure 1: Optimal Bid Function with Loss Aversion

Figure 2: Treatment Effect
Figure 3: Optimal bid function with standard preferences

Figure 4: Impact of change in theta on bids
Figure 5: Estimated bid functions (loss aversion specification)

Figure 6: Estimated demand functions for male subjects
Extract of Instructions to Subjects: Treatment 1

1. The objectives of today’s experiment are to understand your preferences/likes/dislikes for dark chocolate, and to examine how your tasting of a brand of premium dark chocolate translates into how much you would bid for this in an auction.

2. We’ll give details soon, but for now note that for each of you, this is a decision-making situation. In the auction, you don’t compete against each other. You will bid against a computer’s bid; and that bid will be randomly drawn as we’ll explain. This is a real auction: if you win, you get to take a bar of chocolate home, and you must pay the sale price that the auction determines.

3. A quick overview of what now follows. (i) First, we’ll introduce the chocolate. (ii) Next, we’ll describe the auction. (iii) We’ll illustrate the auction with examples. (iv) Then, you’ll taste the chocolate. (v) Then you’ll fill in your auction bids, and some more information. (vi) The outcomes of the auctions will then be determined.

4. This is a premium dark chocolate; it’s an exclusive brand, not widely known yet; we call it Brand X. It has high cocoa content, and the raw chocolate from which it is made is unsweetened Belgian chocolate. Each bar weighs 80 grams.

5. The Auction: After tasting the chocolate, you must write down your bid for an 80 gram bar of the chocolate. Suppose this bid is equal to some number $b$. A computer generates randomly a bid $p$ from a uniform distribution on the positive integers up to 150. This bid is put in a sealed envelope corresponding to the serial number assigned to you. You will choose your bid $b$ without knowing the computer’s bid $p$. Then we will compare $b$ and $p$: if $b$ is greater than or equal to $p$, you win a bar of chocolate and must pay $p$; if $b$ is less than $p$, you do not win the chocolate and do not pay anything. So, the computer’s randomly chosen bid is a randomly chosen sale price.

6. So when you choose a bid, you can determine the probability that you will win. For example if you bid 75, you will win with Probability one-half; if you bid 50, you will win with Probability one-third. If you bid 150, you will win for sure.
I’m now listing on the blackboard a bunch of bids; tell me the probabilities with which these bids will win (go through a variety of low, medium and high bids and win probabilities on the blackboard, with subject participation). Note though, that while you’re more likely to win the higher you bid, you’re likely to have less money left in your pocket if you win: you’ll have to pay more for the chocolate.

7. How much will you have to pay? Suppose you bid 50. If this happens to be greater than the computer’s bid $p$, then we expect the average value of $p$ to be half of that; i.e. 25. Suppose you increase your bid to 60. Conditional on this being a winning bid, the average amount you pay will be 30. More generally, for every one Rupee increase in your bid, you expect your payment, conditional on winning, to increase by one-half of one Rupee. Now we will write down on the blackboard a whole bunch of bids, and ask you what the probabilities of winning are, for these bids, and how much you expect to pay, given these bids.

8. Have any of you seen this way of auctioning? How many of you know of auctions in which if you win, you pay what you bid? (Note: The latter format was known to all participants).

9. Compare the two auction formats with examples and respondent participation. Bring out that a bid $b$ has the same probability of winning in both formats, but conditional on winning, the expected payment in a pay-your-bid auction is higher. Reinforce the different costs of incrementing a bid in these two auction formats. Respondents practice bidding in a fictitious example where the prize is a lunch buffet at a local restaurant.

10. Now please taste the chocolates going around, fill in your bid, and please answer the short questionnaire after that.