The Design of Social Security System: 
Pension System vs. Unemployment Insurance*

Yusuke KINAI†

Graduate School of Economics, Osaka University

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Abstract
This paper considers how the social security system evolves as attribution of voters changes. In our setting, policy determination is based on majority voting and the government has two components in social security policy; pension system and unemployment insurance. That is, when the workers constitute the majority of voters, pension system is supported and when the unemployed is the majority, the unemployment insurance is adopted. Under this setting, employing the concept of structure-induced equilibrium developed by Shepsle (1979), the present paper shows how the contents of the social security system evolves depending on the dynamics of capital accumulation and the unemployment rate, and shows the possibility that the one or other of social security system evaporates in certain instances.

Keywords: Social Security, Pension System vs. Unemployment Insurance, Majority Voting, Structure-induced equilibrium.

JEL Classification: H55, E61, H53.

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†Corresponding to : Graduate School of Economics, Osaka University, 1-7, Toyonaka, Osaka, 560-0043, Japan. E-mail: ykinai@js8.so-net.ne.jp
1 Introduction

Social security system consists of several kinds of policy such as medical care, public sanitation, and social insurance (pension insurance, medical insurance, at-home care insurance, unemployment insurance, and workmen’s compensation insurance) and so forth. Among others, it is pension and unemployment insurance those are of particular importance. In Japan including some countries, we find the increase in the expenditure to the social security is increasing as the population ages and fewer babies are born (See fig. 1.). On the other hand, from fig. 2, we find that the necessity of subsidy to the unemployment (so-called “NEET”, or “Working poor” named in Japan) also remarkably increases, and the expenditure to such a situation (i.e., unemployment insurance) is also vanishingly large. Taking it into consideration that the contribution to both pension and unemployment insurance are increasing, there may emerge the situation in which the government has to prioritize either pension system or the policy for employment although the government should essentially carry out both policy.

Here, we point out the sustainability of the social security system. Although this is not an issue confined to social security policy, the sustainability of economic policy is mainly based on the political factor. In that regard, the determination of policy is based on the diet, in the developed countries where the representative democracy is introduced. Given such a situation, what is important is taking the ratio of power based in all voters into consideration, when we analyze an economic policy in relation to political issue. The aim of this paper is to model such a situation and show that how the scheme of social security varies as time passes from the viewpoint of political economy.

The sketch of our model is as follows: First, there two kinds of households; workers and the unemployed. The former hopes the pay-as-you-go (PAYG) type pension system and the latter hopes the unemployment insurance. In that regard, its decision is based on the majority voting. As times passes, the ratio of the workers and the unemployment varies, and as a result, the contents of social security also varies. At this stage, its choice affects the social welfare. In our setting, policy determination is based on majority voting and the government has two kinds of social security policy; pension and the
unemployment insurance. That is, when the workers constitute the majority of voters, pension system is supported and when the unemployed is the majority, unemployment insurance is adopted. Under such a situation, we show how the contents of the social security system evolves depending on the dynamics of capital accumulation and the unemployment rate, and show the social security system vanishes in certain instances.

### Relationship with the Literature

Here, let us describe the relationship this paper with the past studies in the following two respects. There are some studies which focus on the unemployment in an overlapping generations (hereafter, OLG) model. Roughly speaking, there are two directions; one is introducing the search model and the other is trade union. This paper takes a latter stance. Since Demmel and Keuschnigg (2000) or Corneo and Marquardt (2000), which are the first work that models the behavior of the trade union, there are some studies which models the trade union. For instance, Imoto (2003) extends the model of Corneo and Marquardt (2000) and shows that the existence of trade union may cause the business cycle depending on the value of substitution between volume of employment and wage rate. Kaas and Thadden (2004) propose the another type of wage setting by trade union in a similar model, and Ono (2007) focuses on the interaction between pension and unemployment insurance and derives the unemployment dynamics which is dependent on social security policy. Bräuninger (2005) shows that the unemployment rate is constant under the assumption of endogenous growth model and wage determination through Nash bargaining. What is common to those studies is that the kind of social security system is exogenous, to be more precise, how the policy is chosen is not considered. This paper extends these studies to endogenize the choice of social security system by introducing voting behavior.

On the other hand, since the seminal paper, Meltzer and Richard (1981) or Hu (1982), there are many studies which focus on the social security system in an OLG model in the context of political economy. These studies typically focus on how the ratio of voters varies as time passes and show how the social security system alters. Recently, Hassler, Mora, Storesletten, and Zilibotti (2003) and Conde-Ruiz and Galasso (2005) investigate how the contents of the social security system alters depending on change of the wealth distribution. Unlike these studies, we consider two kinds of redistribution scheme, pension and other redistribution policy as inter and intra-generational redistributive scheme respectively, whereas this paper focuses on unemployment insurance as a role of intra-generational policy.

To summarize, our paper specifically differs from the above papers in the following two respects. First, unlike the past studies under the first part, in our model the policy determination is endogenous by introducing the voting model. In that regard, we focus on the the notion of issue-by-issue voting...
as a way of policy determination. Second, we focus on the unemployment insurance as an intra-generational redistribution scheme, which differs from the second part.

The rest of this paper is organized as follows: §2 sets up the model and we investigate the dynamics of this economy in the section 3. In the section 4, we show how the contents of social security system alters and show the possibility of annihilation of social security. The section 5 is the conclusion.

2 The Model

We consider the infinitely-lived economy which consists of households, firms, trade unions, and the government. Although our model is mainly similar to that of Kaas and Thadden (2004), our model differs from theirs, in the following two respects. The first point is the wage determination through bargaining and the second point is that the determinant of social security system is focused in our model. Households live two periods: young and old period. The population growth rate is $\mu$, that is, $N_{t+1} = (1 + \mu)N_t$. The structure of the model is depicted in fig. 3, which summarizes the intra-temporal flow of goods to avoid complication.

2.1 Behaviors of Each Agent

2.1.1 Households

Households live two periods in a closed-economy without bequest motive. Dynasties derive utility from consumption in young and old period. For simplicity, preferences of the dynasty’s cohort that remain alive at $t$ period is described by the following additively separable function:

Here, $l_t$ denotes the employment rate defined by $l_t \equiv \frac{L_t}{N_t}$, which can be interpreted as the probability
of obtaining jobs in youth period. Then, the objective function of households is written as

\[
\max_{c_t,c_{t+1}} U^i(\cdot) = E_t[u(c_t^v) + \frac{1}{1+\rho} u(c_{t+1}^o)]
\]

\[
= l_t[u(c_t^v) + \frac{1}{1+\rho} u(c_{t+1}^o)] + (1-l_t)[u(c_t^u) + \frac{1}{1+\rho} u(c_{t+1}^u)]
\]

where \( \rho \) denotes the discount factor and \( i = \{e,u\} \). \( c_t^v \) and \( c_t^o \) denote the consumption in young and old, respectively. Two subscripts “\( e \)” and “\( u \)” denote the employed and the unemployed, respectively.

We then specify the utility function as \( u(\cdot) = \ln c \).

The budget constraints of the worker and the unemployed are respectively shown as follows:

- **The Case of Workers:**
  When they are young, they work and divide after-tax labor-income into saving, contribution to pension and consumption. When they are old, they consume saving and pension.

  \[
c_t^v + s_t^e = (1 - \tau_t - \theta_{uw})w_t, \quad c_{t+1}^e = R_{t+1}s_t^e + d_{t+1}^e, \quad (2)
\]

  \( \tau \) and \( \theta_{uw} \) are the contribution of the pension unemployment insurance, respectively. \( s_t \) and \( d_{t+1} \) denote the saving and pension which the old receive. \( w_t \) and \( R_{t+1} \) denote wage rate and rental rate of capital stock. Maximization of the utility function of the employed under the constraint, eq.(2) yields,

  \[
c_t^v = \frac{1}{2+\rho} \left\{ (1 - \theta_{uw} - \tau_t)w_t + \frac{d_{t+1}^e}{R_{t+1}} \right\}, \quad (3a)
\]

  \[
c_{t+1}^e = \frac{1}{2+\rho} \left\{ (1 - \theta_{uw} - \tau_t)w_t + \frac{d_{t+1}^e}{R_{t+1}} \right\}, \quad (3b)
\]

  \[
s_t^e = \frac{1}{2+\rho} \left\{ (1 - \theta_{uw} - \tau_t)w_t - (1 + \rho) \frac{d_{t+1}^e}{R_{t+1}} \right\}. \quad (3c)
\]

- **The Case of the unemployed:**
  When they are young, people receives unemployment insurance (\( b_t \)) and divide it into saving, contribution to pension (\( d_t^u \)) and consumption, whereas, When old, they consume saving and pension.

  \[
c_t^u + s_t^u = (1 - \tau_t)b_t, \quad c_{t+1}^u = R_{t+1}s_t^u + d_{t+1}^u. \quad (4)
\]

Maximization of the utility function of the unemployed under the constraint, eq.(4) yields,

  \[
c_t^u = \frac{1}{2+\rho} \left\{ (1 - \tau) b_t + \frac{d_{t+1}^u}{R_{t+1}} \right\}, \quad (5a)
\]

  \[
c_{t+1}^u = \frac{1}{2+\rho} \left\{ (1 - \tau_t) b_t + \frac{d_{t+1}^u}{R_{t+1}} \right\}, \quad (5b)
\]

  \[
s_t^u = \frac{1}{2+\rho} \left\{ (1 - \tau) b_t - (1 + \rho) \frac{d_{t+1}^u}{R_{t+1}} \right\}. \quad (5c)
\]

Here, we assume \( d_t^e > d_t^u \) in order to eliminate the trivial case in which all households choose unemployment.
2.1.2 Firms

We assume that factor markets are perfectly competitive and that firms maximize their profits. Labor and capital stock are used for production; production technology is assumed to be neo-classical product function with constant return to scale, \( Y_t = F(K_t, L_t) \), where \( Y_t, K_t \) and \( L_t \) respectively represent output in aggregate terms, capital stock and the number of the workers at \( t \) period. \(^4\) Firms’ profit maximization problem is written as

\[
\Pi_t = F(K_t, L_t) - R_t K_t - (1 + \theta_{ft}) w_t L_t,
\]

where \( \theta_{ft} \) denotes contribution to unemployment insurance and pension which the firms burden, respectively. We then specify the production function as Cob-Douglas type, \( F(K_t, L_t) = K_t^\alpha L_t^{1-\alpha} \). Then, FOCs are derived as

\[
\frac{\partial \Pi_t}{\partial K_t} = 0 \iff R_t = \alpha K_t^{\alpha-1} L_t^{1-\alpha}, \tag{6a}
\]

\[
\frac{\partial \Pi_t}{\partial L_t} = 0 \iff (1 + \theta_{wt}) w_t = (1 - \alpha) K_t^\alpha L_t^{-\alpha}. \tag{6b}
\]

Defining \( \hat{k}_t \equiv \frac{K_t}{N_t} = \frac{K_t L_t}{N_t L_t} = k_t l_t \), we obtain

\[
R_t = \alpha \hat{k}_t^{\alpha-1} l_t^{1-\alpha}, \tag{7a}
\]

\[
w_t = \frac{(1 - \alpha) \hat{k}_t^\alpha}{1 + \theta_{wt}}, \tag{7b}
\]

If \( L_t = N_t \) (perfect employment) that is, \( l_t = 1 \), the wage rate is derived as follows:

\[
(1 + \theta_{wt}) \tilde{w}_t = (1 - \alpha) \hat{k}_t^\alpha, \tag{8}
\]

where \( \tilde{w}_t \) denotes the wage rate at perfect employment.

2.1.3 Trade Union

Following Corneo and Marquardt (2000) or Ono (2007), the wage is determined by the monopolistic trade union. The purpose of the trade union is to keep both high wage and low unemployment rate at the same time. Following Imoto (2003) who extends the model of Corneo and Marquardt, let us define the behavior of the trade union. The problem of the trade union is:

\[
w_t \equiv \arg \max_{w_t} W(\cdot) \equiv [\gamma(w_t - \tilde{w}_t)^{-\sigma} + (1 - \gamma)(l_t)^{-\sigma}]^{-\frac{1}{\sigma}}, \quad \sigma \in (-1, \infty), \text{ and } \gamma \in (0, 1), \tag{9}
\]

under the constraint eq.(7b). The type of bargaining is “Right-to-Management”. That is, the firms accept the wage requested by the trade union and then decide the amount of employment so as to maximize their profit, on the other hand, the trade union requests the wage based on that condition, so

\(^4\) Although Ono (2007) adopts the Grossman and Yanagawa (1993) type production function, we adopt the neo-classical growth model because our concern is not directed to the effect on growth rate.
as to maximize its objective function, eq.(9). The trade union decides the wage rate so as to maximize their objective function for given $\bar{w}_t$ treated as reference wage rate. The first order condition for this problem is written as:

$$\frac{\partial W(\cdot)}{\partial w_t} = 0 \iff -\gamma \sigma (w_t - \bar{w}_t)^{-\sigma-1} + \frac{\sigma (1 - \gamma)}{\alpha} \left( \frac{1 - \alpha}{1 + \theta_t} \right)^{\frac{1}{\alpha}} k_t \sigma w_t^{\frac{\sigma}{1-\alpha}} - 1 = 0, \quad (10)$$

Denoting $L \equiv \frac{\alpha T}{V_1} (w_t - \bar{w}_t)^{-(1+\sigma)}$ and $R \equiv w_t^{\sigma-1}$, where $V_t = (1 - \gamma) \left( \frac{1 - \alpha}{1 + \theta_t} \right)^{\frac{1}{\alpha}} k_t$, the determination of the wage rate $w_t$ is shown as in fig. 4.

In what follows, we assume that the following equation (second order conditions) holds:

$$\frac{\partial^2 W(\cdot)}{\partial w_t^2} < 0$$

2.1.4 The Government

Finally, let us describe the behavior of the government. The government has two kinds of redistribution scheme: PAYG-type pension system (the inter-generational redistribution scheme) and unemployment insurance (the intra-generational redistribution scheme). The budget constraint under each scheme is balanced and written as follows:

- PAYG-type pension system (the inter-generational redistribution scheme)

In aggregate, the budget constraint under this scheme is written as

$$L_t \times d_{t+1}^L + (N_t - L_t) \times d_{t+1}^u = \tau_{t+1} L_{t+1} w_{t+1} + \tau_{t+1} (N_{t+1} - L_{t+1}) b_{t+1},$$
where $b_t$ denotes the benefit from unemployment insurance. The term of left side of the above equation denotes the entitlement of pension. The first and second term of the right side denote contribution to pension of workers, and the unemployed, respectively. Dividing both sides of the above equation with $N_t$ yields,

$$l_t d_{t+1}^w + (1-l_t) d_{t+1}^u = (1+\mu)\left\{ \tau_{t+1} w_{t+1} l_{t+1} + \tau_{t+1} b_{t+1}(1-l_{t+1}) \right\}$$  \hfill (11)

- unemployment insurance (the intra-generational redistribution scheme)

Similarly, in aggregate, the budget constraint is written as

$$(N_t - L_t) b_t = \theta_w t L_t w_t + \theta_f L_t w_t$$

The left side of the above equation denotes the entitlement of unemployment insurance and the first and second term of the right side denote contribution to unemployment insurance of workers and firms, respectively. Dividing both sides of the above equation with $N_t$ yields,

$$b_t = \frac{l_t}{1-l_t}(\theta_w t + \theta_f) w_t.$$ \hfill (12)

Note that which policy is chosen is dependent on the voters’ movement. That is, if the young unemployment is the majority, the unemployment insurance is supported, whereas, the old and young workers constitutes the majority, the PAYG-type pension system is supported. So, let us investigate the transitional change of voters in the next section.

### 2.2 Timing of Decision Making

Here, let us summarize the sequence of decision-making (or political process). The sequence of decision making is also depicted in fig. 5.

Stage 1. At the $t$th period, a new generation is born. Note that there are two possibilities that households can be both employed and unemployed at this stage.

Stage 2. Households vote over the policy variables of social security system; the contribution to pension ($\tau$) and unemployment insurance ($\theta_w$).

Stage 3. Then, firms decide volume of employment and go into production. At this stage, households are divided into two cases, the employed and the unemployed.

Stage 4. The government determines which policy is adopted, that is, the contents of social security system is determined, based on the result of voting. In other words, the amount of contribution to pension or unemployment insurance is also determined as $\{\tau, \theta_w\}$. At the same time, $\theta_f$ is also determined.

Stage 5. The $t+1$th generation is newly born.

### 2.3 Market Equilibrium

We finally formulate equilibrium conditions for each market.
Stage.1  
a new generation is born

Stage.2  
Households vote over the policy.

Stage.3  
Firms decide volume of employment and go into production.

Stage.4  
The government determines which policy is adopted.

Stage.5  
\{ \tau_t, \theta_{\text{w,t}} \} is determined.

Figure 5  Sequence of Decision Making in t-th period.

- Commodity market
  In aggregate, we can state this condition as \( C_t^e + C_t^{ow} + C_t^{yu} + C_t^{ou} + K_{t+1} = Y_t \), where \( C_t^i \) denotes the aggregate consumption of type \( i \) in \( t \) period. Dividing both sides of this equation with \( N_t \) yields
  \[
  l_t e_t^e + (1-l_t)c_t^{yu} + \frac{l_{t-1}}{1+\mu} c_t^{oe} + \frac{1-l_{t-1}}{1+\mu} c_t^{ou} + (1+\mu)k_{t+1} = y_t, \tag{13}
  \]
  where \( y_t \equiv \frac{Y_t}{N_t} \).

- Capital market
  In aggregate, we can state as \( K_{t+1} = \varepsilon Y_t = L_t s_t^e + (N_t - L_t) s_t^u \), where \( \varepsilon \) is national saving rate.\(^5\)
  Therefore, we can write as
  \[
  (1+\mu)k_{t+1} = l_t s_t^e + (1-l_t) s_t^u. \tag{14}
  \]
  This equation determines the dynamics of capital accumulation in this economy.

- Labor market
  In this market, the demand of the labor should equal to the supply of the labor. So, combining the solution of eq.(6b) and that of eq.(10) yields the labor market equilibrium condition.
  \[
  L_t = N_t l_t \tag{15}
  \]
  The left and right side of the above equation respectively means the labor demand and labor supply. Fig.6 depicts the situation of labor market equilibrium. In that figure, the heavy and middle line respectively denote the labor demand curve and the indifferent curve. The combination \((\hat{w}_t, \hat{l}_t)\) denotes the wage and volume of employment at full employment. The diremption between \( l_t^* \) (labor supply curve) and \( \hat{l}_t \) denotes the amount of unemployment.

Finally, let us define the economic equilibrium.

**Definition 1 (Economic Equilibrium)** An economic equilibrium is a sequence \( \{c_t^i, c_{t+1}^i, s_t^i\}_{t=1}^{\infty}, i \in \{e, u\} \) that accords with the following condition.

\(^5\) Note that \( K_t \) is a constant which does not depend on policy variables of the social security system at \( t \) periods or \( t + 1 \). That is,

\[
K_t = L_t s_{t-1}^e(\tau_{t-1}, \theta_{\text{w,t-1}}) + (N_t - L_t) s_{t-1}^u(\tau_{t-1}, \theta_{\text{w,t-1}}).
\]

Therefore, we can treat the policy variables as given and treat state variable as fixed. This point is related to the remark in pp.15.
(i) Given the sequence \( \{ \tau_t, \theta_{wt}, \theta_{ft} \}_{t=1}^{\infty} \), each agent (the employed or the unemployed) determines policy variables that maximize their individual utility. That is, the optimal policy variables meet the following maximization problem:

\[
\max \ln(c^y_i) + \frac{1}{1+\rho} \ln(c^o_i), \quad i \in (e,u)
\]

(ii) The budget constraints of pension and unemployment insurance are balanced in each period.

(iii) Firms maximize their profit.

(iv) The condition for the trade union’s wage setting holds.

(v) Finally, commodity market clears, that is, eq. (13) holds.

3 Analysis

In the section, we consider the equilibrium dynamics treating the policy variable \( \tau_t \) and \( \theta_{wt} \) as constant, in other words, we assume the government can commit to the policy once determined. The justification of this assumption is explained in the next section.

3.1 The Dynamics of Capital Accumulation

From eqs. (3c), (5c), and (14), considering the case in which \( \tau = \tau_t = \tau_{t+1} \) and \( \theta_{wt} = \theta_{wt+1} = \theta_{wt+1} \), let us derive the dynamics of \( k_t \) as follows:\(^6\):

\[
\hat{k}_{t+1} = \frac{1}{1+\mu} \left\{ s^f_t \times l_t + s^y_t \times (1-l_t) \right\} \\
= \frac{\alpha(1 - (1 + \theta_{wt}) \tau_t + (1 - \tau_t) \theta_{ft})(1 + \theta_{ft})}{(1 + \mu)(1 + \theta_{ft}) \left(2 + \rho\right) + (1 + \rho)(\tau_t(1 + \theta_{wt}) + \theta_{ft})} \hat{k}_t \delta l_t^{1-\alpha}.
\]

We can rewrite the above equation as \( \hat{k}_{t+1} = \frac{1}{1+\mu} \epsilon y_t \), and \( \epsilon = \frac{\alpha(1 - (1 + \theta_{wt}) \tau_t + (1 - \tau_t) \theta_{ft})(1 + \theta_{ft})}{(1 + \mu)(1 + \theta_{ft}) \left(2 + \rho\right) + (1 + \rho)(\tau_t(1 + \theta_{wt}) + \theta_{ft})} \) denotes the national saving rate. Here, the subscript, \( t \) is omitted. From this equation, we find the

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\(^6\) For calculation details, see appendix A.2.
increase in $\tau$ and $\theta_w$ causes decrease in the capital stock per capita at $t + 1$ period due to the decrease in saving rate.

### 3.2 The Dynamics of the Employment Rate

Next, we have to investigate the dynamics of the employment rate. From eq. (10) and (16), we can derive the following equation:

$$l^\sigma(1+\alpha)(1-l^\alpha)^{-(1+\sigma)} = \frac{1 - \gamma}{\alpha\gamma} \left( \frac{1 - \alpha}{1 + \theta_f} \right)^{\sigma} \hat{k}^\alpha. \quad (17)$$

This equation describes the relationship $\hat{k}$ and $l$.

Considering the labor and capital market equilibrium condition, eqs. (15) and (14), the above equation can be rewritten as

$$\frac{l^{1+\alpha}}{(1-l^\alpha)^{1-\sigma}} = \frac{\varepsilon}{1 + \mu} \left( \frac{1 - \gamma}{\alpha\gamma} \left( \frac{1 - \alpha}{1 + \theta_f} \right)^{\sigma} \right) \frac{l^\alpha}{1 - l^\alpha} \left(1 - l^\alpha\right)^{1-\sigma}. \quad (18)$$

This equation can be rewritten as follows:

$$\frac{l^{1+\alpha}}{1-l^\alpha} = A \frac{l^2}{1 - l^\alpha} \quad (19)$$

where

$$A \equiv \frac{\varepsilon}{1 + \mu} \left( \frac{1 - \gamma}{\alpha\gamma} \left( \frac{1 - \alpha}{1 + \theta_f} \right)^{\sigma} \right).$$

This equation implicitly shows the relationship with $l_{t+1}$ and $l_t$. From this equation, we can show the relationship as

$$l_{t+1} = \phi(l_t). \quad (20)$$

This equation determines the dynamics of the employment rate. Depending on the value of $\sigma$, we have three cases. Fig.7(a) ~ 7(c) depict the dynamics patterns of the employment rate. If the employment rate is less than the half of population, the unemployment constitute the majority of this economy.

Incidentally, we can derive the equilibrium at the steady state which holds $\tilde{k}_t = \tilde{k}_{t+1} = \tilde{k}^*$ or $l_t = l_{t+1} = l^*$. $\tilde{k}^*$ and $l^*$ are respectively shown as follows:

$$\tilde{k}^* = \frac{\alpha(1 - (1 + \theta_w)\tau + (1 - \tau)\theta_f)(1 + \theta_f)}{(1 + \mu)(1 + \theta_f)((2 + \rho) + (1 + \rho)(\tau(1 + \theta_w + \theta_f)))} \left(\tilde{k}^*\right)^{\alpha} \left(l^*\right)^{1-\alpha},$$

$$\tilde{l}^* = \frac{l^*}{1 - l^*} = A \frac{l^2}{1 - l^\alpha} \left(1 - l^\alpha\right)^{2\alpha}.$$
Proposition 1

If the contribution rate of pension and unemployment insurance which the worker burden and the growth rate of population increases, the capital per capita at the steady state decreases. Therefore, if the trade union prefers the complementary between wage rate and employment rate, the employment rate decreases. On the other hand, in the substantial preference, the employment rate increases.

4 Politico-Economic Equilibria

Given the discussion in the previous section, we proceed analysis by endogenizing policy choice. We then consider the voting behavior regarding pension and unemployment insurance. We first assume that

A 1. Voting is held in each period, which means issue-by-issue voting under direct democracy.
A 2. Voting takes place simultaneously on contribution to pension and unemployment insurance.
A 3. Voters consist of the young (the employed and unemployed) and the old those are alive in the same period.
A 4. The policy determination is based on majority voting.
A 5. The voting is repeated among successive generations of voters.

4.1 The Case with Commitment

To characterize the equilibria of this voting game, first, we assume the government has commitment technology; that is, \( \theta_{wt} = \theta_{wt+1} = \theta_w \) and \( \tau_t = \tau_{t+1} = \tau \), following Poutvaara (2006), Conde-Ruiz and Galasso (2005) and so forth. In other words, we assume that each generation those are alive in the same period consider \( \tau_t \) and \( \theta_{wt} \) chosen through election will be in place over its whole lifetime. From another angle, this assumption can be regarded as once-and-for-all voting or static expectation. Under this case, voters determine the constant sequence of the parameters of the welfare state. Under the existence of commitment, we can proceed the analysis in a similar way to the case of static analysis.

It is known that generally, no Condorcet winner exists in voting over multiple issues such as a combination of two kinds of policy, without imposing additional conditions on voter’s preference. To avoid such a problem, following Conde-Ruiz and Galasso (2005), we adopt the concept of a structure-induced equilibrium which is developed by Shepsle (1979). We then investigate the preferences to each policy variable. The indirect utility functions of the worker and the unemployed are respectively derived as follows:

- the employed:
  \[
  V^e(\cdot) = \ln \left( \frac{1 + \rho}{2 + \rho} \right) + \frac{2 + \rho}{1 + \rho} \left[ \ln(1 - \tau_t - \theta_{wt}) + \frac{d_{t+1}}{R_{t+1}} \right] \tag{21}
  \]

- the unemployed:
  \[
  V^u(\cdot) = \ln \left( \frac{1 + \rho}{2 + \rho} \right) + \frac{2 + \rho}{1 + \rho} \left[ \ln(1 - \tau_t b_t) + \frac{d_{t+1}}{R_{t+1}} \right] \tag{22}
  \]

Therefore, the indirect utility function is written as

\[
E[U(\cdot)] = l_t V^e(\cdot) + (1 - l_t) V^u(\cdot)
\]

\[
= l_t \left\{ \ln \left( \frac{1 + \rho}{2 + \rho} \right) + \frac{2 + \rho}{1 + \rho} \left[ \ln(1 - \tau_t - \theta_{wt}) + \frac{d_{t+1}}{R_{t+1}} \right] \right\}
+ (1 - l_t) \left\{ \ln \left( \frac{1 + \rho}{2 + \rho} \right) + \frac{2 + \rho}{1 + \rho} \left[ \ln(1 - \tau_t b_t) + \frac{d_{t+1}}{R_{t+1}} \right] \right\} \tag{23}
\]

We then have the following reaction functions of each agent by solving the following first order

---

7) Some readers may wonder whether this setting is considered to be commitment. Regarding this issue, see the remark of pp.15.
8) Regarding this issue, see Persson and Tabellini (2000), for instance.
9) This approach is also adopted in Konishi (2008), Bethencourt and Galasso (2008), Conde-Ruiz and Profeta (2007), Poutvaara (2006) and so forth.
conditions

\[
\frac{\partial E[U(\cdot)]}{\partial \theta_w} = 0 \quad (24a)
\]
\[
\frac{\partial E[U(\cdot)]}{\partial \tau} = 0 \quad (24b)
\]

Here, note that the old is identical in the sense that the desired contribution pension is

\[\tau^*, \text{old} = 1\]

for both employed and unemployed when they are young, because both of two kinds of elder generation desire as much receipt as possible. Therefore, in case that the old is majority, unemployment insurance evaporates and only pension system survives, and as a result, capital accumulation does not proceed. To eliminate such a trivial case, we proceed the analysis assuming that the old desires the same amount of pension as young workers does. Then, it is necessary to consider the ratio of voters. Note that the ratio among young unemployed households, the young and employed households and the old is \(1 - \mu : \mu : \mu\). To support the pension system, the following equation should holds: \(\mu + \frac{1}{\mu + \mu} \geq \frac{1}{2}(1 + \frac{1}{\mu + \mu}) \iff \mu \geq \frac{\mu + 2}{\mu + 2\mu}\). Therefore, it is necessary to classify the analysis into three cases depending on the value of \(\mu\).

Then, the preferences to unemployment insurance and pension is respectively as follows:

- the contribution to unemployment insurance:
  \[
  \theta^*_w = \arg \max_{\theta_w \in [0, 1]} E_t[V(\cdot)] = \begin{cases} \theta_w \text{ that satisfies } (24a) & \text{if } \mu \leq \frac{\mu + 2}{\mu + 2\mu} \\ 0 & \text{otherwise} \end{cases} \quad (25)
  \]

- the contribution to pension:
  \[
  \tau^*_t = \arg \max_{\tau \in [0, 1]} E_t[V(\cdot)] = \begin{cases} \tau \text{ that satisfies } (24b) & \text{if } \mu \geq \frac{\mu + 2}{\mu + 2\mu} \\ 0 & \text{otherwise} \end{cases} \quad (26)
  \]

The optimal solution can be derived as an intersection of the following two kinds of reaction functions:

\[
\theta^*_w = \theta^*_w(\tau) \quad (27a)
\]
\[
\tau^*_t = \tau^*_t(\theta_w) \quad (27b)
\]

Depending on the patterns of intersections, there are three plausible cases. As in the case of fig.8, both the pension and the unemployment insurance are adopted, whereas, either of pension or the unemployment insurance is supported as in the case of fig.9 or fig.10. Case 1 is the corresponding to the situation in which both pension and unemployment insurance survives. Case 2 and 3 show the situation in which either pension system and unemployment insurance survives. Case 2 is the situation in which pension system does and case 3 is the case in which only unemployment insurance survives.
To summarize, Depending on the dynamics of capital accumulation and the unemployment rate, the contents of social security system varies as in following cases.

Case 1. Both pension and unemployment insurance policies survive.
Case 2. Only pension policy survives
Case 3. Only unemployment insurance survives.

Here, let us explain the change of social security system under each case, that is, the case of $\sigma \geq 0$ and $\sigma \in (-1, 0)$ (the case of $\sigma < \alpha$ or $\sigma > \alpha$). First, regarding the former case, until the time when the unemployed is majority, the social security is the same as in case 2 (Only pension policy survives), and after that, the situation which is same as in case 3 (Only unemployment insurance survives.). On

\footnote{For calculation detail, see appendix A.4.}
the other hand, let us explain how the social security system varies under the case of fig. 7(b). Under this regime, both pension and unemployment insurance policies survive as in case 1 when the level of capital accumulation is not so high. After that, as the employment rate rises, it is likely that only unemployment insurance is adopted.

Stage. 1 \( l_t < \frac{\mu + 2}{2 + 2\mu} \): Only unemployment insurance
Stage. 2 \( l_t = \frac{\mu + 2}{2 + 2\mu} \): Both pension and unemployment insurance
Stage. 3 \( l_t > \frac{\mu + 2}{2 + 2\mu} \): Only pension system
Stage. 4 After that only pension system survives.

Turning to the second case, depending on the value of \( l_0 \), both stage 1 and 2 emerge alternately and eventually, either of two policies is adopted.

Stage. 1 \( l_t < \frac{\mu + 2}{2 + 2\mu} \): Only unemployment insurance
Stage. 2 \( l_t = \frac{\mu + 2}{2 + 2\mu} \): Both pension and unemployment insurance
Stage. 3 \( l_t > \frac{\mu + 2}{2 + 2\mu} \): Finally, either pension or unemployment insurance is adopted

Finally, regarding to the third case, the contents varies as follows: both stage 1 and 2 emerge alternately and eventually, either of two policies is adopted. The second and third case is alike, but the pattern of fluctuation differs from each other.

Stage. 1 \( l_t > \frac{\mu + 2}{2 + 2\mu} \): Only pension system is adopted.
Stage. 2 \( l_t < \frac{\mu + 2}{2 + 2\mu} \): Only unemployment insurance is adopted.
Stage. 3 \( l_t = \frac{\mu + 2}{2 + 2\mu} \): Both pension and unemployment insurance
Stage. 4 Finally, either pension or unemployment insurance is adopted.

To summarize the above discussions,

**Proposition 2** The patterns of policy change are summarized as follows:

1. The Case of \(-1 < \sigma < 0\):
   For the last time, only pension system survives.
2. The Case of \(0 < \sigma < 1\), & \(\sigma < 2\alpha\):
   The case in which only unemployment insurance survives and the case in which only pension system survives emerges alternately.
3. The Case of \(0 < \sigma < 1\), & \(\sigma > 2\alpha\):
   The case in which only unemployment insurance survives and the case in which only pension system survives emerges alternately, and eventually, either pension or unemployment insurance is adopted depending on the value of employment rate at the steady state.

Remark Regarding the assumption of commitment, we have some two remarks. First, once the government determines the policy based on the voting, we assume that this policy lasts for periods
when some generation is alive. It is possible to consider this situation as steady state, but strictly speaking, this differs from steady state. The difference between steady state is explained as follows: In case of steady state, all variables are constant through time $t$. Contrastingly, assuming commitment by the government in this paper means that we assume that policy variables are constant while some generation is alive.

Second, some readers may wonder why the tax rate can be treated as constant despite of the existence of state variable. They consider that the tax rate is dependent on state variable, that is, $\tau_t = \tau(k_t)$, and tax rate cannot be treated as constant as long as it is dependent on state variable. We avoid such a question by assuming that the voting is held only once. Our answer to such a question is as follows: From the capital market condition, we have:

$$K_s = L_s s^{\tau}_{t-1}(\tau_{t-1}, \theta_{w,s-1}) + (N_t - L_s)s^{\nu}_{t-1}(\tau_{t-1}, \theta_{w,s-1}).$$

This equation shows that capital at $s$ period is dependent only on the past policy variables. Therefore, we can treat $K_s$ as constant because $K_s$ is not dependent on policy variables at $s$ period and after that periods. Conversely, we find that the policy variables at $s$ period do not depend on state variable. Therefore, we can avoid the effects of the change in the state variables. Most of studies which apply the structure-induced equilibrium (Conde-Ruiz and Galasso (2005), Konishi (2008), Bethencourt and Galasso (2008), for instance) avoid this kinds of criticism by dropping state variables (i.e, capital) from their model. However, it is the case with commitment that the situation in which policy variables are constant while some generation is alive in Conde-Ruiz and Profeta (2007), Poutvaara (2006) and the present paper, as long as capital is not taxed.

4.2 The Case without Commitment

In this subsection, the assumption of commitment over future social security policies is relaxed. Before entering analysis, lets us define the equilibrium concept. Then, we investigate whether each agent has an incentive deviate or not.

First, in the spirit of Krusell, Quadrini, and Rios-Rull (1997), let us define the equilibrium concept (politico-economic equilibrium\(^\text{11}\)).

\textbf{Definition 2 (Politico-Economic Equilibrium)} A (Markov perfect) politico-economic equilibrium is defined as a pair of functions $\{c^{ly}_{i}, c^{lo}_{i}, \tau_{i}, \theta_{wt}\}_{t=1}^{\infty}$ that accords with the following.

(i) Given the sequence $\{\theta_{ft}\}_{t=1}^{\infty}$, each agent (the employed and the unemployed) determines the policy variables that maximize their individual utility. That is, the optimal policy variables meet

\(^{11}\) This concept corresponds to so-called Markov-perfect equilibrium. These conditions are dependent on the relationship between the $t$ and $t + 1$ period. Therefore, this concept meets the Markov property. See also Forni (2005) who focuses on the Markov-perfect equilibrium in an OLG model.
the following maximization problem:

\[
\max_{\tau, \theta} \ln(c_i^{\tau}) + \frac{1}{1 + \rho} \ln(c_i^{\theta}), \quad i \in \{e, u\}, \quad \text{subject to} \quad K_{t+1} = \Psi(K_t, \tau, \theta_{wt})
\]

(ii) The budget constraints of pension and unemployment insurance are balanced in each period.

(iii) Firms maximize their profit.

(iv) The condition of the trade union’s wage setting holds.

(v) Finally, the following markets clear:

- Commodity Market: Eq. (13)
- Capital Market: Eq. (14)
- Labor Market: Eq. (15)

We then formally define the voting game. The public history of the game at period \(t\), \(h_t = \{(\tau_0, \theta_{w0}), (\tau_1, \theta_{w1}), \ldots, (\tau_{t-1}, \theta_{wt-1})\} \subseteq \mathcal{H}_t\) is the sequence of social security system (pension and unemployment insurance). \(\mathcal{H}_t\) is the set of all possible history at time \(t\). An action profile for the employee is, \(\{\tau, b_t\} \in [0, 1] \times [0, 1]\). Analogously, an action for unemployed individual at time \(t\) is \(\{\tau, b_t\} \in [0, 1] \times [0, 1]\).

Then, a strategy for the employee is at \(t\) period is a mapping from the history of the game into the action space, that is, \(\sigma^e : h_t \rightarrow \{\tau, \theta_{wt}\}\). Analogously, a strategy for the unemployed is at \(t\) period is \(\sigma^u : h_t \rightarrow \{\tau, \theta_{wt}\}\). The strategy profile played by both individuals at \(t\) period is denoted by \(\sigma_t \equiv \sigma^e_t \cup \sigma^u_t\).

At \(t\) periods, the objective function for young each player \((i \in \{e, u\})\) is

\[
V_i^t(\sigma_{t_0}^e, \sigma_{t_1}^e, \ldots, \sigma_{t_j}^e, \sigma_{t+1}^u) = V_i^t(\tau, \theta_{wt}, \tau_{t+1}, \theta_{wt+1}).
\]

and regarding old agents,

\[
V_i(\sigma_0, \sigma_1, \ldots, \sigma_t, \sigma_{t+1}) = V_i^t(\tau, \theta_{wt}).
\]

These solutions describe the relationship between the policy at \(t\) period and the one at \(t + 1\) period.

Moreover, let us describe the definition of equilibrium.

**Definition 3 (The Definition of Markovian Structure-Induced Equilibrium)**

1. \(\sigma\) meets the property of Markov perfect equilibrium.
2. For all \(t\), at \(t\) period, the equilibrium outcome associated to \(\sigma_t\) is a structure-induced equilibrium of the static game with commitment.

As contrasted with the analysis in the previous subsection, we assume that the government does not have commitment technology in this subsection. Then, let us define the history of the game \(H_t\) as

\[
H_0^\theta \equiv \{h_t \in \mathcal{H}_t | \theta_{wt} = \theta_{wt*}, \quad t \in \{0, 1, \ldots\}\},
\]

and

\[
H_t^\sigma \equiv \{h_t \in \mathcal{H}_t | \theta_{wk} = 0, \quad k = 0, 1, \ldots, t_0, \text{and} \; \theta_{wt} = 0, \quad t \geq t_0.\}
\]

Moreover, the strategy profile of the employed and the unemployed are respectively denoted as \(\sigma^e_t\) and \(\sigma^u_t\). We then investigate whether each player has an incentive to deviate from solution under
full commitment discussed in the previous subsection. Under this setting, we first verify that the unemployed does not have an incentive to deviate from the strategy. We assume that the unemployed adopts the following strategy, $\theta_{t0,w}^{\text{deviate}} > \theta^*_w$ and $\tau_{t0}^* < \tau_t^{\text{deviate}}$. However, if the employed does not obtain additional payoff by deviation, because the employed punish the employed by reducing payment of contribution to pension, $\tau$, which causes negative effect on the welfare of both agents. Therefore, we find that the unemployed does not have an incentive to deviate from the commitment solution.

Turning to the employed, suppose that the unemployed deviates from equilibrium, that is, they avoid paying contribution to pension. In this case, the workers will punish the unemployed by not paying contribution to unemployment insurance. The unemployed would pay contribution to pension so as to avoid being punished. So, we find that they do not have an incentive to deviate. To summarize, neither both workers nor the unemployed have an incentive to deviate.

From the discussion, we have:

**Proposition 3**

*The strategies discussed in the previous subsection coincide with the ones without commitment. In other words, the strategies with commitment are time-consistent.*

5 Conclusion

This paper shows how the social security system evolves as attribution of voters changes. In our setting, policy determination is based on majority voting and the government has two kinds of social security policy; pension and the unemployment insurance. That is, when the younger workers and the old constitute the majority of voters, pension system is supported and when the young unemployed is the majority, the unemployment insurance is adopted. Under such a situation, we show how the contents of the social security system evolves depending on the dynamics of capital accumulation and the unemployment rate, and show the social security system vanishes in certain instances. This result may explain the future of social security policy in the developed countries including Japan.

Finally, we conclude the present paper by mentioning the problems to be solved in the future. First, when the unemployment insurance (the intra-generational redistribution scheme) is supported, the following condition is needed; extremely highly-population growth and/or unemployment rate. It is necessary to make these parameters down-on-earth. Second, it is necessary to show the possibility that neither of pension nor unemployment insurance are adopted. Although the condition for the existence of such a situation is not derived, the present paper may be absorbing by deriving the possibility that social security system completely vanishes. Finally, we showed the pattern of fluctuation of social security system, but it may be necessary to fortify persuasion of our result by the aid of numerical simulation.
Appendix A

A.1 Proof of Single Peakness

First, let us prove that the objective function of \( U(\cdot) \) meets the property of single-peaking.

A.2 Derivation of eq. (16)

\[
\hat{k}_{t+1} = \frac{1}{1 + \mu} \left[ s^c_t \times l_t + s^u_t \times (1 - l_t) \right]
\]

\[
= \frac{1}{1 + \mu} \left[ \frac{1}{2 + \rho} \left\{ (1 - \theta_{wt} - \tau_l)w_t - (1 + \rho) \frac{d^c_{t+1}}{R_t+1} \right\} \times l_t + \frac{1}{2 + \rho} \left\{ (1 - \tau_l) b_t - (1 + \rho) \frac{d^u_{t+1}}{R_t+1} \right\} \times (1 - l_t) \right]
\]

\[
= \frac{1}{1 + \mu} \frac{1}{2 + \rho} \left\{ (1 - \theta_{wt} - \tau_l)w_t - (1 + \rho) \frac{d^c_{t+1}}{R_t+1} \right\} \times l_t + \frac{1}{2 + \rho} \left\{ (1 - \tau_l) b_t - (1 + \rho) \frac{d^u_{t+1}}{R_t+1} \times (1 - l_t) \right\}
\]

\[
= \frac{1}{1 + \mu} \frac{1}{2 + \rho} \left\{ (1 - \theta_{wt} - \tau_l)w_t l_t + (1 - \tau_l) b_t (1 - l_t) \frac{1 + \rho}{R_t+1} \left\{ (l_t d^c_{t+1}) + ((1 - l_t) d^u_{t+1}) \right\} \right\}
\]

Here, using eqs.(11) and (12), the term \((*)\) and the second term \((**)*\) are respectively rewritten as \((1 - \theta_{wt} - \tau_l)w_t l_t + (1 - \tau_l) b_t (1 - l_t)\) and \((1 + \mu) l_{t+1} w_{t+1} \{ \tau_{t+1} (1 + \theta_{wt+1} + \theta_f) \} \). Moreover, using eq. 7a and 7b, we obtain

\[
\hat{k}_{t+1} = \frac{\alpha (1 - (1 + \theta_{wt}) \tau + (1 - \tau_l) \theta_{ff}) (1 + \theta_{ff})}{(1 + \mu)(1 + \theta_{ff})((2 + \rho) + (1 + \rho)(\tau_l(1 + \theta_{wt} + \theta_{ff})))} \hat{k}_t^{\alpha(1 - \alpha)}
\]

\[
= \varepsilon \hat{k}_t
\]

A.3 Derivation of eq.(20); the dynamics of \( l_t \)

At the steady state, we have

\[
(l)_{t}^{\frac{\sigma - \alpha}{\sigma}} (1 - l_{t}^{\alpha})^{-\frac{\alpha}{\sigma}} = A \tau_{t}^{\frac{\alpha}{\sigma}}.
\]

(28)

Let the right side of the above equation be \( \psi(l) \), we obtain

\[
\psi'(l) = \left( [1 - \frac{\alpha}{\sigma}] - \frac{1 - \sigma}{\sigma} l^{\alpha(1 - l^{\alpha})^{-1}} \right) (l)^{-\frac{\alpha}{\sigma}} (1 - l^{\alpha})^{-\frac{\alpha}{\sigma}}
\]

\[ \square \text{The Case of } 0 < \sigma < \infty \text{ In this case, } \psi'(l) \text{ is positive. By totally differentiating eq.(19), we then have:}
\]

\[
\left. \frac{d l_{t+1}}{d l_t} \right|_{l_{t+1} = l_t = l} = \frac{\psi' \frac{1}{\psi} + \psi'(1 - l)}{\psi' \frac{1}{\psi} + \psi'(1 - l)} < 1
\]

(29)

Therefore, the steady state equilibrium is locally stable.
The Case of $-1 < \sigma < 0$, \& $\sigma < 2\alpha$ We investigate the relationship between $|\phi'(l)|$ and 1. In this case, $\psi'(l) < 0$ and $\psi(0) = \infty$ and $\psi(1) = 0$. We then obtain

$$\frac{dl_{t+1}}{dl_t} \bigg|_{l_{t+1} = l_t} = \frac{\psi \hat{\sigma} + \psi(1-l)\psi'}{\psi \hat{\pi} + \psi(1-l)\psi'}$$

At $l = 1$, we have

$$\frac{dl_{t+1}}{dl_t} \bigg|_{l_{t+1} = l_t = 1} = 0$$

and At $l = 0$, we have by L'Hôpital’s rule,

$$\frac{dl_{t+1}}{dl_t} \bigg|_{l_{t+1} = l_t = 0} = \frac{(\psi \hat{\pi} + \psi(1-l)\psi')'}{(\psi \hat{\pi} + \psi(1-l)\psi')'}$$

Then, we have

$$\frac{\alpha \psi \hat{\pi} + \frac{1}{\alpha} \psi \hat{\pi}^{-1} \psi'(1-l)\psi' + \psi \hat{\pi} (1-l)\psi''}{\alpha \psi \hat{\pi} + \frac{1}{\alpha} \psi \hat{\pi}^{-1} \psi'(1-l)\psi' + \frac{1}{\alpha} \psi \hat{\pi} (1-l)\psi''} > 0$$

So, the dynamics of $l_t$ is depicted as fig.7(b).

The Case of $-1 < \sigma < 0$, \& $\sigma > 2\alpha$ We then have:

$$\frac{dl_{t+1}}{dl_t} \bigg|_{l_{t+1} = l_t} = \frac{\psi \hat{\sigma} + \psi(1-l)\psi'}{\psi \hat{\pi} + \psi(1-l)\psi'}$$

For the interval $0 < l < 1$, there exists $\bar{l}$ such that $\frac{dl_{t+1}}{dl_t} \bigg|_{l_{t+1} = l_t = \bar{l}} = 0$ because $\phi'(\cdot) < 0$. Therefore, For the interval $0 < l < 1$, there exists at least one solution, which satisfies for $[0, \bar{l}]$, $\frac{dl_{t+1}}{dl_t} < 0$ and for $[\bar{l}, 1]$, $\frac{dl_{t+1}}{dl_t} > 0$. Therefore, the dynamics under this case is depicted as fig.7(c).

A.4 Preference on the contribution rate; $\theta_w$, $\tau$

Each type of individual determines the contributions to unemployment insurance, $\theta_w$, and pension, $\tau$, so as to maximize his/her utility. The preferences to $\theta_w$ and $\tau$ are derived by solving following first
order conditions:
\[
\frac{\partial E_t[V_t]}{\partial \tau} = 0
\]
\[
\Leftrightarrow l_t \left\{ (1 - \tau - \theta) \frac{w_t}{R_{t+1}} \frac{\partial R_{t+1}}{\partial \tau} + (1 - l_t) \left( \frac{b_t}{R_{t+1}} \frac{\partial R_{t+1}}{\partial \tau} \right) \right\} + (1 - l_t) \left\{ (2 + \rho) \left( \frac{w_t}{R_{t+1}} - \frac{(1 + \mu) R_{t+1}}{R_{t+1}} (1 + \theta_w + \theta_f) - (1 + n) (1 + \mu) \theta_f \right) \left( \frac{w_{t+1}}{R_{t+1}} \frac{\partial l_{t+1}}{\partial \tau} + \frac{l_{t+1}}{R_{t+1}} \frac{\partial w_{t+1}}{\partial \tau} \right) \right\} + (1 - l_t) \left\{ (2 + \rho) \left( \frac{w_t}{R_{t+1}} - \frac{(1 + \mu) R_{t+1}}{R_{t+1}} (1 + \theta_w + \theta_f) - (1 + n) (1 + \mu) \theta_f \right) \left( \frac{w_{t+1}}{R_{t+1}} \frac{\partial l_{t+1}}{\partial \tau} + \frac{l_{t+1}}{R_{t+1}} \frac{\partial w_{t+1}}{\partial \tau} \right) \right\}
\]
(32a)

\[
\frac{\partial E_t[V_t]}{\partial \theta_w} = 0
\]
\[
\Leftrightarrow l_t \left\{ (1 - \tau - \theta) \frac{w_t}{R_{t+1}} \frac{\partial R_{t+1}}{\partial \theta_w} + (1 - l_t) \left( \frac{b_t}{R_{t+1}} \frac{\partial R_{t+1}}{\partial \theta_w} \right) \right\} + (1 - l_t) \left\{ (2 + \rho) \left( \frac{w_t}{R_{t+1}} - \frac{(1 + \mu) R_{t+1}}{R_{t+1}} (1 + \theta_w + \theta_f) - (1 + n) (1 + \mu) \theta_f \right) \left( \frac{w_{t+1}}{R_{t+1}} \frac{\partial l_{t+1}}{\partial \theta_w} + \frac{l_{t+1}}{R_{t+1}} \frac{\partial w_{t+1}}{\partial \theta_w} \right) \right\} + (1 - l_t) \left\{ (2 + \rho) \left( \frac{w_t}{R_{t+1}} - \frac{(1 + \mu) R_{t+1}}{R_{t+1}} (1 + \theta_w + \theta_f) - (1 + n) (1 + \mu) \theta_f \right) \left( \frac{w_{t+1}}{R_{t+1}} \frac{\partial l_{t+1}}{\partial \theta_w} + \frac{l_{t+1}}{R_{t+1}} \frac{\partial w_{t+1}}{\partial \theta_w} \right) \right\}
\]
(32b)

References


