Inefficient durable-goods monopolies

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Abstract

We identify inefficient equilibria in a standard durable-goods monopoly model with a finite number of buyers. They are the result of high valuation buyers randomizing their purchase date, while deciding whether to buy at a high initial price or wait for a lower price which is offered only once a critical number of buyers has purchased. This buyer war of attrition, sustained by the monopolist’s pricing decision, can significantly delay market clearing and rationalizes unscheduled purchase and price cut dates in monopolized durable-goods markets.

1 Introduction

Durable-goods theory challenges the classic association of monopoly with inefficiency by proposing a captivating idea: that a monopolist, uncommitted to future prices, will clear the market in a proverbial twinkle of an eye. In this paper we show that the model supporting this view also has inefficient equilibria where the market can take a long time to clear.

In these equilibria the timing of purchases and price cuts is stochastic, a common feature of durable-goods markets which the existing theory—reporting deterministic price paths—has left unexplained. Indeed, while buyers facing a high initial price may rightly expect a price cut in the future, these are typically unscheduled and buyers are unable to forecast their timing. "This is life in the technology lane" according to Steve Jobs, "there is always someone who bought a product before a particular cutoff date and misses the new price" he wrote following a $200 price cut on Apple’s i-Phone in September 2007.1

In the equilibria we study here buyers randomize their purchase date, deciding whether to buy at a high initial price or wait to buy at a lower future price which is offered once a critical number of sales is reached. For this reason the precise date at which any individual buyer will purchase is unknown. Eventually, as if succumbing to the temptation, some end up purchasing at the high price, a critical mass is reached, and the monopolist finally brings the price down for the remaining buyers. The market then clears, but this can take a significant time.

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1Another publicized unscheduled price cut was Motorola’s Razer phone, introduced at $499 in early 2005 and priced at $199 six months later. These price cuts are not explained by competition or cost alone.
Coase (1972) was the first to argue that durability can severely limit monopoly power. Once high-valuation buyers have purchased and left the market, a monopolist selling a durable-good will want to lower the price and sell to the remaining buyers as well. Anticipating these price cuts, buyers will be reluctant to accept high prices and to avoid delaying sales the monopolist will prefer to offer a low price at the outset of the game. The idea has been formalized by studying subgame perfect equilibria (SPE) of an infinite-horizon model of complete information: a monopolist, producing at a constant known marginal cost, posts a price in each period and buyers with unit demand simultaneously decide to either accept or reject each price.

With a continuum of buyers this model has a unique equilibrium path in stationary (Markov) strategies. As Coase conjectured, when the time between two consecutive offers converges to zero the opening price converges to the lowest buyer valuation (above cost) and the competitive quantity is sold in a twinkle of an eye (Gul, Sonnenschein, and Wilson, 1986).

With a finite number of buyers this result changes dramatically. In that case each sale has a nonnegligible effect on profits and the monopolist may then credibly condition price reductions on single purchases. If prices can be revised frequently, there exists a stationary SPE in which the monopolist sells in sequence to each buyer at their own valuation—as if eating down the demand curve—and achieves virtually the profits of a perfectly discriminating monopolist (Bagnoli, Salant, and Swierzbinski, 1989 and von der Fehr and Kuhn, 1995).

While the effect of durability on market power is ambiguous, in both cases described above the equilibrium price paths are deterministic and, since all gains from trade are realized almost instantaneously, market outcomes are efficient.

Here we use this same standard setting to study a new source of monopoly inefficiency: buyer attrition. We show that in a market with a finite number of buyers there also exist mixed-strategy SPE where the market clearing date remains bounded away from zero, even as the interval between offers shrinks to zero.

We do this by constructing (both stationary and non-stationary) SPE that can be described as follows: The monopolist initially chooses a high price and waits for a critical number of buyers to purchase before lowering the price to the remaining buyers. As early buyers create a positive externality for the remaining ones, from the high-valuation buyers’ perspective the game resembles a war of attrition and they delay purchases in the hope others will purchase first—and in equilibrium buyers randomize over (conditional) purchasing dates. Such buyer wars of attrition, when sustained by the monopolist’s pricing decision, create inefficient real-time delay and rationalize stochastic purchase and price cut dates.

In the equilibria we study here the monopolist would like to make a one period price cut to increase the probability some high-valuation buyers purchase immediately, and therefore reduce the interest which he otherwise may lose by delaying the remaining sales. Similar to the effect described by Coase, this incentive to cut prices can create an unravelling of prices which reduces the monopolist’s profit.

When strategies depend not only on current demand but also directly on past prices, i.e. are non-stationary, the threat that if the monopolist cuts the price then all players

\footnote{For this reason the outcome is known as Paesman, after a classic computer game.}

\footnote{Buyers’ payoffs are decreasing in time but those who purchase first get a higher price and a lower payoff.}
coordinate in a low profit equilibrium thereafter can be enough to discipline the monopolist’s pricing behavior. The harsh consequence of loosing his reputation makes the monopolist set a high initial price and endure a buyer war of attrition.

Such trigger strategies are ruled out when strategies depend only on current demand, i.e. are stationary. In this case a similar punishment effect can replace reputation. We find that, when buyers hold heterogeneous strategic postures, a small price cut can induce early purchases by soft buyers, leaving the game with only tough buyers and a low continuation profit. The monopolist avoids reaching that state by setting a higher price—to keep a soft buyer in the market until some tough buyers purchase first—and willing enduring a buyer war of attrition with losses through delay.

Given the significant implications of the standard model to regulation and antitrust, identifying and understanding sources of inefficiency in monopolized durable-goods markets has been an important research topic. Authors have studied the role of reputation (Ausubel and Deneckere, 1989), the effect of time varying demand (Sobel, 1991) and challenged technology and informational assumptions—for example accounting for imperfect durability (Deneckere and Liang, 2008), limited capacity (Bulow, 1982 and McAfee and Wiseman, 2008) and private information on cost (Ausubel and Deneckere, 1992).

The present article contributes a new insight to this literature and extends our understanding of the relationship between reputation and inefficiency to markets with a finite number of buyers. Ausubel and Deneckere (1989) showed that in the version with a continuum of buyers outcomes can be inefficient when strategies are non-stationary. In the reputational equilibria they constructed the monopolist retains market power, the price path is deterministic and sales necessarily occur over infinite time—so the market never clears. These equilibria exist only if the lowest buyer valuation does not exceed the monopolist’s cost—the no-gap case.

Here we show that in the same standard setting, but with a finite number of buyers, not only can reputational concerns sustain inefficient stochastic price paths where the market eventually clears—while no-gap becomes irrelevant—, but also that reputation may actually not be necessary for inefficiency since the monopolist may let buyers engage in a war of attrition even if their strategies are stationary.

More generally, the durable-goods monopoly model is a particular bargaining game where one player makes non-discriminating offers to the remaining ones. In bargaining games with exogenous positive externalities there can be real-time delay—even in stationary strategies—since attrition behavior can be sustained by the belief of each player that a coalition not involving themselves will form (see e.g. Gomes, 2005). In our setting externalities arise endogenously from equilibrium play and attrition behavior can be sustained by the belief that some other player may accept the current offer and trigger better future offers (here a price reduction). This belief is consistent because a player makes non-discriminating offers to the remaining ones, which they then simultaneously accept or reject. Non-discrimination can therefore lead to real-time delay in the absence of exogenous externalities.

This mechanism can have implications to formally related problems. For example, in many jurisdictions minority shareholder protection rules out discriminating offers in takeovers and acquisitions. A raider, who can improve the management, may then have no

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4These are genuine strategic postures, not behavioral types.
incentive to incur bidding costs anticipating that current shareholders may fail to tender their shares while trying to free-ride on the expected appreciation—an exogenous externality (see e.g. Grossman and Hart, 1980 and Holmstrom and Nalebuff, 1992). But many times an acquisition will not increase the target’s value—for example, the raider may have a private benefit because it has synergies with the target.\footnote{This situation is formally similar to our game when current shareholders disagree on the target’s value.} Our analysis suggests that also in those cases the possibility of successive tender offers can generate endogenous externalities and therefore efficient takeovers may take a long time to be completed.

The remainder of the article is organized as follows. In section 2 we present the model and in section 3 we briefly look at pure strategies. In section 4—the core of the paper—we focus on mixed strategies to characterize and explain buyer-attrition. We conclude in section 5. (Instructive proofs were included in the text; other proofs were relegated to an appendix.)

2 The model

We consider the standard durable-goods monopoly model with a finite number of buyers. A monopolist seller, indexed by \( m \), can produce any amount of a durable good at a constant marginal cost in any period \( t = 0, 1, 2, \ldots \). There is a set \( N = \{1, \ldots, n\} \) of buyers and each buyer has a valuation \( v(i) > 0 \) for a single unit of the good. \( H = \max \{ v(i) | i \in N \} \), \( L = \min \{ v(i) | i \in N \} \), and \( z \) is the number of buyers with valuation \( v(i) = L \). We set the marginal cost to zero to interpret prices and valuations as net of the cost.

The monopolist cannot make individual price offers, so in each period \( t \) the monopolist posts a common price \( p_t \in R \). Buyers then simultaneously accept the current price (and leave the game) or reject it and continue to the next period—action \( a_t^i = 0 \) denotes a rejection in period \( t \) by buyer \( i \) and \( a_t^i = 1 \) an acceptance; each buyer \( i \) may choose \( a_t^i = 1 \) at most once. There is complete information, so all valuations, prices, and purchases are observed.

The discount factor is \( \delta \equiv e^{-\rho \Delta} \in (0, 1) \), where \( \Delta > 0 \) denotes the real time between two successive offers and \( \rho > 0 \) the common discount rate. The payoff of buyer \( i \) is

\[
    u^i = \begin{cases} 
    \delta^t [v(i) - p_t] & \text{if } a_t^i = 1 \text{ for some } t \\
    0 & \text{if } a_t^i = 0 \text{ for all } t 
    \end{cases},
\]

and the monopolist’s payoff is the discounted revenue

\[
    u^m = \sum_{t=0}^{\infty} \delta^t \left( p_t \sum_{i \in N} a_t^i \right).
\]

A \( t \)-period history \( \eta(t) \) is a list of prices and purchases from period 0 to \( t - 1 \). A pure strategy is a function specifying a player’s action plan at each period for each history prior to that period. Denote the vector of strategies by players other than \( j \) by \( s^{-j} \) and the vector of all strategies by \( s = (s^j, s^{-j}) \). For a given \( s \), player \( j \)’s expected payoff is

\[
    \mu^j(s) \equiv E \left[ u^j | s \right]
\]
and $\mu^j(s|\eta(t))$ denotes player $j$’s expected payoff if after history $\eta(t)$ players behave according to $s$.

A strategy is Stationary Markov, or simply a *stationary strategy*, if it only depends on the payoff relevant history, which in this game is the set $I(t) \subseteq N$ of buyers remaining in the market at $t$.

We study subgame perfect equilibria of this game (SPE) with a particular interest on Stationary Markov Perfect Equilibria—or simply a *stationary equilibria* (SE). A SE is a strategy vector $s^*$ such that for all $I(t)$ and for any alternative strategy $s^j$ satisfies

$$\mu^j(s^j, s^{-j}|I(t)) \geq \mu^j(s^*, s^{-j}|I(t)) \text{ for all } j \in N \cup m.$$ 

In line with the literature we focus on the case where the time between offers $\Delta$ is close to zero and therefore we typically drop the qualifier "for $\Delta$ close to 1". We want to know whether all outcomes are real-time efficient:

**Definition 1.** *Real-time efficiency:* As $\delta \to 1$ all gains from trade are realized, i.e.

$$\lim_{\delta \to 1} \left[ \sum_{i \in N} \mu^i + \mu^m \right] = \sum_{i \in N} \nu(i).$$

The game is formally infinite but it is practically over when all buyers have purchased, i.e. the market has cleared. We now present a first and important lemma (the proof is simple and included in the appendix):

**Lemma 1.** In any SPE the relevant price space is $[L, \infty)$ and the market clears at $t$ if $p_t = L$, i.e. $p_t = L \Rightarrow I(t + 1) = \emptyset$.

So either all $L$-buyers are in the market or the game is over. Also the monopolist never sets the price below $L$ since that would lower the revenue without generating additional sales. Although $L$-buyers use a simple cut-off strategy—buying whenever the price does not exceed $L$—and get no surplus, they affect the monopolist’s cost of waiting which, as we will see, is an important determinant of equilibrium behavior.

## 3 Pure strategies and efficiency

In this section we briefly look at pure strategies and introduce the two outcomes the literature has focused on: the Pacman and Coasian outcomes.

Bagnoli, Salant, and Swierzbinski (1989) showed that the *Pacman* strategy always forms a SE: the monopolist posts a price equal to the highest valuation still in the market and all buyers with that valuation purchase immediately, i.e. for all $I(t) \neq \emptyset$

$$p_t = \max \{v(i) : i \in I(t)\} \text{ and } a_i^t = \begin{cases} 1 \text{ if } p_t \leq v(i) \\ 0 \text{ otherwise} \end{cases}$$
The outcome is real-time efficient since the market clears in at most \( n \) periods and as \( \delta \to 1 \) the monopolist’s profit becomes that of a perfectly discriminating monopolist, i.e.
\[
\lim_{\delta \to 1} \mu^m = \sum_{i \in N} v(i) \quad \text{and} \quad \mu^i = 0 \text{ for all } i \in N.
\]

The Pacman equilibrium is the unique SPE only if every buyer has a valuation which is "large relative to the sum of valuations of all buyers with a lower willingness to pay" (von der Fehr and Kuhn, 1995 pp. 791). If not, additional SPE exist and some may be Coasian.

In a Coasian equilibrium prices are never significantly higher than \( L \), i.e. \( \lim_{\delta \to 1} p^*_I = L \) for all \( I(t) \neq \emptyset \), so any benefit from price discrimination vanishes in the case of frequent offers, i.e.
\[
\lim_{\delta \to 1} \mu^m = nL \quad \text{and} \quad \lim_{\delta \to 1} \mu^i = v(i) - L \text{ for all } i \in N.
\]
Such outcomes are also real-time efficient. If differences in valuations are not too large there always exists a Coasian SE with an opening price \( p^*_0 = L \), which all buyers immediately accept (see Claim 1 in the appendix).

The existence of SE with low profits can support additional SPE equilibria in pure strategies. For example, the threat to revert to a Coasian equilibrium if the monopolist posts a price lower than \( H \) before some date \( t \) can sustain SPE equilibria where the monopolist always posts a price larger than \( H \) before \( t \) and then players coordinate on the Pacman outcome after that date—provided \( t \) is not too large. In a similar way, many (decreasing) price sequences can generate inefficient equilibria where buyers with different valuations purchase at different dates.

These pure strategy reputational equilibria are the counterpart in a finite buyer formulation of the reputational equilibria described by Ausubel and Deneckere (1989) in the model with a continuum of buyers: on the equilibrium path buyers immediately accept the first offer giving them their expected equilibrium payoff, while deviations from the equilibrium price path revert the game to a low profit equilibrium.

In both cases individual buyer decisions depend on the monopolist’s actions but not the other buyers’ individual actions. However only with an uncountable set of buyers can this behavior support an arbitrarily-slow rate of sales—with a finite number of buyers these SPE have a zero probability of trade for some time.

As it should be expected, stationary rules out reputational equilibria and all SE in pure strategies are real-time efficient (see Claim 2 in the appendix).

4 Mixed strategies, attrition and delay

In the remainder of this paper we focus on mixed strategy equilibria. Individual purchase decisions will now depend on both the actions of the monopolist and on those of the remaining buyers. This strategic interdependence can create buyer attrition—a new source of monopoly inefficiency—and lead to outcomes with an arbitrarily-slow (but positive) rate of sales.

The first thing to note is that a mixed strategy equilibrium is not \textit{per se} inefficient: an outcome with a positive rate of sales is real-time inefficient only if, when the interval between offers becomes close to zero, the per period aggregate probability of acceptance
becomes arbitrarily close to zero as well.\(^6\) We find real-time delay in the equilibria we construct below since, at the outset of the game, the probability of making a sale before a certain date converges to an exponential distribution.

We look at symmetric equilibria—buyers with the same valuation use the same strategy—and we let consumers’ valuations take three values: low, medium and high, i.e. \(v(i) \in \{L, M, H\}\) with \(0 < L < M < H\). In this case the payoff relevant history of the game \(I(t)\) can be summarized by

\[ h(t) = (n_t^H, n_t^M, n_t^L), \]

where \(n_t^v \in \mathbb{N}\) is the number of buyers with valuation \(v\) in the market at time \(t\). To simplify notation we henceforth make reference to each state \(h(t)\) in the text and, since stationary play depends on \(h\) alone, we denote equilibrium actions (not the strategies) by \(p_h^*\) and \(a_h^*\).

For ease of exposition we study the simplest games with outcomes where buyers engage in a war of attrition. We start by looking at buyers’ strategic postures (subsection 4.1) and the Coase temptation (subsection 4.2). In these steps we review the forces behind the Pacman and Coasian outcomes and use them to trace delay. We then identify buyer attrition with both non-stationary (subsection 4.3) and stationary strategies (subsection 4.4).

### 4.1 Soft and tough buyers

By Lemma 1 we can restrict our attention to games where \(n_t^L = z\) and move backwards to games with one additional buyer.\(^7\) This additional buyer can hold one of two strategic postures: soft or tough.

**Lemma 2.** The game with one \(H\)-buyer and \(z\) \(L\)-buyers, i.e. \(h(t) = (1, 0, z)\), has two SE if \(z \geq \frac{x}{1-x} = (H - L)/L\): the Pacman and a Coasian with \(p_h^* = L\). The Pacman outcome is unique otherwise. (Similarly for the game \(h(t) = (0, 1, z)\) if \(z \geq \frac{y}{1-y} = (M - L)/L\).)

**Proof.** In the case of \(h(t) = (1, 0, z)\) we look in step 1 at \(p_h > L\) and in step 2 at \(p_h = L\) (the proof for the state \(h(t) = (0, 1, z)\) is analogous).

Step 1: Suppose that in state \(h(t) = (1, 0, z)\) some price \(p > L\) is part of a (mixed) equilibrium price strategy and \(E[p_h]\) is the expected price of this strategy profile. \(L\)-buyers always refuse \(p\) and, since we are considering stationary pricing strategies, the \(H\)-buyer only accepts this price with positive probability if the payoff of accepting \(p\) is higher than the payoff of waiting an additional period, i.e.

\[ H - p \geq \delta(H - E[p_h]). \]

Since for all \(\delta \in (0, 1)\) the \(H\)-buyer accepts with probability 1 any price \(p \leq E[p_h]\), the monopolist will only choose prices \(p_t \geq E[p_h]\). So the monopolist uses a pure strategy: charges one price \(p_h\) which the \(H\)-buyer always accepts.

This \(p_h \notin (L, H)\) since for every \(p_h\) there exists some higher price \(p'\) which the \(H\)-buyer will accept and which generates a higher profit, i.e.

\[ \forall p \in (L, H) \exists p' \in (p, H) : H - p' > \delta(H - p) \land p' + \delta z L > p + \delta z L. \]

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\(^6\)At least in some state reached with positive probability. Suppose not, then the game would transition from state to state in the “twinkle of an eye” until the market clears.

\(^7\)Since in games with \(L\)-buyers the offer \(p_h^* = L\) is immediately accepted by all buyers.
The price \( p_h = H \) however is an equilibrium price—the Pacman outcome—since no such \( p' \) exists and for \( \delta \) sufficiently close to 1 the price \( p_h = H \) generates a profit larger than the non-discriminating profit, i.e.
\[
H + \delta zL > (z + 1)L.
\]

Step 2: Looking at (1) below, the price \( p_h = L \) is also an equilibrium price if the left-hand term—the premium the \( H \)-buyer is willing to pay to avoid a one period delay in consumption when he expects the next period price to be \( L \)—is lower than the right-hand term—the monopolist’s lost interest from delaying the sales to the \( L \)-buyers—, i.e. if
\[
[\{(1 - \delta)H + \delta L] - L] \leq (1 - \delta)Lz,
\]
(1)
because the monopolist prefers to sell immediately to all buyers (profit on the left-hand below) rather than price discriminate against the \( H \)-buyer (profit on the right-hand below):
\[
(z + 1)L \geq [(1 - \delta)H + \delta L] + \delta zL \iff z \geq x.
\]

If however the premium on the \( H \)-buyer is significant, i.e. (1) does not hold, then \( p = L \) cannot be an equilibrium price and it follows from step 1 and Lemma 1 that the Pacman equilibrium is the unique SE. QED

On the one hand, if the difference in valuations is large the monopolist chooses to delay the sales to the low valuations buyers to collect the premium the additional buyer is willing to pay to avoid a one period delay in consumption, even when this buyer expects the next period price to be \( L \). In this case it should be unreasonable to expect a low opening price and therefore the Pacman outcome is the unique SE.

On the other hand, if the difference in valuations is not too large buyers may hold different strategic postures which are associated with distinct outcomes:8

**Definition 2. Soft and tough buyers:** Buyer \( i \) is soft if in a (sub)game with only the \( L \)-buyers and buyer \( i \) the equilibrium outcome is the Pacman, i.e. \( p_h^* = v(i) \), and the buyer is tough if the outcome is Coasian, i.e. \( p_h^* = L \).

### 4.2 A Coase conjecture with tough buyers

We now study a game with two tough \( H \)-buyers, i.e. \( h(t) = (2,0,z) \), so the monopolist reduces the price to \( L \) once a single \( H \)-buyer has purchased. In this setting a Coase conjecture is verified: in the unique symmetric SE the opening price is virtually \( L \). In fact this result holds more generally in games where buyers’ valuations take two values, i.e. any game \( h(t) = (n - z, 0, z) \).

The intuition is the following. For any stationary price strategy with \( p_h^* \) larger than \( L \), the \( H \)-buyers’ payoff structure is similar to a stationary war of attrition: their payoff is decreasing in time but those who purchase first get a lower payoff. In the unique symmetric strategy profile the \( H \)-buyers randomize their acceptance and are indifferent between buying today or waiting to see if some buyer purchases first and buy tomorrow. This potentially delays the first sale, thus making the monopolist lose the interest on subsequent sales.

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8These are strategic postures and not the behavioral types often used in reputation models (see e.g. Abreu and Gul, 2000).
When buyers use stationary strategies a one period price cut at \( t \) has two effects: it reduces the profit on those sales made today but, by increasing \( H \)-buyers’ acceptance rate at \( t \), the monopolist gains the interest in the sales to the remaining buyers which are then expected to be done earlier.

The small loss associated with the former is outweighed by the significant gain on the latter since the probability a buyer purchases at \( t \) will then be strictly positive—without a price reduction this probability is almost zero for \( \delta \) close to 1. Since this is true for any high price, in equilibrium the monopolist will not choose such prices. (Interestingly the latter part of this explanation echoes the intuition for the Coase conjecture in the version of this game with a continuum of buyers—see Gul, Sommerschein, and Wilson (1986) pp. 169.)

**Lemma 3.** When \( H \)-buyers are tough, the play of the unique symmetric SE in state \( h(t) = (2, 0, z) \) is Coasian. It has the following equilibrium actions:

\[
\begin{align*}
    p_h^* & = L \text{ and } a_h^{H*} = a_h^{L*} = 1 \text{ if } z > 2x. \\
    p_h^* & = W \equiv (1 - \delta)H + \delta L, \ a_h^{H*} = 1 \text{ and } a_h^{L*} = 0 \text{ otherwise.}
\end{align*}
\]

Proof. In step 1, 2 and 3 we show by contradiction that there can be no SE where \( p_h^* \in (W, H] \). In step 4 we look at \( p_h^* \in [L, W] \).

Step 1: Suppose in state \( h(t) = (2, 0, z) \) there exists some \( p_h^* \in (W, H] \) when \( p_h^* = L \) in state \( h(t) = (1, 0, z) \). Then the first buyer creates a positive externality for the remaining one in the form of a low price: if \( H \)-buyer \( i \) accepts the offer \( p_h^* \) his payoff is \( \mu^i = H - p_h^* \), while \( H \)-buyer’s \( j \) payoff is \( \mu^j = \delta(H - L) \) if he waits one period for \( p_{t+1} = L \), the \( p_h^* \) in state \( h(t + 1) = (1, 0, z) \).

Thus for any \( p_h^* > W \), from the \( H \)-buyers’ perspective, the game resembles a war of attrition: the returns to buying decrease with time but, at any time, each buyer is better off if the other buys first. For this reason, for a given \( p_h^* \) there are several stationary strategy profiles but a unique symmetric one and it involves mixed strategies.

If the monopolist is expected to offer the equilibrium stationary price \( p_h^* \in (W, H] \) in every period after \( t \), we can let \( q^i(p_t) \) denote the probability of a typical \( H \)-buyer \( i \) accepting an offer \( p_t \) at \( t \) conditional on reaching this period \( t \) by

\[
q^i(p_t) = \Pr(a^i_t(p_t) = 1 \mid p_h^*) \text{ if } v(i) = H.
\]

This conditional probability amounts to a behavioral strategy (the strategy is a probability distribution over all dates). In a mixed-strategy equilibrium each buyer must be indifferent between purchasing today or waiting to see if the other buyer purchases first and purchase tomorrow. \( H \)-buyer \( i \)’s best response function is:

\[
\begin{align*}
    q^i(p_t) & = 1 \text{ if } H - p_t > \\
    q^i(p_t) & \in [0, 1] \text{ if } H - p_t = \delta \left[ q^i(p_t)(H - L) + (1 - q^i(p_t))(H - p_h^*) \right] . \\
    q^i(p_t) & = 0 \text{ if } H - p_t <
\end{align*}
\]
In the symmetric equilibrium, for a given \( p_t \) and stationary price \( p_h^* \in (W, H) \), we have

\[
q^i(p_t) = q^i(p_t) = q(p_t) = \begin{cases} 
1 & \text{if } p_t < W \\
\frac{(H - p_t) - \delta(H - p_h^*)}{\delta(p_h^* - L)} & \in (0, 1) \text{ if } p_t \in [W, (1 - \delta)H + \delta p_h^*] \\
0 & \text{if } p_t > (1 - \delta)H + \delta p_h^* 
\end{cases}
\]  (2)

As expected the value of the equilibrium randomization depends on \( \delta \).

Step 2: When the monopolist uses a stationary price strategy \( p_h^* \in (W, H) \) in state \( h(t) = (2, 0, z) \), for the \( H \)-buyers the game is like a standard stationary war of attrition with the conditional acceptance probability given by

\[
q(p_h^*) = \frac{(1 - \delta)(H - p_h^*)}{\delta(p_h^* - L)}. 
\]

Not only is \( q \) decreasing in \( \delta \), but for \( \delta \) arbitrarily close to 1 each \( H \)-buyer will accept the current offer with a probability arbitrarily close to zero. Formally, we divide \( q(p_h^*) \) by \( \Delta \) and, since \( \delta \equiv e^{-\rho \Delta} \), we have

\[
\lim_{\Delta \to 0} \frac{q(p_h^*)}{\Delta} = \lim_{\Delta \to 0} \frac{(H - p_h^*)}{(p_h^* - L)} \frac{1 - e^{-\rho \Delta}}{\Delta(e^{-\rho \Delta})} = \rho(H - p_h^*)/(p_h^* - L).
\]

That is, in the limit each \( H \)-buyer’s mixed strategy over purchasing dates is characterized by an exponential distribution with parameter \( \rho(H - p_h^*)/(p_h^* - L) \).

If the monopolist always charges the price \( p_h^* \) in this subgame, his expected payoff is \( \mu^m_h(p_h^*) \), the solution to

\[
\mu^m_h(p_h^*) = q(p_h^*)^2 \left[ 2p_h^* + \delta zL \right] + q(p_h^*) (1 - q(p_h^*)) \left[ p_h^* + \delta(z + 1)L \right] + (1 - q(p_h^*))^2 \delta \mu^m(p_h^*)
\]

or

\[
\mu^m_h(p_h^*) = \frac{1}{q(p_h^*) (2 - q(p_h^*))} \left[ q(p_h^*)^2 \left[ 2p_h^* + \delta zL \right] + q(p_h^*) (1 - q(p_h^*)) \left[ p_h^* + \delta(z + 1)L \right] \right].
\]  (3)

Letting the time between offers converge to zero we have

\[
\lim_{\delta \to 1} \mu^m_h(p_h^*) = \alpha(p_h^*) \cdot [p_h^* + (z + 1)L] \text{ where } \alpha(p_h^*) = \frac{2(H - p_h^*)}{2H - p_h^* - L}.
\]

So the monopolist’s expected profit is only a share \( \alpha(p_h^*) \in (0, 1) \) of the profit he would get in the case one \( H \)-buyer accepts \( p_h^* \) immediately.

Step 3: Again we suppose \( p_h^* \in (W, H) \) is a stationary equilibrium price. When buyers’ strategies are stationary the monopolist can increase the acceptance rate at \( t \) by offering a slightly lower price only at \( t \) without affecting future play. He then i) loses the profit on buyers who accept \( p_t \) but ii) gains the additional interest on the sales to all remaining buyers that are expected to be made earlier.

We now show that, for \( \delta \) arbitrarily close to 1, i) is smaller than ii) since a small price-cut has a negligible effect on the revenue from current sales but generates a non-negligible increase in
the acceptance probability of the current period. Therefore it is always optimal to undercut any price $p^*_h \in (W, H]$, which in turn implies that no such price can be part of a SE.

Formally, the monopolist’s payoff when he offers a price $p_t \in (W, (1 - \delta)H + \delta p^*_h]$ is

$$\mu^m_h(p_t) \equiv q(p_t)^2(2p_t + \delta z L) + 2q(p_t)(1 - q(p_t))(p_t + \delta(z + 1)L) + (1 - q(p_t))^2 \delta \mu^m(p^*_h),$$

and with (2) we have that for all $p^*_h \in (W, H]$

$$\lim_{\delta \to 1} \left. \frac{d\mu^m_h(p_t)}{dp_t} \right|_{p_t = p^*_h} = \lim_{\delta \to 1} \left[ \frac{\partial \mu^m_h(p_t)}{\partial p_t} + \frac{\partial \mu^m_h(p_t)}{\partial q(p_t)} \frac{\partial q(p_t)}{\partial p_t} \right]_{p_t = p^*_h}$$

$$= -\frac{2}{p^*_h - L} (1 - \alpha(p^*_h)) [p^*_h + (z + 1)L] < 0 \text{ since } \alpha(p^*_h) < 1.$$ 

Step 4: Now we focus on $p_h \in [L, W]$ which, by Lemma 1, both $H$-buyers accept with probability 1 ($L$-buyers only accept offers which do not exceed $L$). The price which maximizes the seller’s payoff in that range is either $L$ or $W$, depending on the number of $L$-buyers $z$. QED

As mentioned above, the previous result extends to larger games with two valuations. The proof, by induction on $n$, is in the appendix. The intuition is similar and it builds on the fact that $H$-buyers payoff structure is similar to a war of attrition where $n - z$ players compete for $n - z - 1$ prizes.

**Proposition 1.** When $H$-buyers are tough, the unique symmetric SE of a game $h(0) = (n - z, 0, z)$ is Cosian.

In a market with only two types of buyers even a few low-valuation buyers can induce a monopolist to cut its price before all high-valuation buyers have purchased. With no reason to treat similar buyers differently, it may then be reasonable to expect Cosian outcomes if buyers’ strategies depend only on the present market conditions. The monopolist may be able to price discriminate, but only if he is not expected to reduce its price until each and every high-valuation buyer has purchased—the Pacman outcome.

But the Cosian and Pacman outcomes do not exhaust the equilibrium set. Our next objective is to show that the standard model can still uncover additional insights in durable-goods markets.

### 4.3 Buyer attrition with reputation

Having shown that the game $h(0) = (2, 0, z)$ has no symmetric SE with delay, we here relax the stationarity requirement. When buyers also respond to past prices we can construct reputational equilibria of this game with high prices and an arbitrarily-slow real-time rate of sales (we reintroduce stationarity in the next subsection).

In these reputational equilibria the monopolist posts the same price in every period and patiently waits for $H$-buyers to select in a war of attrition who purchases first and only then he reduces the price to clear the market. Moreover, if the monopolist deviates from this price path then players coordinate in an equilibrium with low continuation profits—the Cosian outcome.
By offering a lower price the monopolist looses his reputation of holding to high prices and so even a small price cut will have a large negative effect on profits. This renders a price cut unprofitable and allows the monopolist to sustain a high price at the outset of the game. Let \( k \equiv 2(H - 2L)/L \) and \( q \) denote a probability:

**Proposition 2.** When \( z \in (x, k) \), for \( \hat{p} \in (L, H - (\frac{z+2}{2})L) \) the following actions form a SPE in state \( h(t) = (2, 0, z) \):

\[
\begin{align*}
 p_t &= \hat{p}, \quad q(a^H_t = 1) = \frac{(1-\delta)(H-\hat{p})}{\delta H - L} \quad \text{and} \quad a^L_t = 0 \quad \text{if} \quad h(t) = (2, 0, z) \quad \text{and} \quad p_{t-1} = \hat{p}
\end{align*}
\]

the Coasian outcome of Lemma 3 otherwise.

The outcome is real-time inefficient: in the limit, as \( \delta \to 1 \), each \( H \)-buyer acceptance follows a Poisson process with parameter \( \rho(H - \hat{p})/(\hat{p} - L) \), and the monopolist’s expected profit and market surplus are respectively

\[
\alpha(\hat{p}) \cdot [\hat{p} + (z + 1)L] \quad \text{and} \quad \alpha(\hat{p}) \cdot \sum_{i \in N} v(i) \quad \text{where} \quad \alpha(\hat{p}) = \frac{2(H - \hat{p})}{2H - \hat{p} - L} < 1.
\]

**Proof.** Step 1: When \( z > x \) and \( \delta \) is close to 1, the Coasian outcome described in Lemma 3 is a stationary equilibrium of the subgame \( h(t) = (2, 0, z) \) which gives the monopolist a sure profit (arbitrarily) close to \((z + 2)L\).

This outcome can be used as a punishment if the monopolist ever charges a price \( p_t \neq \hat{p} \) when \( h(t) = (2, 0, z) \). The monopolist adheres to the price path described in this proposition if the sure profit \((z + 2)L\) is less than the resulting equilibrium profit (derived in step 2 of Lemma 3)

\[
\mu^m_t(p^*_h) = \frac{1}{q(p^*_h)(2 - q(p^*_h))} \left[ q(p^*_h)^2 [2p^*_h + \delta z L] + q(p^*_h)(1 - q(p^*_h)) [p^*_h + \delta(z + 1)L] \right]
\]

with

\[
\lim_{\delta \to 1} \mu^m_t(p^*_h) = \alpha(p^*_h) \cdot [z + (z + 1)L] \quad \text{where} \quad \alpha(p^*_h) = \frac{2(H - p^*_h)}{2H - p^*_h - L}.
\]

This is satisfied if \( \hat{p} \in (L, H - \frac{z+2}{2}L) \). This set is nonempty if \( z < k \), while \( z > x \) ensures that \( H \)-buyers are tough.

Step 2: The outcome is real-time inefficient:

\[
\lim_{\delta \to 1} \left[ \sum_{i \in N} \mu^i + \mu^m \right] = \alpha(\hat{p})(2H + zL) < \sum_{i \in N} v(i)
\]

since \( \alpha(\hat{p}) < 1 \) for \( \hat{p} \in (L, H - (\frac{z+2}{2})L) \). QED

Here \( H \)-buyers engage in a war of attrition and as the time between offers converges to zero the cumulative probability of a \( H \)-buyer accepting an offer before date \( t \)—conditional on the other buyer not accepting before—converges to an exponential distribution with parameter \( \rho(H - \hat{p})/(\hat{p} - L) \). Thus a new monopoly distortion remains here in the form of real-time delay which dissipates a share \( 1 - \alpha(p^*_h) \) of the gains from trade.
We find attrition equilibria when two conditions are satisfied: $H$-buyers’ acceptance rate can be made high, i.e. $(H - \tilde{p})/(\tilde{p} - L)$ is high, but the profit made with the sales to low valuation buyers is not too high. In this situation the benefits of setting a high price $\tilde{p}$ and let a war of attrition select the first buyer outweighs the cost of delaying sales to the remaining buyers.\footnote{When the ratio $(H - \tilde{p})/(\tilde{p} - L)$ is low the $H$-buyers’ hazard rate is also low and it can therefore take a long time to make the first sale. Waiting for the $H$-buyers to select in a war of attrition who makes the first purchase would substantially delay the remaining sales and the monopolist therefore chooses to offer a low price and clear the market.}

They share with the reputational equilibria studied by Ausubel and Deneckere (1989) an arbitrarily-slow (but positive) real-time rate of sales, resulting in inefficient delay. They have however different properties: the market eventually clears (even if perhaps after a long-time), they do not rely on no-gap and, more importantly, dates of purchase and price cuts are stochastic.

As in the previous subsection, similar results can be extend to histories with multiple $H$-buyers—for example, decreasing price sequences will create delay between each sale.\footnote{$H$-buyers engage then in a sequence of multiplayer wars of attrition where rewards are monotone in the order in which they purchase, provided deviations from this price path are followed by a reversion to the Coasian equilibrium from Proposition 1.} A similar result also holds when $H$-buyers have small differences in valuations since the purchasing probabilities can be easily changed—keeping each buyer indifferent between purchasing today or waiting to see if the other buyer purchases first and purchase tomorrow.

### 4.4 Buyer attrition without reputation

Interestingly, the properties of the reputational equilibria described above can be obtained even without relaxing the stationarity assumption. So reputation may not be necessary for inefficiency in the standard model with a finite number of buyers.

To understand why we look at the game $h(0) = (2, 1, z)$ But first we still need to consider the subgame $h(t) = (1, 1, z)$. There is a SE where the $M$-buyer accepts an offer before the $H$-buyer.

**Lemma 4.** For $h(t) = (1, 1, z)$, with a tough $H$-buyer and a soft $M$-buyer, the actions $p_h^* = M$, $a_h^M = 1$ and $a_h^H = a_l^H = 0$ form the equilibrium play of a SE in that state.

**Proof.** Suppose that, for $z \geq x$, the outcome of the game $h(t) = (1, 0, z)$ is Coasian and the outcome of the subgame $h(t) = (0, 1, z)$ is the Pacman. We use the one-stage-deviation principle: The $M$-buyer has no advantage in refusing the price $M$ since he would still get the same price in the future while the $H$-buyer should refuse $M$ since he expects a payoff $\delta (H - L) > H - M$ (and for the same reason he should refuse any price $p_t > (1 - \delta)H + \delta L$). The monopolist can deviate to a $p_t \in [L, (1 - \delta)H + \delta L]$ and sell to both the $H$-buyer and the $M$-buyer immediately. However for $\delta$ close to 1 such a deviation would give him a payoff lower than his equilibrium payoff $M + \delta(z + 1)L$. QED

In a model with a finite number of buyers, individual buyer decisions have an effect on the price path and some buyers can benefit from waiting for some other buyer to purchase...
first. For this reason here skimming can fail.\footnote{Skimming means that higher valuation buyers purchase no later than lower valuation buyers. In a model with a continuum of buyers the equilibrium price path there is independent of single buyer decisions and, since delaying a purchase is more costly to high valuation buyers, these will always buy earlier.}

From the previous results, the actions below summarize the equilibrium play of a symmetric SE for each proper payoff relevant history $h(t)$ of the game $h(0) = (2, 1, z)$ when the difference in valuations is not too high, i.e. $z \geq x$, and the $M$-buyer is soft while $H$-buyers are tough.\footnote{Recall that $x \equiv (H - L)/L$ and $W \equiv (1 - \delta)H + \delta L$.}

\begin{align*}
\begin{array}{c|ccccc}
 h(t) & p_h^* & a_h^{H*} & a_h^{M*} & a_h^{L*} \\
(0, 0, z) & L & - & - & 1 \\
(0, 1, z) & M & - & 1 & 0 \\
(1, 0, z) & L & 1 & - & 1 \\
(1, 1, z) & M & 0 & 1 & 0 \\
(2, 0, z) & \begin{cases}
 L & \text{if } z > 2x \\
 W & \text{if } z \in [x, 2x]
\end{cases} & 1 & - & 1 \\
\end{array}
\end{align*}

We now present a stationary attrition equilibrium for the overall game $h(0) = (2, 1, z)$. In this equilibrium the monopolist first sets the price $p_h^* = M$ and $H$-buyers delay purchasing for a random length of time; then one purchases. The $M$-buyer purchases next (at a price equal to his valuation) and then the market clears as the price drops to $L$—the equilibrium play of subgame $h(t) = (1, 1, z)$. So $H$-buyers play a war of attrition where the looser is the first to buy at price $M$ and the winner is the third to buy at price $L$. Let $F \equiv (2H - (z + 2)L)/3$, then:

**Proposition 3.** For $M \in (W, F)$ and $z > x$ there exists a SE such that in state $h(t) = (2, 1, z)$ each $H$-buyer accepts $p_h^* = M$ with probability

\begin{equation}
\frac{(H - M)(1 - \delta)}{\delta[(1 - \delta)H + M - \delta L]},
\end{equation}

while $a_h^{M*} = a_h^{L*} = 0$.

This SE is real-time inefficient: in the limit, as $\delta \to 0$, each $H$-buyer’s acceptance follows a Poisson process with parameter $\rho(H - M)/(M - L)$ and the monopolist’s expected payoff and market surplus are respectively

\begin{equation}
\alpha(M) \cdot (2M + (z + 1)L) \text{ and } \alpha(M) \cdot \sum_{i \in N} v(i) \text{ where } \alpha(\bar{p}) = \frac{2(H - M)}{2H - M - L} < 1.
\end{equation}

Proof. An argument similar to the proof of Lemma 3 shows that a $p_t \in (M, H]$ cannot be part of a symmetric SE of the game.

Consider now $p_t \in (W, M)$. The $M$-buyer accepts this price with probability 1 since no lower price is offered in any other state he is present. $H$-buyers should therefore reject $p_t$ and wait for $p_{t+1} = p_h^* \leq W$ in $h(t + 1) = (2, 0, z)$ (unless $p_t$ is arbitrarily close to $L$, in which case they should accept).
So \( p_t \in (W, M) \) induces a subset of buyers to accept immediately and for \( \delta \) close to 1 this gives the monopolist an assured profit of (approximately)

\[
p_t + (z + 2)L,
\]

which has the supremum for \( p_t = M \) (a price which leaves the \( M \)-buyer indifferent between accepting or rejecting).

Suppose the \( M \)-buyer rejects \( p_t = M \) when \( h(t) = (2, 1, z) \). Why not charge a price slightly lower than \( M \) (which the \( M \)-buyer always accepts) to virtually get \( M + (z + 2)L \) in the twinkle of an eye?

To answer this question we compute the profit when the price is \( M \). If \( p_h^* = M \) in state \( h(t) = (2, 1, z) \), from the \( H \)-buyers’ perspective the game resembles once again a stationary war of attrition. In the only symmetric strategy profile \( H \)-buyers accept the monopolist’s offer with probability (4) and the monopolist gets, in the limit, the expected profit in (5) (a similar proof to step 2 from the proof of Lemma 3)

The monopolist then sets the price \( p_h^* = M \) in state \( h(t) = (2, 1, z) \) if this profit is larger than the supremum of the sure profit (6), i.e.

\[
\alpha(M) \cdot (2M + (z + 1)L) > M + (z + 2)L.
\]

Essentially, the soft \( M \)-buyer makes the profit function discrete at \( p_t = M \) and this discreteness breaks the Coase temptation: when (7) is satisfied a price cut becomes unprofitable and the monopolist can credibly wait out for additional sales before lowering the price.

For \( \delta \) arbitrarily close to 1 the proposed strategies form an equilibrium if \( M \in (W, F) \) and \( z > x \): the upper bound on \( M \) ensures (7) is verified and \( z > x \) ensures the actions in table 1 are part of a SE. Finally \( (W, F) \) is not empty: for a given value of \( H \) we can plot the values of \( L \) as a function of \( z \) for which both conditions are satisfied—for each point \((z, L)\) in that region the range of admissible values of \( M \) is \((W, F)\).

Since the market takes some time to clear the outcome is real-time inefficient (similar to proof proposition 2). QED

In a nutshell, the reason a soft \( M \)-buyer enables the monopolist to sustain a high opening price is the following: At the outset of the game any price lower than \( M \) is accepted by the \( M \)-buyer. His acceptance moves the game to a state where the monopolist can’t resist the Coase temptation and therefore makes a small profit (Lemma 3). The monopolist may avoid reaching that state by not pricing below \( M \), thus keeping the soft \( M \)-buyer in the market and enduring a buyer war of attrition until one of the tough buyers has purchased.\(^{13}\)

In some sense a soft buyer can replace reputation. With non-stationary strategies a small price cut changes the non-payoff relevant history of the game and buyers delay purchases anticipating a punishment path with an even lower price (and profit), i.e. trigger strategies discipline the monopolist’s pricing behavior. In the stationary case a small price cut leads to a purchase, which changes in the payoff relevant history and also moves the game to a state with a low continuation profit. In this case no trigger strategies are used but a similar punishment effect renders a price cut unprofitable.

\(^{13}\)Informational assumptions are critical in the stationary case. Suppose \( H \)-buyers’ valuations are independent draws from a common distribution. Then, if \( H \)-buyers’ valuations are public and differ, the monopolist can offer a price slightly higher than \( M \) at \( t \)—which the buyer with the lower of the two valuations rejects for sure—and induce the other \( H \)-buyer to accept the current offer immediately. In this case buyer attrition is unsustainable—but not if the values of the valuation draws remain private.
5 Conclusion

We studied a standard durable-goods monopoly model with a finite number of buyers. In a simple tractable setting, we showed that inefficient real-time delay and unscheduled price cuts can arise in (both stationary and non-stationary) mixed-strategy SPE where buyers randomize their purchasing date. These properties are distinct from those found in the previous literature, including the reputational equilibria of this same model with a continuum of buyers. For that reason the present work also offers a complementary view—and predictions—of reputation in durable-goods markets.

Most of all, buyer attrition can help us understand price patterns in these markets: many hit goods are first priced high and suddenly, once some buyers have purchased, the price finally drops and the good becomes available to many buyers. We showed that these can be equilibrium features of a standard complete information monopoly model.

Similar qualitative features can also be found in incomplete information settings. For example, the standard model assumes common knowledge of the payoff relevant history. If instead buyers observe only prices and the monopolist additionally knows the number of buyers who purchased in each period, there are perfect-Bayesian equilibria of this game where the monopolist initially charges a high price and high valuation buyers engage in a (unobservable actions) war of attrition. Buyers randomize the timing of purchases, and so over time some buyers leave the market. When a certain number of units has been sold and purchases become less frequent, the monopolist will find it optimal to finally cut its price and sell to all remaining buyers. In turn, the anticipation of this price cut—although at an unknown date—rationalizes the high-valuation buyers’ randomization.

There are many ways of introducing incomplete information in this model and it seems worthwhile to explore buyer attrition under alternative assumptions. In an interesting extension buyers may have private valuations. Suppose there are only two types of buyers—high and low—but each high valuation is independently drawn from a common distribution.\(^\text{14}\) Again we expect perfect-Bayesian equilibria of this perturbed game in which the monopolist posts a high price until a certain number of units is sold and decreases the price thereafter to clear the market.

This related model could explain the frenzies surrounding the launch of many hit goods, which are typically followed by a market slowdown before a large price cut. The reason is that, from the high-valuation buyer’s perspective, this game resembles a generalized war of attrition.\(^\text{15}\) If the monopolist is expected to drop the price to the low valuation after \(k\) units have been sold, then \(k - 1\) high-valuation buyers should purchase almost immediately and only one too many are left competing to buy at lower price.

In our view, once we look beyond the two particular outcomes the literature has focused on—the Coasian and Pacman—, the standard setting can still significantly enhance our understanding of durable-goods markets, providing us with new, alternative, and empirically relevant predictions.

\(^\text{14}\)To be more specific, assume a distribution \(F(v)\) with \(F(H - \varepsilon) = 0, F(H) = 1\), a strictly positive finite derivative everywhere and that \(\varepsilon\) is "small". This contrasts with a setting in which all buyers may have a low valuation, i.e. where \(F(L) = 0\). In the latter case we expect the Coase conjecture to be verified.

\(^\text{15}\)As studied by Bulow and Klemperer (1999).
Appendix

Proof of Lemma 1. We prove the first statement in step 1 and the second in step 2.

Step 1: Denote by $\bar{p}$ the highest price any buyer would accept with probability 1 in all subgames, i.e. $\bar{p} = \sup \{ p : a^i_i = 1 \text{ } \forall i \in I(t) \subseteq N \}$. The monopolist’s equilibrium profit has to be non-negative, so a profit maximizing monopolist will always offer $p_t \geq \bar{p} \geq 0$. For any $s^*$ we have $\mu^i = v(i) - \bar{p}$ for all $i \in N$. Buyer $i$ will also accept with probability 1 any other price $\bar{p}$ such that

$$v(i) - \bar{p} \geq \delta (v(i) - \bar{p}) \iff \bar{p} \leq (1 - \delta) v(i) + \delta \bar{p}.$$ 

Since buyer $i$ refuses prices larger than $v(i)$, $\bar{p} = L$ by the definition of $\bar{p}$. So without loss of generality we can restrict the monopolist’s action space to $p_t \geq L$.

Step 2: Buyers with $v(i) = L$ refuse all prices $p_t > L$ and, by step 1, all buyers accept with probability 1 a price $p_t = L$ when it is offered, i.e. $a^i_t = 1$ for all $i$ since

$$v(i) - L \geq \delta (v(i) - p_t) \text{ for all } v(i) \in [L, H] \text{ and } p_t \geq L. \quad QED$$

Proof of Claim 1. Suppose $p^*_t = L$ for every payoff relevant history $I(t)$. Then, from Lemma 1, if the monopolist offers a price $p_t \geq L$,

$$a^i_t = 1 \text{ for all } i : v(i) - p_t \geq \delta (v(i) - L) \iff p_t \leq v(i)(1 - \delta) + \delta L. \quad (8)$$

By the one period deviation principle, the proposed strategy forms an equilibrium if its profit is higher than the profit of selling at a premium to those $l$ buyers satisfying (8), i.e. if

$$nL \geq l [v(i)(1 - \delta) + \delta L] + \delta [(n - l)L] \iff nL \geq l v(i) \quad (9)$$

Since $l \leq n - z$ and $v(i) \leq H$, (9) is always satisfied if

$$(n - z)H \leq nL \iff \frac{H - L}{L} \leq \frac{z}{n - z},$$

i.e. if the difference between the highest and the lowest valuation is not too high. \textit{QED}

Proof of Claim 2. If we restrict attention to pure strategies which are stationary, a price which all buyers refuse cannot be part of a SE since the expected payoff of all players would be zero from that time on. By Lemma 1, the monopolist can deviate and make a positive profit by charging $L$ and clearing the market immediately.

So in each state a nonempty subset of the remaining buyers has to accept every equilibrium price with probability one. In equilibrium the market always clears in at most $n$ periods and it is therefore real-time efficient. \textit{Q.E.D.}

Proof of Proposition 1. By induction on $n$. Suppose $H$-buyers are tough and when $h(t) = (n - z - 1, 0, z)$ the outcome is Coasian, i.e. $\lim_{t \to 1} p^*_h = L$ (from Lemma 3 this is verified for $n - z - 1 = 2$). We will show by contradiction that when $h(t) = (n - z, 0, z)$ there cannot be a symmetric SE where $\lim_{t \to 1} p^*_h > L$. For simplicity we consider only the limiting case where $\delta$ is arbitrarily close to 1.

Step 1: We work by contradiction, assuming there is a non-Coasian SE for $n - z$ (but not for $n - z - 1$). In this equilibrium the $H$-buyers payoff structure would be similar to a (continuous
time) war of attrition where \( n - z \) players compete for \( n - z - 1 \) prizes. In the unique symmetric strategy profile each \( H \)-buyer’s mixed strategy is represented by an exponential distribution with parameter 
\[
\frac{\delta}{n-z-1}(H - p_h^*)/(p_h^* - L)
\]
and the monopolist’s limiting expected payoff is

\[
\lim_{\delta \to 1} \mu_{h}^{\infty}(p_h^*) = \beta(p_h^*) \cdot [p_h^* + (n - 1)L]\ 
\text{where } \beta(p_h^*) = \frac{(n - z)(H - p_h^*)}{(n - z)H - p_h^* - (n - z - 1)L}.
\]

Step 2: We now consider how buyers would respond to a one period price cut at \( t \). If all \( H \)-buyers except \( i \) accept the current price with probability \( q \), then the payoff of buyer \( i \) if he refuses the current offer will be (approximately)

\[
(1 - q)^{n-z-1} \cdot (H - p_h^*) + (1 - (1 - q)^{n-z-1}) \cdot (H - L)
\]

In a symmetric strategy profile all \( H \)-buyers accept with the same probability \( q \) and in the unique symmetric solution we have:

\[
q(p_t) = \begin{cases} 
1 & \text{if } p_t \leq L \\
\frac{\ln \frac{p_t - L}{H - p_h^*}}{n-z} + 1 & \text{if } p_t \in (L, p_h^*) \\
0 & \text{if } p_t > p_h^*
\end{cases}
\]

That is, the cumulative distribution representing each \( H \)-buyer equilibrium strategy would then contains a mass point \( q(p_t) \) at \( t \) (and an exponential distribution in the remainder of the support).

Moreover for every \( p_t, p_h^* \in (L, H) \) the monopolist’s profit is

\[
\mu_{h}^{\infty}(p_t) = (1 - q(p_t))^{n-z} \cdot \mu_{h}^{\infty}(p_h^*) + \sum_{s=0}^{n-z-1} [(n - z - s)p_t + (z + s)L] \cdot q(p_t)^{n-z-s} \cdot (1 - q(p_t))^s.
\]

Step 3: For every \( p_t \in (L, p_h^*) \) we have (with equality for \( p_t = p_h^* \))

\[
\mu_{h}^{\infty}(p_t) > \phi_h(p_t) \equiv (1 - q(p_t))^{n-z} \mu_{h}^{\infty}(p_h^*) + (1 - (1 - q(p_t))^{n-z})(p_t + (n - 1)L).
\]

We differentiate \( \phi_h(p_t) \) with respect to \( p_t \) and its limit, as \( p_t \to p_h^* \), is

\[
\frac{\partial}{\partial p_t} (1 - q(p_t))^{n-z} \cdot (\beta(p_h^*) - 1) [p_h^* + (n - 1)L] < 0
\]
since the probability of no one buying at \( t \) is increasing in the current price \( p_t \) and \( \beta(p_h^*) < 1 \) for every \( p_h^* \in (L, H) \). Since \( \mu_{h}^{\infty} \) and \( \phi_h \) are continuous and left differentiable at \( p_h^* \), and \( \mu_{h}^{\infty}(p_h^*) = \phi_h(p_h^*) \) but \( \mu_{h}^{\infty}(p_t) < \phi_h(p_t) \) for \( p_t \in (L, p_h^*) \), we have

\[
\frac{\partial \mu_{h}^{\infty}(p_h^*)}{\partial p_t} = \frac{\partial \mu_{h}^{\infty}(p_t)}{\partial p_t} < \frac{\partial \phi(p_h^*)}{\partial p_t} < 0.
\]

So undercutting the proposed equilibrium price \( p_h^* \) provides a profitable deviation to the monopolist when buyers’ response is symmetric. This holds for every \( p_h^* > L \), so when \( H \)-buyers are tough there can be no symmetric SE which is not Coasian. QED
References


