Exchange Rate Pass-Through and Inflation:
A Nonlinear Time Series Analysis

Mototsugu Shintani$^1$  Akiko Terada-Hagiwara$^2$
Tomoyoshi Yabu$^3$

$^1$Vanderbilt University
$^2$Asian Development Bank
$^3$Keio University

August 3 2009 / FESAMES, University of Tokyo
Outline

1. Motivation

2. A Simple Model of Importers

3. STAR Models

4. Empirical Results

5. Conclusion
Outline

1. Motivation

2. A Simple Model of Importers

3. STAR Models

4. Empirical Results

5. Conclusion
Objectives

1. Investigate the relationships between the degree of exchange rate pass-through (ERPT) and inflation using nonlinear time series methods - smooth transition autoregressive (STAR) models
Objectives

1. Investigate the relationships between the degree of exchange rate pass-through (ERPT) and inflation using nonlinear time series methods - smooth transition autoregressive (STAR) models

2. Construct a simple theoretical model of importing firms where ERPT is predicted to be a nonlinear function of the past inflation rate
Objectives

1. Investigate the relationships between the degree of exchange rate pass-through (ERPT) and inflation using nonlinear time series methods - smooth transition autoregressive (STAR) models.

2. Construct a simple theoretical model of importing firms where ERPT is predicted to be a nonlinear function of the past inflation rate.

3. Use monthly US domestic and import price data to estimate the STAR model with several U-shaped transition functions and alternative specification.
Related literature

- Evidence of declining (or time-varying) ERPT

- Evidence of cross-sectional positive correlation of ERPT and inflation
Related literature

- Evidence of declining (or time-varying) ERPT

- Evidence of cross-sectional positive correlation of ERPT and inflation

- Application of STAR models
Theoretical model of ERPT determination

- **Endogeous ERPT (Two approaches)**

- We modify the importer model of Devereux-Yetman(2008)
  Differences are
Theoretical model of ERPT determination

- **Endogenous ERPT (Two approaches)**

- We modify the importer model of *Devereux-Yetman (2008)*

  Differences are
  1. Importers make finite-period contracts as in the Taylor (1980) staggered pricing model instead of infinite horizon Calvo (1983) model
  2. Contract pricing rule is inflation indexation
  3. Select a probability of opting out from the contract
  4. ERPT depends on lagged inflation not steady-state inflation
Outline

1. Motivation

2. A Simple Model of Importers

3. STAR Models

4. Empirical Results

5. Conclusion
Firms (Importers)

- Imports a differentiated good $i$ from abroad and sell monopolistically as in Devereux-Yetman (2008)
- Taylor (1980)-type staggered $N$ period contracts between the importers and the retailers
- Firm makes a contract at $t - j$ faces a demand at $t$

$$C_t(i, t - j) = \left( \frac{P_t(i, t - j)}{P_t(t - j)} \right)^{-\theta} C_t(t - j)$$

where $P_t(t - j) = \left( \int_0^1 P_t(i, t - j)^{1-\theta} di \right)^{1/(1-\theta)}$

- Flexible price solution from maximizing profit

$$P_t(i, t - j)C_t(i, t - j) - (1 + \tau) S_t P_t^* C_t(i, t - j)$$

is

$$\hat{P}_t(i, t - j) = \hat{P}_t = \frac{\theta}{\theta - 1} (1 + \tau) S_t P_t^*$$

or

$$\hat{p}_t = s_t + p_t^* + \mu$$

where $\mu = \ln(\theta/(\theta - 1)) + \ln(1 + \tau)$
Opting out from the contract

- Ball-Mankiw (1994), Devereux-Siu (2007) introduce the possibility of opt out in the two-period Taylor staggered contract model (pay a fixed cost in resetting the price)

- Two subperiods:
  1. $N_1$: inflation indexation rule (contract pricing rule)
     
     \[ \pi_t = p_t - p_{t-1} \text{ with } p_t = N^{-1} \sum_{j=0}^{N-1} \ln P_t(t-j) \]
  2. $N_2 = N - N_1$: optimal (flexible) price with a fixed cost $F$

\[ N \text{ original contract} \]

\[ N_1 \text{ indexation} \quad \text{and} \quad N_2 \text{ flexible} \]
Endogenous choice of opt out probability

- Endogenous choice of $\kappa$ in Calvo-type model (Ball-Mankiw-Romer, 1988, Romer, 1990, Devereux-Yetman, 2008)

- Endogenous choice in Taylor-type model
  - Probability of opt out $1 - \kappa(t)$
  - Probability of stay in the contract $\kappa(t)$

- Marginal cost components (in logs), $s_t$ and $p_t^*$, are assumed to follow random walks

- Given MC process, importers choose $\kappa(t)$ to minimize the expected loss

$$L_t = E_t \left[ \sum_{j=1}^{N-1} (\beta \kappa(t))^{j} (\hat{\rho}_t + j \pi_t - \hat{\rho}_{t+j})^2 \right] + \frac{1 - \kappa(t)}{\kappa(t)} \sum_{j=1}^{N-1} (\beta \kappa(t))^{j} \left( \sum_{\ell=1}^{N-j} \beta^{\ell-1} \right) F$$
Price determination

- Average duration of period in the contract (indexation rule)

\[
E[N_1] = \sum_{\ell=1}^{N-1} (\kappa(t-j))^{\ell} + 1
\]

- Example of price paths

1. If \( \kappa(t) = 1 \) then \( N_1 = N \) and
   \[\{\hat{p}_t, \hat{p}_t + \pi_t, \hat{p}_t + 2\pi_t, \ldots, \hat{p}_t + (N - 1)\pi_t\}\]

2. If \( \kappa(t) = 0 \) then \( N_1 = 1 \) and
   \[\{\hat{p}_t, \hat{p}_{t+1}, \ldots, \hat{p}_{t+(N-1)}\}\]

3. otherwise
   \[\{\hat{p}_t, \hat{p}_t + \pi_t, \ldots, \hat{p}_t + (N_1 - 1)\pi_t, \hat{p}_{t+N_1}, \ldots, \hat{p}_{t+(N-1)}\}\]
ERPT determination

(A) Two-period contract case \((N = 2)\)

- **Expected loss**

\[
L_t = E_t \left[ \beta \kappa(t) (\hat{p}_t + \pi_t - \hat{p}_{t+1})^2 \right] + \beta (1 - \kappa(t)) F \\
= \beta F - \beta (F - \sigma^2 - \pi_t^2) \kappa(t)
\]

- **Solution** \(\kappa(t)\)

\[
\kappa(\pi_t) = \begin{cases} 
1 & \text{if } -\sqrt{F - \sigma^2} \leq \pi_t \leq \sqrt{F - \sigma^2} \\
0 & \text{otherwise}
\end{cases}
\]

\[
p_t = \frac{1}{2} (p_t(t) + p_t(t - 1)) = (s_t + p_t^* + \mu) - \frac{\kappa(\pi_{t-1})}{2} \Delta(s_t + p_t^*) + \frac{\kappa(\pi_{t-1})}{2} \pi_{t-1}
\]

- **Inflation dynamics**

\[
\pi_t = \left(1 - \frac{\kappa(\pi_{t-1})}{2}\right) \Delta(s_t + p_t^*) + \frac{\kappa(\pi_{t-2})}{2} \Delta(s_{t-1} + p_{t-1}^*) + \frac{\kappa(\pi_{t-1})}{2} \pi_{t-1} - \frac{\kappa(\pi_{t-2})}{2} \pi_{t-2}
\]

- **ERPT**

\[
ERPT = 1 - \frac{\kappa(\pi_{t-1})}{2}
\]
Figure 1. Model implied relationship between ERPT and inflation: Two-period contract case ($N = 2$)
(B) Three-period contract case \((N = 3)\)

- **Expected loss**

\[
L_t = E_t \left[ \beta \kappa^{(t)} (\hat{p}_t + \pi_t - \hat{p}_{t+1})^2 + (\beta \kappa^{(t)})^2 (\hat{p}_t + 2 \pi_t - \hat{p}_{t+2})^2 \right] \\
+ \beta (1 - \kappa^{(t)}) (1 + \beta) F + \beta^2 \kappa^{(t)} (1 - \kappa^{(t)}) F \\
= \beta (1 + \beta) F - \beta (F - \sigma^2 - \pi_t^2) \kappa^{(t)} - \beta^2 (F - 2 \sigma^2 - 4 \pi_t^2) (\kappa^{(t)})^2
\]

- **Solution** \(\kappa^{(t)}\)

\[
\kappa(\pi_t) = \frac{-(F - \sigma^2 - \pi_t^2)}{2 \beta (F - 2 \sigma^2 - 4 \pi_t^2)}
\]

- **Inflation dynamics**

\[
\pi_t = \left( 1 - \frac{\kappa(\pi_{t-1}) + \kappa(\pi_{t-2})^2}{3} \right) \Delta(s_t + p_t^*) \\
- \frac{1}{3} (\kappa(\pi_{t-2})^2 - \kappa(\pi_{t-2}) - \kappa(\pi_{t-3})^2) \Delta(s_{t-1} + p_{t-1}^*) + \frac{\kappa(\pi_{t-3})^2}{3} \Delta(s_{t-2} + p_{t-2}^*) \\
+ \frac{\kappa(\pi_{t-1})}{3} \pi_{t-1} + \frac{1}{3} (2 \kappa(\pi_{t-2})^2 - \kappa(\pi_{t-2})) \pi_{t-2} - \frac{2 \kappa(\pi_{t-3})^2}{3} \pi_{t-3}
\]

- **ERPT**

\[
ERPT = 1 - \frac{\kappa(\pi_{t-1}) + \kappa(\pi_{t-2})^2}{3}
\]
Figure 2. Model implied relationship between ERPT and inflation: Three-period contract case \((N = 3)\)
(C) \(N\)-period contract case

- **ERPT**

  \[
  ERPT = 1 - \frac{\sum_{j=1}^{N-1} \kappa (\pi_t - j)^j}{J}
  \]

- Second term \( N^{-1} \sum_{j=1}^{N-1} \kappa (\pi_t - j)^j \) represents the fraction of firms adapting the indexation rule

- ERPT can vary from \(1/N\) to 1

- ERPT is a smooth nonlinear function of inflation, with its dynamics possibly approximated by a U-shaped transition function with a set of lagged inflation rates used as transition variables

- \(\pi_t\) is a function of \(\pi_{t-j}\) for \(j = 1, \ldots, N\) and \(\Delta(s_{t-j} + p^*_{t-j})\) for \(j = 0, \ldots, N - 1\).
Outline

1. Motivation

2. A Simple Model of Importers

3. STAR Models

4. Empirical Results

5. Conclusion
**Exponential STAR (ESTAR) model**

\[
\pi_t = \mu + \sum_{j=1}^{N} \phi_{1,j} \pi_{t-j} + \sum_{j=0}^{N-1} \phi_{2,j} \Delta(s_{t-j} + p^*_{t-j}) \\
+ \left( \sum_{j=1}^{N} \phi_{3,j} \pi_{t-j} + \sum_{j=0}^{N-1} \phi_{4,j} \Delta(s_{t-j} + p^*_{t-j}) \right) G(z_t; \gamma) + \varepsilon_t
\]

- **exponential transition function**

\[
G(z_t; \gamma) = 1 - \exp\{-\gamma z_t^2\}, \quad \gamma > 0
\]

- **Transition variable** \( z_t = d^{-1} \sum_{j=1}^{d} \pi_{t-j} \)

- **Error term** \( \varepsilon_t \sim \text{i.i.d.}(0, \sigma^2_{\varepsilon}) \)

- **ERPT** = \( \phi_{2,0} + \phi_{4,0} G(z_t; \gamma) \)
1. Motivation
2. A Simple Model of Importers
3. STAR Models
4. Empirical Results
5. Conclusion

ESTAR model
Transition function

\[ \gamma \uparrow \]
Double (Three-regime) logistic STAR (D-LSTAR) model

\[ \pi_t = \mu + \sum_{j=1}^{N} \phi_{1,j} \pi_{t-j} + \sum_{j=0}^{N-1} \phi_{2,j} \Delta(s_{t-j} + p_{t-j}^*) \]

\[ + \left( \sum_{j=1}^{N} \phi_{3,j} \pi_{t-j} + \sum_{j=0}^{N-1} \phi_{4,j} \Delta(s_{t-j} + p_{t-j}^*) \right) G(z_t; \gamma_1, \gamma_2, c) \]

- double logistic transition function

\[ G(z_t; \gamma_1, \gamma_2, c) = \left( 1 + \exp\{-\gamma_1(z_t - c)\} \right)^{-1} \]

\[ + \left( 1 + \exp\{\gamma_2(z_t + c)\} \right)^{-1}, \quad \gamma_1, \gamma_2 > 0 \]

- \textit{ERPT} = \phi_{2,0} + \phi_{4,0}G(z_t; \gamma_1, \gamma_2, c)

- Symmetric DLSTAR model if \( \gamma_1 = \gamma_2 \)

- Asymmetric DLSTAR model if \( \gamma_1 \neq \gamma_2 \)
Symmetric DLSTAR model

Transition function

\[ \gamma \uparrow \\
(\gamma_1 = \gamma_2) \]
Asymmetric DLSTAR model
Transition function

$\gamma_1 > \gamma_2$
1. Motivation

2. A Simple Model of Importers

3. STAR Models

4. Empirical Results

5. Conclusion
Data

- Two main variables
  1. **Domestic price inflation** $\pi_t$: US producer price index from *IFS*
  2. **Changes in marginal costs (nominal exchange rate and import price)** $\Delta(s_t + p_t^*)$: US import price index from *IFS* (US dollar prices paid by US importers)

- Monthly series from January 1975 to December 2007

- Estimation Strategy
  1. Conduct linearity test against STAR models
  2. Search for the best $d$ in $z_t = d^{-1} \sum_{j=1}^{d} \pi_{t-j}$ that minimizes SSR from NLLS
  3. Use general-to-specific approach (suggested by van Dijk-Teräsvirta-Franses, 2002), sequentially remove the lagged variables for which the $t$ statistics are less than 1.0 in absolute value (starting from $J = 13$)
Figure 3. Producer price index inflation
A.0. Linearity test against ESTAR model (Saikkonen-Luukkonen, 1988)

- ESTAR model

\[ y_t = x_t \phi_1 + G(z_t; \gamma, c)x_t \phi_2 \]

where exponential transition function

\[ G(z_t; \gamma, c) = 1 - \exp\{-\gamma(z_t - c)^2\}, \quad \gamma > 0 \]

- Third-order Taylor series approximation of \( G(z_t; \gamma, c) \) with respect to \( \gamma \) evaluated at \( \gamma = 0 \)

\[ G(z_t; \gamma, c) \approx \gamma(z_t - c)^2 \]

- Auxiliary regression

\[ y_t = x_t \beta_1 + x_t z_t \beta_2 + x_t z_t^2 \beta_3 + e_t \]

- Linearity test \( \rightarrow H_0 : \beta_2 = \beta_3 = 0 \rightarrow LM_1 \)
Table 1. LM-type tests for STAR nonlinearity

$H_0$ vs. $H_1$: Linear vs. ESTAR

Transition Variable ($z_t = d^{-1} \sum_{j=1}^{d} \pi_{t-j}$)

<table>
<thead>
<tr>
<th>Test</th>
<th>$d = 1$</th>
<th>$d = 2$</th>
<th>$d = 3$</th>
<th>$d = 4$</th>
<th>$d = 5$</th>
<th>$d = 6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$LM_1$</td>
<td>4.93</td>
<td>4.12</td>
<td>3.89</td>
<td>3.58</td>
<td>3.03</td>
<td>2.36</td>
</tr>
<tr>
<td>($p$-values)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>$LM_1^*$</td>
<td>347.1</td>
<td>341.1</td>
<td>341.5</td>
<td>344.6</td>
<td>346.9</td>
<td>344.9</td>
</tr>
<tr>
<td>($p$-values)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
</tbody>
</table>

$LM_1$: LM tests by Saikkonen-Luukkonen(1988)

Table 2. Nonlinear estimation results

A: ESTAR parameter estimates

\[
\pi_t = 0.099 + 0.123 \pi_{t-1} + 0.200 \pi_{t-3} - 0.081 \pi_{t-4} + 0.336 \Delta(s_t + p_t^*) \\
+ 0.093 \Delta(s_{t-1} + p_{t-1}^*) + 0.074 \Delta(s_{t-4} + p_{t-4}^*) + 0.039 \Delta(s_{t-5} + p_{t-5}^*) \\
+ \left[ 0.752 - 1.352 \pi_{t-5} + 0.664 \Delta(s_t + p_t^*) - 0.569 \Delta(s_{t-2} + p_{t-2}^*) \\
- 0.300 \Delta(s_{t-4} + p_{t-4}^*) \right] G(z_t; \hat{\gamma}) + \hat{\varepsilon}_t,
\]

\[
G(z_t; \hat{\gamma}) = 1 - \exp \left\{ -0.160 \left( \frac{1}{3} \sum_{j=1}^{3} \pi_{t-i} \right) / 0.477 \right\}
\]

\[ R^2 = 0.606, \; se = 0.476, \; obs = 396, \; LM(1) = [0.146], \; LM(1-12) = [0.189] \]
Figure 4. ERPT against transition variable: ESTAR model
Figure 5. ERPT over time: ESTAR model
A.1. Linearity test against DLSTAR model

- (Asymmetric) DLSTAR model

\[ y_t = x_t' \phi_1 + G(z_t; \gamma_1, \gamma_2, c)x_t' \phi_2 \]

where double logistic transition function

\[ G(z_t; \gamma_1, \gamma_2, c) = \left(1 + \exp\{-\gamma_1(z_t - c)\}\right)^{-1} \]
\[ + \left(1 + \exp\{\gamma_2(z_t + c)\}\right)^{-1}, \quad \gamma_1, \gamma_2 > 0 \]

- Third-order Taylor series approximation of \( G(z_t; \gamma_1, \gamma_2, c) \) with respect to \( \gamma_1 \) and \( \gamma_2 \) evaluated at \( \gamma_1 = \gamma_2 = 0 \)

\[ G(z_t; \gamma_1, \gamma_2, c) \approx \left[\frac{\gamma_1}{4} (z_t - c) - \frac{\gamma_1^3}{48} (z_t - c)^3\right] + \left[-\frac{\gamma_2}{4} (z_t + c) + \frac{\gamma_2^3}{48} (z_t + c)^3\right] \]

- Auxiliary regression for symmetric case

\[ y_t = x_t' \beta_1 + x_t' z_t^2 \beta_3 + e_t. \]

when \( \gamma_1 = \gamma_2 \)

- Linearity test \( \rightarrow H_0 : \beta_3 = 0 \rightarrow LM_2 \)
### Table 1 (continued). LM-type tests for STAR nonlinearity

**H₀ vs. H₁: Linear vs. Symmetric DLSTAR**

Transition Variable \( z_t = d^{-1} \sum_{j=1}^{d} \pi_{t-j} \)

<table>
<thead>
<tr>
<th>Test</th>
<th>( d = 1 )</th>
<th>( d = 2 )</th>
<th>( d = 3 )</th>
<th>( d = 4 )</th>
<th>( d = 5 )</th>
<th>( d = 6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( LM_2 )</td>
<td>4.86</td>
<td>3.07</td>
<td>4.01</td>
<td>3.20</td>
<td>3.23</td>
<td>3.14</td>
</tr>
<tr>
<td>( p )-values</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>( LM_2^* )</td>
<td>372.9</td>
<td>377.2</td>
<td>370.8</td>
<td>371.8</td>
<td>373.1</td>
<td>369.4</td>
</tr>
<tr>
<td>( p )-values</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
</tbody>
</table>
Table 2 (continued). Nonlinear estimation results

B: Symmetric DLSTAR parameter estimates

\[ \pi_t = 0.098 + 0.208\pi_{t-1} + 0.159\pi_{t-3} - 0.101\pi_{t-5} + 0.349 \Delta(s_t + p_t^*) + 0.075\Delta(s_{t-1} + p_{t-1}^*) - 0.070\Delta(s_{t-2} + p_{t-2}^*) + 0.066\Delta(s_{t-5} + p_{t-5}^*) + \]
\[ + 0.350\Delta(s_{t-2} + p_{t-2}^*) - 0.534\Delta(s_{t-4} + p_{t-4}^*) - 0.356\Delta(s_{t-5} + p_{t-5}^*) \]
\[ G(z_t; \hat{\gamma}, \hat{c}) + \hat{e}_t, \]

\[ G(z_t; \hat{\gamma}, \hat{c}) = \left( 1 + \exp \left\{ -5.130 \left( \pi_{t-1} - \frac{1.474}{21.283} \right) / 0.686 \right\} \right)^{-1} \]
\[ + \left( 1 + \exp \left\{ 5.130 \left( \pi_{t-1} + \frac{1.474}{21.283} \right) / 0.686 \right\} \right)^{-1} \]

\[ R^2 = 0.654, \ se = 0.448, \ obs = 396, \ LM(1) = [0.040], \ LM(1-12) = [0.242] \]
Figure 6. ERPT against transition variable: Symmetric DLSTAR model
Figure 7. ERPT over time: Symmetric DLSTAR model
A.1 (continued). Linearity test against DLSTAR model

- **Asymmetric DLSTAR model**
  \[ y_t = x_t' \phi_1 + G(z_t; \gamma_1, \gamma_2, c)x_t' \phi_2 \]

  where double logistic transition function
  \[ G(z_t; \gamma_1, \gamma_2, c) = \left( 1 + \exp\{-\gamma_1(z_t - c)\}\right)^{-1} \]
  \[ + \left( 1 + \exp\{\gamma_2(z_t + c)\}\right)^{-1}, \quad \gamma_1, \gamma_2 > 0 \]

- **Third-order Taylor series approximation of** \( G(z_t; \gamma_1, \gamma_2, c) \) \n  with respect to \( \gamma_1 \) and \( \gamma_2 \) evaluated at \( \gamma_1 = \gamma_2 = 0 \)

  \[ G(z_t; \gamma_1, \gamma_2, c) \approx \left[ \frac{\gamma_1}{4}(z_t - c) - \frac{\gamma_1^3}{48}(z_t - c)^3 \right] + \left[ -\frac{\gamma_2}{4}(z_t + c) + \frac{\gamma_2^3}{48}(z_t + c)^3 \right] \]

- **Auxiliary regression for asymmetric case**

  \[ y_t = x_t' \beta_1 + x_t' z_t \beta_2 + x_t^2 z_t \beta_3 + x_t^3 z_t \beta_4 + e_t \]

  when \( \gamma_1 \neq \gamma_2 \)

- **Linearity test** \( H_0 : \beta_2 = \beta_3 = \beta_4 = 0 \rightarrow LM_3 \)


Table 1 (continued). LM-type tests for STAR nonlinearity

H₀ vs. H₁: Linear vs. Asymmetric DLSTAR

Transition Variable \( z_t = d^{-1} \sum_{j=1}^{d} \pi_{t-j} \)

<table>
<thead>
<tr>
<th>Test</th>
<th>( d = 1 )</th>
<th>( d = 2 )</th>
<th>( d = 3 )</th>
<th>( d = 4 )</th>
<th>( d = 5 )</th>
<th>( d = 6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( LM_3 )</td>
<td>5.14</td>
<td>4.04</td>
<td>3.57</td>
<td>2.95</td>
<td>2.68</td>
<td>1.98</td>
</tr>
<tr>
<td>( p )-values</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>( LM_3^* )</td>
<td>351.8</td>
<td>355.3</td>
<td>354.1</td>
<td>357.7</td>
<td>358.2</td>
<td>358.4</td>
</tr>
<tr>
<td>( p )-values</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
</tbody>
</table>
Table 3. Nonlinear estimation results

Asymmetric DLSTAR parameter estimates

\[
\pi_t = 0.095 + 0.270 \pi_{t-1} + 0.153 \pi_{t-3} - 0.105 \pi_{t-5} + 0.341 \Delta(s_t + p_t^*) \\
+ 0.062 \Delta(s_{t-1} + p_{t-1}^*) - 0.078 \Delta(s_{t-2} + p_{t-2}^*) + 0.064 \Delta(s_{t-5} + p_{t-5}^*) + \\
\left[ -0.198 \pi_{t-1} - 0.510 \pi_{t-5} + 1.001 \pi_{t-6} + 0.659 \Delta(s_t + p_t^*) \\
- 0.338 \Delta(s_{t-1} + p_{t-1}^*) + 0.417 \Delta(s_{t-2} + p_{t-2}^*) - 0.298 \Delta(s_{t-4} + p_{t-4}^*) \\
- 0.482 \Delta(s_{t-5} + p_{t-5}^*) \right] G(z_t; \gamma_1, \gamma_2, \hat{c}_1, \hat{c}_2) + \hat{\epsilon}_t,
\]

\[
G(z_t; \gamma_1, \gamma_2, \hat{c}_1, \hat{c}_2) = \left( 1 + \exp \left\{ -55.253 \left( \pi_{t-1} - 1.293 \right) / 0.686 \right\} \right)^{-1} \\
+ \left( 1 + \exp \left\{ 5.762 \left( \pi_{t-1} + 1.591 \right) / 0.686 \right\} \right)^{-1}
\]

\[ R^2 = 0.663, \ se = 0.443, \ obs = 396, \ LM(1) = [0.073], \ LM(1-12) = [0.247] \]
Figure 8. ERPT against transition variable: Asymmetric DLSTAR model
1. Motivation
2. A Simple Model of Importers
3. STAR Models
4. Empirical Results
5. Conclusion

Figure 9. ERPT over time: Asymmetric DLSTAR model
A.2. Specification test among ESTAR, symmetric DLSTAR, asymmetric DLSTAR

- Tests not available in the literature
- Auxiliary regression for asymmetric DLSTAR model ($\gamma_1 \neq \gamma_2$)
  \[ y_t = x'_t \beta_1 + x'_t z_t \beta_2 + x'_t z_t^2 \beta_3 + x'_t z_t^3 \beta_4 + e_t \]
- Auxiliary regression for symmetric DLSTAR model ($\gamma_1 = \gamma_2$)
  \[ y_t = x'_t \beta_1 + x'_t z_t^2 \beta_3 + e_t. \]
- Test symmetric DLSTAR against asymmetric DLSTAR
  \[ H_0 : \beta_2 = \beta_4 = 0 \rightarrow LM_4 \]
- Auxiliary regression for ESTAR model
  \[ y_t = x'_t \beta_1 + x'_t z_t \beta_2 + x'_t z_t^2 \beta_3 + e_t \]
- Test ESTAR against asymmetric DLSTAR \[ H_0 : \beta_4 = 0 \rightarrow LM_5 \]
- Test symmetric DLSTAR against ESTAR \[ H_0 : \beta_2 = 0 \rightarrow LM_6 \]
Table 4. LM-type tests for STAR model selection

| Transition Variable \( z_t = \frac{1}{d} \sum_{j=1}^d \pi_{t-j} \) | \( H_0 \) vs. \( H_1 \) | Test Statistics | \( d = 1 \) | \( d = 2 \) | \( d = 3 \) | \( d = 4 \) | \( d = 5 \) | \( d = 6 \) |
|-----------------|-----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Symmetric DLSTAR | \( LM_4 \)       |                | 3.86           | 3.42           | 2.84           | 2.47           | 2.17           | 1.26           |                |
|                  |                 |                | (0.00)         | (0.00)         | (0.00)         | (0.00)         | (0.01)         | (0.19)         |                |
| vs. Asymmetric DLSTAR | \( LM_4^* \) | \( 358.2 \)       | 348.6          | 350.9          | 356.2          | 360.3          | 361.3          |                   |                |
|                  |                 |                | (0.00)         | (0.00)         | (0.00)         | (0.00)         | (0.00)         | (0.00)         |                |
| ESTAR            | \( LM_5 \)       |                | 4.41           | 3.24           | 2.52           | 1.56           | 1.81           | 1.19           |                |
|                  |                 |                | (0.00)         | (0.00)         | (0.01)         | (0.10)         | (0.05)         | (0.29)         |                |
| vs. Asymmetric DLSTAR | \( LM_5^* \) | \( 374.1 \)       | 376.1          | 371.3          | 371.8          | 372.4          | 369.2          |                   |                |
|                  |                 |                | (0.00)         | (0.00)         | (0.00)         | (0.00)         | (0.00)         | (0.00)         |                |
| Symmetric DLSTAR | \( LM_6 \)       |                | 2.97           | 3.34           | 3.02           | 3.30           | 2.47           | 1.32           |                |
|                  |                 |                | (0.00)         | (0.00)         | (0.00)         | (0.00)         | (0.01)         | (0.20)         |                |
| vs. ESTAR        | \( LM_6^* \)     |                | 377.0          | 377.0          | 373.6          | 373.1          | 372.6          | 369.8          |                |
|                  |                 |                | (0.00)         | (0.00)         | (0.00)         | (0.00)         | (0.00)         | (0.00)         |                |

Note: Lag length is \( N = 13 \). LM is to test for the symmetric DLSTAR model against the asymmetric DLSTAR model, LM* is to test for the ESTAR model against the asymmetric DLSTAR model, LM is to test for the symmetric DLSTAR model against the ESTAR model. In addition, the heteroskedasticity-robust variants of the LM, LM*, LM are denoted as LM*, LM*, LM*, respectively. (See Granger and Teräsvirta, 1993). Figures in parentheses below LM statistics denote their p-values.
Outline

1. Motivation

2. A Simple Model of Importers

3. STAR Models

4. Empirical Results

5. Conclusion
Conclusion

1. We showed that the STAR models offer a very convenient framework in examining the relationship between the ERPT and inflation

2. Under a simple theoretical model of importing firms, ERPT is predicted to be a nonlinear function of the past inflation rate

3. Empirical applications to monthly US domestic and import price data revealed the nonlinearity and suggested that low level ERPT are associated with the lowered and stable inflation